PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 14

Conductors

Conductors

Basic properties of a conductor

- Materials which permit the flow of electric charges through them.

- Metallic conductors have free electrons which serve as the charge carrier.

- In liquid conductors, the ions serve as the charge carriers.

- A perfect (or an ideal) conductor would be a material containing an unlimited supply of completely free charges. This ideal scenario, of course, is not achieved in real life. However, many substances come very close to being a perfect conductor, e.g.metals.

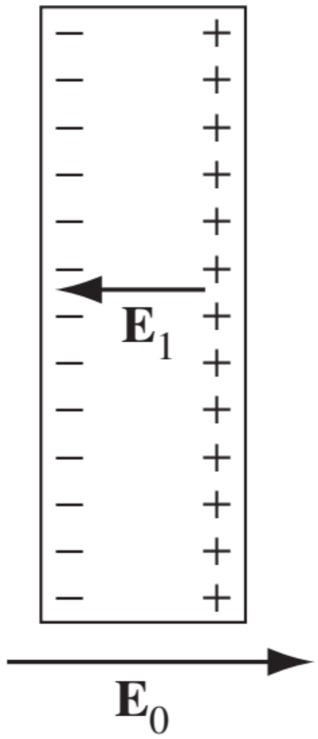
In the following, when we mention ‘Conductor’, we will be referring to an ideal conductor.

Conductors

<u>Basic electrostatic properties of ideal conductor</u>

- **E = 0, inside the conductor**

An ideal conductor has an unlimited supply of free electrons. Once it is placed in an electric field, the free electrons are driven opposite to the direction of the electric field towards the edge of the conductor, resulting in accumulation of **induced charge** on the surface. The effect of this induced surface charge density is such that electric field generated by it inside that material exactly cancels the external electric field resulting in a net zero field inside the conductor.



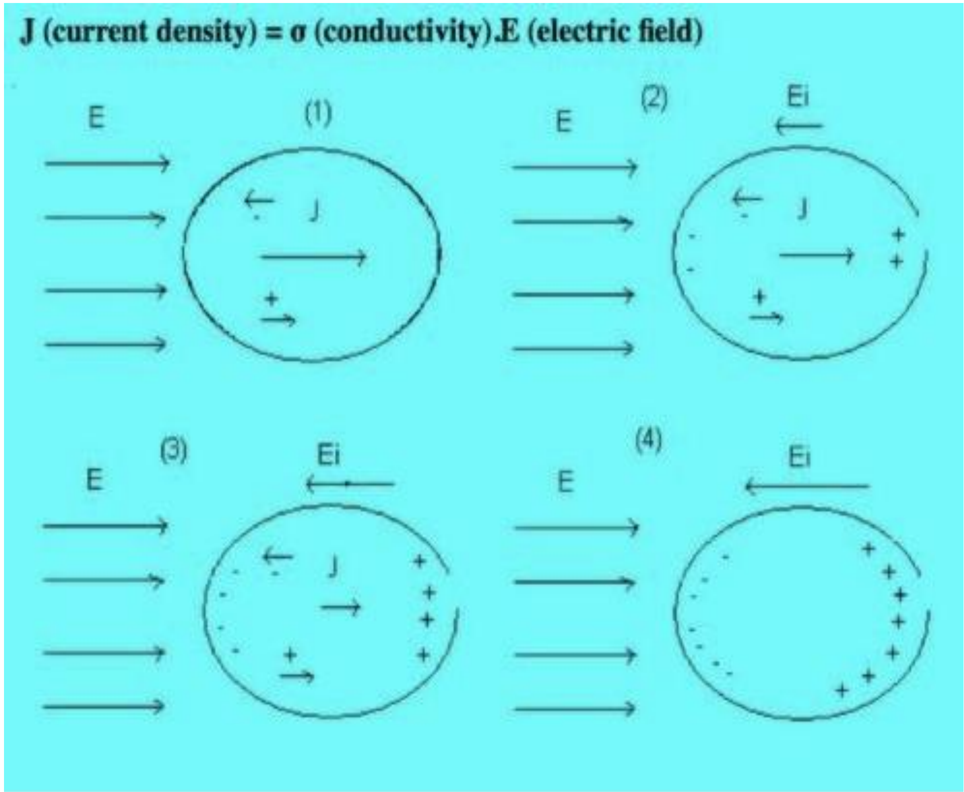
$$\boldsymbol{E} = \boldsymbol{E}_0 - \boldsymbol{E}_1 = \mathbf{0}$$

