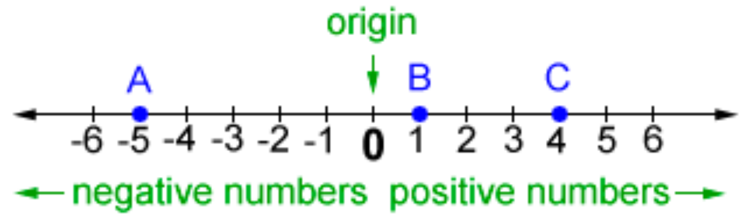
PHY101: Introduction to Physics I

Monsoon Semester 2024

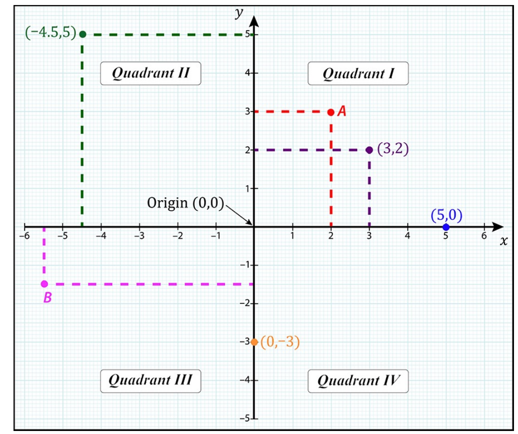
Lecture 3

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

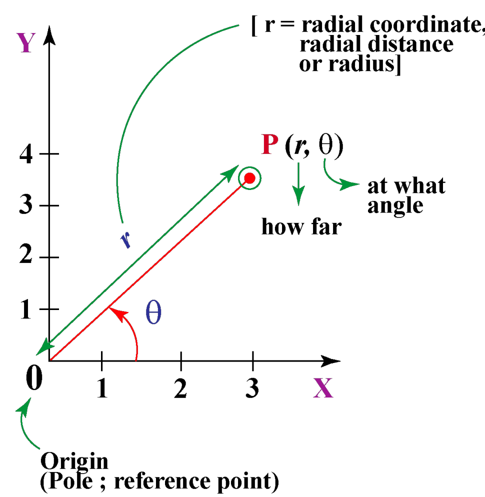


- Co-ordinate systems
- 1D-coordinate system



- 2D-coordinate system

Cartesian Coordinate System



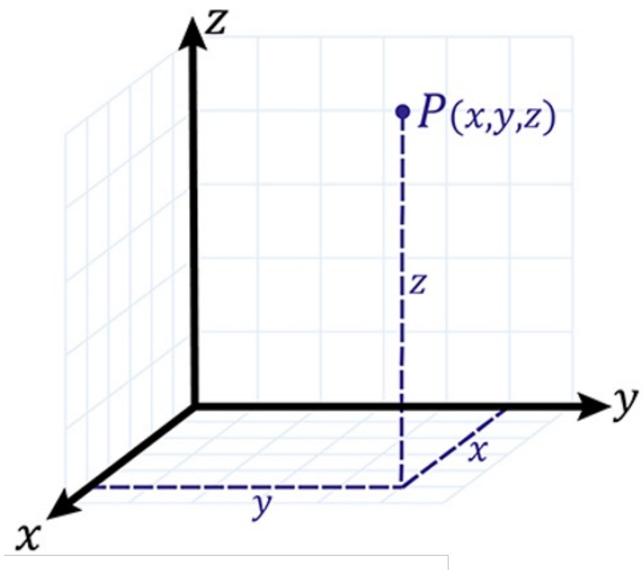
Polar Coordinate System

This Lecture

3D-coordinate system

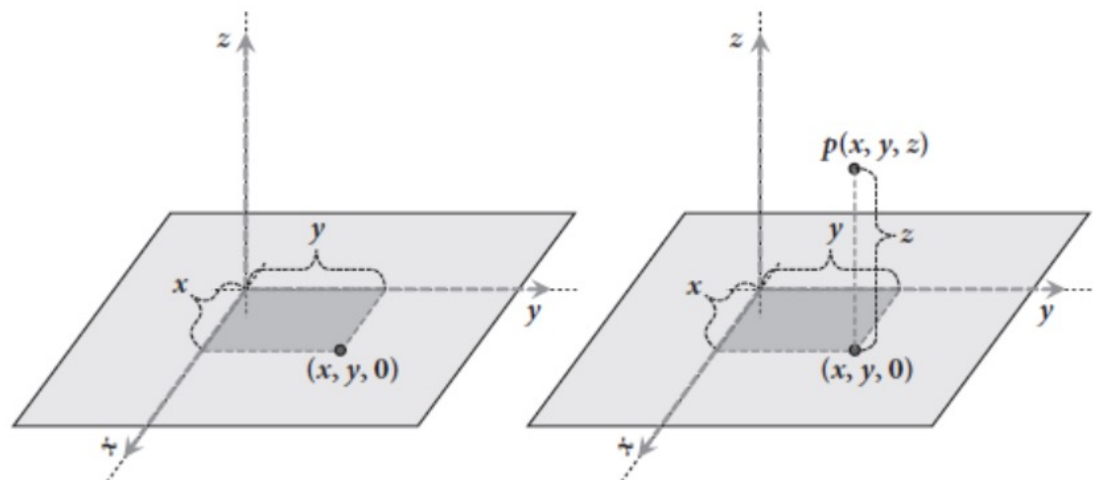
(Cartesian, cylindrical polar and spherical polar)

Coordinate system in 3 Dimension (3D)



Cartesian coordinates system

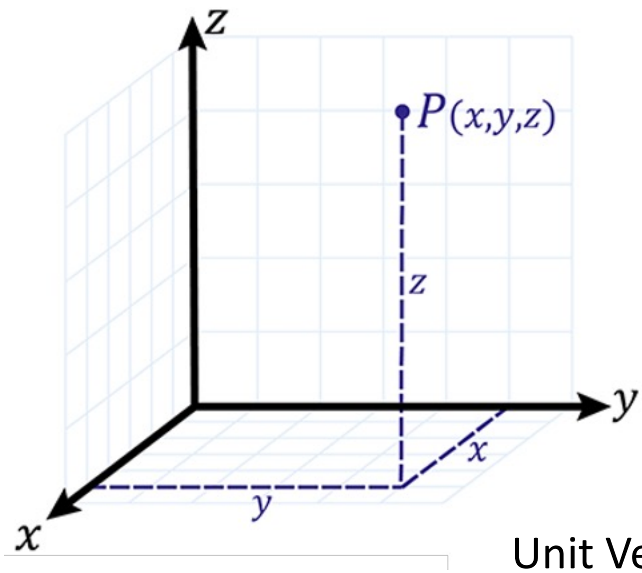
In a three-dimensional Cartesian coordinate system, the position of the point in space needs to be described by three coordinates (x,y,z)



Ref:https://math.libretexts.org/Bookshelves/Calculus/Calculus_%28OpenStax%29/12%3A_Vectors_in_Space/12.02%3A_Vectors_in_Three_Dimensions

