Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 14

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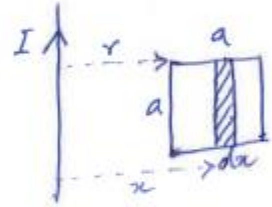
1. 1. Suppose a square loop of side *a* is placed in the plane of a long straight wire carrying current *I*. The nearest side of the loop is at a distance *r* from the wire. Find the magnetic flux through the loop. If someone pulls the loop directly away from the wire at a constant speed, what would be the EMF generated in the loop? What is the value of EMF generated when the loop is pulled parallel to the wire?

Sol The magnetic induction at a distance x from the wire is $B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{x}$

which is in a direction normal to the plane of the loop. The flux through an elementary area $a\,dx$ within the loop is

$$d\phi = B\,a\,dx = \frac{\mu_0}{2\pi} \cdot Ia \frac{dx}{x}$$



Therefore, total flux through the loop is

$$\phi = \frac{\mu_0 I a}{2\pi} \int_r^{r+a} \frac{dx}{x} = \frac{\mu_0 I a}{2\pi} \ln \frac{r+a}{r}$$

The induced EMF in the loop is

$$\mathcal{E} = -\frac{d\phi}{dt} = -\frac{d\phi}{dr}\frac{dr}{dt} = -v\frac{d\phi}{dr}$$

$$= -\frac{\mu_0 I a v}{2\pi}\left[\frac{1}{r+a} - \frac{1}{r}\right]$$

$$= \frac{\mu_0 I v a^2}{2\pi r (r+a)}$$

When the loop is pulled parallel to the wire the flux does not change and hence $\mathcal{E} = 0$.

2. The intensity of sunlight reaching the Earth surface is about 1300 W/m^2. Calculate the strength of electric and magnetic fields of the incoming sunlight.

Sol The time average Poynting vector is

$$\langle \vec{S} \rangle = \frac{1}{2} Re(\vec{E} \times \vec{H})$$

Assuming harmonic variations

$$\vec{E} = \vec{E}_0 e^{-i\omega t} \quad \text{and} \quad \vec{H} = \vec{H}_0 e^{-i\omega t}$$

$$S_{av} = |\langle \vec{S} \rangle| = \frac{1}{2} E_0 \cdot H_0 = \frac{E_0}{\sqrt{2}} \cdot \frac{H_0}{\sqrt{2}}$$

$$= E_{rms} \cdot H_{rms}$$

$$B = \mu_0 H = \frac{E}{c} \quad \text{or} \quad H_{rms} = \frac{E_{rms}}{c\mu_0}$$

$$\therefore \; S_{av} = \frac{E_{rms}^2}{c\mu_0}$$

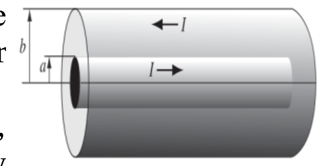
$$\therefore \; E_{rms} = \sqrt{c\mu_0 \cdot S_{av}}$$

$$= \sqrt{3 \times 10^8 \times 4\pi \times 10^{-7} \times 1300} \; V/m$$

$$= 700 \; V/m$$

$$\therefore \; B_{rms} = \frac{E_{rms}}{c} = \frac{700}{3 \times 10^8} \; T = 2.33 \times 10^{-6} \; T$$

3. A long coaxial cable carries current I. The current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder of radius b, as shown in the figure.



Calculate the power (energy per unit time) transported down the cables, assuming the two conductors are held at potential difference V and carry current I (down one and back up the other).

Solution: According to Ampère's law, the field between the cylinders is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}}.$$

$$\left.\begin{aligned} \mathbf{E} &= \frac{\lambda}{2\pi\epsilon_0}\frac{1}{s}\,\hat{\mathbf{s}} \\ \mathbf{B} &= \frac{\mu_0 I}{2\pi}\frac{1}{s}\,\hat{\boldsymbol{\phi}} \end{aligned}\right\} \mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{\lambda I}{4\pi^2\epsilon_0}\frac{1}{s^2}\,\hat{\mathbf{z}};$$

$$P = \int \mathbf{S}\cdot d\mathbf{a} = \int_a^b S\,2\pi s\,ds = \frac{\lambda I}{2\pi\epsilon_0}\int_a^b \frac{1}{s}\,ds = \frac{\lambda I}{2\pi\epsilon_0}\ln(b/a).$$

$$\text{But } V = \int_a^b \mathbf{E}\cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0}\int_a^b \frac{1}{s}\,ds = \frac{\lambda}{2\pi\epsilon_0}\ln(b/a), \text{ so } \boxed{P = IV.}$$

4. A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in the figure below, and consider the capacitor is charging.

a) Find the electric and magnetic fields in the gap, as functions of the distance s from the axis and the time t. (Assume the charge is zero at t = 0.)
b) Find the energy density u_{em} and the Poynting vector S in the gap. Note especially the direction of **S**. Check that $\frac{\partial u}{\partial t} = -\boldsymbol{\nabla}.\mathbf{S}$ is satisfied.
c) Determine the total energy in the gap, as a function of time. Calculate the total power flowing into the gap, by integrating the Poynting vector over the appropriate surface. Check that the power input is equal to the rate of increase of energy in the gap. (Do it for a volume of radius b < a well inside the gap).

