28/02/2025 | 2 Absent

Spring 2025

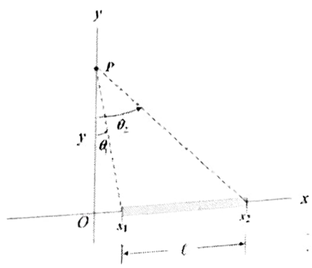
Shiv Nadar Institution of Eminence
Department of Physics

28/02/2025

PHY102 Introduction to Physics-II **Midterm Exam Full Marks = 25** Time: **2 hrs**

Answer all questions. *Marks have been indicated alongside each question. No part marking will be awarded without any explanation.*

Q1. A non-conducting rod of length ℓ with a uniform charge density, λ, has a total charge Q. The rod is lying along the x-axis, as illustrated in the **Figure** below. Compute the electric field at a point P, located at a distance y off the axis of the rod. **[5]**



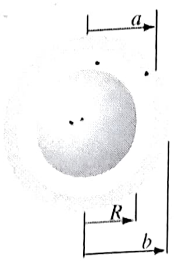
Q2. A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta) = k\frac{R}{r^2}(R-2r)sin\theta$$

where k is a constant and r and θ are the usual spherical coordinates. Find the mono and dipole terms in the expression of the potential for points on the z axis, far from the sphere. Will the value of the dipole term change if the origin is shifted? Explain. **[4+1]**

Q3. A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as in **Figure** below). The shell carries no net charge.

(a) Find the surface charge density σ at R, at a, and at b. **[1.5]**
(b) Find the potential at the center, using infinity as the reference point. **[1]**
(c) Now the outer surface is touched to a grounding wire, which drains off charge and lowers its potential to zero (same as at infinity). Find capacitance of the system now? **[2.5]**



Q4. (a) Consider a spherical charge distribution with a volume charge density

$$\rho(r) = \begin{cases} \rho_0 \cdot \dfrac{a}{r} & for\ 0 < r < a \\ 0 & for\ r > a \end{cases}$$

Show that the total electrostatic energy of the system is $Q^2/6\pi\varepsilon_0 a$, where Q is the total charge in the system. **[3]**

(b) A parallel plate air capacitor has the plates of area A each.

(i) Assuming the charge Q on it is kept constant, find the work done against the electrical forces to increase the plate separation from d_1 to d_2. **[1]**

(ii) Assuming that the potential difference V across it is maintained constant, find the change in energy of the system. **[1]**

Q5. (a) Show that the solution of Laplace's equation of electric potential V at any point $P(x, y, z)$ due to a point charge $+q$ placed at a distance z along the z-axis in a 3D case is given by

$$V(x, y, z) = V_{center} = \frac{1}{4\pi R^2} \oiint V da$$

where $P(x, y, z)$ refers to the center of a sphere of radius R and $'da'$ corresponds to a surface element of that sphere. Consider the situation when $z > R$ only. **[3]**

(b) A point charge $(-2q)$ and another point charge $(+q)$ is placed at a distance 'd' and '$3d$' from the origin above the $x - y$ plane. If the $x - y$ plane has an infinitely long grounded conductor ($V = 0$), calculate the force on $+q$ charge. [Hint: You may apply the concept of first uniqueness theorem and results of a point charge in front of a grounded conductor (image charges).]
[2]

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$\frac{q}{\varepsilon_0}$ $-\frac{\rho}{\varepsilon_0}$ $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$ $\nabla^2 V = \frac{\rho}{\varepsilon_0}$

$E \cdot da = \frac{q}{\varepsilon_0}$ $\frac{d^2 V}{dr^2} + \frac{d^2 V}{d\theta^2}$

$-\frac{q}{\frac{4}{3}\pi R^3 \varepsilon_0}$ $\frac{\rho}{\varepsilon_0}$ $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$\frac{1}{R^2}\oint V da \frac{Q}{4\pi\varepsilon_0 d}$ $-\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$\frac{1}{8\pi r^3}$ $\frac{1}{r^3} = \frac{-1}{4\pi R^2}$ $+2\frac{1}{4\pi\varepsilon_0}\frac{x}{r^3} = \frac{q}{4\pi R^2 \varepsilon_0}$