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FINAL EXAM

Applied Linear Algebra (MAT161)

Date: 1st May, 2025 **Total marks = 65** **Total time = 2 hours**

There are total **7** questions. Answer the questions in serial order.

Q1) Consider the set { e^x, e^{2x}, $e^x - e^{2x}$ } of solutions of the linear differential equation $y''' - 6y'' + 11y' - 6y = 0$. Test for linear independence of this set using the Wronskian. **(5 marks)**

Q2) Let $T : R^3 \rightarrow R^3$ be represented by $T(v) = \text{proj}_u v$, where u = (0,1,2). Find the standard matrix for T. **(5 marks)**

Q3) Solve the system of first-order linear differential equation:

$$y'_1 = y_1 + 2y_2$$
$$y'_2 = 2y_1 + y_2$$

(10 marks)

Q4) Find the least squares quadratic polynomial for the following data points:
$(0,2), (1,3/2), (2, 5/2), (3,4)$.
(Hint: The reduced form of the given matrix can be used

$$\begin{bmatrix} 4 & 6 & 14 & 10 \\ 6 & 14 & 36 & \frac{37}{2} \\ 14 & 36 & 98 & \frac{95}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & \frac{39}{20} \\ 0 & 1 & 0 & -\frac{4}{5} \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$$

). **(10 marks)**

Q5) A population has the characteristics listed below.

(a) A total of 80% of the population survives its first year. Of that 80%, 25% survives the second year. The maximum life span is 3 years.
(b) The average number of offspring for each member of the population is 3 the first year, 6 the second year, and 3 the third year.

The population now consists of 150 members in each of the three age classes. How many members will there be in each age class in 1 year? In 2 years? **(10 marks)**

Q6) For what value(s) of a does the matrix **(3 + 5 + 2 = 10 marks)**

$$A = \begin{pmatrix} 0 & 1 \\ a & 1 \end{pmatrix}$$

have the characteristics listed below?
(a) A has an eigenvalue of multiplicity 2.
(b) A has −1 and 2 as eigenvalues.
(c) A has real eigenvalues.

Q7) Consider the linear Transformation: $T: R^2 \rightarrow R^3$ such that $T(x,y) = (-x, y, x + y)$. Let v = (0,1).
Consider a basis B = { (1,1), (1, -1)} for R^2 and B' = { (0,1,0), (0, 0, 1), (1, 0, 0 } for R^3.
Find T(v) by using (a) the standard matrix and (b) the matrix relative to B and B'. **(5 + 10 = 15 marks)**