The field of the induced charges *tends to cancel the original field*

Conductors

Basic electrostatic properties of ideal conductor

- **E = 0, inside the conductor**



Look at the adjoining picture

In **part (1)** all the electric field (let the direction be along +ve x axis) is penetrating through the material causing a current density J (current/area) in the same direction.

In **part (2)** as a result of the current in the material there happens to be an electric field opposing the external electric field so now only part of the external electric field is penetrating and the magnitude of the current density J got smaller. The electric field inside the material is still in the +x direction with a magnitude equal to $\mathbf{E} - \mathbf{E}_i$.

In **part (3)** the same action continues as in part (2) and J gets smaller and **E**i gets bigger. As a result the electric field penetrating through the material got even more smaller than part (2).

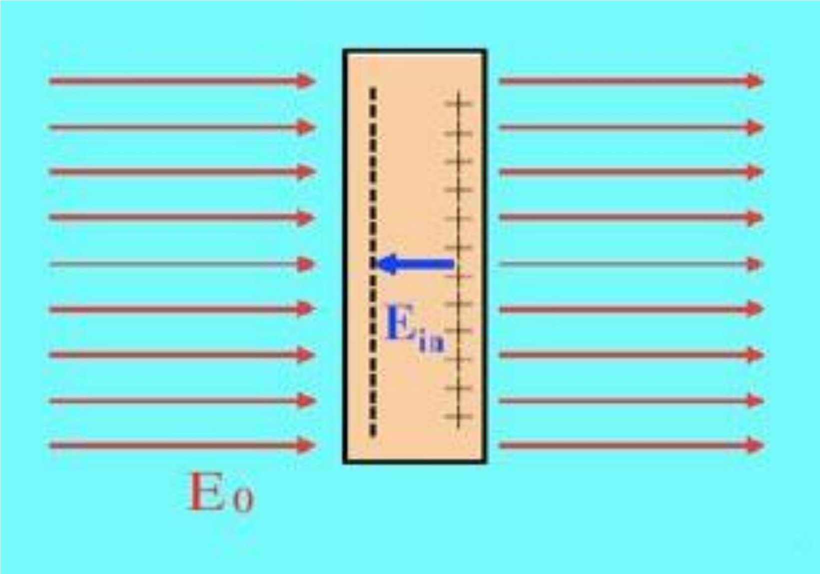
In part (4) $\mathbf{E} = \mathbf{E}_i$ which means there is no more electric field in the material and the current density is zero. We can see that the negative charges are collected on the left side and positive charges on the right that creates a potential difference and the electric field **E**i. We have to remember that and electric field is from positive to negative, the external electric field attracted the negative charges to its source and repelled the positive charges away from its source.

