PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 21

Department of Physics, School of Natural Sciences,
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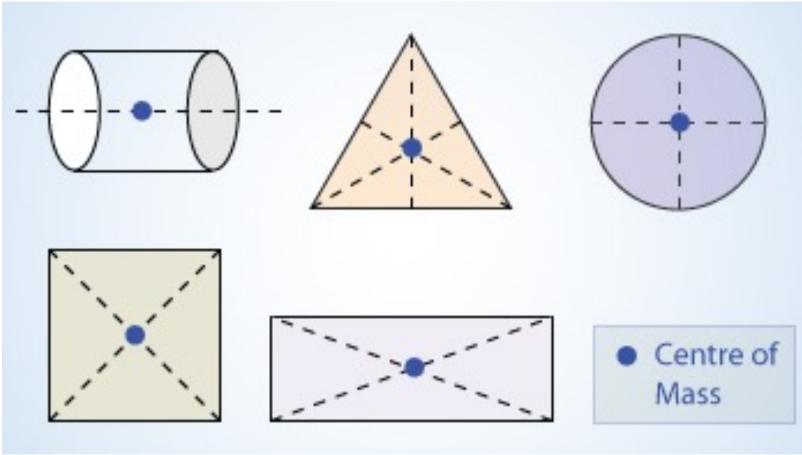
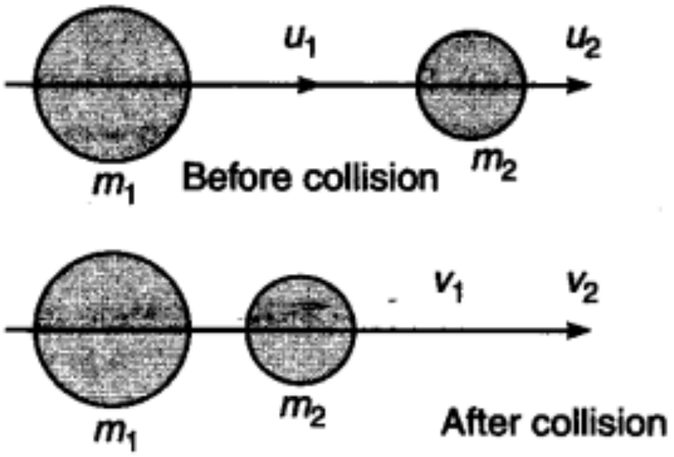
Previous Lecture

Centre of mass
Momentum
Impulse

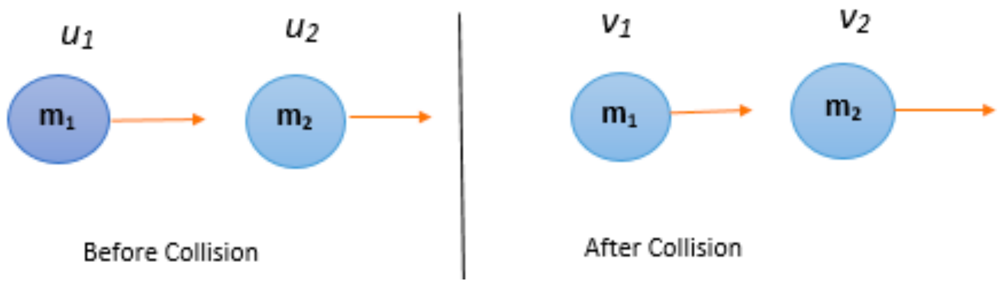


This Lecture

Collison in 1D



Collision in 1D

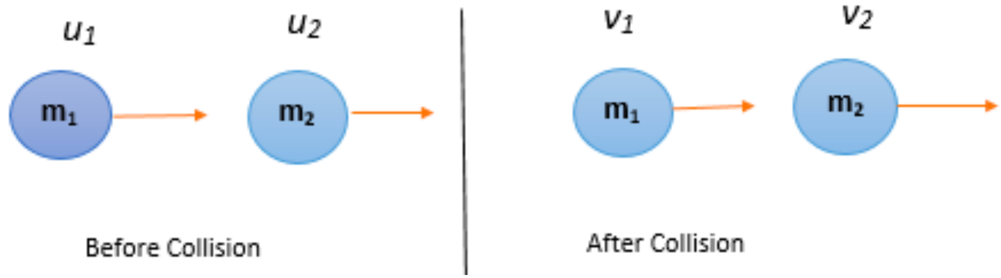


In a collision, the forces applied by the particles on each other are internal forces.

