PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 5

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

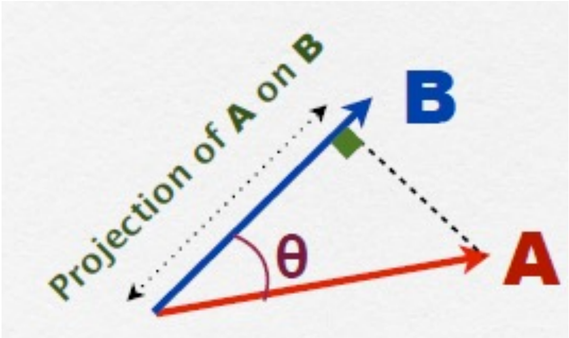
Incremental length, surface, and volume element

Introduction to scalars and vectors

This Lecture

Properties of vectors

Geometrical addition, multiplication of vectors etc.



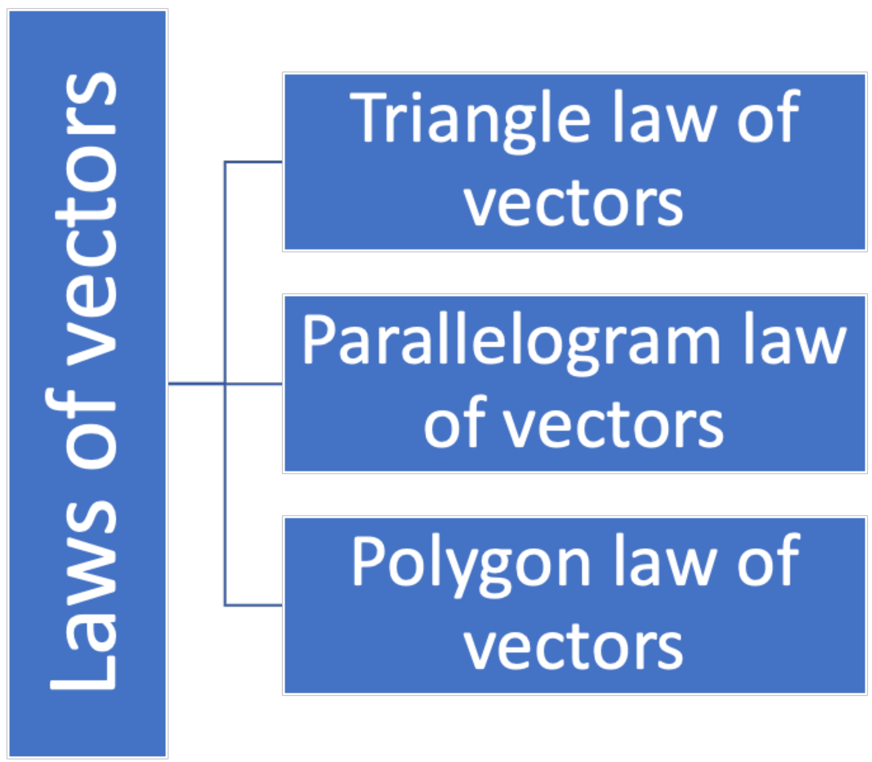
Vectors

Properties

Addition and Subtraction:

- Vectors can be added and subtracted.

