- Voltage division allows us to calculate what fraction of the total voltage across a series string of resistors is dropped across any one resistor (or group of resistors).
- Current division allows us to calculate what fraction of the total current into a parallel string of resistors flows through any one of the resistors.

READING FURTHER

A discussion of the principles of conservation of energy and conservation of charge, as well as Kirchhoff's laws, can be found in

R. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics.* Reading, Mass.: Addison-Wesley, 1989, pp. 4-1, 4-7, and 25-9.

A very detailed discussion of grounding practices consistent with the 1996 National Electrical Code® can be found in

J. F. McPartland and B. J. McPartland, *McGraw-Hill's National Electrical Code® Handbook,* 22nd ed. New York: McGraw-Hill, 1996, pp. 337–485.

EXERCISES

3.1 Nodes, Paths, Loops, and Branches

1. Redraw the circuit of Fig. 3.42, consolidating nodes into the minimum number possible.

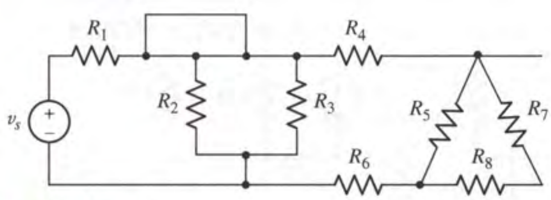


FIGURE 3.42

2. In the circuit of Fig. 3.42, count the number of (*a*) nodes; (*b*) branches.
3. In Fig. 3.43,
   (*a*) How many nodes are there?
   (*b*) How many branches are there?
   (*c*) If we move from *A* to *B* to *E* to *D* to *C* to *B*, have we formed a path? A loop?
4. In Fig. 3.44,
   (*a*) How many nodes are there?
   (*b*) How many branches are there?
   (*c*) If we move from *B* to *F* to *E* to *C*, have we formed a path? A loop?

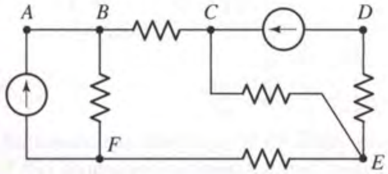


FIGURE 3.44

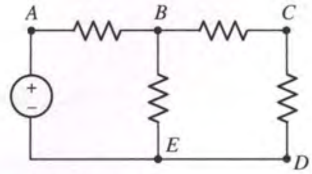


FIGURE 3.43

5. Referring to the circuit depicted in Fig. 3.43,
   (*a*) If a second wire is connected between points *E* and *D* of the circuit, how many nodes does the new circuit have?
   (*b*) If a resistor is added to the circuit so that one terminal is connected to point *C* and the other terminal is left floating, how many nodes does the new circuit have?
   (*c*) Which of the following represent loops?
      (*i*) Moving from *A* to *B* to *C* to *D* to *E* to *A*.
      (*ii*) Moving from *B* to *E* to *A*.
      (*iii*) Moving from *B* to *C* to *D* to *E* to *B*.
      (*iv*) Moving from *A* to *B* to *C*.
      (*v*) Moving from *A* to *B* to *C* to *B* to *A*.

3.2 Kirchhoff's Current Law

6. (*a*) Determine the current labeled i_z in the circuit shown in Fig. 3.45. (*b*) If the resistor carrying 3 A has a value of 1 Ω, what is the value of the resistor carrying −5 A?

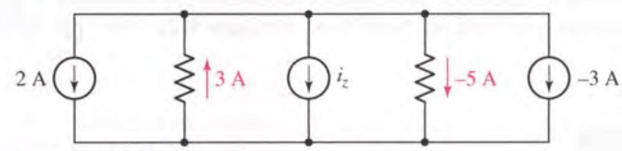


FIGURE 3.45

7. Find i_x in each of the circuits in Fig. 3.46.

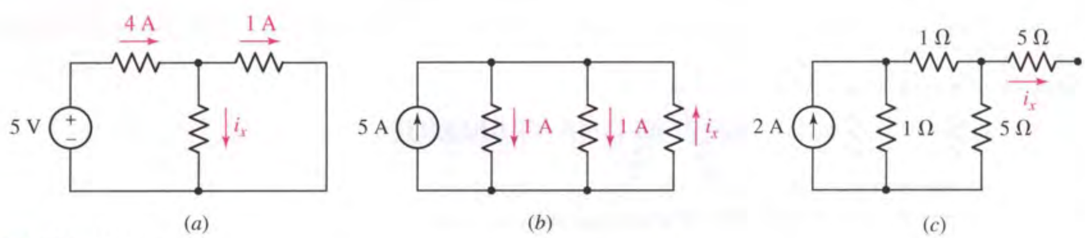


FIGURE 3.46

8. Referring to Fig. 3.47,
   (*a*) Find i_x if $i_y = 2$ A and $i_z = 0$ A. (*b*) Find i_y if $i_x = 2$ A and $i_z = 2\,i_y$.
   (*c*) Find i_z if $i_x = i_y = i_z$.

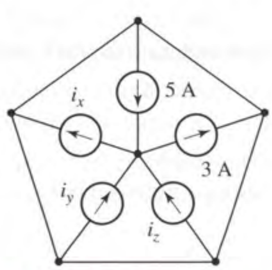


FIGURE 3.47

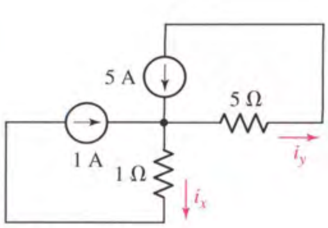


FIGURE 3.48

9. Find i_x and i_y in the circuit of Fig. 3.48.
10. A 100 W light bulb, a 60 W light bulb, and a 40 W light bulb are connected in parallel to each other and to a standard North American household 115 V supply. Compute the current flowing through each light bulb and the total current delivered by the voltage supply.

11. The digital multimeter (DMM) is a device commonly used to measure voltages. It is equipped with two leads (usually red for the positive reference and black for the negative reference) and an LCD display. Let's suppose a DMM is connected to the circuit of Fig. 3.46*b* with the positive lead at the top node and the negative lead on the bottom node. Using KCL, explain why we would ideally want a DMM used in this way to have an infinite resistance as opposed to zero resistance.

