Passive Filters

A filter is a frequency selective circuit/device.
- it allows certain frequencies to pass through, but block unwanted frequencies.

1. **Passive Filters:** only R, L ,C as elements (no active device**)**

2. **Active Filters:** R, L, C and Amplifier

In this course we are limited to passive filters only.

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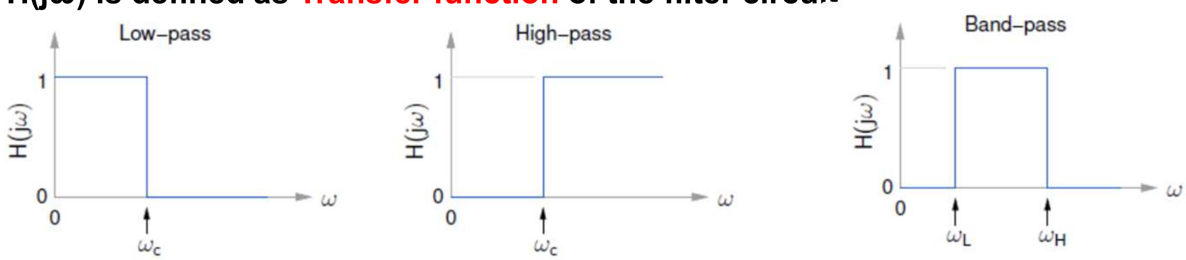
Low Pass, High Pass, and Band-pass filters

H(jω) is defined as Transfer function of the filter circuit



Ideal transfer function of various filters

General Form of Transfer function of three filters, respectively

L.P

$$H(j\omega) = \frac{K}{1 + j\omega/\omega_c}$$

H.P

$$H(j\omega) = \frac{K}{1 - j\omega_c/\omega}$$

B.P

$$H(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$\omega_c =$ cut-off frequency

The maximum value of $|H(j\omega)| = |K|$ is called the filter gain.

ω_o is the central frequency
ω_L and ω_H are lower and higher cutoff

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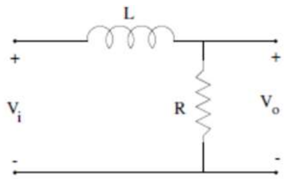
Low Pass filter

R-L Circuit



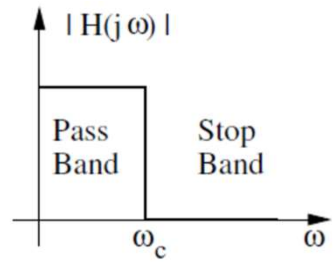
$$V_o = \frac{R}{R + j\omega L} V_i$$

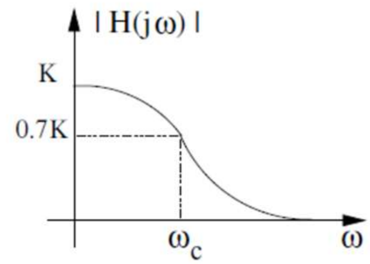
$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L}$$

$$= \frac{1}{1 + j(\omega L / R)}$$

$$H(j\omega) = \frac{1}{1 + j\omega/\omega_c}$$

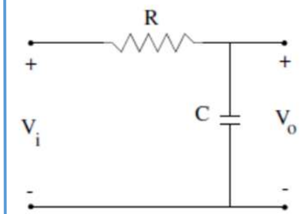
$$\omega_c = \frac{R}{L}$$





$$|H(j\omega)|_{\omega=\omega_c} = \frac{1}{\sqrt{2}}\, |H(j\omega)|_{max} = \frac{1}{\sqrt{2}}$$

R-C Circuit



$$V_o = \frac{1/(j\omega C)}{R + 1/(j\omega C)} V_i$$

$$= \frac{1}{1 + j(\omega RC)} V_i$$

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$H(j\omega) = \frac{1}{1 + j\omega/\omega_c}$$

$$\omega_c = 1/RC$$

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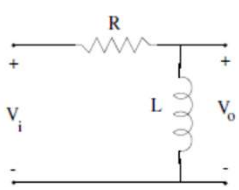
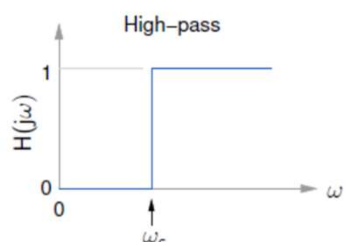
High Pass filter

