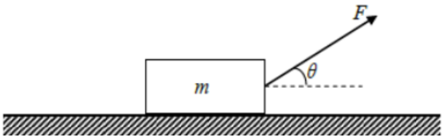
Tutorial 4 Solutions

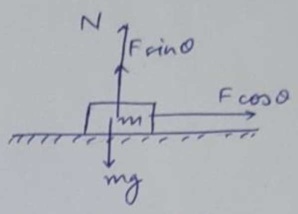
Q1: A block is placed at rest on horizontal surface. The coefficient of friction between the block and the surface is μ_s . It is pulled with a force F at an angle θ with the horizontal plane as shown. Find the value of θ at which minimum force is required to move the block.



Applying Newton's 2nd Law,

$$N + F\sin\theta = mg \quad \text{——— (i)}$$

$$F\cos\theta = f_{max} = \mu_s N \quad \text{——— (ii)}$$



Free Body Diagram.

& from eq^n (1)

$$N = mg - F\sin\theta$$

Put it in eq^n (2) we have.

$$F\cos\theta = \mu_s(mg - F\sin\theta)$$

$$= \mu_s mg - \mu_s F\sin\theta$$

$$\Rightarrow F(\cos\theta + \mu_s\sin\theta) = \mu_s mg \quad \text{——— (iii)}$$

$$\Rightarrow F = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} \quad \text{——— (3)}$$

For F to be minimum $\frac{dF}{d\theta} = 0$

so differentiate eq^n (iii) wrt θ using chain rule.

$$\frac{dF}{d\theta}(\cos\theta + \mu_s\sin\theta) + F\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0$$

$$\frac{dF}{d\theta}(\cos\theta + \mu_s\sin\theta) + F(-\sin\theta + \mu_s\cos\theta) = 0$$

since $\frac{dF}{d\theta} = 0 \Rightarrow -\sin\theta + \mu_s\cos\theta = 0$

$$\Rightarrow \tan\theta = \mu_s \Rightarrow \boxed{\theta = \tan^{-1}\mu_s}$$

Q2: Consider an automobile moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tires and the road is μ_s , what is the shortest distance in which the automobile can be stopped?

Using final speed $v = 0$ in $v^2 = {v_0}^2 + 2ax$

We obtain $x = -{v_0}^2/2a$ ……………..(1)

where negative sign indicates that a points in the negative x-direction.

Applying 2nd law of motion to the x-component of the motion

$-f_s = ma = (W/g)a$

That gives $a = - g(f_s/W)$ ……….(2)

From the y-components we obtain

$N - W = 0 \rightarrow N = W$

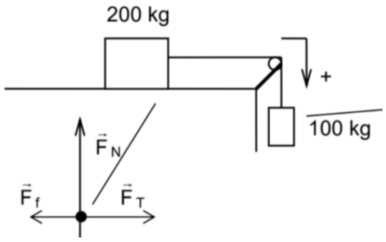
Hence, $\mu_s = \frac{f_s}{N} = f_s/W$ and thus from Eq. 2 we found

$$a = -\mu_s g$$

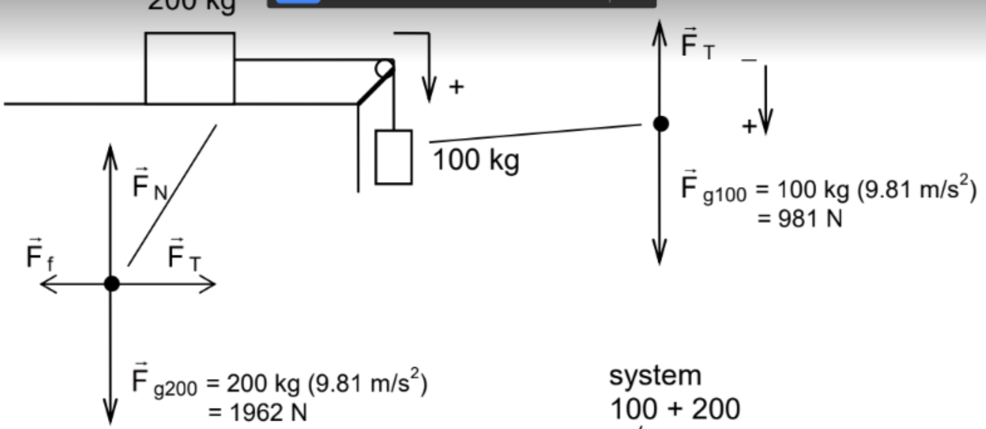
Then from Eq. (1) we can find the distance of stopping which is

$$x = -\frac{{v_0}^2}{2a} = \frac{{v_0}^2}{2g\mu_s}$$

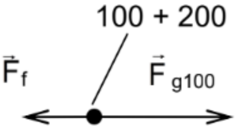
Q3: A 200 kg mass rests on a surface which has a coefficient of friction of 0.25. It is connected to a 100 kg mass over a pulley as shown in the diagram below. When the masses are released what is the resulting tension in the rope?



