Circuit Analysis Techniques

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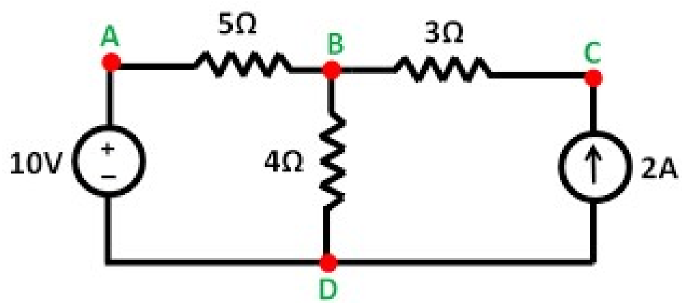
Few Terms:

- Branch
- Path
- Loop
- Mesh
- Node



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Circuit Analysis

1. Mesh Analysis

   Assuming Mesh currents write KVL using Ohm's Law

2. Nodal Analysis

   Assuming Node Voltages write KCL using Ohm's Law

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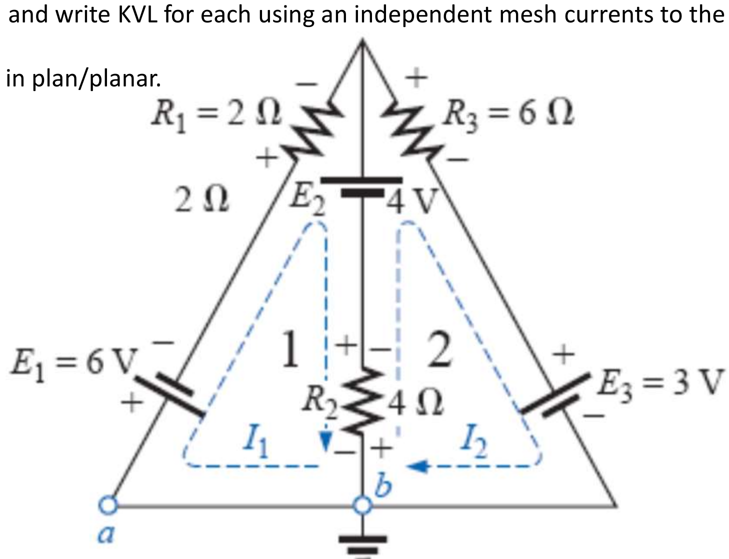
Find the branch currents of the network using Mesh Analysis ?

Identify the total no. of Mesh and write KVL for each using an independent mesh currents to the each one of them.
Make sure the given circuit is in plan/planar.



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Steps 1 and 2 are as indicated in the circuit. [Identify the Mesh and assign an independent current to them]

Step 3: Kirchhoff's voltage law is applied around each closed loop:

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point *a*)

$-6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point *b*)

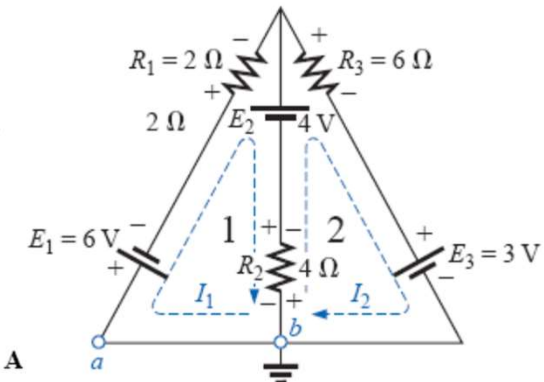
$\underline{-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0}$

which are rewritten as

$$\left.\begin{array}{r} -10 - 4I_1 - 2I_1 + 4I_2 = 0 \\ +\ 1 + 4I_1 - 4I_2 - 6I_2 = 0 \end{array}\right\} \begin{array}{r} -6I_1 + \ \ 4I_2 = +10 \\ +4I_1 - 10I_2 = -1 \end{array}$$

or, by multiplying the top equation by -1, we obtain

$$\begin{aligned} 6I_1 - \ \ 4I_2 &= -10 \\ 4I_1 - 10I_2 &= -1 \end{aligned}$$



Step 4:

$$I_1 = \frac{\begin{vmatrix} -10 & -4 \\ -1 & -10 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ 4 & -10 \end{vmatrix}} = \frac{100 - 4}{-60 + 16} = \frac{96}{-44} = \mathbf{-2.182\ A}$$

$$I_2 = \frac{\begin{vmatrix} 6 & -10 \\ 4 & -1 \end{vmatrix}}{-44} = \frac{-6 + 40}{-44} = \frac{34}{-44} = \mathbf{-0.773\ A}$$

