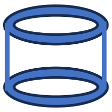
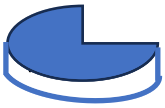
# Practice Problems

Q1 Calculate moment of inertia of the following objects:

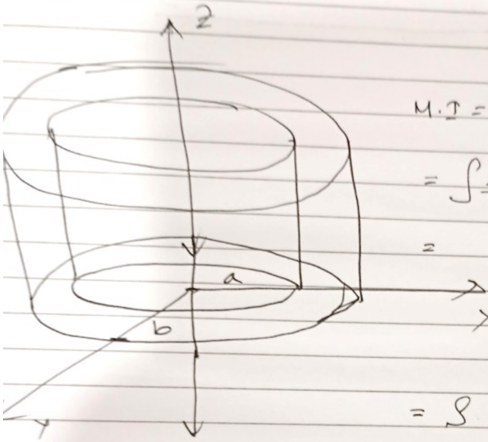
1.  Wrt z axis at center of the horizontal face. Inner radius=a, outer radius= b, height =h

2.  Wrt z axis at center of the horizontal face. Inner radius=a, height =h,one quadrant of the material missing

Soln:

(a)Moment of inertia=$\int dm\, r^2 = \int \rho dV r^2 (\rho = density)$

... coordinate system, $r, \phi, z$




$$M.I = \int \rho\, dV\, r^2$$

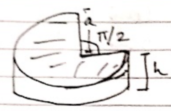
$$= \int \rho\, r\, dr\, d\phi\, dz$$

$$= \rho \int_a^b r\, dr \int_0^{2\pi} d\phi \int_0^h dz$$

$$= \rho\, \frac{b^2 - a^2}{2} \times 2\pi \times h$$

$$\boxed{I = \rho\, \pi (b^2 - a^2) h}$$

(b)




$$M.I = \int \rho \, dV \, r^2$$

$$= \rho \int r \, dr \, d\phi \, dz$$

$$= \rho \int_0^a r \, dr \int_0^{2\pi - \pi/2} d\phi \int_0^h dz$$

$$= \rho \times \frac{a^2}{2} \times \frac{3\pi}{2} \times h$$

$$\boxed{M.I = \frac{\rho \pi a^2 h}{4}}$$

Q2. One mole of a monatomic perfect gas initially at temperature $T_o$ expands from volume $V_o$ to $2V_o$ (a) at constant temperature, (b) at constant pressure. Calculate the work of expansion and the heat absorbed by the gas in each case.

**Solution:**

(a) At constant temperature $T_0$, the work is

$$W = \int_A^B p dV = RT_0 \int_{V_0}^{2V_0} dV/V = RT_0 \ln 2 \ .$$

As the change of the internal energy is zero, the heat absorbed by the gas is

$$Q = W = RT_0 \ \ln 2 \ .$$

(b) At constant pressure $p$, the work is

$$W = \int_{V_0}^{2V_0} p dV = pV_0 = RT_0 \ .$$

The increase of the internal energy is

$$\Delta U = C_v \Delta T = \frac{3}{2} R \Delta T = \frac{3}{2} p \Delta V = \frac{3}{2} p V_0 = \frac{3}{2} RT_0 \ .$$

Thus the heat absorbed by the gas is

$$Q = \Delta U + W = \frac{5}{2} RT_0 \ .$$

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Q3. A compressor designed to compress air is used instead to compress helium. It is found that the compressor overheats. Explain this effect, assuming the compression is approximately adiabatic and the starting pressure is the same for both gases.

**Solution:**

The state equation of ideal gas is

$$pV = nRT \ .$$

The equation of adiabatic process is

$$p \left( \frac{V}{V_0} \right)^{\gamma} = p_0 \ ,$$

where $\gamma = c_p / c_v, p_0$ and $p$ are starting and final pressures, respectively, and $V_0$ and $V$ are volumes. Because $V_0 > V$ and $\gamma_{\mathrm{He}} > \gamma_{\mathrm{Air}}$ ($\gamma_{\mathrm{He}} = 7/5; \gamma_{\mathrm{Air}} = 5/3$), we get

$$p_{\mathrm{He}} > p_{\mathrm{Air}} \quad \text{and} \quad T_{\mathrm{He}} > T_{\mathrm{Air}} \ .$$

Q4 Two identical bodies have internal energy U = NCT, with a constant C. The values of N and C are the same for each body. The initial temperatures of the bodies are $T_l$ and $T_2$, and they are used as a source of work by connecting them to a Carnot heat engine and bringing them to a common final temperature $T_f$.