Solution:



$$|\vec{F}_f| = \mu|\vec{F}_N| \longleftarrow |\vec{F}_N| = |\vec{F}_{g200}|$$
$$|\vec{F}_f| = 0.25(1962\text{N})$$
$$|\vec{F}_f| = 490.5\text{N}$$



From the free body diagram for the system note that the force acting against F_{g100} is F_f on the table

$$\vec{F}_{NET} = \vec{F}_{g100} + \vec{F}_f$$
$$\vec{F}_{NET} = (+981\text{N}) + (-490.5\text{N})$$
$$\vec{F}_{NET} = +490.5\text{N}$$

$$\vec{a} = \frac{\vec{F}_{NET}}{m}$$
$$\vec{a} = \frac{+490.5\text{N}}{100\text{kg} + 200\text{kg}}$$
$$\vec{a} = +1.635 \, \text{m}/\text{s}^2$$

Calculate tension. (I chose the 100 kg mass).

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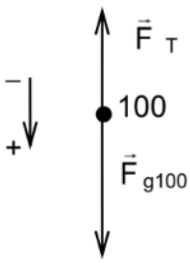


$$\vec{F}_{NET100} = \vec{F}_T + \vec{F}_{g100}$$
$$\vec{F}_T = \vec{F}_{NET100} - \vec{F}_{g100}$$
$$\vec{F}_T = (+163.5\text{N}) - (+981\text{N})$$
$$\vec{F}_T = -817.5\text{N}$$
$$\boxed{|\vec{F}_T| = 818\text{N}}$$

$$\vec{F}_{NET100} = m\vec{a}$$
$$\vec{F}_{NET100} = 100\text{kg}(+1.635 \, \text{m}/\text{s}^2)$$
$$\vec{F}_{NET100} = +163.5\text{N}$$

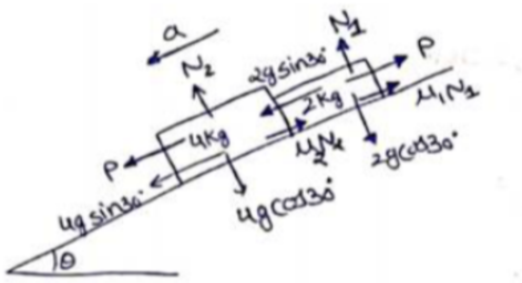
Q4: Figure shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2.0 kg and the incline is μ_1 and that between the block of mass 4.0 kg and the incline is μ_2 . Calculate the acceleration of the 2.0 kg block if,

(a)μ_1 = 0.20 and μ_2 = 0.30,

(b) μ_1 = 0.30 and μ_2 = 0.20. Take g =10 ms $^{-2}$

Solution:

Solution: Consider P as the contact force as shown in the free body diagram and resolve all the forces acting on the system.



$N_1 = 4g\cos 30$

$N_1 = 4\times 10\times\frac{\sqrt{3}}{2} = 20\sqrt{3}$ N

for 4Kg block

$P + 4g\sin 30 - \mu_2 N_2 = ma$

$P + 4\times 10\times\frac{1}{2} - (0.3)(20\sqrt{3}) = 4a$

$P = 4a + 6\sqrt{3} - 20 \quad -(i)$

For 2Kg block

$N_1 = 2g\cos 30 = 10\sqrt{3}$

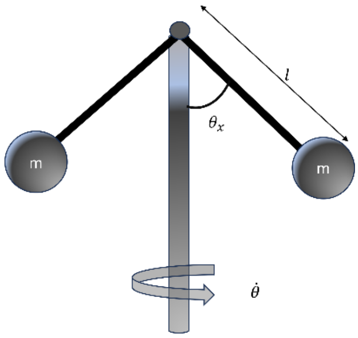
$2g\sin 30 - P - \mu_1 N_1 = 2a$

$P = 2a + 10 + 2\sqrt{3} \quad -(ii)$

On solving (i) and (ii)

