PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 2

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Coordinate Systems

Coordinate system

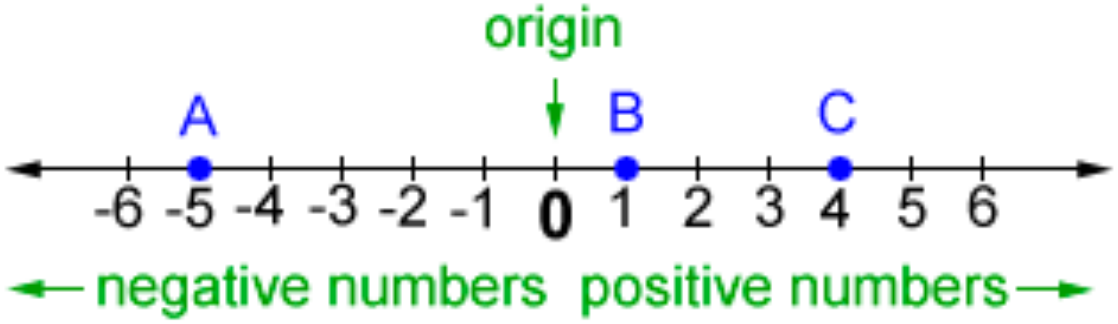
Unification of '**Algebra**' and '**Geometry**'

Cartesian coordinates

René Descartes

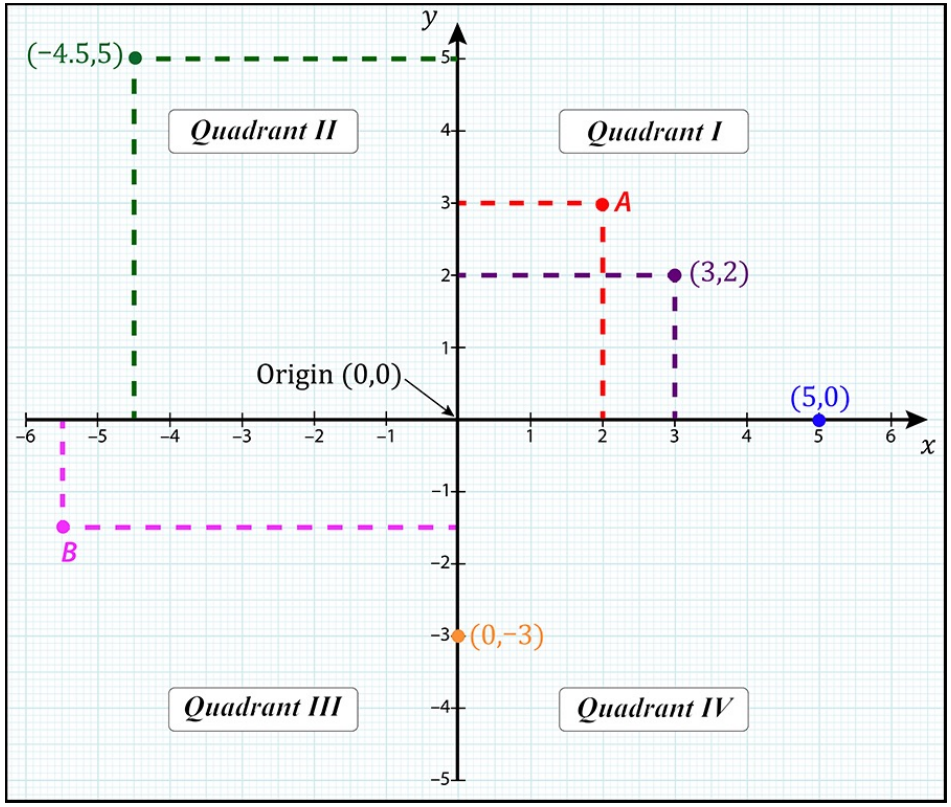
French Philosopher and Mathematician
(1596 – 1650)

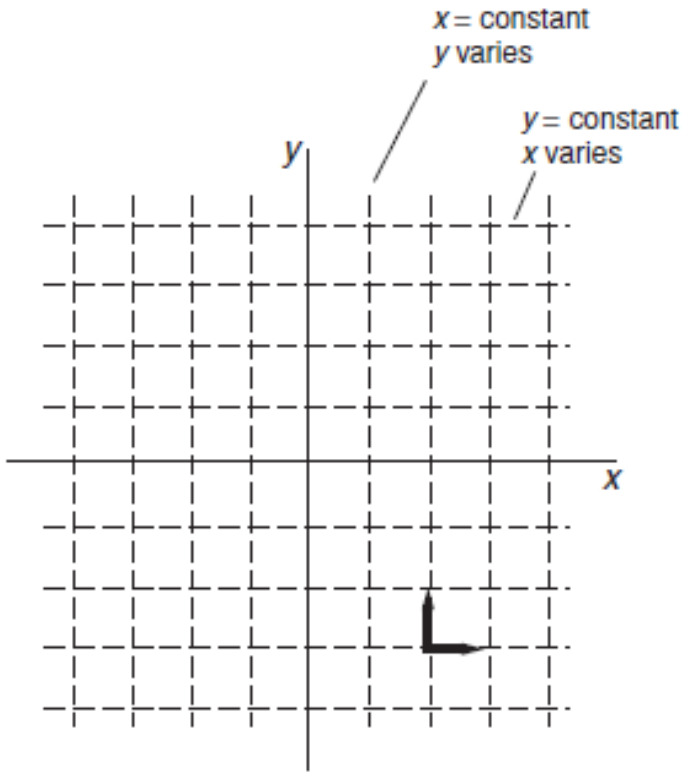
Coordinate system in 1 Dimension (1D)



