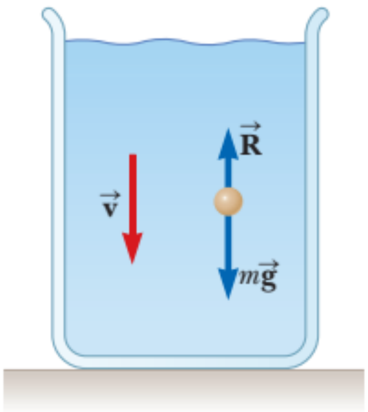
PHY101: Introduction to Physics I

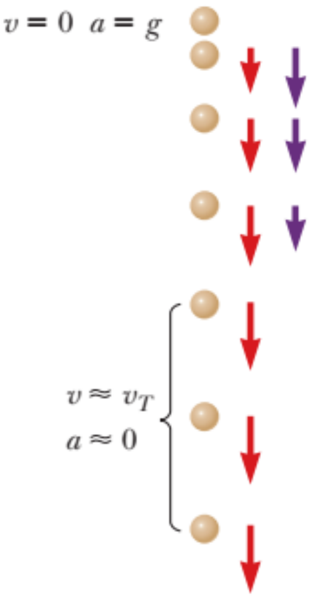
Monsoon Semester 2024
Lecture 13

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

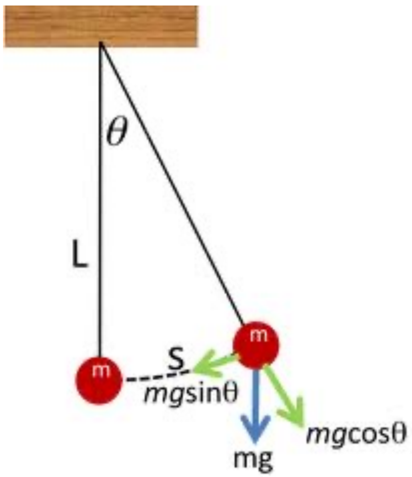
Type of forces
Viscous force





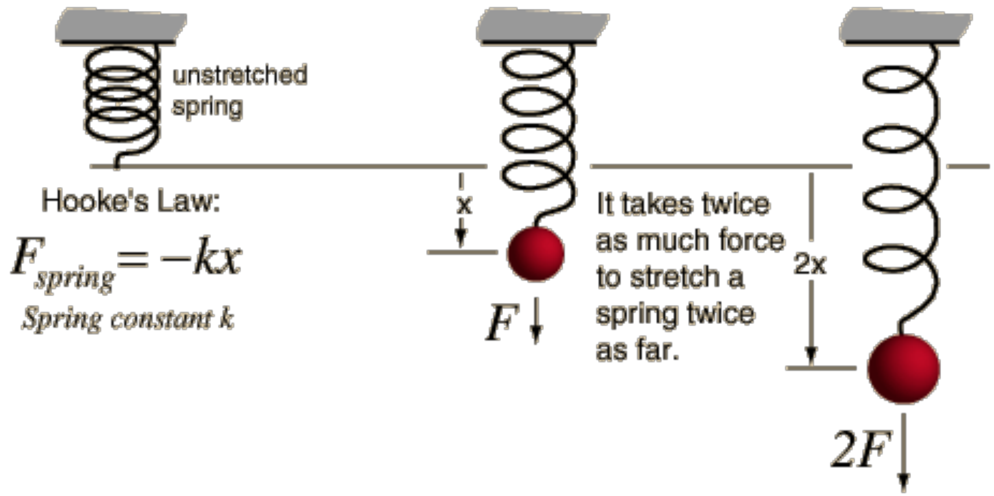
This Lecture

Restoring forces
Inverse square forces
Work-Energy Theorem



Linear restoring force

The restoring force experienced depends linearly on the displacement.



Hooke's law:

$$F = -kx$$

Almost all restoring forces can be approximated by a linear dependence when the displacement involved from the equilibrium is small, e.g., the restoring force on the pendulum for small angular displacement ϑ from equilibrium is approximately proportional to ϑ.

