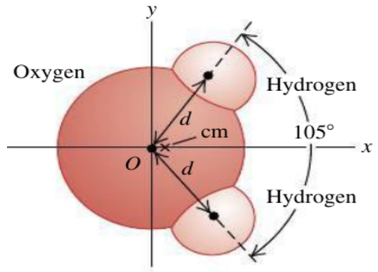
Tutorial 8

PHY 101 Solutions

Q1: The figure shows a simple model of a water molecule. The oxygen–hydrogen separation is d=9.57× 10^{-11} m. Each hydrogen atom has mass 1.0 u, and the oxygen at.om has mass 16.0 u. Find the position of the centre of mass. Nearly all the mass of each atom is concentrated in its nucleus, whose radius is only about 10^{-5} times the overall radius of the atom. Hence we can safely represent each atom as a point particle. We can choose our coordinate system such that the x-axis lie along the molecule's symmetry axis as shown in the figure.



Solution:

The oxygen atom is at x = 0, y = 0. The x-coordinate of each hydrogen atom is dcos(105°/2) and the y-coordinates are

± dsin(105°/2).

$$x_{\mathrm{cm}} = \frac{\left[\begin{array}{c}(1.0\ \mathrm{u})(d\cos 52.5°) + (1.0\ \mathrm{u})(d\cos 52.5°) \\ + (16.0\ \mathrm{u})(0)\end{array}\right]}{1.0\ \mathrm{u} + 1.0\ \mathrm{u} + 16.0\ \mathrm{u}} = 0.068d$$

$$y_{\mathrm{cm}} = \frac{\left[\begin{array}{c}(1.0\ \mathrm{u})(d\sin 52.5°) + (1.0\ \mathrm{u})(-d\sin 52.5°) \\ + (16.0\ \mathrm{u})(0)\end{array}\right]}{1.0\ \mathrm{u} + 1.0\ \mathrm{u} + 16.0\ \mathrm{u}} = 0$$

Substituting $d = 9.57 \times 10^{-11}$ m, we find

$$x_{\mathrm{cm}} = (0.068)(9.57 \times 10^{-11}\ \mathrm{m}) = 6.5 \times 10^{-12}\ \mathrm{m}$$

Note: The center of mass is much closer to the oxygen atom (located at the origin) than to either hydrogen atom because the oxygen atom is much more massive. The center of mass lies along the molecule's axis of symmetry. If the molecule is rotated 180° around this axis, it looks exactly the same as before. The position of the center of mass can't be affected by this rotation, so it must lie on the axis of symmetry.

Q2. Question: Two cars are moving towards each other on a straight road. Car A has a mass of 1500 kg and is moving at a speed of 20 m/s, while Car B has a mass of 1000 kg and is moving at a speed of 15 m/s. The two cars collide head-on and come to a complete stop after the collision. Assume the collision is perfectly inelastic (they stick together after the collision). Calculate the velocity of the combined mass immediately after the collision (before they come to rest).

Solution:

Velocity after the collision:

In a perfectly inelastic collision, momentum is conserved. Therefore, the total momentum before the collision is equal to the total momentum after the collision.

Initial momentum of Car A= $mA \cdot vA = 1500kg \cdot 20m/s = 30000mkg/s$

Initial momentum of Car B=$mB \cdot (-vB) = 1000kg \cdot (-15)m/s = -15000kgm/s$

Total initial momentum=$30000kgm/s - 15000kgm/s = 15000kgm/s$

Since they stick together after the collision, the final momentum is given by the combined mass ($mA+mB$) and the velocity vf

Final momentum=$(mA+mB) \cdot vf$

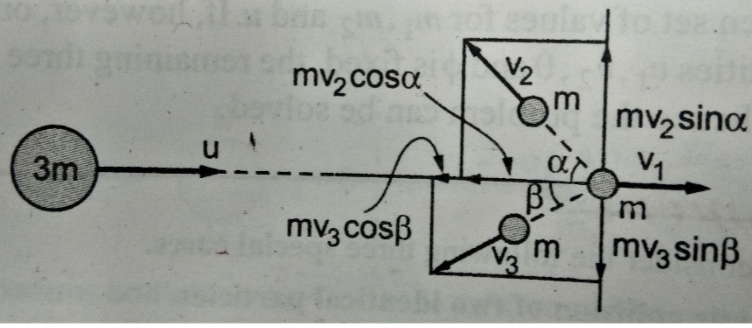
$15000kgm/s = (1500kg+1000kg) \cdot vf$

$vf = 6m/s$

Q3. A shell flying with a velocity u (mass '3m') bursts into three identical fragments (mass 'm') so that the kinetic energy of the system increases k times. What maximum velocity can one of the fragments attain?

Solution:

Solution : If the mass of the shell is $3m$, mass of each of the fragments is m. *It should be understood that the particle with the maximum velocity travels in the forward direction*, [Fig. 10.8].



