PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 28

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Previous Lecture

Translation and rotation

This Lecture

Kinetic theory of gases

Atoms

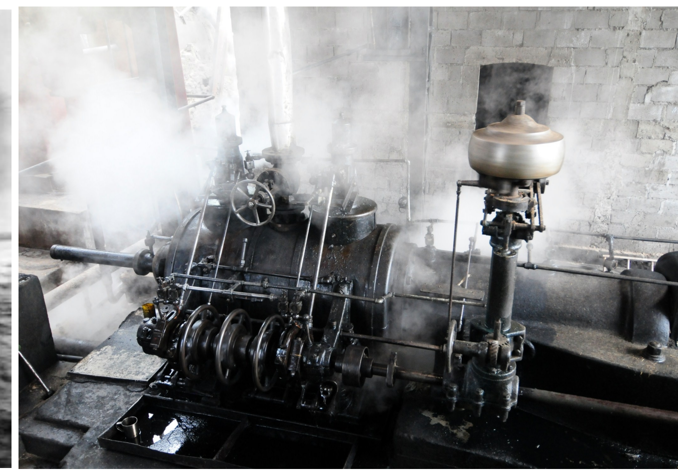
- The concept of an atom as an indivisible component of matter goes back to Indian and Greek philosophers. The word comes from átomos (Greek: ἄτομος), which means "uncuttable“.
- The idea of atoms was used in 17th and 18th century to explain chemical properties of various substances.
- However, people thought that even if atoms really existed, they were far too small to see even under the most powerful microscopes. Speculating about them was not really of much use.
- Thus, their existence was debatable till the mid 19th century.
- However, somewhere around this time (mid 19th century) whether the atom is real, became a question of utmost importance.

Why ?

Atoms

- The reason was STEAM & Steam powered machines: **Industrial revolution**.





- It became necessary to understand the dynamics of the constituents of steam (or a gas in general) to improve the machinery and hence efficiency.

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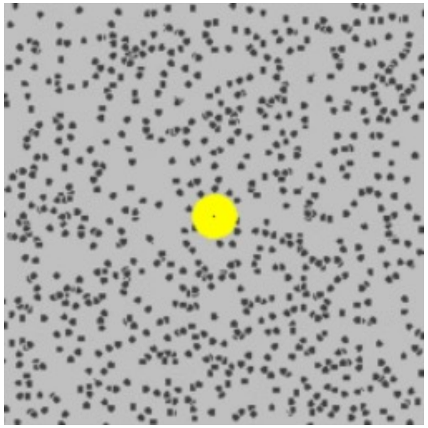
Atoms

- **L. E. Boltzmann** (1844 - 1906) paved the way for development of Statistical Mechanics, which explains and predicts how the properties of atoms (microscopic) determine the physical properties (macroscopic) of matter.
- Boltzmann's kinetic theory of gases seemed to presuppose the reality of atoms and molecules.
- He had to face strong opposition from the philosophers and physicists of that era who argued that atoms were a mere mathematical convenience rather than real physical objects.

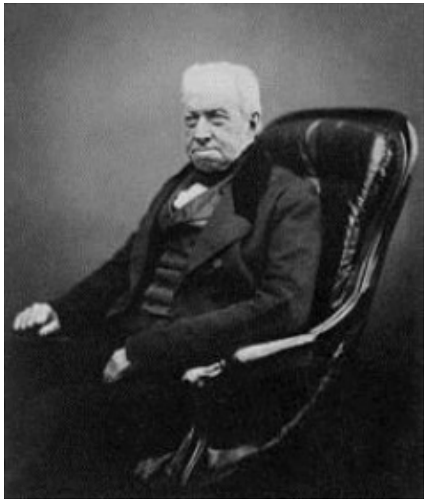
Ludwig Eduard Boltzmann

Brownian motion

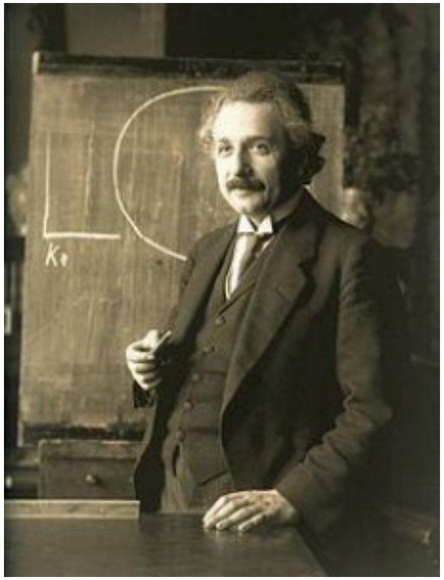
- In 1827, the botanist **Robert Brown**, looking through a microscope at particles found in pollen grains in water, noted that the particles moved through the water but was not able to determine the mechanisms that caused this motion.



- **Albert Einstein** in 1905 successfully explained the phenomenon by considering the atoms as real objects and was also able to estimate their size.