Coordinate system in 2 Dimension (2D)

Cartesian coordinates





I- quadrant (+, +)
II-quadrant (-, +)
III-quadrant (-, -)
IV-quadrant (+, -)

Rectangular coordinates / Orthogonal coordinates

Ref: https://www.skillsyouneed.com/num/cartesian-coordinates.html

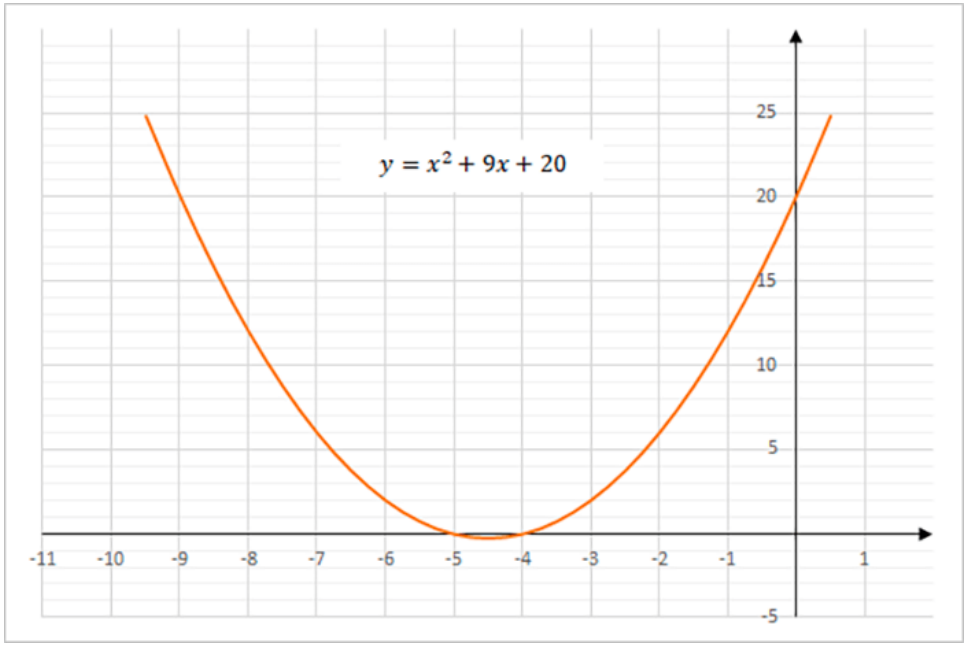
Coordinate system in 2 Dimension (2D)