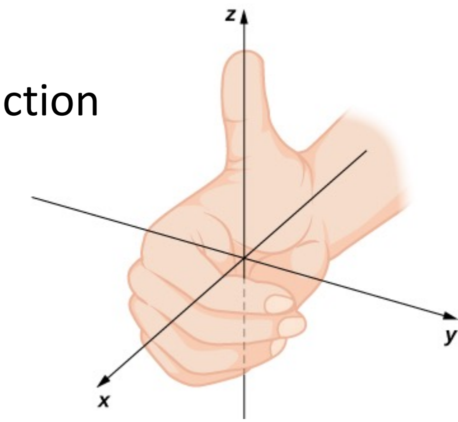
Coordinate system in 3 Dimension (3D)

Cartesian coordinates system

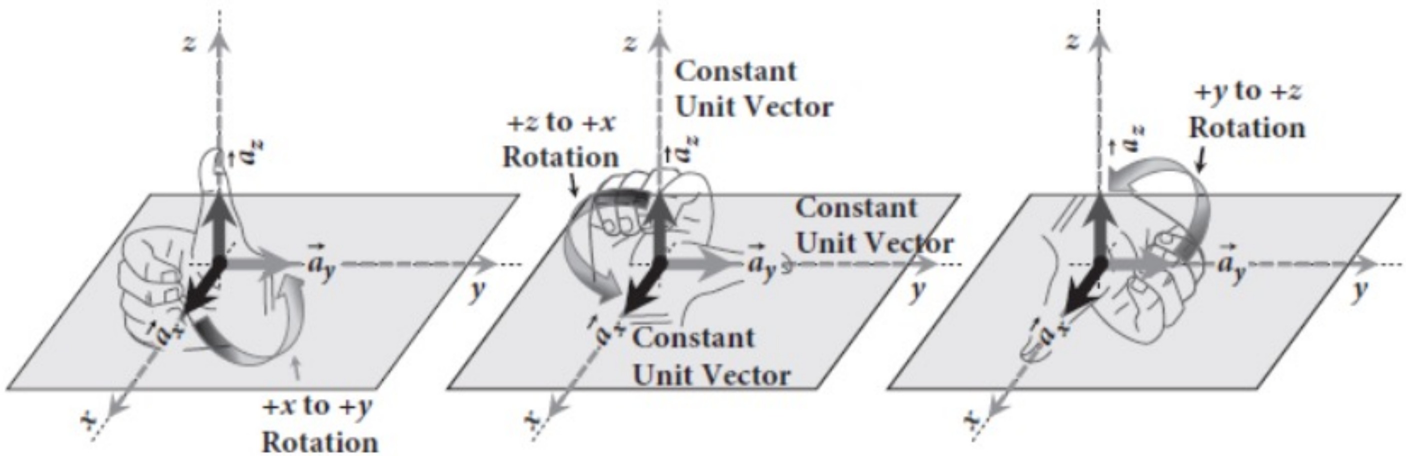


In a three-dimensional Cartesian coordinate system, the position of the point in space needs to be described by three coordinates (x,y,z)

Right-hand approach provides the direction



Unit Vectors ⟶ $(\overrightarrow{a_x}, \overrightarrow{a_y}, \overrightarrow{a_z})$ or $(\hat{\imath}, \hat{\jmath}, \hat{k})$



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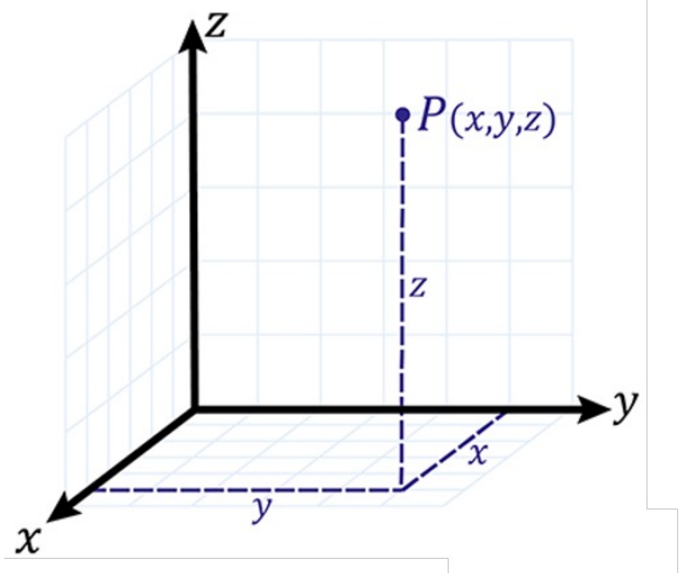
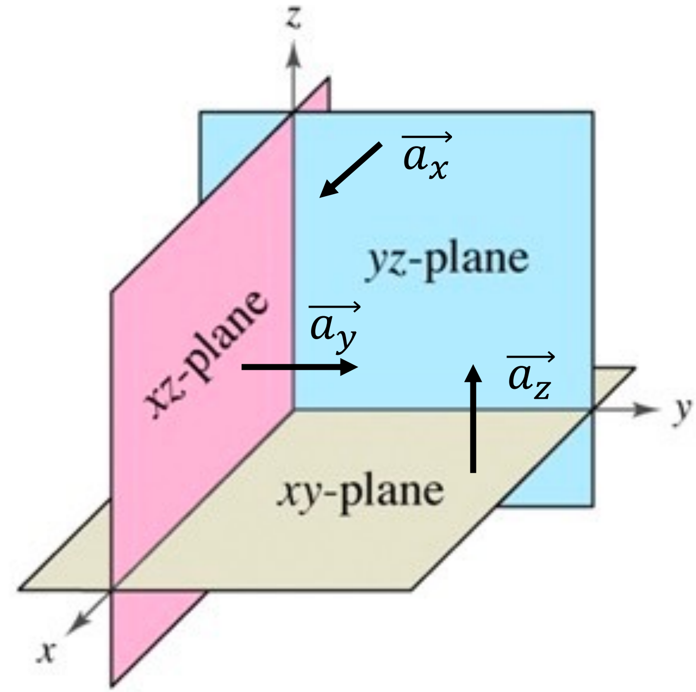
Coordinate system in 3 Dimension (3D)



Cartesian coordinates system

In a three-dimensional Cartesian coordinate system, the position of the point in space needs to be described by three coordinates (x,y,z)



The three unit vectors are **orthonormal**

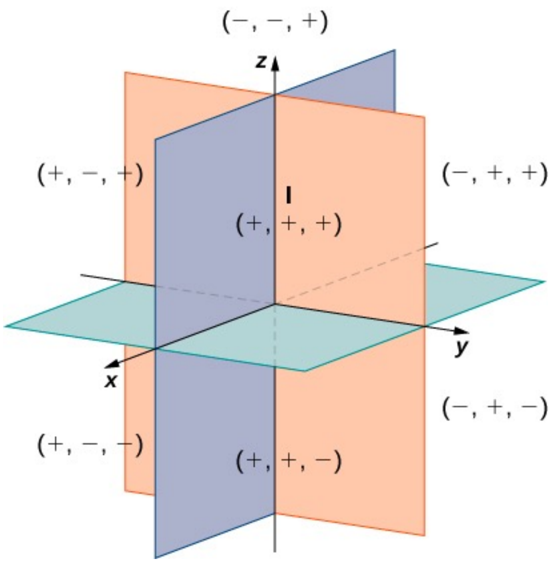
(A set of vectors is orthonormal if every vector in has magnitude 1 and the set of vectors are mutually orthogonal)