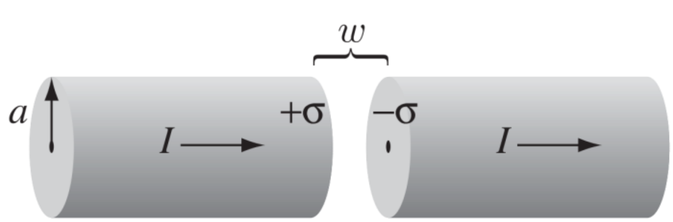


Fig. 1

Solution

Q1.

a)

$$\text{(a) } \mathbf{E} = \frac{\sigma}{\epsilon_0}\,\hat{\mathbf{z}};\ \sigma = \frac{Q}{\pi a^2};\ Q(t) = It \ \Rightarrow\ \mathbf{E}(t) = \boxed{\frac{It}{\pi\epsilon_0 a^2}\,\hat{\mathbf{z}}.}$$

$$B\,2\pi s = \mu_0\epsilon_0\frac{\partial E}{\partial t}\pi s^2 = \mu_0\epsilon_0\frac{I\pi s^2}{\pi\epsilon_0 a^2} \ \Rightarrow\ \mathbf{B}(s,t) = \boxed{\frac{\mu_0 I s}{2\pi a^2}\,\hat{\boldsymbol{\phi}}.}$$

b)

$$\text{(b) } u_{\text{em}} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left[\epsilon_0\left(\frac{It}{\pi\epsilon_0 a^2}\right)^2 + \frac{1}{\mu_0}\left(\frac{\mu_0 I s}{2\pi a^2}\right)^2\right] = \boxed{\frac{\mu_0 I^2}{2\pi^2 a^4}\left[(ct)^2 + (s/2)^2\right].}$$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0}\left(\frac{It}{\pi\epsilon_0 a^2}\right)\left(\frac{\mu_0 I s}{2\pi a^2}\right)(-\hat{\mathbf{s}}) = \boxed{-\frac{I^2 t}{2\pi^2\epsilon_0 a^4}\,s\,\hat{\mathbf{s}}.}$$

$$\frac{\partial u_{\text{em}}}{\partial t} = \frac{\mu_0 I^2}{2\pi^2 a^4}2c^2 t = \frac{I^2 t}{\pi^2\epsilon_0 a^4};\quad -\boldsymbol{\nabla}\cdot\mathbf{S} = \frac{I^2 t}{2\pi^2\epsilon_0 a^4}\boldsymbol{\nabla}\cdot(s\,\hat{\mathbf{s}}) = \frac{I^2 t}{\pi^2\epsilon_0 a^4} = \frac{\partial u_{\text{em}}}{\partial t}.$$

c)

$$\text{(c) } U_{\text{em}} = \int u_{\text{em}} w 2\pi s\,ds = 2\pi w\frac{\mu_0 I^2}{2\pi^2 a^4}\int_0^b[(ct)^2 + (s/2)^2]s\,ds = \frac{\mu_0 w I^2}{\pi a^4}\left[(ct)^2\frac{s^2}{2} + \frac{1}{4}\frac{s^4}{4}\right]\Bigg|_0^b$$

$$= \boxed{\frac{\mu_0 w I^2 b^2}{2\pi a^4}\left[(ct)^2 + \frac{b^2}{8}\right].}\ \text{Over a surface at radius } b\text{: } P_{\text{in}} = -\int\mathbf{S}\cdot d\mathbf{a} = \frac{I^2 t}{2\pi^2\epsilon_0 a^4}\left[b\,\hat{\mathbf{s}}\cdot(2\pi b w\,\hat{\mathbf{s}})\right] = \boxed{\frac{I^2 w t b^2}{\pi\epsilon_0 a^4}.}$$

$$\frac{dU_{\text{em}}}{dt} = \frac{\mu_0 w I^2 b^2}{2\pi a^4}2c^2 t = \frac{I^2 w t b^2}{\pi\epsilon_0 a^4} = P_{\text{in}}.\ \checkmark\ \text{(Set } b = a \text{ for } \textit{total.}\text{)}$$