Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 8

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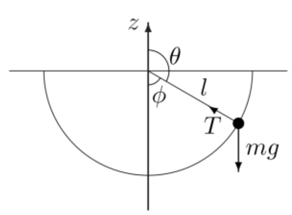
1. The statement that “the induced dipole moment of an atom is proportional to the external field” is a “rule of thumb,” and not a fundamental law --- it is easy to concoct exceptions in theory. Suppose, for example, the charge density of the electron cloud were proportional to the distance from the center, out to a radius R. To what power of E = |**E**| would p = |**p**| be proportional in that case? Here, **E** is the external electric field and **p** is the induced dipole moment.

Solution:

$\rho(r) = Ar$. Electric field (by Gauss's Law): $\oint \mathbf{E}\cdot d\mathbf{a} = E\left(4\pi r^2\right) = \frac{1}{\epsilon_0} Q_{\text{enc}} = \frac{1}{\epsilon_0}\int_0^r A\bar{r}\, 4\pi\bar{r}^2\, d\bar{r}$, or $E = \dfrac{1}{4\pi r^2}\dfrac{4\pi A}{\epsilon_0}\dfrac{r^4}{4} = \dfrac{Ar^2}{4\epsilon_0}$. This “internal” field balances the external field $\mathbf{E}$ when nucleus is “off-center” an amount d: $ad^2/4\epsilon_0 = E \;\Rightarrow\; d = \sqrt{4\epsilon_0 E/A}$. So the induced dipole moment is $p = ed = 2e\sqrt{\epsilon_0/A}\sqrt{E}$. Evidently $\boxed{p \text{ is proportional to } E^{1/2}.}$

2. An ideal electric dipole is situated at the origin, and points in the *z* direction. An electric charge is released from rest at a point in the *xy* plane. Show that it swings back and forth in a semi-circular arc, as though it were a pendulum supported at the origin.



$$\mathbf{F} = q\mathbf{E} = \frac{qp}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$

Now consider the pendulum: $\mathbf{F} = -mg\,\hat{\mathbf{z}} - T\,\hat{\mathbf{r}}$, where $T - mg\cos\phi = mv^2/l$ and (by conservation of energy) $mgl\cos\phi = (1/2)mv^2 \Rightarrow v^2 = 2gl\cos\phi$ (assuming it started from rest at $\phi = 90°$, as stipulated). But $\cos\phi = -\cos\theta$, so $T = mg(-\cos\theta) + (m/l)(-2gl\cos\theta) = -3mg\cos\theta$, and hence

$$\mathbf{F} = -mg(\cos\theta\,\hat{\mathbf{r}} - \sin\theta\,\hat{\boldsymbol{\theta}}) + 3mg\cos\theta\,\hat{\mathbf{r}} = mg(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$

This total force is such as to keep the pendulum on a circular arc, and it is identical to the force on q in the field of a dipole, with $mg \leftrightarrow qp/4\pi\epsilon_0 l^3$. Evidently q also executes semicircular motion, as though it were on a tether of fixed length l.

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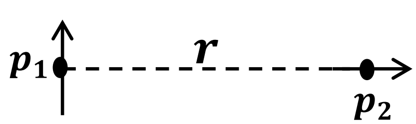
3. $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ are (perfect) dipoles a distance r apart. What is the torque on p_1 due to p_2? What is the torque on $\boldsymbol{p_2}$ due to $\boldsymbol{p_1}$? In each case, torque refers to the torque on each dipole at its center.



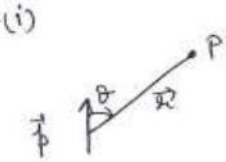
1.

