Lec-7
Network Theorems

- Superposition Theorem
- Thevenin's Theorem and
- Norton's Theorem
- Maximum Power Transfer Theorem

1

SUPERPOSITION THEOREM

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.

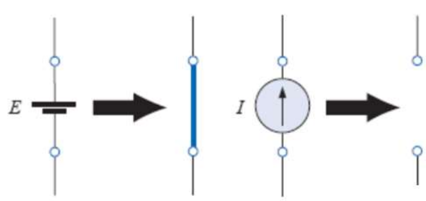

FIG. 9.1
Removing the effects of ideal sources.

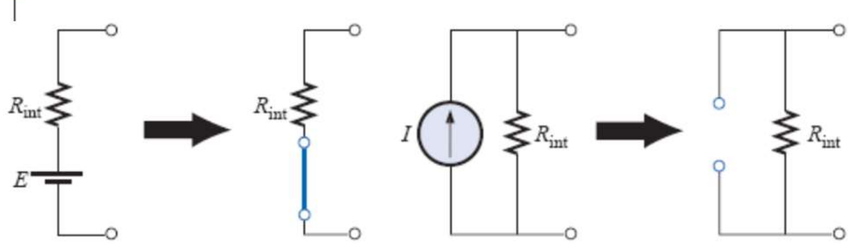

FIG. 9.2
Removing the effects of practical sources.

2

Not Valid for Power Calculation

The total power delivered to a resistive element must be determined using the total current through or the total voltage across the element and cannot be determined by a simple sum of the power levels established by each source.

$$I_T^2 = (I_1 + I_2)^2 = I_1^2 + I_2^2 + 2I_1I_2$$

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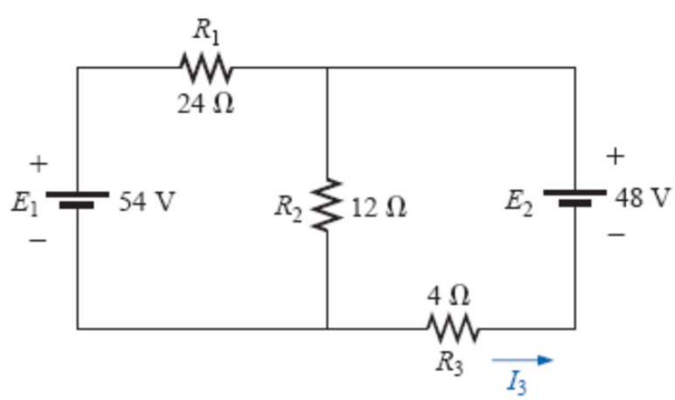
Determine the current through the 4 Ohm?



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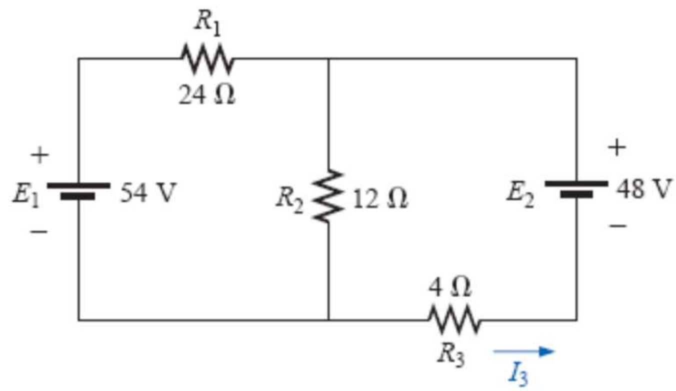

$$I_3 = I''_3 - I'_3 = 4\text{ A} - 1.5\text{ A} = \mathbf{2.5\text{ A}}$$

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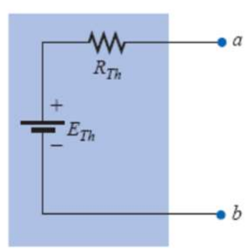
THEVENIN'S THEOREM

Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor,



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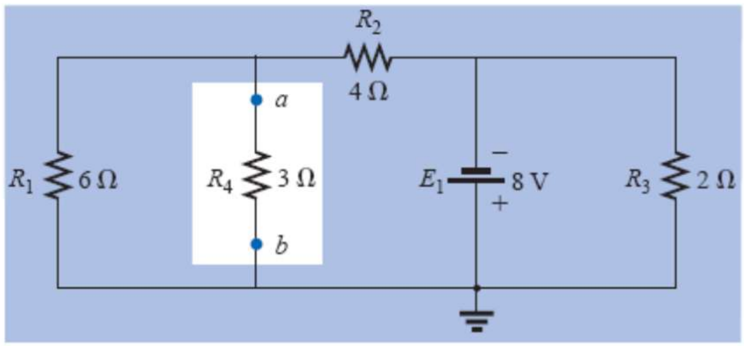
Draw Thevenin's equivalent ckt taking R4 as load

