PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 30

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Previous Lecture

Kinetic theory of gases - continued
Photon gas

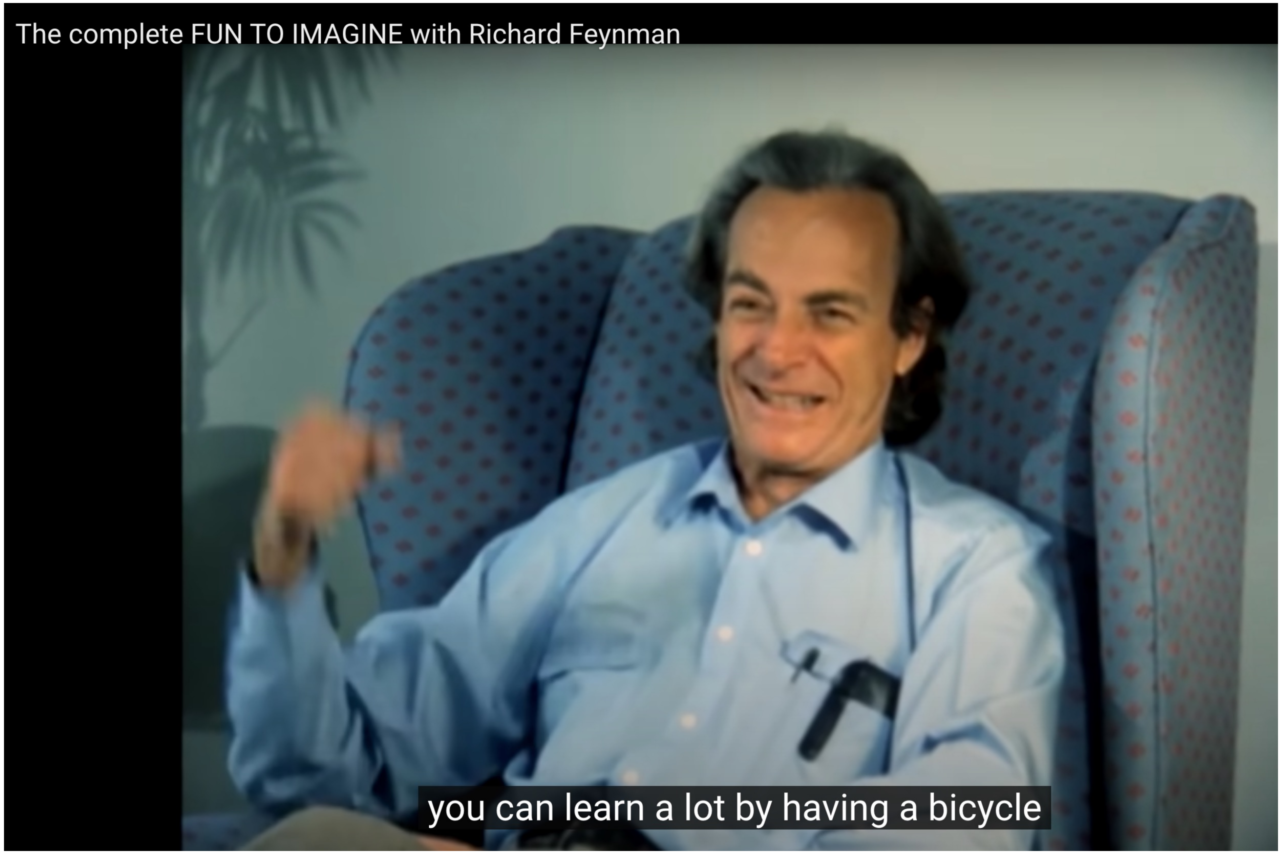
This Lecture

Involvement of temperature
in case of the ideal gas

So far during the lectures we did not talk about the effect of temperature in motion of gas atoms

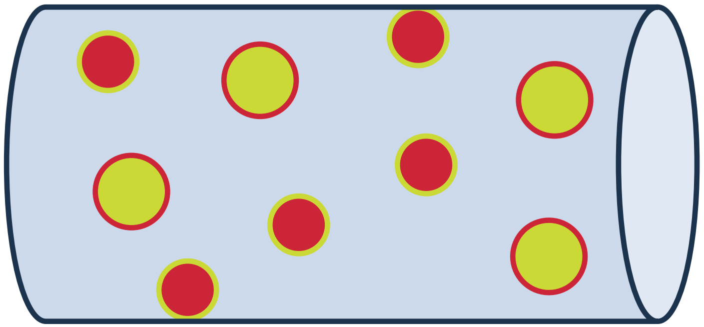
"I'd pump up the tires that the pump would get hot ……as the pump handle comes down and the atoms are coming up against it and bouncing off, it's moving in, the ones that are coming off have a bigger speed than the ones that are coming in so that as it comes down and each time they collide, it speeds them up and so they are hotter. **When you compress the gas it heats**!"