12. A local restaurant has a neon sign constructed from 12 separate bulbs; when a bulb fails, it appears as an infinite resistance and cannot conduct current. In wiring the sign, the manufacturer offers two options (Fig. 3.49). From what you've learned about KCL, which one should the restaurant owner select? Explain.

FIGURE 3.49

13. In the circuit of Fig. 3.50,
    (*a*) Calculate v_y if $i_z = -3$ A.
    (*b*) What voltage would need to replace the 5 V source to obtain $v_y = -6$ V if $i_z = 0.5$ A?

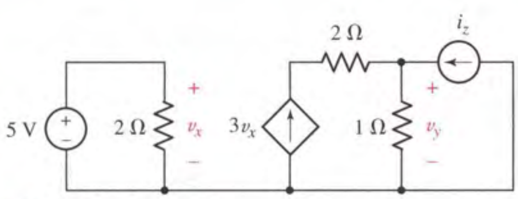


FIGURE 3.50

14. Referring to the circuit in Fig. 3.51*a*,
    (*a*) If $i_x = 5$ A, find v_1 and i_y. (*b*) If $v_1 = 3$ V, find i_x and i_y.
    (*c*) What value of i_s will lead to $v_1 \neq v_2$?

15. Find *R* and *G* in the circuit of Fig. 3.51*b* if the 5 A source is supplying 100 W and the 40 V source is supplying 500 W.

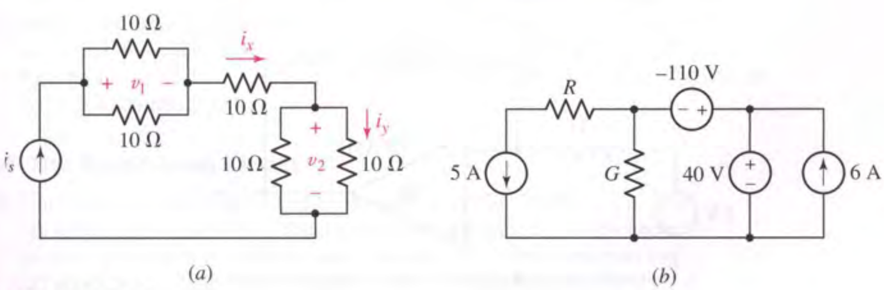


FIGURE 3.51

3.3 Kirchhoff's Voltage Law

16. In the circuits of Fig. 3.52*a* and *b*, determine the current labeled *i*.

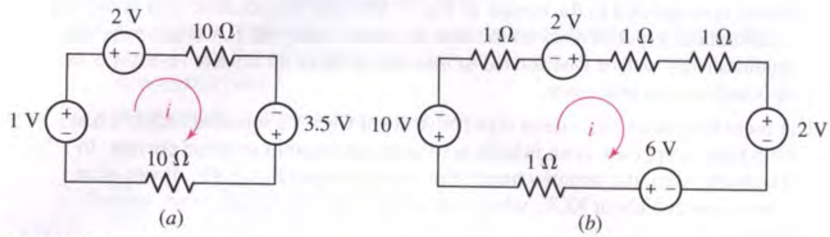


FIGURE 3.52

17. Calculate the value of *i* in each circuit of Fig. 3.53.

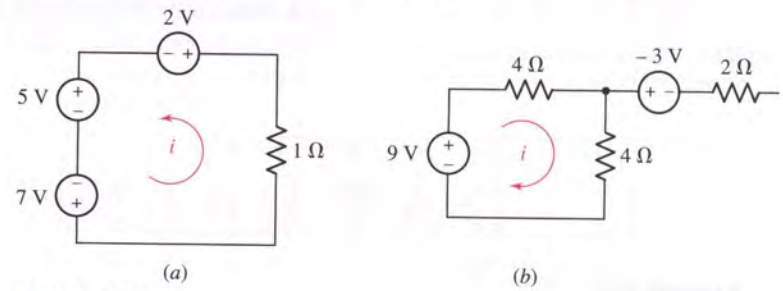


FIGURE 3.53

18. Consider the simple circuit shown in Fig. 3.54. Using KVL, derive the expressions

$$v_1 = v_s \frac{R_1}{R_1 + R_2} \quad \text{and} \quad v_2 = v_s \frac{R_2}{R_1 + R_2}$$

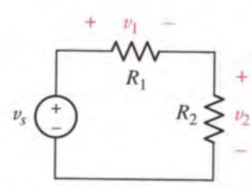


FIGURE 3.54

19. The circuit shown in Fig. 3.55 includes a device known as an op amp. This device has two unusual properties in the circuit shown: (1) $V_d = 0$ V, and (2) no current can flow into either input terminal (marked "−" and "+" inside the symbol), but it *can* flow through the output terminal (marked "OUT"). This seemingly impossible situation—in direct conflict with KCL—is a result of power leads to the device that are not included in the symbol. Based on this information, calculate V_{out}. (Hint: two KVL equations are required, both involving the 5 V source.)

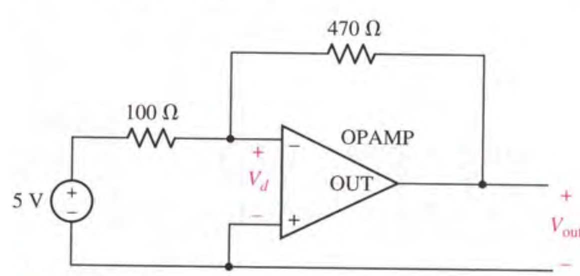
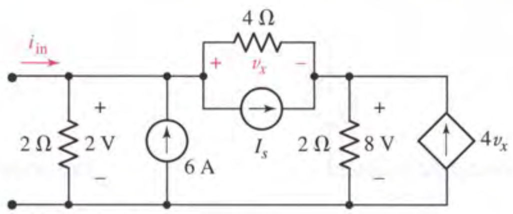


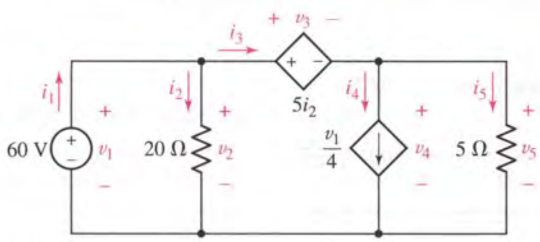
FIGURE 3.55

20. Use Ohm's and Kirchhoff's laws on the circuit of Fig. 3.56 to find (*a*) v_x; (*b*) i_{in}; (*c*) I_s; (*d*) the power provided by the dependent source.