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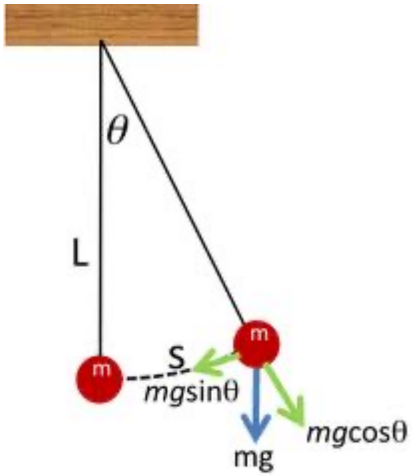
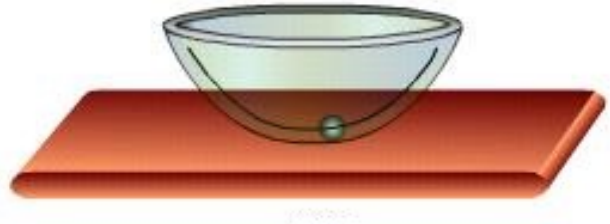
Restoring Force

Restoring force is a variable force that gives rise to an equilibrium (stable) in a physical system. If the system is perturbed away from the stable equilibrium, the restoring force tends to bring the system back toward it.



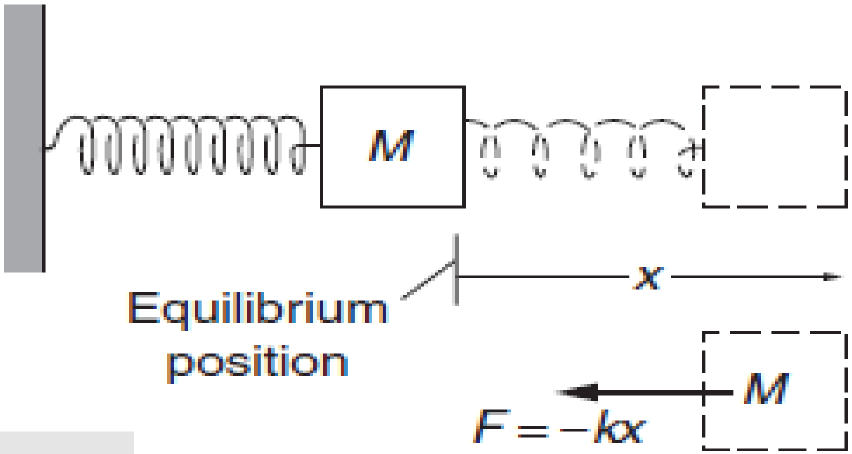
A simple pendulum.
The bob experiences a nonlinear restoring force $F = -mg \sin\theta$



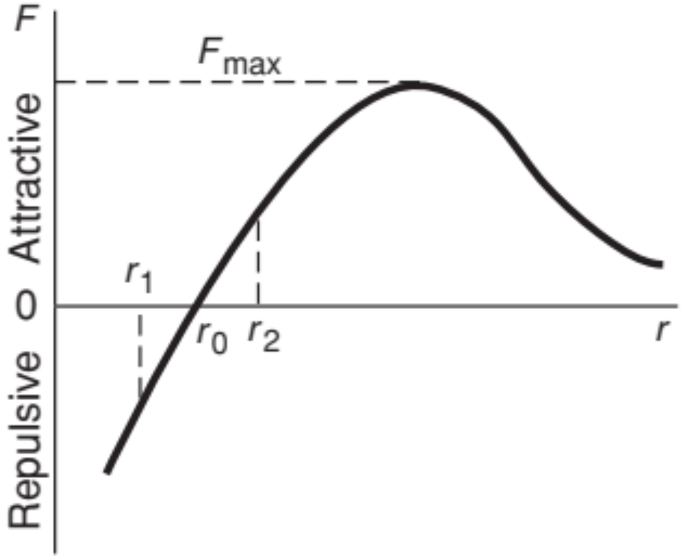
A ball in a bowl. Similar to the pendulum, this also experiences a nonlinear restoring force. However, for small displacements from the equilibrium position a linear approximation can be invoked in **almost** all these systems.

