PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 29

THE FIELD OF A MAGNETIZED OBJECT
Physical Interpretation of Bound Currents
THE AUXILIARY FIELD H
Boundary Conditions
Linear Media
Ferromagnetism

Magnetization

In the presence of a magnetic field, matter becomes **magnetized**.

Two mechanisms that account for this magnetic polarization:

(1) **Paramagnetism** (the dipoles associated with the spins of unpaired electrons experience a torque tending to line them up parallel to the field).

(2) **Diamagnetism** (the orbital speed of the electrons is altered in such a way as to change the orbital dipole moment in a direction opposite to the field).

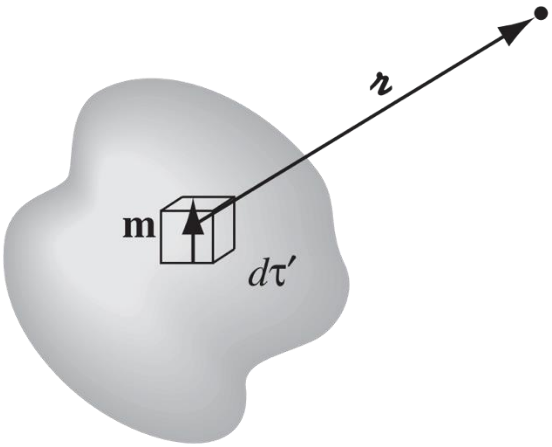
$$\mathbf{M} \equiv \textit{magnetic dipole moment per unit volume.}$$

M is called the magnetization

Bound Currents

You have a piece of magnetized material- magnetic dipole moment per unit volume $\boldsymbol{M}$ is given

What field does this object produce?



Each volume element element dτ’ contains dipole moment $\boldsymbol{M}d\tau'$- potential due to dipole moment ‘m’

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\boldsymbol{\imath}}}{\imath^2}$$

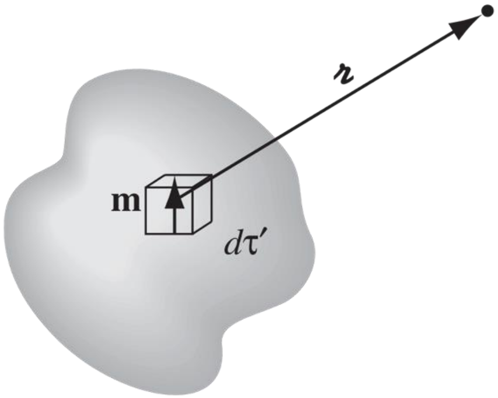
Magnetization **(M)** is dipole moment per unit volume

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{M}(\mathbf{r}')\times\hat{\boldsymbol{\imath}}}{\imath^2}d\tau'$$

Dipole potential of volume element

THE FIELD OF A MAGNETIZED OBJECT

Bound Currents



For the entire object:

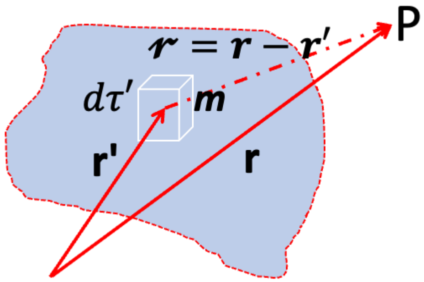
$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{M}(\mathbf{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\, d\tau'$$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \left[\mathbf{M}(\mathbf{r}') \times \left(\boldsymbol{\nabla}'\frac{1}{\mathscr{r}}\right)\right] d\tau'$$

$$[\boldsymbol{\mathscr{r}} = \boldsymbol{r} - \boldsymbol{r}']$$

$$\boldsymbol{\nabla}\left(\frac{1}{\mathscr{r}}\right) = -\boldsymbol{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = -\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

(integration must contain primed variables)



$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\boldsymbol{\nabla}' \times \mathbf{M}(\mathbf{r}')]\, d\tau' - \int \boldsymbol{\nabla}' \times \left[\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}\right] d\tau'\right\}$$