Cartesian coordinates

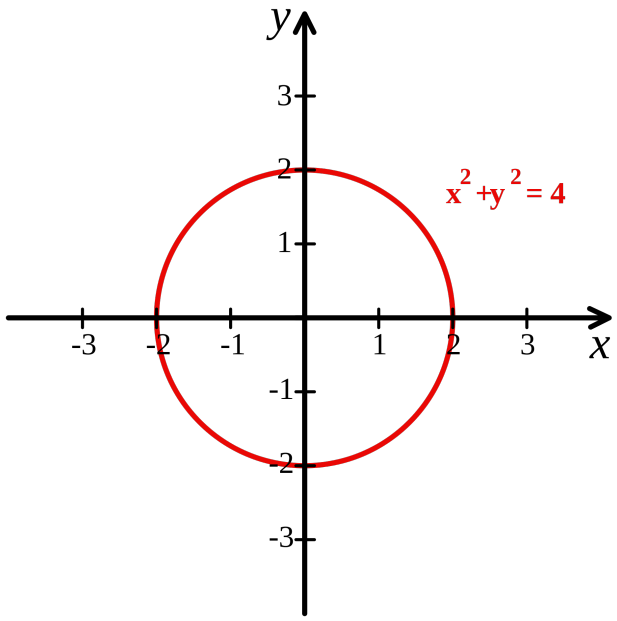
Applications of Cartesian Coordinates

Geometric representation of algebric equations ⟶ **y = f(x)**

Characteristic curve of the quadratic equation



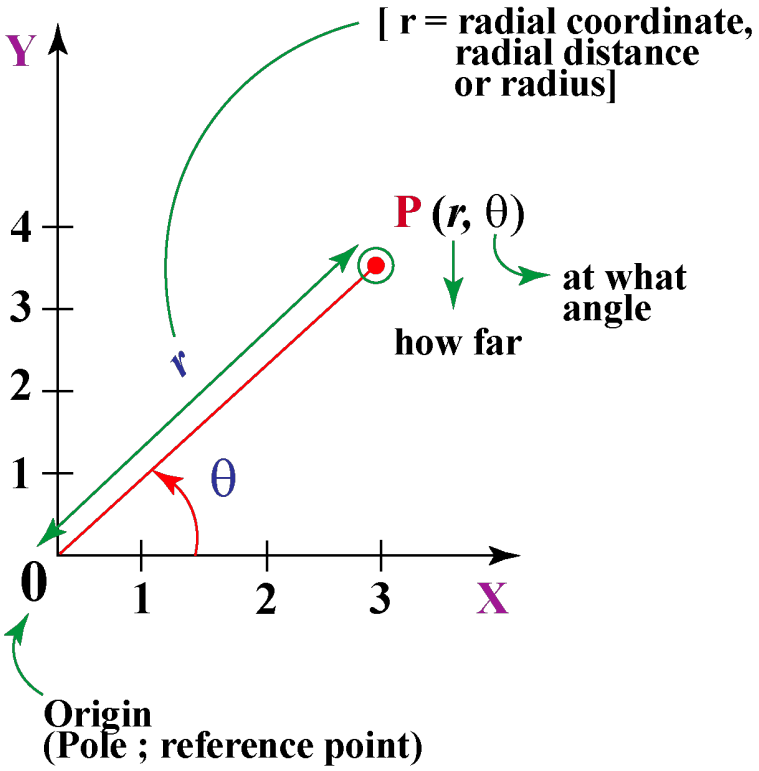
Circle of radius 2, centered at the origin



Ref: https://www.skillsyouneed.com/num/cartesian-coordinates.html, https://en.wikipedia.org/wiki/Cartesian_coordinate_system

Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)



In the polar coordinate system, each point on a plane has a unique distance from a reference point and a specific angle from a reference direction. A polar coordinate is specified by its radial coordinate **r** and its angular coordinate **θ**.

Ref: https://en.wikipedia.org/wiki/File:CircularCoordinates.svg, https://www.cuemath.com/geometry/polar-coordinates/, https://www.cuemath.com/geometry/polar-coordinates/#what-are-polar-coordinates

Coordinate system in 2 Dimension (2D)

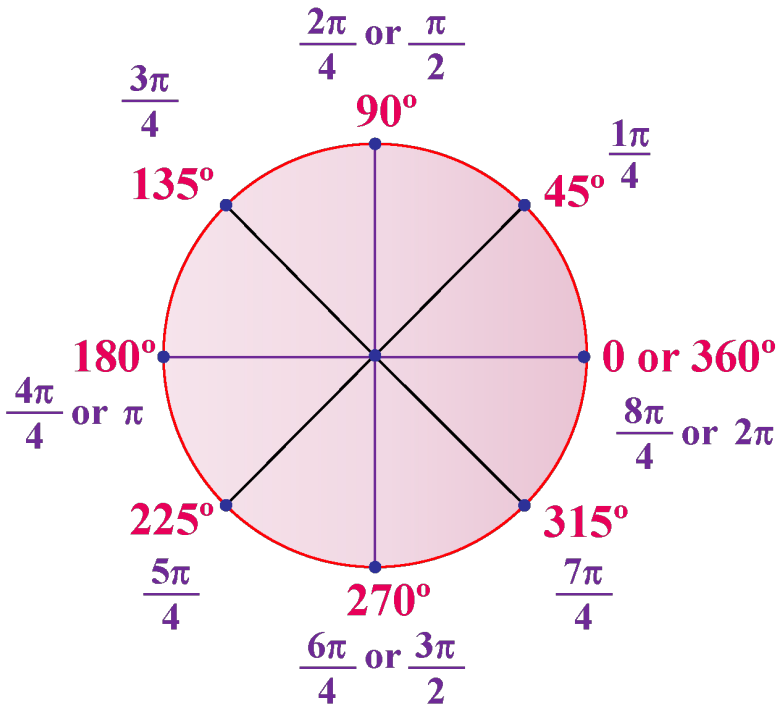
Plane polar coordinates (r, θ)

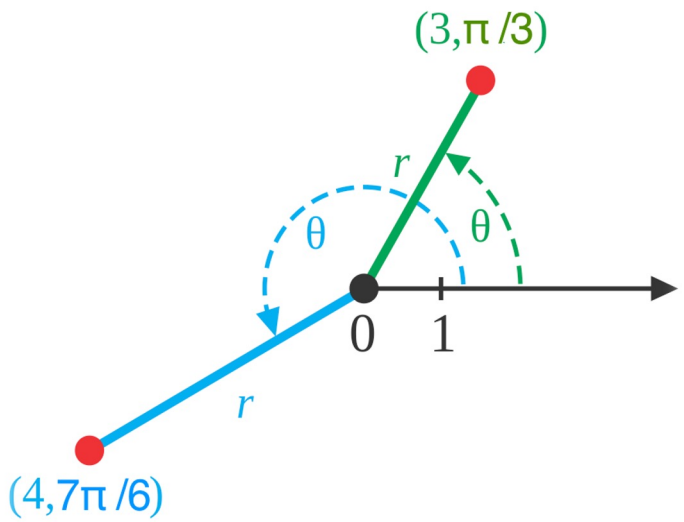
How to Plot Points Using Polar Coordinates?