Applying the law of conservation of momentum along horizontal,

$$3mu = mv_1 - mv_2 \cos\alpha - mv_3 \cos\beta$$

or $$v_1 = 3u + v_2 \cos\alpha + v_3 \cos\beta \qquad ...(i)$$

and along vertical,

$$mv_2 \sin\alpha = mv_3 \sin\beta$$

or $$v_2 \sin\alpha = v_3 \sin\beta \qquad ...(ii)$$

For v_1 to be maximum, from eqn. (i), $\alpha = \beta = 0°$.

From eqn. (ii), since $\alpha = \beta, v_2 = v_3 = v$ (say)

From eqn. (i), $$v_1 = 3u + 2v$$

or $$v = \frac{v_1 - 3u}{2} \qquad ...(iii)$$

From the law of conservation of energy,

$$\frac{1}{2}(3m)u^2 = \frac{1}{k}\left[\frac{1}{2}mv_1^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2\right]$$

or $$3ku^2 = v_1^2 + 2v^2 \qquad ...(iv)$$

From eqns. (iii) and (iv),

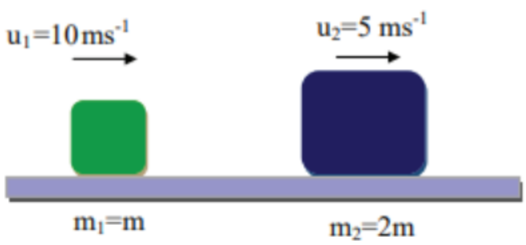
$$3ku^2 = v_1^2 + 2\left(\frac{v_1 - 3u}{2}\right)^2$$

or $$v_1^2 - 2uv_1 + u^2(3-2k) = 0$$

or $$v_1 = \frac{2u + \sqrt{4u^2 - 4u^2(3-2k)}}{2}$$

or $$v_1 = u[1 + \sqrt{2(k-1)}]$$ (neglecting negative root)

Q4. A lighter particle moving with a speed of 10 m s-1 collides with an object of double its mass moving in the same direction with half its speed. Assume that the collision is a one dimensional elastic collision. What will be the speed of both particles after the collision?



Solution:

Initial part of the solution is for the reference of the scholars taking tutorials:

Elastic collisions in one dimension

Consider two elastic bodies of masses m_1 and m_2 moving in a straight line (along positive x direction) on a frictionless horizontal surface as shown in figure 4.16.

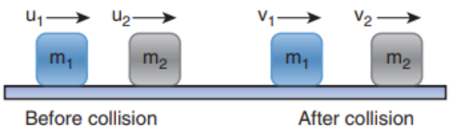


Figure 4.16 Elastic collision in one dimension

Mass	Initial velocity	Final velocity
Mass m_1	u_1	v_1
Mass m_2	u_2	v_2

In order to have collision, we assume that the mass m_1 moves faster than mass m_2 i.e., $u_1 > u_2$. For elastic collision, the total linear momentum and kinetic energies of the two bodies before and after collision must remain the same.

	Momentum of mass m_1	Momentum of mass m_2	Total linear momentum
Before collision	$p_{i1} = m_1u_1$	$p_{i2} = m_2u_2$	$p_i = p_{i1} + p_{i2}$ $p_i = m_1u_1 + m_2u_2$
After collision	$p_{f1} = m_1v_1$	$p_{f2} = m_2v_2$	$p_f = p_{f1} + p_{f2}$ $p_f = m_1v_1 + m_2v_2$

From the law of conservation of linear momentum,

Total momentum before collision (p_i) = Total momentum after collision (p_f)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad (4.46)$$

$$\text{Or } m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad (4.47)$$

Further,

	Kinetic energy of mass m_1	Kinetic energy of mass m_2	Total kinetic energy
Before collision	$KE_{i1} = \frac{1}{2}m_1u_1^2$	$KE_{i2} = \frac{1}{2}m_2u_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_i = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$
After collision	$KE_{f1} = \frac{1}{2}m_1v_1^2$	$KE_{f2} = \frac{1}{2}m_2v_2^2$	$KE_i = KE_{i1} + KE_{i2}$ $KE_f = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$

For elastic collision,

Total kinetic energy before collision KE_i = Total kinetic energy after collision KE_f

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad (4.48)$$

After simplifying and rearranging the terms,

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

Using the formula $a^2 - b^2 = (a+b)(a-b)$, we can rewrite the above equation as

$$m_1(u_1+v_1)(u_1-v_1)=m_2(v_2+u_2)(v_2-u_2) \quad (4.49)$$

Dividing equation (4.49) by (4.47) gives,

$$\frac{m_1(u_1+v_1)(u_1-v_1)}{m_1(u_1-v_1)}=\frac{m_2(v_2+u_2)(v_2-u_2)}{m_2(v_2-u_2)}$$

$$u_1+v_1=v_2+u_2 \quad \text{Rearranging,} \quad (4.50)$$
$$u_1-u_2=v_2-v_1$$

Equation (4.50) can be rewritten as

$$u_1-u_2=-(v_1-v_2)$$

This means that for any elastic head on collision, the relative speed of the two elastic bodies after the collision has the same magnitude as before collision but in opposite direction. Further note that this result is independent of mass.

