Resonance

SERIES RESONANT CIRCUIT

PARALLEL RESONANT CIRCUIT

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Series Resonance

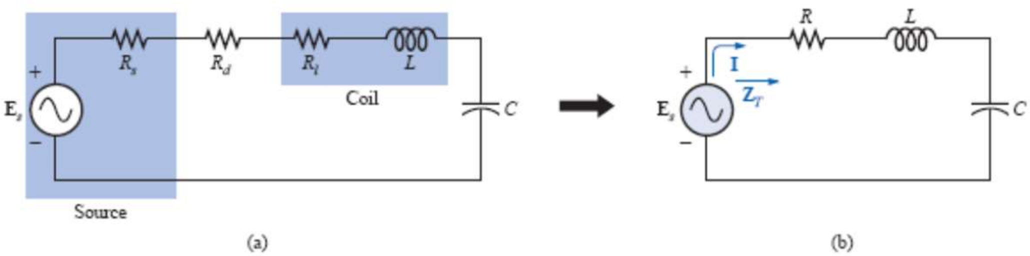

FIG. 20.2
Series resonant circuit.

$$\boxed{R = R_s + R_l + R_d}$$

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$$X_L = X_C$$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

and

$$\boxed{\omega_s = \frac{1}{\sqrt{LC}}}$$

or

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}}$$

f = hertz (Hz)
L = henries (H)
C = farads (F)

The current through the circuit at resonance is

$$\mathbf{I} = \frac{E \angle 0°}{R \angle 0°} = \frac{E}{R} \angle 0°$$

which you will note is the maximum current for the circuit .

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Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance:

$$\left.\begin{aligned} \mathbf{V}_L &= (I \angle 0°)(X_L \angle 90°) = IX_L \angle 90° \\ \mathbf{V}_C &= (I \angle 0°)(X_C \angle -90°) = IX_C \angle -90° \end{aligned}\right\} \begin{matrix}180° \\ \text{out of} \\ \text{phase}\end{matrix}$$

and, since $X_L = X_C$, the magnitude of V_L equals V_C at resonance; that is,

$$\boxed{V_{L_s} = V_{C_s}} \qquad \textbf{(20.6)}$$

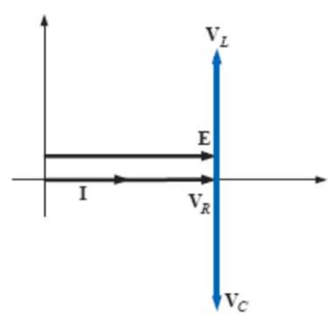

Figure 20.3, a phasor diagram of the voltages and current, clearly indicates that the voltage across the resistor at resonance is the input voltage, and **E,** and **I,** and $\mathbf{V}_R$ are in phase at resonance.

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The power triangle at resonance (Fig. 20.4) shows that the total apparent power is equal to the average power dissipated by the resistor since $Q_L = Q_C$. The power factor of the circuit at resonance is

$$F_p = \cos\theta = \frac{P}{S}$$

and

$$\boxed{F_{p_s} = 1}$$

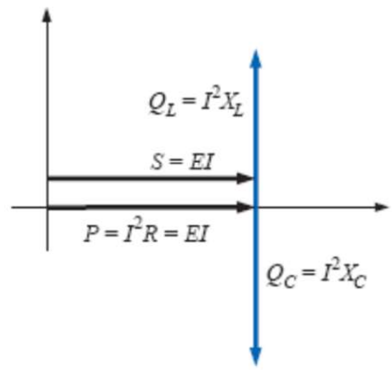

FIG. 20.4
Power triangle for the series resonant circuit at resonance.

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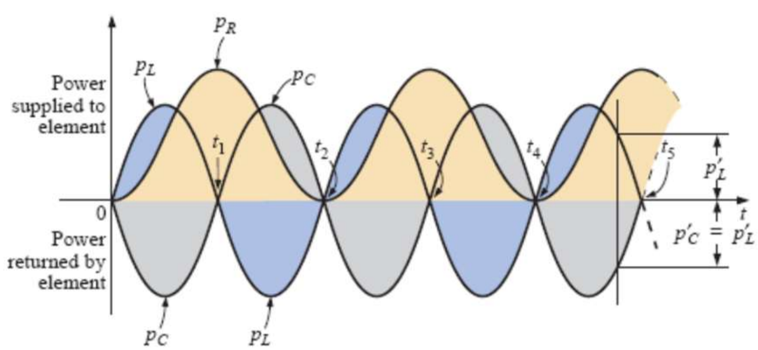

FIG. 20.5
Power curves at resonance for the series resonant circuit.