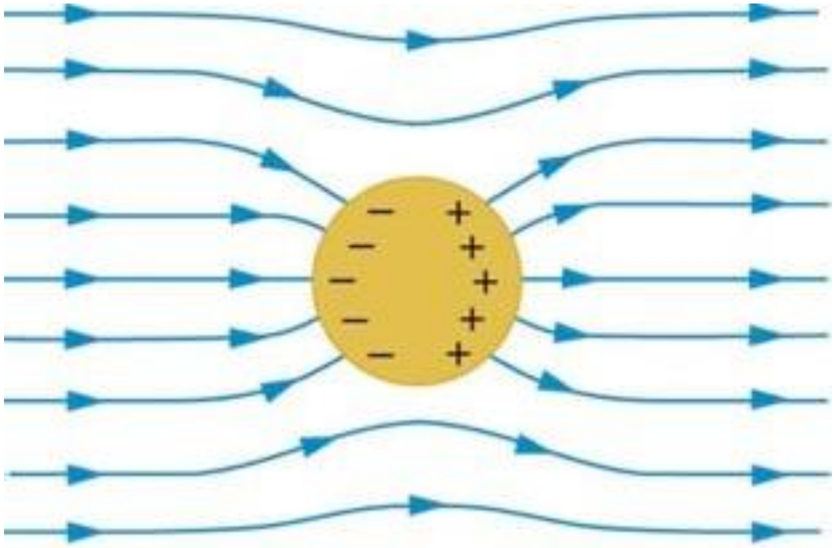
This entire process is practically instantaneous.

Conductors

Basic electrostatic properties of ideal conductor

- **E = 0, inside the conductor**



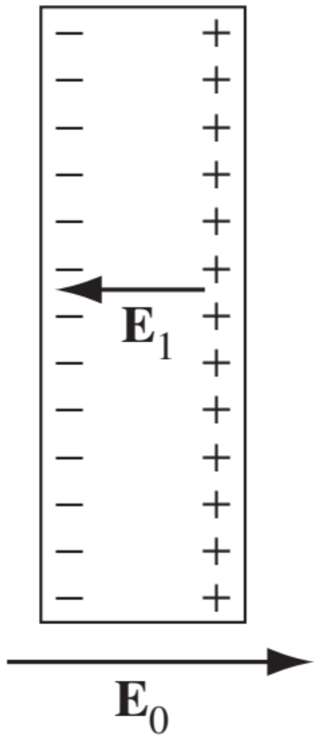


Conductors

Basic electrostatic properties of ideal conductor

- **ρ = 0 inside a conductor.**

This follows from Gauss's law. Since the electric field **E**=0, it is implied that the charge density **ρ** is zero.



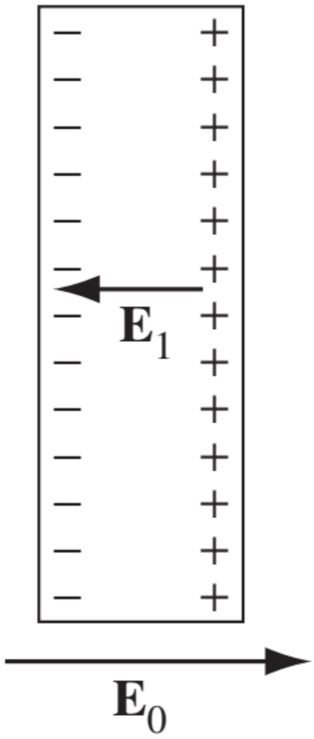
In the equilibrium the charges would have redistributed themselves to give net zero electric field, **E**=0, inside the conductor. There is still charge around, but exactly as much positive as negative, resulting in a net zero chargedensity.

Conductors

Basic electrostatic properties of ideal conductor

- ***Any excess charge resides on the surface only.***

This follows from the last result. Since the net charge density inside the conductor is zero, there cannot be any excess charge inside it, as it would lead to a nonzero $\boldsymbol{\rho}$.



Any free dynamical system seeks a configuration which minimizes its potential energy. The above behavior of charge is consistent with the potential energy minimization. The electrostatic energy is minimum if the charge is spread over the surface.

Conductors

Basic electrostatic properties of ideal conductor

- **A conductor is an equipotential.**

Since $\mathbf{E}$=0 inside the conductor, we have for potentials V($\mathbf{a}$) and V($\mathbf{b}$) at two points $\mathbf{a}$ and $\mathbf{b}$ inside the conductor:

$$V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\Rightarrow V(\mathbf{a}) = V(\mathbf{b}).$$

Since $\mathbf{a}$ and $\mathbf{b}$ are arbitrary points inside the conductor, if follows that the potential is same at all the points inside the conductor. Also, by continuity the same potential exists at the surface also.