The current in the 4-Ω resistor and 4-V source for loop 1 is

$$\begin{aligned} I_1 - I_2 &= -2.182\text{ A} - (-0.773\text{ A}) \\ &= -2.182\text{ A} + 0.773\text{ A} \\ &= \mathbf{-1.409\ A} \end{aligned}$$

revealing that it is 1.409 A in a direction opposite (due to the minus sign) to I_1 in loop 1.

Cramer's Rule for Three Equations in Three Unknowns

The solution to the system

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

is given by $x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$, and $z = \frac{D_z}{D}$, where

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \qquad D_x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix},$$

$$D_y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \qquad D_z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix},$$

provided that $D \neq 0$.

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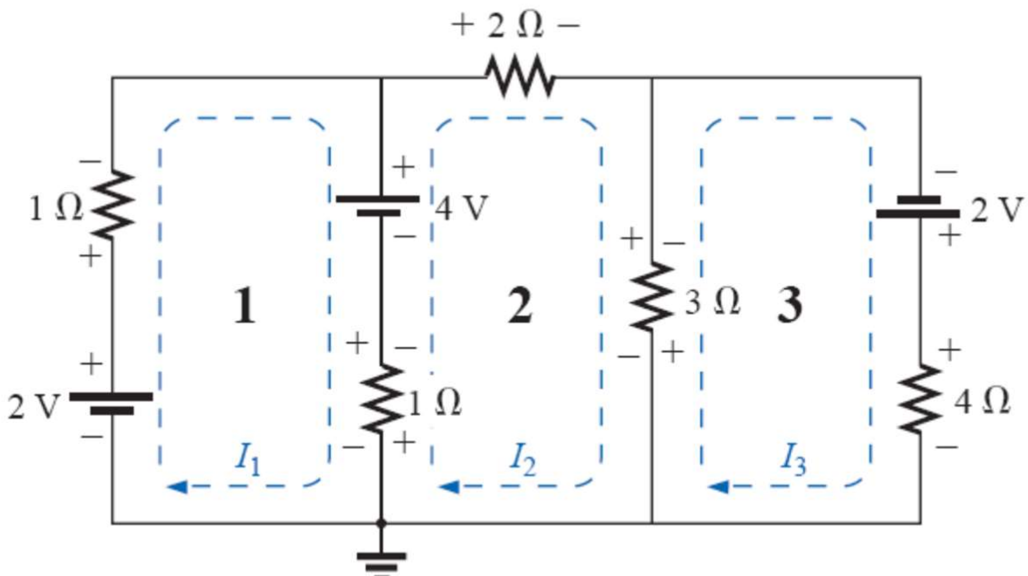
Write the mesh equations for the network



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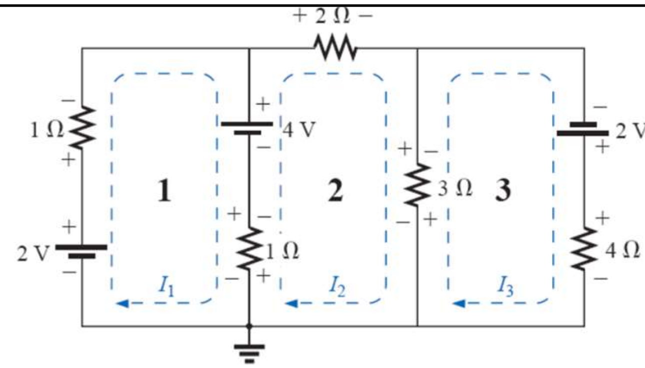

I_1 does not pass through an element mutual with I_3.

$$I_1: \quad (1\ \Omega + 1\ \Omega)I_1 - (1\ \Omega)I_2 + 0 = 2\ \text{V} - 4\ \text{V}$$
$$I_2: \quad (1\ \Omega + 2\ \Omega + 3\ \Omega)I_2 - (1\ \Omega)I_1 - (3\ \Omega)I_3 = 4\ \text{V}$$
$$I_3: \quad (3\ \Omega + 4\ \Omega)I_3 - (3\ \Omega)I_2 + 0 = 2\ \text{V}$$

I_3 does not pass through an element mutual with I_1.

