Tutorial 5: PHY101

(MONSOON 2024)

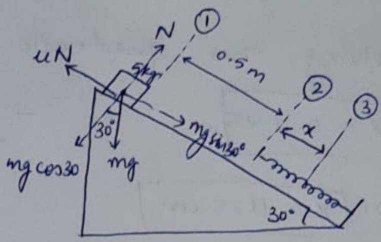
Solutions

Question 1: A block of mass 5 kg resting on a 30 degree inclined plane is released. The block after travelling a distance of 0.5 m along the inclined plane hits a spring of stiffness 15N/cm. Find the maximum compression of the spring. Assume the coefficient of friction between the block and inclined plane as 0.2.

solution:

given- $m = 5kg$, $\mu = 0.2$

$k = 15\ N/cm = 15 \dfrac{N}{10^{-2} m} = 1500\ N/m$



Position 1: $v_1 = 0$ $T_1 = 0$

2: $v_2 = ?$ $T_2 = ?$

3: $v_3 = 0$ $T_3 = 0$

⊛ Apply work-Energy Principle b/w position ① & ②, i.e

$W_{12} = T_2 - T_1$ —— ⓐ

and for that we need corrosponding forces that are acting on the system.

∴ $N = mg\cos 30 = 5 \times 9.8 \times \cos 30° = 42.48N$

$f = \mu N = 0.2 \times 42.48\ N = 8.496\ N.$

∴ eq^n ⓐ gives us. (if F & displacement are in same dirc^n then + & vice versa.)

$(+mg\sin 30°).(0.5) - (\mu N).0.5 = \frac{1}{2} \times m v_2^2 - 0$

$\Rightarrow 5 \times 9.8 \times \sin 30° \times 0.5 - 8.496 \times 0.5 = \frac{1}{2} \times 5 \times v_2^2$

$\Rightarrow 24.53 \times 0.5 - 4.248 = \frac{5}{2} v_2^2$

$\Rightarrow v_2^2 = 3.2068 \Rightarrow \boxed{v_2 = 1.79\ ms^{-1}}$ This is the velocity by which block hits the spring.

Now, we have $v_2 = 1.79\ ms^{-1}$ so $T_2 = \frac{1}{2} m v_2^2$.

⊛ Applying work-Energy Principle b/w position ② & ③.

also as the block hits the spring it starts to compress it say after x-distance we have max. compression.

so the workdone due to spring force we have will be given as.

$W_{sp} = \frac{1}{2} k (0^2 - x^2) = -\frac{1500}{2} x^2 = -750 x^2\ J$

∴ $W_{23} = T_3 - T_2$

$+(mg\sin 30).x - \mu N x - 750x^2 = 0 - \frac{1}{2} \times 5 \times (1.79)^2.$

$\Rightarrow 24.53x - 8.496x - 750x^2 = -8.01$

$\Rightarrow 750x^2 - 16.034x - 8.01 = 0$

solving this quadratic eqn we have

$\boxed{x = 0.1145\,m}$ or -0.0932 → Not acceptable.

or $\boxed{x = 11.5\ cm}$

Q2. If $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$ evaluate work done $\int \vec{F}.\overrightarrow{dr}$ along the curve C in xy plane given by the equation $y = 2x^2$ in the limit (0,0) to (1,2)

Since the integration is performed in the xy plane ($z=0$), we can take $\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j}$. Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C (3xy\,\mathbf{i} - y^2\,\mathbf{j})\cdot(dx\,\mathbf{i} + dy\,\mathbf{j})$$

$$= \int_C 3xy\,dx - y^2\,dy$$

First Method. Let $x=t$ in $y=2x^2$. Then the parametric equations of C are $x=t,\ y=2t^2$. Points (0,0) and (1,2) correspond to $t=0$ and $t=1$ respectively. Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_{t=0}^{1} 3(t)(2t^2)\,dt - (2t^2)^2\,d(2t^2) = \int_{t=0}^{1} (6t^3 - 16t^5)\,dt = -\frac{7}{6}$$

Second Method. Substitute $y = 2x^2$ directly, where x goes from 0 to 1. Then

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_{x=0}^{1} 3x(2x^2)\,dx - (2x^2)^2\,d(2x^2) = \int_{x=0}^{1} (6x^3 - 16x^5)\,dx = -\frac{7}{6}$$

Note that if the curve were traversed in the opposite sense, i.e. from (1,2) to (0,0), the value of the integral would have been 7/6 instead of $-7/6$.