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The **quality factor** Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor **at resonance**;

$$\boxed{Q_s = \frac{\text{reactive power}}{\text{average power}}}$$

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

The lower the level of dissipation for the same reactive power, the larger the *Qs* factor and the more concentrated and intense the region of resonance.

$$\boxed{Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}}$$

If the resistance R is just the resistance of the coil (R_l), we can speak of the Q of the coil, where

$$\boxed{Q_{\text{coil}} = Q_l = \frac{X_L}{R_l}} \qquad R = R_l \qquad \textbf{(20.10)}$$

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If we substitute

$$\omega_s = 2\pi f_s$$

and then

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

into Eq. (20.9), we have

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right)L$$
$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}}$$

and

$$\boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

providing Q_s in terms of the circuit parameters.

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$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \qquad \text{(at resonance)}$$

and $$\boxed{V_{L_s} = Q_s E}$$

or $$V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

and $$\boxed{V_{C_s} = Q_s E}$$

Qs is usually greater than 1, the voltage across the capacitor or inductor of a series resonant circuit can be significantly greater than the input voltage.

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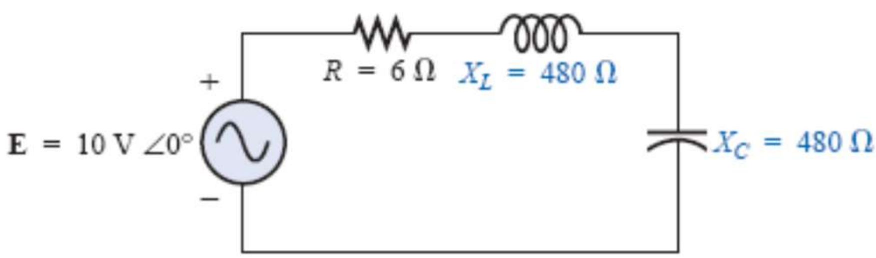

$$Q_s = \frac{X_L}{R} = \frac{480\ \Omega}{6\ \Omega} = 80$$

$$V_L = V_C = Q_s E = (80)(10\ \text{V}) = \mathbf{800\ V}$$

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$f < f_s$:	network capacitive; **I** leads **E**
$f > f_s$:	network inductive; **E** leads **I**
$f = f_s$:	network resistive; **E** and **I** are in phase

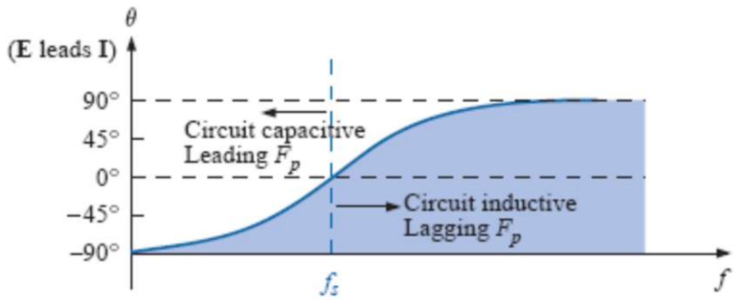

FIG. 20.13
Phase plot for the series resonant circuit.

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SELECTIVITY

Band frequencies, cutoff frequencies, or **half-power frequencies.**

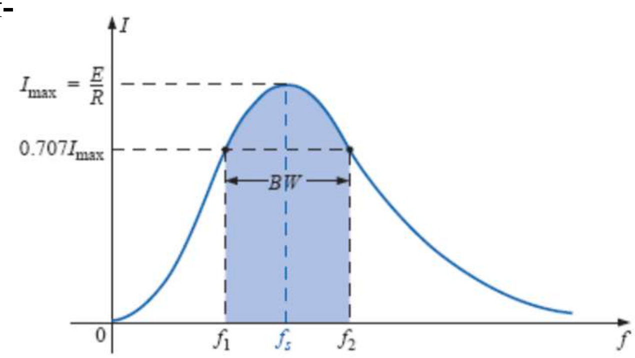

FIG. 20.14
I versus frequency for the series resonant circuit.

The range of frequencies between the f_1 and f_2 is referred to as the **bandwidth** (abbreviated *BW*) of the resonant circuit.

