

PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 20

Electric Fields in Matter

Induced Dipoles

THE FIELD OF A POLARIZED OBJECT

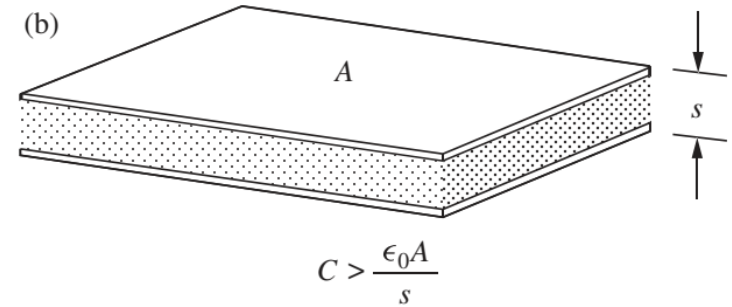
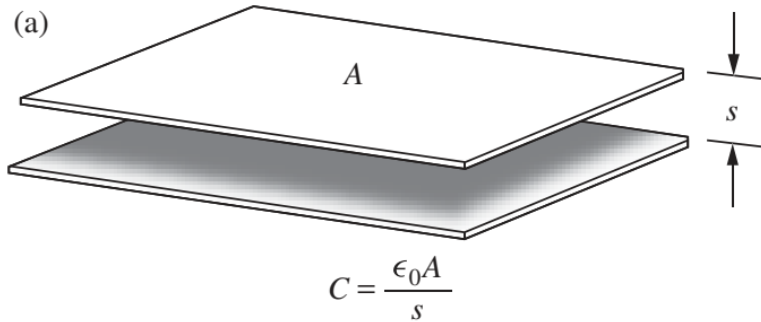
Physical Interpretation of bound charges



Electric Fields in Matter

Dielectrics

Faraday discovered that the capacitance of such a capacitor is *increased* when an insulator is put between the plates.



$$C = \frac{Q}{\phi_{12}}$$

If the insulator completely fills the space between the plates, the capacitance is increased by a factor κ which depends only on the nature of the insulating material.

Insulating materials are also called **dielectrics**; the factor κ is then a property of the dielectric and is called the **dielectric constant**.

The dielectric constant of a vacuum is, of course, **unity**.