Rectangular coordinates / Orthogonal coordinates

Ref:https://math.libretexts.org/Bookshelves/Calculus/Calculus_%28OpenStax%29/12%3A_Vectors_in_Space/12.02%3A_Vectors_in_Three_Dimensions

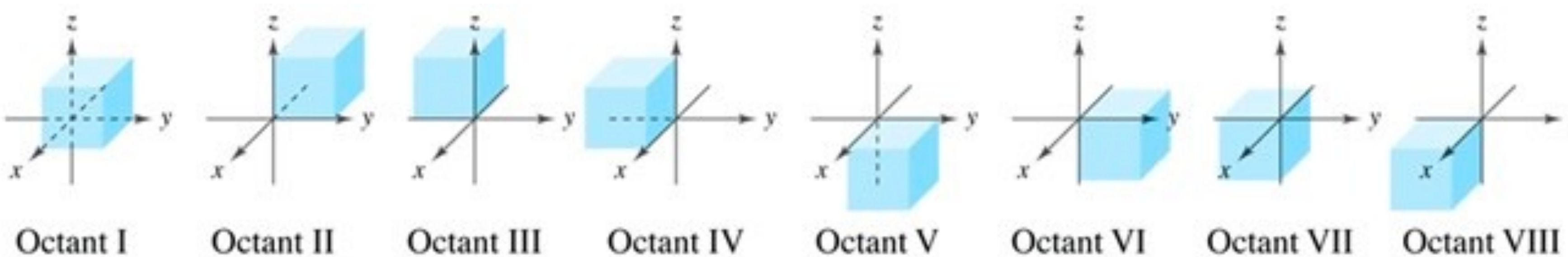
Coordinate system in 3 Dimension (3D)

Octants



Cartesian coordinates system

Octants / Co-ordinates	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−



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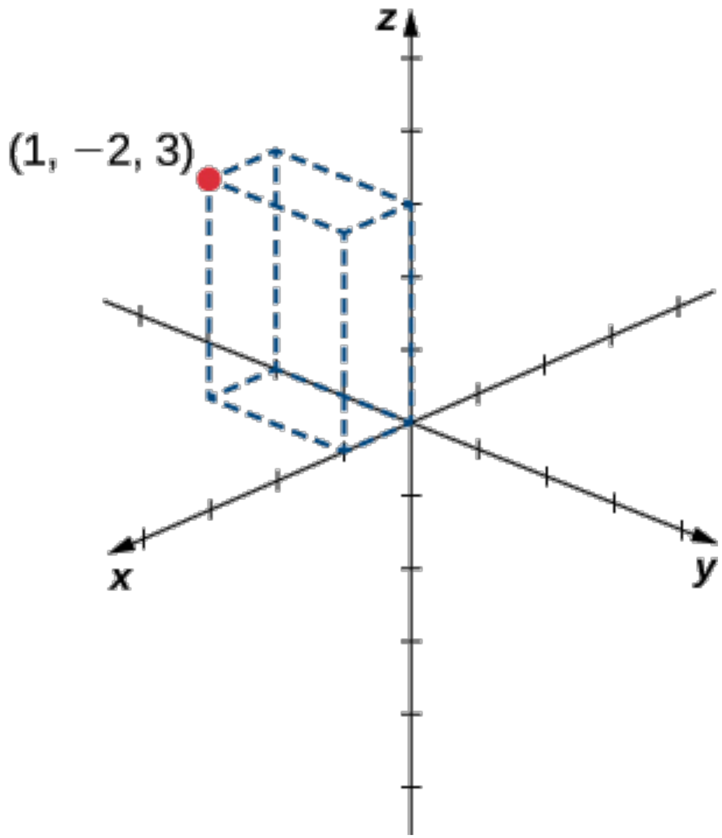
Coordinate system in 3 Dimension (3D)

Cartesian coordinates system

Q1. Sketch the point (1,−2,3) in three-dimensional space.



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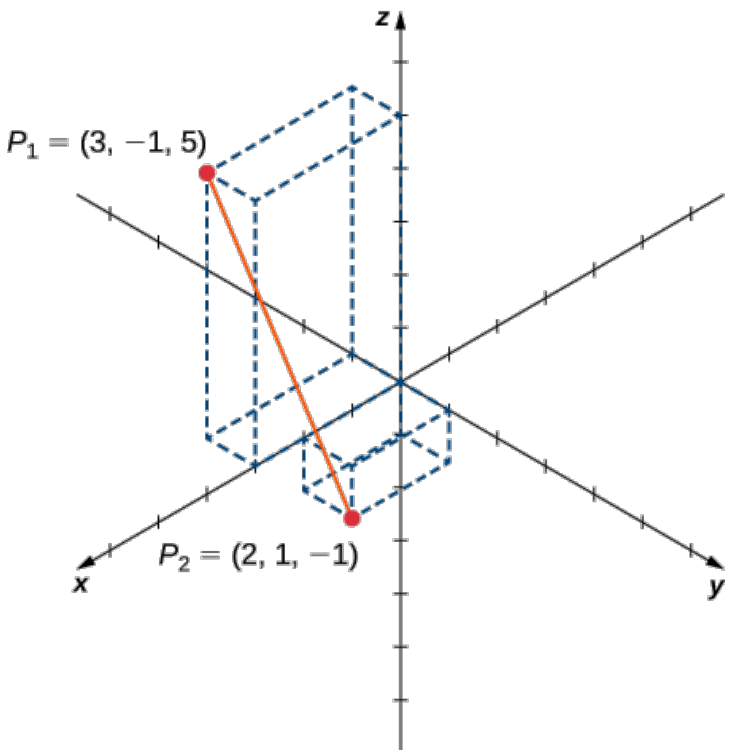
Coordinate system in 3 Dimension (3D)

Cartesian coordinates system

Q2. Find the distance between points P_1=(3,−1,5) and P_2=(2,1,−1).



