PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 15

Conductors

"…..In order therefore to appreciate the requirements of the science [of electromagnetism], the student must make himself familiar with a considerable body of most intricate mathematics, the mere retention of which in the memory materially interferes with further progress……". *:- James Clerk Maxwell [1855]*

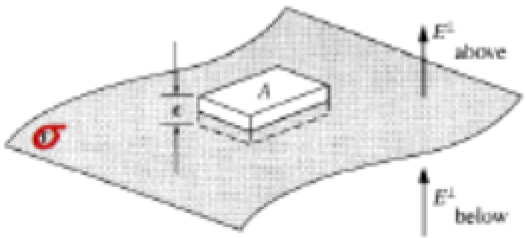
This Lecture

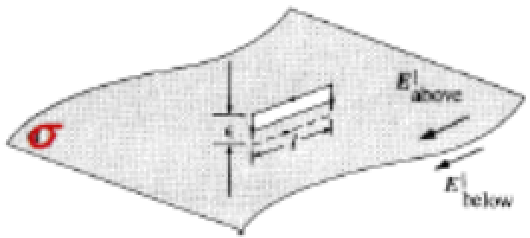
Surface Charges and Force on a Conductor

Surface Charges and Force on a Conductor

Remember that the boundary condition for E at any interface in general was

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{1}{\epsilon_0}\sigma \qquad \mathbf{E}^{\parallel}_{\text{above}} = \mathbf{E}^{\parallel}_{\text{below}} \longrightarrow \mathbf{E}_{\text{above}} - \mathbf{E}_{\text{below}} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}$$





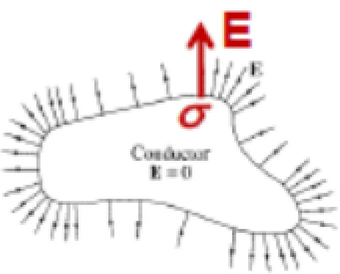
In the particular case of a conductor,

The field inside a conductor is zero, $\mathbf{E}_{\text{below}} = 0$

$\longrightarrow$ The **field immediately *outside*** is

$$\boxed{\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}}}$$

(→ Always normal to the surface)



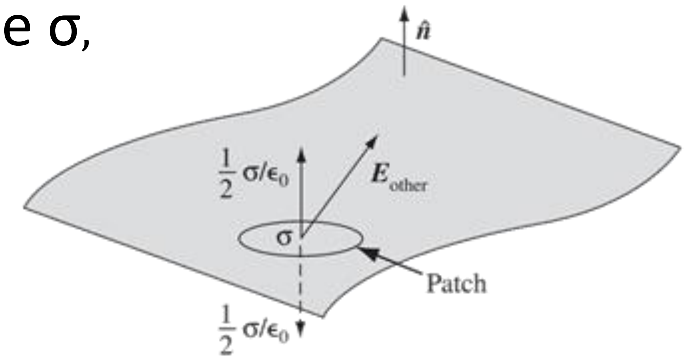
In terms of potential, $\mathbf{E} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \longrightarrow \boxed{\sigma = -\epsilon_0 \frac{\partial V}{\partial n}}$

→ **Surface charge on a conductor can be determined from E or *V*.**

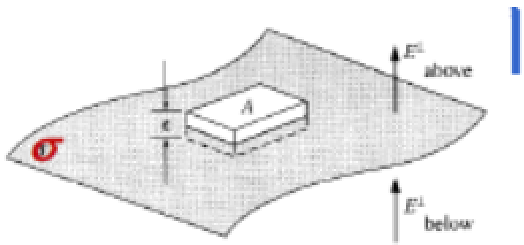
Surface Charge and force on a Conductors

Force per unit area acting on a surface charge σ,

$$\boldsymbol{f} = \sigma \boldsymbol{E}$$



$$\mathbf{E}_{above} - \mathbf{E}_{below} = \frac{\sigma}{\epsilon_0}\hat{\mathbf{n}} \longrightarrow E^{\perp}_{above} - E^{\perp}_{below} = \frac{1}{\epsilon_0}\sigma$$



How much force exerts on the surface charge?

