In electrostatic work done to form an arrangement of charge is given as

$$W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau \qquad \text{—(i)}$$

In magnetostatic work done to flow current in circuit is given as

$$W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau \qquad \text{—(ii)}$$

Energy stored in electromagnetic field

$$U = \frac{1}{2\mu_0}\int_{\text{all space}} B^2 d\tau + \frac{\epsilon_0}{2}\int E^2 d\tau$$

$$U = \frac{1}{2}\int\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) d\tau \qquad \text{—(iii)}$$

Now energy per unit volume

$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) \qquad \text{—(iv)}$$

Suppose we have 'ρ' and '$\vec{J}$' and they produce $\vec{E}$ and $\vec{B}$ resp.
Let us move a charge in electromagnetic field in electromagnetic field by 'dl' for a time dt.

Work done to displace this charge

$$dW = \vec{F} \cdot d\vec{l}$$

$$\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$$

$$dW = q(\vec{v} \times \vec{B} + \vec{E}) \cdot d\vec{l} \qquad \because d\vec{l} = \vec{v}t$$

$$= q\vec{E} \cdot \vec{v}\,dt + \cancel{\vec{v} \times \vec{B} \cdot \vec{v}}^{\,0}\,dt$$

$$dW = \vec{E} \cdot (q\vec{v})\,dt \qquad \text{— (1)}$$

$$\vec{J} = \rho\vec{v} \qquad \rho = \frac{\text{charge}}{\text{vol}} \qquad q = \rho\,d\tau$$

$$\vec{v} = \frac{\vec{J}}{\rho}$$

$$dW = \left(\vec{E} \cdot \cancel{\rho}\,d\tau \cdot \frac{\vec{J}}{\cancel{\rho}}\right) dt$$

$$dW = \vec{E} \cdot \vec{J}\,d\tau\,dt$$

$$\frac{dW}{dt} = (\vec{E} \cdot \vec{J})\,d\tau$$

$$\boxed{\frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{J})\,d\tau}$$

→ rate at which work is done to move charge in volume $d\tau$

— (2)

$\vec{E} \cdot \vec{J}$ = work done per unit time per unit volume

= power delivered per unit volume.

Now, $\vec{E}\cdot\vec{J}$ in terms of field only.

From maxwell equation
4th eqn
$$\nabla\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{E}\cdot(\nabla\times\vec{B}) = \mu_0\left(\vec{E}\cdot\vec{J} + \epsilon_0 E\cdot\frac{\partial\vec{E}}{\partial t}\right)$$

Product rule,
$$\vec{E}\cdot(\nabla\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{B})$$

$$\vec{B}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{B}) = \mu_0\left(\vec{E}\cdot\vec{J} + \epsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t}\right)$$

We know, Faradays law,
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{B}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \nabla\cdot(\vec{E}\times\vec{B}) = \mu_0\left(\vec{E}\cdot\vec{J} + \epsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{E}\cdot\vec{J} = \frac{1}{\mu_0}\left[-\vec{B}\frac{\partial\vec{B}}{\partial t} - \nabla\cdot(\vec{E}\times\vec{B})\right] - \epsilon_0\vec{E}\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{\mu_0}\left(\vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right) - \epsilon_0\left(\vec{E}\frac{\partial\vec{E}}{\partial t}\right) - \frac{1}{\mu_0}(\nabla\cdot\vec{E}\times\vec{B})$$

$$\therefore\ \vec{B}\cdot\frac{\partial\vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B})$$

$$= \vec{B}\cdot\frac{\partial\vec{B}}{\partial t} + \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

$$= 2\vec{B}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{2\mu_0}\frac{\partial}{\partial t}(\vec{B}\cdot\vec{B}) - \frac{\epsilon_0}{2}\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{2\mu_0}\frac{\partial}{\partial t}B^2 - \frac{\epsilon_0}{2}\frac{\partial}{\partial t}E^2 - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})$$

$$\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left[\frac{1}{\mu_0}B^2 + \epsilon_0 E^2\right] - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})$$

