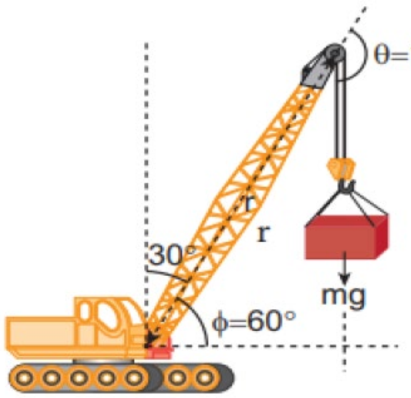
Tutorial 10 Solutions

PHY 101

Q1. A crane has an arm length of 20 m inclined at 30^{o} with the vertical. It carries a container of mass of 2 ton suspended from the top end of the arm. Find the torque produced by the gravitational force on the container about the point where the arm is fixed to the crane. [Given: 1 ton = 1000 kg; neglect the weight of the arm. g = 10 ms^{-2}]



Solution

The force F at the point of suspension is due to the weight of the hanging mass.

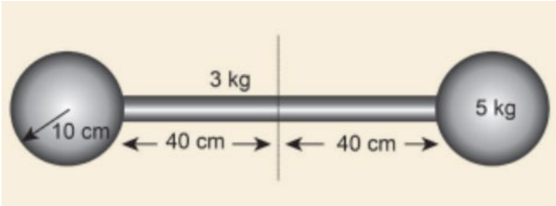
$$F = mg = 2 \times 1000 \times 10 = 20000 \text{ N};$$
$$\text{The arm length, } r = 20 \text{ m}$$

The angle (θ) between the arm length (r) and the force (F) is, $\theta = 150^{\circ}$

The torque (τ) about the fixed point of the arm is,

$$\tau = rF \sin\theta$$
$$\tau = 20 \times 20000 \times \sin\left(150^{\circ}\right)$$
$$= 400000 \times \sin\left(90^{\circ} + 60^{\circ}\right)$$
$$[\text{here, } \sin\left(90^{\circ} + \theta\right) = \cos\theta]$$
$$= 400000 \times \cos\left(60^{\circ}\right)$$
$$= 400000 \times \frac{1}{2} \quad \left[\cos 60^{\circ} = \frac{1}{2}\right]$$
$$= 200000 \text{ N m}$$
$$\tau = 2 \times 10^{5} \text{ N m}$$

Q2. Find the moment of inertia about the geometric centre of the given structure made up of one thin rod connecting two similar solid spheres as shown in Figure.



Solution

The structure is made up of three objects; one thin rod and two solid spheres.

The mass of the rod, M = 3 kg and the total length of the rod, ℓ = 80 cm = 0.8 m

The moment of inertia of the rod about its center of mass is, $I_{rod} = \frac{1}{12}M\ell^2$

$$I_{rod} = \frac{1}{12} \times 3 \times (0.8)^2 = \frac{1}{4} \times 0.64$$

$$I_{rod} = 0.16 \, kg \, m^2$$

The mass of the sphere, M = 5 kg and the radius of the sphere, R = 10 cm = 0.1 m

The moment of inertia of the sphere about its center of mass is, $I_C = \frac{2}{5}MR^2$

The moment of inertia of the sphere about geometric center of the structure is,

$I_{sph} = I_C + Md^2$

Where, d = 40 cm + 10 cm = 50 cm = 0.5 m

$$I_{sph} = \frac{2}{5}MR^2 + Md^2$$

$$I_{sph} = \frac{2}{5} \times 5 \times (0.1)^2 + 5 \times (0.5)^2$$

$$I_{sph} = (2 \times 0.01) + (5 \times 0.25) = 0.02 + 1.25$$

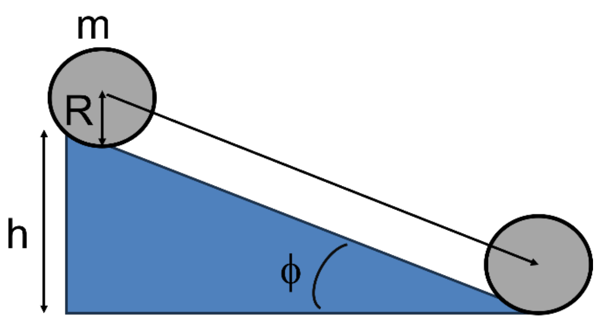
$$I_{sph} = 1.27 \, kg \, m^2$$

As there are one rod and two similar solid spheres we can write the total moment of inertia (I) of the given geometric structure as, $I = I_{rod} + (2 \times I_{sph})$

