1. The equation of motion of a damped oscillator is

$$m\,\ddot{x} \;+\; \gamma\,\dot{x} \;+\; m\omega_0^2\,x \;=\; 0 \qquad \textbf{\textit{or}} \qquad \ddot{x} \;+\; \frac{\gamma}{m}\,\dot{x} \;+\; \omega_0^2\,x \;=\; 0\;,$$

which has three classes of solution:

(i) $x(t) = e^{-\gamma\,t/(2m)}\;[C_1\,t + C_2]$

(ii) $x(t) = A_0\,e^{-\gamma\,t/(2m)}\,\sin(\omega\,t + \phi_0)$ *with* $\omega \equiv \sqrt{\omega_0^2 \;-\; \gamma^2/(4m^2)}$

(iii) $x(t) = e^{-\gamma\,t/(2m)}\;[B_1\,e^{q\,t} + B_2\,e^{-q\,t}]$ *with* $q \equiv \sqrt{\gamma^2/(4m^2) \;-\; \omega_0^2}$

(a) What is the physical quantity represented by each term in the equation of motion?

(b) For each of the solutions (i)–(iii): name the type of damping that is described; give the criterion that determines whether the solution applies to a particular system; and sketch a typical $x(t)$ curve illustrating the main features of the motion.

2. A damped oscillator with mass 2 kg has the equation of motion

$$2\,\ddot{x} \;+\; 12\,\dot{x} \;+\; 50\,x \;=\; 0\;,$$

where x is the displacement from equilibrium, measured in metres.

(a) What are the damping constant and the natural angular frequency for this oscillator?

(b) What type of damping is this? Is the motion still oscillatory and periodic? If so, what is the oscillation period?

(c) For what value of the damping constant would this system, if displaced, return as quickly as possible to equilibrium? What would the equation of motion then be?

3. A simple pendulum is made from a ping-pong ball with a mass of 10 grams, attached to a 60-cm length of thread with negligible mass. The force of air resistance on the ball is $F_{\text{air}} = -\gamma\dot{s}$, in which $\gamma = 0.016$ kg s^{-1}.

(a) Show that the pendulum is underdamped. Find the angular frequency ω and the period T of oscillation, and compare to the natural (undamped) ω_0 and T_0.

(b) How long does it take for the amplitude of the pendulum's swing to decrease by a factor of 1000? By what factor does the mechanical energy decrease in this time?

(c) If a pendulum made with the same ping-pong ball were to be critically damped by air resistance, what would its length have to be?