<u>Tutorial Questions and Solutions</u>

<u>1)</u> Use the method of dimensions to obtain the form of the dependance of the lift force per unit wingspan on an aircraft wing of width (in the direction of motion) L, moving with velocity v through the air density ρ, on the parameters L, v, ρ.

<u>Ans:</u>

Let us call the lift per unit wingspan Φ, and write

$$\Phi = kL^{\alpha}v^{\beta}\rho^{\gamma},$$

where k, α, β and γ are dimensionless constants. Since the dimensions of force are MLT^{-2}, the dimensions of Φ are MT^{-2}. Thus

$$MT^{-2} = L^{\alpha}L^{\beta}T^{-\beta}M^{\gamma}L^{-3\gamma}.$$

So by equating the terms in M, $\gamma = 1$.
By equating the terms in T, $-\beta = -2$ so $\beta = 2$.
By equating the terms in L, $\alpha + \beta - 3\gamma = 0$, therefore $\alpha = 1$.
Thus we may write

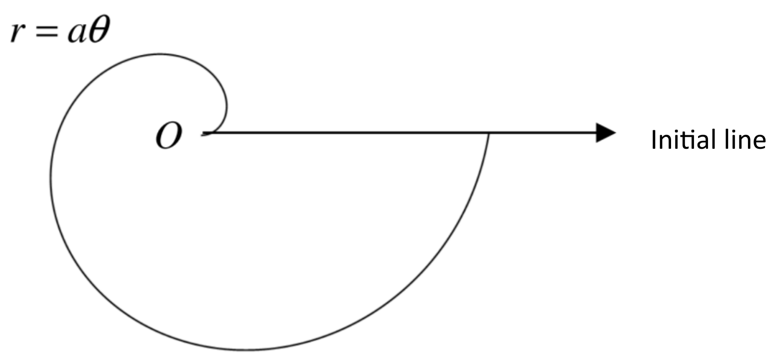
$$\Phi = kLv^2\rho.$$

<u>2)</u> The figure below shows a spiral curve with the polar equation.

$$r = a\,\theta\,, \qquad 0 \leq \theta \leq 2\pi$$

where a is a positive constant.

Find the area of the finite region bounded by the spiral and the initial line.



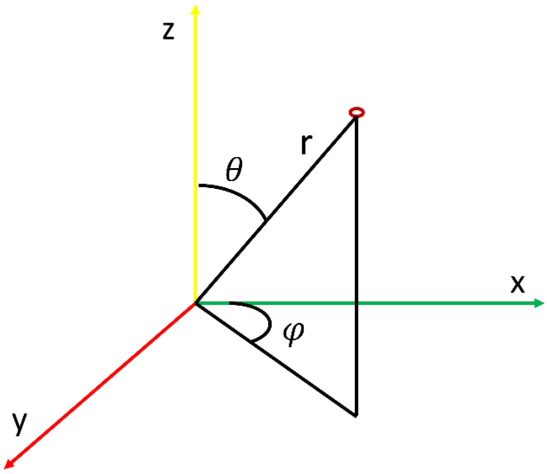
Ans:

USING THE STANDARD FORMULA FOR POLAR AREA

$$\text{AREA} = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta = \int_{\theta=0}^{\theta=2\pi} \frac{1}{2}(a\theta)^2 \, d\theta = \frac{1}{2}a^2 \int_0^{2\pi} \theta^2 \, d\theta$$

$$= \frac{1}{2}a^2 \left[\frac{1}{3}\theta^3\right]_0^{2\pi} = \frac{1}{6}a^2\left[8\pi^3 - 0\right] = \frac{4}{3}\pi^3 a^2$$

3) Represent the spherical coordinate location B $(4, \pi/3, \pi/6)$ in rectangular coordinate system. Locate the point on the figure.



Ans:

$$(\boldsymbol{r},\ \boldsymbol{\theta}, \boldsymbol{\varphi}) \rightarrow (\boldsymbol{x},\ \boldsymbol{y},\ \boldsymbol{z})$$

$$\mathbf{x} = \mathbf{r} \sin(\boldsymbol{\theta}) \cos(\boldsymbol{\varphi})$$

$$\mathbf{y} = \mathbf{r} \sin(\boldsymbol{\theta}) \sin(\boldsymbol{\varphi})$$

$$\mathbf{z} = \mathbf{r} \cos(\boldsymbol{\theta})$$

$$\mathbf{x = 2\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{6}\right) = 2\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{2}}$$

$$\mathbf{y = 2\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{6}\right) = 2\times\frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{2}}$$

z=2 cos(π/3)=2 ×1/2=1

4. Convert (-1, -1) into polar coordinates.

Let us first get r

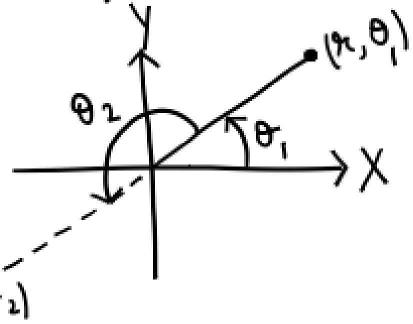
$$r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(-1)^2} = \sqrt{2}$$

Now let's get θ,

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{-1}{-1}\right)$$

$$= \tan^{-1}\tan(\pi/4) = \pi/4$$

But this value of θ is not the correct answer as it belongs to the first quadrant whereas the point in question $(-1,-1)$ corresponds to the third quadrant.



From adjacent figure, we know (r, θ_1) and $(-r, \theta_2)$ where $\theta_2 = \theta_1 + \pi$ represent same point in a polar graph.

So, correct value of θ will be $\pi/4 + \pi = \frac{5\pi}{4}$ which lies in third quadrant

Answer $\left(\sqrt{2}, \frac{5\pi}{4}\right)$ ✓