

SNU - MAT103 - Monsoon 2024  
MidSem Exam: Sept 28th, 10:00 -11:30 pm  
5 problems, 10 marks each, total: 50 marks

### Instructions

All instructions received from the controller of examinations must be followed. In addition to that you must follow these.

- Write your name on the question paper as well.
- Write your section number (T1, T2, etc.) on the top right corner of your answer sheet. If you miss this step, **you will loose 2 marks**.
- **Bathroom breaks:** One student from the room at a time, and a maximum of two minutes of break allowed. The student will hand over the answer sheet to the invigilator before leaving the room. The invigilator will note down the IN and OUT time of the student on the answer sheet. Time spent out more than two minutes **will attract penalty of 2 marks for every extra minutes stayed out**.

Version: B

1. Sketch the domain of the function  $f(x, y) = \sqrt{x-3} + \sqrt{y^2-3}$ . And evaluate and expression for  $\frac{\partial f}{\partial y}$ .
2. Suppose we are in 2d plane. The temperature at any point  $(x, y)$  is given by the formula  $T(x, y) = \frac{1}{1+x^2+y^2}$ . Imagine a bug is crawling on this plane. The bug's position at time  $t$  is  $(x, y) = (3 + \cos t, 2 + \sin t)$ . Find the rate the change of temperature observed by the bug as it is passing through the point  $(3, 3)$ .
3. (a) Given a function  $f(x, y)$ , a point  $P = (a, b)$ , and a direction vector  $\hat{v}$ , what is the meaning of Directional Derivative of  $f$  at point  $P$  in the direction of  $\hat{v}$ ? And what is the working formula to evaluate this directional derivative?  
(b) Suppose we are a function  $f(x, y)$ , a point  $P = (a, b)$ . Among all possible directions, in which direction is the directional derivative maximum? Why?  
(c) Suppose  $f(x, y) = 200 + 0.02x^2 - 0.001y^3$ . If you start at point  $(80, 60)$  and start moving in a straight line towards the origin  $(0, 0)$ , what will be the observed rate of change of  $f$ ? Is  $f$  increasing or decreasing? If we wanted to move in the direction in which the function value increases fastest, which direction we would choose? Sketch this direction.
4. (a) Given two vectors  $\mathbf{u}$  and  $\mathbf{v}$ , explain the meaning of **scalar projection** and **vector projection** of  $\mathbf{u}$  onto  $\mathbf{v}$ . Then evaluate these projections for  

$$\mathbf{u} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}, \quad \mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k}.$$
  
(b) Write an integral expression which represents the length the space curve given by the vector function. No need to evaluate this integration.  

$$\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}, \quad 0 \leq t \leq 1.$$
5. Suppose we have two resistors  $R_1$  and  $R_2$  in parallel. Then the total resistance is given by the formula  

$$R = \frac{R_1 R_2}{R_1 + R_2}$$
  
The resistances are measured in ohms as  $R_1 = 25\Omega$ , and  $R_2 = 40\Omega$ , with a possible error of 1% in each case. **Using differentials**, estimate the error in the calculated value of total resistance  $R$ .