Now

$$\frac{dW}{dt} = \int_v (\vec{E}\cdot\vec{J})\, d\tau$$

$$\frac{dW}{dt} = \int_v \left[-\frac{1}{2}\frac{\partial}{\partial t}\left(\frac{B^2}{\mu_0} + \epsilon_0 E^2\right) - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})\right] d\tau$$

$$\frac{dW}{dt} = -\frac{1}{2}\int_v \frac{\partial}{\partial t}\left(\frac{B^2}{\mu_0} + \epsilon_0 E^2\right) d\tau - \frac{1}{\mu_0}\int \nabla\cdot(\vec{E}\times\vec{B})\, d\tau$$

$$\boxed{\frac{dW}{dt} = -\frac{d}{dt}\underbrace{\int \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau}_{\text{\# First term: energy stored in the field}} - \underbrace{\frac{1}{\mu_0}\oint_S (\vec{E}\times\vec{B})\cdot d\vec{a}}_{\text{\# Second term:}}}$$

Poynting theorem

First term:

$\frac{d}{dt}(U) =$ Time rate at which the energy of the field change in volume 'v'

$$U = \int_v \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$$

Second term:

Time rate at which the energy crossing the surface 'S' of volume v or boundary change

LHS:

Power delivered into the system.

State the Poynting Theorem - Work energy theorem in electrodynamics

Time rate at which energy is changing in volume 'V' plus the time rate at which the energy enters the surface 'S' or boundary is equal to power delivered into the fields

In second term,

$$\frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})$$

energy per unit time per unit area

$$\frac{dW}{dt} = \frac{1}{\mu_0}\oint(\vec{E}\times\vec{B})\cdot d\vec{a}$$

⇒ work done per unit time per unit area

$$\frac{1}{\mu_0}\vec{E}\times\vec{B} \longrightarrow \text{pointing vector}$$

$$\frac{dW}{dt} = -\frac{d}{dt}\int u\,d\tau - \oint S\cdot da$$

In region where there are no charge

$$\frac{dW}{dt} = 0$$

$$\frac{d}{dt}\int u\,d\tau = -\oint \vec{S}\cdot d\vec{a}$$

$$\int \frac{\partial u}{\partial t}\,d\tau = -\oint \vec{S}\cdot d\vec{a}$$

$$\int \frac{\partial u}{\partial t}\,d\tau = -\int(\nabla\cdot\vec{S})\,d\tau$$

$$\int\left(\frac{\partial u}{\partial t} + \nabla\cdot\vec{S}\right)d\tau = 0$$

$$\frac{\partial u}{\partial t} + \nabla\cdot\vec{S} = 0$$ Law of conservation of electromagnetic energy

equation of continuity for 'u'

$$\boxed{\nabla\cdot\vec{S} = -\frac{\partial u}{\partial t}}$$

What is a wave?

A wave is a disturbance that propagates through a medium transferring energy and momentum from one point to another without permanent displacement of medium itself.

① should be of the form $f(z,t) = g(z \pm vt)$

② $$\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

③ $f(z,t)$ is finite $\forall$ z and t

General sinusoidal wave can be given as:

$$\boxed{f(z,t) = A\cos[k(z-vt)+\delta]}$$

$0 \leq \delta \leq 2\pi$

$k \to$ wave number

$$\boxed{\lambda \text{ (wavelength)} = \frac{2\pi}{k}}$$

$$T = \frac{2\pi}{kv}$$

angular freq : $\boxed{\omega = 2\pi f = kv}$

$f(z,t) = A\cos(kz - \omega t + \delta) \to$ travelling to right

$-vt \to$ right

Travelling to left

$+vt \to$ left

$$f(z,t) = A\cos(kz + \omega t - \delta)$$

$$f(z,t) = g(z-vt) + h(z+vt)$$

↳ most general solution

Complex notation

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\text{Re}[e^{i\theta}] = \cos\theta$$

$$f(z,t) = A\cos(kz - \omega t + \delta) = \text{Re}[e^{i(kz - \omega t + \delta)}]$$

$$\tilde{f}(z,t) = \tilde{A}e^{i(kz - \omega t)} = Ae^{i\delta}e^{i(kz - \omega t)}$$

$$\tilde{A} = Ae^{i\delta}$$

$$f(z,t) = \text{Re}[\tilde{f}(z,t)]$$

Maxwell eqns in space where there are no charge or current.

