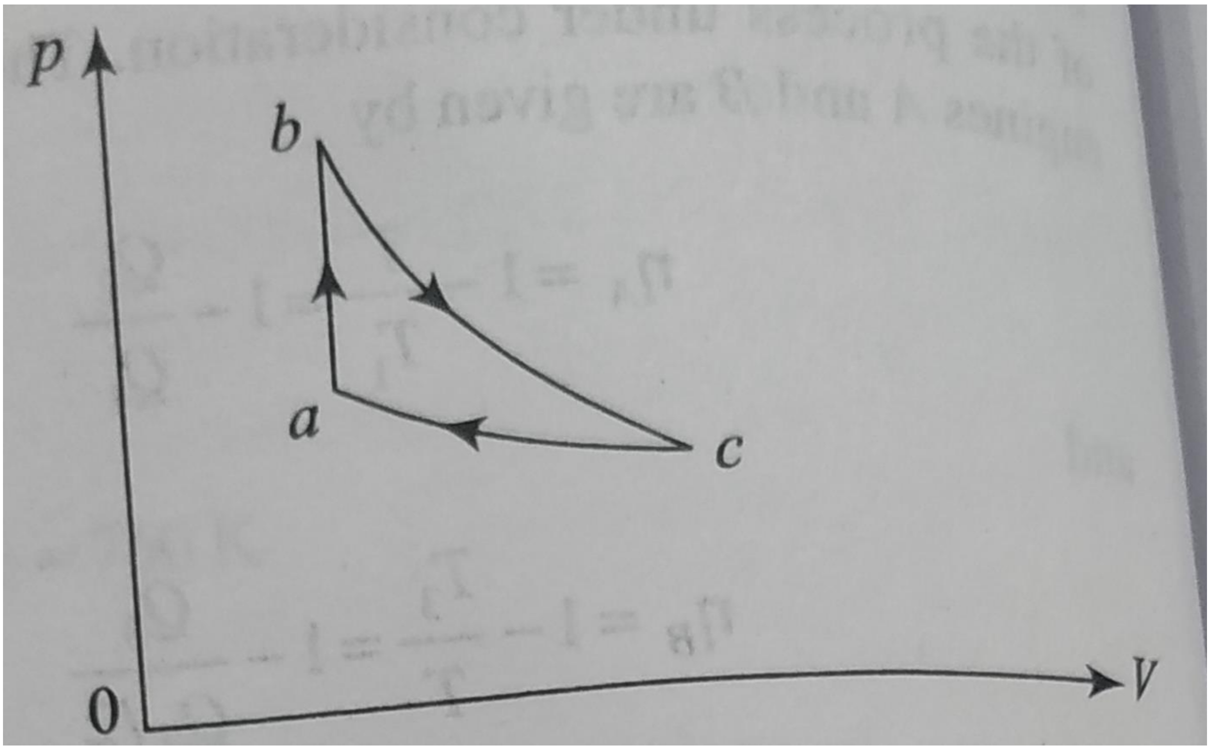
Q) An ideal monoatomic gas occupies 2 liters at 30k and $5 \times 10^{-3} pa$.The internal energy of the gas is taken to be zero at this point. It undergoes the following changes.

a) The temperature is raised to 300k at constant volume.

b) The gas is then expanded adiabatically till it attains the initial temperature.

c)Finally, it is compressed isothermally. Calculate the efficiency of the cycle.

Sol:



Refer to fig. which is indicator diagram for the processes under consideration. Here

$$a \rightarrow b: isochoric\ comperssion$$

$$b \rightarrow c: adabatic\ expansion$$

$$c \rightarrow a: isothermal\ compression$$

At a, internal energy is zero. To calculate the number of moles of the gas, we use the number of the gas. We use the equation of state for an ideal gas:

$$PV = nRT$$

Here V=2 liter=$2 \times 10^{-3}m^3, P = 5 \times 10^2 Nm^{-2}\ and\ T = 30K$. Using these values, we get $(5 \times 10^2 Nm^{-2}) \times (2 \times 10^{-3}m^3 = nR \times (30K)$

Or $30nR = 1Nmk^{-1}$

Hence, the number of moles o the gas can be expressed in terms of R:

$$n = \frac{1}{30R} JK^{-1}$$

To calculate the efficiency, we need to know the work done as well as heat absorbed. So, we consider the three processes one by one.