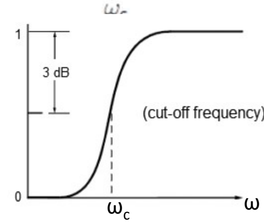


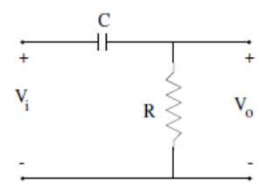
$$H(j\omega) = \frac{V_o}{V_i} = \frac{j\omega L}{R + j\omega L}$$

$$H(j\omega) = \frac{1}{1 - j\omega_c/\omega}$$

$$\omega_c = \frac{R}{L}$$







$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + 1/(j\omega C)}$$

$$= \frac{1}{1 - j(1/\omega RC)}$$

$$H(j\omega) = \frac{1}{1 - j\omega_c/\omega}$$

$$\omega_C = 1/RC.$$

$$|H(j\omega)|_{\omega=\omega_c} = \frac{1}{\sqrt{2}}\, |H(j\omega)|_{max} = \frac{1}{\sqrt{2}}$$

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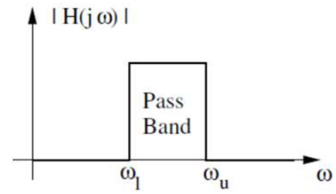
Band Pass Filter

1. R-L-C resonance

2. Cascading Low-Pass and High-Pass filters



Performance Parameters

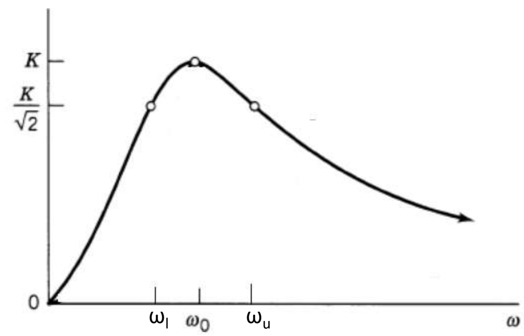
ω_l : Lower cut-off frequency; $|H(j\omega)|_{db}$

ω_u : Upper cut-off frequency;

$\omega_0 \equiv \sqrt{\omega_l \omega_u}$: Center frequency;

$B \equiv \omega_u - \omega_l$: Band width;

$Q \equiv \dfrac{\omega_0}{B}$: Quality factor.



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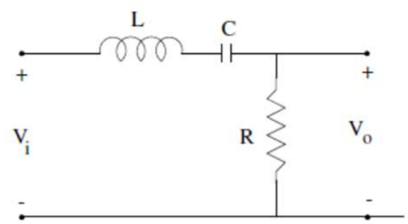
Band Pass Filter

Series RLC Band-pass filters

$$H(j\omega) = \frac{V_o}{V_i} = \frac{R}{R + j\omega L + 1/(j\omega C)}$$

$$H(j\omega) = \frac{R}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)}$$



At $\omega=\omega_o$ Transfer function $H(j\omega)$ will be real => $\omega_0 L - \dfrac{1}{\omega_0 C} = 0$ ➔ $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$$H(j\omega = j\omega_0) = \frac{R}{R} = 1$$

The cut-off frequencies can then be found by setting:

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$1 + \left(\frac{\omega_c L}{R} - \frac{1}{\omega_c RC}\right)^2 = 2$$

which can be solved to find ω_u and ω_l.

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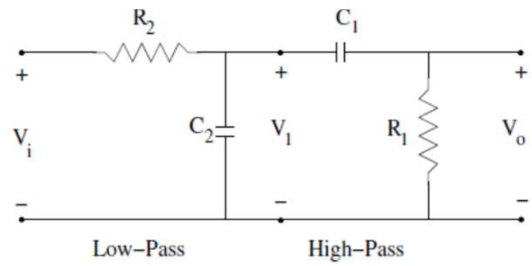
Band Pass Filter

Cascading Low-Pass and High Pass Filters

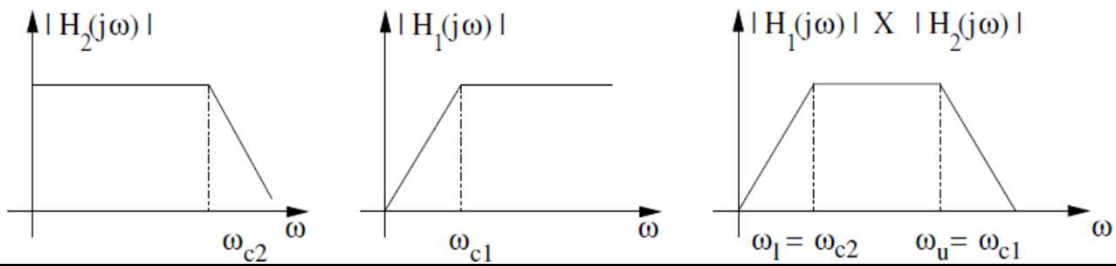
Equivalent Transfer function is multiple of both



$$H(j\omega) = H_1(j\omega) \times H_2(j\omega) = \frac{1}{1 + j\omega/\omega_{c2}} \times \frac{1}{1 - j\omega_{c1}/\omega}$$