Image Sources:
http://www.ic.sunysb.edu/Class/phy141md/lib/exe/fetch.php?media=phy141:lectures:simplependulum.png
http://images.tutorvista.com/content/rigid-body/stable-equilibrium.jpeg
http://en.wikipedia.org/wiki/Hooke%27s_law#mediaviewer/File:HookesLawForSpring-English.png

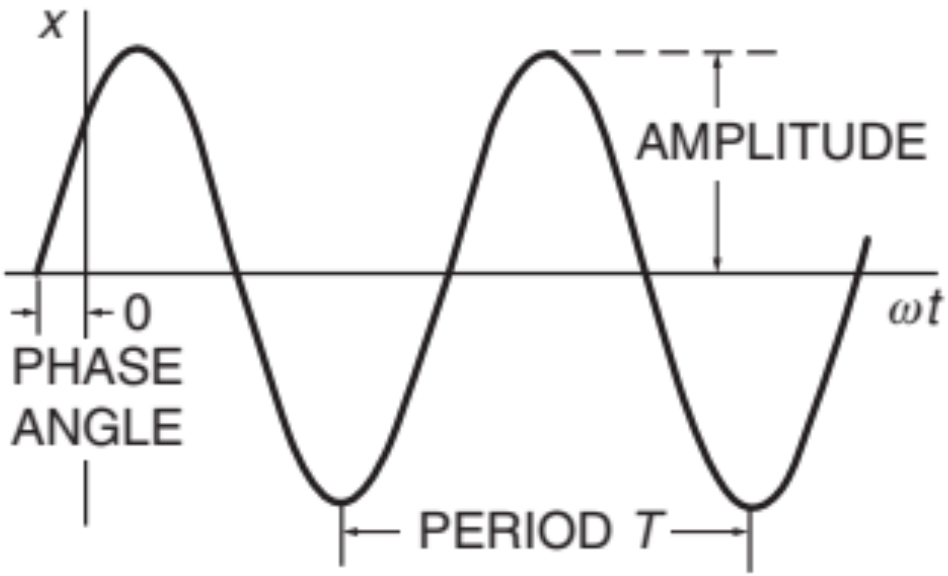
For a linear restoring force:

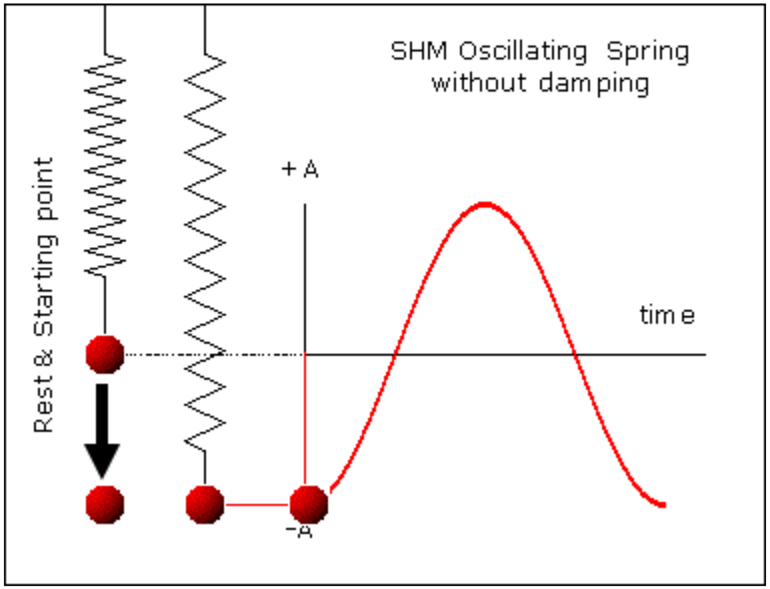


The restoring force: $F = -kx$



$$\Rightarrow \ddot{x} + \omega^2 x = 0, \qquad \omega = \sqrt{\frac{k}{m}}$$



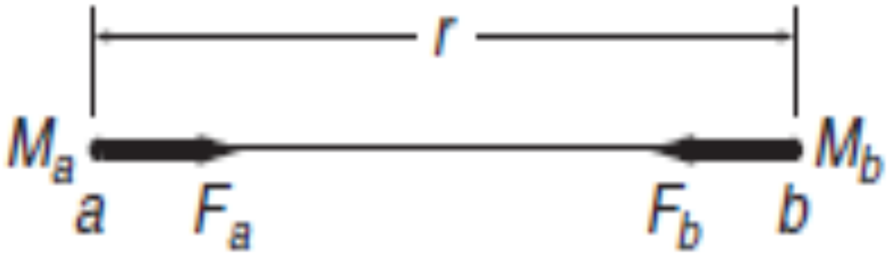


$$x = C\sin(\omega t + \phi)$$

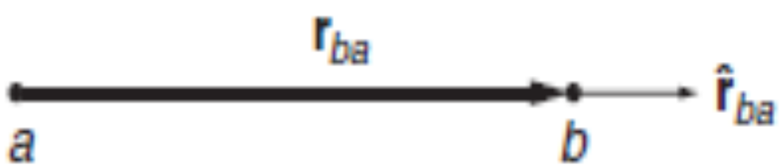
$$\boxed{x = x_0 \sin(\omega t + \pi/2)}$$