From $a \to b$,the process is isochoric and no work is done:

$\delta W_{a\to b} = 0$

$$Q_{a\to b} = nC_V(T_b - T_a)$$

For a Monoatomic gas,$C_v = \frac{3}{2}R$

$$Q_{a\to b} = \frac{1JK^{-1}}{30R} \times \frac{3}{2}R \times (300 - 30)K = 13.5J$$

And $U_b - U_a = 13.5J$

But $U_a = 0 \quad implies\ \ U_b = 13.5J$

From $b \to c$,the processes adiabatic and no heat is exchanged:

$$\delta Q_{a\to b} = 0$$

From the first law, we can write

$$W_{a\to b} = - \Delta U = -(U_c - U_b)$$

Since T_c=30K and U is a function of only temperature for an ideal gas, we note that $U_c = 0\ since U_a = 0$

$$W_{a\to b} = U_b = 13.5J$$

For $c \to a$,

$$W_{a\to b} = nRTln\frac{V_a}{V_c} = -nRTln\left(\frac{V_c}{V_B}\right)$$

Since $V_a = V_b$

To calculate the ration $\frac{V_a}{V_c}$,we note that points c and b are located on the same adiabat. So, we can write

$$TV^{\gamma-1} = constant$$

$$T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$$

$$\left(\frac{V_c}{V_b}\right)^{\gamma-1} = \frac{T_b}{T_c}$$

Since **γ** = 1.67, we get

$\left(\frac{V_c}{V_b}\right)^{0.67}$=$\frac{300K}{30K} = 10$

$$\frac{V_c}{V_b} = 10^{1/0.67} = 10^{1.4925} = 31.08$$

Hence

$$W_{b\to c} = -\frac{1JK^{-1}}{30R} \times R \times (30K)\ln(31.08)$$

=-3.4J

Therefore $Q_{b\to c,} = -3.4J$ and $U_a = U_c = 0$

Hence, the net work done =13.5J-3.4J=10.1J

And heat absorbed=13.5J

$$\eta = \frac{10.1J}{13.5J} = 0.748 = 74.8\%$$

QUESTION:

a) A Carnot cycle operates as a heat engine between two bodies of equal heat capacities until their temperatures becomes equal. If the initial temperatures of the bodies are T_1 and T_2 respectively and $T_1 > T_2$, then what is their common temperature?

b) The internal energy E(T) of a system at a fixed volume is found to depend on the temperature T as E(T)=a$T^2 + bT^4$. Then what will be the entropy S(T), as a function of temperature?

SOLUTION:

a) For Carnot cycle, ΔS=0

ΔS=C$\int_{T_1}^{T} \frac{dT}{T} + C \int_{T_2}^{T} \frac{dT}{T}$

$$\ln\left(\frac{T}{T_1}\right) + \ln\left(\frac{T}{T_2}\right) = 0$$

$$\ln\left(\frac{T^2}{T_1 T_2}\right) = 0$$

$$\frac{T^2}{T_1 T_2} = 1$$

$$T = \sqrt{T_1 T_2}$$

b) $E(T) = \mathrm{a}T^2 + bT^4$

$$C_v = \frac{dE}{dT} = 2aT + 4bT^3$$

$$Tds = C_v dT$$

$$\int dS = \int C_v \frac{dT}{T}$$

$$S = 2aT + \frac{4bT^3}{3}$$

Question:

A real has an efficiency of 33%. The engine has a work output of 24 J per cycle.

(a)How much heat energy is extracted from the high-temperature reservoir per cycle?

(b) How much heat energy is discharged into the low-temperature reservoir per cycle?

Answer:

To obtain the amount of heat energy extracted from the higher temperature $|Q_H|$, substitute 33% for ε and 24 J for ΔW in the equation.

$\varepsilon = \Delta W/|Q_H|$

$33/100 = 24\ \text{J}/|Q_H|$

Or $|Q_H| = (2400/33)\ \text{J}$

$= 72.73\ \text{J}$

To obtain the heat energy is discharged into the low-temperature reservoir per cycle $|Q_L|$, substitute 72 J for $|Q_H|$ and 33% for ε in the equation.

$\varepsilon = 1 - |Q_L|/|Q_H|$

$33/100 = 1 - |Q_L|/72.73\ \text{J}$

$|Q_L|/72.73\ \text{J} = 1 - 33/100$

$= 67/100$

$|Q_L| = (67/100) \times 72.73\text{J}$

$= 48.72\ \text{J}$