General form for writing polar coordinates is **P (r,θ)**

Angles in polar coordinates are expressed in either degrees or radians.

360°=2π radians





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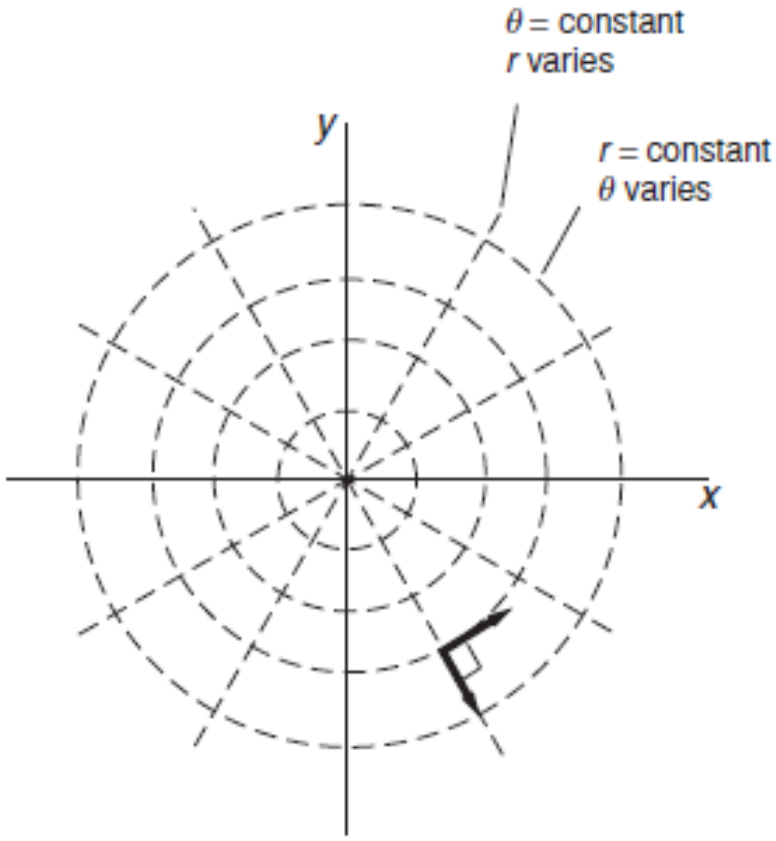
Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)

How to Plot Points Using Polar Coordinates?

Orthogonal coordinates



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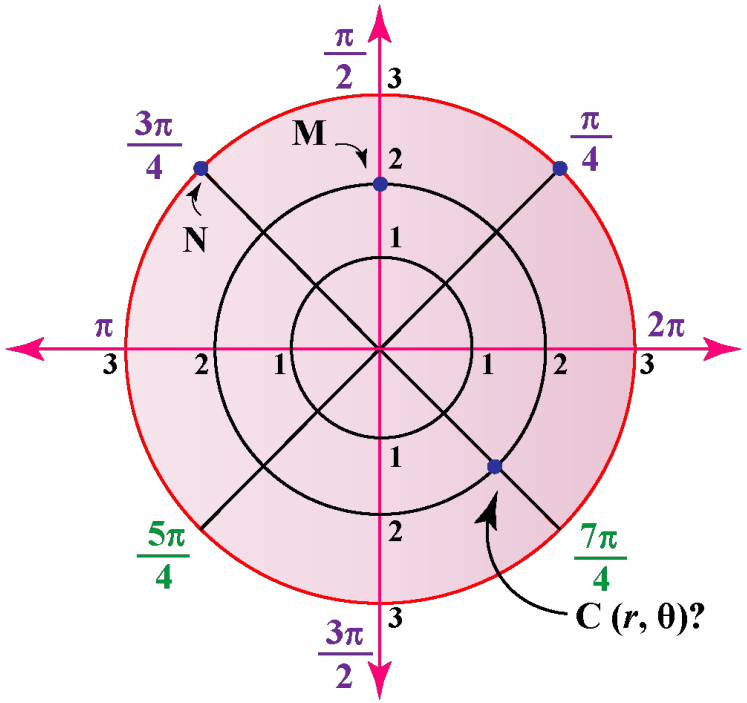
Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)

How to Plot Points Using Polar Coordinates?

Q1. Plot the points M (2, $\frac{\pi}{2}$) and N (3, $\frac{3\pi}{4}$) on a polar coordinate plane?