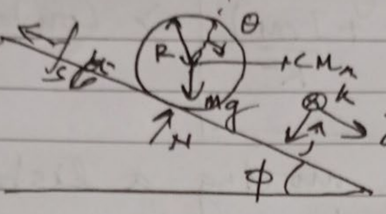
$$I = (0.16) + (2 \times 1.27) = 0.16 + 2.54$$
$$I = 2.7\,\mathrm{kg\,m^2}$$

Q3.A wheel is rolling down a inclined plane with coefficient of friction f_s and angle ϕ without slipping as shown in the figure. If I_{CM} is the moment of inertia , R the radius of the wheel, m the mass of the wheel and h the distance it dropped from its initial position in the vertical direction and v_{CM} is the translational velocity of the centre of mass, then show using the kinematics of rotational and translational motion

$$v_{CM} = \sqrt[2]{\frac{2mgh}{(m + \frac{I_{CM}}{R^2})}}$$



Wheel rolling down inclined plane.



$a_x = a_{cm}$

Along Plane ($\hat{i}$) $\Rightarrow$ $\vec{F} = m\vec{a}$

$\Rightarrow mg\sin\phi - f_s = m a_x$ — (1)

Torque eqn $\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$

Torque in this case is generated due to friction

again. $R\hat{j} \times (-\hat{i} f_s) = I_{cm}\alpha$ $[\vec{R} \times \vec{f_s}$ $\pi/2$ $= R f_s \sin\theta]$

Torque equation

Along $\hat{k}$ $R f_s = I_{cm}\alpha.$ — (2)

out of plane direction

Now, $v_{cm} = R\omega$

and $a_{cm} = a_x = R\alpha$ (3)

Unknown.

$f_s, a_x, \alpha.$

$$mg\sin\phi - \frac{I_{cm}\alpha}{R} = ma_x \quad \text{[From eqn (2)]}$$

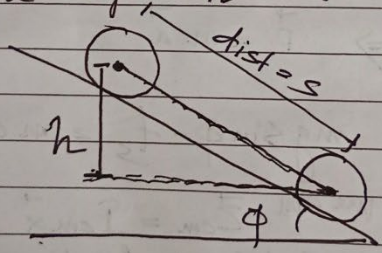
$\Rightarrow$ using $a_x = R\alpha$.

$$mg\sin\phi - \frac{I_{cm}}{R^2}a_x = ma_x$$

$$\Rightarrow a_x\left(1+\frac{I_{cm}}{R^2}\right) = mg\sin\phi$$

$$\Rightarrow \boxed{a_x = \frac{mg\sin\phi}{\left(1+\frac{I_{cm}}{R^2}\right)}} \Rightarrow \text{Constant} = a_x = a_{cm}$$

If our body is moving a distance s and drops to a height h



From kinematical eqn of the Centre of mass.

$$s = x_{cm} = \frac{1}{2}a_{cm}t^2$$

$$v_{cm} = a_{cm}t$$

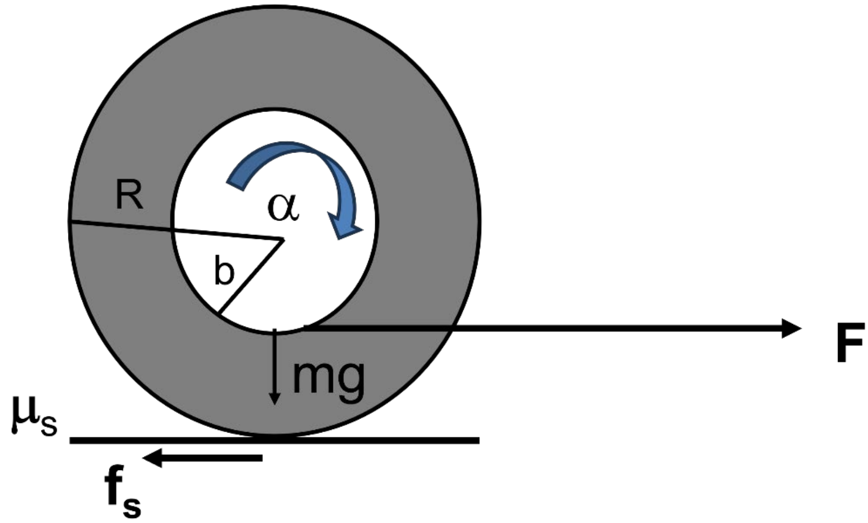
$$v_{cm} = \sqrt{2sa_{cm}}$$

$s = \frac{h}{\sin\phi}$ as $\sin\phi = \frac{h}{s}$

$$v_{cm} = \left[2 \times \frac{h}{\cancel{\sin\phi}} \times \frac{mg \cancel{\sin\phi}}{\left(m + \frac{I_{CM}}{R^2}\right)} \right]^{1/2}$$

$$v_{cm} = \left[\frac{2mgh}{\left(m + \frac{I_{CM}}{R^2}\right)} \right]^{1/2}$$