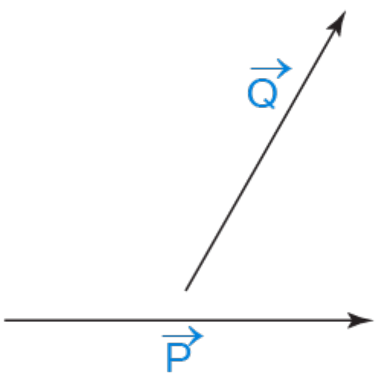
Geometrical Addition of vectors

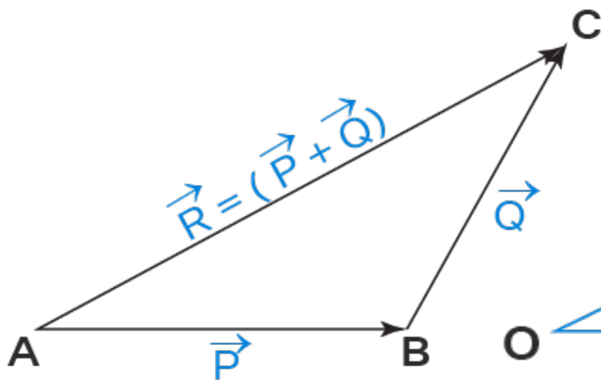


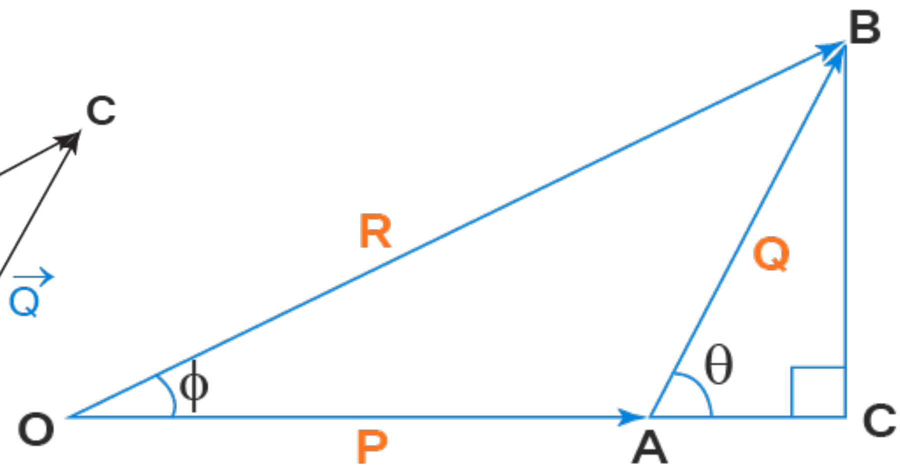
Vectors

Triangle law

Formulas for its magnitude and direction of $\vec{R}$







$\vec{R} = (\vec{P} + \vec{Q})$

$\vec{AC} = (\vec{AB} + \vec{BC})$

$$|R| = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$$

$$\varphi = \tan^{-1}[(Q \sin \theta)/(P + Q \cos \theta)]$$

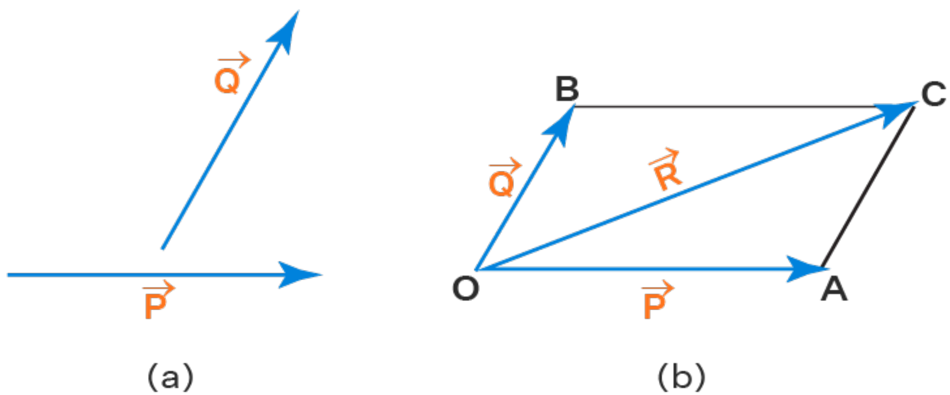
Ref: 1) https://www.cuemath.com/calculus/triangle-law-of-vector-addition/