Can you also identify the polar coordinate of point **C**?

$$(2, \frac{7\pi}{4})$$

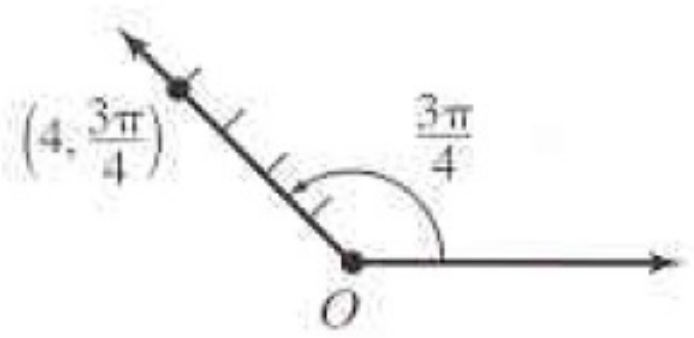
Ref: https://en.wikipedia.org/wiki/File:CircularCoordinates.svg, https://www.cuemath.com/geometry/polar-coordinates/

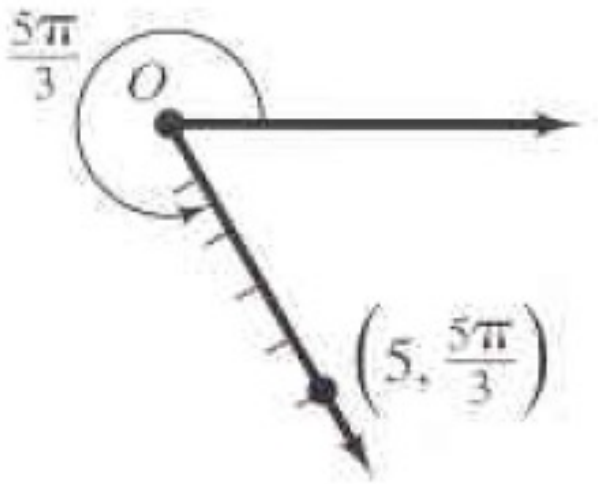
Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)

How to Plot Points Using Polar Coordinates?

Q2. Plot $(4, \frac{3\pi}{4})$ and $(5, \frac{5\pi}{3})$ on a polar coordinate plane?





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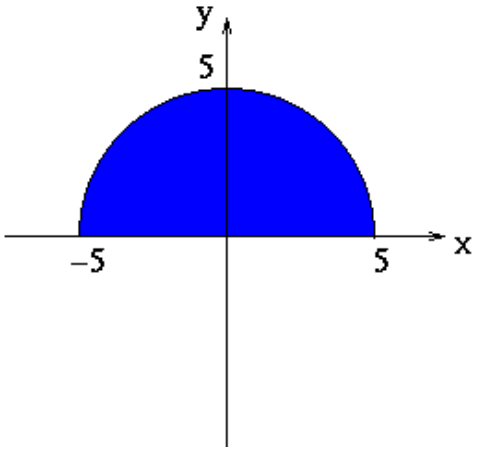
Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)

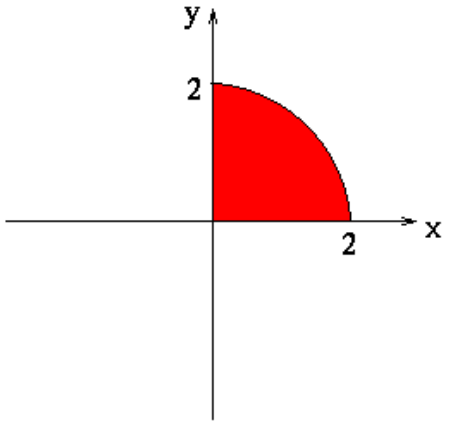
How to Plot Points Using Polar Coordinates?

Q3. How to plot the intervals on a polar coordinate plane?

$$0 \le r \le 5, \qquad 0 \le \theta \le \pi$$



$$0 \le r \le 2, \qquad 0 \le \theta \le \frac{\pi}{2}$$



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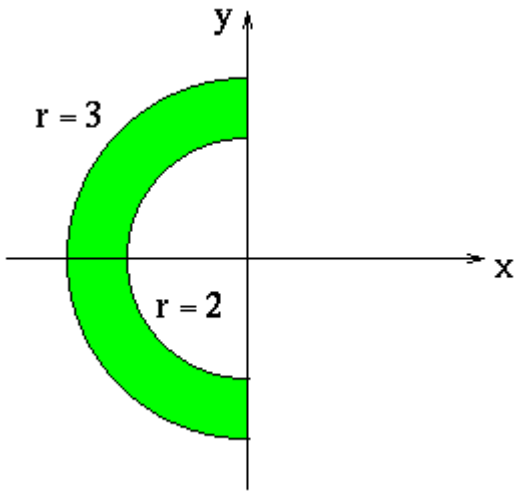
Coordinate system in 2 Dimension (2D)

Plane polar coordinates (r, θ)

How to Plot Points Using Polar Coordinates?