In the absence of the external forces total linear momentum of the particles do not change. This is the Law of Conservation of Linear momentum.

Since Linear Momentum is a vector quantity, the law of conservation of momentum implies that the momentum in every direction is conserved.

Collision in 1D



Based on the conservation of Kinetic Energy (KE)

Types of collisions

Elastic Collision

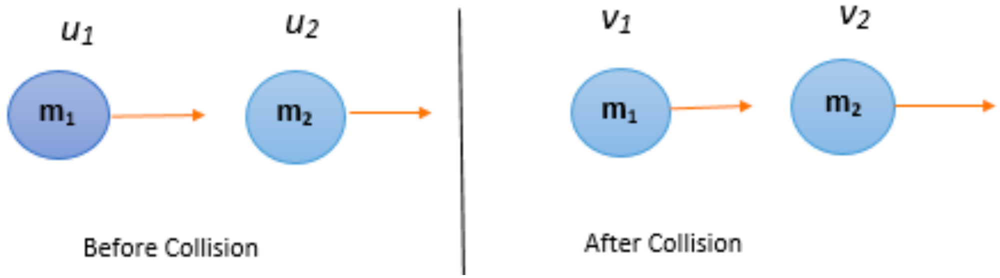
Kinetic energy conserved

Inelastic Collision

Kinetic energy is not conserved

In both cases linear momentum is conserved

Collision in 1D

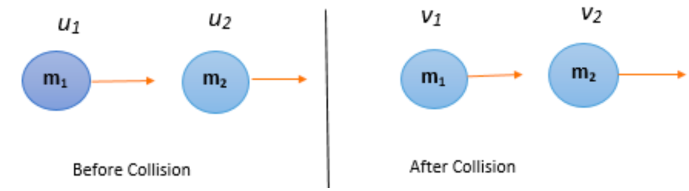


Coefficient of restitution (e)

- To describe the **elasticity** or "**bounciness**" of a collision between two objects.
- It quantifies how much **kinetic energy is conserved** in a collision.
- The coefficient of restitution is defined as the ratio of the relative speed after a collision to the relative speed before the collision.

$$e = \frac{\text{relative speed after collision}}{\text{relative speed before collision}} = \left|\frac{v_2 - v_1}{u_2 - u_1}\right|$$

Collision in 1D



Types of collisions

Elastic Collision

Kinetic energy conserved

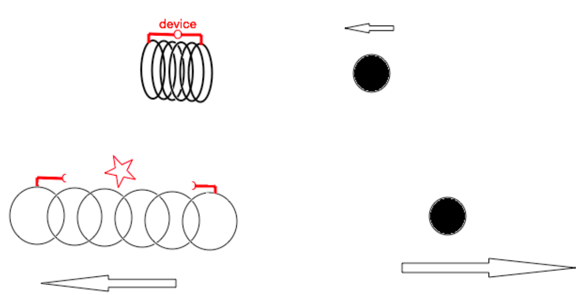
$$e = 1$$

Inelastic Collision

Kinetic energy is not conserved

$e < 1$ Kinetic energy is smaller after the collision.

$e = 0$ **Completely inelastic:** Kinetic energy is smaller, and the objects stick together, after the collision.

