PHY101: Introduction to Physics I

Monsoon Semester 2024
Lecture 11

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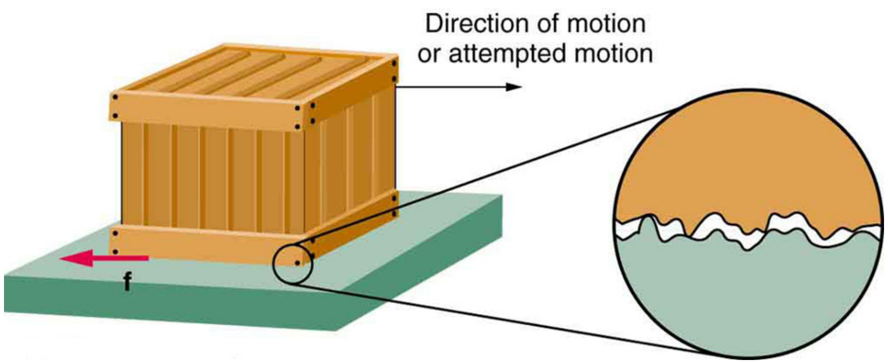
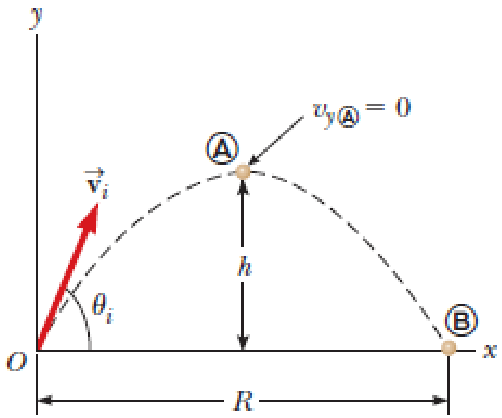
Previous Lecture

