Assignment-1

GATE Problems

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Question

The Chromatic Number of the following graph is

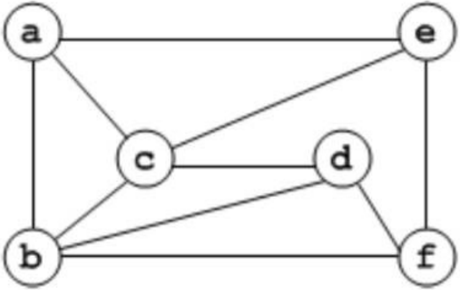

Figure: Question

Solution

The **chromatic number** of a graph is the smallest number of colors needed to color the vertices of the graph so that no two adjacent vertices share the same color.

Steps to Calculate Chromatic Number

1. We color first vertex with the first color.
2. For the remaining (V-1) vertices we do the following one by one:
3. We color the currently picked vertex with the lowest numbered color if the color has not been used to color any of its adjacent vertices.
4. If it has been used, then we choose the next least numbered color.
5. If all the previously used colors have been used, then we assign a new color to the currently picked vertex.

For given graph

We color first vertex with the first color.

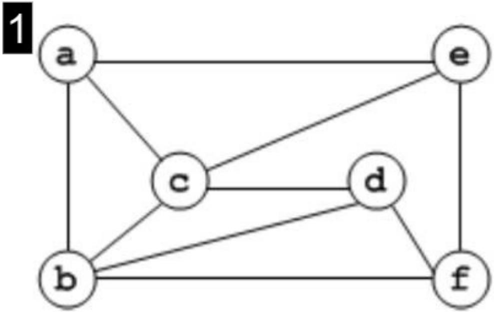

Figure: STEP 1

We assign color to the vertices which share an edge with the first vertex.

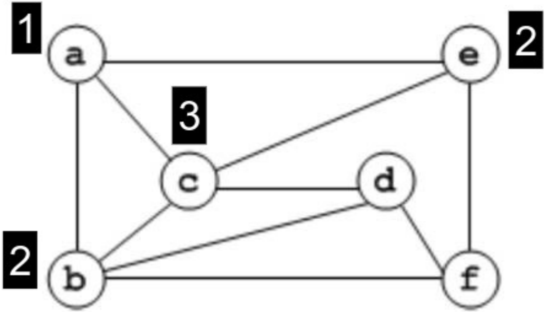

Figure: STEP 2

We assign color to the remaining vertices.

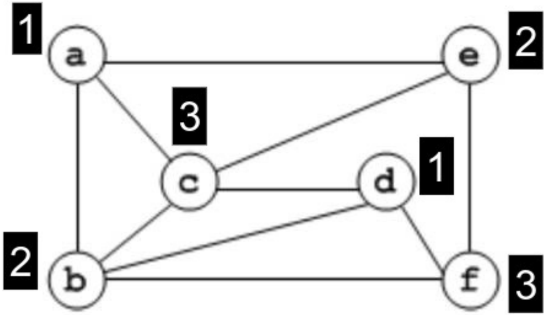

Figure: STEP 3

The number of colours used is 3. Hence the Chromatic Number is 3.

How to implement Graph as Adjacency List?

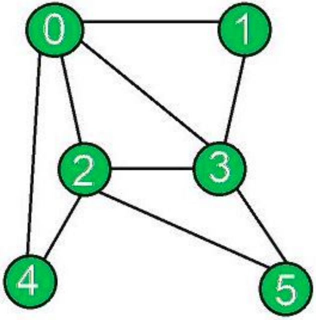

Figure: Graph

How to implement Graph as Adjacency Matrix?

This is the adjacency matrix for the graph $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$

How to implement Graph as Adjacency Matrix?

1. Let the number of vertices be N.
2. Create an N$\times$N matrix.
3. Let all the rows and columns be initialised as 0s.
4. Then for each edge between any 2 vertices we mark the respective row-column index as 1.
5. For example if vertices 1 and 2 have an edge between them we mark the cell of row 1, column 2 and cell of column 1, row 2 as 1 to show that an edge exists.