**Solution:**

(a) The internal energy is $U = NCT$. Thus $dQ_1 = NCdT_1$ and $dQ_2 = NCdT_2$. For a Carnot engine, we have $\frac{dQ_1}{T_1} = -\frac{dQ_2}{T_2}$. Hence

$$\frac{dT_1}{T_1} = -\frac{dT_2}{T_2}.$$

Thus $\int_{T_1}^{T_f} \frac{dT_1}{T_1} = -\int_{T_2}^{T_f} \frac{dT_2}{T_2}$, $\quad \ln\frac{T_f}{T_1} = -\ln\frac{T_f}{T_2}$,

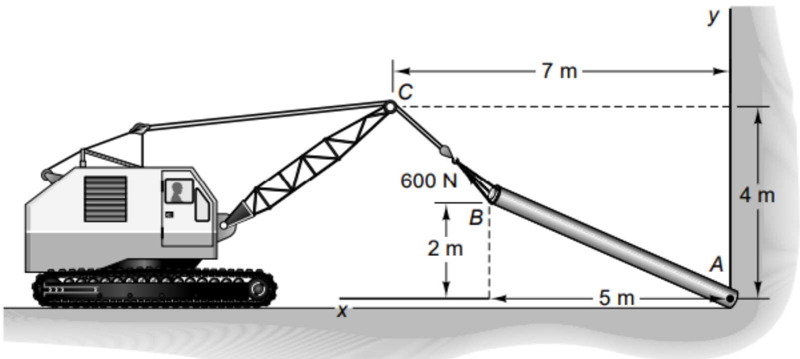
Therefore $T_f = \sqrt{T_1 T_2}$.

(b) Conservation of energy gives

$$W = (U_1 - U) - (U - U_2) = U_1 + U_2 - 2U$$
$$= NC(T_1 + T_2 - 2T_f).$$

Q5

A crane lifts the end of a drain pipe. If the crane provides a force of 6000-N acting along the line $CB$, calculate the moment created about the point $A$ using the coordinate system indicated.




Sol:

Solution: First calculate the component form of **F**. A unit vector along $BC$ is

$$\hat{\mathbf{e}}_{BC} = \frac{(7-5)\hat{\mathbf{i}} + (4-2)\hat{\mathbf{j}}}{\sqrt{(7-5)^2 + (4-2)^2}} = .707(\hat{\mathbf{i}} + \hat{\mathbf{j}}).$$

Thus

$$\mathbf{F} = (6000)(.707)(\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 4243(\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{ N}.$$

The vector

$$\mathbf{r}_A = 5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$$

so that

$$\mathbf{M}_A = (5\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \times (4243)(\hat{\mathbf{i}} + \hat{\mathbf{j}}) = -8485\hat{\mathbf{k}} + 21,213\hat{\mathbf{k}} = \underline{12,727\hat{\mathbf{k}}\text{ Nm}},$$

clockwise about $A$. ■

Q6 . One mole of a gas is contained in a cube of side 0.2m If these molecules, each of mass 5 $\times 10^{-26}$kg, move with translational speed 483$ms^{-1}$ , calculate the pressure exerted by the gas on the sides of the cube.

Sol:One mole of a gas is contained in a cube of side 0.2m If these molecules, each of mass kg, move

with translational speed 483m, calculate the pressure exerted by the gas on the sides of the cube.