Contact force
Friction



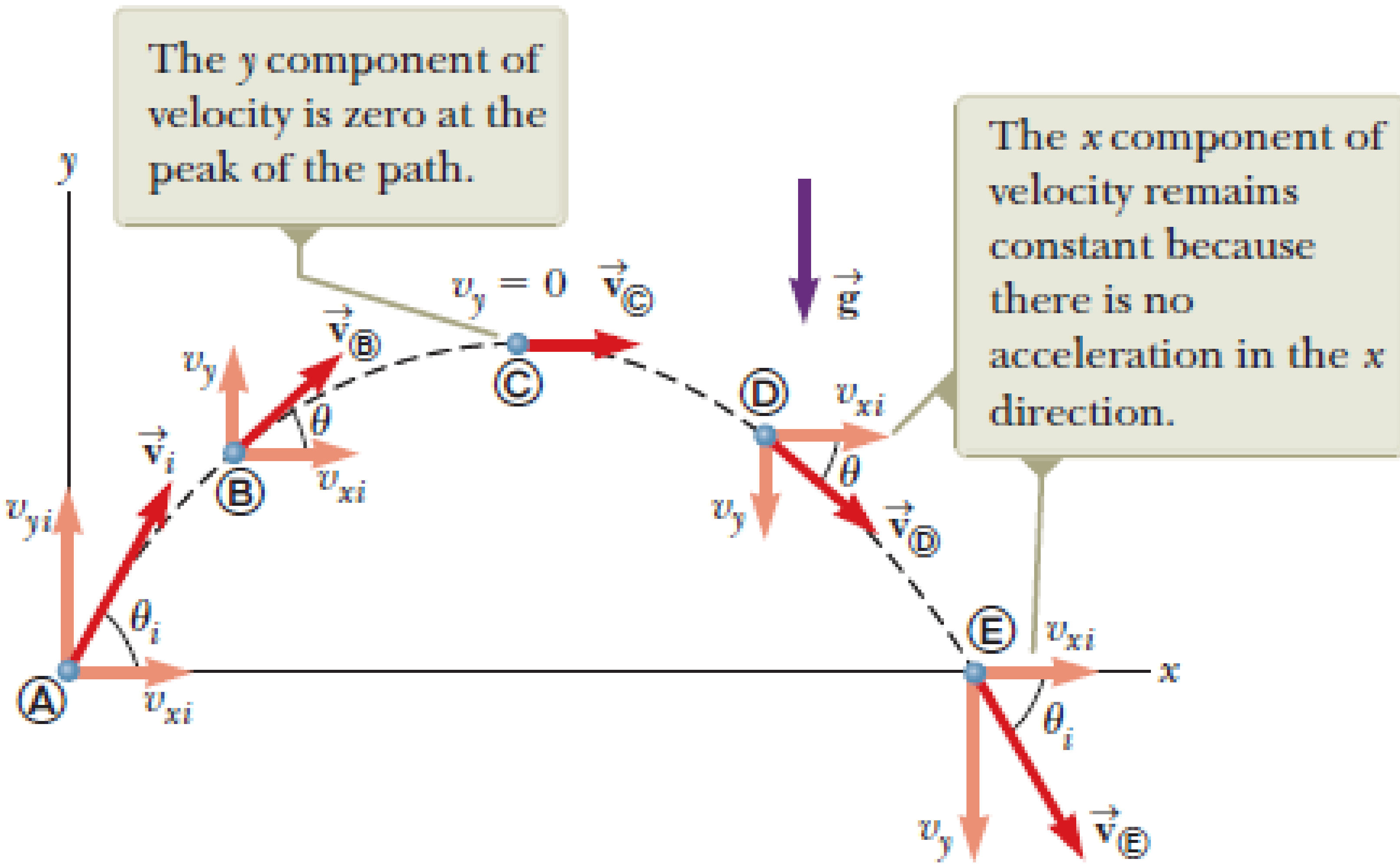


This Lecture

Projectile motion
Type of forces

Projectile Motion

A particle projected initially with a speed at an elevation angle under the influence of uniform gravitational field. The magnitude and direction of the velocity components will change. Under this condition the trajectory of the particle forms a parabola.



$\Rightarrow$ Free fall acceleration is constant

$\Rightarrow$ No air resistance

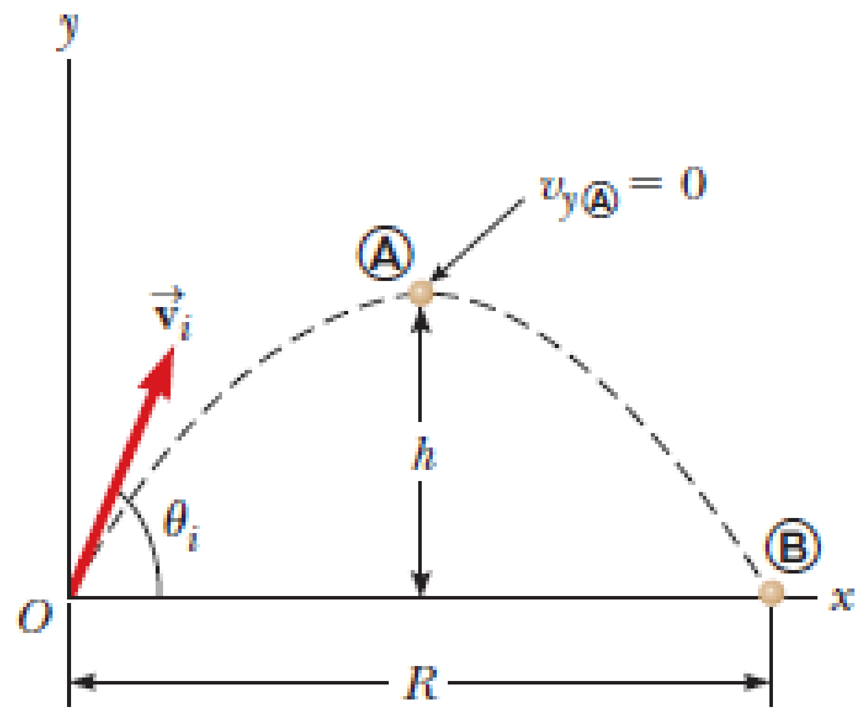
$\Rightarrow$ Path of projectile is parabolic.