- Richard Feynman (during one of his interviews)

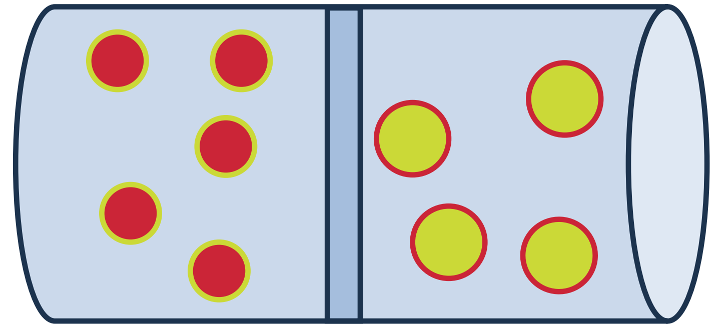


Link: The complete FUN TO IMAGINE with Richard Feynman

The dynamics of monoatomic gases for two different environments



Case 1



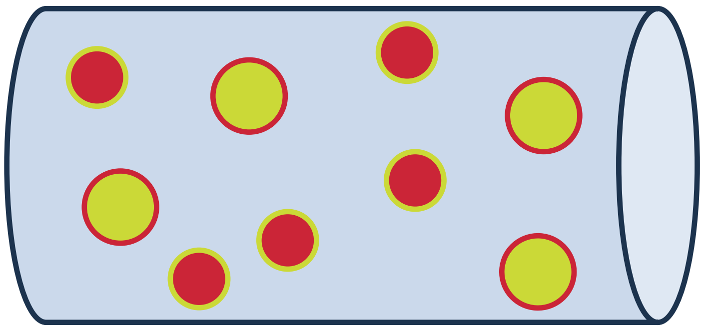
Case 2

- **System**: A single cylinder contains two types of monoatomic gases mixed up
- **Observation:** The average kinetic energies of the two species are equal in equilibrium. This means that on average the heavier ones will move slower than the lighter ones.

- **System**: A single cylinder contains two types of monoatomic gases separated by a piston.
- **Observation**: However, even if we have a situation when two different gases are separated with the aid of a piston, in equilibrium the average kinetic energies of the two gases become equal.

How is energy distribution taking place for both the cases and what is the involvement of temperature?

The dynamics of monoatomic gases mixed together



Case 1

The letters with arrow or with the bold fonts presents vector quantities

- Suppose the mass and velocities of the two types of gas particles are m_1, $\mathbf{v}_1$ and m_2, $\mathbf{v}_2$ respectively.
- The velocities of the center of mass is $\mathbf{V}_{CM}$

$$\vec{V}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

- Here relative velocity in CM frame, $\mathbf{w} = \mathbf{v}_1 - \mathbf{v}_2$

- The process is that the Center of Mass (CM) is moving and in the CM frame there is a relative velocity w, and the molecules collide and come off in some new direction. The CM keeps on moving without any change.

- It is intuitive that there is no correlation between the V_{CM} and w. Their directions are independent, thus the cosine of the angle between them is zero on average.

$$\langle \vec{w} \cdot \vec{V}_{CM} \rangle = 0$$

The dynamics of monoatomic gases and their energies

$$\langle \vec{w} \cdot \vec{V}_{CM} \rangle = 0$$

$$\Rightarrow \left\langle (\vec{v}_1 - \vec{v}_2) \cdot \left(\frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} \right) \right\rangle = 0$$

$$\Rightarrow \langle (\vec{v}_1 - \vec{v}_2) \cdot (m_1\vec{v}_1 + m_2\vec{v}_2) \rangle = 0$$

$$\Rightarrow \langle m_1\vec{v}_1 \cdot \vec{v}_1 + m_2\vec{v}_1 \cdot \vec{v}_2 - m_1\vec{v}_2 \cdot \vec{v}_1 - m_2\vec{v}_2 \cdot \vec{v}_2 \rangle = 0$$

$$\Rightarrow \langle m_1 v_1^2 - m_2 v_2^2 + (m_2 - m_1)\vec{v}_1 \cdot \vec{v}_2 \rangle = 0$$

$$\Rightarrow \langle m_1 v_1^2 \rangle - \langle m_2 v_2^2 \rangle + (m_2 - m_1)\langle \vec{v}_1 \cdot \vec{v}_2 \rangle = 0$$