Solution: The change in the momentum of the gaseous molecule between any two successive collisions with a wall of the container will be

$\Delta p_x = 2mv_x = 2 \times (5\times10^{-26}$ kg$) \times (483ms^{-1}) = 4.83\times10^{-23}$ Ns

The time between successive collision on the same face

$\Delta t = \frac{2L}{V_x} =- \frac{2\times0.2m}{483ms^{-1}} = 8.3\times10^{-4}s$

Hence the rate of change of momentum of one molecule

$\frac{\Delta p_x}{\Delta t} = \frac{4.83\times10^{-23}\text{ Ns}}{8.3\times10^{-4}s} = 0.582\times10^{-19}$ N

Therefore, the total force exerted by all the molecules of the gas on a wall is

$f_x = (0.582\times10^{-19}$ N$) \times (6\times10^{23}) = 3.49 \times 10^4$ N

Hence average pressure exerted by all the molecules of the gas on the walls of the container

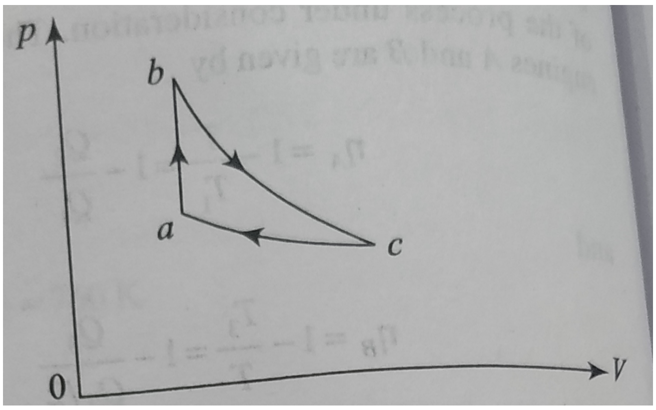
$P = \frac{3.49\times10^4\text{ N}}{3\times4\times10^{-2}\,m^2} = 2.9 \times 10^5\,Nm^{-2}$

Q7 An ideal monoatomic gas occupies 2 liters at 30k and $5 \times 10^{-3}pa$.The internal energy of the gas is taken to be zero at this point. It undergoes the following changes.

a) The temperature is raised to 300k at constant volume.

b) The gas is then expanded adiabatically till it attains the initial temperature.

c)Finally, it is compressed isothermally. Calculate the efficiency of the cycle.



Refer to fig. which is indicator diagram for the processes under consideration. Here

$$a \to b: isochoric\ comperssion$$

$$b \to c: adabatic\ expansion$$

$$c \to a: isothermal\ compression$$

At a, internal energy is zero. To calculate the number of moles of the gas, we use the number of the gas. We use the equation of state for an ideal gas:

$$PV = nRT$$

Here V=2 liter=$2 \times 10^{-3} m^3, P = 5 \times 10^2 Nm^{-2}\ and\ T = 30K$. Using these values, we get
$(5 \times 10^2 Nm^{-2}) \times (2 \times 10^{-3} m^3 = nR \times (30K)$

Or $30nR = 1Nmk^{-1}$

Hence, the number of moles o the gas can be expressed in terms of R:

$$n = \frac{1}{30R} JK^{-1}$$

To calculate the efficiency, we need to know the work done as well as heat absorbed. So, we consider the three processes one by one.

From $a \to b$,the process is isochoric and no work is done:

$\delta W_{a \to b} = 0$

$$Q_{a \to b} = nC_V(T_b - T_a)$$

For a Monoatomic gas,$C_v = \frac{3}{2}R$

$$Q_{a \to b} = \frac{1JK^{-1}}{30R} \times \frac{3}{2}R \times (300 - 30)K = 13.5J$$