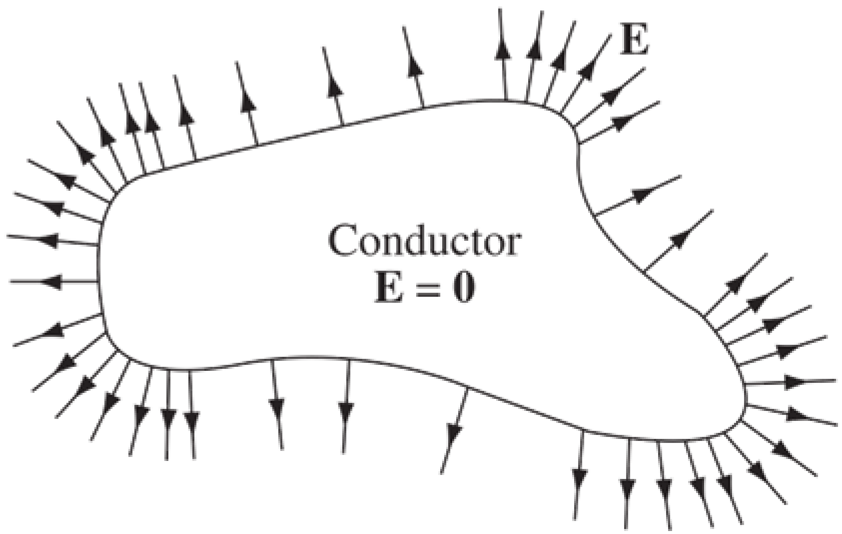
Thus a conductor is an equipotential.

Conductors

Basic electrostatic properties of ideal conductor

- **E** is perpendicular to the surface, just outside a conductor.

If not so, the tangential component of **E** would make the charges to flow across the surface, and eventually the charges would have spread out it such a way that the tangential component dies off.

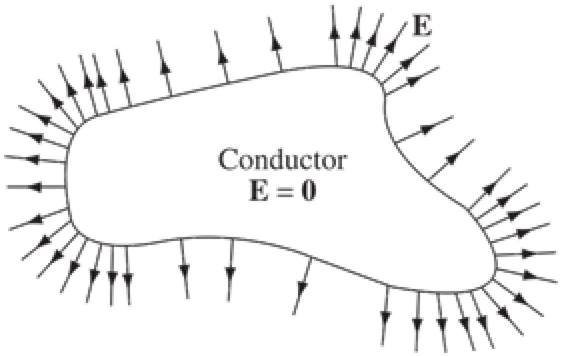


Note that the perpendicular **E** cannot lead to flow of charge, as it is confined to the conducting object —it cannot jump off the surface

Conductors

Basic electrostatic properties of ideal conductor

To summarize:



- **E** = 0 inside a conductor.
- ρ = 0 inside a conductor.
- Any excess charge resides on the surface only.
- A conductor is an equipotential object.
- Just outside the a conductor, the electric field is perpendicular to the surface. ($\mathbf{E} = -\nabla\mathbf{V}$)
- The component of the filed normal to the surface of the conductor is related to the charge density on the surface.

$$\nabla.\mathbf{E} = \frac{\rho}{\varepsilon_0} \rightarrow \mathbf{SI} \qquad \nabla.\mathbf{E} = 4\pi\rho \rightarrow \mathbf{cgs}$$

Conductors

Basic electrostatic properties of ideal conductor

To summarize:

- The potential distribution in the electrostatic filed in the conductor has the following remarkable property:

The potential function $\boldsymbol{V}(\mathbf{x}, \mathbf{y}, \mathbf{z})$ can take maximum and minimum values only at the boundaries of the regions where there is a field.
If a test charge q introduced into the field cannot be in stable equilibrium, since there is no point at which its potential energy (qV) would have minimum.