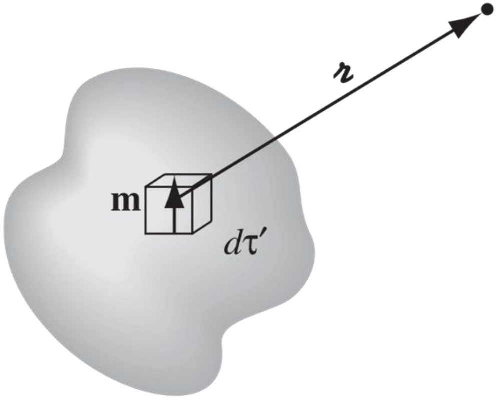
THE FIELD OF A MAGNETIZED OBJECT

Bound Currents

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\,d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times d\mathbf{a}']$$

Vector identity

$$\int_{v} (\nabla \times \boldsymbol{v})d\tau' = -\int_{S} \boldsymbol{v} \times \boldsymbol{da}$$



Bound Currents

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\,d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times d\mathbf{a}']$$

The first term looks just like the potential of a volume current,

$$\boxed{\mathbf{J}_b = \nabla \times \mathbf{M},}$$

the second looks like the potential of a surface current

$$\boxed{\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}},}$$

where $\hat{\mathbf{n}}$ is the normal unit vector.

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\,d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\,da'.$$

Bound Currents

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\, d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\, da'.$$

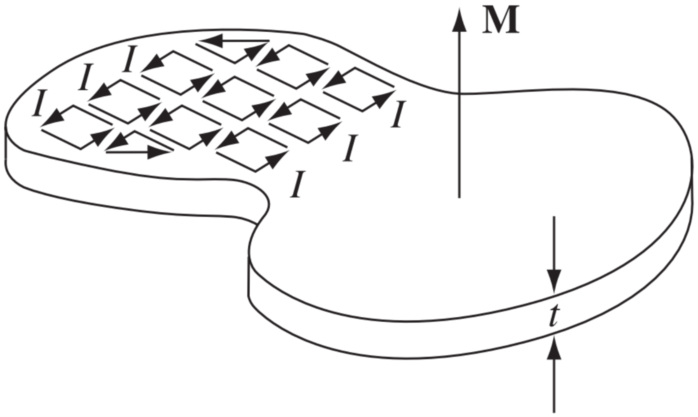
The potential (and hence also the field) of a magnetized object is the same as would be produced by a volume current $\mathbf{J}_b = \nabla \times \mathbf{M}$ throughout the material, plus a surface current $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$ on the boundary.

Instead of finding contribution of all infinitesimal dipoles, we first determine the bound currents and then find the field they produce.

Compare with $\sigma_b = \boldsymbol{P}.\hat{\boldsymbol{n}}$ and $\rho_b = -\boldsymbol{\nabla}.\boldsymbol{P}$

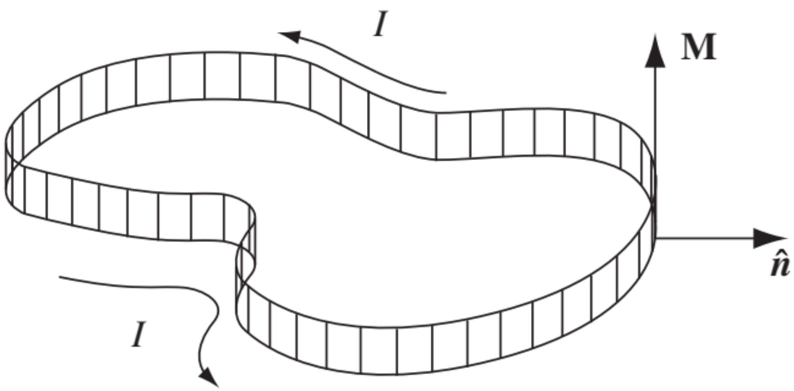
Physical Interpretation of Bound Currents

How do bound currents arise physically?