Code

1. We use an adjacency List to create the Graph.
2. The Code
3. Time Taken:0.000053s
4. We use an adjacency Matrix to create the Graph.
5. The Code
6. Time Taken:0.000073s

PART-2

Solving a Circuit Problem using Graph Theory

Solving a circuit problem using Graph Theory

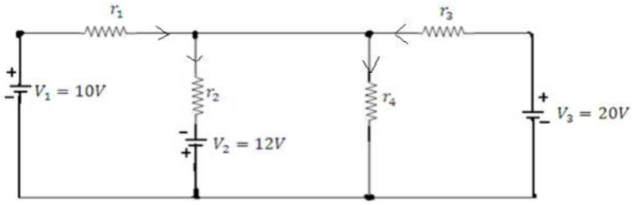

Figure: Question

$r_1 = 2, r_2 = 5, r_3 = 1, r_4 = 10$

Fundamental Tie Set Matrix (Fundamental Loop Matrix)

1. We define the edges of the graph and remove all the elements like voltage source, resistor, capacitor from the circuit and the corresponding graph is formed.
2. The graph has loops, we define and number the edges and the non overlapping loops and give a direction for current flow in each loop.
3. We then form a matrix with n (number of edges) rows and m (number of loops) columns.
4. We check if an edge is a part of the loop or not, if it is not then we assign 0 in the respective cell.
5. If edge is part of the loop, we check the direction of loop and current flow. If they coincide we assign the value 1 in the cell, else -1.
6. This is the Fundamental Tie Set Matrix, denoted by B.

KVL & KCL

1. KVL states that for a closed loop series path the algebraic sum of all the voltages around any closed loop in a circuit is equal to zero.
2. In the matrix form it can be represented as $B\ V_b{=}0$ where V_b vector represents the total voltage drop in each branch of the circuit(edge).
3. $V_b = V_s + Z_b I_b$ where V_s, Z_b, I_b are matrices and vectors
4. $V_s{=}$ Voltage Source, $I_b{=}$ Current in that branch, $Z_b{=}$ Impedance of that branch.
5. $B(V_s + Z_b I_b) = 0$
6. $B(Z_b I_b) = -BV_s$
7. From KCL we know that $I_b = B^T I_L$ where B^T is the transpose of B and I_L is the current in each loop.
8. So, $BZ_b B^T I_L = -BV_s$

Solution for given Circuit

1. The graph for the given circuit can be represented as:

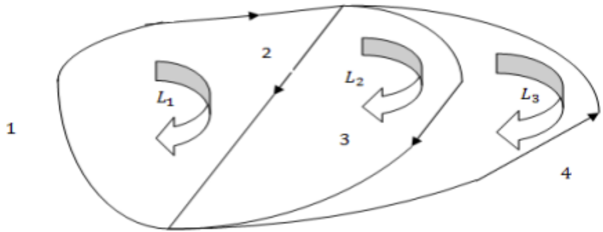

Figure: Loops and Current Flow

Solution for given Circuit

1. We have to define the parameters

$$B= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$Z_b = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}$$

Solution for given Circuit

$$\mathrm{V}_s = \begin{bmatrix} 10 \\ 12 \\ 0 \\ 20 \end{bmatrix}$$

$$\mathrm{I}_L = \begin{bmatrix} \mathrm{I}_1 \\ \mathrm{I}_2 \\ \mathrm{I}_3 \end{bmatrix}$$

Solution for given Circuit

1. We put these parameters in the equation : $BZ_bB^TI_L = -BV_s$
2. We can solve this using pen and paper or using a basic code.
3. The Code
4. The current values are $I_L = \begin{bmatrix} \text{-3.72A} \\ \text{-0.8A} \\ \text{1.74A} \end{bmatrix}$
5. The negative sign indicates that our assumption of current flow direction was wrong.