Proof $\rightarrow$ $\boldsymbol{Earnshaw"s\ theorem}$

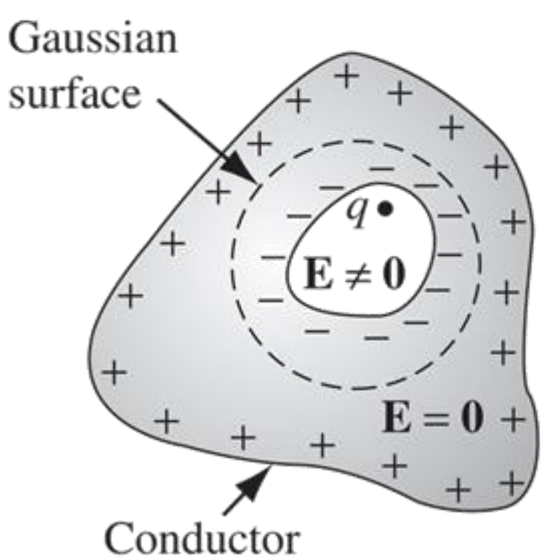
Conductors

Cavity inside the conductor

In all of the preceding discussions when we referred to inside of the conductor it meant inside the solid portion of the conductor.

If there is a cavity inside the conductor with some charge q in it, then the electric field **inside the cavity** is not zero.

However, this charge inside the cavity induces a net charge of opposite sign and equal magnitude (non uniform in general) smeared over the inner surface. This induced charge then kills off the electric field for points external to the cavity but internal to the conductor's outer surface. This can be seen easily by drawing a Gaussian surface enclosing the cavity, but lying within the conductor, for it encloses net zero charge.



Conductors

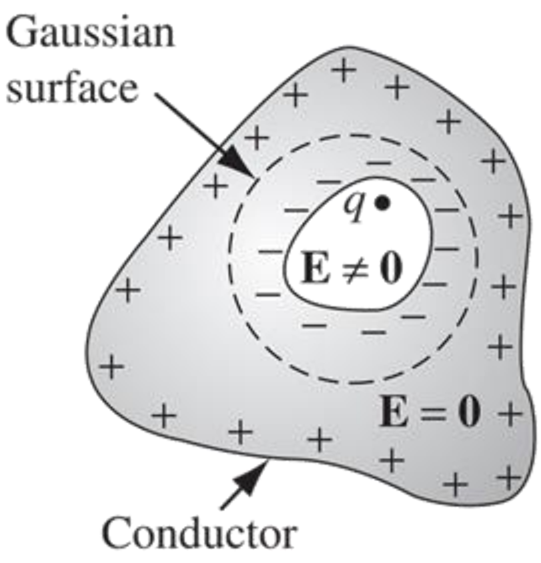
Cavity inside the conductor

OR

arguing the other way we can say that since the field in bulk of the conductor is zero and hence at the Gaussian surface is zero, the net charge enclosed must be zero.

For a point outside the conductor, however, the information about presence of this charge is "communicated" via the compensating charge which appears on the surface of the conductor to balance the charge on the cavity wall. This induced charge on the surface is equal in sign and magnitude to the charge within the cavity.

An external point will experience a net electric field because of this induced charge on the surface. The effect would be same if a net charge ***q*** were supplied to an otherwise neutral conductor from outside.