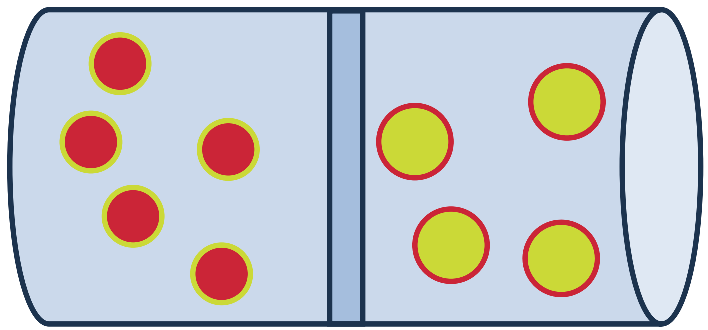
- $\vec{v}_1$ and $\vec{v}_2$ are also uncorrelated, so $\langle \vec{v}_1 \cdot \vec{v}_2 \rangle = 0$ giving,

$$\langle m_1 v_1^2 \rangle - \langle m_2 v_2^2 \rangle = 0$$

$$\Rightarrow \left\langle \frac{1}{2} m_1 v_1^2 \right\rangle = \left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

The average kinetic energies of the two species are equal in equilibrium

The dynamics of physically separated monoatomic gases



Case 2

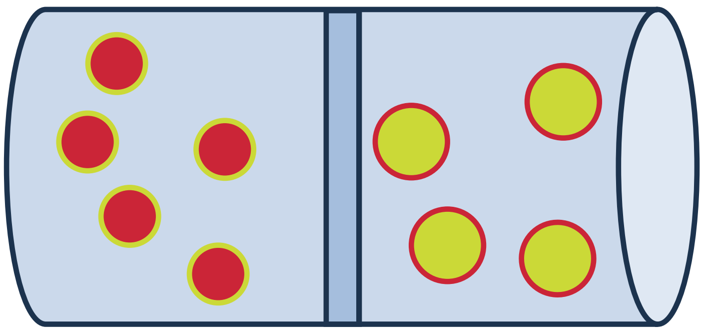
- In the two apartments of the cylinder, let the number of atoms per unit volume be n_1 and n_2 respectively.

- If we leave the system for long enough, it will come to equilibrium: The pressures become equal on both sides of the piston.

$$P = \frac{1}{3} nm\langle v^2 \rangle = \frac{2}{3} n \left\langle \frac{1}{2} mv^2 \right\rangle.$$

$$n_1 \left\langle \frac{1}{2} m_1 v_1^2 \right\rangle = n_2 \left\langle \frac{1}{2} m_2 v_2^2 \right\rangle$$

In previous lectures the relationships between the pressure and the velocity is established. **This equality can be satisfied even if we have large density with small velocities on one side and small density with large velocities on the other side.**



Observation: Even if we have a situation when two different gases are separated in a box with the aid of a piston in equilibrium the average kinetic energies of the two gases become equal.

Justification:

- The energy redistribution takes *via* the piston in this case.
- **We can use this observation to define temperature when we have two gases at the same temperature, the mean kinetic energies are equal**.

The dynamics of monoatomic gases and the role of temperature

- Owing to the direct relationship with the internal energy, we can define the scale of temperature so that it is linearly proportional to the internal energy.
- The relation is

$$U = \frac{3}{2} k_B T. \qquad \textit{(Monatomic case)}$$

- Here $k_B (= 1.38 \times 10^{-23} JK^{-1})$ is the Boltzmann constant. 3/2 is a factor for convenience.

  (*It will have relation to the degrees of freedom of the gas molecule*.)

- The kinetic energy associated with motion in the any particular direction is $\frac{1}{2} k_B T$. Thus for three directions we have $\frac{3}{2} k_B T$.

The dynamics of monoatomic gases and the Ideal gas law

- For a monatomic gas we had derived the following relation

$$PV = \frac{2}{3}N\left\langle \frac{1}{2}mv^2 \right\rangle,$$

where N is the total number of particles.

- We defined the temperature using

$$\left\langle \frac{1}{2}mv^2 \right\rangle = \frac{3}{2}k_B T.$$

- Combining these two we obtain,

$$PV = Nk_B T.$$

- Usually the number of molecules is very large, thus one uses the Avogadro number N_0 $(= 6.02 \times 10^{23}: \mathit{One\ Mole})$ to express it. So, $N = \mathcal{N} N_0$, where $\mathcal{N}$ is the number of moles. Thus

$$PV = \mathcal{N} N_0 k_B T = \mathcal{N} RT$$

- $R = N_0 k_B$ is referred to as the Universal Gas constant.

Numerical problem related to the Ideal gas law

Question: A cylinder contains 12 L of oxygen at 20º C and 15 atm. The temperature is raised to 35º C and the volume is reduced to 8.5 L. What is the final pressure of the gas in the atmosphere? Assume that the gas is ideal gas.

[Book: Fundamentals of Physics (Chapter 20)]

$$PV = \mathcal{N} N_0 k_B T = \mathcal{N} R T$$

Thus, $nR = p_iV_i/T_i = p_fV_f/T_f$

Therefore, $p_f = p_iV_i\, T_f\,/T_i\, V_f$

All the variables in both side of this relation should be converted into one standard unit system

Thus, T_i = (273 + 20) K = 293 K

T_f = (273 +35) K = 308 K

Therefore, p_f = 22 atm.