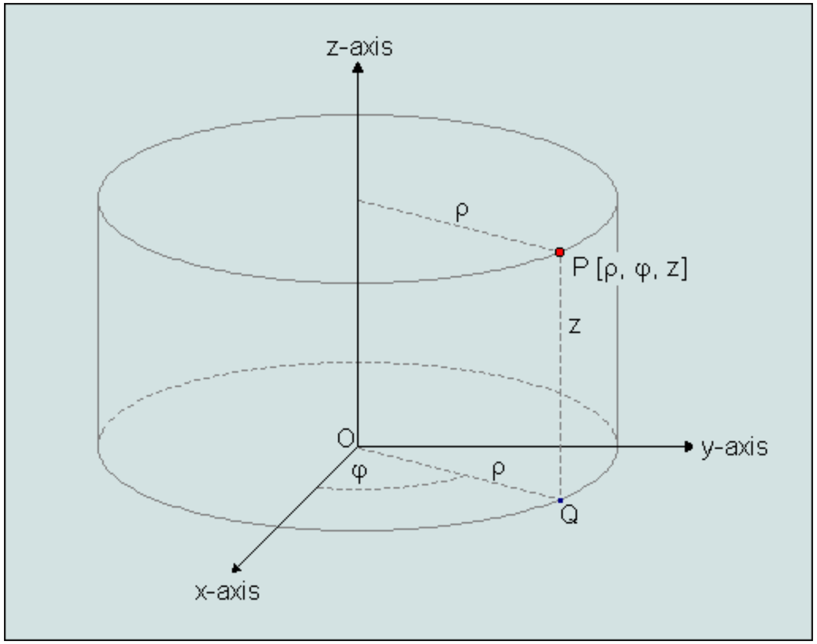
$$\begin{aligned} d(P_1, P_2) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \\ &= \sqrt{(2-3)^2 + (1-(-1))^2 + (-1-5)^2} \\ &= \sqrt{(-1)^2 + 2^2 + (-6)^2} \\ &= \sqrt{41}. \end{aligned}$$

Ref:https://math.libretexts.org/Bookshelves/Calculus/Calculus_%28OpenStax%29/12%3A_Vectors_in_Space/12.02%3A_Vectors_in_Three_Dimensions

Coordinate system in 3 Dimension (3D)

Cylindrical polar coordinates system

Cylindrical coordinates can be thought of as an extension of the polar coordinates.



A point P in the cylindrical coordinate system is represented by three numbers (ρ, φ, z).

$$\rho \rightarrow radius,$$
$$\varphi \rightarrow azhimuthal\ angle$$

to describe the position of the projection of a point onto the $\boldsymbol{xy}$ **plane**

Cylindrical coordinates are useful for describing situations with azimuthal symmetry, such as the motion along the surface of a cylinder.

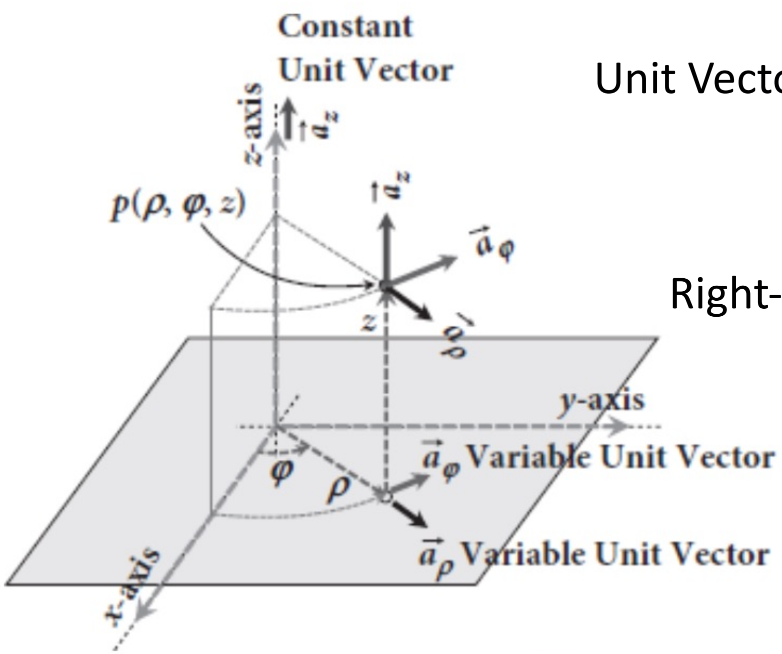
Ranges

$$0 \leq \rho \leq \infty$$
$$0 \leq \varphi \leq 2\pi$$
$$-\infty \leq z \leq \infty$$

Ref: http://www.vias.org/comp_geometry/math_coord_cylinder_3d.htm

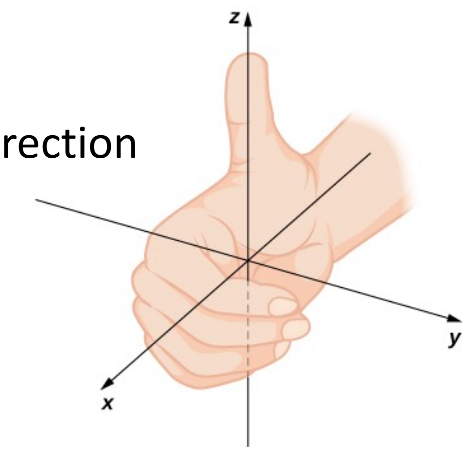
Coordinate system in 3 Dimension (3D)

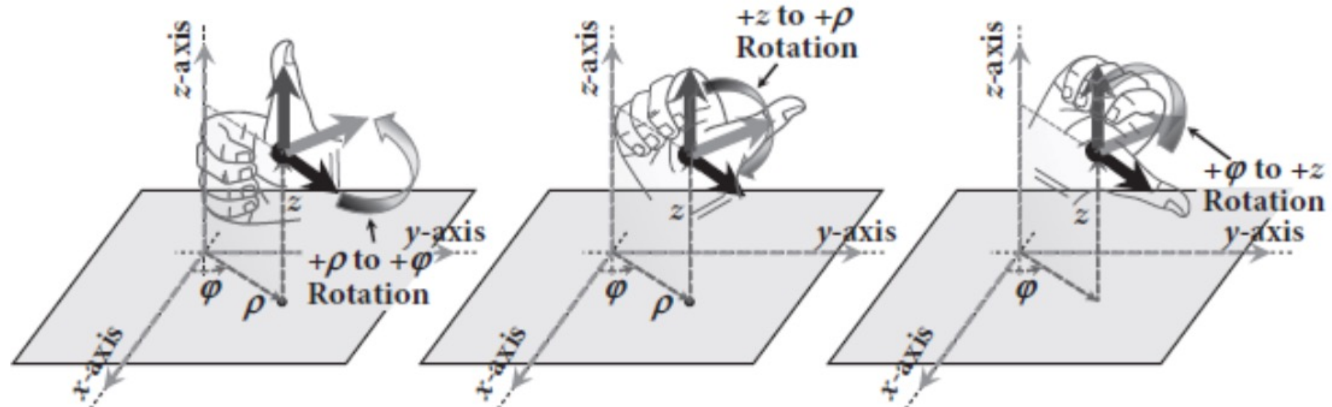
Cylindrical polar coordinates system



Unit Vectors $\longrightarrow$ $(\overrightarrow{a_\rho}, \overrightarrow{a_\varphi}, \overrightarrow{a_z})$ or $(\hat{\rho}, \hat{\varphi}, \hat{z})$