Q4.A YoYo of mass m is pulled along the plane with a string in the horizontal direction. It experiences a frictional force $\mathbf{f_s}$ (coefficient of static friction of the surface being μ_s)when it is being pulled with force **F.** It has a inner radius b and a outer radius R shown in the figure. What is the maximum magnitude of pulling force F for which the YoYo will roll without slipping. (assume g=acceleration due to gravity and I_{CM} is the moment of inertia of the YoYo.



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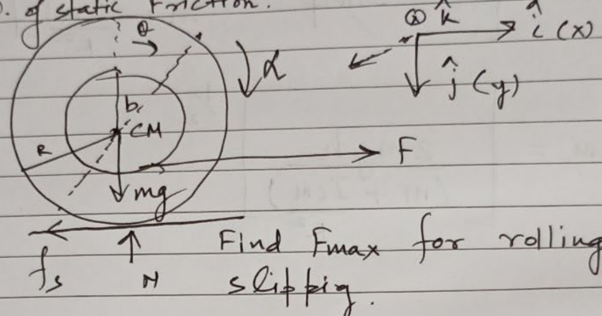
Q A yoyo of mass m is pulled along a plane with friction (f_s Force) using the string. It has a inner radius b and outer radius R. If it is pulled with a force F in the figure shown. What is the maximum magnitude of pulling force F for which the yoyo will roll without slipping. (I_{CM} = Mom. of Inertia)

μ_s = Coeff. of static Friction.



Find F_{max} for rolling without slipping.

As we are pulling static friction is non zero in order to keep the centre of mass accelerating.

$\boxed{a = R\alpha}$ ⇒ We need some torque to produce non zero α which will come from the frictional force.

So, $\vec{F} = m\vec{a}$

In x direction,

$$\hat{i} \rightarrow \quad \vec{F} - f_s = ma \quad —— (1)$$

Torque eqn. $\vec{\tau}_{CM} = I_{cm}\vec{\alpha}$

Now The trorque is ~~done~~ due to 2 forces, one is friction f_s being applied on the outer radius R and the pulling force $\vec{F}$ being applied on the inner radius b.

[Here $f_s \perp R$]

$$\vec{\tau}_F = \vec{r} \times \vec{F} \sim rF\sin\theta \quad [\text{Here } F \perp b]$$

$$\vec{\tau}_{f_s} = Rf_s \quad \sim rF \qquad \sin\theta = 1$$

$\Rightarrow$ Torque equation :-

$$f_s R - bF = I_{CM}\alpha \quad — (2)$$

and, $a = R\alpha$ — (3)

So we need to solve for F.

From (3) $\Rightarrow$ $\alpha = \frac{a}{R}$ — (4)

From (1) $\Rightarrow$ $\frac{F - f_s}{m} = a$ — (5)

So using eqn (4) in equation (2)

$$\Rightarrow f_s R - bF = I_{CM} \times \frac{a}{R}$$

$$\Rightarrow f_s R - bF = I_{CM} \frac{(F - f_s)}{mR}$$

$$\Rightarrow f_s R + f_s \frac{I_{CM}}{mR} = bF + \frac{I_{CF} F}{mR}$$

$$\Rightarrow f_s \left(R + \frac{I_{CM}}{mR}\right) = F\left(b + \frac{I_{CM}}{mR}\right)$$

$$\Rightarrow \boxed{F = \frac{f_s\left(R + \frac{I_{CM}}{mR}\right)}{\left(b + \frac{I_{CM}}{mR}\right)}}$$

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So when we are pulling the string, the harder we pull the bigger is the coefficient of static friction. So to evaluate what is the highest force one can exert so that no slippage is possible, we need to know the maximum value of static friction possible, ie,

$$(f_s)_{max}$$

If $\mu_s \rightarrow$ Coeficient of static friction.

$$f_s = \mu_s N$$

$$\Rightarrow (f_s)_{max} = \mu_s mg$$

So,

$$\boxed{F_{max} = \mu_s mg \frac{\left(R + \frac{I_{CM}}{mR}\right)}{\left(b + \frac{I_{CM}}{mR}\right)}}$$