Vectors

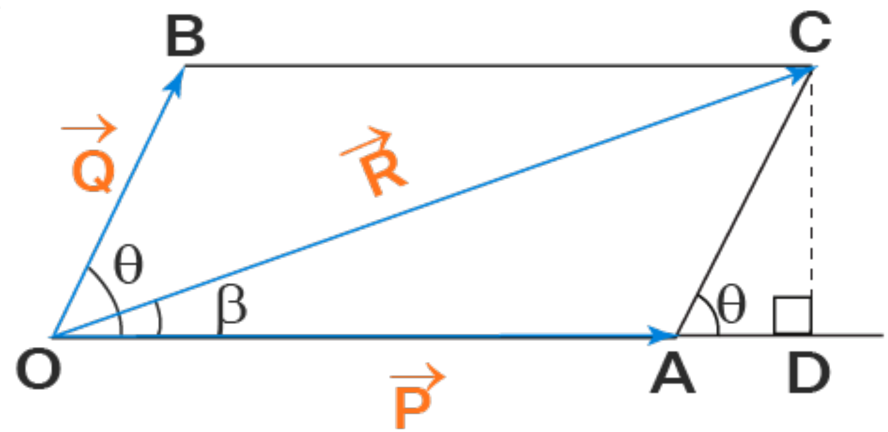
Parallelogram law

Formulas for its magnitude and direction of $\vec{R}$

Parallelogram law of vector addition



$\vec{R} = (\vec{P} + \vec{Q})$



$|R| = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$

$\beta = \tan^{-1}[(Q \sin \theta)/(P + Q \cos \theta)]$

Ref: 1) https://www.cuemath.com/calculus/parallelogram-law-of-vector-addition/

Properties

Vectors

Addition and Subtraction:

Geometrical Addition of vectors

Example 1: Two forces of magnitudes **4N** and **7N** act on a body and the angle between them is **45°**. Determine the magnitude and direction of the resultant vector with the 4N force.

Parallelogram law

Solution:

$|\mathbf{R}| = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}$

$= \sqrt{(4^2 + 7^2 + 2\times4\times7 \cos 45°)}$

$= \sqrt{(16 + 49 + 56/\sqrt{2})}$

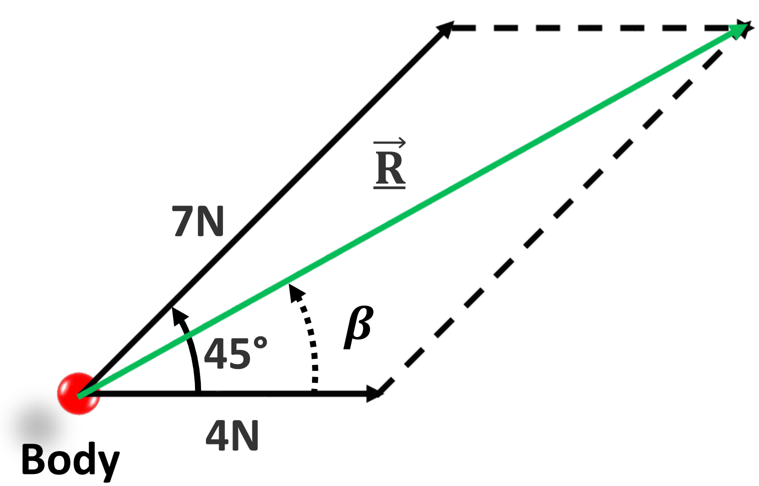
$= \sqrt{(65 + 56/\sqrt{2})}$

$\approx$ 10.22 N

$\beta = \tan^{-1}[(7 \sin 45°)/(4 + 7 \cos 45°)]$

$= \tan^{-1}[(7/\sqrt{2})/(4 + 7/\sqrt{2})]$

$\approx 28.95°$



Answer: The magnitude is approximately 12 N and the direction is 28.95°.