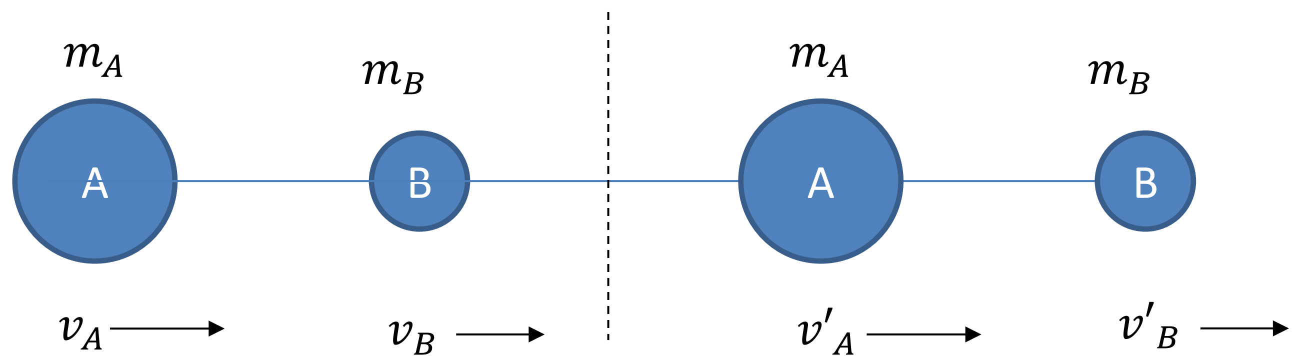


$e > 1$ **Super-elastic:** Kinetic energy is larger after the collision (e.g., an explosion).

Collision in 1D (Laboratory Frame of Reference)

Consider two balls A, B. Mass m_A , m_B, velocity v_A and v_B before collision

velocity v'_A and v'_B after collision



Before Collision **After Collision**

If their centers lie on the line of the path then the force applied by the balls on one another will lie on the same line and the balls will not go out of line after collision. This is called **Head on Collision.**

Inelastic Collision

Applying **conservation of linear momentum** and **conservation of KE** for elastic collision energy.

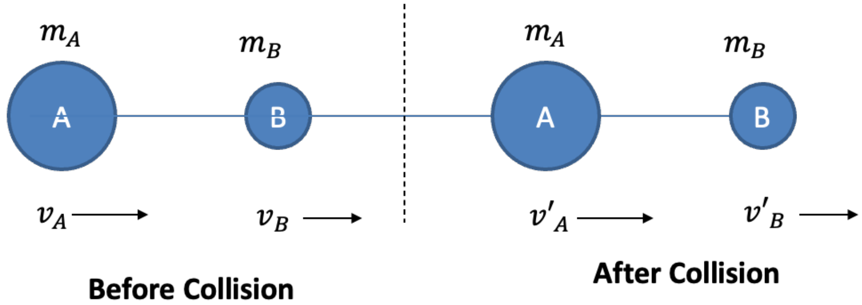
$$m_A\boldsymbol{v_A} + m_B\boldsymbol{v_B} = m_A\boldsymbol{v'_A} + m_B\boldsymbol{v'_B}$$

$$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A v'^2_A + \frac{1}{2}m_B v'^2_B$$

Collision in 1D (Laboratory Frame of Reference)

Consider two balls A, B. Mass m_A , m_B, velocity v_A and v_B before collision

velocity v'_A and v'_B after collision



$$m_A \boldsymbol{v_A} + m_B \boldsymbol{v_B} = m_A \boldsymbol{v'_A} + m_B \boldsymbol{v'_B}$$

$$m_A(\boldsymbol{v_A} - \boldsymbol{v'_A}) = m_B(\boldsymbol{v'_B} - \boldsymbol{v_B})$$

$$m_A(v_A^2 - v'^2_A) = m_B(v'^2_B - v_B^2)$$

$$m_A(v_A - v'_A)(v_A + v'_A) = m_B(v'_B - v_B)(v'_B + v_B)$$

$$(v_A + v'_A) = (v'_B + v_B)$$

$$(v_A - v_B) = (v'_B - v'_A)$$

$$Initial\ relative\ velocity = -Final\ relative\ velocity$$

Collision in 1D (Center of Mass Frame of Reference)