Position vector as a function of time,

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \tfrac{1}{2}\vec{\mathbf{g}} t^2$$

Velocity components,

$$v_{xi} = v_i \cos\theta_i \qquad v_{yi} = v_i \sin\theta_i$$



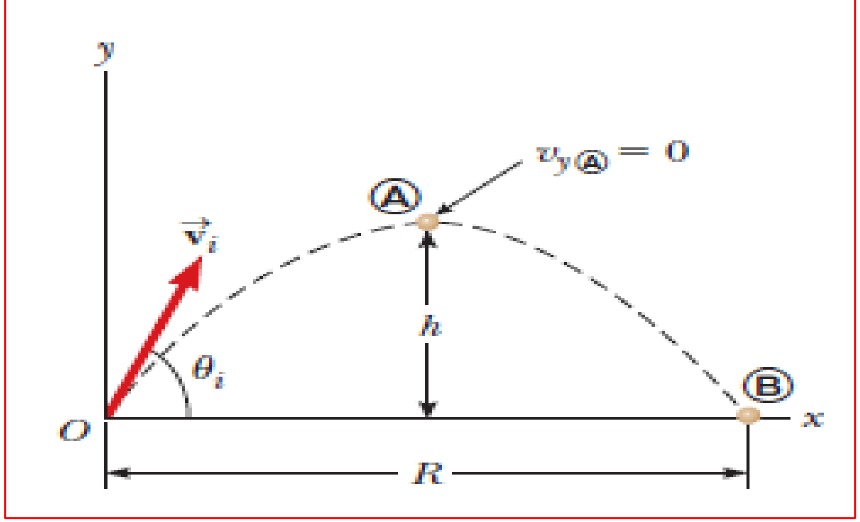
If there is no gravitational field, how the trajectory would look like?

=> Straight line along v_i.

The horizontal and vertical components of projectile's motion are independent to each other.

$\Rightarrow$ Horizontal motion: Under constant velocity.

$\Rightarrow$ Vertical motion: Under constant acceleration (-g)



Time to reach at peak:

$$v_{yf} = v_{yi} + a_y t$$

$$0 = v_i \sin\theta_i - g t_{A}$$

$$t_{A} = \frac{v_i \sin\theta_i}{g}$$

Maximum height

$$h = (v_i \sin\theta_i)\frac{v_i \sin\theta_i}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

Range:

$$R = v_{xi} t_{B} = (v_i \cos\theta_i) 2 t_{A}$$

$$= (v_i \cos\theta_i)\frac{2 v_i \sin\theta_i}{g} = \frac{2 v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Homework:

For a given velocity, what should be the angle to achieve maximum range?

$$R = v_{xi} t_{Ⓑ} = (v_i \cos\theta_i) 2 t_{Ⓐ}$$

$$= (v_i \cos\theta_i) \frac{2 v_i \sin\theta_i}{g} = \frac{2 v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Problem 1:

Suppose a large rock is ejected from a volcano, as illustrated in the figure, with a speed of $25.0\ \text{m/s}$ and at an angle $35°$ above the horizontal. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path.