$\omega_{c1} = 1/(R_1C_1) \qquad \omega_{c2} = 1/(R_2C_2)$

$$H(j\omega) = \frac{1}{(1 + j\omega/\omega_{c2})(1 - j\omega_{c1}/\omega)} = \frac{1}{(1 + \omega_{c1}/\omega_{c2}) + j(\omega/\omega_{c2} - \omega_{c1}/\omega)} \quad \text{………….(1)}$$



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Eq. (1) cont…….

$$H(j\omega) = \frac{1}{(1 + j\omega/\omega_{c2})(1 - j\omega_{c1}/\omega)} = \frac{1}{(1 + \omega_{c1}/\omega_{c2}) + j(\omega/\omega_{c2} - \omega_{c1}/\omega)}$$

Assuming,

$$K = \frac{1}{1 + \omega_{c1}/\omega_{c2}} \qquad Q = \frac{\sqrt{\omega_{c1}/\omega_{c2}}}{1 + \omega_{c1}/\omega_{c2}} \qquad \omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

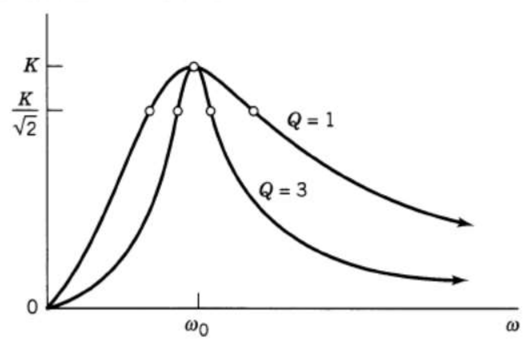
Where $\omega_{c1} = 1/(R_1C_1)$

$\omega_{c2} = 1/(R_2C_2)$

The transfer function for a second-order band-pass filter can be written as

$$H(j\omega) = \frac{K}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

$|H(j\omega)|_{db}$



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The applications of bandpass filters include the following.

•These filters are extensively applicable to wireless transmitters & receivers.

•The main purpose of the filter in the transmitter is to limit the BW of the output signal to the selected band for the communication.

•The best application of this filter is audio signal processing, wherever a specific range of sound frequencies is necessary though removing the rest.

•These filters are applicable in sonar, instruments, medical, and **Seismology** applications

•These filters involve **communication systems** for choosing a particular signal from a variety of signals.

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- **Self study: Bode Plot as H.W.**
- **Few next slides are additional ref. material**
- **To be discussed in Tutorials with assignment Problems**

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Bode Plots and Decibel

The voltage transfer function of a two-port network is usually expressed in Bel:
(and/or the ratio of output to input

$$\text{Number of Bels} = \log_{10}\left(\frac{P_o}{P_i}\right) \quad \text{or} \quad \text{Number of Bels} = 2\log_{10}\left|\frac{V_o}{V_i}\right| \qquad \text{because } P \propto V^2$$

Bel is a large unit and decibel (dB) is usually used:

$$\text{Number of decibels} = 20\log_{10}\left|\frac{V_o}{V_i}\right| \quad \text{or} \quad \left|\frac{V_o}{V_i}\right|_{dB} = 20\log_{10}\left|\frac{V_o}{V_i}\right|$$

using dB definition, 3 dB difference between maximum gain and gain at the cut-off frequency:

$$20\log|H(j\omega_c)| - 20\log|H(j\omega)|_{max} = 20\log\left[\frac{|H(j\omega_c)|}{|H(j\omega)|_{max}}\right] = 20\log\left(\frac{1}{\sqrt{2}}\right) \approx -3\text{ dB}$$

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Additional Ref. for slop of bode curve dB/Octave and dB/decade

In most cases our transfer function is a voltage or current ratio, so we will use $20\log|H(j\omega)|$ to compute the magnitude in dB. Some important dB conversions to remember are summarized below:

$\lvert\mathbf{H}\rvert$	$\lvert\mathbf{H}\rvert_{dB}$
1	$20\log(1) = 0$ dB
$\sqrt{2}$	$20\log(\sqrt{2}) = 10\log 2 = 3$ dB
2	$20\log(2) = 6$ dB
4	$20\log(4) = 12$ dB
5	$20\log(5) = 14$ dB
10	$20\log(10) = 20$ dB

A logarithmic scale like the dB scale prove to be a great advantage when dealing with circuit transfer functions, which are always of the form of a rational polynomial function as in (1.2).