Consider a thin slab of uniformly magnetized material, with the dipoles represented by tiny current loops.

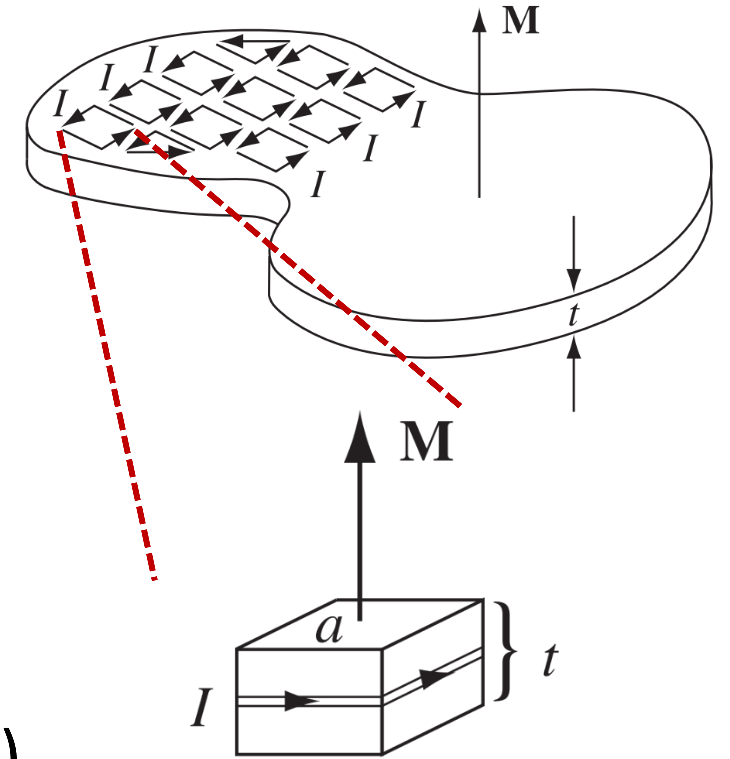
Internal currents cancel- at the edge there is no cancelling. Whole thing is a single ribbon of current I *flowing along the boundary (bound surface current)*



What is this current, in terms of **M**?

Physical Interpretation of Bound Currents

Consider each of the tiny loops has area **a** and thickness **t**



Dipole moment

$$m = Mat$$

In terms of the circulating current I

$$m = Ia$$

Therefore

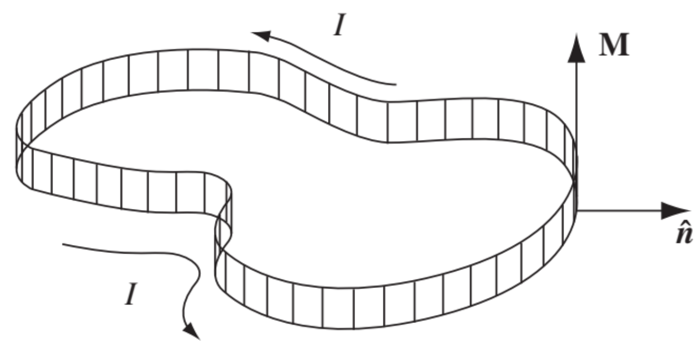
$$I = Mt$$

(M **uniform**)

