Tutorial 12

PHY-101

Q1.An ideal diatomic gas, with rotation but no oscillation, undergoes an adiabatic compression. Its initial pressure and volume are 1.20 atm and 0.200 m^3 . Its final pressure is 2.40 atm. How much work is done by the gas?

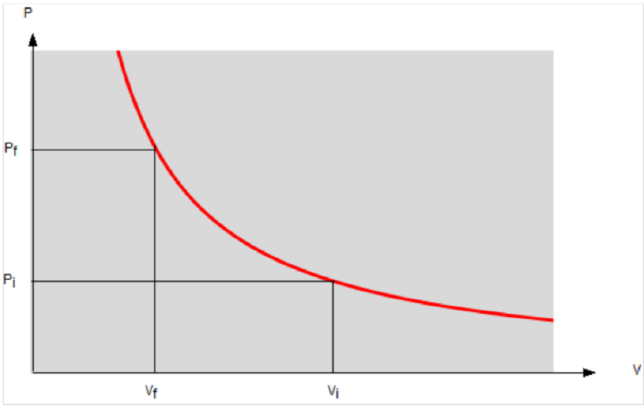
Hint: γ= CP/CV For diatomic gas CP = 7R/2; CV = 5R/2

Sol:

Solution

$C_P = 7R/2$
$C_V = 5R/2$
$\gamma = C_P / C_V = 7/5 = 1.40$

$P_f = 2.40$ atm.
$P_i = 1.20$ atm
$V_i = 0.2\ m^3$
Adiabatic process



$$PV^{\gamma} = P_i V_i^{\gamma} = P_f V_f^{\gamma}$$

$$\frac{V_f}{V_i} = \left(\frac{P_i}{P_f}\right)^{1/\lambda} = 2^{-1/\gamma}$$

$$V_f = 2^{-1/1.40} V_i = 0.122 m^3$$

The work done on the system is

$$\Delta W = -\int_{V_i}^{V_f} P dV = -\int_{V_i}^{V_f} P_i V_i^{\gamma} V^{-\gamma} dV$$

$$= -\frac{P_i V_i^{\gamma}}{1-\gamma}[V_f^{1-\gamma} - V_i^{1-\gamma}]$$

$$= -\frac{P_f V_f^{\gamma}}{1-\gamma} V_f^{1-\gamma} + \frac{P_i V_i^{\gamma}}{1-\gamma} V_i^{1-\gamma}$$

$$= -\frac{P_f V_f}{1-\gamma} + \frac{P_i V_i}{1-\gamma}$$

$$= \frac{1}{\gamma - 1}(P_f V_f - P_i V_i)$$

$$= -1.3312 \times 10^4 J$$

Q2. A 2.5 mol sample of helium gas (a monoatomic ideal gas) is contained in a 10.0 L vessel at 300K.

- Calculate the pressure of the gas using the Ideal Gas Law.
- Calculate the average kinetic energy of each helium atom.

Find the root mean square (RMS) speed of the helium atoms. (Helium has a molar mass of 4.00g/mol=0.004 kg/mol).

Solution: Step 1: Calculate the Pressure

Using the Ideal Gas Law:

PV=nRT, we can solve for P

$$P = \frac{nRT}{V}$$

Given values:

- $n = 2.5$ mol,
- $R = 8.314\,\mathrm{J/(mol\backslash cdotpK)}$,
- $T = 300\,\mathrm{K}$,
- $V = 10.0\,\mathrm{L} = 0.010\,\mathrm{m^3}$ (converting liters to cubic meters for SI units).

Step 2: Calculate the Average Kinetic Energy per Atom

The average kinetic energy per atom in a monoatomic gas is given by:

$$E_{avg} = \frac{3}{2}k_B T$$

Where $k_B = 1.38 \times 10^{-23}\,\mathrm{J/K}$.