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$$P_{\max} = I_{\max}^2 R$$

and $$P_{\text{HPF}} = I^2 R = (0.707 I_{\max})^2 R = (0.5)(I_{\max}^2 R) = \frac{1}{2} P_{\max}$$

Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is,

$$\boxed{P_{\text{HPF}} = \frac{1}{2} P_{\max}} \qquad \textbf{(20.17)}$$

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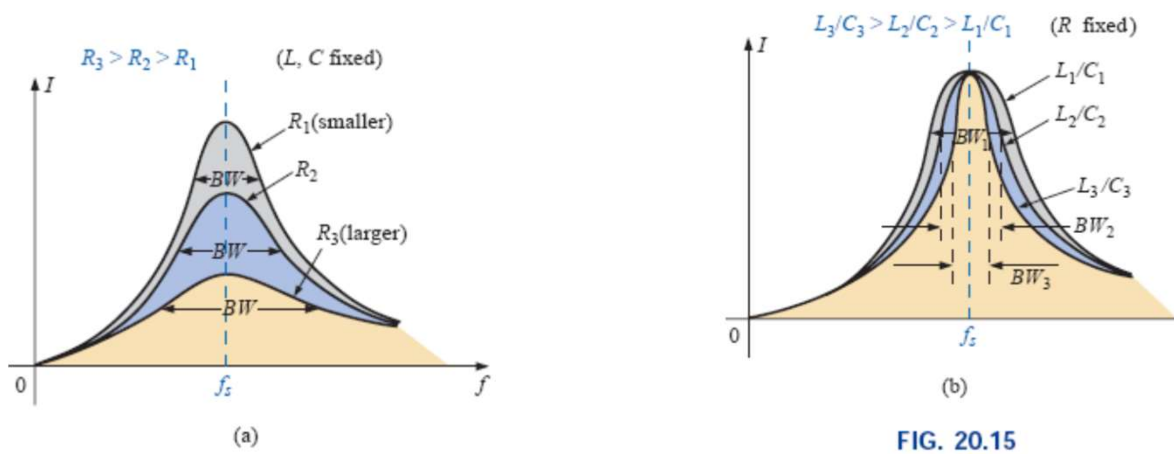

FIG. 20.15
Effect of R, L, and C on the selectivity curve for the series resonant circuit.

A small Q_s, therefore, is associated with a resonant curve having a large bandwidth and a small selectivity, while a large Q_s indicates the opposite.

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PARALLEL RESONANT CIRCUIT

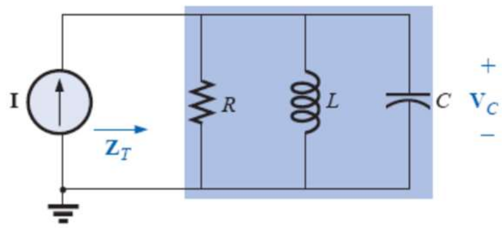

FIG. 20.21
Ideal parallel resonant network.

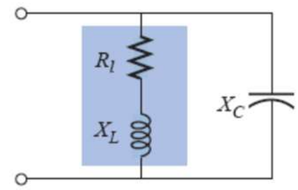

FIG. 20.22
Practical parallel L-C network.

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Find a parallel network equivalent (at the terminals) for the series *R-L* branch

$$\mathbf{Z}_{R\text{-}L} = R_l + j\,X_L$$

and
$$\mathbf{Y}_{R\text{-}L} = \frac{1}{\mathbf{Z}_{R\text{-}L}} = \frac{1}{R_l + j\,X_L} = \frac{R_l}{R_l^2 + X_L^2} - j\,\frac{X_L}{R_l^2 + X_L^2}$$
$$= \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{j\,X_{Lp}}$$

with
$$\boxed{R_p = \frac{R_l^2 + X_L^2}{R_l}} \qquad \textbf{(20.24)}$$

and
$$\boxed{X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}} \qquad \textbf{(20.25)}$$

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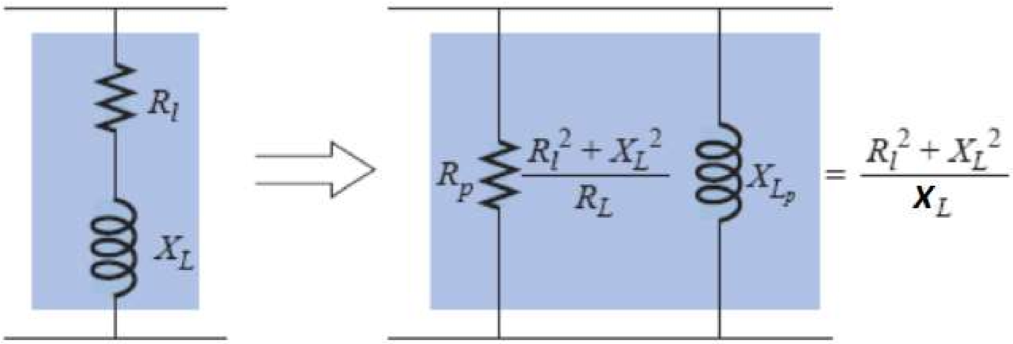

FIG. 20.23
Equivalent parallel network for a series R-L combination.