Right-hand approach provides the direction





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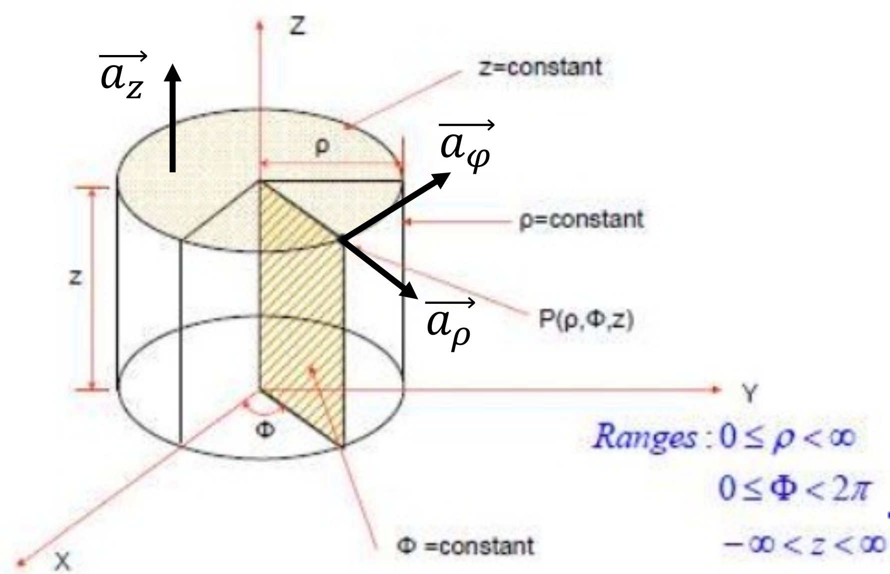
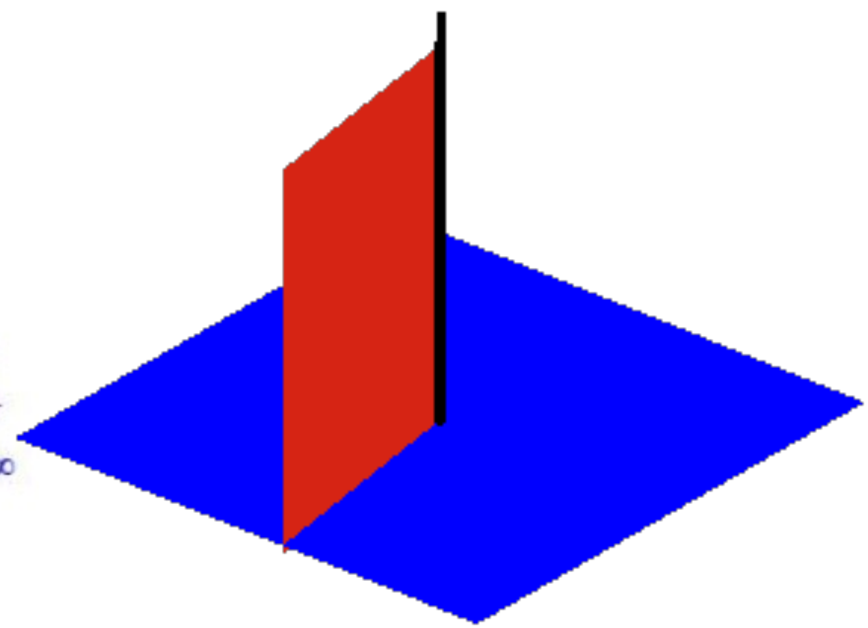
Coordinate system in 3 Dimension (3D)

Cylindrical polar coordinates system





The three unit vectors are **orthonormal**

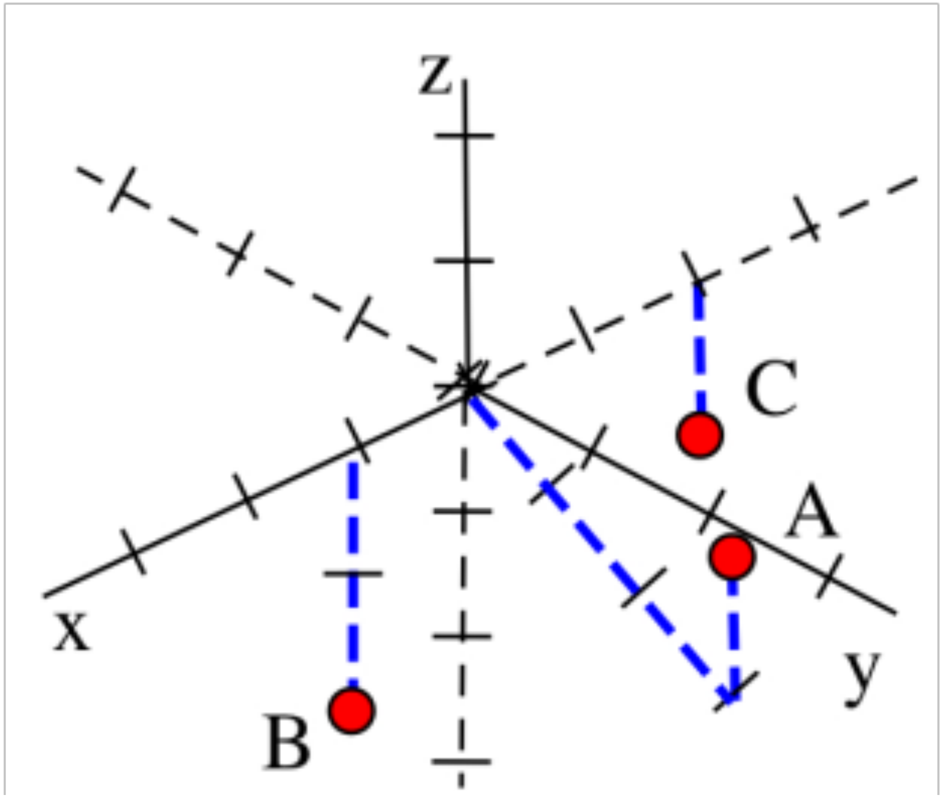
Rectangular coordinates / Orthogonal coordinates

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Cylindrical polar coordinates system

Q1. Plot the points given by the cylindrical coordinates **A**(3, π/3, 1), **B**(1, 0°, –2), and **C**(2, 180°, –1).



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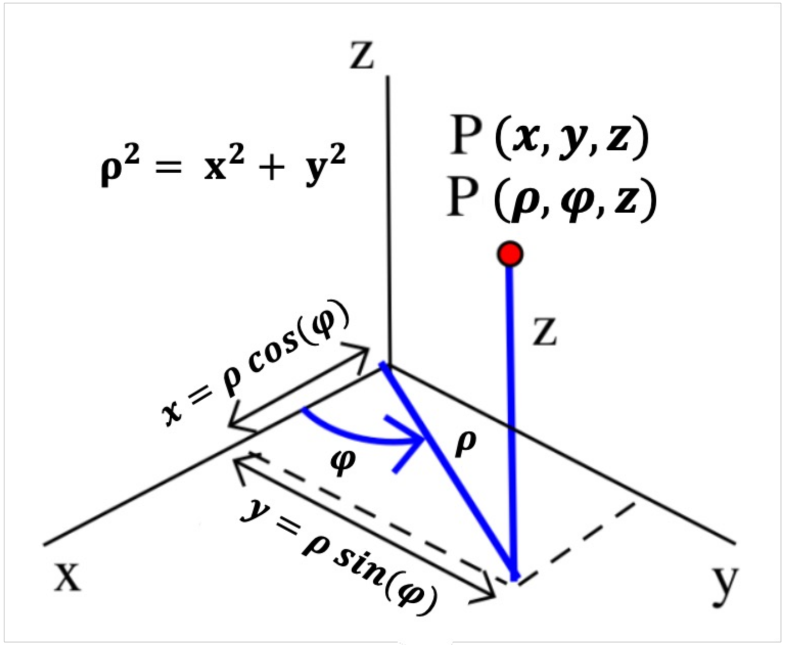
Coordinate system in 3 Dimension (3D)