$\vec{p}_1 \uparrow - - - \overset{r}{-} - - - \rightarrow \vec{p}_2$

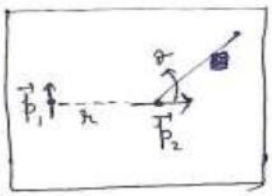
<u>To calculate torque on $\vec{p}_1$ due to $\vec{p}_2$:</u>

Field due to a dipole aligned along z-axis is given by

$$E = \frac{p}{4\pi\epsilon_0 r^3}\left[2\cos\theta\,(\hat{r}) + \sin\theta\,(\hat{\theta})\right] \quad \text{—(i)}$$



Therefore $\vec{E}_{\vec{p}_2}$ (at pos. of $\vec{p}_1$) =



$$= \frac{2p_2\cos\pi}{4\pi\epsilon_0 r^3}(\hat{r}) + \frac{p_2\sin\pi}{4\pi\epsilon_0 r^3}\hat{\theta}$$

$$= \frac{2p_2}{4\pi\epsilon_0 r^3}(-\hat{r}) \quad \text{(pointing along right)}$$

Torque at $\vec{p}_1$, $\vec{N}_{p_1} = \vec{p}_1 \times \vec{E}_{p_2} = \frac{2p_1p_2}{4\pi\epsilon_0 r^3}$ (pointing into the page)

<u>To calculate torque on $\vec{p}_2$ due to $\vec{p}_1$:</u>

$\vec{E}_{\vec{p}_1}$ due to $\vec{p}_1$ (at pos. of $\vec{p}_2$) $= \frac{p_1(\sin\pi/2)\,\hat{\theta}}{4\pi\epsilon_0 r^3}$ [according to (i) with $\theta = \pi/2$]

$$= \frac{p_1(\hat{\theta})}{4\pi\epsilon_0 r^3}$$

(pointing downwards at pos. of $\vec{p}_2$)

Torque $\vec{N}_{p_2}$ experienced by $\vec{p}_2$

$$= \vec{p}_2 \times \vec{E}_{p_1} = \frac{p_1p_2}{4\pi\epsilon_0 r^3} \quad \text{(pointing into the page)}$$

[illegible]

4. A "pure" dipole p is situated at the origin, pointing in the z-direction.
(i) What is the force on a point charge q at $(a, 0, 0)$ (Cartesian coordinates)
(ii) What is the force on q at $(0, 0, a)$?
(iii) How much work does it take to move q from $(a, 0, 0)$ to $(0, 0, a)$?

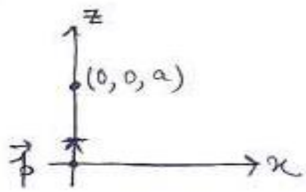
Thus at $(a, 0, 0)$, field is $\vec{E} = \dfrac{-p}{4\pi\epsilon_0 a^3}(\hat{z})$

Force on a charge particle is $q\vec{E}$

$$= \frac{-pq}{4\pi\epsilon_0 a^3}(\hat{z})$$

(b) Force on q at $(0, 0, a)$

$\vec{E}$ due to $\vec{p}$ at $(0, 0, a)$ given by



(i) with $\theta = 0$

$$\vec{E} = \frac{2p}{4\pi\epsilon_0 a^3}(\hat{r}) \text{ . Here } \hat{r} = \hat{z}$$

$$= \frac{2p}{4\pi\epsilon_0 a^3}(\hat{z}) \qquad \therefore \boxed{\vec{F} = \frac{2pq}{4\pi\epsilon_0 a^3}\hat{z}}$$

(c) Work it takes to move q from $(a, 0, 0)$ to $(0, 0, a)$

$$= q\left[V(0, 0, a) - V(a, 0, 0)\right]$$

But $V = \dfrac{\vec{p}\cdot\hat{r}}{4\pi\epsilon_0 r^2}$ [potential at $\vec{r}$ due to $\vec{p}$ at origin]

$$= \frac{p\cos\theta}{4\pi\epsilon_0 r^2} \qquad V(0, 0, a) \rightarrow \theta = 0$$
$$V(a, 0, 0) \rightarrow \theta = \pi/2$$

$$W = q\left[\frac{p}{4\pi\epsilon_0 a^2} - 0\right] = \frac{pq}{4\pi\epsilon_0 a^2}$$

3. We have to find an approximate potential ... from the origin