Two related terms we will use in our discussion of frequency response plots are "decade" and "octave". A decade change in frequency is a factor of ten. So, for example, 1 kHz is a decade *above* 100 Hz and a decade *below* 10 kHz. An "octave" is a factor of two, so similarly 1 kHz is an octave above 500 Hz and an octave below 2 kHz.

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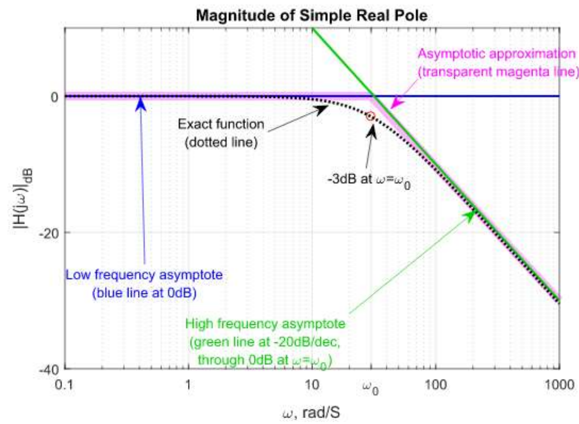
How to use transfer function to plot bode plot.

1. To draw a piecewise linear approximation, use the low frequency asymptote up to the break frequency, and the high frequency asymptote thereafter.

2. The resulting asymptotic approximation is shown highlighted in transparent magenta.
3. The maximum error between the asymptotic approximation and the exact magnitude function occurs at the break frequency and is approximately -3 dB.



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Bode plots are plots of $|H(j\omega)|_{dB}$ (magnitude) and $\angle H(j\omega)$ (phase) versus frequency in a semi-log format (*i.e.*, ω axis is a log axis). Bode plots of first-order low-pass RL filters are shown below (W denotes ω_c).

Low Pass filter $H(j\omega) = \dfrac{1}{1 + j\omega/\omega_c}$

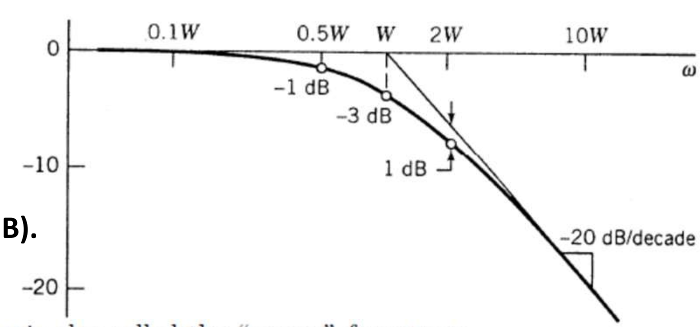
At high frequencies, $\omega/\omega_c \gg 1$, $|H(j\omega)| \approx \dfrac{1}{\omega/\omega_c}$ ……A

$$|H(j\omega)|_{dB} = 20\log\left[\frac{1}{\omega/\omega_c}\right] = 20\log(\omega_c) - 20\log(\omega)$$

which is a straight line with a slope of -20 dB/decade in the Bode plot.

At low frequencies, $\omega/\omega_c \ll 1$, $|H(j\omega)| \approx 1$ ……B

➔ $|H(j\omega)|_{dB} = 0$



$\omega = \omega_c$ =W is the intersection of these two Asymptotic values (A and B).

the cut-off frequency is also called the "corner" frequency.

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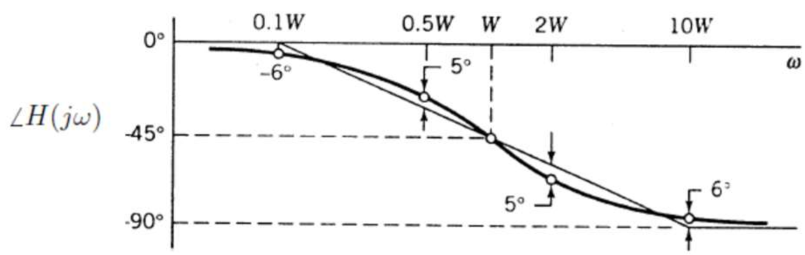
The behavior of the phase of $H(j\omega)$ can be found by examining $\angle H(j\omega) = -\tan^{-1}(\omega/\omega_c)$

low frequencies, $\omega/\omega_c \ll 1$, $\angle H(j\omega) \approx 0$

at high frequencies, $\omega/\omega_c \gg 1$, $\angle H(j\omega) \approx -90°$

At cut-off frequency, $\angle H(j\omega) \approx -45°$.