Conversion Between Cylindrical and Cartesian Coordinates



From Cylindrical to Cartesian
$x = \rho cos\phi$
$y = \rho sin\phi$
$z = z$

From Cartesian to Cylindrical
$\rho = \sqrt{x^2 + y^2}$
$\phi = tan^{-1}\frac{y}{x}$
$z = z$

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Conversion Between Cylindrical and Cartesian Coordinates

Q1. Write the cylindrical coordinate location **A** (2, π/6, 3) in the rectangular coordinate system

Trigonometric Table

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θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

$$\mathbf{x} = \boldsymbol{\rho}\ \mathbf{cos}(\boldsymbol{\varphi}) = \mathbf{2}\ \mathbf{cos}\left(\frac{\pi}{6}\right) = \mathbf{2}\left(\frac{\sqrt{3}}{2}\right) \approx \mathbf{1.73}$$

$$\mathbf{y} = \boldsymbol{\rho}\ \mathbf{sin}(\boldsymbol{\varphi}) = \mathbf{2}\ \mathbf{sin}\left(\frac{\pi}{6}\right) = \mathbf{2}\left(\frac{1}{2}\right) = \mathbf{1.00}$$

The rectangular coordinates of **A** is approximately **(1.73, 1.00, 3.00)**

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Conversion Between Cylindrical and Cartesian Coordinates

Q2. Write the rectangular coordinate location **B** (3, 4, 2) in the cylindrical coordinate system.

$$\boldsymbol{\rho^2 = x^2 + y^2 = 3^2 + 4^2 = 25}$$
$$\boldsymbol{\rho = 5}$$

$$\boldsymbol{\rho^2 = x^2 + y^2}$$

$$\boldsymbol{\tan\varphi = \frac{y}{x} = \frac{4}{3}}$$

$$\boldsymbol{\tan\varphi = \frac{y}{x}}$$

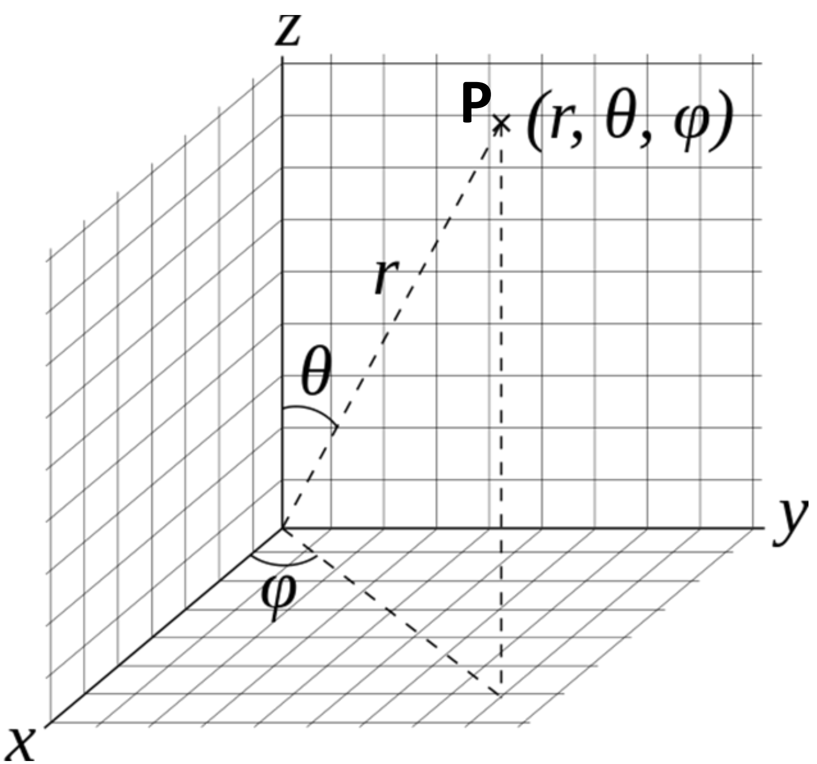
$$\boldsymbol{\varphi = tan^{-1}\left(\frac{4}{3}\right)} = (53.13^{\circ})$$

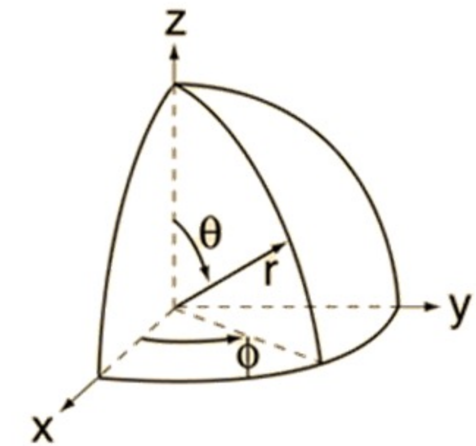
The cylindrical coordinates of **B** is approximately **(5, 53.13° , 2)**

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Spherical coordinates system





In spherical coordinates, a point P is described by the radius $\boldsymbol{r}$, the *polar angle* $\boldsymbol{\theta}$, and the *azimuthal angle* $\boldsymbol{\phi}$.

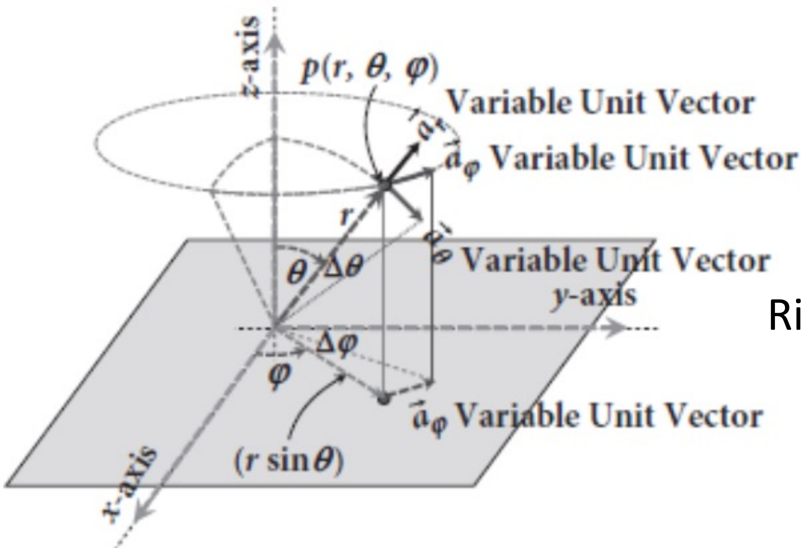
Ranges

$$0 \leq r \leq \infty$$
$$0 \leq \theta \leq \pi$$
$$0 \leq \varphi \leq 2\pi$$

Ref: http://en.wikipedia.org/wiki/File:Coord_system_CA_0.svg, http://upload.wikimedia.org/wikipedia/commons/thumb/4/4f/3D_Spherical.svg/558px-3D_Spherical.svg.png

