Tutorial 9 Solutions

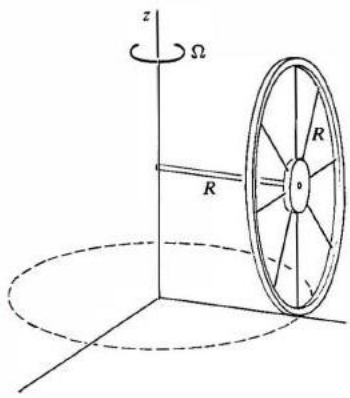
PHY 101

Q1. A thin hoop of mass M and radius R rolls without slipping about the z axis. It is supported by an axle of length R through it's centre, as shown. The hoop circles around the z axis with angular velocity Ω.

(a) What is the instantaneous angular velocity ω of the hoop?

(b) What is the angular momentum L of the hoop? Is L parallel to ω?

(Note: the moment of inertia of the hoop for an axis along its diameter is $\frac{1}{2}MR^2$.)



Soln:

8.1 *Rolling hoop*

(a)

$$\omega_s = \frac{v}{R} = \frac{\Omega R}{R} = \Omega$$

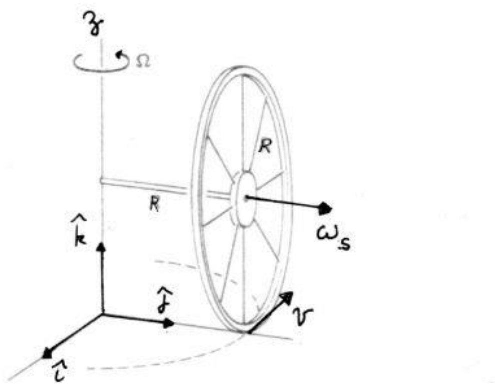
$$\boldsymbol{\omega} = \boldsymbol{\omega}_s + \boldsymbol{\Omega} = \Omega(\hat{\mathbf{j}} + \hat{\mathbf{k}})$$

(b)

$$\mathbf{L} = \mathbf{L}_s + \mathbf{L}_\omega = I_s\boldsymbol{\omega}_s + I_z\boldsymbol{\Omega}$$

$$I_s = MR^2 \qquad I_z = I_0 + MR^2 = \frac{3}{2}MR^2$$

$$\mathbf{L} = MR^2\left(\boldsymbol{\omega}_s + \frac{3}{2}\boldsymbol{\Omega}\right) = MR^2\,\Omega(\hat{\mathbf{j}} + \tfrac{3}{2}\hat{\mathbf{k}})$$

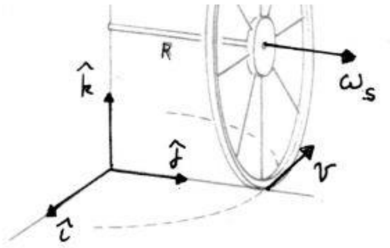


(b)

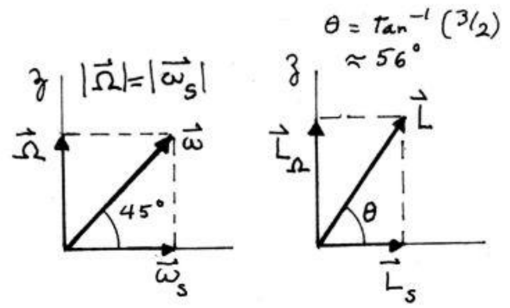
$$\mathbf{L} = \mathbf{L_s} + \mathbf{L}_\omega = I_s\omega_s + I_z\mathbf{\Omega}$$

$$I_s = MR^2 \qquad I_z = I_0 + MR^2 = \frac{3}{2}MR^2$$

$$\mathbf{L} = MR^2\left(\omega_s + \frac{3}{2}\mathbf{\Omega}\right) = MR^2\,\Omega(\hat{\mathbf{j}} + \tfrac{3}{2}\hat{\mathbf{k}})$$



The lower sketches show that ω and $\mathbf{L}$ are not parallel. We treated $\mathbf{L}$ as the angular momentum of a body with moment of inertia from the parallel axis theorem. $\mathbf{L}$ can also be viewed as the sum of orbital angular momentum $MR^2\mathbf{\Omega}$ plus spin angular momentum $(1/2)MR^2\mathbf{\Omega}$.