Vectors

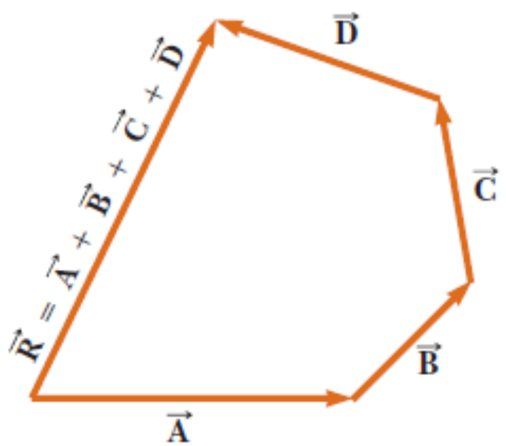
Properties

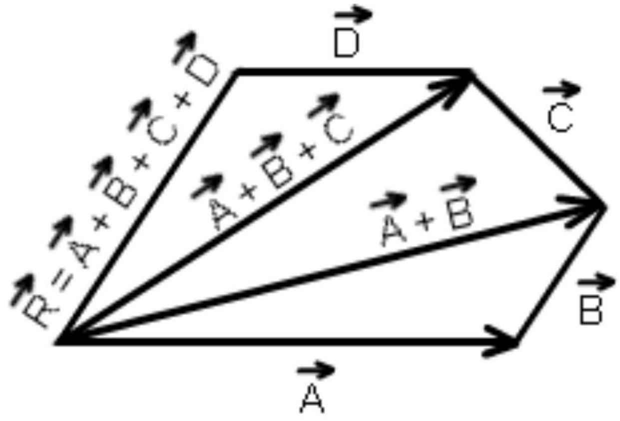
Addition and Subtraction:

Geometrical Addition of vectors

Polygon law

It is stated that if a number of vectors can be represented in magnitude and direction by the sides of a polygon taken in the same order, then their resultant can be represented in magnitude and direction by the closing side of a polygon taken in the opposite order.





Vectors

Properties

Addition and Subtraction:

Properties of vector addition (symbolic representation)

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

Commutative law

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

Associative law

$$c\left(d\vec{A}\right) = (cd)\vec{A}$$

$$(c + d)\vec{A} = c\vec{A} + d\vec{A}$$

$$c(\vec{A} + \vec{B}) = c\vec{A} + c\vec{B}$$

Distributive law

Ref: 1) An Introduction to Mechanics" by Kleppner and Kolenkow (McGrawHill, 1973)

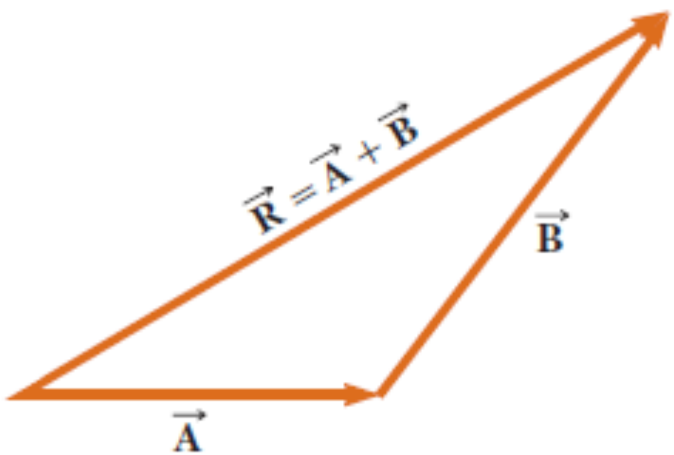
Vectors

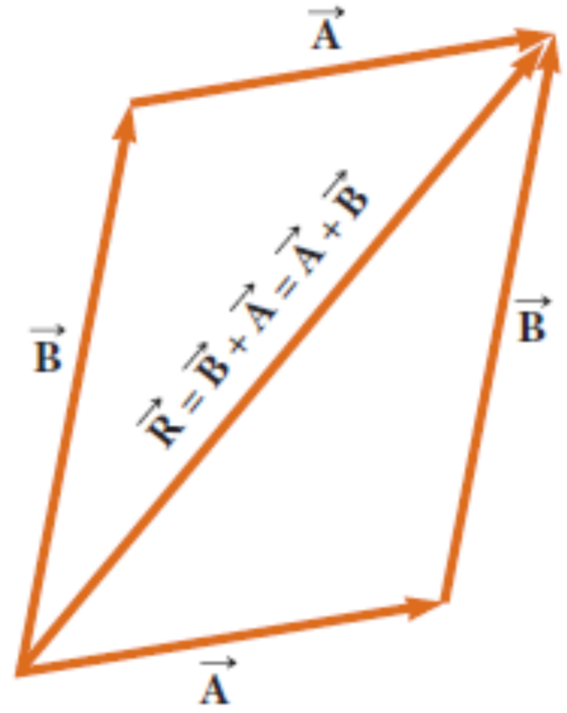
Properties

Addition and Subtraction:

Geometric construction of commutative law of addition

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$





Ref: 1) Physics for Scientists and Engineers with Modern Physics” by Serway & Jewett (Thomson Learning, Inc., 2008)