➔ Because the electric field is ***discontinuous*** at a surface charge, so which value are we supposed to use: $\boldsymbol{E}_{above}$, $\boldsymbol{E}_{below}$, or something in between?

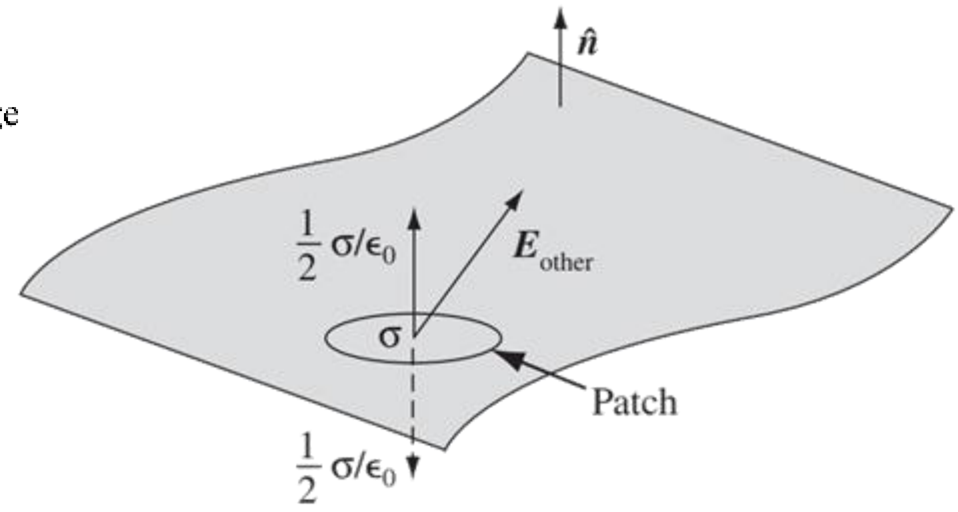
Answer $\boldsymbol{E}_{avg} = \frac{(\boldsymbol{E}_{above} + \boldsymbol{E}_{below})}{2}$; $\boldsymbol{f} = \sigma \boldsymbol{E}_{avg}$

Surface Charge and force on a Conductors

Consider a patch on the sheet of charge- we would like to calculate the force/area on this patch due to electric field from other regions, i.e $\mathbf{E}_{\text{other.}}$ $\mathbf{E}_{\text{other}}$ suffers no discontinuity.

$$\mathbf{E}_{\text{above}} = \mathbf{E}_{\text{other}} + \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}}, \qquad \mathbf{E}_{\text{below}} = \mathbf{E}_{\text{other}} - \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}},$$

$$\mathbf{E}_{\text{other}} = \frac{1}{2}(\mathbf{E}_{\text{above}} + \mathbf{E}_{\text{below}}) = \mathbf{E}_{\text{average}}$$



Surface Charge and force on a Conductors

For a conductor

We know from boundary conditions

$$\boldsymbol{E}_{above} - \boldsymbol{E}_{below} = \frac{\sigma}{\epsilon_0}\widehat{\boldsymbol{n}}$$

$$E_{inside} = 0 = E_{below}$$ (electric field inside a conductor is zero)

This gives the field immediate outside the conductor to be

$$\boldsymbol{E}_{above} = \frac{\sigma}{\epsilon_0}\widehat{\boldsymbol{n}}$$

$$\boldsymbol{E}_{avg} = \frac{\frac{\sigma}{\epsilon_0} + 0}{2} = \frac{\sigma}{2\epsilon_0}\widehat{\boldsymbol{n}}$$

$$\boldsymbol{f} = \sigma\boldsymbol{E}_{avg} = \frac{\sigma^2}{2\epsilon_0}\widehat{\boldsymbol{n}}$$

This is the *electrostatic pressure* on the charged surface- it tends to draw the conductor in the direction of the field

In terms of electric field, this is given by $P = \dfrac{\epsilon_0 E^2}{2}$

Capacitance

Consider two conductors with charges $+Q$ on one, and $-Q$ on the other. As we found the electric potential is constant over a conductor. Therefore we can talk about the potential difference between the two conductors without any ambiguity.

$$V = V_+ - V_- = -\int_{(-)}^{(+)} \mathbf{E} \cdot d\mathbf{l}.$$

The distribution of charge on the conductors would be, in general complicated, depending on their shape. However, in any case $\mathbf{E}$ is proportional to Q.

