Tutorial 13 Solutions

PHY-101

Q1.A Carnot engine whose low temperature reservoir is at 280 K has an efficiency of 40 %. It is desired to increase this to 50 %. By how many degrees must the temperature of the low temperature reservoir be decreased if that of the high temperature reservoir remains constant.

Sol:

lower temperature, T_2 = 280 K
initial efficiency of machine, η_1 = 40 $\%$
final efficiency of machine, η_2 = 50 $\%$
higher temperature, T_1 = *x (say)*
final lower temperature, T_2' = ?
decrease in lower temperature, $T_2 - T_2'$ = ?

Now,

$$
\begin{aligned}
\eta_1 &= \left(1 - \frac{T_2}{T_1}\right) \times 100\% \\
40 &= \left(1 - \frac{280}{x}\right) \times 100 \\
\frac{2}{5} &= 1 - \frac{280}{x} \\
\frac{280}{x} &= \frac{3}{5} \\
x &= \frac{280 \times 5}{3} \\
\therefore x &= 466.67\,\mathrm{K}
\end{aligned}
$$

Then,

$$\eta_2 = \left(1 - \frac{T_2'}{T_1}\right) \times 100\%$$

$$50 = \left(1 - \frac{T_2'}{466.67}\right) \times 100$$

$$0.5 = \left(1 - \frac{T_2'}{466.67}\right)$$

$$\frac{T_2'}{466.67} = 0.5$$

$$\therefore T_2' = 233.33\text{ K}$$

Thus, the lower temperature should be decreased by $T_2 - T_2'$ = 280 - 233.33 K = 46.67 K.

Q2. Calculate the increase in entropy when 50g ice at -10°C is converted to steam at 100°C .Given that specific heat capacity of ice is 2090 $Jkg^{-1}K^{-1}$, specific heat capacity of water is 4180 $Jkg^{-1}K^{-1}$ and latent heat of steam is 2.26×10^6 Jkg^{-1}.

Sol:

Solution: **(a)** The change in entropy when 50 g ice at –10°C is heated to 0°C is given by

$$\Delta S_1 = mc \int_{263}^{273} \frac{dT}{T}$$

$$= (50 \times 10^{-3}\text{ kg}) \times (2090\text{ J kg}^{-1}\text{K}^{-1}) \log_e \left(\frac{273\text{ K}}{263\text{ K}}\right)$$

$$= 50 \times 2.09 \times 2.3026 \times \log_{10} \left(\frac{273\text{ K}}{263\text{ K}}\right) \text{J K}^{-1}$$

$$= 50 \times 2.09 \times 2.3026 \times 0.0162\text{ J K}^{-1}$$

$$= 3.90\text{ J K}^{-1}$$

(b) The change in entropy when 50 g ice at 0°C is converted into water 0°C is given by

$$\Delta S_2 = \frac{\delta Q}{T} = \frac{mL}{T} = \frac{(50 \times 10^{-3}\text{ kg}) \times (3.35 \times 10^5\text{ J kg}^{-1})}{273\text{ K}}$$

$$= \frac{50 \times 3.35}{2.73}\text{ J K}^{-1}$$

$$= 61.35\text{ J K}^{-1}$$

(c) The change in entropy when 50 g water is heat from 0°C to 100°C is given by

$$\Delta S_3 = mc \int_{273}^{373} \frac{dT}{T}$$

$$= (50 \times 10^{-3}\ \text{kg}) \times (4.18 \times 10^{3}\ \text{J kg}^{-1}\ \text{K}^{-1}) \times 2.3026 \times \log_{10}\left(\frac{373\ \text{K}}{273\ \text{K}}\right)$$

$$= 50 \times 4.18 \times 2.3026 \times 0.1355\ \text{JK}^{-1}$$

$$= 65.21\ \text{J K}^{-1}$$

(d) The change in entropy when 50 g water at 100°C is converted into steam at the same temperature is