Step 3: Calculate the RMS Speed

The RMS speed is calculated by:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

where:

- $M = 0.004\,\mathrm{kg/mol}$,
- $T = 300\,\mathrm{K}$,
- $R = 8.314\,\mathrm{J}/(\mathrm{mol}\backslash\mathrm{cdotp}\,\mathrm{K})$.

Calculating these values, we have:

1. Pressure of the gas: $P = 623{,}550\,\mathrm{Pa}$.
2. Average kinetic energy of each helium atom: $E_{avg} = 6.21 \times 10^{-21}\,\mathrm{J}$.
3. Root mean square (RMS) speed of helium atoms: $v_{rms} = 1367.7\,\mathrm{m/s}$.

Q3. Find the total internal energy of 96 gm of oxygen gas at 27^{o}C.

Sol:

Q Find the total internal energy of 96g of Oxygen gas is at 27°C.

Sol:- $T = 273 + 27 = 300\,K$

$n = \frac{96g}{32g/mol} = 3$ mole

Oxygen is a diatomic molecule, So its degree of freedom

$\Rightarrow 3N - R = 3 \times 2 - 1$

$= 5$

Total internal energy $\Rightarrow U = \frac{f}{2} nRT$

$= \frac{5}{2} \times 3 \times 8.314 \times 300$

$= 18{,}301.5\,J$

Q4. (a) 60 Joule(J) of work is done on a gas and the gas loses 150J of heat to the surrounding. What is the change in internal energy?

(b)A gas starts with 200J of internal energy, while 180J of heat is added to the gas while the gas does 70J of work. What is the final internal energy of the gas?

(c)If 40J of work is done on a gas, the internal energy goes down by 150J. What was the value of heat added to the gas?

Sol:

DATE

(a) $\Delta U = \Delta Q - W$

Heat added to the system

Work done by the gas (system)

$\Delta U = -150\,J - (-60\,J)$

Negative sign because work is done on the gas, so total internal energy has to increase.

$\Rightarrow \Delta U = -150\,J + 60\,J$

$\Rightarrow \boxed{\Delta U = -90\,J}$

(b) $\Delta U = +180\,J - (+70\,J)$

↳ Work done by the gas.

$\Rightarrow \Delta U = +110\,J$

$\Rightarrow U_f - U_i = 110\,J$

$\Rightarrow U_f = 110\,J + U_i$

$\Rightarrow U_f = 110\,J + 200\,J$

$\Rightarrow U_f = 310\,J$.

(c) Internal energy goes down.

$\Rightarrow \Delta U = -150\,J$

$\Rightarrow -150\,J = \Delta Q - W$

$\Rightarrow -150\,J = \Delta Q - (-40\,J)$

$\Rightarrow \Delta Q = -150\,J - 40\,J$

$\Rightarrow \boxed{\Delta Q = -190\,J}$ → Lot of heat is taken away from the system

Q5. When 1.3KJ of heat is added to a balloon ,its volume increases from 4 x 10^6 to 4.5 x 10^6 under constant pressure of 105 kPa. Calculate change in internal energy.

Q5 (a) when 1.3×10^5 kJ of heat is added to a baloon, it's volume increases from 4×10^6 lit to 4.5×10^6 lit under a constant pressure of 105 kPa. Calculate ΔU (in kJ)

Soln:

$\Delta U = \Delta Q - W$ → Work done by the system.

$\Rightarrow \Delta U = 1.3\times10^5\,kJ - (P\Delta V) \quad (W = P\Delta V)$

$= 1.3\times10^5\,kJ - (105\times10^3\,pa\,[4.5\times10^6 - 4\times10^6])$

$\Delta U = 1.3\times10^8\,J - (105\times10^3\,Pa)\times(0.5\times10^6)\,\frac{m^3}{10^3} \times \left(\frac{1\,m^3}{1000\,L}\right)$ → Conversion to cancel the L

$= [1.3\times10^2 - 52.5]\times10^6\,J$

$= [130 - 52.25]\times10^6\,J$

$= 7.75\times10^7\,J$