SHM

1. An object is in simple harmonic oscillation about the position $x = 0$ with a period of 2 seconds and an amplitude of 4 cm. At time $t = 0$ its position is $x = +2$ cm. Write its displacement x, velocity $\dot{x}$, and acceleration $\ddot{x}$ as functions of time.

2. Given $x(t) = A\,\sin(\omega\,t + \phi_0)$ for the displacement from equilibrium in simple harmonic oscillation, determine the phase constant ϕ_0 for each of the following conditions at $t = 0$:

(a) $x(0) = A$ (b) $x(0) = -A$
(c) $x(0) = 0$ and $\dot{x}(0) < 0$ (d) $x(0) = 0$ and $\dot{x}(0) > 0$
(e) $x(0) = A/2$ and $\dot{x}(0) > 0$ (f) $x(0) = A/2$ and $\dot{x}(0) < 0$

3. An object of mass $m = 25$ g on a frictionless flat surface is attached to the right-hand end of a horizontal spring with $k = 0.4$ N m^{-1}. At time $t = 0$ the object is located 10 cm to the right of its equilibrium position and has a velocity $\dot{x} = 40$ cm s^{-1} towards the right. Knowing that $x(t) = A\sin(\omega t + \phi_0)$, find

(a) the angular frequency ω of the oscillation; (b) the period T; (c) the frequency f; (d) the amplitude A and phase constant ϕ_0; (e) the position x and velocity $\dot{x}$ at time $t = \pi/8$; (f) the maximum velocity $\dot{x}_{\rm max}$, and the position x at which $\dot{x} = \dot{x}_{\rm max}$; (g) the maximum acceleration $\ddot{x}_{\rm max}$, and the position x at which $\ddot{x} = \ddot{x}_{\rm max}$.

4. A simple pendulum, consisting of a 100-gram bob at the end of a string of length $L = 1$ m, is hanging straight down in equilibrium. Another 100-gram mass moving horizontally at a speed of $+2$ m s^{-1} hits the bob of the pendulum at $t = 0$ and sticks to it, which starts the pendulum swinging.

(a) Calculate the angular frequency ω, given that $g = 9.81$ m s^{-2}.

(b) Use the conservation of momentum to find the initial linear velocity, $\dot{s}(0)$, of the pendulum bob, and from this find the initial angular velocity $\dot{\theta}(0) = \dot{s}(0)/L$.

(c) From the information above, and assuming that the angular displacement as a function of time is of the form $\theta(t) = \theta_{\rm max}\,\sin(\omega\,t + \phi_0)$, find the phase constant ϕ_0 and the angular amplitude $\theta_{\rm max}$. Confirm that the amplitude you find is small enough that the underlying approximation $\sin\theta \simeq \theta$ is a good one, and find the first time $t > 0$ at which $\theta = \theta_{\rm max}$.