Q4. How to plot the intervals on a polar coordinate plane?

$$2 \le r \le 3, \qquad \frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$$

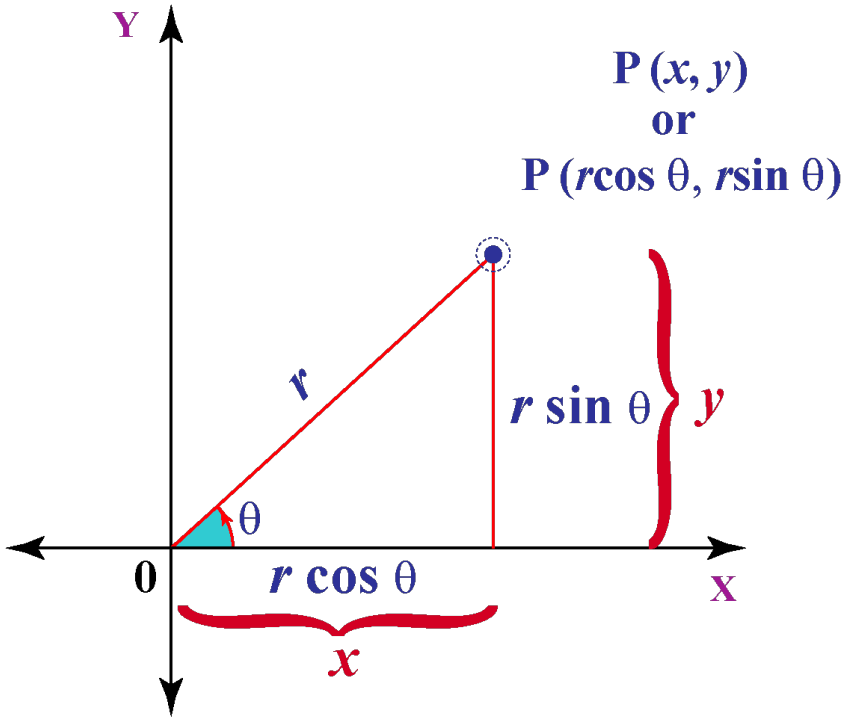


Ref: https://en.wikipedia.org/wiki/File:CircularCoordinates.svg, https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Conversion Between Polar and Cartesian Coordinates

How to Convert Polar Coordinates into Cartesian Coordinates?



Right-angled triangle
Hypotenuse: r

Base: $x = r\,cos\theta$
Height: $y = r\,sin\theta$

Ref: https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Conversion Between Polar and Cartesian Coordinates

How to Convert Cartesian Coordinates into Polar Coordinates?

Q1: What will be Cartesian coordinates for polar coordinates (10,30°)?

Solution: For x coordinate, we will use the cosine function

$$x = r \times \cos\theta$$

For y coordinate, we will use the sine function, $y = r \times \sin\theta$
Putting $r = 10$

$$\cos 30 = \frac{\sqrt{3}}{2}$$
$$x = 10 \times \sqrt{3}/2$$
$$x = 5\sqrt{3}$$
$$x = 5 \times 1.732$$
$$x = 8.66$$

Trigonometric Table

θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

For y we will use sine function

$$\sin 30^\circ = \frac{1}{2}$$
$$y = r \times \sin\theta$$
$$y = 10 \times \frac{1}{2}$$
$$y = 5$$

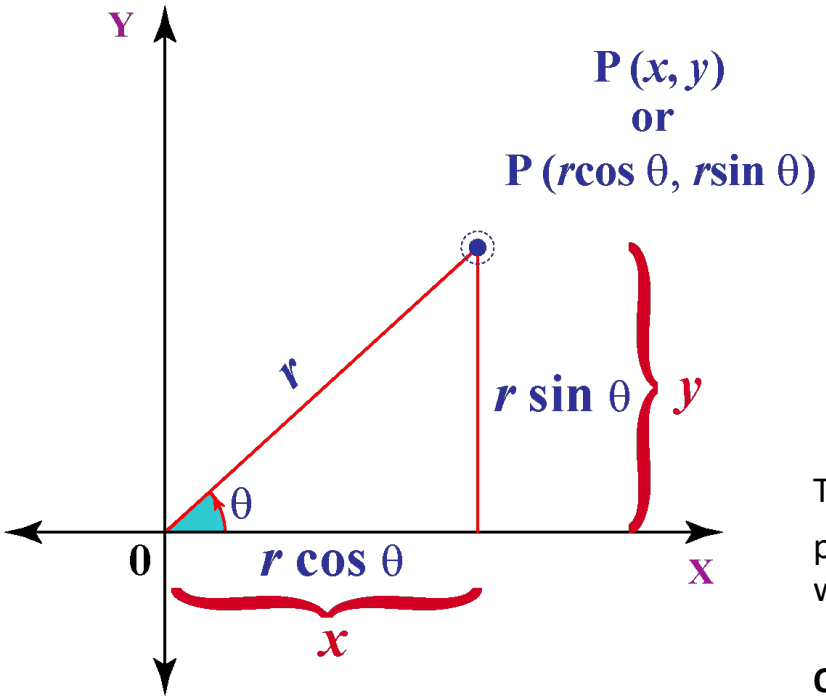
The polar coordinates $(10, 30^\circ)$ are almost exactly $(8.66, 5)$ in Cartesian coordinates.

