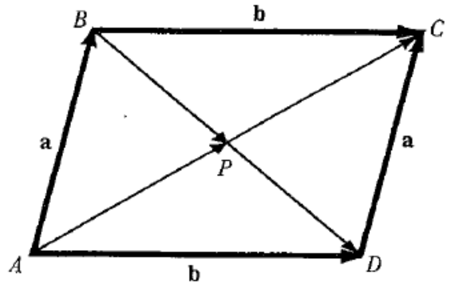
`PRACTICE PROBLEMS FOR MIDSEM

PHY 101

Q1. If r_1=2i-j+k , r_2=i+3j+2k, r_3=-2i+j-3k, r_4=3i+2j+5k then find scalars such that $r_4=ar_1+br_2+cr_3$.

Q2. Prove that diagonals of a parallelogram bisect each other using properties of vectors.



Q3. Given a scalar field defined by $\varphi(x,y,z) = 3x^2z - xy^3 + 5$. Find φ at the points

(a) (0,0,0) , (1,-2,2) , (-1,-2,-3)

(b) If $\vec{V}(x,y,z) = \vec{\nabla}\varphi(x,y,z) = \frac{\delta}{\delta x}(\varphi)\hat{\imath} + \frac{\delta}{\delta y}(\varphi)\hat{\jmath} + \frac{\delta}{\delta z}(\varphi)\hat{k}$,where $\frac{\delta}{\delta a}$ is the partial derivative wrt the variable a($\frac{\delta}{\delta x}f(x)g(y)h(z) = g(y)h(z)\frac{\delta}{\delta x}f(x)$,true for other variables y and z)

then find the value of $\vec{V}(x,y,z)$ at the same points given in (a)

Q4. Find the angle vector $\vec{A} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ makes with the coordinate axes.

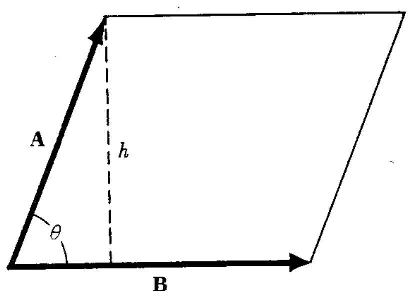
Q5.

If $\mathbf{A} = 2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\mathbf{B} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, find ($a$) $\mathbf{A} \times \mathbf{B}$, ($b$) $\mathbf{B} \times \mathbf{A}$, ($c$) $(\mathbf{A} + \mathbf{B}) \times (\mathbf{A} - \mathbf{B})$.

Find the unit vector in the direction of each vectors in (a), (b)and (c)

Q6. (a)Prove that area of a parallelogram of sides $\vec{A}$ and $\vec{B}$ is $|\overrightarrow{A \times B}|$

(b)Similarly Prove that area of a triangle of sides $\vec{A}$ and $\vec{B}$ is $\frac{1}{2}|\overrightarrow{A \times B}|$



(c) Find the area of the triangle having vertices at $P(1, 3, 2)$, $Q(2, -1, 1)$, $R(-1, 2, 3)$.

Q7. Use the method of dimensions to obtain the form of the dependance of the lift force per unit wingspan on an aircraft wing of width (in the direction of motion) L, moving with velocity v through the air density ρ, on the parameters L, v, ρ.

Q8. Speed of waves v on a string depend on its mass m, length l, force τ by the equation

$$v = m^a l^b \tau^c$$

Find the values of a,b and c using dimensional analysis. Write down the final form of the equation.

Q9. Convert (-1, -1) into polar coordinates.

Q10. Two vectors A and B have equal magnitudes of 10 units. Vector A makes an angle of 30 degrees with the positive x-axis, while vector B makes an angle of 45 degrees with the positive y-axis. Calculate the dot product and cross product of vectors A and B.

Q.11 A particle sliding along a radial groove in a rotating turntable has polar coordinates at time t given by r = ct, θ = Ωt, where c and Ω are positive constants. Find the velocity and acceleration vectors of the particle at time t and find the speed of the particle at time t. Deduce that, for t > 0, the angle between the velocity and acceleration vectors is always acute.