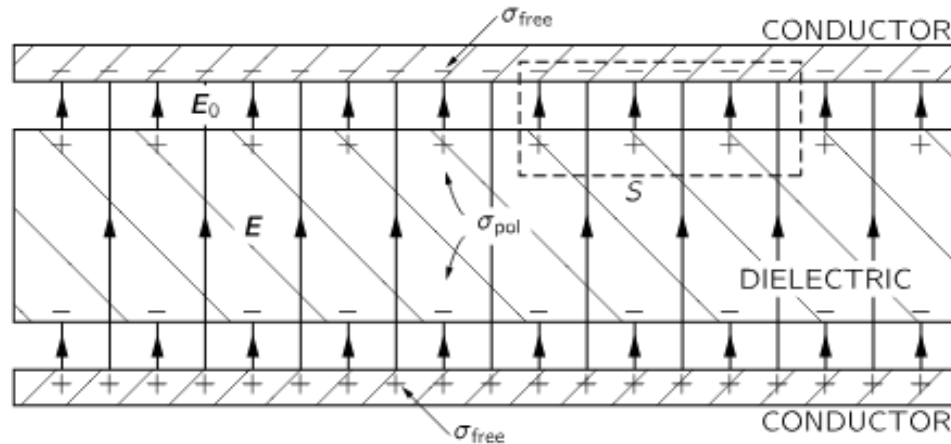
Dielectrics

Dielectric constants of various substances

Substance	Conditions	Dielectric constant (κ)
Air	gas, 0 °C, 1 atm	1.00059
Methane, CH ₄	gas, 0 °C, 1 atm	1.00088
Hydrogen chloride, HCl	gas, 0 °C, 1 atm	1.0046
Water, H ₂ O	gas, 110 °C, 1 atm	1.0126
	liquid, 20 °C	80.4
Benzene, C ₆ H ₆	liquid, 20 °C	2.28
Methanol, CH ₃ OH	liquid, 20 °C	33.6
Ammonia, NH ₃	liquid, −34 °C	22.6
Mineral oil	liquid, 20 °C	2.24
Sodium chloride, NaCl	solid, 20 °C	6.12
Sulfur, S	solid, 20 °C	4.0
Silicon, Si	solid, 20 °C	11.7
Polyethylene	solid, 20 °C	2.25–2.3
Porcelain	solid, 20 °C	6.0–8.0
Paraffin wax	solid, 20 °C	2.1–2.5
Pyrex glass 7070	solid, 20 °C	4.00

Dielectrics

Parallel-plate capacitor with some charges on the surfaces of the conductors



$$C = \frac{\epsilon_0 A}{d}$$

$$Q = CV$$

If we put a piece of insulating material like lucite or glass between the plates, we find that the capacitance is larger.

That means, of course, that the voltage is lower for the same charge.

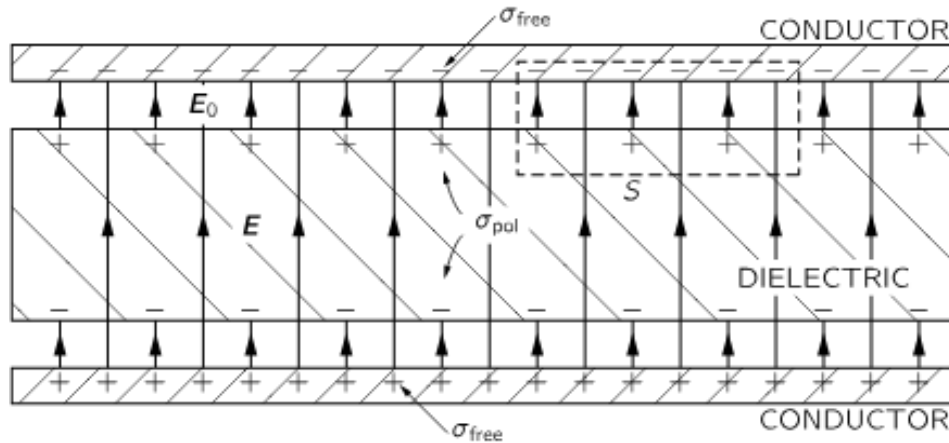
But the voltage difference is the integral of the electric field across the capacitor; so we must conclude that inside the capacitor, the electric field is reduced even though the charges on the plates remain unchanged.

Consider the Gaussian surface S shown by broken lines.

Since the electric field is reduced with the dielectric present, we conclude that the net charge inside the surface must be lower than it would be without the material. There is only one possible conclusion, and that is that there must be positive charges on the surface of the dielectric. Since the field is reduced but is not zero, we would expect this positive charge to be smaller than the negative charge on the conductor.

Dielectrics

Parallel-plate capacitor with some charges on the surfaces of the conductors



$$C = \frac{\epsilon_0 A}{d}$$

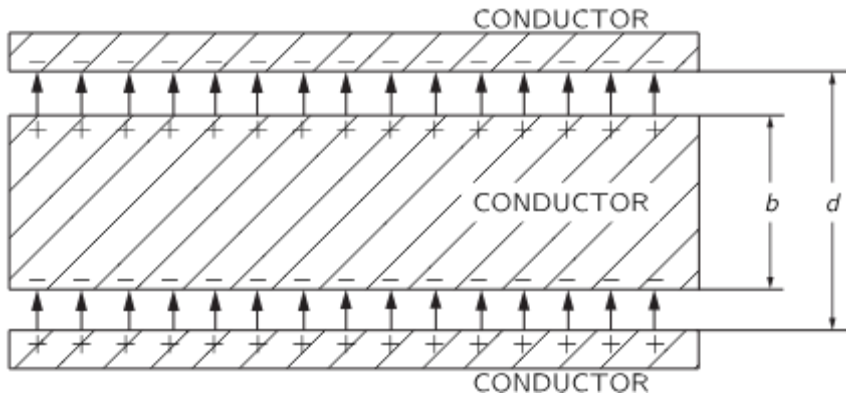
$$Q = CV$$

Consider the Gaussian surface S shown by broken lines.