Consider two balls A, B. Mass m_A , m_B, velocity v_A and v_B before collision
velocity v'_A and v'_B after collision

What is the velocity of center of mass?

$$V_c = \frac{(m_A v_A + m_B v_B)}{(m_A + m_B)} = \frac{(m_A v'_A + m_B v'_B)}{(m_A + m_B)}$$

What is the velocity of particles in **center of mass frame** ?

$$v_A = V_c + v_{Ac} \Rightarrow \quad v_{Ac} = -V_c + v_A \;, \qquad v_B = V_c + v_{Bc} \Rightarrow \quad v_{Bc} = -V_c + v_B$$

Collision in 1D (Center of Mass Frame of Reference)

Consider two balls A, B. Mass m_A , m_B, velocity v_A and v_B before collision
velocity v'_A and v'_B after collision

Total momentum in the Center of Mass reference frame is zero

Applying the Law of conservation in Center of Mass frame, we have

$$m_A \boldsymbol{v_{Ac}} + m_B \boldsymbol{v_{Bc}} = m_A \boldsymbol{v'_{Ac}} + m_B \boldsymbol{v'_{Bc}} = 0 \quad , \quad \frac{1}{2} m_A v_{Ac}^2 + \frac{1}{2} m_B v_{Bc}^2 = \frac{1}{2} m_A v'^2_{Ac} + \frac{1}{2} m_B v'^2_{Bc}$$

$$m_A \boldsymbol{v_{Ac}} + m_B \boldsymbol{v_{Bc}} = 0 \Rightarrow m_A \boldsymbol{v_{Ac}} = - m_B \boldsymbol{v_{Bc}} \ ,$$

$$m_A \boldsymbol{v'_{Ac}} + m_B \boldsymbol{v'_{Bc}} = 0 \Rightarrow m_A \boldsymbol{v'_{Ac}} = - m_B \boldsymbol{v'_{Bc}}$$

Collision in 1D (Center of Mass Frame of Reference)

From last slide

Putting the value of $\boldsymbol{v_{Ac}}$ and $\boldsymbol{v'_{Ac}}$ in energy equation we trivially get $v_{Bc}^2 = v_{Bc}'^2$ and $v_{Ac}^2 = v_{Ac}'^2$

$$v_{Bc}^2 = v_{Bc}'^2 \quad \text{and } v_{Ac}^2 = v_{Ac}'^2$$

What does this suggest ?

Magnitude of $|\boldsymbol{v_{Ac}}| = |\boldsymbol{v'_{Ac}}|$ and magnitude of $|\boldsymbol{v_{Bc}}| = |\boldsymbol{v'_{Bc}}|$

⇒ In center of mass reference frame, particles collide and just reverse their velocity.

$$\Rightarrow \boldsymbol{v'_{Ac}} = -\boldsymbol{v_{Ac}} \qquad \text{and} \qquad \boldsymbol{v'_{Bc}} = -\boldsymbol{v_{Bc}}$$

We also have $\quad \boldsymbol{v'_{A}} = \boldsymbol{V_c} + \boldsymbol{v'_{Ac}} \Rightarrow \boldsymbol{v'_{A}} = \boldsymbol{V_c} - \boldsymbol{v_{Ac}} = \boldsymbol{V_c} - (-\boldsymbol{V_c} + \boldsymbol{v_A}),$

$$\Rightarrow \boldsymbol{v'_{A}} = 2\boldsymbol{V_c} - \boldsymbol{v_A} \qquad \text{Similarly} \qquad \boldsymbol{v'_{B}} = 2\boldsymbol{V_c} - \boldsymbol{v_B}$$

See how simple is a collision if we involve center of mass velocity in one dimension .

Collision in 1D

Example: A ball of mass m kept stationary on a friction less table collide head on with a ball of same mass moving with velocity v. What will be final the velocities of the balls after collision ?