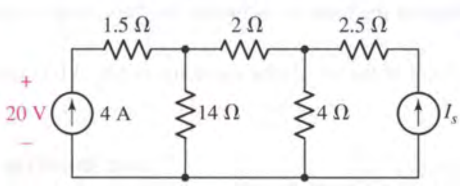
■ **FIGURE 3.56**

21. (*a*) Use Kirchhoff's and Ohm's laws in a step-by-step procedure to evaluate all the currents and voltages in the circuit of Fig. 3.57. (*b*) Calculate the power absorbed by each of the five circuit elements and show that the sum is zero.

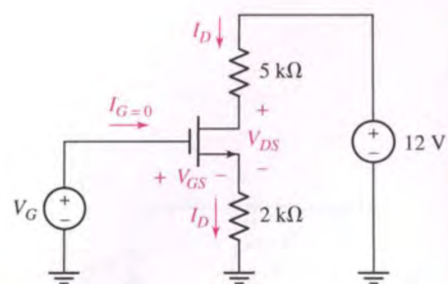


■ **FIGURE 3.57**

22. With reference to the circuit shown in Fig. 3.58, find the power absorbed by each of the seven circuit elements.



■ **FIGURE 3.58**



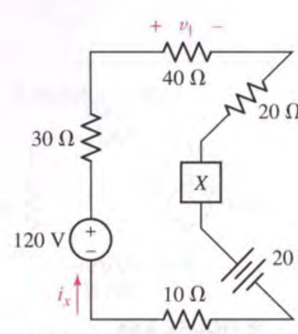
■ **FIGURE 3.59**

23. A certain circuit contains six elements and four nodes, numbered 1, 2, 3, and 4. Each circuit element is connected between a different pair of nodes. The voltage v_{12} (+ reference at first-named node) is 12 V, and $v_{34} = -8$ V. Find v_{13}, v_{23}, and v_{24} if v_{14} equals (*a*) 0; (*b*) 6 V; (*c*) −6 V.

24. Refer to the transistor circuit shown in Fig. 3.59. Keep in mind that although we do not know the current-voltage relationship for the device, it still obeys both KCL and KVL. (*a*) If $I_D = 1.5$ mA, compute V_{DS}. (*b*) If $I_D = 2$ mA and $V_G = 3$ V, compute V_{GS}.

3.4 The Single-Loop Circuit

25. Find the power being absorbed by element X in Fig. 3.60 if it is a (*a*) 100 Ω resistor; (*b*) 40 V independent voltage source, + reference on top; (*c*) dependent voltage source labeled $25i_x$, + reference on top; (*d*) dependent voltage source labeled $0.8v_1$, + reference on top; (*e*) 2 A independent current source, arrow directed upward.



■ **FIGURE 3.60**

26. Find i_1 in the circuit of Fig. 3.61 if the dependent voltage source is labeled: (*a*) $2v_2$; (*b*) $1.5v_3$; (*c*) $-15i_1$.

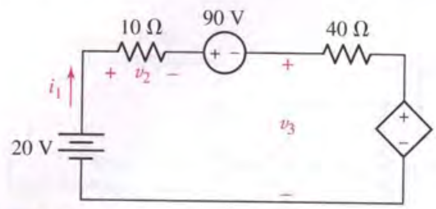


FIGURE 3.61

27. Refer to the circuit of Fig. 3.61 and label the dependent source $1.8v_3$. Find v_3 if (*a*) the 90 V source generates 180 W; (*b*) the 90 V source absorbs 180 W; (*c*) the dependent source generates 100 W; (*d*) the dependent source absorbs 100 W of power.

28. For the battery charger modeled by the circuit of Fig. 3.62, find the value of the adjustable resistor R so that: (*a*) a charging current of 4 A flows; (*b*) a power of 25 W is delivered to the battery (0.035 Ω and 10.5 V); (*c*) a voltage of 11 V is present at the terminals of the battery (0.035 Ω and 10.5 V).

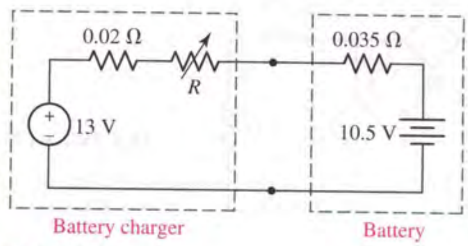


FIGURE 3.62

29. The circuit of Fig. 3.62 is modified by installing a dependent voltage source in series with the battery. Break the top wire, place the + reference at the right and let the control be $0.05i$, where i is the clockwise loop current. Find this current and the terminal voltage of the battery, including the dependent source, if $R = 0.5\ \Omega$.

30. Find the power absorbed by each of the six circuit elements in Fig. 3.63, and show that they sum to zero.

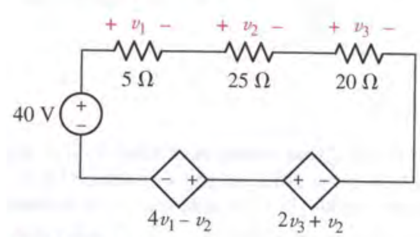


FIGURE 3.63

31. For the circuit of Fig. 3.64,
   (*a*) Determine the resistance R that will result in the 25 kΩ resistor absorbing 2 mW.
   (*b*) Determine the resistance R that results in the 12 V source delivering 3.6 mW to the circuit.
   (*c*) Replace the resistor R with a voltage source such that no power is absorbed by either resistor; draw the circuit, indicating the voltage polarity of the new source.