Calculate: I (R4)



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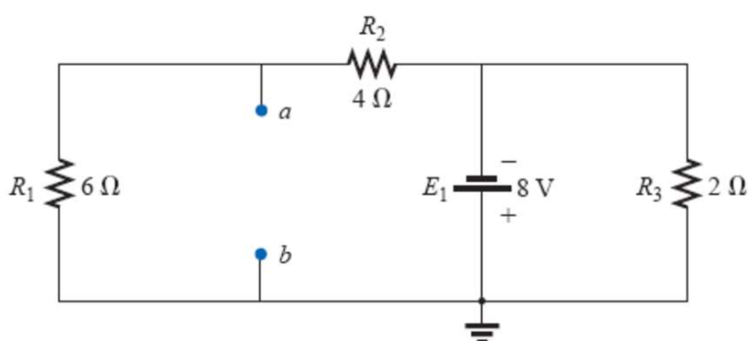
1. Identifying the terminals of particular interest for the network (Removing Load)



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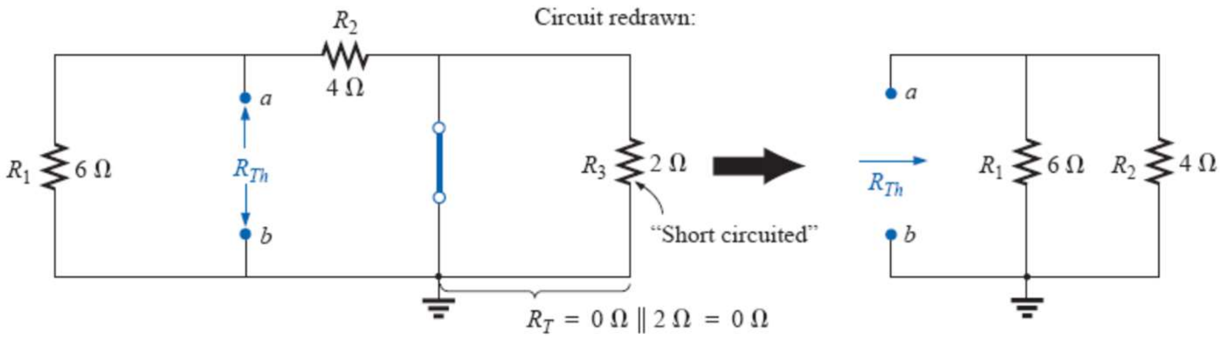
***2. Calculate R_{Th}** by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.)*



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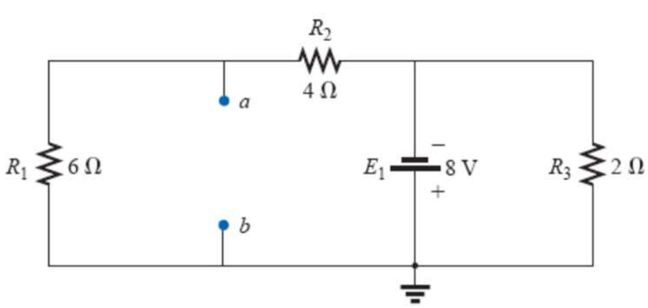
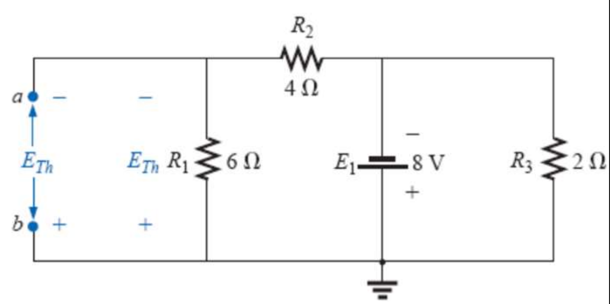
$$R_{Th} = R_1 \parallel R_2 = \frac{(6\ \Omega)(4\ \Omega)}{6\ \Omega + 4\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

3. Determining E_{Th} for the network

Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.