Inverse square law force

Gravity



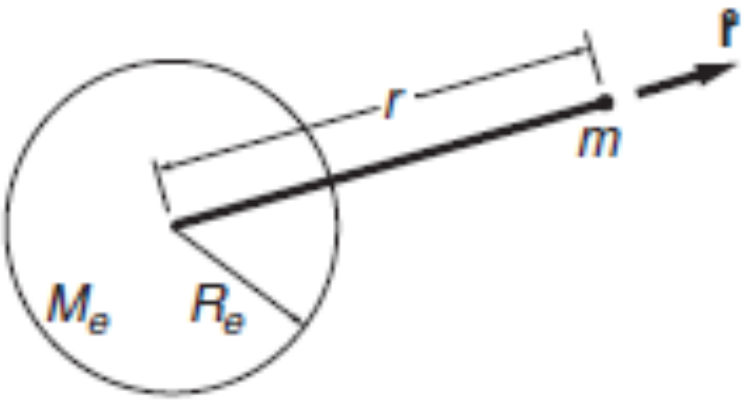
$$|\mathbf{F}_{b,a}| = \frac{GM_aM_b}{r^2}.$$



$$\mathbf{F}_{b,a} = -\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{b,a}.$$

$$\mathbf{F}_{a,b} = +\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{b,a} = -\frac{GM_aM_b}{r^2}\hat{\mathbf{r}}_{a,b} = -\mathbf{F}_{b,a}$$

The <u>negative sign</u> indicates that the force on particle *b* is directed toward particle *a*, that is, the force is <u>attractive</u>.



$$\mathbf{F} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}} \qquad r \geq R_e$$

The Acceleration Due to Gravity

and the acceleration due to gravity is $\mathbf{a} = \frac{\mathbf{F}}{m} = -\frac{GM_e}{R_e^{\,2}}\hat{\mathbf{r}}.$

Coulomb Force

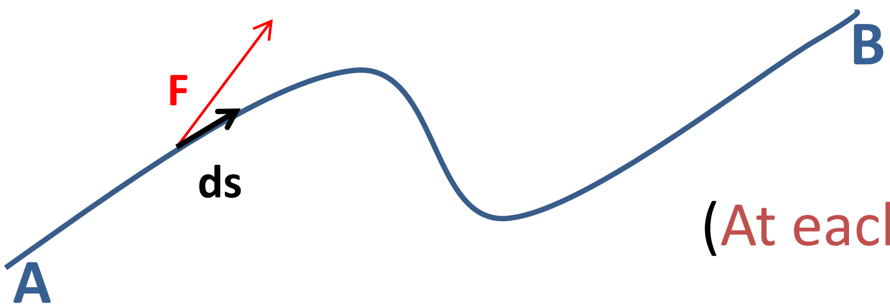
$$\mathbf{F} \propto \frac{Qq\,\hat{r}}{r^2}$$

Work done

When multiple forces act on a body, the total work done can be calculated in two ways:

1. $W = W_1 + W_2 + \cdots = \overrightarrow{F_1}.\vec{s} + \overrightarrow{F_2}.\vec{s} + \cdots$
2. $W = \left(\overrightarrow{F_1} + \overrightarrow{F_2} + \cdots\right).\vec{s} = \vec{F}.\vec{s}$

Let's consider a particle moving in a curved path and subjected to a force that varies both in magnitude and direction.



$$dW = \vec{F}.d\vec{s}$$

(At each point $\overrightarrow{ds}$ is tangent to the path)