And $U_b - U_a = 13.5J$

But $U_a = 0 \quad implies\ \ U_b = 13.5J$

From $b \to c$,the processes adiabatic and no heat is exchanged:

$$\delta Q_{a \to b} = 0$$

From the first law, we can write

$$W_{a \to b} = - \vartriangle U = -(U_c - U_b)$$

Since $T_c$=30K and U is a function of only temperature for an ideal gas, we note that $U_c =$
$0\ since U_a = 0$

$$W_{a \to b} = U_b = 13.5J$$

For $c \to a$,

$$W_{a \to b} = nRTln\frac{V_a}{V_c} = -nRTln\left(\frac{V_c}{V_B}\right)$$

Since $V_a = V_b$

To calculate the ration $\frac{V_a}{V_c}$,we note that points c and b are located on the same adiabat. So, we can write

$$TV^{\gamma-1} = constant$$

$$T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$$

$$\left(\frac{V_c}{V_b}\right)^{\gamma-1} = \frac{T_b}{T_c}$$

Since **γ** = 1.67, we get

$\left(\frac{V_c}{V_b}\right)^{0.67}$=$\frac{300K}{30K} = 10$

$$\frac{V_c}{V_b} = 10^{1/0.67} = 10^{1.4925} = 31.08$$

Hence

$$W_{b\to c} = -\frac{1JK^{-1}}{30R} \times R \times (30K)\ln(31.08)$$

=-3.4J

Therefore $Q_{b\to c,} = -3.4J$ and $U_a = U_c = 0$

Hence, the net work done =13.5J-3.4J=10.1J

And heat absorbed=13.5J

$$\eta = \frac{10.1J}{13.5J} = 0.748 = 74.8\%$$

Q8.The internal energy E(T) of a system at a fixed volume is found to depend on the temperature T as E(T)=a$T^2 + bT^4$. Then what will be the entropy S(T), as a function of temperature?

Sol:

$$E(T) = \mathrm{a}T^2 + bT^4$$

$$C_v = \frac{dE}{dT} = 2aT + 4bT^3$$

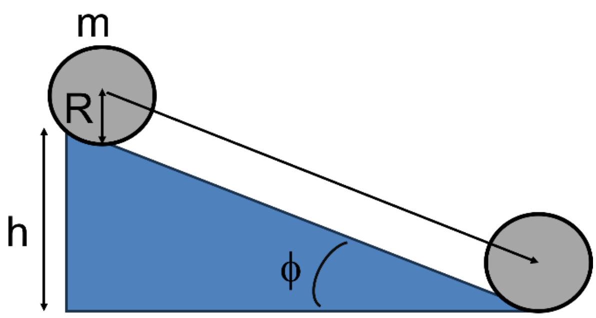
$$Tds = C_v dT$$

$$\int dS = \int C_v \frac{dT}{T}$$

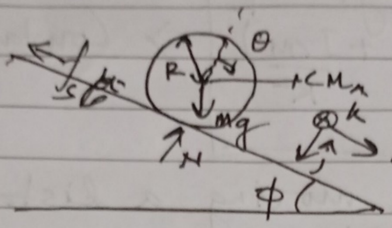
$$S = 2aT + \frac{4bT^3}{3}$$

Q9.A wheel is rolling down a inclined plane with coefficient of friction $f_s$ and angle ϕ without slipping as shown in the figure. If $I_{CM}$ is the moment of inertia , R the radius of the wheel, m the mass of the wheel and h the distance it dropped from its initial position in the vertical direction and $v_{CM}$ is the translational velocity of the centre of mass, then show using the kinematics of rotational and translational motion

$$v_{CM} = \sqrt[2]{\frac{2mgh}{(m + \frac{I_{CM}}{R^2})}}$$



Wheel rolling down inclined plane.




$a_x = a_{cm}$

Along Plane ($\hat{i}$) $\Rightarrow$ $\vec{F} = m\vec{a}$

$\Rightarrow mg\sin\phi - f_s = m a_x$ — (1)

Torque eqn $\vec{\tau}_{cm} = I_{cm}\vec{\alpha}$

Torque in this case is generated due to friction

again, $R\hat{j} \times (-\hat{i} f_s) = I_{cm}\alpha$ $[\vec{R} \times \vec{f_s} = R f_s \sin\theta]$, $\theta = \pi/2$

Torque equation

Along $\hat{k}$ $R f_s = I_{cm}\alpha$. — (2)

out of plane direction

Now, $v_{cm} = R\omega$

and $a_{cm} = a_x = R\alpha$ (3)

Unknown.

