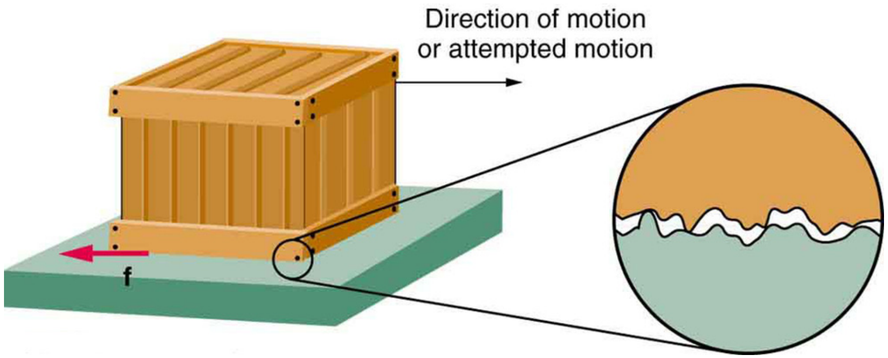
PHY101: Introduction to Physics I

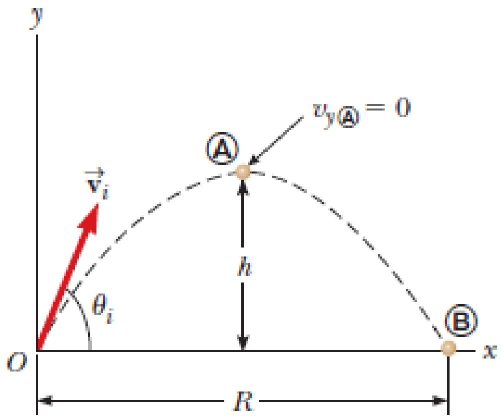
Monsoon Semester 2023
Lecture 12

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

Contact force
Friction
Projectile motion



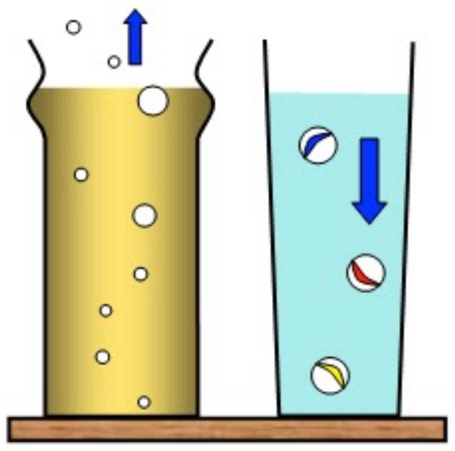


This Lecture

Viscous force

Viscous force

- Viscous force is experienced by a body when it moves through a fluid (liquid or gas).



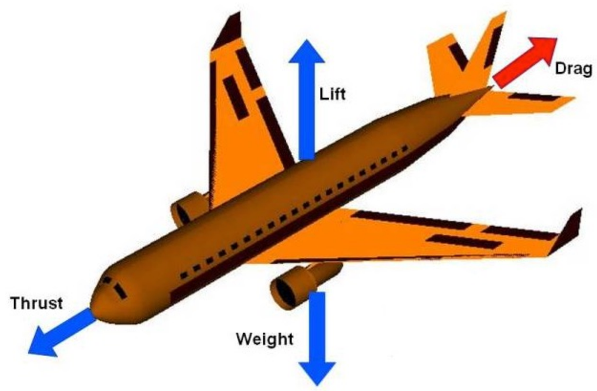


Image sources:
http://www.schoolphysics.co.uk/age16-19/Properties%20of%20matter/Viscosity/text/Stokes_law/images/1.png
http://www.freevector.com/site_media/preview_images/FreeVector-Paragliding-Vectors.jpg
http://www.grc.nasa.gov/WWW/k-12/airplane/Images/drag1.jpg

Viscous force

- **Viscous force** is experienced by a body when it moves through a fluid (liquid or gas).
- **At low velocities**, it has a simple velocity dependence: **It's proportional to the velocity** ($F \propto v$).
- For objects moving **at high speeds** through air, such as airplanes, skydivers, cars, and baseballs, the resistive force is reasonably well modeled as **proportional to the square of the speed** ($F \propto v^2$).
- However, in general, at higher velocities, other effects due to turbulence occur and the total drag force can have complicated velocity dependence.