Since the electric field is reduced with the dielectric present, we conclude that the net charge inside the surface must be lower than it would be without the material. There is only one possible conclusion, and that is that there must be positive charges on the surface of the dielectric. Since the field is reduced but is not zero, we would expect this positive charge to be smaller than the negative charge on the conductor. So the phenomena can be explained if we could understand in some way that when a dielectric material is placed in an electric field there is positive charge induced on one surface and negative charge induced on the other.

Dielectrics

Parallel-plate capacitor with conducting plate



The induced charges reduce the field in the conductor to zero.

The field in the rest of the space is the same as it was without the conductor, because it is the surface density of charge divided by ϵ_0 ; but the distance over which we have to integrate to get the voltage (the potential difference) is reduced.

$$V = \frac{\sigma}{\epsilon_0} (d - b)$$

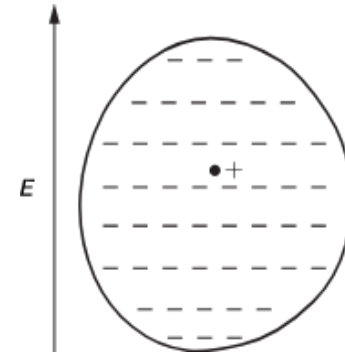
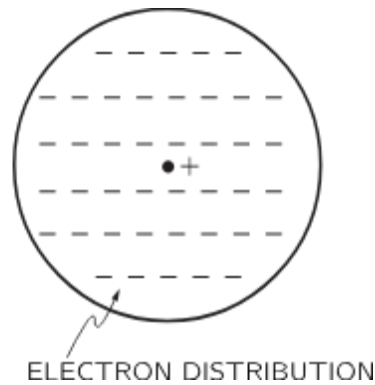
$$C = \frac{\epsilon_0 A}{d[1 - (b/d)]}$$

The capacitance is increased by a factor that depends upon **(b/d)**, the proportion of the volume which is occupied by the conductor.

Induced Dipoles

In dielectrics, by contrast, all charges are attached to specific atoms or molecules—they're on a tight leash, and all they can do is move a bit within the atom or molecule.

There are actually two principal mechanisms by which electric fields can distort the charge distribution of a dielectric atom or molecule: stretching and rotating.



In each atom there are charges q separated by a distance δ , so that $q\delta$ is the dipole moment per atom.

$$p = q\delta$$

If there are N atoms per unit volume, there will be a *dipole moment per unit volume* equal to $Nq\delta$. This dipole moment per unit volume will be represented by a vector, \mathbf{P} .

$$\mathbf{P} = Nq\delta$$

Induced Dipoles

In general, \mathbf{p} will vary from place to place in the dielectric. However, at any point in the material, \mathbf{p} is proportional to the electric field \mathbf{E} .