The surface current

$$K_b = \frac{I}{t} = M$$

$$\boxed{\boldsymbol{K_b} = \boldsymbol{M} \times \hat{\boldsymbol{n}}}$$



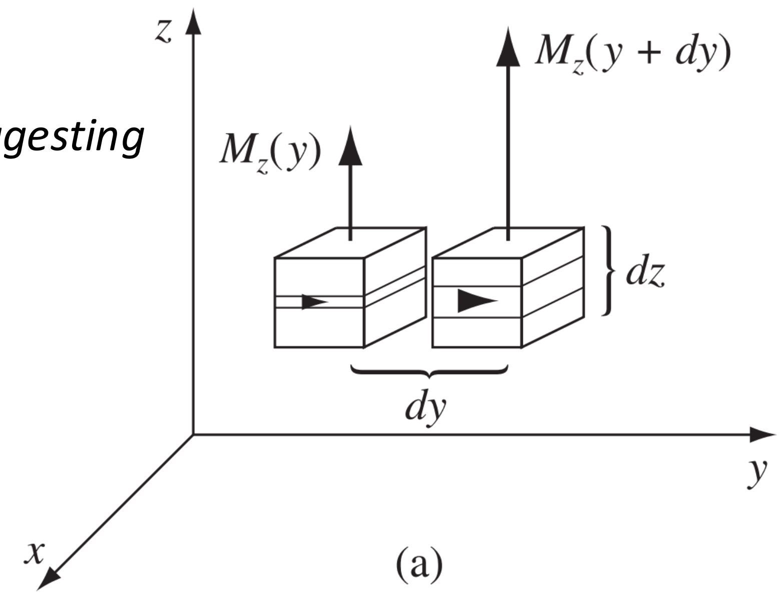
(This expression also records the fact that there is *no* current on the top or bottom surface of the slab; here **M** is parallel to $\hat{\mathbf{n}}$, so the cross product vanishes.)

Physical Interpretation of Bound Currents

In case on ***non-uniform magnetization***, internal currents do not cancel.

Two chunks of magnetized material- non-uniformity in magnetization is along z-axis

"The larger arrow on the one to the right suggesting greater magnetization at that point"



On the surface where they join, there is a net current along x-axis

$$I_x = [M_z(y + dy) - M_z(y)] \times dz$$
$$= \frac{\partial M_z}{\partial y} dydz$$

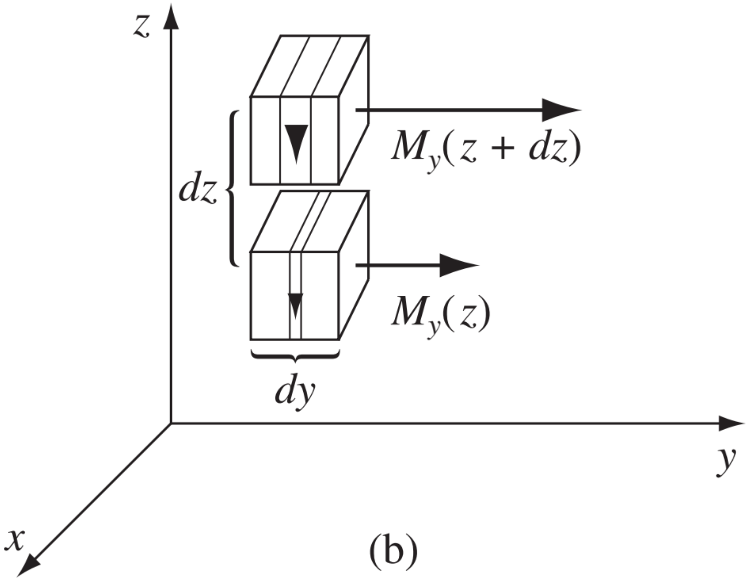
$$\boxed{\frac{I_x}{dydz} = (J_b)_x = \frac{\partial M_z}{\partial y}}$$

Physical Interpretation of Bound Currents

In case on ***non-uniform magnetization***, internal currents do not cancel.

Similarly, non-uniformity of M along y-direction leads to

$$\boxed{(J_b)_x = -\frac{\partial M_y}{\partial z}}$$



The volume current density along x-direction is

$$(J_b)_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}$$

In general,

$$\boxed{\boldsymbol{J_b} = \boldsymbol{\nabla} \times \boldsymbol{M}}$$

Incidentally, like any other steady current, $\boldsymbol{J_b}$ should obey the **conservation law**

$$\nabla \cdot \mathbf{J}_b = 0$$

THE AUXILIARY FIELD H

Effect of magnetization is to establish bound currents $\boldsymbol{J_b} = \boldsymbol{\nabla} \times \boldsymbol{M}$ within the material and $\boldsymbol{K_b} = \boldsymbol{M} \times \boldsymbol{\hat{n}}$ on the surface.