For low velocities :

The flow of air will be **laminar**.

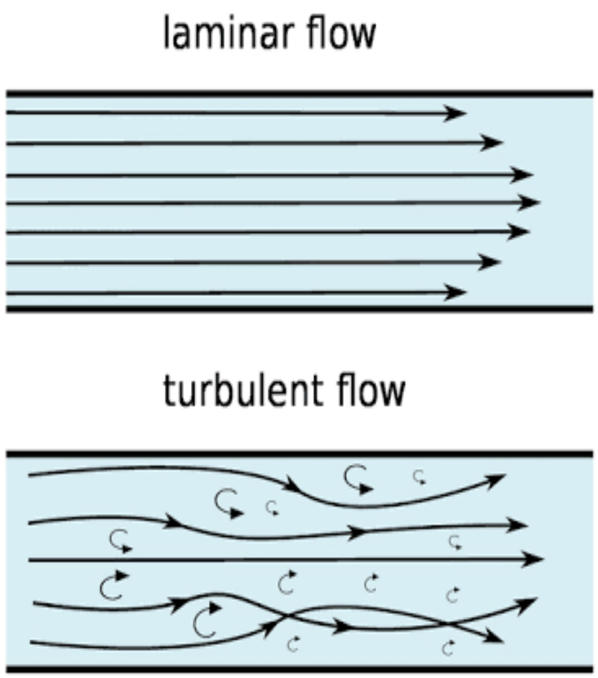
At high velocities:

Turbulence in medium.

(a) For a given quantity of intercepted air, the change in momentum is proportional to the velocity ($p \propto v$)

(b) The amount of intercepted air per unit time is proportional to the velocity ($n \propto v$)

=> The friction force must be quadratic in the velocity.



Viscous force

- For linear dependence on the velocity, the viscous force can be written

$$\mathbf{F}_\mathbf{v} = -k\mathbf{v}. \quad \Rightarrow \quad v = v_0 e^{-\frac{k}{m}t}$$

Here *k* is a positive constant which depends on the fluid and the geometry of the body.

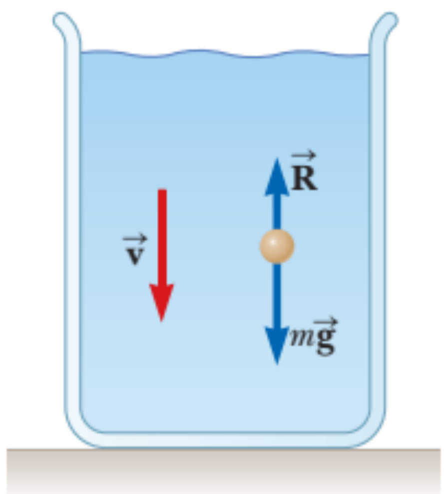
- $\mathbf{F}_\mathrm{v}$ is always along the line of motion, because it is proportional to **v**. Consequently, it can change the speed but not the direction (*and therefore we may consider a scalar equation instead of a vector equation*).

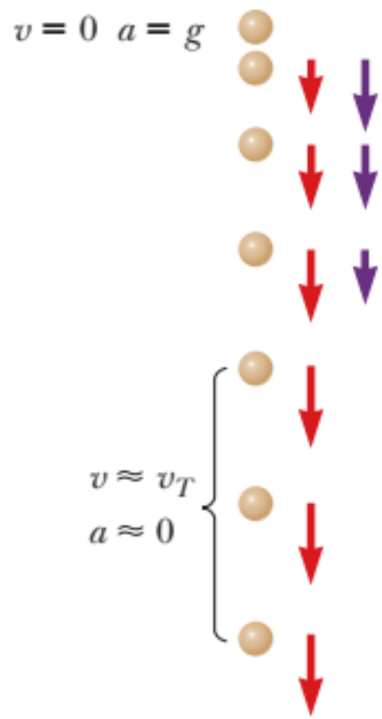
- The negative sign in the above relation signifies that $\mathbf{F}_\mathrm{v}$ opposes the motion.