(i) $\nabla \cdot \vec{E} = 0$

(ii) $\nabla \cdot \vec{B} = 0$

(iii) $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

(iv) $\nabla \times \vec{B} = \mu_0 \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$

deduce apply curl to eqn iii

$$\nabla \times (\nabla \times \vec{E}) = \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

theorem: $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\underbrace{\nabla(\nabla \cdot \vec{E})}_{0} - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E}$$

$$\boxed{\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Similarly

$$\boxed{\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

They are of the form

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$$\frac{1}{v^2} = \mu_0 \epsilon_0$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

Electromagnetic waves should obey the equation

$$\nabla^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{and} \quad \nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\tilde{B}(z,t) = \tilde{B}_0 e^{i(kz-\omega t)} \qquad \tilde{E}(z,t) = \tilde{E}_0 e^{i(z-\omega t)}$$

From $\nabla \cdot \vec{E} = 0$ and $\nabla \cdot \vec{B} = 0$

→ that $\vec{E} \perp$ to direction of propogation of wave
$\vec{B} \perp$ " "

From $\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$ → $\vec{E}$ and $\vec{B}$ are mutually $\perp$
and they are in phase

$$\boxed{E_0 B_0 = c} \qquad \boxed{EB = c}$$

$$u = \frac{1}{2}\left(\epsilon_0 \vec{E}^2 + \frac{1}{\mu_0} B^2\right) = \epsilon_0 E^2$$

$$B^2 = \frac{E^2}{c^2} = \mu_0 \epsilon_0 E^2$$

$$\boxed{\epsilon_0 E^2 = \frac{B^2}{\mu_0}}$$

→ thus energy distribution of electric and magnetic field are same

Energy flux density (energy per unit area per unit time) is given by Poynting vector

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

For monochromatic plane wave propagating in z direction.

$$S = c\,\epsilon_0 E_0^2 \cos^2(kz - \omega t + \delta)\,\hat{z} = cu\hat{z}$$

$$\vec{E} = E_0 \cos(kz - \omega t + \delta)\,\hat{x}$$

$$\vec{B} = B_0 \cos(kz - \omega t + \delta)\,\hat{y}$$

$$S = \frac{E_0 B_0}{\mu_0} \cos^2(kz - \omega t + \delta)\,\hat{z}$$

$$S = \frac{E_0 \times E_0}{c\mu_0} \times \cos^2(kz - \omega t + \delta)\,\hat{z}$$

$$S = c\,\epsilon_0 E^2 \cos^2(kz - \omega t + \delta)\,\hat{z}$$

$$\boxed{S = cu\hat{z}}$$

energy density

$$u = \epsilon_0 E^2$$

$$E = E_0 \cos(kz - \omega t + \delta)$$

Energy carried by EMW per unit time per unit area

$$\boxed{\vec{S} = cu\hat{k}}$$

They also carry momentum

$$\vec{p} = \frac{\vec{S}}{c^2} = \frac{cu\hat{k}}{c^2}$$

$$\boxed{\vec{p} = \frac{u\hat{k}}{c}}$$

$$\overline{\cos^2 \omega t} = \frac{1}{2}$$

Average energy density : $\langle u \rangle = \frac{1}{2}\epsilon_0 E_0^2 \rightarrow \epsilon \quad S = cu$

$$p = \frac{\bar{u}}{c}$$

Average poynting vector : $\langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2 \hat{z} = c\bar{u}$

Average momentum: : $\langle p \rangle = \frac{\bar{u}}{c} = \frac{1}{2c}\epsilon_0 E_0^2 = \frac{\bar{u}}{c}$