If the particle moves from point A to B:

$$W = \int_A^B \vec{F}.\overrightarrow{ds}$$

Work and Energy Theorem

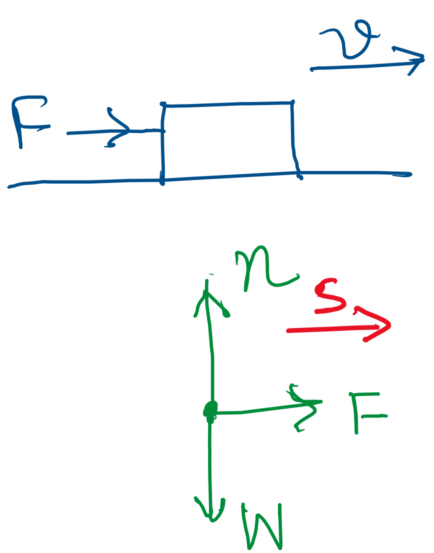
Statement : $W = K_f - K_i$

K = kinetic energy

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system .

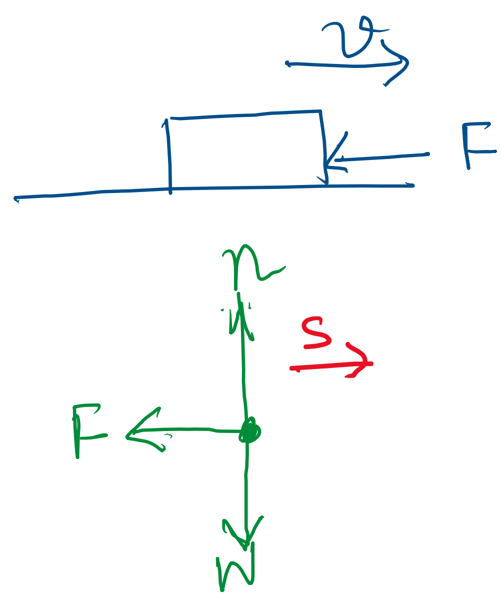
The work–kinetic energy theorem indicates that the speed of a system ***increases*** if the net work done on it is ***positive*** because the final kinetic energy is greater than the initial kinetic energy. The speed ***decreases*** if the net work is ***negative*** because the final kinetic energy is less than the initial kinetic energy.

Consider a block sliding on a frictionless surface:



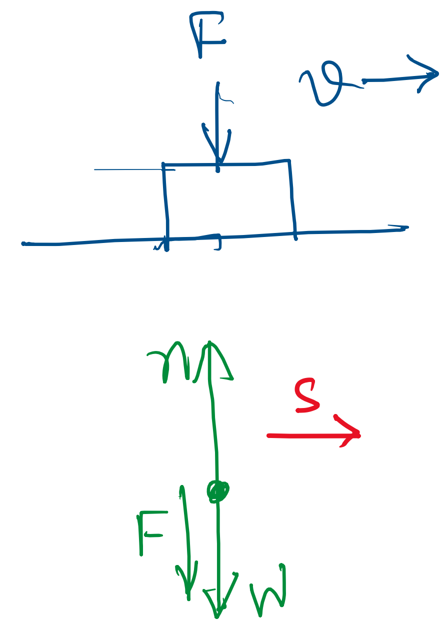
$W > 0$

the block speeds up



$W < 0$

the block slows down



$W = 0$

no change in speed

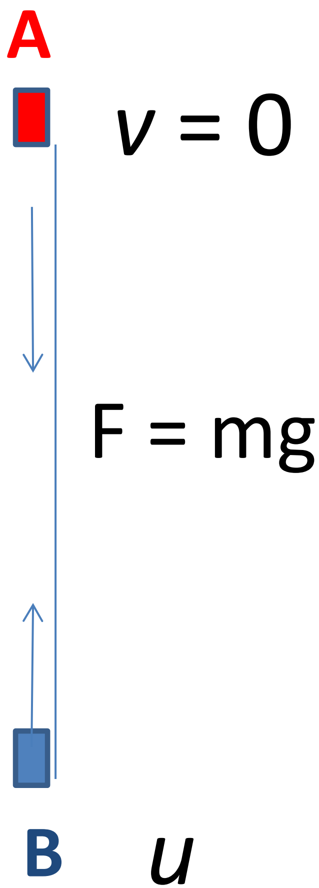
Example 1:

$$v_A = u \qquad s = \frac{v^2 - u^2}{2a} \qquad v_B = v$$

$$W = F.\boldsymbol{s} = \boldsymbol{ma}.\boldsymbol{s} = \boldsymbol{ma}\frac{v^2 - u^2}{2a} = K_B - K_A$$