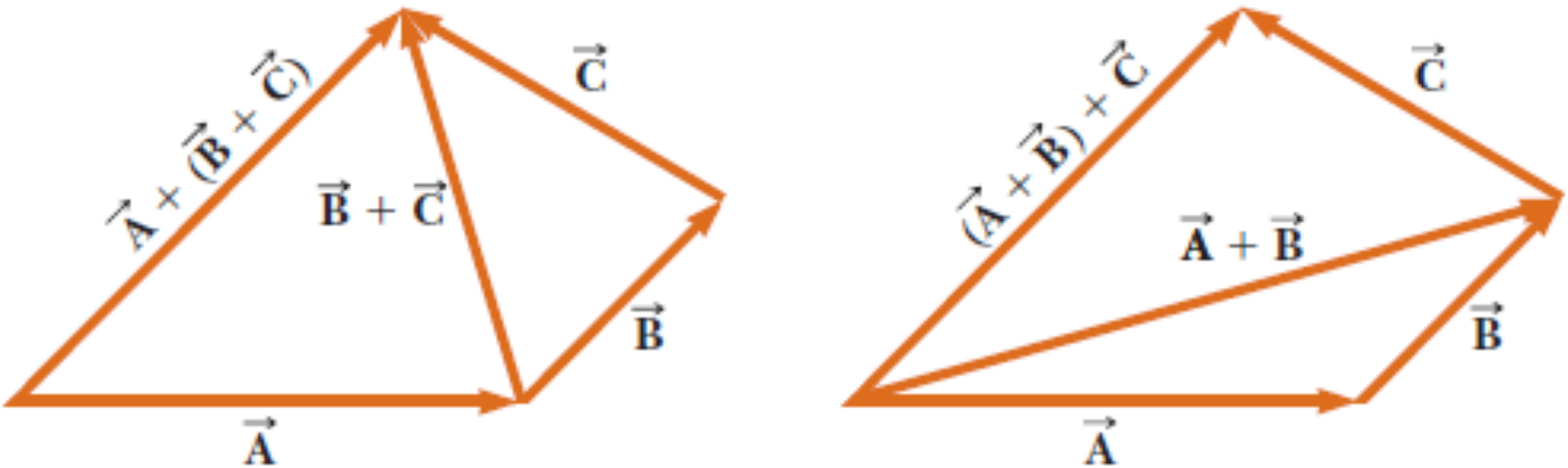
Scalar and Vectors

Properties

Addition and Subtraction:

Geometric construction of associative law of addition

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$



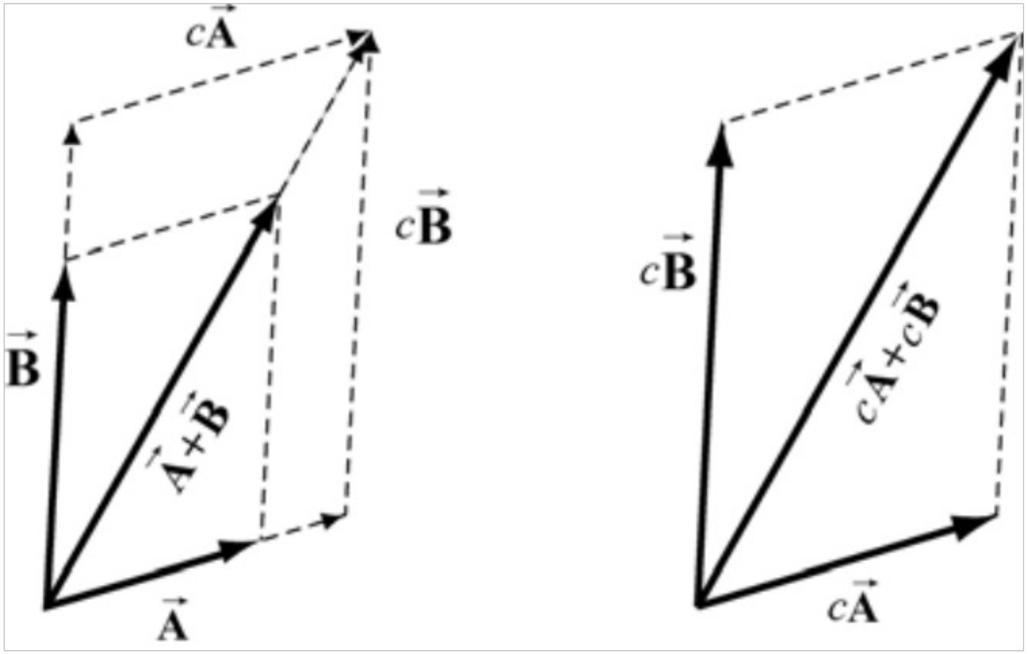
Ref: 1) Physics for Scientists and Engineers with Modern Physics” by Serway & Jewett (Thomson Learning, Inc., 2008)