Intensity and Radiation Pressure

Intensity: Power per unit area = $\langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2$

Radiation Pressure: $P = \frac{1}{A}\frac{\Delta p}{\Delta t} = \frac{1}{2}\epsilon_0 E_0^2 = \frac{I}{c}$

$$\boxed{P = \frac{I}{c}}$$

↳ speed of light

On a perfect absorber

↳ $P = \frac{I}{c}$

On a perfect reflector

$P = \frac{2I}{c}$

Superposition of two sinusoidal waves

$$x_1(t) = a_1 \cos(\omega t + \theta_1)$$
$$x_2(t) = a_2 \cos(\omega t + \theta_2)$$

By superposition principle, resulting displacement =

$$x(t) = x_1(t) + x_2(t)$$
$$= a_1 \cos(\omega t + \theta_1) + a_2 \cos(\omega t + \theta_2)$$

Using $\quad \cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= a_1(\cos\omega t \cos\theta_1 - \sin\omega t \sin\theta_1) + a_2(\cos\omega t \cos\theta_2 - \sin\omega t \sin\theta_2)$$

$$\cos\omega t(a_1\cos\theta_1 + a_2\cos\theta_2) - \sin\omega t(a_1\sin\theta_1 + a_2\sin\theta_2)$$

Let
$$A\cos\phi = a_1\cos\theta_1 + a_2\cos\theta_2 \quad \text{—(i)}$$
$$A\sin\phi = a_1\sin\theta_1 + a_2\sin\theta_2 \quad \text{—(ii)}$$

$$x(t) = A\cos\omega t\cos\phi - A\sin\omega t\sin\phi$$
$$x(t) = A\cos(\omega t + \phi)$$

This is the resultant wave

Now $\quad (i)^2 + (ii)^2$

$$A^2(\cos^2\phi + \sin^2\phi) = a_1^2\cos^2\theta_1 + a_2^2\cos^2\theta_2 + 2a_1a_2\cos\theta_1\cos\theta_2 + a_1^2\sin^2\theta_1 + a_2^2\sin^2\theta_2 + 2a_1a_2\sin\theta_1\sin\theta_2$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2)$$

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1 - \theta_2)$$

$$\boxed{A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}}$$

phase difference b/w the two waves

Now (iii) ÷ (iv)

$$\boxed{\tan\theta = \frac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2}}$$ → phase difference

For constructive interference,

Amplitude should be maximum,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$$

For A to be maximum $\cos\phi = 1$

ie the phase diffrence btw the two wave should be $\cos\phi = 1$

$\phi = 2n\pi$ for $\cos\phi = 1$ $n = 0, 1, 2, 3 \ldots$

For destructive interference,

$\cos\phi = -1$

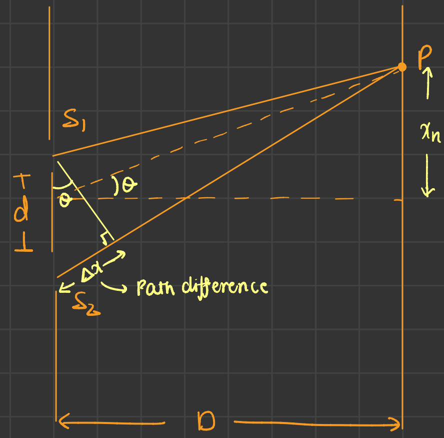
so $\phi = (2n-1)\frac{\pi}{2}$ $n = 1, 2, 3, 4 \ldots$

Conditions for constructive and destructive interference

For constructive, $\phi = 2n\pi$ where $n = 0, 1, 2, \ldots$

For destructive $\phi = (2n-1)\pi$ where $n = 1, 2, 3, 4 \ldots$

Youngs Double slit experiment



$$\sin\theta = \frac{\Delta x}{d}$$

$$\tan\theta = \frac{x_n}{D}$$

For small angles, $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$

so $$\frac{\Delta x}{d} = \frac{x_n}{D}$$

$$\Delta x = \frac{x_n d}{D}$$

so, Path difference,

$$\boxed{\Delta x = \frac{x_n d}{D}}$$

For constructive interference $\Delta x = n\lambda$, where $n = 0, 1, 2, 3 \ldots$