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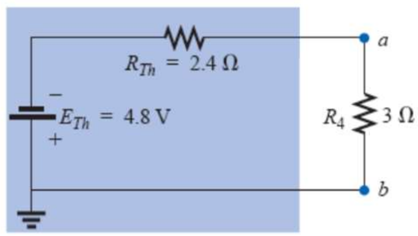
Applying the voltage divider rule,

$$E_{Th} = \frac{R_1 E_1}{R_1 + R_2} = \frac{(6\ \Omega)(8\ \text{V})}{6\ \Omega + 4\ \Omega} = \frac{48\ \text{V}}{10} = \mathbf{4.8\ V}$$

4. Substituting the Thévenin equivalent circuit for the network external to the resistor R4



Calculate: I (R4)= 0.88 A

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Thevenin's equivalent resistance can also be calculated from:

$$R_{Th} = V_{OC} / I_{SC}$$

Where:

V_{oc} is open ckt voltage

I_{sc} is short ckt current

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NORTON'S THEOREM

Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.

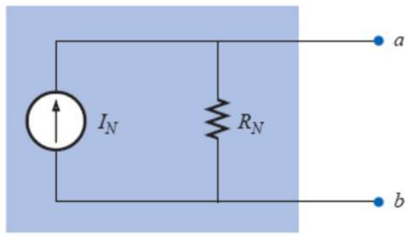

FIG. 9.58
Norton equivalent circuit.

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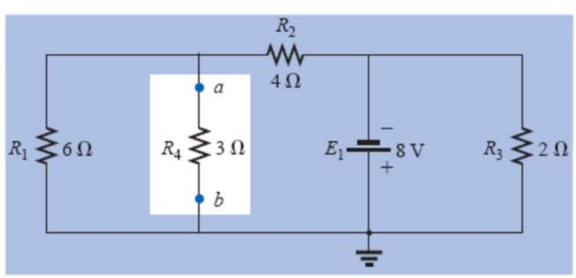
Draw Norton's Equivalent Ckt for given network being R4 as load



Calculate: I (R4)= ?, using Norton's Theorem

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1. Remove that portion of the network across which the Norton equivalent circuit is to be found.

2. Mark the terminals of the remaining two-terminal network.

R_N:
3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals.

$$R_N = R_{Th}$$

I_N:
4. Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

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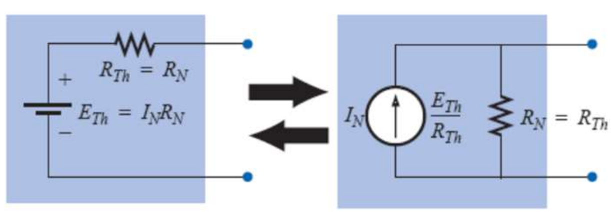
Converting between Thévenin and Norton equivalent circuits



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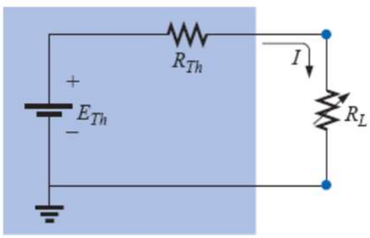
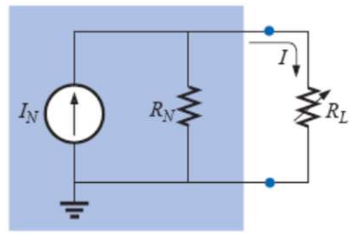
MAXIMUM POWER TRANSFER THEOREM

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thévenin resistance of the network as "seen" by the load.





$R_L = R_{Th}$

$R_L = R_N$

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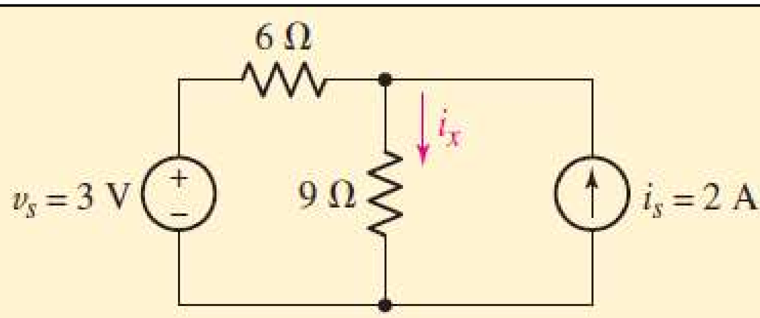