(Field due to magnetization is just the field produced by these currents.)

$$\boxed{\boldsymbol{J} = \boldsymbol{J_b} + \boldsymbol{J_f}}$$

J_f is the free current- it can flow through wires imbedded in the magnetized material, or though the material itself in case it is a conductor,

Ampere's law

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}_f + (\nabla \times \mathbf{M})$$

$$\nabla \times \left(\frac{1}{\mu_0}\mathbf{B} - \mathbf{M}\right) = \mathbf{J}_f$$

$$\boxed{\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}.}$$

$$\boxed{\boldsymbol{\nabla} \times \boldsymbol{H} = \boldsymbol{J_f} \qquad \oint \boldsymbol{H.dl} = (I_f)_{enc}}$$

(Ampere's law in terms of $\boldsymbol{H}$)

THE AUXILIARY FIELD H

- ***H*** plays a role in magnetostatics analogous to ***D*** in electrostatics

- Just as ***D*** allowed us to write Gauss's law in terms of free charge alone, ***H*** permits us to express Ampere's law in terms of free current alone- and free current is what we can control directly.

- Bound current, like bound charges comes when the material gets magnetized

Boundary Conditions

$$\boxed{\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}.}$$

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}.$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}.$$

$$\mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{\mathbf{n}}.$$

$$\mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$$

Linear media: Magnetic susceptibility

In paramagnetic and diamagnetic materials, the magnetization is sustained by the field; when **B** is removed, **M** disappears. For most substances the magentization is proportional to the field (LINEAR MEDIA), provided the field is not too strong.

It is customary to express this linear relationship using **H** instead of **B**:

$$\mathbf{M} = \chi_m \mathbf{H}.$$

The constant of proportionality χ_m is called the **magnetic susceptibility**. It is a dimensionless quantity that depends on the substance considered. It is small and positive (signifying weak attraction) for paramagnets and small and negative for diamagnets (signifying weak repulsion).

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.6×10^{-4}	Oxygen	1.9×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.1×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.8×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.8×10^{-4}
Carbon Dioxide	-1.2×10^{-8}	Liquid Oxygen ($-200°$ C)	3.9×10^{-3}
Hydrogen	-2.2×10^{-9}	Gadolinium	4.8×10^{-1}

Linear media: Magnetic permeability

We have

$$\mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}.$$

Therefore,

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$
$$\Rightarrow \mathbf{B} = \mu_0(\mathbf{H} + \chi_m \mathbf{H})$$
$$\Rightarrow \mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H}.$$

Evidently, for linear media, **B** is also proportional to **H**, such that

$$\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} = \mu\mathbf{H}.$$

μ is called the permeability of the material:

$$\mu = \mu_0(1 + \chi_m).$$

In vacuum there is no matter to magnetize, therefore magnetic susceptibility χ_m is zero, and permeability is μ_0 (the permeability of vacuum/free space).

Ferromagnetism

- In linear media the atomic (magnetic) dipoles get aligned due to external magnetic fields. Ferromagnetism refers to nonlinear behavior in which no external field is required to sustain the magnetisation – the alignment is "frozen in".

- Like paramagnetism, ferromagnetism involves the magnetic dipoles associated with the spins of unpaired electrons. However, in addition, there is presence of interaction between nearby dipoles.

- In a ferromagnet, each dipole prefers to point in the same direction as its neighbors. The interaction is so strong as to align almost all the unpaired electrons. The origin of this correlation is of quantum mechanical nature.