For destructive interference $\Delta x = (2n-1)\frac{\lambda}{2}$ where $n = 1, 2, 3 \ldots$

Position of Bright fringes

$$\frac{x_n d}{D} = n\lambda$$

$$\boxed{x_n = \frac{n\lambda D}{d}}$$

Position of dark fringes

$$\frac{x_n d}{D} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{x_n = \frac{(2n-1)\lambda D}{2d}}$$

We know

phase difference $= \frac{2\pi}{\lambda} \times$ path diff

$$\boxed{\phi = \frac{2\pi}{\lambda}\Delta x}$$

Coherent sources

Two sources which vibrate at a constant phase difference and emit light of same frequency are called coherent sources.

If the phase difference would vary rapidly, then the interference pattern would change rapidly and no stationary pattern would occur and the interference pattern wouldn't be visible

We know that $I \propto A^2$

if we consider two waves such that

$x_1(t) = A\cos(\omega t - \theta_1)$

$x_2(t) = A\cos(\omega t - \theta_2)$

Then their $A^2 = 2A^2 + 2A^2\cos\phi$ where $\phi = \theta_1 \sim \theta_2$

$= 2A^2(1+\cos\phi)$ we know $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$

$= 2A^2 \times 2\cos^2\frac{\phi}{2}$

$= 4A^2\cos^2\frac{\phi}{2}$

intensity of each wave = $I_0 = A^2$

so the intensity of the wave formed after their superposition = $4I_0\cos^2\frac{\phi}{2}$

$$\boxed{I = 4I_0\cos^2\frac{\phi}{2}}$$

For incoherent sources the net intensity of the superimposed wave is simply the sum of both intensity.

For coherent sources

$$\boxed{I = 4I_0\cos^2\frac{\phi}{2}}$$

For minimum intensity (destructive interference) $\boxed{\phi = (2n-1)\pi}$ $n = 1,2,3,4\ldots$

For maximum intensity (constructive interference) $\boxed{\phi = 2n\pi}$ $n = 0,1,2,3,\ldots$

For incoherent sources : Phase difference changes rapidly with time

$$\boxed{I = 4I_0\left\langle\cos^2\frac{\phi}{2}\right\rangle}$$ → we take average

we know $\langle\cos^2\theta\rangle = \frac{1}{2}$

varies btw 0 and 1 and the observer will see the average of this

$= 4I_0 \times \frac{1}{2} = 2I_0^2$

$\boxed{I = 2I_0}$ → we will see a brightly uniformly lit screen

Fringe width

It is the distance between two consecutive dark or bright fringe

Let us take two consecutive bright fringes

We know

$$x_n = \frac{n\lambda D}{d}$$

For nth and (n+1)th bright fringe

$$x_{nth} = \frac{n\lambda D}{d} \qquad x_{n+1} = \frac{(n+1)\lambda D}{d}$$

$$\beta = x_{n+1} - x_n = \frac{n\lambda D}{\cancel{d}} + \frac{\lambda D}{d} - \frac{n\lambda D}{\cancel{d}}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

→ same for both dark and bright fringes

→ are dependent on other variables

Intensity distribution

We know that $I \propto A^2 \Rightarrow I = KA^2$

We know the amplitude of two super imposed wave is given by

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi \qquad \phi = \theta_1 \sim \theta_2$$

$$I = Ka_1^2 + Ka_2^2 + 2Ka_1a_2\cos\phi$$

$$I_1 = Ka_1^2 \qquad I_2 = Ka_2^2$$

$$\sqrt{I_1I_2} = Ka_1a_2$$

So

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi}$$

For constructive interference $\cos\phi = 1$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}$$

$$\boxed{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2}$$

For destructive interference $\cos\phi = -1$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}$$