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Mesh Analysis If current source is present in the circuit:

Method 1: **Super-Mesh Analysis**

Method 2: Assume voltage across current source (Vx) then apply mesh analysis, use node to relate mesh current and source current.

Method 3: If possible convert all current sources into Voltage sources then use mesh analysis **(Source conversion:** convert the current source to a voltage source **if a parallel resistor is present)**

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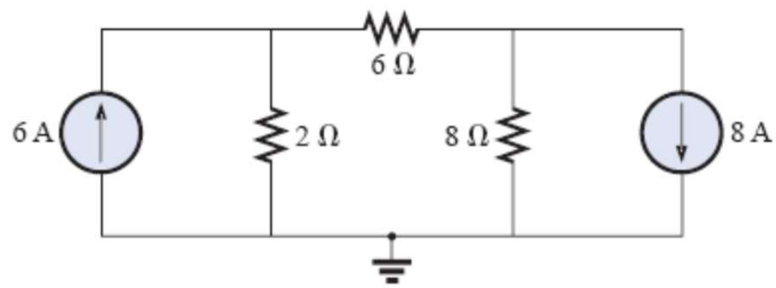
Super Mesh Approach

Find the current through 6 Ohm Resistor.



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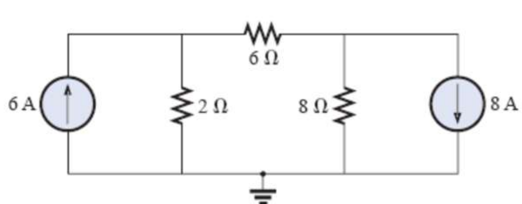
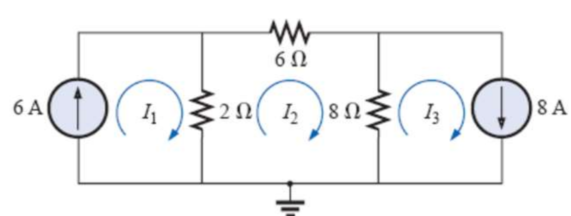
Ckt having current Sources: **at the periphery of the network**

Super-Mesh Analysis:

1. Assign a mesh current to each independent loop, including the current sources, as if they were resistors or voltage sources.





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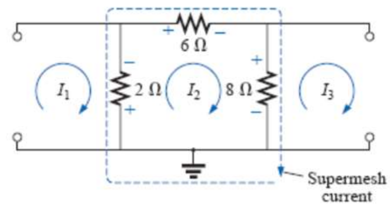
2. Remove current sources (replace with open-circuit equivalents, mentally),

Any resulting path, including two or more mesh currents, is said to be the path of a *supermesh* current.

3. Apply KVL to all the remaining independent paths of the network using the mesh currents



$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$
$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$
$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$
$$2I_1 - 16I_2 + 8I_3 = 0$$

4. **Relate the chosen mesh currents** of the network **to the independent current sources** of the network, and solve for the mesh currents.

$$I_1 = 6\text{ A}$$
$$I_3 = 8\text{ A}$$

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$$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\ A}$$
$$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\ A}$$
$$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\ A}$$

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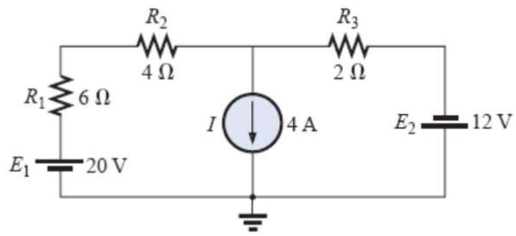
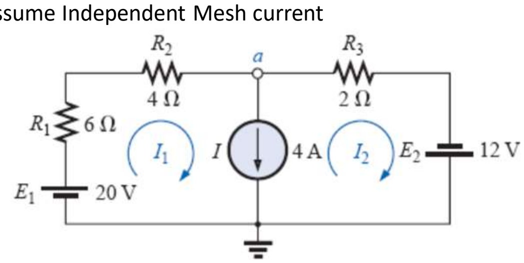
(II) Super Mesh: Current source is common to two meshes