Let the moving balls be A and stationary ball be B,

Given that mass of the balss $m_A = m_B = m$

And $\boldsymbol{v_A = v}$, $\boldsymbol{v_B = 0}$ $\Rightarrow \boldsymbol{V_c} = \dfrac{(m_A v_A + m_B v_B)}{(m_A + m_B)} = \dfrac{mv}{m+m} = \dfrac{v}{2}$

We also have $\boldsymbol{v'_A = 2V_c - v_A = 0}$ and $\boldsymbol{v'_B = 2V_c - v_B}$

$\Rightarrow \boldsymbol{v'_A = 2V_c - v_A = 0}$ and $\boldsymbol{v'_B = 2V_c - v_B = v}$

$\Rightarrow$ After collision moving ball stops and the second ball start moving with same velocity .

How will this collision look like in Center of Mass reference frame ?

Collision in 1D

Completely **inelastic** collision

If in a completely inelastic collision particles get stick with one another, What will be the final velocity?

If particle gets stick with one another $\boldsymbol{v'}_{\boldsymbol{A}} = \boldsymbol{v'}_{\boldsymbol{B}}$ (Lab frame)

$$\boldsymbol{V_c} = \frac{(m_A \boldsymbol{v_A} + m_B \boldsymbol{v_B})}{(m_A + m_B)} = \frac{(m_A \boldsymbol{v'_A} + m_B \boldsymbol{v'_B})}{(m_A + m_B)}$$

$$\boldsymbol{V_c} = \frac{(m_A \boldsymbol{v'_A} + m_B \boldsymbol{v'_B})}{(m_A + m_B)} = \frac{(m_A \boldsymbol{v'_A} + m_B \boldsymbol{v'_A})}{(m_A + m_B)} = \boldsymbol{v'}_{\boldsymbol{A}} \frac{(m_A + m_B)}{(m_A + m_B)}$$

$$\boldsymbol{v'_B} = \boldsymbol{v'}_{\boldsymbol{A}} = \boldsymbol{V_c}$$

See momentum is still conserved **but not the energy**

What is the energy loss ?

$$\left(\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2\right) - \left(\frac{1}{2} m_A v_A'^2 + \frac{1}{2} m_A v_B'^2\right)$$

$$= \left(\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2\right) - \frac{1}{2}(m_A + m_B)\, \boldsymbol{V_c}^2$$

Collision in 1D

Kinetic energy in lab frame (K_L): $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}m_A\ (\boldsymbol{V_c} + \boldsymbol{v_{Ac}})^2 + \frac{1}{2}m_B\ (\boldsymbol{V_c} + \boldsymbol{v_{Bc}})^2$

$$K_L = \frac{1}{2}(m_A + m_B)\ \boldsymbol{V_c}^2 + \frac{1}{2}m_A v_{Ac}^2 + \frac{1}{2}m_B v_{Bc}^2 + \boldsymbol{V_c}\cdot(m_A\boldsymbol{v_{Ac}} + m_B\boldsymbol{v_{Bc}})$$

$$K_L = K_{Lc} + \frac{1}{2}m_A v_{Ac}^2 + \frac{1}{2}m_B v_{Bc}^2 + \boldsymbol{V_c}\cdot\boldsymbol{0}$$

Remind: K_{LC} is the K. E. of centre of mass in lab frame
Total momentum in C. M. frame = 0

$K_L = K_{Lc} + K_c \quad \Rightarrow K_c = K_L - K_{Lc}$ (before collision)

K_C is the K. E. in C. M. frame

$\Rightarrow K'_c = K'_L - K'_{Lc}$ (after collision)

In an **<u>elastic collision</u>** there is no loss of kinetic energy $\Rightarrow K'_c = K_c \Rightarrow \frac{K'_c}{K_c} = 1$

Coefficient of restitution = 1

In a completely inelastic collision particle gets stuck with one another and we don't have any kinetic energy in the Center of Mass reference frame.

$\Rightarrow K'_c = 0 \Rightarrow \frac{K'_c}{K_c} = 0$ Note this is the maximum loss possible.