Ref: https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Conversion Between Polar and Cartesian Coordinates

How to Convert Cartesian Coordinates into Polar Coordinates?



Convert from rectangular to polar coordinates

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\frac{y}{x}$$

The value of $\boldsymbol{tan^{-1}}(\frac{y}{x})$ for converting Cartesian coordinates to polar coordinates may need to be adjusted as per the quadrant in which the point lies:

Quadrant I: Use the calculated value (with sign)
Quadrant II: Add 180° to the calculated value (with sign)
Quadrant III: Add 180° to the calculated value (with sign)
Quadrant IV: Add 360° to the calculated value (with sign)

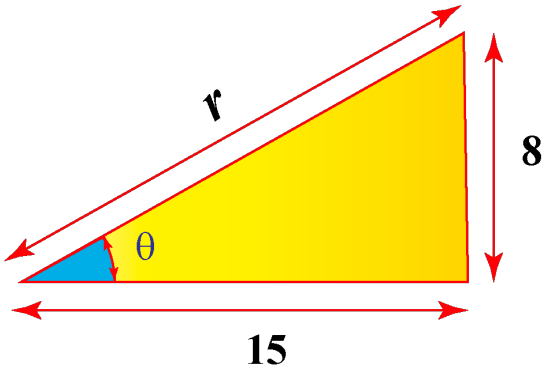
Ref: https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Conversion Between Polar and Cartesian Coordinates

How to Convert Cartesian Coordinates into Polar Coordinates?

Q1: What is (15, 8) in polar coordinates?



Using Pythagoras theorem to find the long side r (the hypotenuse) we get:

$$r^2 = (15^2 + 8^2)$$

$$r = \sqrt{(15^2 + 8^2)}$$

$$r = \sqrt{(225 + 64)}$$

$$r = \sqrt{(289)} = 17$$

Using the tangent function to find the angle:

$$\tan(\theta) = \frac{8}{15}$$

$$\theta = \tan^{-1}\left(\frac{8}{15}\right) = 28.07^\circ$$

The point $(15, 8)$ is $(17, 28.07^\circ)$ in polar coordinates.

Ref: https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Conversion Between Polar and Cartesian Coordinates

How to Convert Cartesian Coordinates into Polar Coordinates?

Q2: What will be the polar coordinates for the point (−4,-5)?

Solution:

P is in quadrant III.