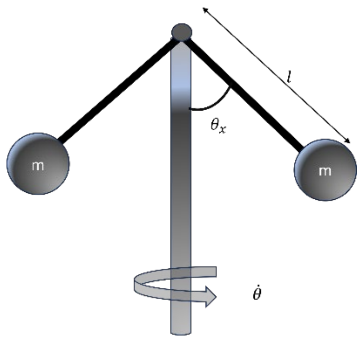
Q.12 A light rope fixed at one end of a wooden clamp on the ground passes over a tree branch and hangs on the other side. It makes an angle of 30° with the ground. A man weighing (60 kg) wants to climb up the rope. The wooden clamp can come out of the ground if an upward force greater than 360 N is applied to it. Find the maximum acceleration in the upward direction with which the man can climb safely. Neglect friction at the tree branch. Take g = 10 m/s^2.

Q.13 A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the same ball?

Q.14 Consider an automobile moving along a straight horizontal road with a speed v_0. If the coefficient of static friction between the tires and the road is μ_s, what is the shortest distance in which the automobile can be stopped?

Q.15 A small body was launched up an inclined plane set at an angle θ= 150 against the horizontal. Find the coefficient of friction if the time of the ascent of the body is β = 2 times less than the time of its descent.

Q.16 A device consists of mass of equal magnitude m tethered to a central shaft as shown in the figure. At a constant rotational speed of the central shaft the masses will be at a constant angle θ_x wrt to the central shaft. Considering length of the tethers are l and acceleration due to gravity g.



(a) Rate of spinning of the shaft is $\omega = \dot{\theta} = \sqrt{\frac{g}{l\ Cos\theta_x}}$

(b) If we want to spin it exactly at 60 rpm, what will be the angle $\theta_x$if m=0.5kg and l=1m

Q.17 Considering the identities:

$$(a)\quad \frac{d}{du}(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\cdot\frac{d\mathbf{B}}{du} + \frac{d\mathbf{A}}{du}\cdot\mathbf{B}, \qquad (b)\quad \frac{d}{du}(\mathbf{A}\times\mathbf{B}) = \mathbf{A}\times\frac{d\mathbf{B}}{du} + \frac{d\mathbf{A}}{du}\times\mathbf{B}$$

If $\mathbf{A} = 5t^2\mathbf{i} + t\mathbf{j} - t^3\mathbf{k}$ and $\mathbf{B} = \sin t\ \mathbf{i} - \cos t\ \mathbf{j}$, find $(a)\ \frac{d}{dt}(\mathbf{A}\cdot\mathbf{B})$, $(b)\ \frac{d}{dt}(\mathbf{A}\times\mathbf{B})$, $(c)\ \frac{d}{dt}(\mathbf{A}\cdot\mathbf{A})$.

Q.18 Acceleration of a particle at any time $t \geq 0$ is given by:

$$\vec{a} = \frac{d\vec{v}}{dt} = 12Cos\ (2t)\hat{\imath} + 8Sin(2t)\hat{\jmath} + 16t\hat{k}$$

If $\vec{v} = 0$ and $\vec{r} = 0$ at t=0(initial conditions), calculate the value of $\vec{v}$ and $\vec{r}$ at any time t.

(hint: compute the constant of integration using the initial conditions given in the problem)

Q19.If $\vec{F} = 3xy\hat{\imath} - y^2\hat{\jmath}$ evaluate work done $\int \vec{F}.\overrightarrow{dr}$ along the curve C in xy plane given by the equation $y = 2x^2$ in the limit (0,0) to (1,2).

Q.20 A block of mass 5 kg resting on a 30 degree inclined plane is released. The block after travelling a distance of 0.5 m along the inclined plane hits a spring of stiffness 15N/cm. Find the maximum compression of the spring. Assume the coefficient of friction between the block and inclined plane as 0.2.