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A practical current source having an internal resistance Rs will result in the network

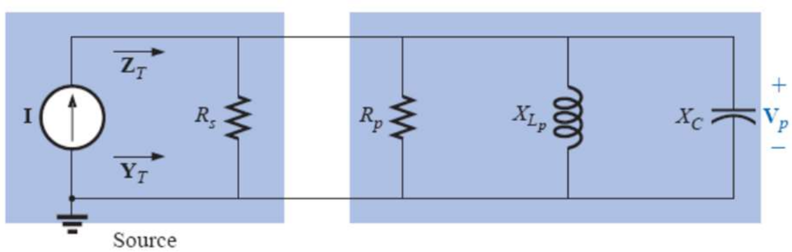

FIG. 20.24
Substituting the equivalent parallel network for the series R-L combination of Fig. 20.22.

If we define the parallel combination of R_s and R_p by the notation

$$\boxed{R = R_s \parallel R_p} \qquad \textbf{(20.26)}$$

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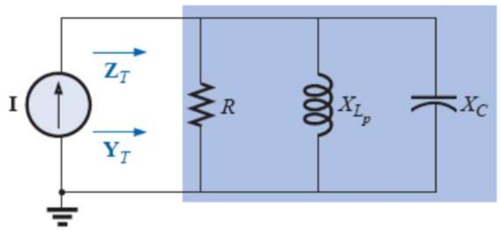

FIG. 20.25
Substituting $R = R_s \| R_p$ for the network of Fig. 20.24.

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Unity Power Factor, f_p

At resonance $X_{Lp} = X_C$

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}$$

where f_p is the resonant frequency of a parallel resonant circuit (for $F_p = 1$) and f_s is the resonant frequency as determined by $X_L = X_C$ for series resonance. Note that unlike a series resonant circuit, the resonant

Since the factor $\sqrt{1 - (R_l^2 C/L)}$ is less than 1, f_p is less than f_s. Recognize also that as the magnitude of R_l approaches zero, f_p rapidly approaches f_s.

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Maximum Impedance, f_m

$$Z_{T_m} = R \parallel X_{L_p} \parallel X_C$$

$f = f_m$

If R=R$_l$

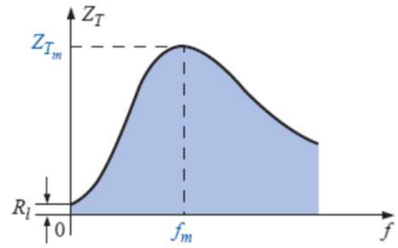

FIG. 20.26
Z_T versus frequency for the parallel resonant circuit.

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

$$f_s > f_m > f_p$$

Since the voltage across parallel elements is the same,

$$V_C = V_p = IZ_T$$

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Cutoff Frequencies:

- Each cutoff frequency as the frequency at which the input impedance is 0.707 times its maximum value.
- Since the maximum value is the equivalent resistance R , the cutoff frequencies will be associated with an impedance equal to 0.707R.

$$\mathbf{Z} = \frac{1}{\frac{1}{R}\left[1 + jR\left(\omega C - \frac{1}{\omega L}\right)\right]} = \frac{R}{\sqrt{2}}$$

Magnitude of **Z** = Mod **Z**

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

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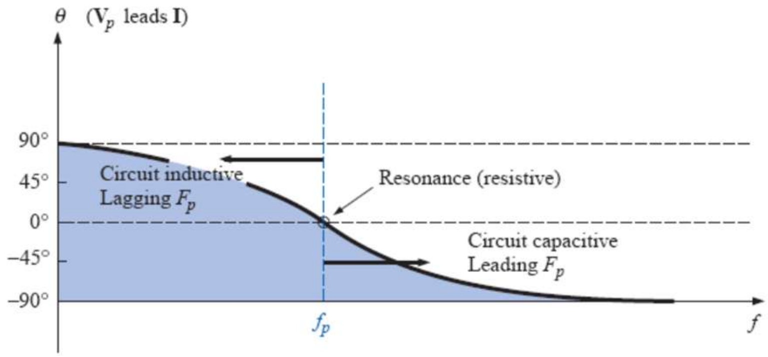

FIG. 20.29
Phase plot for the parallel resonant circuit.

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Thanks

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