$a = 2.7\, m/s^2$

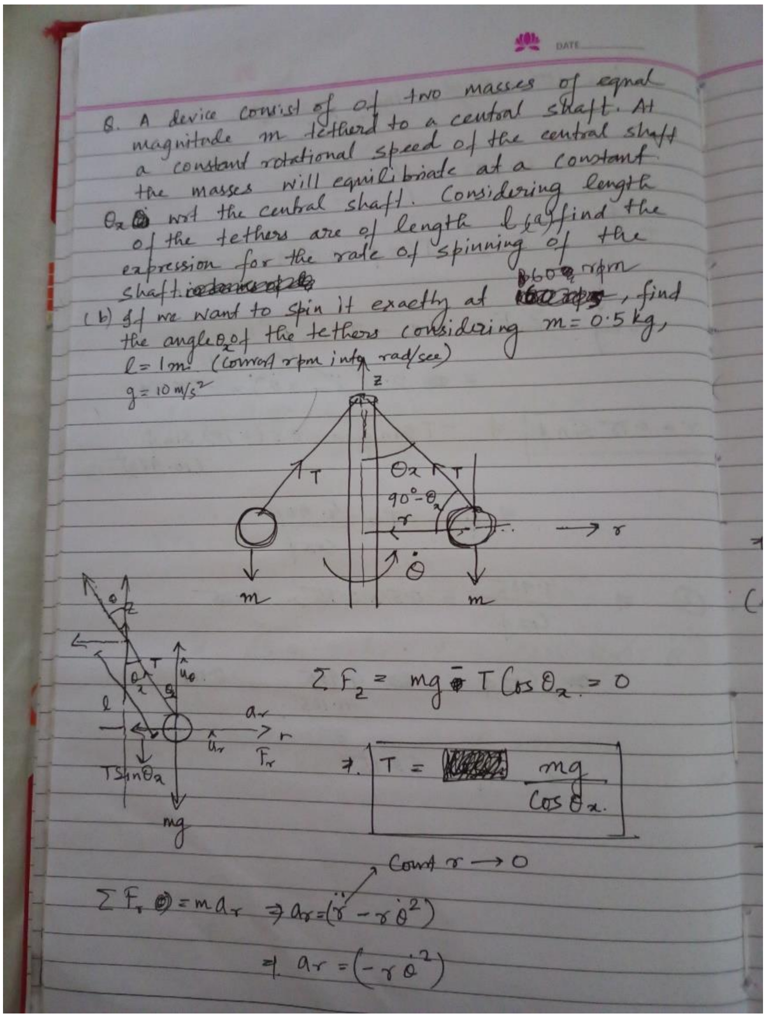
Q5: A device consists of mass of equal magnitude m tethered to a central shaft as shown in the figure. At a constant rotational speed of the central shaft the masses will be at a constant angle θ_x wrt to the central shaft. Considering length of the tethers are l and acceleration due to gravity g.



(a) Rate of spinning of the shaft is $\omega = \dot{\theta} = \sqrt{\frac{g}{l\ Cos\theta_x}}$

(b) If we want to spin it exactly at 60 rpm, what will be the angle θ_x if m=0.5kg and l=1m

Solution:



Q. A device consist of of two masses of equal magnitude m tetherd to a central shaft. At a constant rotational speed of the central shaft the masses will equilibriate at a constant θ_x wrt the central shaft. Considering length of the tethers are of length l (a) find the expression for the rate of spinning of the shaft.

(b) If we want to spin it exactly at 60 rpm, find the angle θ_x of the tethers considering $m = 0.5$ kg, $l = 1$ m. (convert rpm into rad/sec)

$g = 10\ m/s^2$

$$\Sigma F_z = mg - T\cos\theta_x = 0$$

$$\Rightarrow \boxed{T = \frac{mg}{\cos\theta_x}}$$

Const $r \to 0$

$$\Sigma F_r = m a_r \Rightarrow a_r = (\ddot{r} - r\dot{\theta}^2)$$

$$\Rightarrow a_r = (-r\dot{\theta}^2)$$

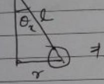
T goes opposite to $\vec{F}_r$

$\Rightarrow -T\sin\theta_x = m\,a_r$

$\Rightarrow -\dfrac{\cancel{m}g\sin\theta_x}{\cos\theta_x} = \cancel{m}\times(-r\dot{\theta}^2)$

$\Rightarrow \cancel{+}\,g\tan\theta_x = \cancel{+}\,r\dot{\theta}^2$

$\Rightarrow g\dfrac{\cancel{\sin}\theta_x}{\cos\theta_x} = l\,\cancel{\sin}\theta_x\,\dot{\theta}^2$



$\Rightarrow \dot{\theta}^2 = \dfrac{g}{l\cos\theta_x}$

$\dfrac{r}{l} = \sin\theta_x \quad \Rightarrow \quad \boxed{\dot{\theta} = \sqrt{\dfrac{g}{l\cos\theta_x}}}$

$\Rightarrow r = l\sin\theta_x$

(b) ~~360 rpm = 360 Revolution / 60×60 sec.~~ $\quad$ 1 rpm = $\dfrac{1\text{ revolution}}{1\text{ min }(60\text{ sec})}$

~~⇒ 1 Rev/sec = 2π rad/sec~~

~~⇒ 360 rpm = 360×2π rad / 60×60 sec~~

$\dot{\theta} = 2\pi$ rad/sec.