$f_s, a_x, \alpha$.

$$mg\sin\phi - \frac{I_{cm}\alpha}{R} = ma_x \quad \text{[From eqn (2)]}$$

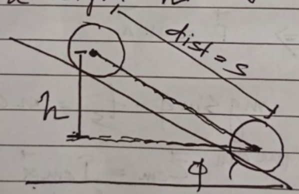
$\Rightarrow$ using $a_x = R\alpha$.

$$mg\sin\phi - \frac{I_{cm}}{R^2}a_x = ma_x$$

$$\Rightarrow a_x\left(1 + \frac{I_{cm}}{R^2}\right) = mg\sin\phi$$

$$\Rightarrow \boxed{a_x = \frac{mg\sin\phi}{\left(1 + \frac{I_{cm}}{R^2}\right)}} \Rightarrow \text{Constant} = a_x = a_{cm}$$

If our body is moving a distance $s$ and drops to a height $h$




From kinematical eqn of the Centre of mass.

$$s = x_{cm} = \frac{1}{2}a_{cm}t^2$$

$$v_{cm} = a_{cm}t$$

$$v_{cm} = \sqrt{2s\,a_{cm}}$$

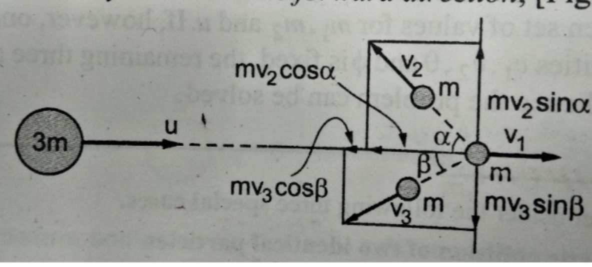
$$s = \frac{h}{\sin\phi} \quad \text{as} \quad \sin\phi = \frac{h}{s}$$

$$v_{cm} = \left[ 2 \times \frac{h}{\cancel{\sin\phi}} \times \frac{mg\,\cancel{\sin\phi}}{\left(m + \frac{I_{CM}}{R^2}\right)} \right]^{1/2}$$

$$v_{cm} = \left[ \frac{2mgh}{\left(m + \frac{I_{CM}}{R^2}\right)} \right]^{1/2}$$

3. A shell flying with a velocity u (mass '3m') bursts into three identical fragments (mass 'm') so that the kinetic energy of the system increases k times. What maximum velocity can one of the fragments attain?

**Solution :** If the mass of the shell is $3m$, mass of each of the fragments is $m$. *It should be understood that the particle with the maximum velocity travels in the forward direction,* [Fig. 10.8].



Applying the law of conservation of momentum along horizontal,

$$3mu = mv_1 - mv_2 \cos\alpha - mv_3 \cos\beta$$

or $$v_1 = 3u + v_2 \cos\alpha + v_3 \cos\beta \quad \text{...(i)}$$

and along vertical,

$$mv_2 \sin\alpha = mv_3 \sin\beta$$

or $$v_2 \sin\alpha = v_3 \sin\beta \quad \text{...(ii)}$$

For $v_1$ to be maximum, from eqn. (i), $\alpha = \beta = 0°$.

From eqn. (ii), since $\alpha = \beta, v_2 = v_3 = v$ (say)

From eqn. (i), $$v_1 = 3u + 2v$$

or $$v = \frac{v_1 - 3u}{2} \quad \text{...(iii)}$$

From the law of conservation of energy,

$$\frac{1}{2}(3m)u^2 = \frac{1}{k}\left[\frac{1}{2}mv_1^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2\right]$$

or $$3ku^2 = v_1^2 + 2v^2 \quad \text{...(iv)}$$

From eqns. (iii) and (iv),

$$3ku^2 = v_1^2 + 2\left(\frac{v_1 - 3u}{2}\right)^2$$

or $$v_1^2 - 2uv_1 + u^2(3-2k) = 0$$

or $$v_1 = \frac{2u + \sqrt{4u^2 - 4u^2(3-2k)}}{2}$$

(neglecting negative root)

or $$v_1 = u[1 + \sqrt{2(k-1)}]$$