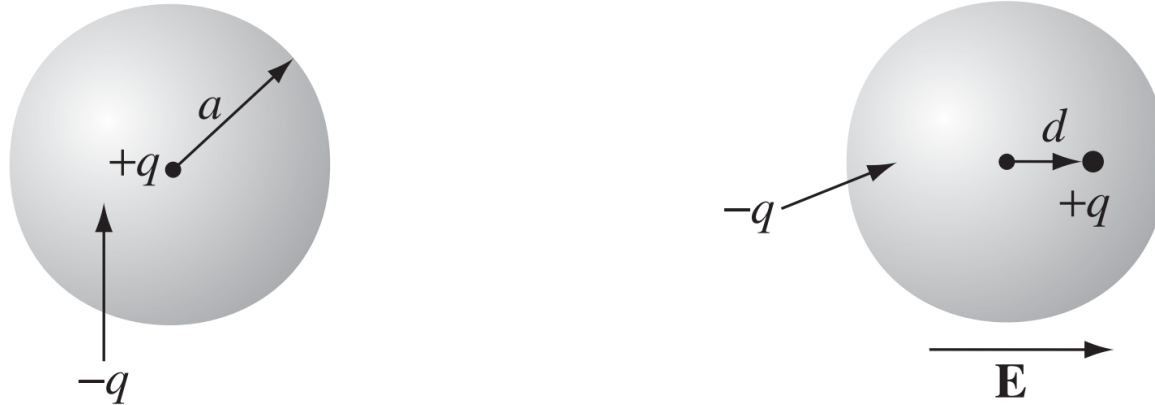
$$\mathbf{p} = \alpha \mathbf{E}$$

The constant of proportionality α is called *atomic polarizability*. The constant of proportionality, which depends on the ease with which the electrons are displaced, will depend on the kinds of atoms in the material. Its value depends on the detailed structure of the atom in question.

H	He	Li	Be	C	Ne	Na	Ar	K	Cs
0.667	0.205	24.3	5.60	1.67	0.396	24.1	1.64	43.4	59.4

Induced Dipoles

A primitive model for an atom consists of a point nucleus (+q) surrounded by a uniformly charged spherical cloud (−q) of radius a. Calculate the atomic polarizability of such an atom.



At the point where the external field pushing the nucleus to the right exactly balances the internal field pulling it to the left: $\mathbf{E} = \mathbf{E}_e$, where \mathbf{E}_e is the field produced by the electron cloud.

The field at a distance d from the center of a uniformly charged sphere is $E_e = \frac{1}{4\pi\epsilon_0} \frac{qd}{a^3}$

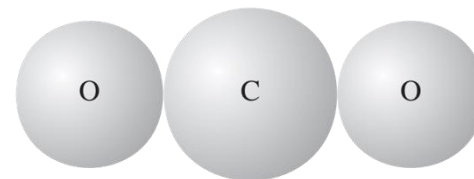
At equilibrium, $E = \frac{1}{4\pi\epsilon_0} \frac{qd}{a^3}$, or $p = qd = (4\pi\epsilon_0 a^3) E$

The atomic polarizability is therefore

$$\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 v, \quad \text{where } v \text{ is the volume of the atom.}$$

Induced Dipoles

For molecules such as carbon dioxide:



$$\mathbf{p} = \alpha_{\perp} \mathbf{E}_{\perp} + \alpha_{\parallel} \mathbf{E}_{\parallel}$$

The induced dipole moment may not even be in the same direction as \mathbf{E} .

For a completely asymmetrical molecule, the most general linear relation between \mathbf{E} and \mathbf{p}

$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$

$$p_y = \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z$$

$$p_z = \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z$$

The set of nine constants α_{ij} constitute the **polarizability tensor** for the molecule. Their values depend on the orientation of the axes you use, though it is always possible to choose “principal” axes such that all the off-diagonal terms (α_{xy} , α_{zx} , etc.) vanish, leaving just three nonzero polarizabilities: α_{xx} , α_{yy} , and α_{zz} .

THE FIELD OF A POLARIZED OBJECT

In each atom there are charges q separated by a distance δ , so that $q\delta$ is the dipole moment per atom.

$$p = q\delta$$

If there are N atoms per unit volume, there will be a *dipole moment per unit volume* equal to $Nq\delta$. This dipole moment per unit volume will be represented by a vector, \mathbf{P} .

$$\mathbf{P} = Nq\delta$$

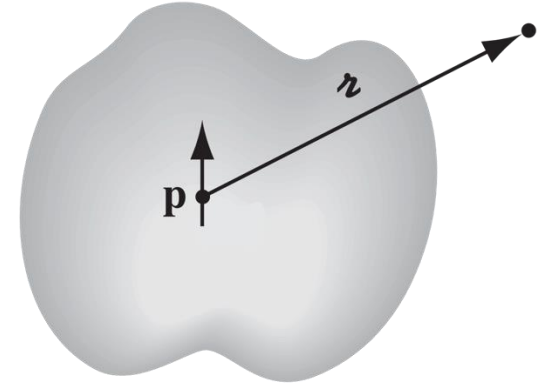
What is the field produced by this object (not the field that may have caused the polarization, but the field the polarization itself causes)?