$$\boxed{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

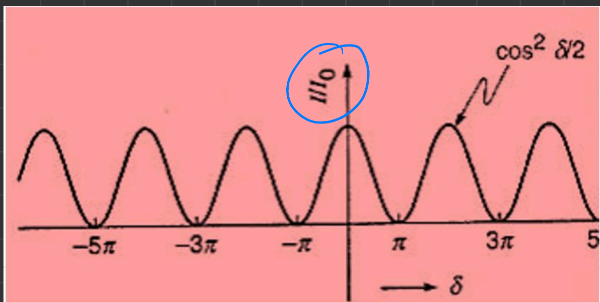
If the holes are illuminated by different light sources, (incoherent source)

$$I = I_1 + I_2$$

If there are coherent source for which $I_1 = I_2 = I_0$

$$I = 4I_0 \cos^2\frac{\phi}{2} \qquad \phi = \theta_1 \sim \theta_2$$

Graph of I/I_0 vs ϕ

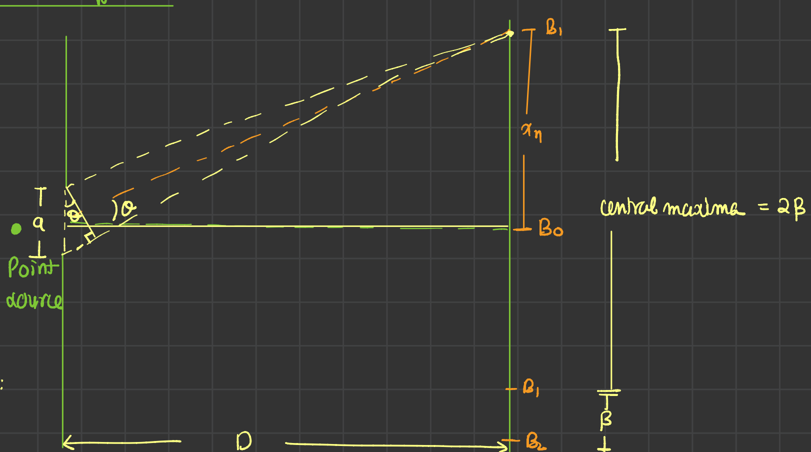


I/I_0 vs ϕ

Diffraction : Single slit interference

It is the bending of light around the sharp edges of the obstacle or aperture in the geometrical shadow region. This phenomenon is known as diffraction

Fresnel diffraction



Condition:

$a \approx \lambda$

Similarly,

$$\sin\theta = \frac{\Delta x}{a} \qquad \tan\theta = \frac{x_n}{D}$$

path difference

angular position

$$\boxed{\Delta x = a\sin\theta}$$

width of slit

For dark fringes

$\Delta x = n\lambda \qquad n = 1, 2, 3, 4$

$a\sin\theta = n\lambda$

If θ is small

$\theta = \frac{n\lambda}{a}$ and $\theta = \frac{x_n}{D}$

$$\boxed{x_n = \frac{n\lambda D}{a}}$$

<u>For bright fringes</u>

$\Delta x = (2n+1)\frac{\lambda}{2} \qquad n = 1, 2, 3, 4 ..$

$a\sin\theta = (2n+1)\frac{\lambda}{2}$

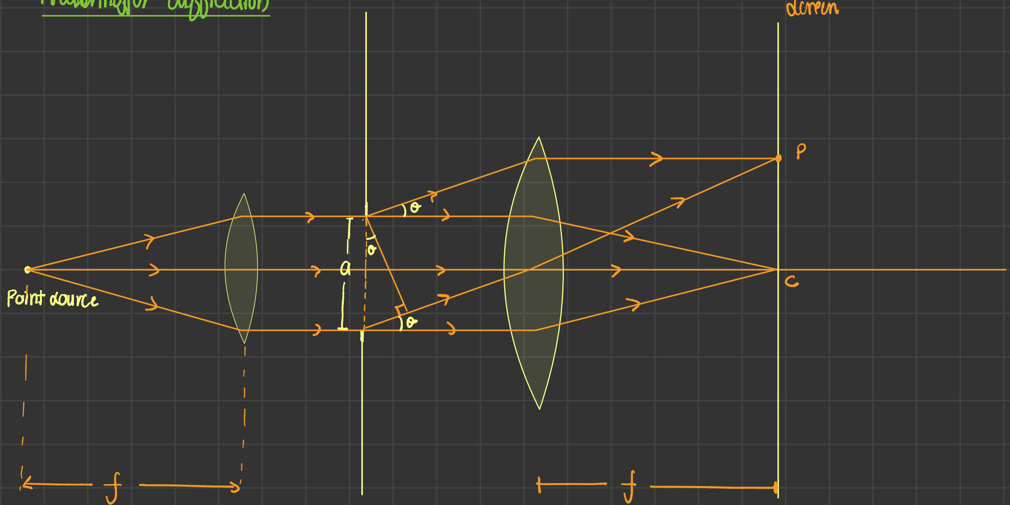
If θ is small

in radians

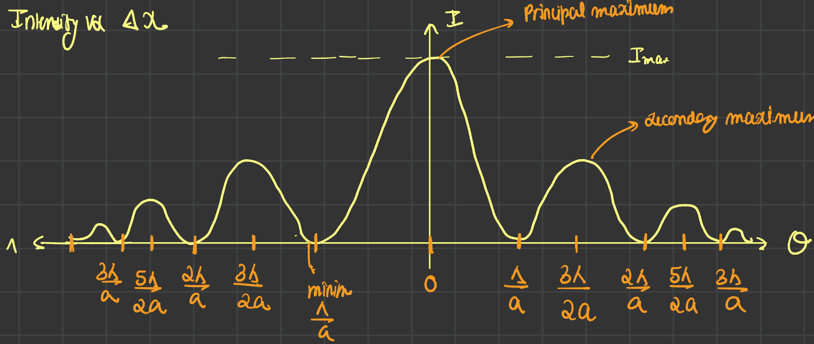
$\theta = (2n+1)\frac{\lambda}{2a} = \frac{x_n}{D}$

$$\boxed{x_n = (2n+1)\frac{\lambda D}{2a}}$$

<u>Fraunhoffer diffraction</u>



Intensity vs Δx



bright

$\Delta x = (2n+1)\frac{\lambda}{2}$

$n = 1, 2, 3, 4$

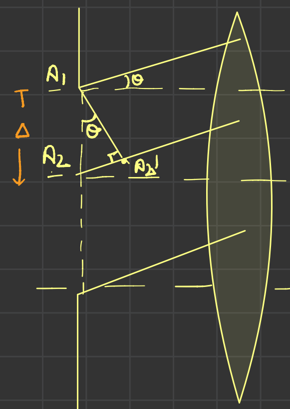
$\Delta x = a\sin\theta$

Now we want to calculate intensity distribution on the screen (focal plane on the lens)

$a = (n-1) \times$ distance btw two sources

└ no of sources

width of slit.



$a = (n-1)\Delta$

From the triangle

path difference

$\sin\theta = \frac{A_2 A_2'}{\Delta}$

$A_2 A_2' = \Delta\sin\theta$

We know $\phi = \frac{2\pi}{\lambda} \times$ path diff

$\phi = \frac{2\pi}{\lambda} \cdot \Delta\sin\theta$

Field at P due to A_1

$x_1(t) = a\cos\omega t$

Then field at P due to A_2

$x_2(t) = a_1\cos(\omega t - \phi)$

and so on

there are n-1 sources

$$\vec{E} = a[\cos\omega t + \cos(\omega t-\phi) + \cdots + \cos(\omega t-(n-1)\phi)]$$

net resultant wave

$$\cos\omega t + \cos(\omega t-\phi) + \cdots + \cos(\omega t-(n-1)\phi) = \frac{\sin(n\phi/2)}{\sin(\phi/2)}\cos\left[\omega t - \frac{1}{2}(n-1)\phi\right]$$