Q3. A uniform chain of length l and mass m overhangs a smooth table with its two third part lying on the table. Find the kinetic energy of the chain as it completely slips off the table.

Solution:

Let us take the zero of potential energy at the table. Consider a part *dx* of the chain at a depth *x* below the surface of the table. The mass of this part is $dm = (m/l)dx$ and hence its potential energy is

$$-\left(\frac{m}{l}dx\right)\mathrm{gx}$$

The potential energy of the $l/3$ of the chain that overhangs is

$$U_1 = \int_0^{l/3}\left(-\frac{m}{l}\right)gxdx$$

$$= -\left[\frac{m}{l}g\left(\frac{x^2}{2}\right)\right]_0^{l/3} = -\frac{1}{18}mgl$$

This is also the potential energy of the full chain in the initial position because the part lying on the table has zero potential energy. The potential energy of the chain when it completely slips off the table is

$$U_2 = \int_0^l -\frac{m}{l}gxdx = -\frac{1}{2}mgl$$

The loss in potential energy = $(-\frac{1}{18}mgl) - (-\frac{1}{2}mgl) = \frac{4}{9}mgl$

This should be equal to the gain in the kinetic energy. But the initial kinetic energy is zero. Hence, the kinetic energy of the chain as it completely slips off the table is $\frac{4}{9}mgl$

Q4 A Spherical object with a radius of 5 cm is moving through a viscous fluid. The fluid's viscosity is 0.1 Pa·s, and the object is moving with a constant velocity of 0.02 m/s. Calculate the viscous force acting on the object

To calculate the viscous force acting on the spherical object moving through a viscous fluid, you can use Stokes' law, which describes the viscous force experienced by a sphere moving through a viscous medium at low Reynolds numbers. The formula for viscous force (F_v) is given by:

$$F_v = 6\pi\eta r v$$

Where:

- F_v is the viscous force.
- η (eta) is the dynamic viscosity of the fluid (given as 0.1 Pa·s).
- r is the radius of the sphere (given as 5 cm, or 0.05 m).
- v is the velocity of the sphere relative to the fluid (given as 0.02 m/s).

Now, let's plug in the values and calculate the viscous force:

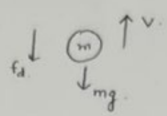
$$F_v = 6\pi(0.1 Pa\cdot s)(0.05 m)(0.02 m/s)$$

Calculate the result:

$$F_v = 6\pi(0.1)(0.05)(0.02)\text{N}$$

So, the viscous force acting on the spherical object moving through the viscous fluid is approximately 0.01884 Newtons.

Q5. A ball is thrown upward with an initial velocity Vo from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where m is the mass of the ball, v is its instantaneous velocity and γ is a constant). Calculate time taken by the ball to rise to its zenith.



$f_d = m\gamma v^2$

$f_{net} = mg + m\gamma v^2$

$ma = mg + m\gamma v^2$

$a = g + \gamma v^2$

But this ball will be deaccelerated.

Therefore, $-a = g + \gamma v^2$

$$-\frac{dv}{dt} = g + \gamma v^2$$

$$\int_{v_0}^{0} \frac{dv}{g + \gamma v^2} = -\int_0^T dt$$

$$\frac{1}{\gamma}\int_{v_0}^{0} \frac{dv}{\frac{g}{\gamma} + v^2} = -T$$

$$\frac{1}{\gamma}\int_{v_0}^{0} \frac{dv}{\left(\sqrt{\frac{g}{\gamma}}\right)^2 + v^2} = -T$$

$$\frac{1}{\gamma}\int_{0}^{v_0} \frac{dv}{\left(\sqrt{\frac{g}{\gamma}}\right)^2 + v^2} = T$$

using formula.

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}(x/a)$$

$$\frac{1}{\gamma}\sqrt{\frac{\gamma}{g}}\tan^{-1}\left(v\sqrt{\frac{\gamma}{g}}\right)\Bigg|_0^{v_0} = T$$

$$\boxed{\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(v_0\sqrt{\frac{\gamma}{g}}\right) = T}$$