Q2. A bowling ball of mass 4.0kg, a moment of inertia of 1.6×10^{-2} kgm^2 and a radius of 0.10m. If it rolls down the lane without slipping at a linear speed of 4msec^{-1}, what is its total energy?

Solution

The total (kinetic) energy of an object which rolls without slipping is given by

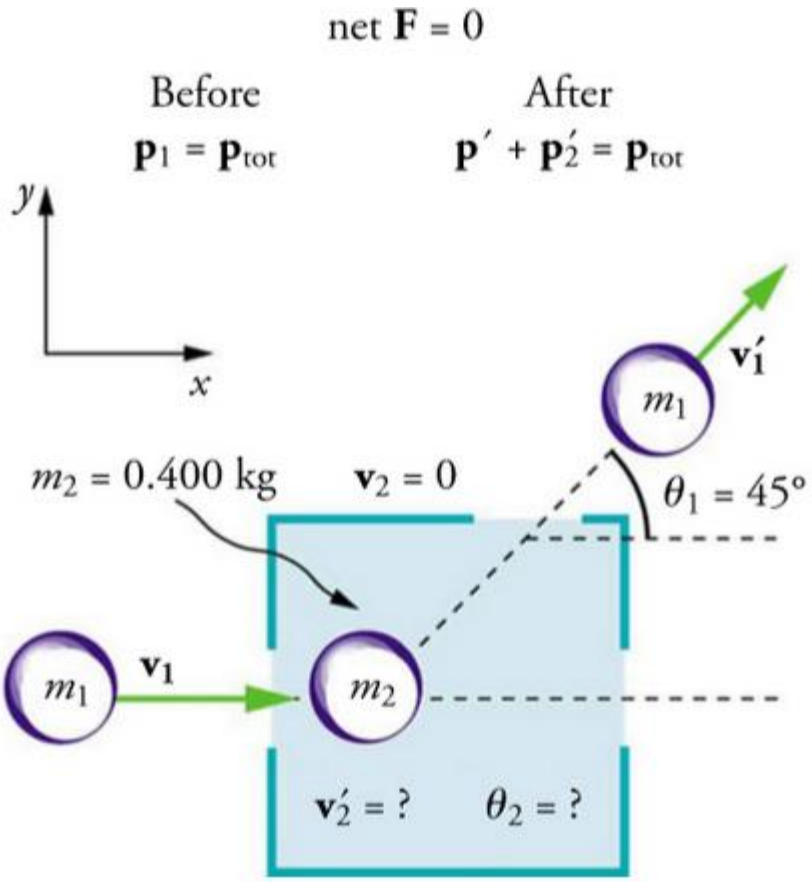
$$\omega = \frac{v_{\mathrm{CM}}}{R} = \frac{(4.0\,\frac{\mathrm{m}}{\mathrm{s}})}{(0.10\,\mathrm{m})} = 40.0\,\frac{\mathrm{rad}}{\mathrm{s}}$$

$$\begin{aligned} K_{\mathrm{roll}} &= \tfrac{1}{2}I_{\mathrm{CM}}\omega^2 + \tfrac{1}{2}Mv_{\mathrm{CM}}^2 \\ &= \tfrac{1}{2}(1.6\times10^{-2}\,\mathrm{kg\cdot m^2})(40.0\,\tfrac{\mathrm{rad}}{\mathrm{s}})^2 + \tfrac{1}{2}(4.0\,\mathrm{kg})(4.0\,\tfrac{\mathrm{m}}{\mathrm{s}})^2 \\ &= 44.8\,\mathrm{J} \end{aligned}$$

Q3. A 0.250-kg object (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object with mass of 0.400 kg (m_2). The 0.250-kg object emerges from the room at an angle of 45.0º with its incoming direction. The speed of the 0.250-kg object is originally 2.00 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and ϑ_2) of the 0.400-kg object after the collision.

Momentum is conserved because the surface is frictionless. The coordinate system shown in the figure is one in which m_2 is originally at rest and the initial velocity is parallel to the *x*-axis, so that conservation of momentum along the *x*- and *y*-axes is applicable.

Everything is known in these equations except v'_2 and ϑ_2, which are precisely the quantities we wish to find. We can find two unknowns because we have two independent equations: the equations describing the conservation of momentum in the *x*- and *y*-directions.