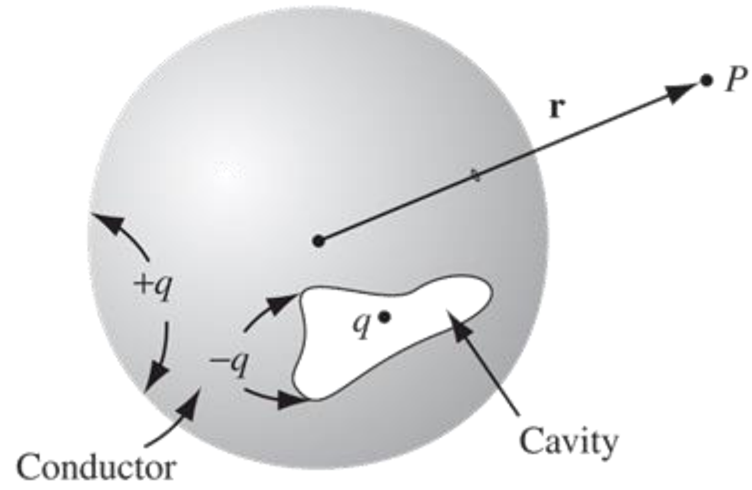


The nature of the cavity is never revealed to the world outside the conductor! However, information about the charge (if any) inside the cavity does get communicated to the outside world.

Conductors

Cavity inside the conductor

P 1. Consider a conducting sphere of radius R with a cavity of an arbitrary shape within it and enclosing a charge q. What will be the electric field at some point P distant r from the center of the shown sphere?



The conductor conceals from us all information concerning the nature of the cavity, revealing only the total charge it contains. The charge $+q$ induces an opposite charge $-q$ on the wall of the cavity which distributes itself in such a way that its field cancels that of q, for all points exterior to the cavity. Since the conductor carries no net charge, this leaves +q to distribute itself uniformly over the surface of the sphere. For points outside the sphere, the only thing that survives is the field of the leftover +q, uniformly distributed over the outer surface.

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0 r^2}\hat{\boldsymbol{r}}$$

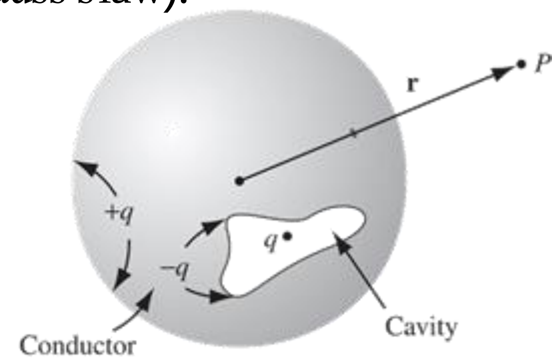
Conductors

Cavity inside the conductor

If there is no charge inside the cavity, then the field within the cavity is zero.

If this were not the case, then any field line would have to begin and end on the cavity wall, going from a plus charge to a minus charge. This is because field inside the cavity must come from the charges not outside the cavity (Gauss's law).

But we can imagine this field line to be part of a closed loop, the rest of which is entirely inside the conductor (where **E**=0). That would lead to a net nonzero value for the line integral of E over this closed loop, which in not possible in view of conservative nature of E field.

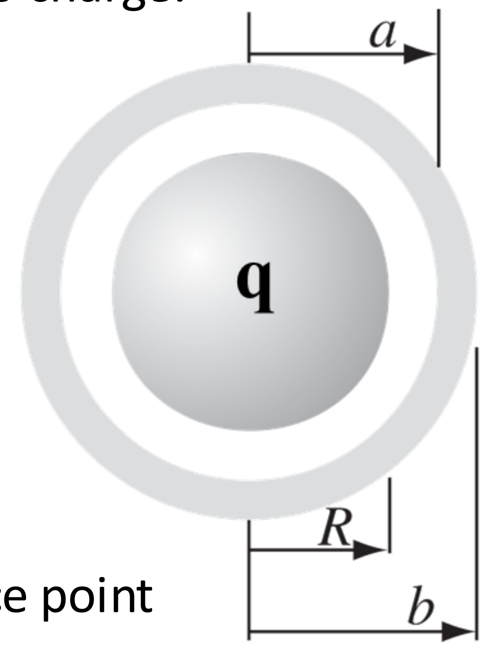


Thus, **E must be zero within an empty cavity**, and in fact there is no charge on the surface of the cavity in the equilibrium.