Properties

Addition and Subtraction:

Geometric construction of distributive law of vector addition

$$c(\vec{A} + \vec{B}) = c\vec{A} + c\vec{B}$$



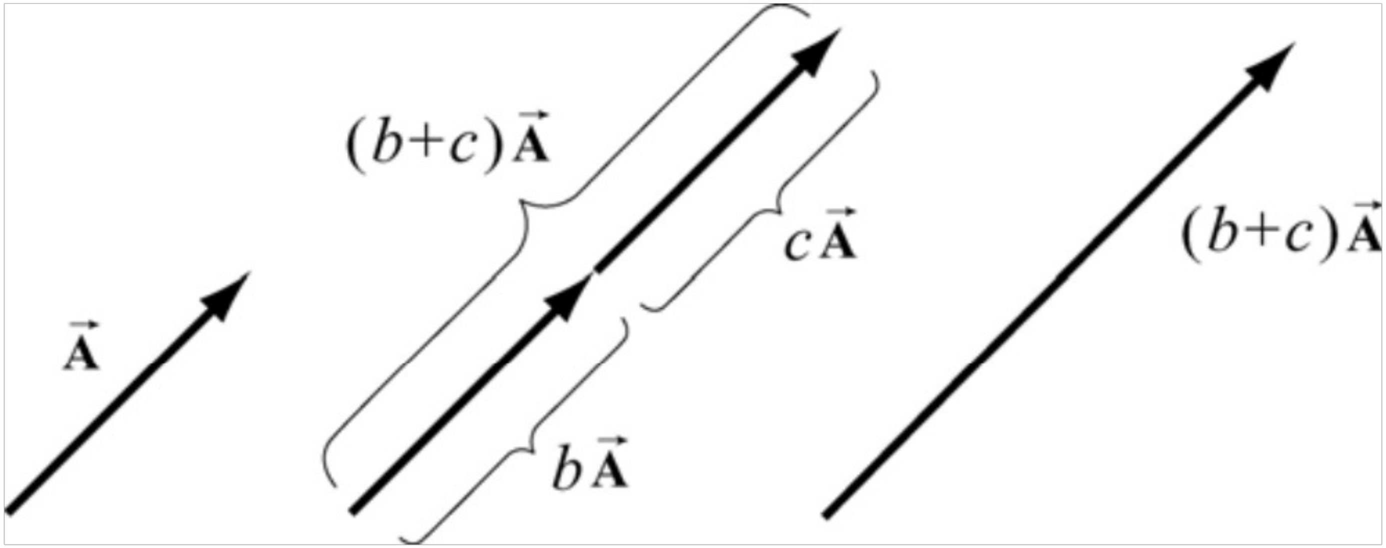
Ref: 1) MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Physics, Review A: Vector Analysis

Properties

Addition and Subtraction:

Geometric construction of distributive law of Scalar Addition

$$(b + c)\vec{A} = b\vec{A} + c\vec{A}$$

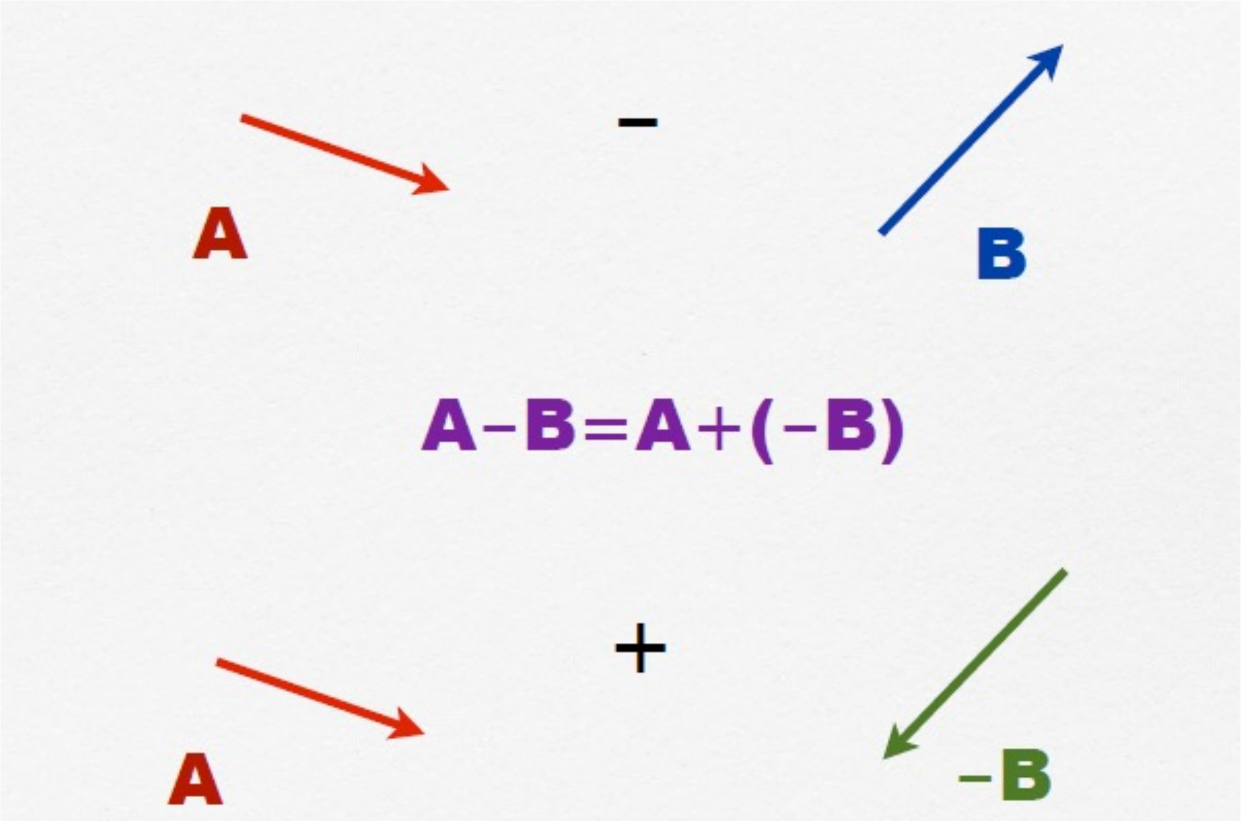


Ref: 1) MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Physics, Review A: Vector Analysis

Properties

Addition and Subtraction:

Geometrical Subtraction



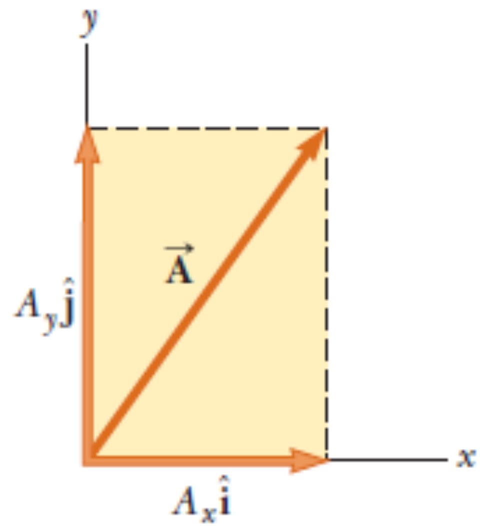
Ref: 1) Physics for Scientists and Engineers with Modern Physics" by Serway & Jewett (Thomson Learning, Inc., 2008)