Capacitance

This follows from the Coulomb's law:

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} d\tau'.$$

If ρ is doubled, so is E, etc. Moreover, doubling of Q (and $-Q$) leads to doubling of ρ. And since V is proportional to E, it follows that V is proportional to Q.

$$V \propto Q$$

The constant of proportionality for a given arrangement defines the capacitance of the system:

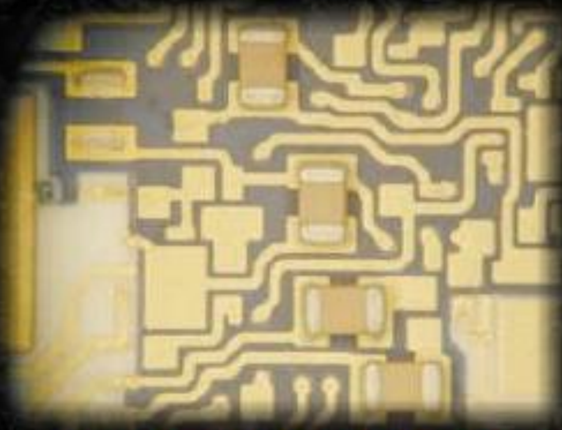
$$C = \frac{Q}{V}.$$

Capacitance & Capacitors

Capacitance is a purely geometrical quantity, determined by the sizes, shapes and separation of the two conductors constituting the capacitor.

Its SI unit is Coulomb per volt, or Farad (F).

Various kinds of capacitor



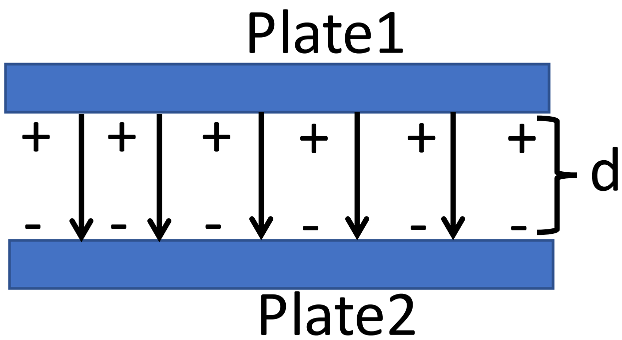
Ceramic chip capacitors inside a microcircuit package

A supercapacitor

Image Sources: http://www.mds975.co.uk/Images/radios/capacitors01.jpg; http://nepp.nasa.gov/WHISKER/experiment/exp5/hybrid-chip-caps.jpg; http://lerablog.org/wp-content/uploads/2013/04/supercapacitor.jpg

Parallel plate capacitor

Put equal and opposite charges on two metal plates – charges spread out uniformly on the inner surface of the plates – plates will have surface charge density +σ and -σ



Field between the plates is $\frac{\sigma}{\epsilon_0}$ Field outside the plates is zero (Why?)

$V = \phi_1 - \phi_2$= potential difference between the plates. ϕ_1, ϕ_2 are the potentials on plates 1 and 2

V = work done per unit charge to carry a small charge from one plate to another

$$V = Ed = \frac{\sigma}{\epsilon_0} d = \frac{Q}{A\epsilon_0} d$$

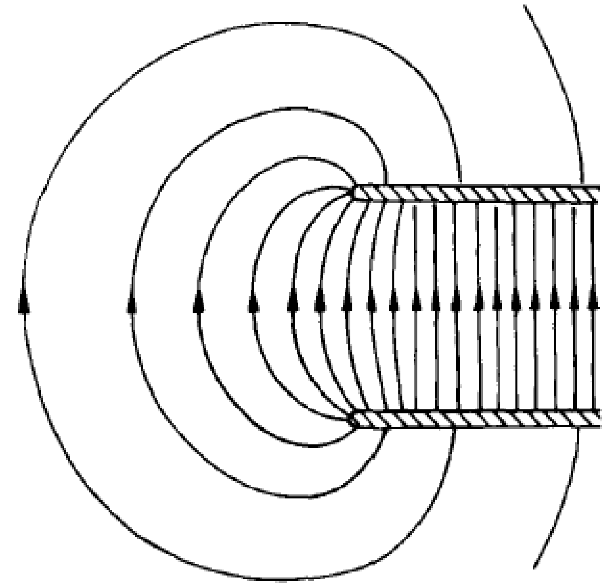
(Voltage drop is proportional to the charge)