The value of r can be calculated as

$$r = \sqrt{(-4^2 + (-5)^2)}$$

$$r = \sqrt{(16 + 25)}$$

$$r = \sqrt{(41)} = 6.4$$

Angular coordinate is

$$\theta = \tan^{-1}\left(\frac{-5}{-4}\right)$$

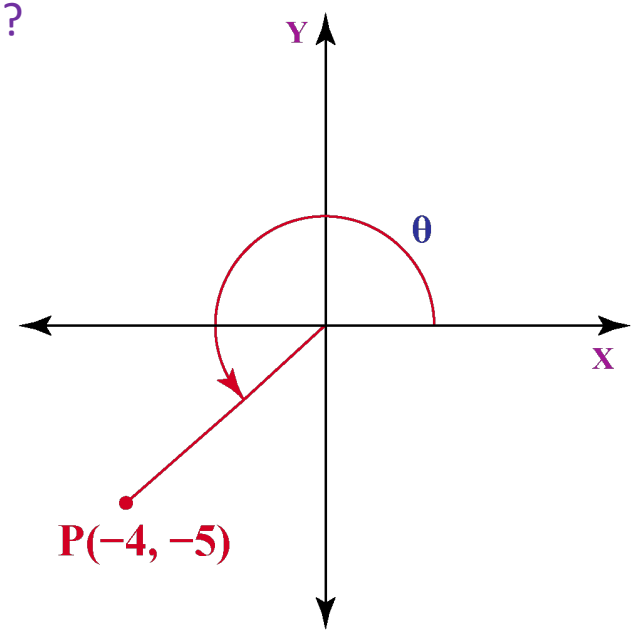
$$\theta = \tan^{-1}(1.25)$$

The calculator value for $tan^{-1}(1.25)$ is $51.34°$

For quadrant III we will addd $180°$ to the calculated value

$$\theta = 51.34° + 180° = 231.34°$$

So the polar coordinates for the point $(-4, -5)$ are 6.4, 231.34°



Quadrant III: Add 180° to the calculated value

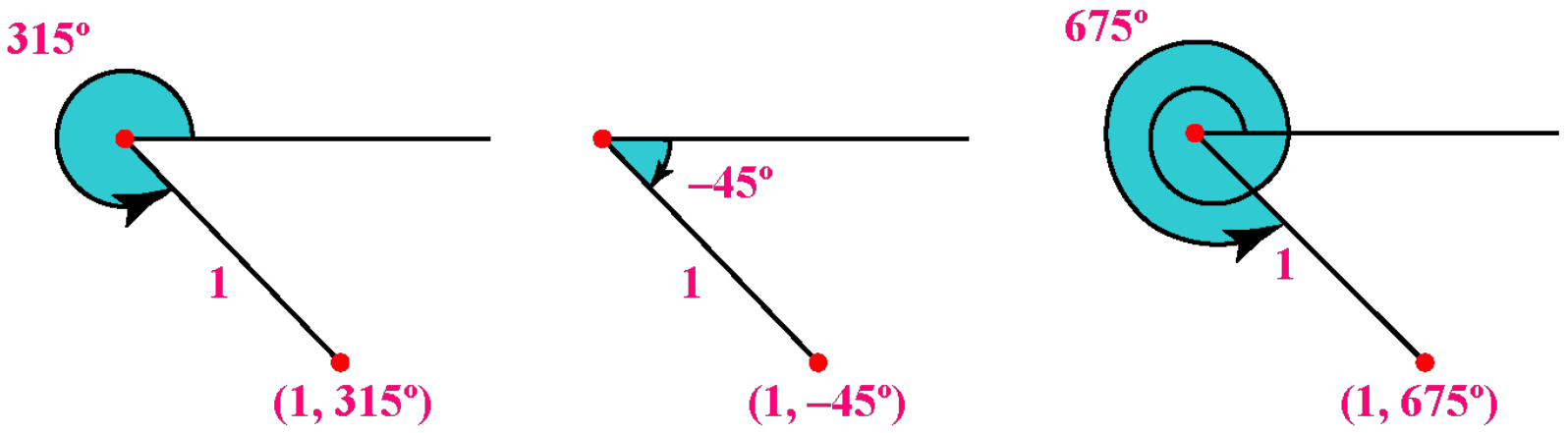
Ref: https://www.cuemath.com/geometry/polar-coordinates/

Coordinate system in 2 Dimension (2D)

Are polar coordinates unique?

Polar coordinates are not unique. Every point has infinitely polar coordinates that are not unique. However, Rectangular/ Cartesian coordinates are unique.

E1: the polar coordinates (1, 315°), (1,−45°) and (1, 675°) all represent the same point



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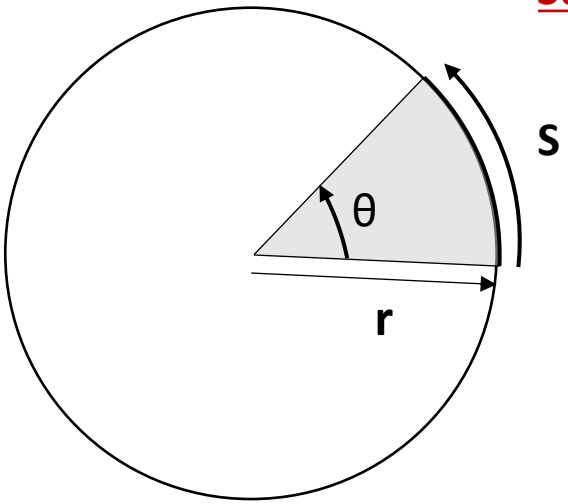
Coordinate system in 2 Dimension (2D)

Surface element in Polar Coordinates

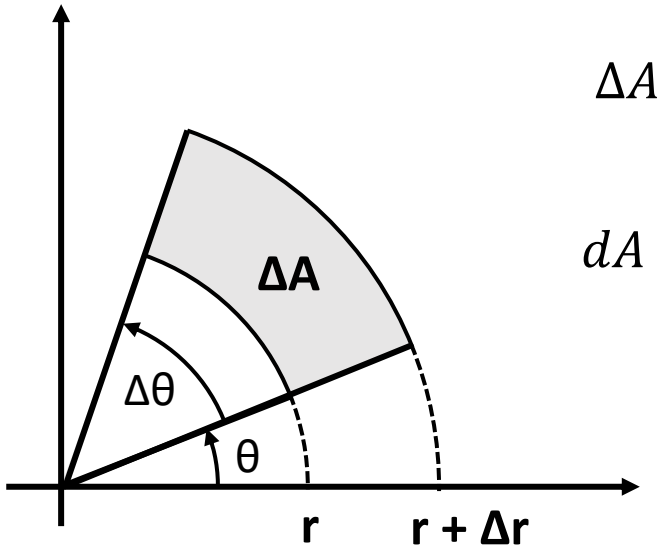


s

Length of the arc = s = θ/360 × 2πr

360°=2π radians

Then **s = rθ**



$$\Delta A \approx length \times width = (\Delta r)(r\Delta\theta)$$

$$dA = (dr)(rd\theta) = rdrd\theta$$

Ref: https://www.cuemath.com/geometry/polar-coordinates/

Next lecture:

3D coordinate systems