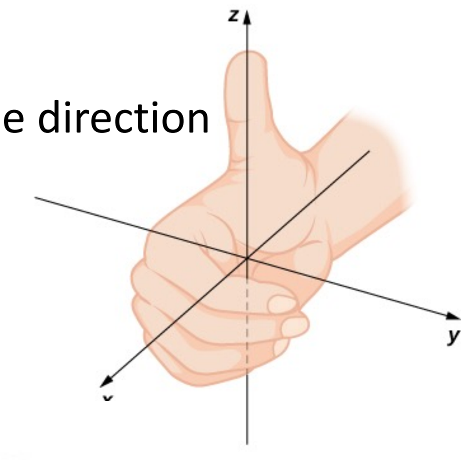
Coordinate system in 3 Dimension (3D)

Spherical coordinates system

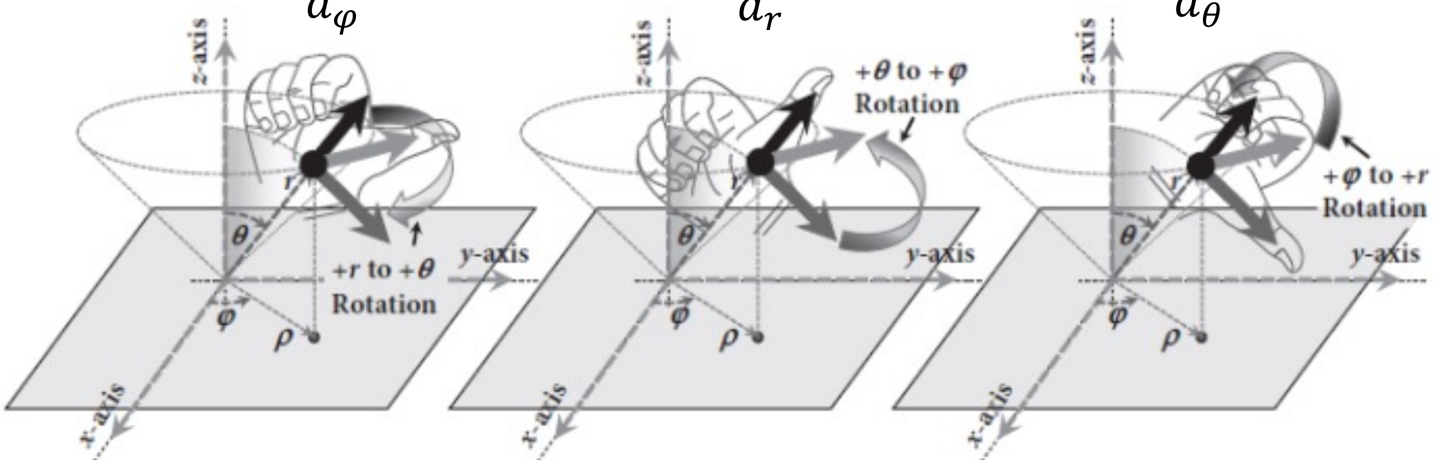


Unit Vectors ⟶ $(\overrightarrow{a_r}, \overrightarrow{a_\theta}, \overrightarrow{a_\varphi})$or $(\hat{r}, \hat{\theta}, \hat{\varphi})$

Right-hand approach provides the direction



$\overrightarrow{a_\varphi}$ $\overrightarrow{a_r}$ $\overrightarrow{a_\theta}$



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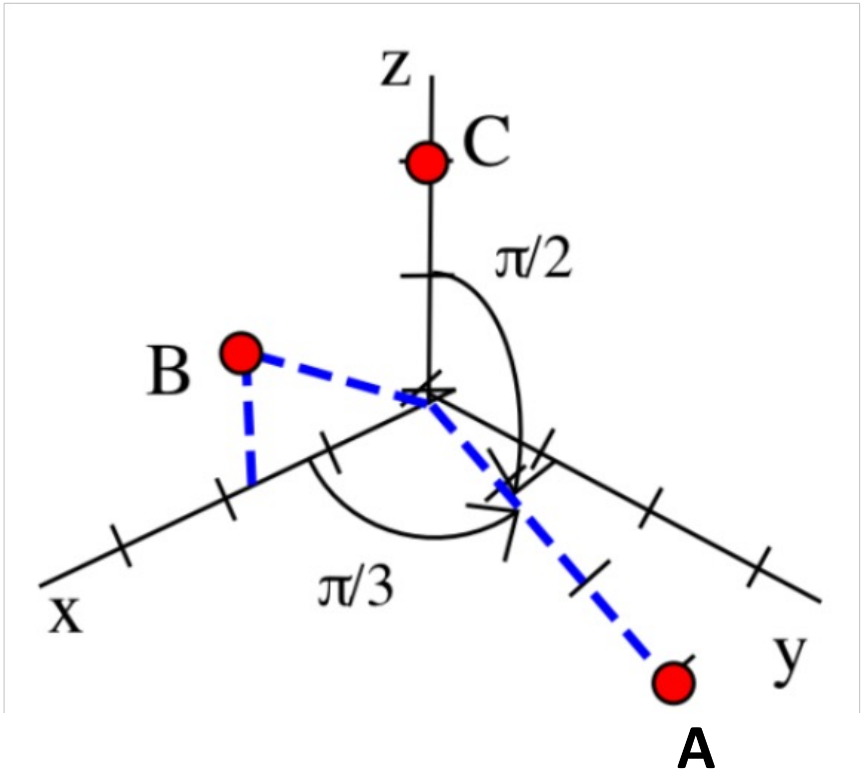
Coordinate system in 3 Dimension (3D)

Cylindrical coordinates system

Q1. Plot the points given by the spherical coordinates **A**(3, π/2, π/3), **B**(2, π/3, 0), and **C**(2, 0º, 0º).