Vectors

Base Vectors

Components of Vector in 2D

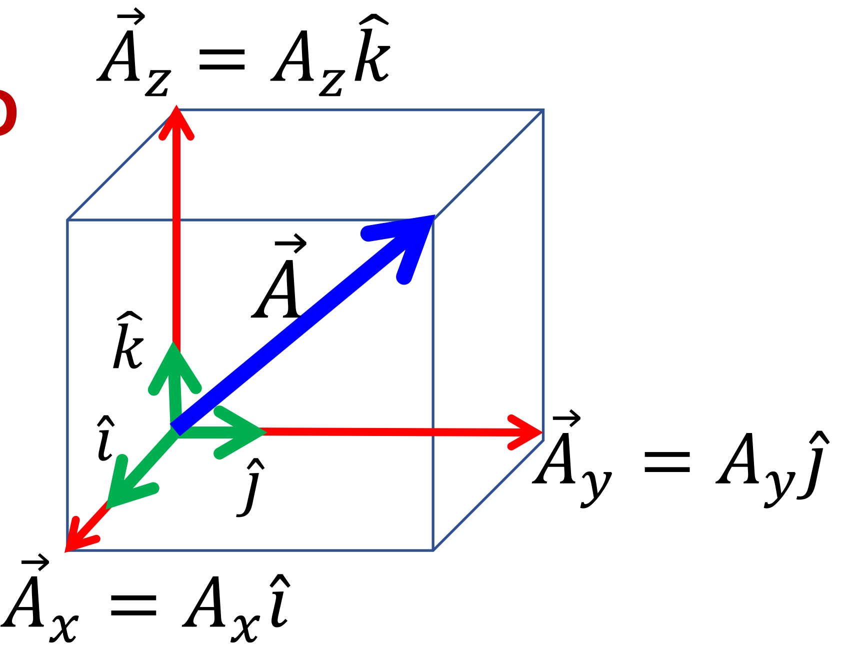
$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}$$



Components of Vector in 3D

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$$

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$



$\hat{\imath}, \hat{\jmath}, \hat{k}$ are the base vectors which are a set of orthogonal unit vectors

Next Lecture

Dot product and cross product of vectors

Reserved Slides

Vectors

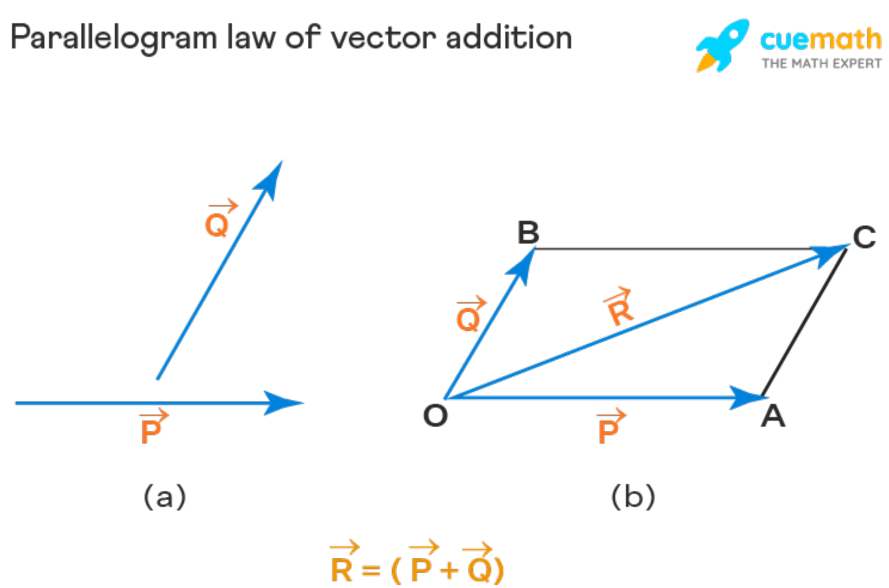
Properties

Addition and Subtraction:

Geometrical Addition of vectors

Parallelogram law

Two vectors can be arranged as adjacent sides of a parallelogram such that their tails attach with each other and the sum of the two vectors is equal to the diagonal of the parallelogram whose tail is the same as the two vectors



- **Step 1:** Draw the vectors **P** and **Q** such that their tails touch each other.
- **Step 2:** Complete the parallelogram by drawing the other two sides.
- **Step 3:** The diagonal of the parallelogram that has the same tail as the vectors **P** and **Q** represents the sum of the two vectors. i.e., **P** + **Q** = **R.**

Ref: 1) *https://www.cuemath.com/calculus/parallelogram-law-of-vector-addition/*