We need to find θ_x

~~$\Rightarrow 2\pi = \sqrt{\dfrac{g}{l\cos\theta_x}}$~~

(b) 1 rpm = $\dfrac{1\text{ revolution}}{1\text{ min}}$ $\qquad \dfrac{360^\circ}{2\pi}$

$\Rightarrow$ 1 rpm = $\dfrac{1\text{ revolution}}{60\text{ sec.}}$

$\Rightarrow$ 60 rpm = $\dfrac{60\text{ revolution}}{60\text{ sec.}}$

$\Rightarrow$ 60 rpm = 1 revolution/sec.

1 revolution $\cong 2\pi$ rad.

$\Rightarrow$ 60 rpm = 2π rad/sec = $\dot{\theta}$

We need to find θ_x

$\Rightarrow 2\pi = \sqrt{\dfrac{g}{l\cos\theta_x}}$

$\Rightarrow \left(\dfrac{4\pi^2 l}{g}\right) = \dfrac{1}{\cos\theta_x}$

$\Rightarrow \cos\theta_x = \dfrac{g}{4\pi^2 l}$

$\Rightarrow \theta_x = \cos^{-1}\left[\dfrac{10}{4\pi^2\times 1}\right]$

$\Rightarrow \theta_x = 1.314$ rad $\quad$ [∵ 2π rad = 360°]

$= 75.35^\circ$

Q6: 2. In the arrangement of Fig. 1 the masses m_0, m_1 and m_2 of bodies are equal, the masses of the pulley and the threads are negligible, and there is no friction in the pulley. Find the acceleration a with which the body m_0 comes down, and the tension of the thread binding together the bodies m_1 and m_2, if the coefficient of friction between these bodies and the horizontal surface is equal to k.

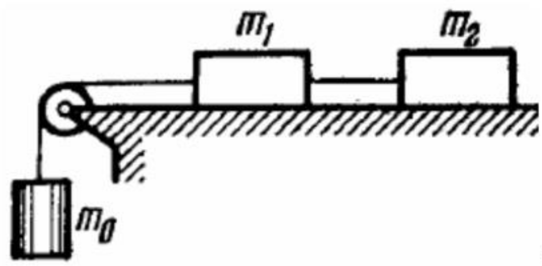
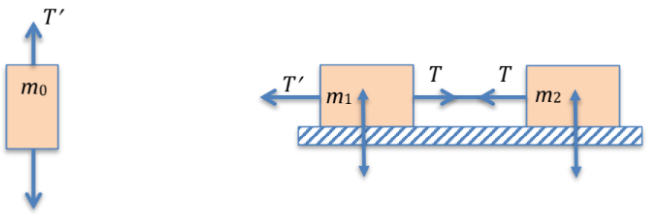


Fig. 1

Q2.



Equation of motion for vertical component

$m_0g - T' = m_0a$ …………..(1)

The horizontal forward motion of the bodies of mass m_1 and m_2 is

$T' - k(m_1g) - k(m_2g) = (m_1 + m_2)a$

$T' - k(m_1 + m_2)g = (m_1 + m_2)a$ ……………(2)

Substituting T′ from Eq. (1) to Eq. (2)

$m_0g - m_0a - k(m_1 + m_2)g = (m_1 + m_2)a$

$m_0g - k(m_1 + m_2)g = (m_0 + m_1 + m_2)a$

$a = \{m_0g - k(m_1 + m_2)g\}/(m_0 + m_1 + m_2)$

On the body of mass m_2 one forward force T and one backward frictional force $k(m_2g)$ are acting. Therefore, the equation of motion of m_2 would be

$T - k\,m_2g = m_2a$

$T = m_2a + k\,m_2g$

$$= m_2\left[\frac{m_0g - k(m_1+m_2)g}{m_0+m_1+m_2}\right] + km_2g$$

$$= \frac{(1+k)m_1m_2g}{m_0+m_1+m_2}$$