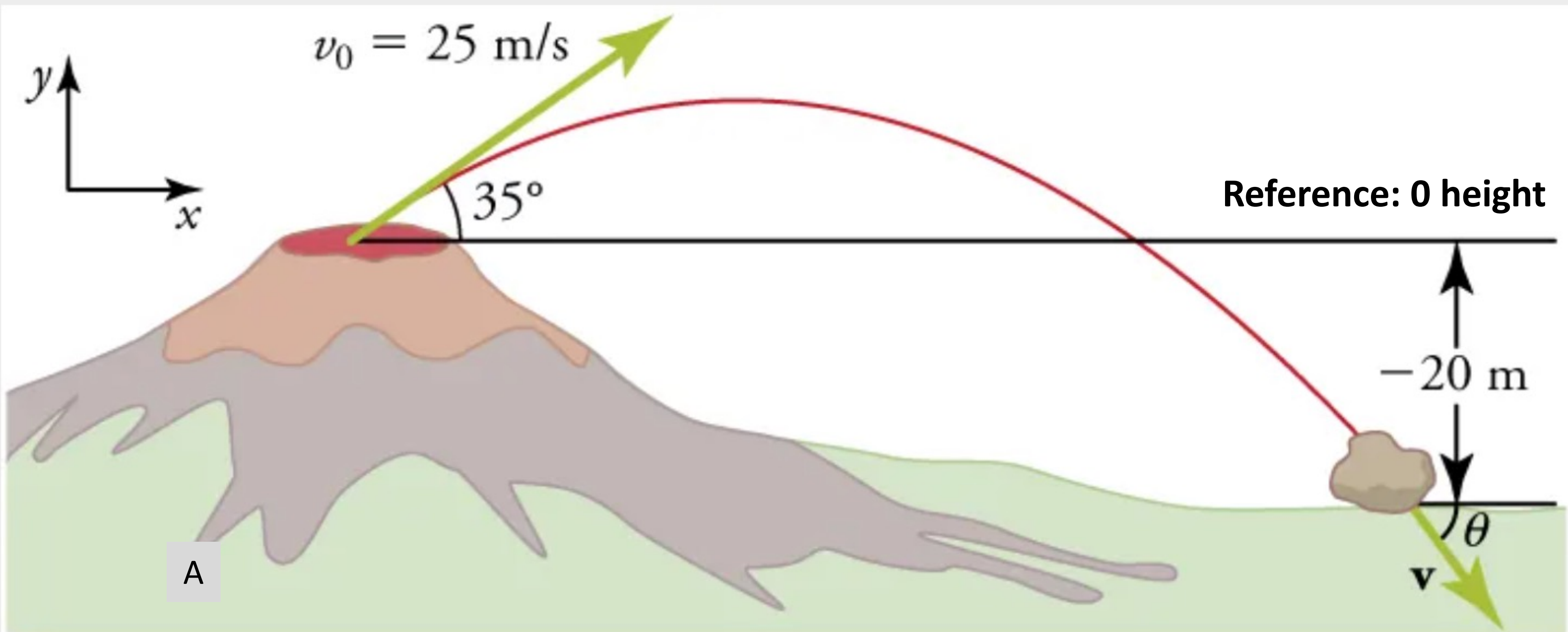


Figure The diagram shows the projectile motion of a large rock from a volcano.

Solution:

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0\mathbf{y}} t - \frac{1}{2}\mathbf{g}t^2 .$$

If we take the initial position $\mathbf{y}_0$ to be zero, then the final position is $\mathbf{y} = -20.0\ \text{m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta_0 = (25.0\ \text{m/s})(\sin 35^\circ) = 14.3\ \text{m/s}.$$

Substituting known values yields

$$-20.0\ \text{m} = (14.3\ \text{m/s})t - \left(4.90\ \text{m/s}^2\right) t^2 .$$

Rearranging terms gives a quadratic equation in t

$$\left(4.90\ \mathrm{m/s^2}\right) t^2 - (14.3\ \mathrm{m/s})\, t - (20.0\ \ \mathrm{m}) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are a = 4.90, b = –14.3, and c = –20.0. Its solutions are given by the quadratic formula