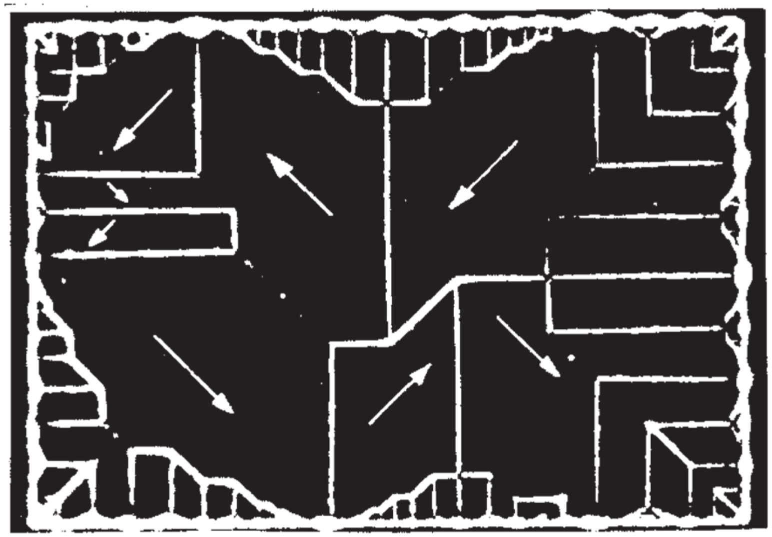
- They have a tendency to get strongly attracted in an external magnetic field.

- Not all ferromagnets will exhibit powerful magnet like behavior. This is because the alignment occurs in relatively small patches, called domains. These domains themselves are randomly oriented.

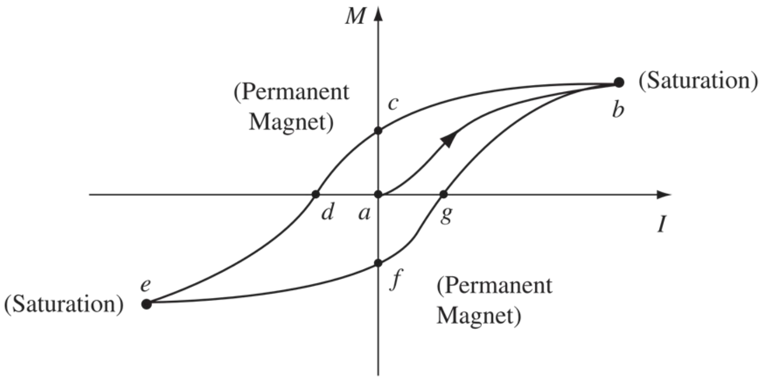
- When kept in external magnetic field, these domains tend to align parallel to the applied field. If the external field is switched off, even then many domains may remain aligned, leading to a permanent magnetism.

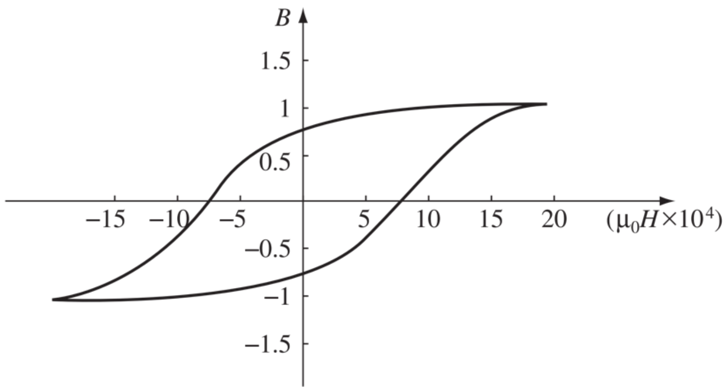
Ferromagnetism

Hysteresis loop



Ferromagnetic domains. (Photo courtesy of R. W. DeBlois)





The dipoles within a given domain line up parallel to one another. Random thermal motions tend to break this ordering. If temperature gets too high, the alignment is destroyed and the substance loses its ferromagnetic behavior, and starts to behave like a paramagnetic substance. There's a ferro-to-para transition if the temperature crosses certain value, called the Curie point. For iron this transition takes place at 770°C.

Watch this: https://youtu.be/haVX24hOwQI