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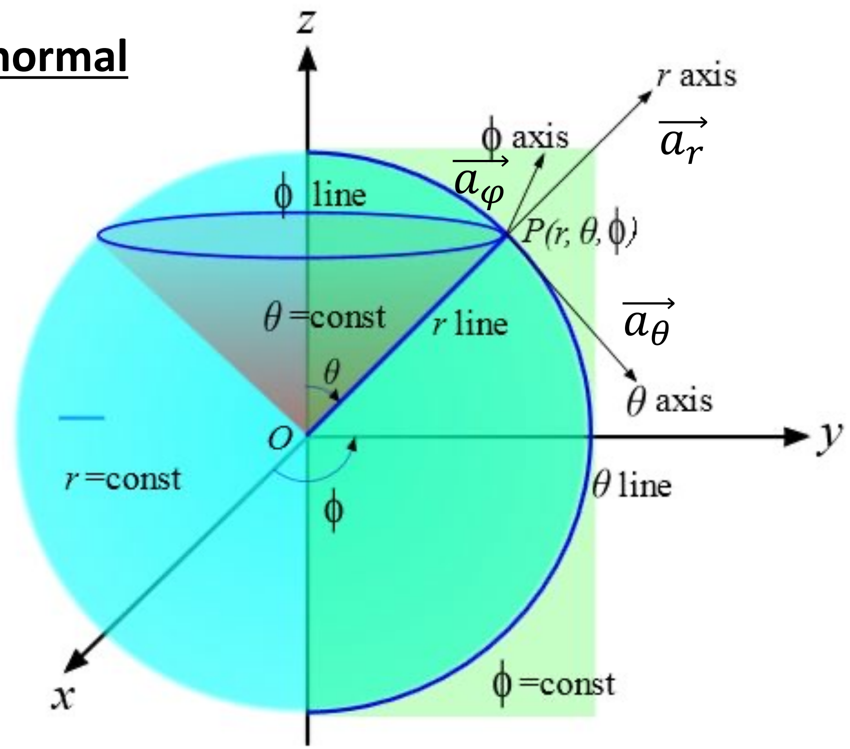
Coordinate system in 3 Dimension (3D)

Spherical coordinates system

The three unit vectors are **orthonormal**



Rectangular coordinates / Orthogonal coordinates

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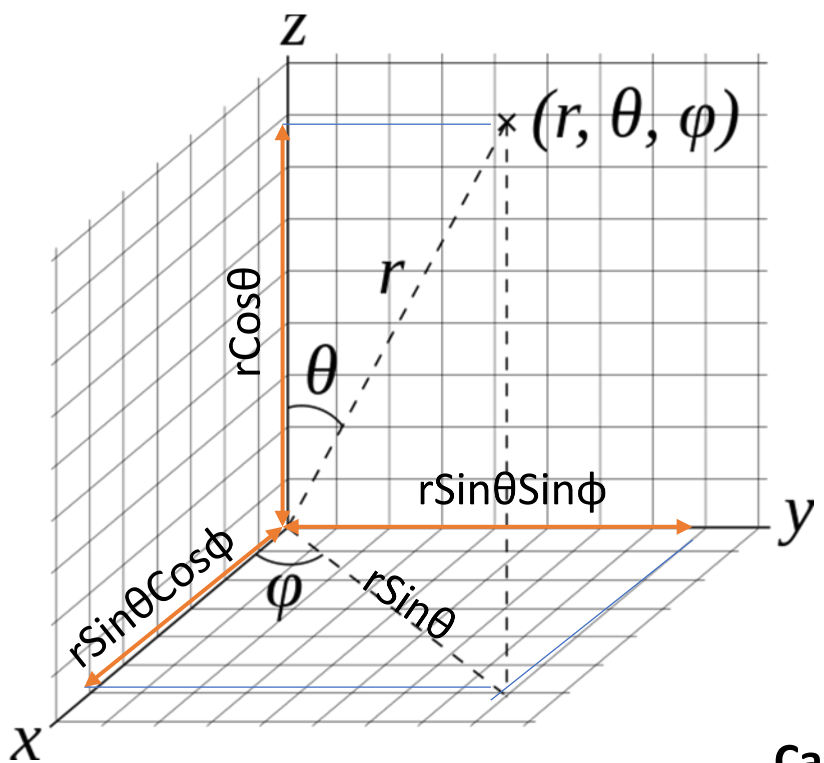
Coordinate system in 3 Dimension (3D)

Conversion Between Spherical and Cartesian Coordinates



Spherical to Cartesian

$$x = r \sin\theta \cos\phi$$

$$y = r \sin\theta \sin\phi$$

$$z = r \cos\theta$$

Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$

$$\phi = \tan^{-1}(y/x)$$

Ref: http://en.wikipedia.org/wiki/File:Coord_system_CA_0.svg, http://upload.wikimedia.org/wikipedia/commons/thumb/4/4f/3D_Spherical.svg/558px-3D_Spherical.svg.png

Coordinate system in 3 Dimension (3D)

Conversion Between Spherical and Cartesian Coordinates

Q1. Write the rectangular coordinate location **A** (3, 6, 2) in the spherical coordinate system.

$$(x, y, z) \rightarrow (r, \theta, \varphi)$$

$$r^2 = x^2 + y^2 + z^2$$

$$\tan\varphi = \frac{y}{x}$$

$$z = r\cos(\theta)$$

$$r^2 = x^2 + y^2 + z^2 = 3^2 + 6^2 + 2^2 = 49$$

$$r = 7$$

$$\tan\varphi = \frac{y}{x} = \frac{6}{3} = 2$$

$$\varphi = tan^{-1}(2) \approx (63.4^{\circ})$$

$$\theta = cos^{-1}\left(\frac{z}{r}\right) \approx (73.4^{\circ})$$

The cylindrical coordinates of **A** is approximately $\mathbf{(5, 73.4^{\circ}, 63.4^{\circ})}$

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Conversion Between Spherical and Cartesian Coordinates

Q2. Write the spherical coordinate location **B**(2, π/6, π/4) in the rectangular coordinate system.

$$\mathbf{x} = 2 \sin\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{4}\right) = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\mathbf{y} = 2 \sin\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{4}\right) = 2 \times \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\mathbf{z} = 2 \cos\left(\frac{\pi}{6}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$(r, \theta, \varphi) \rightarrow (x, y, z)$$

$$\mathbf{x} = \mathbf{r} \sin(\theta) \cos(\varphi)$$

$$\mathbf{y} = \mathbf{r} \sin(\theta) \sin(\varphi)$$

$$\mathbf{z} = \mathbf{r} \cos(\theta)$$

Trigonometric Table

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θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

The cylindrical coordinates of **B** is approximately $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{3}\right)$

Ref: https://en.wikipedia.org/wiki/Cylindrical_coordinate_system

Coordinate system in 3 Dimension (3D)

Conversion Between Cartesian, Cylindrical, and Spherical Coordinates

Cylindrical–Cartesian	Spherical–Cartesian	Cylindrical–Spherical
$\rho = \sqrt{x^2+y^2}$ $\varphi = \tan^{-1}(y/x)$ $z = z$	$r = \sqrt{x^2+y^2+z^2}$ $\theta = \cos^{-1}(z/r)$ $\varphi = \tan^{-1}(y/x)$	$\rho = r\sin\theta$ $\varphi = \varphi$ $z = r\cos\theta$
$x = \rho\cos\varphi$ $y = \rho\sin\varphi$ $z = z$	$x = r\sin\theta\cos\varphi$ $y = r\sin\theta\sin\varphi$ $z = r\cos\theta$	$r = \sqrt{\rho^2+z^2}$ $\theta = \cos^{-1}(z/r)$ $\varphi = \varphi$

Ref: http://en.wikipedia.org/wiki/File:Coord_system_CA_0.svg, http://upload.wikimedia.org/wikipedia/commons/thumb/4/4f/3D_Spherical.svg/558px-3D_Spherical.svg.png

Next lecture

Incremental length, surface, and volume element

Scalars and Vectors