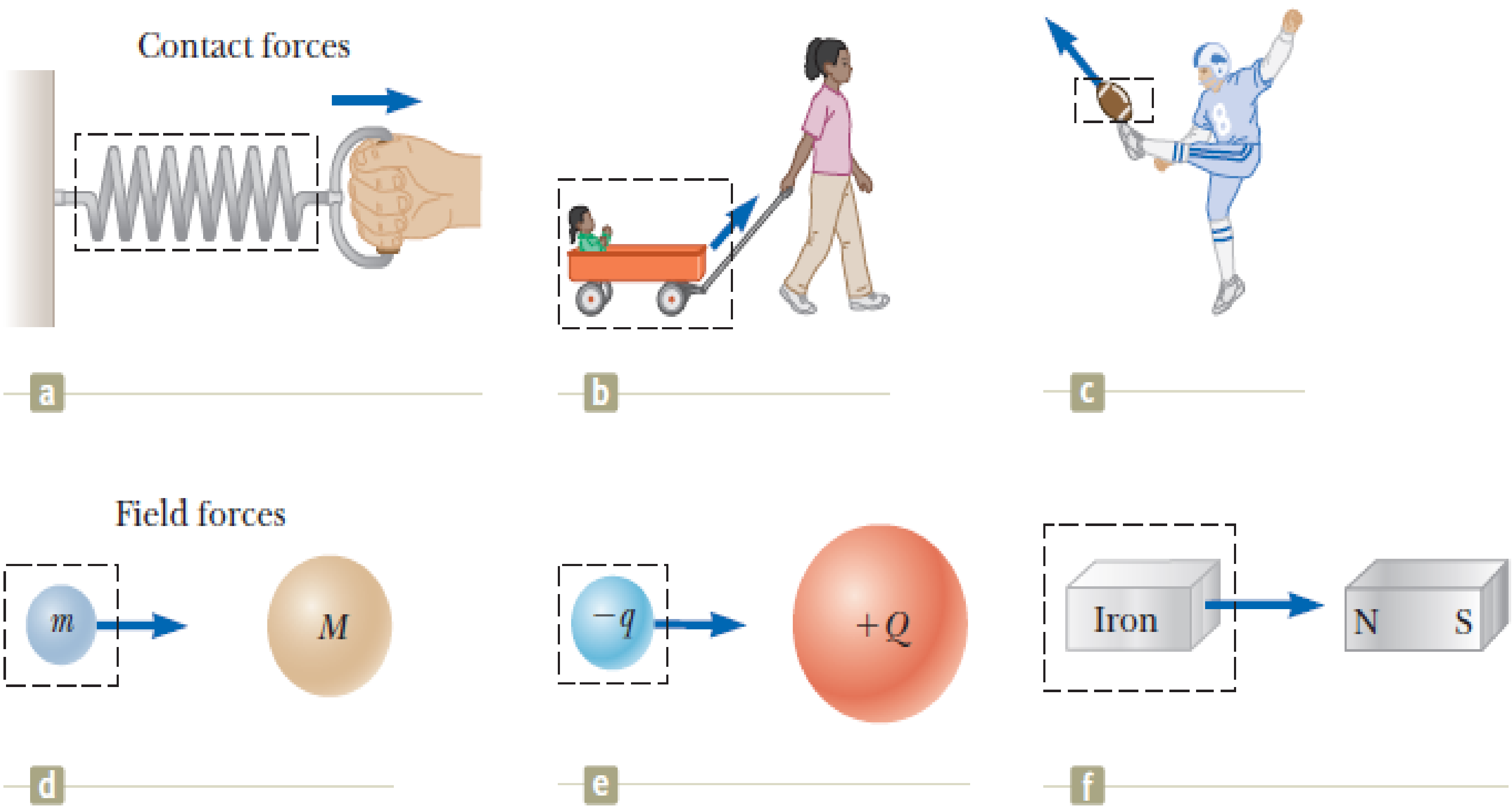
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions t = 3.96 and t = –1.03. You may verify these solutions as an exercise. The time is t = 3.96 s or –1.03 s. The negative value of time implies an event before the start of motion, so we discard it. Therefore,

$$t = 3.96\ \ \mathrm{s}.$$

Type of forces

⇒ A mean of interaction with an object
⇒ May/may not cause motion in an object
⇒ Contact force & Field force



Fundamental forces are field forces

1. **Gravitational force:** between any objects
2. **Electromagnetic force:** between electric charges, magnetic poles
3. **Strong force:** between subatomic particles
4. **Weak force:** in radioactive decay process

Position dependent force

Consider a situation when the applied force can be obtained in terms of the position, viz.,

$$F = F(x)$$

Thus, Newton's second law gives

$$m\frac{dv}{dt} = F(x)$$

Using $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$, we get

$$mv\frac{dv}{dx} = F(x)$$

$$mv\frac{dv}{dx} = F(x)$$

$$\Rightarrow mvdv = F(x)dx$$

Suppose at $x = x_a$, $v = v_a$ and at $x = x_b$, $v = v_b$

$$\Rightarrow m\int_{v_a}^{v_b} vdv = \int_{x_a}^{x_b} F(x)dx$$

$$\Rightarrow \frac{1}{2}m(v_b^2 - v_a^2) = \int_{x_a}^{x_b} F(x)dx$$

=> Change of kinetic energy = Work done

Next lecture

Viscous force