Example 2:

Object thrown upward



$$W = F.\boldsymbol{h} = |\boldsymbol{mg}||\boldsymbol{h}|\ \boldsymbol{cos}\,\pi$$

$$= -mgh$$

$$\text{Change in KE} = K_A - K_B$$

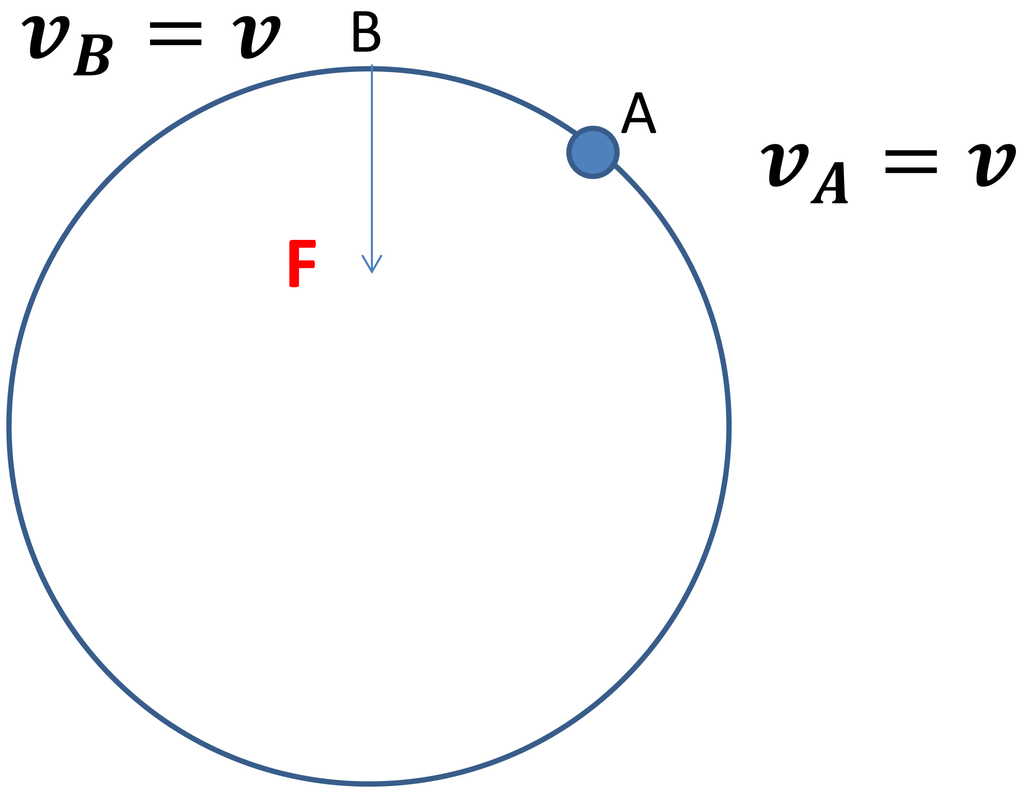
$$= -\frac{m\,u^2}{2}$$

$$= -mgh$$

$$= W$$

$v^2 = u^2 - 2gh$

Example 3 : Moving on a Circular Path

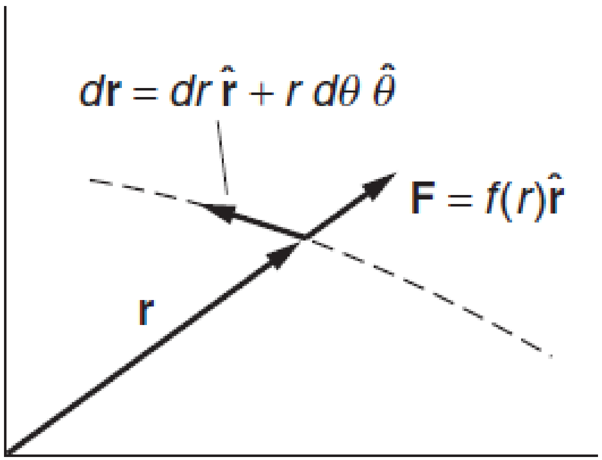


$$W = \int_A^B \mathbf{F} . d\mathbf{s} = 0$$ **Why ?**

At any point force is perpendicular to displacement.