Rewriting the above equation for v_1 and v_2,

$$v_1=v_2+u_2-u_1 \quad (4.51)$$

Or

$$v_2=u_1+v_1-u_2 \quad (4.52)$$

To find the final velocities v_1 and v_2:

Substituting equation (4.52) in equation (4.47) gives the velocity of m_1 as

$$m_1(u_1-v_1)=m_2(u_1+v_1-u_2-u_2)$$

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - u_2 - u_2)$$

$$m_1(u_1 - v_1) = m_2(u_1 + v_1 - 2u_2)$$

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - 2m_2u_2$$

$$m_1u_1 - m_2u_1 + 2m_2u_2 = m_1v_1 + m_2v_1$$

$$(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$$

$$\text{or } v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad (4.53)$$

Similarly, by substituting (4.51) in equation (4.47) or substituting equation (4.53) in equation (4.52), we get the final velocity of m_2 as

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 \quad (4.54)$$

Case 1: When bodies has the same mass i.e., $m_1 = m_2$,

$$\text{equation (4.53)} \Rightarrow v_1 = (0)\ u_1 + \left(\frac{2m_2}{2m_2}\right)u_2$$

$$v_1 = u_2 \quad (4.55)$$

$$\text{equation (4.54)} \Rightarrow v_2 = \left(\frac{2m_1}{2m_1}\right)u_1 + (0)u_2$$

$$v_2 = u_1 \quad (4.56)$$

The equations (4.55) and (4.56) show that in one dimensional elastic collision, when two bodies of equal mass collide after the collision their velocities are exchanged.

Main solution

Let the mass of the fi rst body be m which moves with an initial velocity, $u_1 = 10 \text{ m s}^{-1}$.

Therefore, the mass of second body is 2m and its initial velocity is $u_2 = ½ u_1 = ½(10 \text{ms}^{-1})$

Then, the fi nal velocities of the bodies can be calculated from the equation (4.53) and equation (4.54)

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_1 = \left(\frac{m - 2m}{m + 2m}\right)10 + \left(\frac{2 \times 2m}{m + 2m}\right)5$$

$$v_1 = -\left(\frac{1}{3}\right)10 + \left(\frac{4}{3}\right)5 = \frac{-10 + 20}{3} = \frac{10}{3}$$

$$v_1 = 3.33 \text{ ms}^{-1}$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m}{m + 2m}\right)10 + \left(\frac{2m - m}{m + 2m}\right)5$$

$$v_2 = \left(\frac{2}{3}\right)10 + \left(\frac{1}{3}\right)5 = \frac{20 + 5}{3} = \frac{25}{3}$$

$$v_2 = 8.33 \text{ms}^{-1}$$

As the two speeds $v_1$and v_2 are positive, they move in the same direction with the velocities, 3.33 m s^{-1} and 8.33 m s^{-1} respectively.

Q5.A bullet of mass 50 g is fired from below into a suspended object of mass 450 g. The object rises through a height of 1.8 m with bullet remaining inside the object. Find the speed of the bullet. Take g = 10 ms^{-2}.

Solution:

Perfect inelastic collision

In a perfectly inelastic or completely inelastic collision, the objects stick together permanently aft er collision such that they move with common velocity. Let the two bodies with masses m_1 and m_2 move with initial velocities u_1 and u_2 respectively before collision. Aft er perfect inelastic collision both the objects move together with a common velocity v as shown in Figure (4.17).

Since, the linear momentum is conserved during collisions,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$$

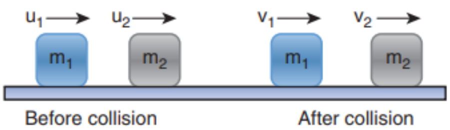


Figure 4.17 Perfect inelastic collision in one dimension

	Velocity		Linear momentum	
	Initial	Final	Initial	Final
Mass m_1	u_1	v	m_1u_1	m_1v
Mass m_2	u_2	v	m_2u_2	m_2v
Total			$m_1u_1 + m_2u_2$	$(m_1 + m_2)$ v

The common velocity can be computed by

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \qquad (4.63)$$

$m_1 = 50$ g = 0.05 kg; $m_2 = 450$ g = 0.45kg

The speed of the bullet is u_1. The second body is at rest $u_2 = 0$. Let the common velocity of the bullet and the object after the bullet is embedded into the object is v.

$$v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)}$$

$$v = \frac{0.05u_1 + (0.45\times 0)}{(0.05+0.45)} = \frac{0.05}{0.50}u_1$$

The combined velocity is the initial velocity for the vertical upward motion of the combined bullet and the object. From second equation of motion,

$$v = \sqrt{2gh}$$

$$v = \sqrt{2\times 10\times 1.8} = \sqrt{36}$$

$$v = 6\,ms^{-1}$$

Substituting this in the above equation, the value of u_1 is

$$6 = \frac{0.05}{0.50}u_1 \quad \text{or} \quad u_1 = \frac{0.50}{0.05}\times 6 = 10\times 6$$

$$u_1 = 60\,ms^{-1}$$