THE FIELD OF A POLARIZED OBJECT

Why not chop the material up into infinitesimal dipoles and integrate to get the total?

For a single dipole \mathbf{p}

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$



a dipole moment $\mathbf{p} = \mathbf{P} d\tau'$ in each volume element $d\tau'$

Total potential

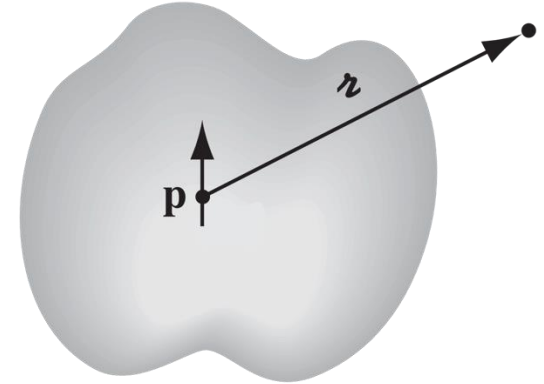
$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\mathbf{P}(\mathbf{r}') \cdot \hat{\mathbf{r}}}{r^2} d\tau'$$

THE FIELD OF A POLARIZED OBJECT

We have

$$\nabla' \left(\frac{1}{r} \right) = -\frac{\hat{\mathbf{r}}}{r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \int_V \mathbf{P} \cdot \nabla' \left(\frac{1}{r} \right) d\tau'$$



Integrating by parts

$$V = \frac{1}{4\pi\epsilon_0} \left[\int_V \nabla' \cdot \left(\frac{\mathbf{P}}{r} \right) d\tau' - \int_V \frac{1}{r} (\nabla' \cdot \mathbf{P}) d\tau' \right]$$

from the divergence theorem

$$V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{r} \mathbf{P} \cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} (\nabla' \cdot \mathbf{P}) d\tau'$$

THE FIELD OF A POLARIZED OBJECT

$$V = \frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{r} \mathbf{P} \cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} (\nabla' \cdot \mathbf{P}) d\tau'$$

The first term looks like the potential of a surface charge

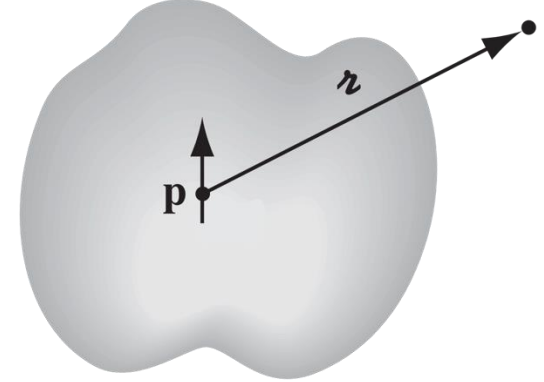
$$\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}}$$

the second term looks like the potential of a volume charge

$$\rho_b \equiv -\nabla \cdot \mathbf{P}.$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma_b}{r} da' + \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho_b}{r} d\tau'.$$

The potential (and hence also the field) of a polarized object is the same as that produced by a volume charge density $\rho_b = -\nabla \cdot \mathbf{P}$ and a surface charge density $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$



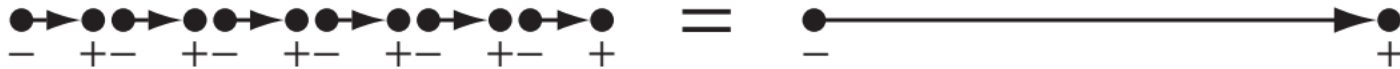
Instead of integrating the contributions of all the infinitesimal dipoles, we could first find those **bound charges**, and then calculate the fields they produce.