$$\Delta S_4 = \frac{\delta Q}{T} = \frac{mL_{vap}}{T} = \frac{(50 \times 10^{-3}\ \text{kg}) \times (2.26 \times 10^{6}\ \text{J kg}^{-1})}{373\ \text{K}}$$

$$= \frac{50 \times 2.26 \times 10^{3}}{373}\ \text{JK}^{-1}$$

$$= 302.94\ \text{J K}^{-1}$$

Therefore, the total change (i.e., increase) in entropy is obtained by adding the values for each change:

$$\Delta S = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4$$

$$= (3.90 + 61.35 + 65.21 + 302.94)\ \text{J K}^{-1}$$

$$= 433.4\ \text{J K}^{-1}.$$

Q3. A 1.5m^3 of insulated rigid tank contains 2.7 kg of CO_2 at 100kPa. Paddle work is done on the system until the pressure rises to 150 kPa. What is the entropy change of CO_2 in this process. Cv=0.657 KJKg^{-1}K^{-1}

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Q3 soln, Change is occuring at constant volume.

$\Delta S = m\, C_v \ln (T_2/T_1)$

so we need the ratio T_2/T_1

Using the ideal gas equation,

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$V_1 = V_2$ (as tank is rigid)

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

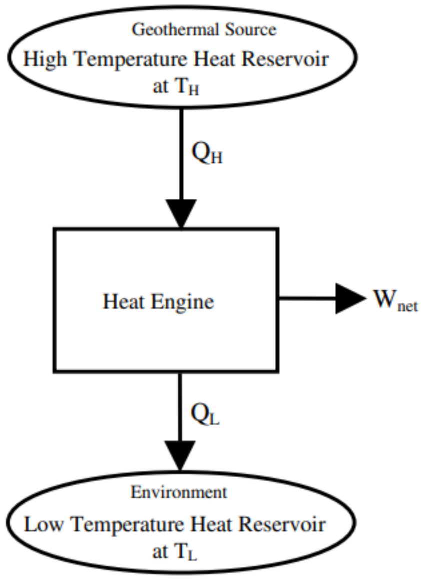
$$\Rightarrow \boxed{\frac{P_1}{P_2} = \frac{T_1}{T_2}} = \frac{150}{100} = 1.5$$

$$\Rightarrow \Delta S = (2.7\ \text{kg}) \left(0.657 \frac{\text{kJ}}{\text{kg K}}\right) \ln 1.5$$

$$\Rightarrow \Delta S = 0.72 \frac{\text{kJ}}{\text{K}}$$

Q4. An innovative way of power generation involves the utilization of geothermal energy, the energy of hot water that exists naturally underground (hot springs), as the heat source. If a supply of hot water at 140°C is discovered at a location where the environmental temperature is 20°C, determine the maximum thermal efficiency a geothermal plant built at that location can have. If the power output of the plant is to be 5 MW, what is the minimum mass flow rate of hot water needed? C_p=4.197$Jkg^{-1}K^{-1}$

We begin by sketching our device interactions



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Q4. The maximum thermal efficiency will occur when the heat engine operates at carnot cycle.

$$\eta_{th} = \eta_{carnot} = 1 - \frac{T_L}{T_H} = 1 - \frac{20+273}{140+273} = 0.291$$

The minimum mass flow rate of the hot water should correspond to maximum thermal efficiency

$$\therefore \quad \frac{d}{dt}(Q_H)_{min} = \frac{5000 \text{ kW}}{0.291} = 17208 \text{ kW/sec}$$

Now from first Law,

$$Q = \left(\frac{dm}{dt}\right)(\text{Heat}_{out} - \text{Heat in})$$

$$\Rightarrow \quad \frac{dm}{dt} = \frac{Q}{(C_p T_{out} - C_p T_{in})}$$

as the work is done by the system,

$Q = -\cancel{Q}\ 17208$

$$\therefore \quad \frac{dm}{dt} = \frac{-17208}{4.1978(20-140)}$$

$$= 34.2 \text{ kg/sec.}$$