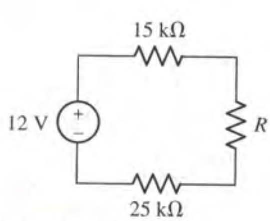
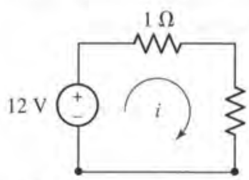
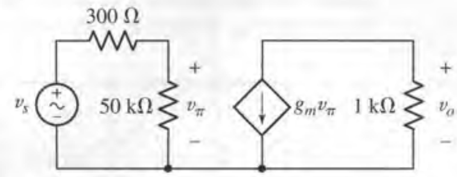


FIGURE 3.64

32. Referring to Table 2.4, if the bottom wire segment in the circuit of Fig. 3.65 is 22 AWG solid copper and 3000 ft long, compute the current i.



■ **FIGURE 3.65**



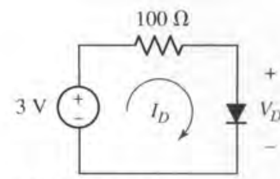
■ **FIGURE 3.66**

33. In Fig. 3.66, if $g_m = 25 \times 10^{-3}$ siemens and $v_s = 10 \cos 5t$ mV, find $v_o(t)$.

34. Kirchhoff's laws apply whether or not Ohm's law applies to a particular element. The *I-V* characteristic of a diode, for example, is given by

$$I_D = I_S\left(e^{V_D/V_T} - 1\right)$$

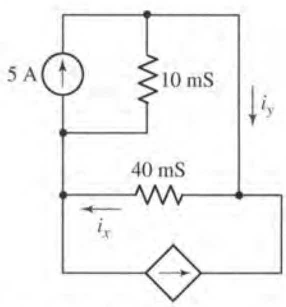
where $V_T = 27$ mV at room temperature and I_S can vary from 10^{-12} to 10^{-3} A. In the circuit of Fig. 3.67, use KVL/KCL to obtain V_D if $I_S = 3\ \mu$A. (*Note: This problem results in a transcendental equation, requiring an iterative approach to obtaining a numerical solution. Most scientific calculators will perform such a function.*)



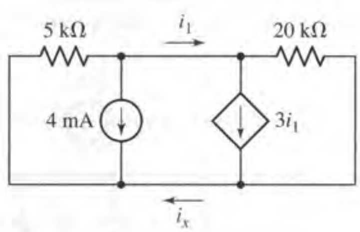
■ **FIGURE 3.67**

3.5 The Single-Node-Pair Circuit

35. Find the power absorbed by each circuit element of Fig. 3.68 if the control for the dependent source is (*a*) $0.8i_x$; (*b*) $0.8i_y$. In each case, demonstrate that the absorbed power quantities sum to zero.



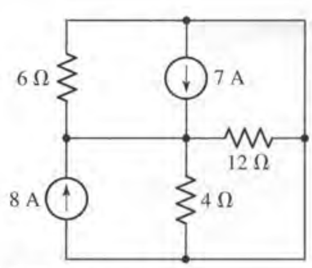
■ **FIGURE 3.68**



■ **FIGURE 3.69**

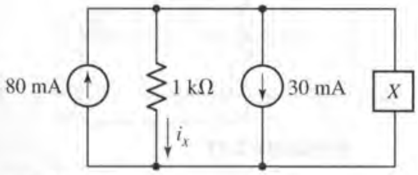
36. Find i_x in the circuit of Fig. 3.69.

37. Find the power absorbed by each element in the single-node-pair circuit of Fig. 3.70, and show that the sum is equal to zero.



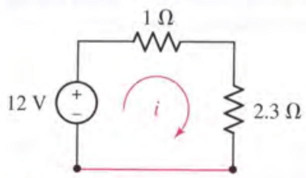
■ **FIGURE 3.70**

38. Find the power absorbed by element X in the circuit of Fig. 3.71 if it is a (*a*) 4 kΩ resistor; (*b*) 20 mA independent current source, reference arrow downward; (*c*) dependent current source, reference arrow downward, labeled $2i_x$; (*d*) 60 V independent voltage source, + reference at top.

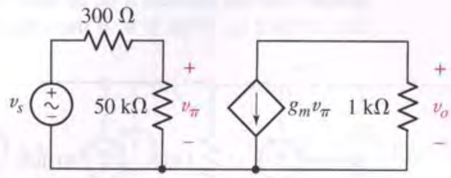


■ **FIGURE 3.71**

32. Referring to Table 2.4, if the bottom wire segment in the circuit of Fig. 3.65 is 22 AWG solid copper and 3000 ft long, compute the current i.



■ **FIGURE 3.65**



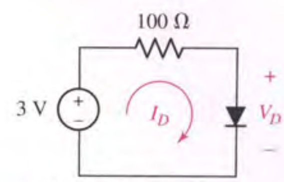
■ **FIGURE 3.66**

33. In Fig. 3.66, if $g_m = 25 \times 10^{-3}$ siemens and $v_s = 10 \cos 5t$ mV, find $v_o(t)$.

34. Kirchhoff's laws apply whether or not Ohm's law applies to a particular element. The *I-V* characteristic of a diode, for example, is given by

$$I_D = I_S \left(e^{V_D/V_T} - 1\right)$$

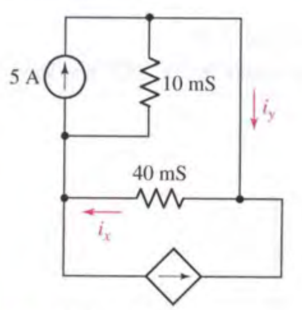
where $V_T = 27$ mV at room temperature and I_S can vary from 10^{-12} to 10^{-3} A. In the circuit of Fig. 3.67, use KVL/KCL to obtain V_D if $I_S = 3\ \mu$A. (*Note: This problem results in a transcendental equation, requiring an iterative approach to obtaining a numerical solution. Most scientific calculators will perform such a function.*)



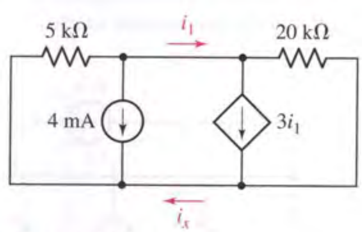
■ **FIGURE 3.67**

3.5 The Single-Node-Pair Circuit

35. Find the power absorbed by each circuit element of Fig. 3.68 if the control for the dependent source is (*a*) $0.8i_x$; (*b*) $0.8i_y$. In each case, demonstrate that the absorbed power quantities sum to zero.