Assume Independent Mesh current



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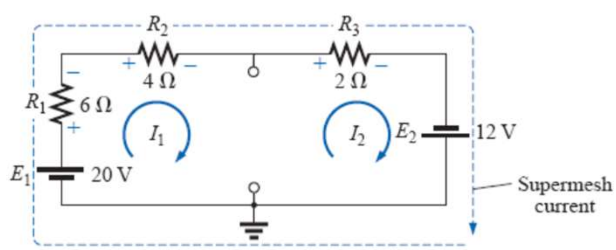
Eliminate the Current source Defining the supermesh current



Applying Kirchhoff's law:

$$20\text{ V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\text{ V} = 0$$

$$10I_1 + 2I_2 = 32$$

Relate the mesh currents and the current source using Kirchhoff's current law:

$$I_1 = I + I_2$$

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NODAL ANALYSIS : Assuming Node Voltages write KCL using Ohm's Law

The nodal analysis method is applied as follows:

1. *Determine the number of nodes within the network.*
2. *Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law. In other words, for each node, don't be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.*
4. *Solve the resulting equations for the nodal voltages.*

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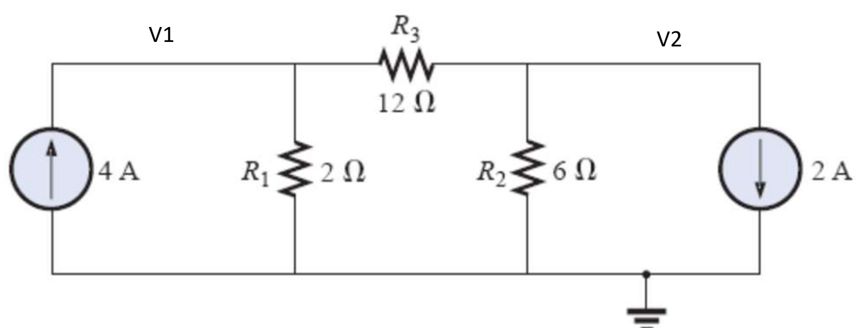
Calculate Nodal Voltages, the magnitude & direction of current through R3



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ANS: V_1=6 V, V_2= -6 V, $I(R_3)$= 1 A

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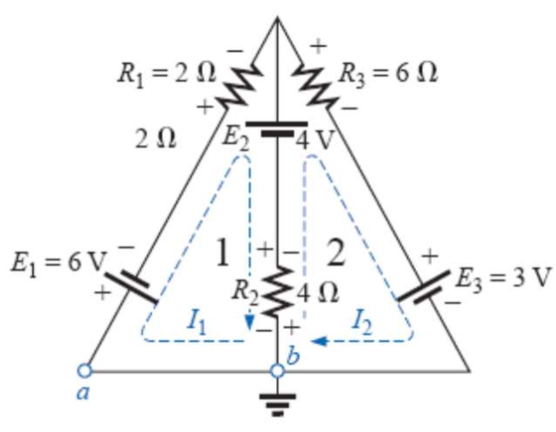
Find the branch currents of the network using Nodal Analysis ?



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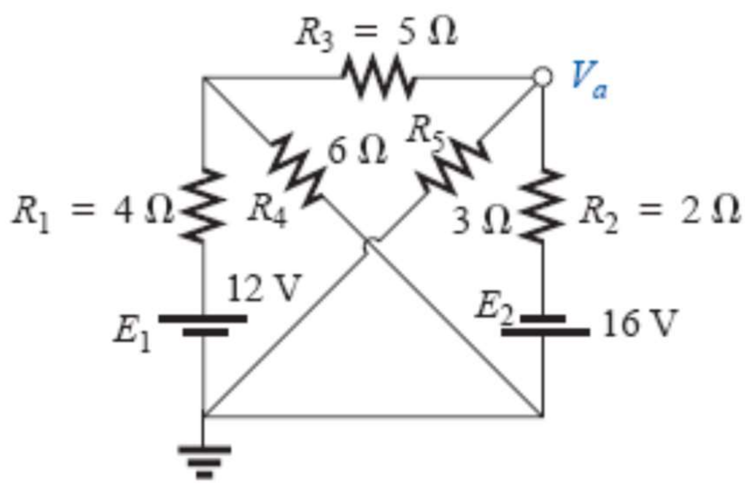
Determine Va, Using Nodal Analysis:
Mesh Analysis is valid only for ckts that can be drawn in a plane