Solving $m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2$ for $v'_2 \cos\theta_2$ and $0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2$ for $v'_2 \sin\theta_2$ and taking the ratio yields an equation (in which θ_2 is the only unknown quantity. Applying the identity $\left(\tan\theta = \frac{\sin\theta}{\cos\theta}\right)$, we obtain:

$$\tan\theta_2 = \frac{v'_1 \sin\theta_1}{v'_1 \cos\theta_1 - v_1}. \tag{8.68}$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50\text{ m/s})(0.7071)}{(1.50\text{ m/s})(0.7071) - 2.00\text{ m/s}} = -1.129. \tag{8.69}$$

Thus,

$$\theta_2 = \tan^{-1}(-1.129) = 311.5^\circ \approx 312^\circ. \tag{8.70}$$

Angles are defined as positive in the counter clockwise direction, so this angle indicates that m_2 is scattered to the right in Figure 8.11, as expected (this angle is in the fourth quadrant). Either equation for the x- or y-axis can now be used to solve for v'_2, but the latter equation is easiest because it has fewer terms.

$$v'_2 = -\frac{m_1}{m_2} v'_1 \frac{\sin\theta_1}{\sin\theta_2} \qquad 8.71$$

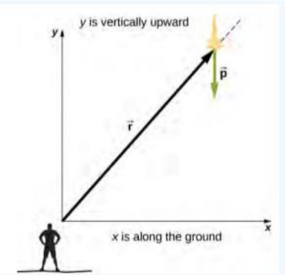
Entering known values into this equation gives

$$v'_2 = -\left(\frac{0.250\ \mathrm{kg}}{0.400\ \mathrm{kg}}\right)(1.50\ \mathrm{m/s})\left(\frac{0.7071}{-0.7485}\right). \qquad 8.72$$

Thus,

$$v'_2 = 0.886\ \mathrm{m/s}. \qquad 8.73$$

Q4. Q4. A meteor enters Earth's atmosphere as shown in fig. and is observed by someone on the ground before it burns up in the atmosphere. The vector $\vec{r}$ = 25 km $\hat{\imath}$ + 25 km $\hat{\jmath}$ gives the position of the meteor with respect to the observer. At the instant the observer sees the meteor, it has linear momentum $\vec{p}$ = (15.0 kg)(−2.0 km/s $\hat{\jmath}$), and it is accelerating at a constant 2.0 m/s^2 ($-\hat{\jmath}$) along its path, which for our purposes can be taken as a straight line. What is the angular momentum of the meteor about the origin, which is at the location of the observer?



We resolve the acceleration into x- and y-components and use the kinematic equations to express the velocity as a function of acceleration and time. We insert these expressions into the linear momentum and then calculate the angular momentum using the cross-product. Since the position and momentum vectors are in the xy-plane, we expect the angular momentum vector to be along the z-axis. To find the torque, we take the time derivative of the angular momentumThe meteor is entering Earth's atmosphere at an angle of 90.0° below the horizontal, so the components of the acceleration in the x- and y-directions are

$$a_x = 0,\ a_y = -2.0\ m/s^2.$$

We write the velocities using the kinematic equations.

$$v_x = 0,\ v_y = (-2.0 \times 10^3\ m/s) - (2.0\ m/s^2)t.$$

a. The angular momentum is

$$\begin{aligned} \vec{l} = \vec{r} \times \vec{p} &= (25.0\ km\ \hat{i} + 25.0\ km\ \hat{j}) \times (15.0\ kg)(0\hat{i} + v_y\hat{j}) \\ &= 15.0\ kg[25.0\ km(v_y)\hat{k}] \\ &= 15.0\ kg\{(2.50 \times 10^4\ m)[(-2.0 \times 10^3\ m/s) - (2.0\ m/s^2)t]\hat{k}\}. \end{aligned}$$

At t = 0, the angular momentum of the meteor about the origin is

$$\vec{l}_0 = 15.0\ kg[(2.50 \times 10^4\ m)(-2.0 \times 10^3\ m/s)\hat{k}] = 7.50 \times 10^8\ kg \cdot m^2/s(-\hat{k}).$$

This is the instant that the observer sees the meteor.