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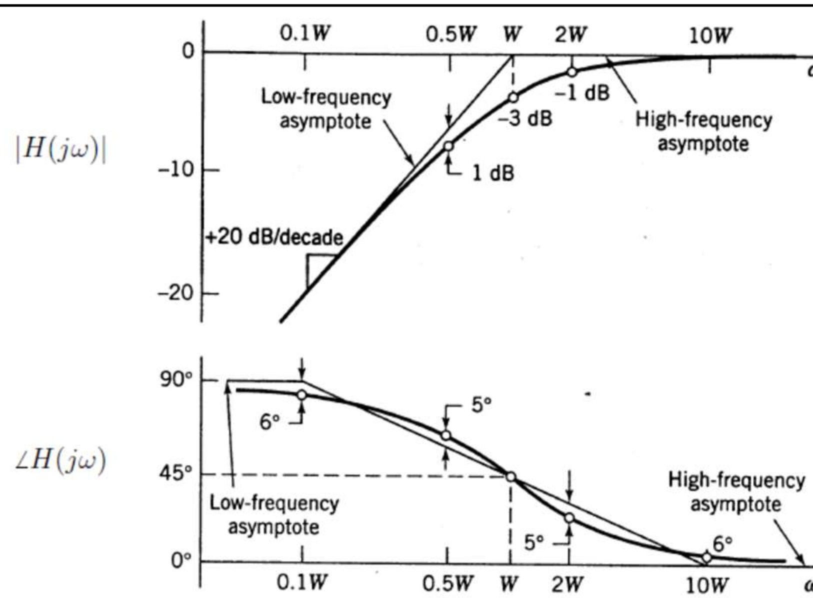
Similarly for High pass filter, Bode Plots:

$$H(j\omega) = \frac{1}{1 - j\omega_c/\omega}$$



At low frequencies, $\omega/\omega_c \ll 1$, $|H(j\omega)| \propto \omega$ (a +20dB/decade line) and $\angle H(j\omega) = 90°$
At high frequencies, $\omega/\omega_c \gg 1$, $|H(j\omega)| \propto 1$ (a line with a slope of 0) and $\angle H(j\omega) = 0°$

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Solved Example for Hint

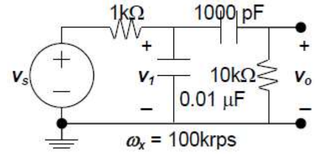
For the given circuit Find the following:

a) The Transfer Function $H(j\omega) = V_o / V_s$, its Magnitude $|H(j\omega)|$ and Phase angle θ as functions of ω.

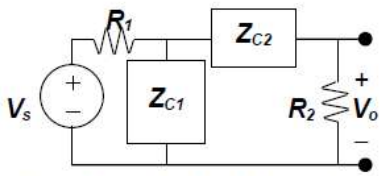
b) The values of $|H(j\omega)|$ and θ at $\omega = \omega_x$ (the value of ω_x is specified in each problem).

c) The asymptotic expressions ($\omega \to 0$ and $\omega \to \infty$) for $|H(j\omega)|$ and hence the slopes of the low-frequency and high-frequency asymptotes (in **dB / octave**).

d) Infer the nature of the frequency response:
low-pass / high-pass / band-pass / band-stop.



Sol. **[Consistent set of units used: voltage – V, current – mA, *R* – kΩ, *C* – μF, *L* – H, ω – krps]**



$R_1 = 1$, $C_1 = 0.01$, $C_2 = 0.001$, $R_2 = 10$.

(a) $H(j\omega) = j\omega C_2R_2/[(1-\omega^2C_1C_2\, R_1R_2) + j\omega(C_1R_1 + C_2R_1 + C_2R_2)]$.
$\Rightarrow |H(j\omega)| = \omega / \sqrt{[(100 - 0.01\omega^2)^2 + 4.41\omega^2]}$
and $\theta = 90^o - \tan^{-1}[2.1\omega / (100 - 0.01\omega^2)]$.

(b) $\omega_x = 100 \Rightarrow |H(j\omega_x)| = 10 / 21$ and $\theta_x = 0$.

(c) $|H(j\omega)|_{\omega \to 0} = 0.01\omega \Rightarrow 6$ dB/octave and
$|H(j\omega)|_{\omega \to \infty} = 100 / \omega \Rightarrow -6$ dB/octave.

(d) **Band pass** characteristic with centre frequency ω_x.

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Thanks

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