Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 6

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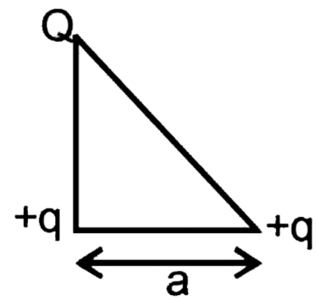
1. Three charges, Q, +q and +q, are placed at the vertices of a right-angled isosceles triangle as shown. Find Q if the net electrostatic potential energy of the configuration is zero.



Solution:

Here we have $\frac{Qq}{a} + \frac{q^2}{a} + \frac{Qq}{a\sqrt{2}} = 0$

$$\therefore\ Q = -\frac{q\sqrt{2}}{\sqrt{2}+1} = -\frac{2q}{2+\sqrt{2}}$$

Q1. A spherical conducting shell of radius a, centered at the origin, has a potential field

$$V = \begin{cases} V_0 & r \le a \\ \\ \dfrac{V_0 a}{r} & r > a \end{cases}$$

with the zero reference at infinity. Find an expression for the stored energy that this potential represents?

Solution

$$\mathbf{E} = -\nabla V = \begin{cases} 0 & r < a \\ (V_0 a/r^2)\mathbf{a}_r & r > a \end{cases}$$

(D=$\varepsilon_0 E$)

$$W_E = \frac{1}{2}\int \epsilon_0 E^2\, dv = 0 + \frac{\epsilon_0}{2}\int_0^{2\pi}\int_0^{\pi}\int_a^{\infty}\left(\frac{V_0 a}{r^2}\right)^2 r^2 \sin\theta\, dr\, d\theta\, d\phi = 2\pi\epsilon_0 V_0^2 a$$

Note that the total charge on the shell is, from Gauss' law,

$$Q = DA = \left(\frac{\epsilon_0 V_0 a}{a^2}\right)(4\pi a^2) = 4\pi\epsilon_0 V_0 a$$

while the potential at the shell is $V = V_0$. Thus, $W_E = \frac{1}{2}QV$, the familiar result for the energy stored in a capacitor (in this case, a spherical capacitor with the other plate of infinite radius).

3. Consider two concentric spherical shells of radii a and b. Suppose the inner one carries a charge q, and the outer one a charge −q (both of them uniformly distributed over the surface). Calculate the energy of this configuration?

Solution : ***Method 1***

(a) $W = \frac{\epsilon_0}{2}\int E^2\,d\tau.$ $\quad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\ (a < r < b)$, zero elsewhere.

$$W = \frac{\epsilon_0}{2}\left(\frac{q}{4\pi\epsilon_0}\right)^2\int_a^b\left(\frac{1}{r^2}\right)^2 4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0}\int_a^b\frac{1}{r^2} = \boxed{\frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right).}$$

Method 2

(b) $W_1 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{a}$, $W_2 = \frac{1}{8\pi\epsilon_0}\frac{q^2}{b}$, $\quad \mathbf{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\,\hat{\mathbf{r}}\ (r > a)$, $\mathbf{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{-q}{r^2}\,\hat{\mathbf{r}}\ (r > b)$. So $\mathbf{E}_1\cdot\mathbf{E}_2 = \left(\frac{1}{4\pi\epsilon_0}\right)^2\frac{-q^2}{r^4}$, $(r > b)$, and hence $\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 q^2\int_b^\infty\frac{1}{r^4}4\pi r^2 dr = -\frac{q^2}{4\pi\epsilon_0 b}$.

$W_{\text{tot}} = W_1 + W_2 + \epsilon_0\int \mathbf{E}_1\cdot\mathbf{E}_2\,d\tau = \frac{1}{8\pi\epsilon_0}q^2\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{b}\right) = \frac{q^2}{8\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right).\checkmark$