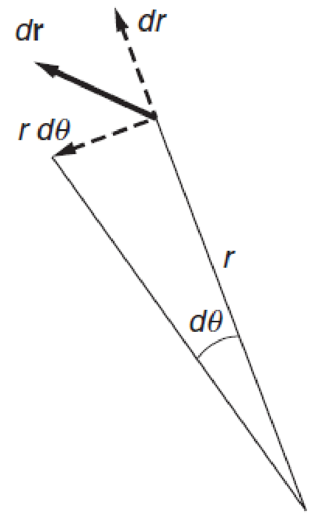
Work by a central force:

=> Radial force that depend only on the distance from the origin.



Take a particle moving from $\vec{r}_a$ to $\vec{r}_b$ under a force $\vec{F} = f(r)\hat{r}$.

The particle moves in a plane. The position vector is:



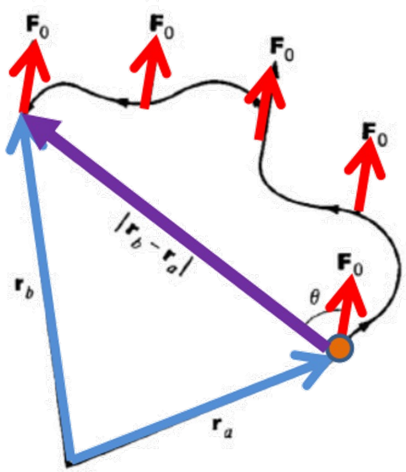
$$d\mathbf{r} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\theta}.$$

$$\begin{aligned} W_{ba} &= \oint_a^b \mathbf{F}\cdot d\mathbf{r} \\ &= \oint_a^b f(r)\hat{\mathbf{r}}\cdot(dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}}) \\ &= \int_a^b f(r)\,dr. \end{aligned}$$

$$W_{ba} = \int_a^b f(r)\,dr.$$

Work done is same as going from a to b following a straight line (same as one dimensional case, **no θ dependence**!).

=> Under the central force work done depends only on the end points, and not on the particular path followed.

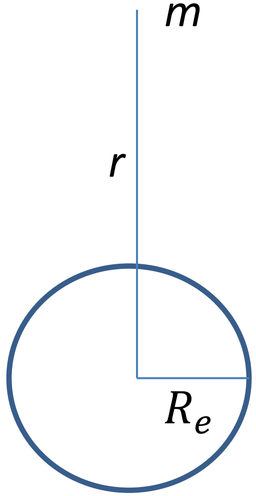


$$W_{ba} = F_0 \cos\theta \left|\vec{r}_b - \vec{r}_a\right|$$

From this observation, $W_{ba} + W_{ab} = 0$

=> Work done by a central force around a closed path is zero.

An object under earth's gravity

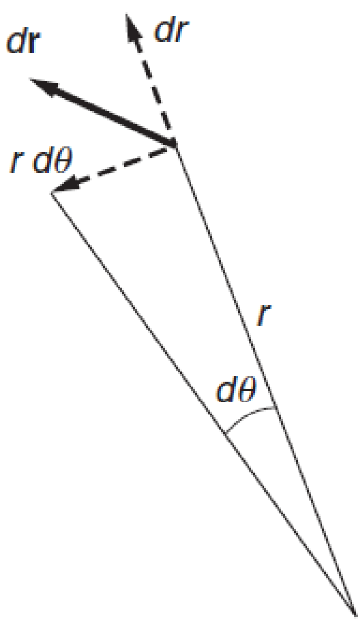


Neglecting the air resistance,

$$\mathbf{F} = -\frac{GM_e m}{r^2}\hat{\mathbf{r}}$$

Element of displacement in the plane

$$d\mathbf{r} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}}.$$



On surface of earth, $mg = \dfrac{GM_e m}{{R_e}^2}$

$$GM_e m = mgR_e^2$$

$$\mathbf{F}\cdot d\mathbf{r} = -mg\frac{{R_e}^2}{r^2}\hat{\mathbf{r}}\cdot(dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}})$$

$$= -mg\frac{{R_e}^2}{r^2}dr. \quad \because \hat{\mathbf{r}}\cdot\hat{\boldsymbol{\theta}} = 0$$