■ **FIGURE 3.68**

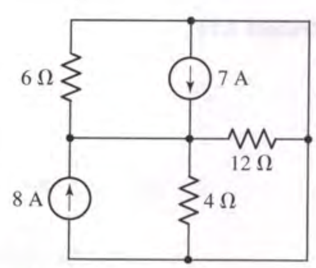


■ **FIGURE 3.69**

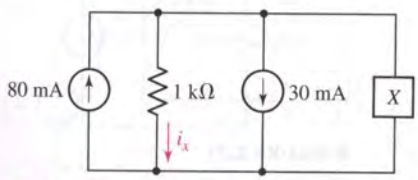
36. Find i_x in the circuit of Fig. 3.69.

37. Find the power absorbed by each element in the single-node-pair circuit of Fig. 3.70, and show that the sum is equal to zero.

38. Find the power absorbed by element X in the circuit of Fig. 3.71 if it is a (*a*) 4 kΩ resistor; (*b*) 20 mA independent current source, reference arrow downward; (*c*) dependent current source, reference arrow downward, labeled $2i_x$; (*d*) 60 V independent voltage source, + reference at top.



■ **FIGURE 3.70**



■ **FIGURE 3.71**

39. (*a*) Let element X in Fig. 3.72 be an independent current source, arrow directed upward, labeled i_s. What is i_s if none of the four circuit elements absorbs any power? (*b*) Let element X be an independent voltage source, + reference on top, labeled v_s. What is v_s if the voltage source absorbs no power?

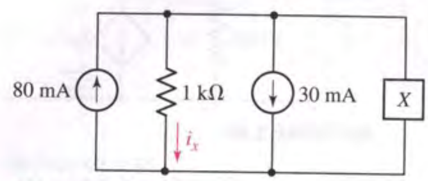


FIGURE 3.72

40. (*a*) Apply the techniques of single-node-pair analysis to the upper right node in Fig. 3.73 and find i_x. (*b*) Now work with the upper left node and find v_8. (*c*) How much power is the 5 A source generating?

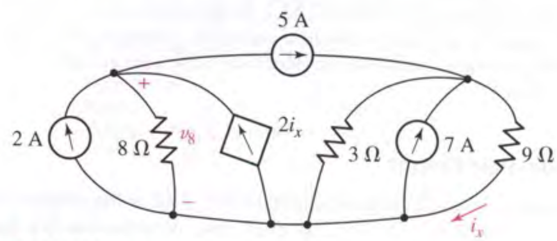


FIGURE 3.73

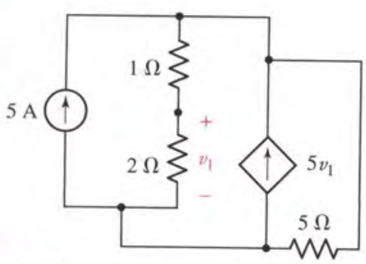


FIGURE 3.74

41. Find the power absorbed by the 5 Ω resistor in Fig. 3.74.

42. Compute the power supplied by each element shown in Fig. 3.75, and show that their sum is equal to zero.

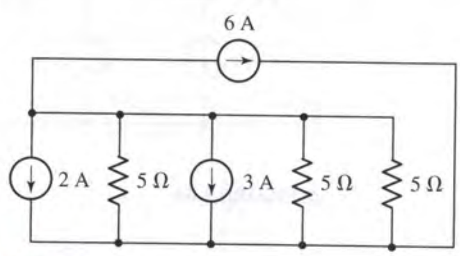


FIGURE 3.75

43. Referring to Table 2.4, how many miles of 28 AWG solid copper wire is required for the labelled wire segment of Fig. 3.76 to obtain $i_1 = 5$ A?

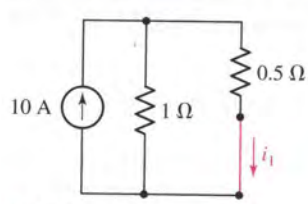


FIGURE 3.76

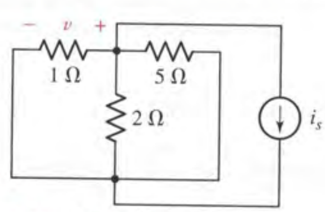
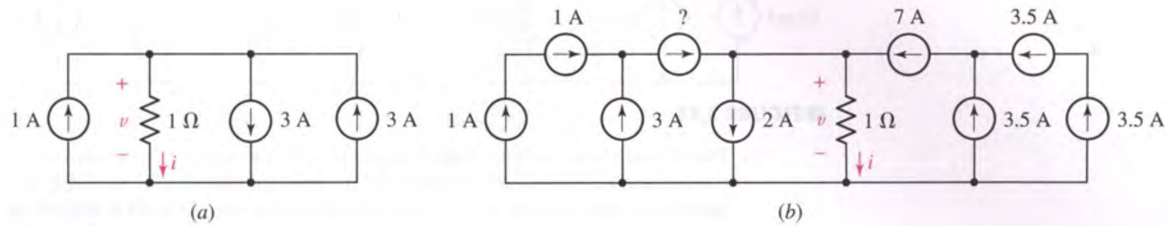


FIGURE 3.77

44. In the circuit of Fig. 3.77, if $v = 6$ V, find i_s.

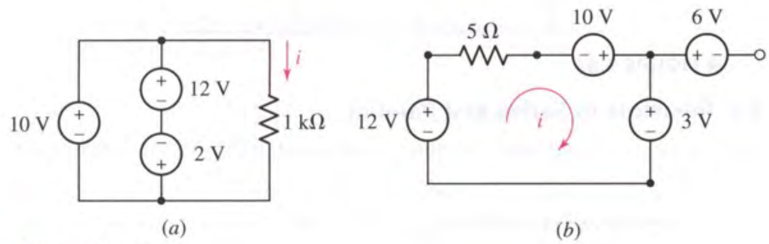
3.6 Series and Parallel Connected Sources

45. Using combinations of sources, compute i for both circuits in Fig. 3.78.



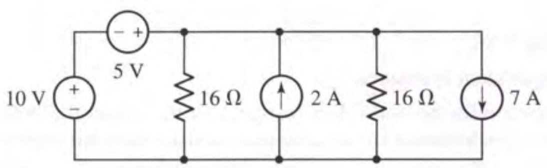
■ **FIGURE 3.78**

46. Compute v for each of the circuits in Fig. 3.78 by first combining sources.
47. Compute the current labeled i in each of the circuits in Fig. 3.79.