Robert Brown(1773-1858)



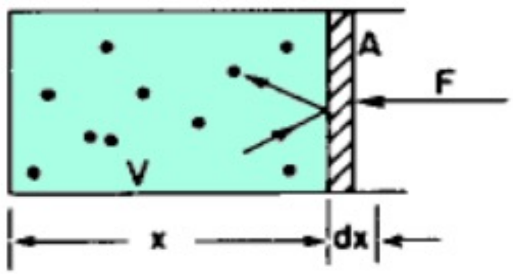
Albert Einstein (1879-1955)

Assumptions of kinetic theory of gases

- Gases are composed of a large number of particles that behave like **hard, spherical objects** in a state of constant, random motion.
- These particles **move in a straight line** until they collide with another particle or the walls of the container.
- These particles are much smaller than the distance between particles. Most of the **volume of a gas is therefore empty space**.
- There is **no additional interaction** (other than when they collide) between gas particles or between the particles and the walls of the container.
- Collisions between gas particles or collisions with the walls of the container are perfectly **elastic**.

Kinetic theory of gases

- Consider a box with a frictionless piston filled with some gas.
- We are interested in finding out the force on the piston due to the particles (atoms/molecules) constituting the gas.



- This force, however, is not localized at a single point but rather distributed over the entire area of the piston.
- A convenient way to measure it would be to talk about force per unit area, i.e., **Pressure** :

$$P = \frac{F}{A}$$

Kinetic theory of gases

- Consider a particle which has a **mass *m* and velocity $\boldsymbol{v}$**. If the x-component of the velocity is v_x , then when the atom hits the piston (elastic collision), **this component gets reversed**.
- The change in momentum is

$$\triangle P = m\left((-v_x) - (v_x)\right)$$

(-ve sign represents the loss in momentum)

- Momentum delivered to the piston because of this single collision $= 2mv_x$.
- For simplicity let us assume that all the atoms have the same velocity. (We will generalize this to the case of unequal velocities soon).
- Let us consider a small time interval Δt. In this interval, only the particles which lie within the distance $v_x \Delta t$ from the wall will be able to hit the wall. Others won't be able to reach the wall in Δt.

Kinetic theory of gases

- If A is the area of the piston, then the particles which lie within the volume $Av_x\Delta t$ will be able to hit the piston.
- If n is the number of particles per unit volume:

$$n = \frac{N}{V} \,,$$

- The number of particles that hit the wall in time Δt is:

$$nAv_x\Delta t$$

- Thus, total momentum imparted to the piston in this interval is

$$= (nAv_x\Delta t)(2mv_x)$$

Kinetic theory of gases

- The force on the piston is therefore:

$$F = \frac{(nAv_x\Delta t)(2mv_x)}{\Delta t} = 2nmv_x^2 A$$

(The result does not change if we take the limit $\Delta t \to 0$).

- Hence, the pressure is:

$$P = \frac{F}{A} = 2mnv_x^2\,.$$

- Now let us generalize to arbitrary velocities for the particles. However, we are considering identical particles, so masses are same for all.

- For that we need to replace v_x^2 by the average velocity in the x-direction. So,

$$v_x^2 \to \frac{1}{2}\left\langle v_x^2 \right\rangle$$

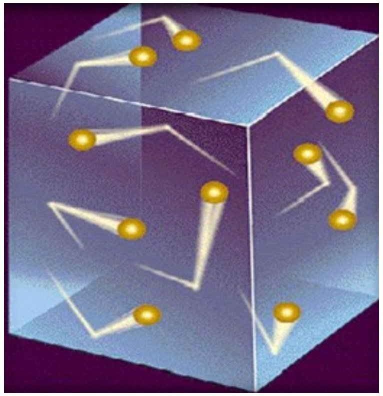
Kinetic theory of gases

- The factor of half must be introduced because $\langle v_x^2 \rangle$ counts contribution from both v_x and $-v_x$, whereas we are focusing on v_x only.
- Thus,

$$P = 2mnv_x^2 \to mn\langle v_x^2 \rangle$$

- But now there is nothing special about the x-direction, we might as well consider y and z directions. Since there is no preferred direction for the particles, for the averages we must have:

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle \ .$$



- Now if v^2 is the velocity squared of the particles (in general different for all), then $v^2 = v_x^2 + v_y^2 + v_z^2$.

Kinetic theory of gases

- Take the average,

$$\left\langle v^2 \right\rangle = \left\langle v_x^2 \right\rangle + \left\langle v_y^2 \right\rangle + \left\langle v_z^2 \right\rangle = 3 \left\langle v_x^2 \right\rangle \ .$$

- Thus, finally we have

$$P = 2mnv_x^2 \rightarrow mn\left\langle v_x^2 \right\rangle \rightarrow \frac{1}{3} nm \left\langle v^2 \right\rangle$$

- $\left\langle v^2 \right\rangle$ is the mean-square velocity:

$$\left\langle v^2 \right\rangle = \frac{1}{N} \sum_{j=1}^{N} v_j^2 = \frac{1}{N} \sum_{j=1}^{N} \left(v_{j,x}^2 + v_{j,y}^2 + v_{j,z}^2 \right)$$

- Here *N* represents the total number of particles and *j* denotes the j-th particle.