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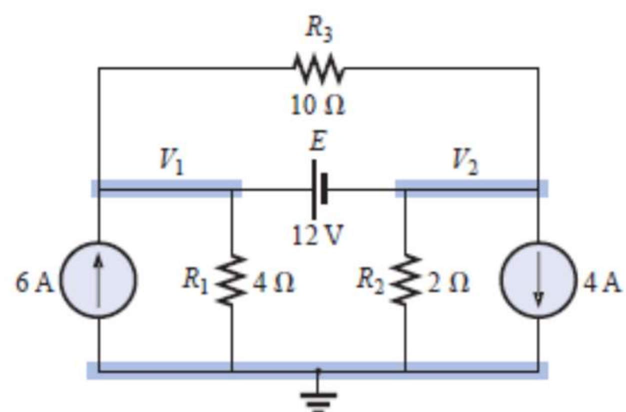
Nodal Analysis (Continue…)

Concept of Super Node: when two nodes are connected with voltage source

Determine the Nodal Voltages V_1 and V_2



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when applying Kirchhoff's current law, as shown below:

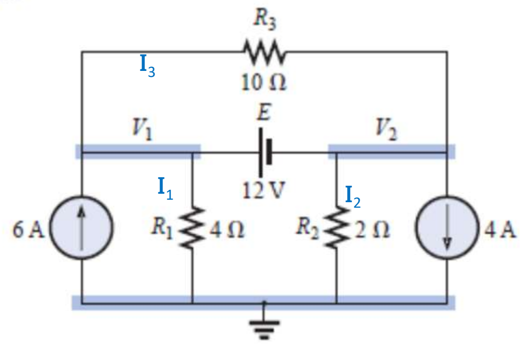
$$\Sigma I_i = \Sigma I_o$$

$$6\text{ A} + I_3 = I_1 + I_2 + 4\text{ A} + I_3$$

or $$I_1 + I_2 = 6\text{ A} - 4\text{ A} = 2\text{ A}$$

Then $$\frac{V_1}{R_1} + \frac{V_2}{R_2} = 2\text{ A}$$

and $$\frac{V_1}{4\ \Omega} + \frac{V_2}{2\ \Omega} = 2\text{ A}$$



Relating the defined nodal voltages to the independent voltage source, we have

$$V_1 - V_2 = E = 12\text{ V}$$

which results in two equations and two unknowns:

$$\begin{aligned} 0.25V_1 + 0.5V_2 &= 2 \\ V_1 - \ \ 1V_2 &= 12 \end{aligned}$$

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Substituting:

$$V_1 = V_2 + 12$$

$$0.25(V_2 + 12) + 0.5V_2 = 2$$

and $$0.75V_2 = 2 - 3 = -1$$

so that $$V_2 = \frac{-1}{0.75} = \mathbf{-1.333\ V}$$

and $$V_1 = V_2 + 12\text{ V} = -1.333\text{ V} + 12\text{ V} = \mathbf{+10.667\ V}$$

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Circuits with Dependent Sources

Nodal Analysis

Determine the value of v and the power supplied by the independent current source in Fig. 3.17.

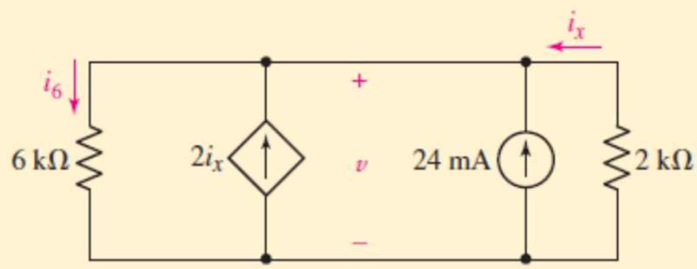

FIGURE 3.17 A voltage v and a current i_6 are assigned in a single-node-pair circuit containing a dependent source.