$$C = \frac{Q}{V} = \frac{\epsilon_0 A}{d}$$

Such a *proportionality between Q and V* is found if there is a 'plus' charge on one and 'minus' charge on the other

Edge effects of parallel plate capacitor

Field does not quit suddenly at edges- total charge is not σA, there is a little correction at the edges

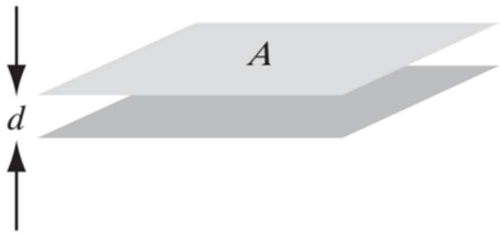


Exact calculation of field taking into account the fringing effect of fields at edges is complicated. Calculation shows that *charge density rises near the edges* of the plates- *capacitance should be more* than what we have calculated

A good approximation of capacitance is obtained if we use $C = \frac{\epsilon_0 A}{d}$ with the area one would get if the plates were extended artificially by a distance 3/8 of the separation distance between the plates

Capacitors

Example 2.10 Parallel-plate capacitor



$$\sigma = Q/A$$

$$\mathbf{E} = (1/\epsilon_0)Q/A$$

$$V = E \cdot d = \frac{Q}{A\epsilon_0}d \longrightarrow C = \frac{A\epsilon_0}{d}$$

Example 2.11 Two concentric spherical metal shells

$$\mathrm{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

$$V = -\int_b^a \mathbf{E}\cdot d\mathbf{l} = -\frac{Q}{4\pi\epsilon_0}\int_b^a \frac{1}{r^2}\,dr = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$\longrightarrow C = \frac{Q}{V} = 4\pi\epsilon_0\frac{ab}{(b-a)}$$

Problem 2.39 Two coaxial metal cylindrical tubes

$$\rho_l = \frac{Q}{L} \to E = \frac{Q}{2\pi\varepsilon rL}\hat{r} \to V = -\int_{r=b}^{r=a}\left(\frac{Q}{2\pi\varepsilon rL}\hat{r}\right)\cdot(\hat{r}dr) = \frac{Q}{2\pi\varepsilon L}\ln\left(\frac{b}{a}\right)$$

$$\to C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon L}{\ln\left(\frac{b}{a}\right)}$$

Capacitors

What good is the capacitor- it is good for _storing charge_

If you try to store a charge on a ball, its potential rises rapidly as you charge it up- (it may be so high that charge begins to escape into the air by way of sparks)

If you put the same charge on a condenser whose *capacity is very high*, *voltage* developed across the condenser would be *small*.

In applications in electronic circuits, it is useful to have something which can absorb or deliver large quantities of ***charge*** _without_ changing its ***potential*** much- a capacitor just does that.

In computers, a condenser is used to get a *specified change in voltage* in response to a *particular change in charge*

Work done in charging a capacitor

To charge up a capacitor, you have to remove electrons from the 'positive plate' and carry them to the 'negative plate'. In doing so, you fight against the electric field. How much work does it take to charge the capacitor to a final value Q?

Suppose the instantaneous value of charge is 'q'. Work done to transport the next piece of charge 'dq' is $dW = V(q)dq$

$$dW = V(q)dq = \frac{q}{C}dq$$

Total work to go from q = 0 to q = Q

$$W = \int_0^Q \left(\frac{q}{C}\right) dq = \frac{1}{2}\frac{Q^2}{C}$$

$$W = \frac{1}{2}CV^2$$