a formula

so the resultant wave has amplitude

$$\boxed{E_0 = a\frac{\sin(n\phi/2)}{\sin(\phi/2)}}$$

Resultant wave is

$$\boxed{E = E_0\cos\left(\omega t - \frac{1}{2}(n-1)\phi\right)}$$

Putting the limit $n\to\infty$ and $\Delta\to 0$ i.e. there is continuous distribution of sources

such that $(n-1)\Delta \to b$

We know $\phi = \frac{2\pi}{\lambda} \times$ path diff

$$\phi = \frac{2\pi}{\lambda}\times\Delta\sin\theta$$

multiplying n on both side

$$n\phi = \frac{2\pi}{\lambda}\underbrace{n\Delta}_{a}\sin\theta$$

$$n\phi = \frac{2\pi}{\lambda}a\sin\theta$$

i.e. $\frac{n\phi}{2} = \frac{\pi a\sin\theta}{\lambda}$ or $\phi = \frac{2\pi a\sin\theta}{n\lambda} = \frac{2\beta}{n}$

Substituting,

$$\phi = \frac{2\pi a \sin\theta}{n\lambda}$$

in $E_0 = \frac{a \sin(n\phi/2)}{\sin(\phi/2)} \approx \frac{a\sin(n\phi/2)}{\phi/2}$

$$E_0 \approx \frac{a\sin\left(\frac{\cancel{n}}{\cancel{2}} \times \frac{\cancel{2}\pi a \sin\theta}{\cancel{n}\lambda}\right)}{\frac{\cancel{2}\pi a \sin\theta}{n\lambda \cancel{2}}} = n a \frac{\sin(\pi a \sin\theta/\lambda)}{(\pi a \sin\theta/\lambda)}$$

$$\beta = \frac{\pi a \sin\theta}{\lambda}$$

$$E_0 = \frac{na \sin\beta}{\beta} = \frac{A\sin\beta}{\beta}$$

$$\boxed{E = \frac{A\sin\beta}{\beta}\cos(\omega t - \beta)}$$

We know $I = KA^2$

intensity at $\theta = 0°$

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2}}$$

Minima

For intensity to be minimum,

$$\boxed{a\sin\theta = n\lambda}$$

For maximum

$$\frac{dI}{d\beta} = 0 = I_0 \left(\frac{\beta^2 \, 2\sin\beta . \cos\beta - \sin^2\beta . 2\beta}{\beta^4} \right)$$

$$= I_0 \left(\frac{2\sin\beta\cos\beta}{\beta^2} - \frac{2\sin^2\beta}{\beta^3} \right) = 0$$

$$\frac{2\sin\beta\cos\beta}{\cancel{\beta^2}} = \frac{2\sin^2\beta}{\cancel{\beta^3}\,\beta}$$

$$2\sin\beta\cos\beta - \frac{2\sin^2\beta}{\beta} = 0$$

$$2\sin\beta\left(\cos\beta - \frac{\sin\beta}{\beta}\right) = 0$$

$$2\sin\beta\cos\beta\left(1 - \frac{\tan\beta}{\beta}\right) = 0$$

$$\sin(2\beta)\left(1 - \frac{\tan\beta}{\beta}\right) = 0$$

$\sin(2\beta) = 0$ $\quad 1 - \frac{\tan\beta}{\beta} = 0$

$\beta = n\pi$ when $n = 0, 1, 2, 3, \ldots$

$\beta = 0$ (Brightest spot)

For dark fringes $\boxed{\beta = n\pi}$

$\tan\beta = \beta$ gives us secondary maxima

On solving $\beta = 1.43\pi$ & $\beta = 2.46\pi$ and so on

these values are slightly less than $\frac{3\pi}{2}$, $\frac{5\pi}{2}$

ie $(2n+1)\frac{\pi}{2}$.

Problem 1 A parallel beam of light is incident normally on a narrow slit of width 0.2 mm. The Fraunhofer diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens whose focal length is 20 cm. Calculate the distance between the first two minima and the first two maxima on the screen. Assume that $\lambda = 5 \times 10^{-5}$ cm and that the lens is placed very close to the slit.