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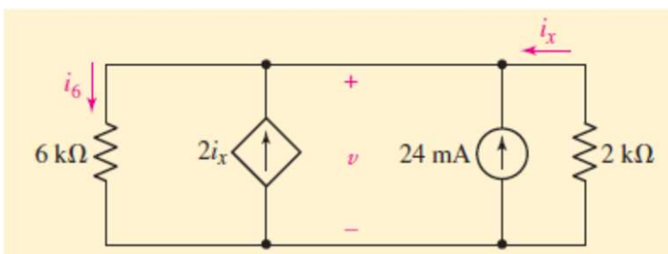

By KCL, the sum of the currents leaving the upper node must be zero, so that

$$i_6 - 2i_x - 0.024 - i_x = 0 \qquad \text{Eq. 1}$$

We next apply Ohm's law to each resistor:

$$i_6 = \frac{v}{6000} \quad \text{and} \quad i_x = \frac{-v}{2000} \qquad \text{Eq. 2}$$

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Solving Eq. 1 and 2

$$\frac{v}{6000} - 2\left(\frac{-v}{2000}\right) - 0.024 - \left(\frac{-v}{2000}\right) = 0$$

and so $v = (600)(0.024) = 14.4$ V.

Any other information we may want to find for this circuit is now easily obtained, usually in a single step. For example, the power supplied by the independent source is $p_{24} = 14.4(0.024) = 0.3456$ W (345.6 mW).

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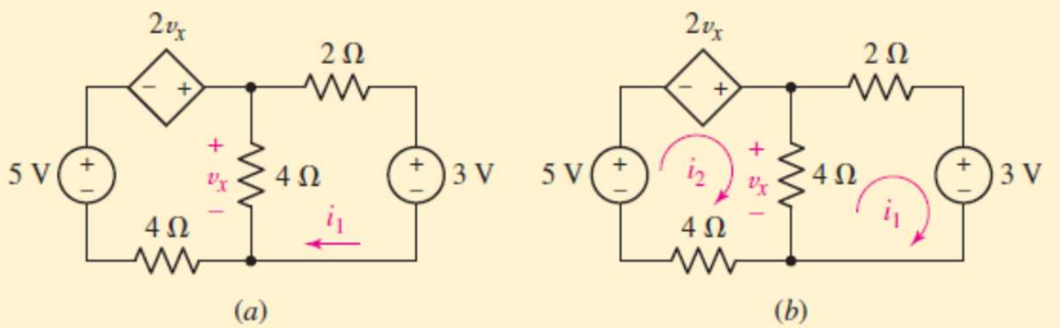
Mesh Analysis with Dependent Sources

Determine the current i_1 in the circuit of Fig. 4.22*a*.



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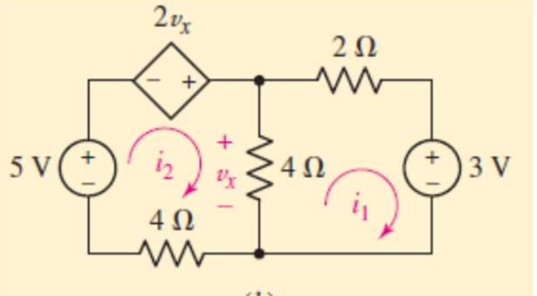

(b)

For the left mesh, KVL now yields

$$-5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0$$ Eq. 1

and for the right mesh we find the same as before, namely,

$$4(i_1 - i_2) + 2i_1 + 3 = 0$$ Eq. 2

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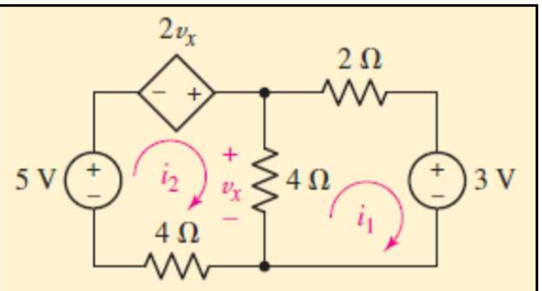
$$v_x = 4(i_2 - i_1)$$ Eq. 3



(b)

Solving Eq. 1 and 3 $$4i_1 = 5$$

we find that $i_1 = 1.25$ A.

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