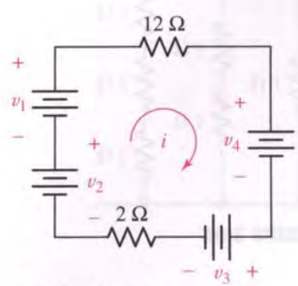
■ **FIGURE 3.79**

48. Compute the power absorbed by each element of the circuit shown in Fig. 3.80, and verify that their sum is zero.

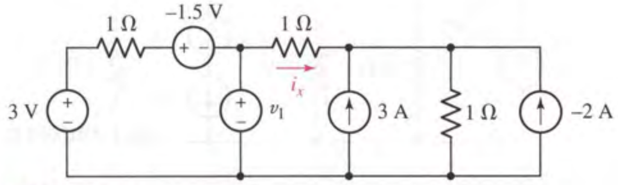


■ **FIGURE 3.80**

49. For the circuit in Fig. 3.81, compute i if:
    (*a*) $v_1 = v_2 = 10$ V and $v_3 = v_4 = 6$ V.
    (*b*) $v_1 = v_3 = 3$ V and $v_2 = v_4 = 2.5$ V.
    (*c*) $v_1 = -3$ V, $v_2 = 1.5$ V, $v_3 = -0.5$ V, and $v_4 = 0$ V.
50. In the circuit of Fig. 3.82, choose v_1 to obtain a current i_x of 2 A.



■ **FIGURE 3.81**



■ **FIGURE 3.82**

51. Find the voltage v in the circuit of Fig. 3.83.

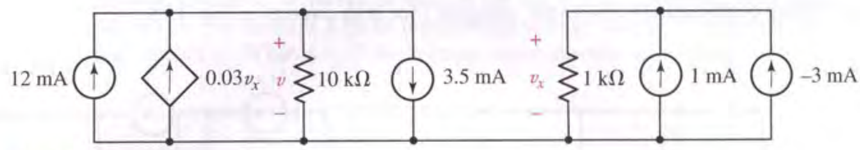


FIGURE 3.83

52. The circuit shown in Fig. 3.84 contains several examples of independent current and voltage sources connected in series and in parallel. (*a*) Find the power absorbed by each source. (*b*) To what value should the 4 V source be changed to reduce the power supplied by the −5 A source to zero?

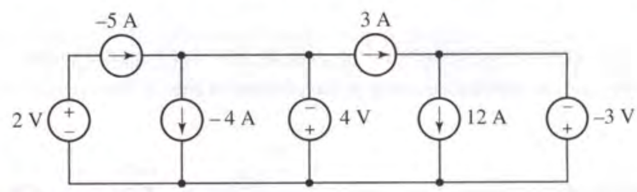


FIGURE 3.84

3.7 Resistors in Series and Parallel

53. Compute the equivalent resistance as indicated in Fig. 3.85 if each resistor is 1 kΩ.

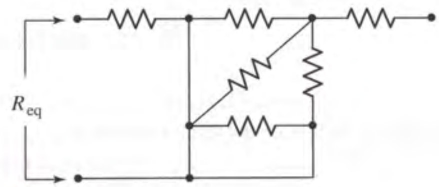


FIGURE 3.85

54. For the circuit in Fig. 3.86,
    (*a*) Compute the equivalent resistance.
    (*b*) Derive an expression for the equivalent resistance if the circuit is extended using *N* branches, each branch having one more resistor than the branch to its left.

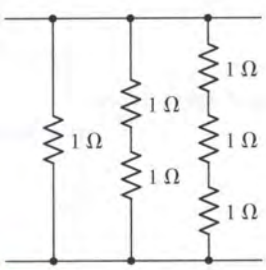


FIGURE 3.86

55. Given three 10 kΩ resistors, three 47 kΩ resistors, and three 1 kΩ resistors, find a combination (not all resistors need to be used) that yields:
    (*a*) 5 kΩ
    (*b*) 57,333 Ω
    (*c*) 29.5 kΩ

56. Simplify the networks in Fig. 3.87 using resistor and source combinations.

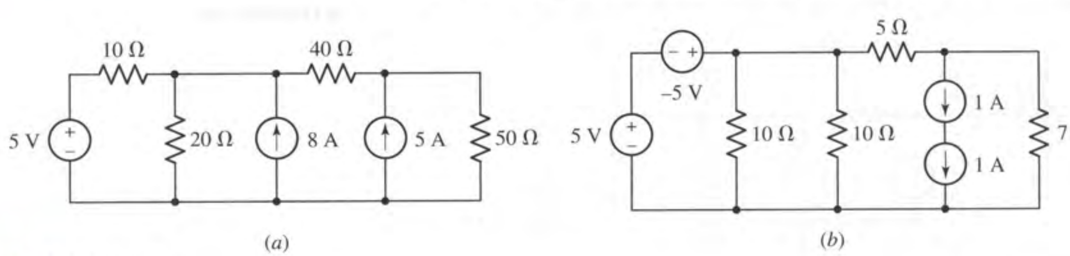


FIGURE 3.87

65. The ***Wheatstone bridge*** (Fig. 3.95) is one of the most well-known electrical circuits and is used in resistance measurement. The resistor with an arrow through its symbol (R_3) is a variable resistor, sometimes referred to as a potentiometer; its value can be changed by simply rotating a knob. The ammeter, symbolized by a circle with a diagonal arrow in the center, measures the current through the center wire. We assume this ammeter to be ideal, so that it has zero internal resistance.