- Note that $\tau = m/k$ has the dimension of time. The value of τ for a given problem decides the time-scale of the problem.

- When $t = \tau$, the velocity reduces to $1/e$ (≈ 0.37) of its initial value v_0. τ is referred to as the characteristic time for the system.

Particle falling under gravity with air resistance





$$\vec{\mathbf{R}} = -b\vec{\mathbf{v}}$$

$$a = \frac{dv}{dt} = g - \frac{b}{m}v$$

- Initially when $v = 0$, the magnitude of the resistive force is also zero and the acceleration of the sphere is simply *g*.
- As *t* increases, the magnitude of the resistive force increases and the acceleration decreases.

- The acceleration approaches zero when the magnitude of the resistive force approaches the sphere's weight so that the net force on the sphere is zero.
- In this situation, the speed of the sphere approaches its **terminal speed** v_T.

$$g - \frac{b}{m} v_T = 0 \qquad v_T = \frac{mg}{b}$$

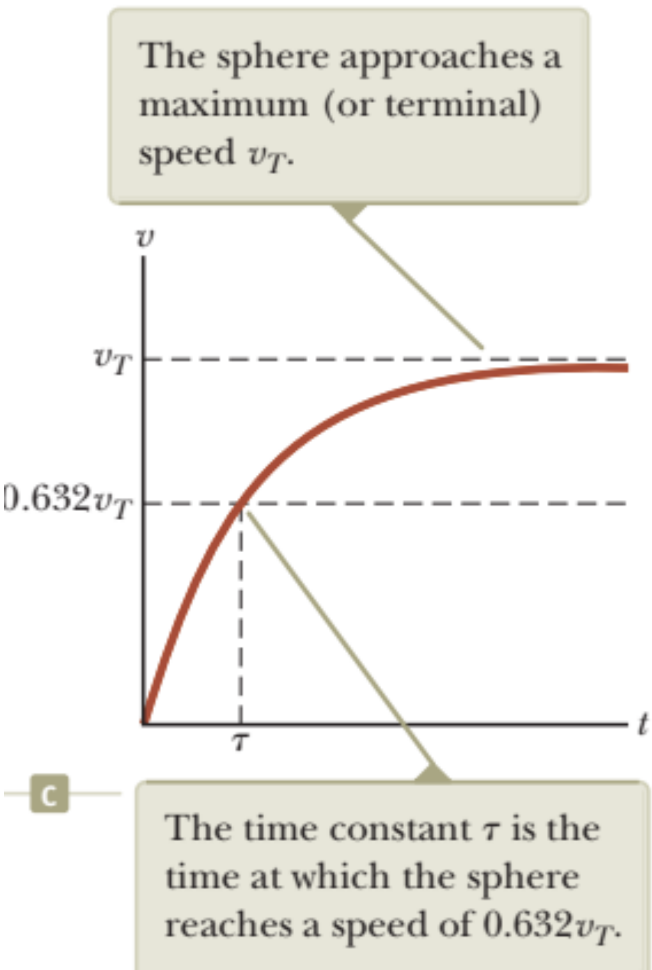
$$\boxed{v = v_0 e^{-\frac{k}{m}t}}$$

At any instant, the velocity is:

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

Time constant

At t = τ $\quad v = 0.632 v_T$



- For an object falling through the air at high speeds, like a skydiver, the fluid resistance is equal to Dv^2.
- The value of *D* depends on the shape and size of the body and on the density of the air.
- The terminal velocity is given by

$$v_T = \sqrt{\frac{mg}{D}}$$

Image Source:
https://encrypted-tbn0.gstatic.com/images?q=tbn:ANd9GcTVm-Yx5HzzMiVXshH5L_toIfhFOf9luKj6Wg&usqp=CAU

Next lecture:
Restoring forces