Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 01

1. Find the volume of a parallelepiped whose edges are given by
$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$, $\vec{B} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, and $\vec{C} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$, $\vec{B} = \hat{i} - 2\hat{j} + 2\hat{k}$, $\vec{C} = 3\hat{i} - \hat{j} - 2\hat{k}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \hat{i}(4) - \hat{j}(5) + \hat{k}(-7)$$

$$= 4\hat{i} - 5\hat{j} - 7\hat{k}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (4\hat{i} - 5\hat{j} - 7\hat{k}) \cdot (3\hat{i} - \hat{j} - 2\hat{k}) = 12 + 5 + 14 = \underline{31}$$

2. Find the projection of $\vec{\mathbf{F}} = (y - 1)\hat{\imath} + 2x\hat{\jmath}$ on $\vec{\mathbf{B}} = 5\hat{\imath} - \hat{\jmath} + 2\hat{k}$ at the point (2,2,1)

$\vec{F} = (y-1)\hat{i} + 2x\hat{j}$, $\vec{B} = 5\hat{i} - \hat{j} + 2\hat{k}$

$$\vec{F}_{(2,2,1)} = \hat{i} + 4\hat{j}$$

$$\text{Proj. of } \vec{F} \text{ on } \vec{B} = \frac{\vec{F} \cdot \vec{B}}{|\vec{B}|} = \frac{(\hat{i} + 4\hat{j}) \cdot (5\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{5^2 + (-1)^2 + 2^2}}$$

$$= \frac{5 - 4}{\sqrt{30}} = \underline{\frac{1}{\sqrt{30}}}$$

3. Given the two displacements
   D = (6 **i** + 3 **j** − 1 **k**) m and **E** = (4 **i** - 5 **j** − 8 **k**) m, find the magnitude of the displacement 2**D-E**.

   F = 2 **D** - **E** = (8 **i** + 11 **j** − 10 **k**) m

   The magnitude of **F** = | **F**| = $\sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(8\,m)^2 + (11\,m)^2 + (-10\,m)^2} =$

   $16.9\,m$

4. Find the angle between the vectors **A**=$\hat{i}+\hat{k}$ and **B**=$\hat{j}+\hat{k}$

Q) Find the angle between the vectors $\vec{A} = \hat{i} + \hat{k}$ and
$\vec{B} = \hat{j} + \hat{k}$

Ans $\vec{A}\cdot\vec{B} = 1.0 + 0.1 + 1.1 = 1$ — (1)

Since $\vec{A}\cdot\vec{B} = AB\cos\theta = \sqrt{2}\sqrt{2}\cos\theta = 2\cos\theta$ — (2)

from (1) and (2)

$2\cos\theta = 1 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta = 60^\circ}$

5. Let **C**=**A-B** and calculate the dot product of **C** with itself.

Let $\vec{C} = \vec{A} - \vec{B}$, and calculate the dot product of C with itself.

Ans $\vec{C}\cdot\vec{C} = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B}) = \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$

$\Rightarrow \boxed{C^2 = A^2 + B^2 - 2AB\cos\theta}$

This is called the Laws of cosines.

6. Find the magnitude of two vectors A and B, having the same magnitude such that the nagle between them is 60º and their scaler product is ½.

Given: $|\vec{a}| = |\vec{b}|$ and angle θ (say) between $\vec{a}$ and $\vec{b}$ is 60° and their scalar (*i.e.,* dot) product $= \frac{1}{2}$

i.e., $\quad \vec{a}\cdot\vec{b} = \frac{1}{2}$

$\Rightarrow \quad |\vec{a}|\,|\vec{b}|\cos\theta = \frac{1}{2} \qquad [\because \ \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta]$

Putting $|\vec{b}| = |\vec{a}|$ (given) and $\theta = 60^\circ$ (given), we have

$|\vec{a}|\,|\vec{a}|\cos 60^\circ = \frac{1}{2} \qquad \Rightarrow \quad |\vec{a}|^2\left(\frac{1}{2}\right) = \frac{1}{2}$

Multiplying by 2, $\quad |\vec{a}|^2 = 1 \qquad \Rightarrow \quad |\vec{a}| = 1 \qquad$ ...(*i*)

($\because$ Length of a vector is never negative)

$\therefore \quad |\vec{b}| = |\vec{a}| = 1 \qquad$ [By (*i*)]

$\therefore \quad |\vec{a}| = 1$ and $|\vec{b}| = 1$.

7. Find the area of a triangle with vertices A(1,1,2), B(2,3,5) and C(1,5,5)

Vertices of ΔABC are A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

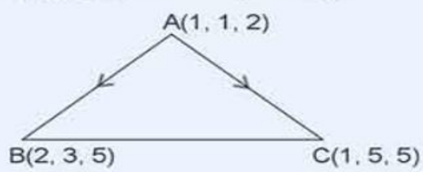
$\therefore$ Position Vector (P.V.) of point A is $(1, 1, 2) = \hat{i} + \hat{j} + 2\hat{k}$

P.V. of point B is (2, 3, 5)

$= 2\hat{i} + 3\hat{j} + 5\hat{k}$

P.V. of point C is (1, 5, 5)

$= \hat{i} + 5\hat{j} + 5\hat{k}$



$\therefore \quad \overrightarrow{AB}$ = P.V. of point B − P.V. of point A

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$

$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - \hat{j} - 2\hat{k} \quad = \hat{i} + 2\hat{j} + 3\hat{k}$

and $\overrightarrow{AC}$ = P.V. of point C − P.V. of point A

$= \hat{i} + 5\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k}) = \hat{i} + 5\hat{j} + 5\hat{k} - \hat{i} - \hat{j} - 2\hat{k}$

$= 0\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \quad \overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$= \hat{i}(6 - 12) - \hat{j}(3 - 0) + \hat{k}(4 - 0) = -6\hat{i} - 3\hat{j} + 4\hat{k}$

We know that **area of triangle ABC**

$$= \frac{1}{2}\,|\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AC}}| = \frac{1}{2}\sqrt{36 + 9 + 16} \quad | \sqrt{x^2 + y^2 + z^2}$$

$$= \frac{1}{2}\sqrt{61} \text{ sq. units.}$$