Operation is simple. The values of R_1, R_2, and R_3 are known, and the value of R is desired. Resistor R_3 is adjusted until $i_m = 0$; in other words, until no current flows through the ammeter. At this point the bridge is said to be "balanced."

Using KCL and KVL, show that $R = \frac{R_2}{R_1} R_3$. (Hints: The value of V_s is irrelevant; with $i_m = 0$, $i_1 = i_3$ and $i_2 = i_R$; and there is no voltage dropped across the ammeter.)

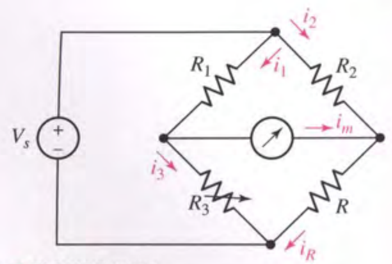


FIGURE 3.95

66. The circuit of Fig. 3.96 consists of several resistors connected in a series string. Use voltage division to calculate how much voltage is dropped across the smallest resistor and across the largest resistor, respectively.

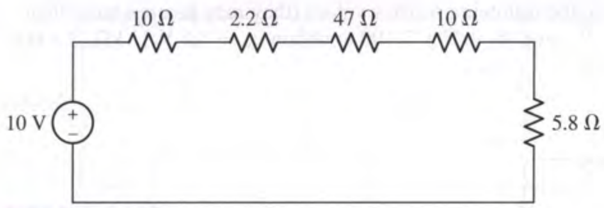


FIGURE 3.96

67. Employ voltage division to calculate the voltage across the 47 kΩ resistor of Fig. 3.97.

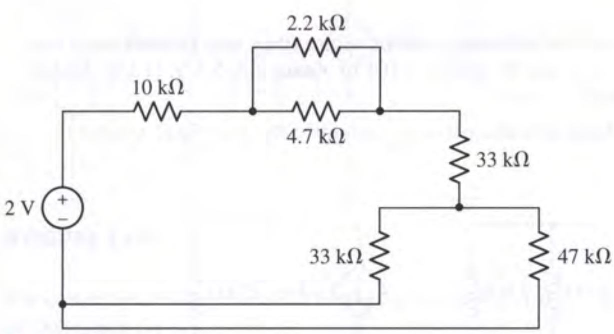


FIGURE 3.97

68. Referring to the circuit depicted in Fig. 3.98, use current division to calculate the current flowing downward through (*a*) the 33 Ω resistor; (*b*) the rightmost 134 Ω resistor.

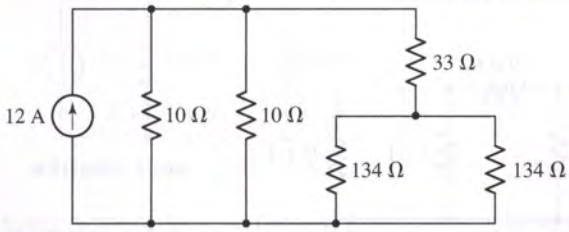
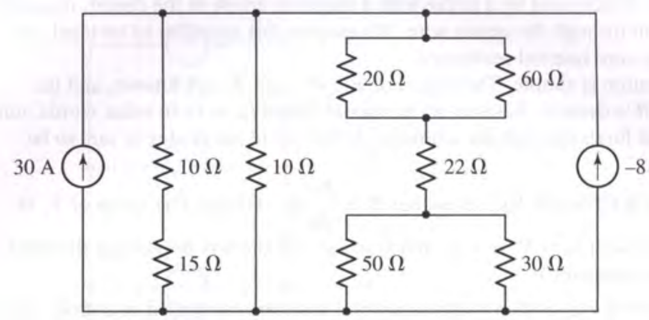


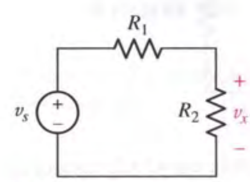
FIGURE 3.98

69. It appears that despite the large number of components in the circuit of Fig. 3.99, only the voltage across the 15 Ω resistor is of interest. Use current division to assist in calculating the correct value.

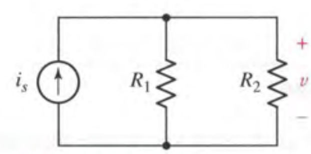


■ **FIGURE 3.99**

70. Choosing from the following resistor values (they may be used more than once), set v_s, R_1, and R_2 in Fig. 3.100 to obtain $v_x = 5.5$ V. [1 kΩ, 3.3 kΩ, 4.7 kΩ, 10 kΩ]



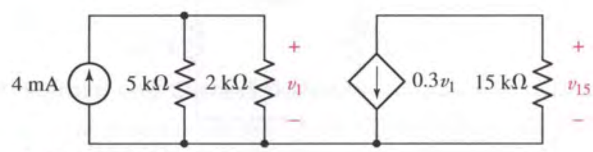
■ **FIGURE 3.100**



■ **FIGURE 3.101**

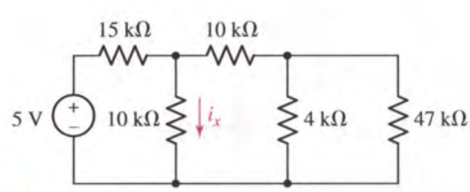
71. Choosing from the following resistor values (they may be used more than once), set i_s, R_1, and R_2 in Fig. 3.101 to obtain $v = 5.5$ V. [1 kΩ, 3.3 kΩ, 4.7 kΩ, 10 kΩ]