(a)

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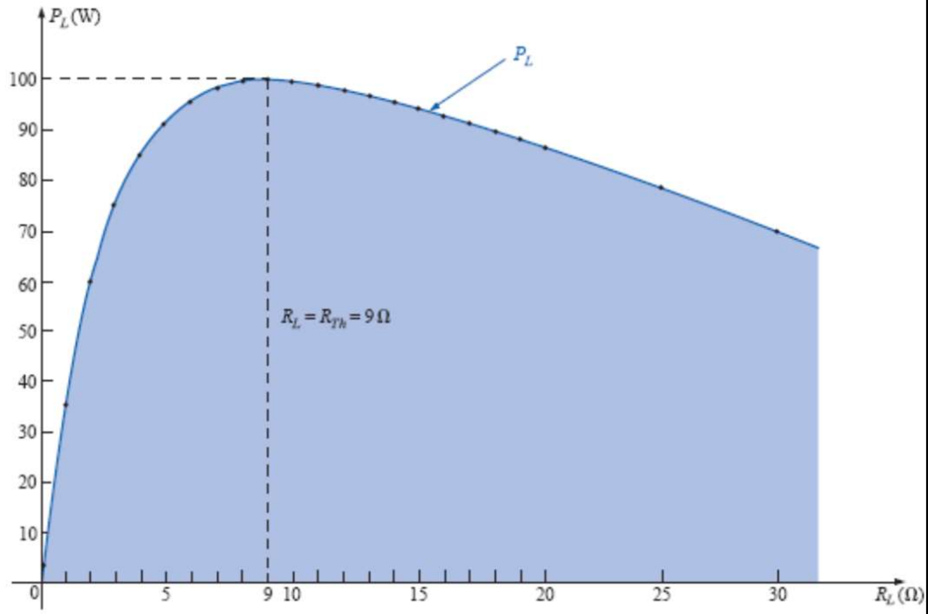
$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

$$P_L = I^2 R_L = \left(\frac{E_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$P_L = \frac{E_{Th}^2 R_L}{(R_{Th} + R_L)^2}$$



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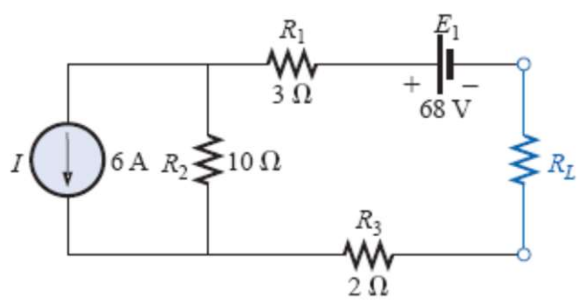
Find the value of R_L for maximum power to R_L, and determine the maximum power.



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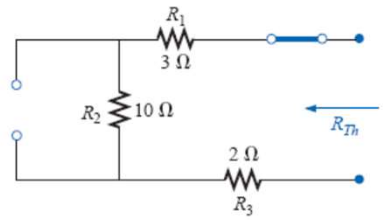
$$R_{Th} = R_1 + R_2 + R_3 = 3\ \Omega + 10\ \Omega + 2\ \Omega = 15\ \Omega$$

$$R_L = R_{Th} = \mathbf{15\ \Omega}$$



$$V_1 = V_3 = 0\ \text{V}$$

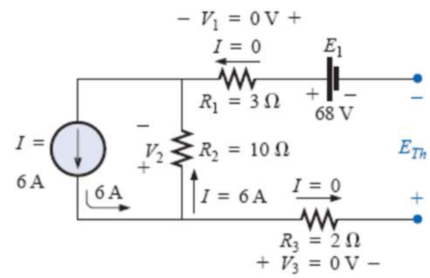
and $$V_2 = I_2R_2 = IR_2 = (6\ \text{A})(10\ \Omega) = 60\ \text{V}$$

Applying Kirchhoff's voltage law,

$$\Sigma_{\circlearrowright} V = -V_2 - E_1 + E_{Th} = 0$$

and $$E_{Th} = V_2 + E_1 = 60\ \text{V} + 68\ \text{V} = 128\ \text{V}$$

Thus, $$P_{L_{\max}} = \frac{E_{Th}^2}{4R_{Th}} = \frac{(128\ \text{V})^2}{4(15\ \Omega)} = \mathbf{273.07\ W}$$



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Thanks

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