Physical Interpretation of bound charges

Concept of bound charges emerged from the manipulation of the integral

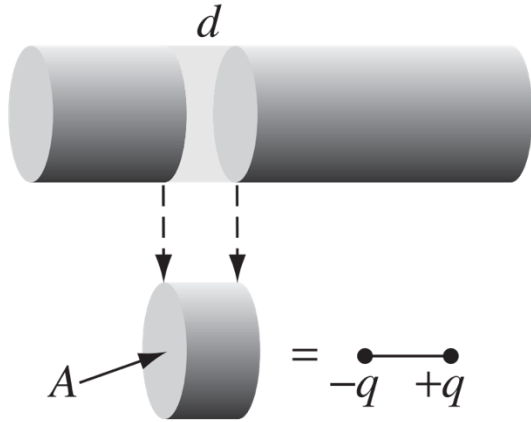
$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}$$

It was found field due to a polarized object was due to field produced by a certain distributions of ρ_b and σ_b . They represent genuine *accumulations of charges*.



Bound charges at the *ends* due to a *string of dipoles*- bound charges cannot be removed

Physical Interpretation of bound charges

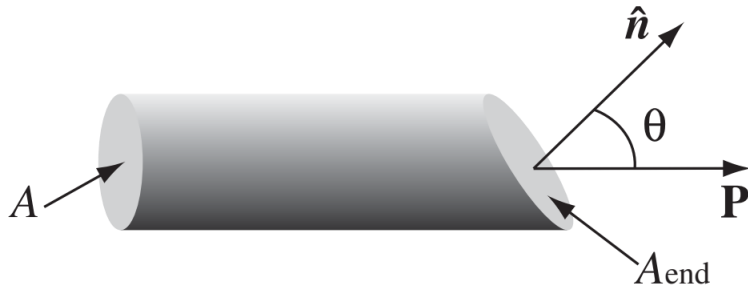


To calculate the *amount of bound charge from a given polarization*, consider a tube of dielectric material parallel to \mathbf{P} .

The dipole moment of the chunk of length ' d ' is $\mathbf{P}(\mathbf{A}d)$.

In terms of ' q ' at the ends, the same dipole moment can be written as qd which means $q=PA$.

Surface charge density, $\sigma_b = \frac{q}{A} = P$ when ends are sliced perpendicularly.



For an oblique cut, the *charge* is still the same, but $A = A_{\text{end}} \cos \theta$

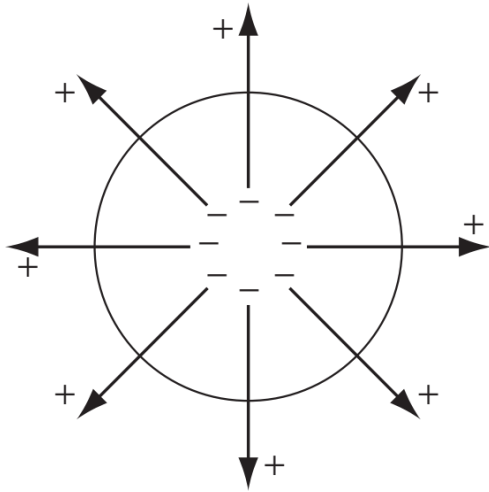
$$\sigma_b = \frac{q}{A_{\text{end}}} = P \cos \theta = \mathbf{P} \cdot \hat{\mathbf{n}}$$

(when ends are sliced obliquely)

Physical Interpretation of bound charges

Effect of *polarization* is to paint a bound *charge* $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$ over the surface of the material.

If *polarization is non-uniform*, there are *accumulations of bound charges* within the material and also over the surface. A diverging \mathbf{P} always results in pile up of negative charge



Net bound charge in a given volume ($\int \rho_b d\tau$) is equal and opposite to the amount that has been pushed out through the surface ($\mathbf{P} \cdot \hat{\mathbf{n}}$ per unit area)

$$\int_V \rho_b d\tau = - \oint_S \mathbf{P} \cdot d\mathbf{a} = - \int_V (\nabla \cdot \mathbf{P}) d\tau$$

$$\rho_b = -\nabla \cdot \mathbf{P}$$