72. Determine the power dissipated by (absorbed by) the 15 kΩ resistor in Fig. 3.102.



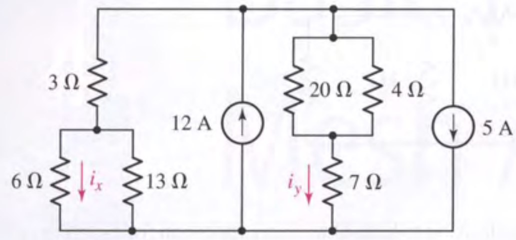
■ **FIGURE 3.102**

73. For the circuit in Fig. 3.103, determine i_x, and compute the power dissipated by (absorbed by) the 15 kΩ resistor.



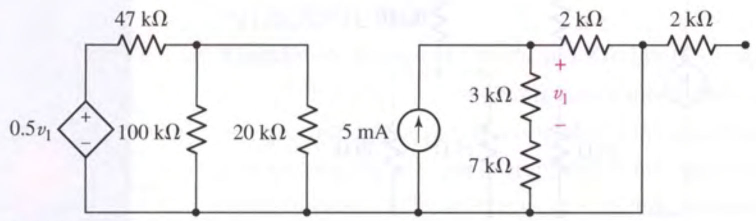
■ **FIGURE 3.103**

74. For the circuit in Fig. 3.104, find i_x, i_y, and the power dissipated by (absorbed by) the 3 Ω resistor.



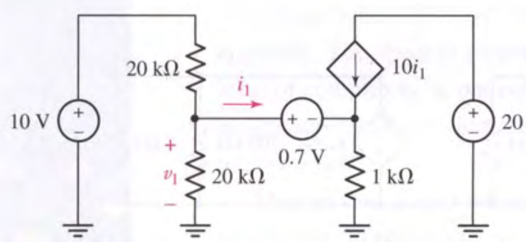
■ **FIGURE 3.104**

75. What is the power dissipated by (absorbed by) the 47 kΩ resistor in Fig. 3.105?

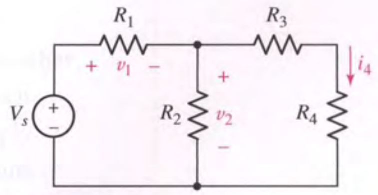


■ **FIGURE 3.105**

76. Explain why voltage division cannot be used to determine v_1 in Fig. 3.106.



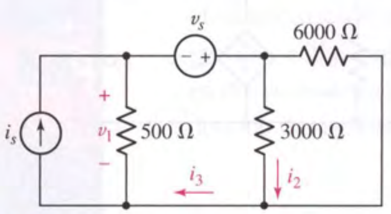
■ **FIGURE 3.106**



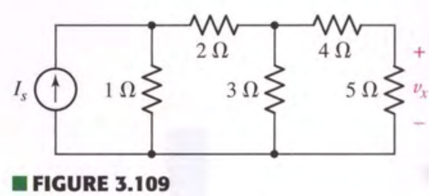
■ **FIGURE 3.107**

77. Use current and voltage division on the circuit of Fig. 3.107 to find an expression for (*a*) v_2; (*b*) v_1; (*c*) i_4.

78. With reference to the circuit shown in Fig. 3.108: (*a*) let $v_s = 40$ V, $i_s = 0$, and find v_1; (*b*) let $v_s = 0$, $i_s = 3$ mA, and find i_2 and i_3.



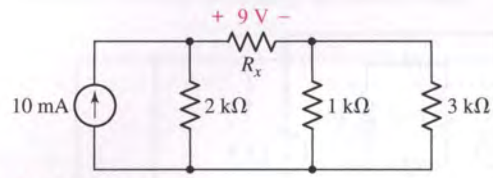
■ **FIGURE 3.108**



■ **FIGURE 3.109**

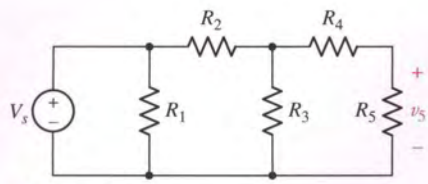
79. In Fig. 3.109: (*a*) let $v_x = 10$ V and find I_s; (*b*) let $I_s = 50$ A and find v_x; (*c*) calculate the ratio v_x/I_s.

80. Determine how much power is absorbed by R_x in the circuit of Fig. 3.110.



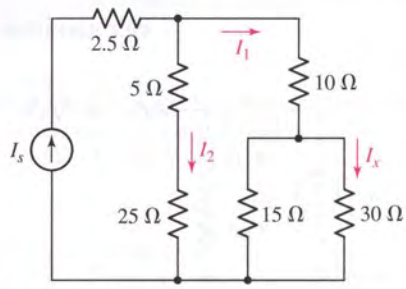
■ **FIGURE 3.110**

81. Use current and voltage division to help obtain an expression for v_5 in Fig. 3.111.



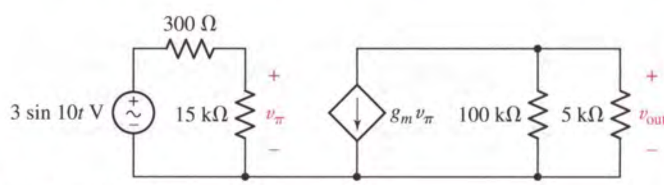
■ **FIGURE 3.111**

82. With reference to the circuit of Fig. 3.112, find (*a*) I_x if $I_1 = 12$ mA; (*b*) I_1 if $I_x = 12$ mA; (*c*) I_x if $I_2 = 15$ mA; (*d*) I_x if $I_s = 60$ mA.



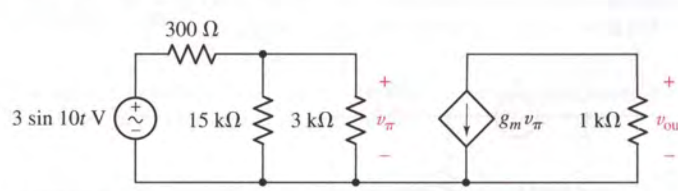
■ **FIGURE 3.112**

83. The circuit in Fig. 3.113 is a commonly used equivalent circuit used to model the ac behavior of a MOSFET amplifier circuit. If $g_m = 4$ m$\mho$, compute v_{out}.



■ **FIGURE 3.113**

84. The circuit in Fig. 3.114 is a commonly used equivalent circuit used to model the ac behavior of a bipolar junction transistor amplifier circuit. If $g_m = 38$ m$\mho$, compute v_{out}.



■ **FIGURE 3.114**