The same principle applies to a **FaradayCage***.

Conductors

A metal sphere of radius R. carrying charge q, is surrounded by a thick concentric metal shell (inner radius 'a', outer radius 'b') The shell carries no charge.



(a) Find the surface density σ at 'R', at 'a', and at 'b'

$$\text{(a)}\ \boxed{\sigma_R = \frac{q}{4\pi R^2}\,;\ \sigma_a = \frac{-q}{4\pi a^2}\,;\ \sigma_b = \frac{q}{4\pi b^2}.}$$

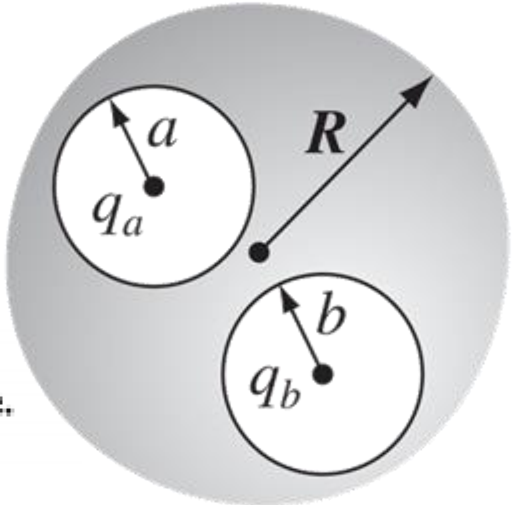
(b) Find the potential at the center, using infinity as the reference point

$$\text{(b)}\ V(0) = -\int_\infty^0 \mathbf{E}\cdot d\mathbf{l} = -\int_\infty^b \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_b^a (0)dr - \int_a^R \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_R^0 (0)dr = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{q}{b}+\frac{q}{R}-\frac{q}{a}\right).}$$

(c) The outer surface is touched to the grounding wire, which lowers its potential to zero (same as infinity). How do your answers to (a) and (b) change?

$$\text{(c)}\ \boxed{\sigma_b \to 0}\ \text{(the charge "drains off")};\ V(0) = -\int_\infty^a (0)dr - \int_a^R \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right)dr - \int_R^0 (0)dr = \boxed{\frac{1}{4\pi\epsilon_0}\left(\frac{q}{R}-\frac{q}{a}\right).}$$

Conductors



(a) Find the surface charges σ_a, σ_b, and σ_R.

(a) $\boxed{\sigma_a = -\frac{q_a}{4\pi a^2};}$ $\boxed{\sigma_b = -\frac{q_b}{4\pi b^2};}$ $\boxed{\sigma_R = \frac{q_a + q_b}{4\pi R^2}.}$

(b) What is the field outside the conductor?

(b) $\boxed{\mathbf{E}_{\text{out}} = \frac{1}{4\pi\epsilon_0}\frac{q_a + q_b}{r^2}\hat{\mathbf{r}},}$ where $\mathbf{r}$ = vector from center of large sphere.

(c) What is the field within each cavity?

(c) $\boxed{\mathbf{E}_a = \frac{1}{4\pi\epsilon_0}\frac{q_a}{r_a^2}\hat{\mathbf{r}}_a, \quad \mathbf{E}_b = \frac{1}{4\pi\epsilon_0}\frac{q_b}{r_b^2}\hat{\mathbf{r}}_b,}$ where $\mathbf{r}_a$ ($\mathbf{r}_b$) is the vector from center of cavity a (b).

(d) What is the force on q_a and q_b?

(d) $\boxed{\text{Zero.}}$

(e) Which of these answers would change if a third charge, q_c, were brought near the conductor?

(e) σ_R changes (but not σ_a or σ_b); $\mathbf{E}_{\text{outside}}$ changes (but not $\mathbf{E}_a$ or $\mathbf{E}_b$); force on q_a and q_b still zero.