PHY101: Introduction to Physics I

Monsoon Semester 2024
Lecture 8

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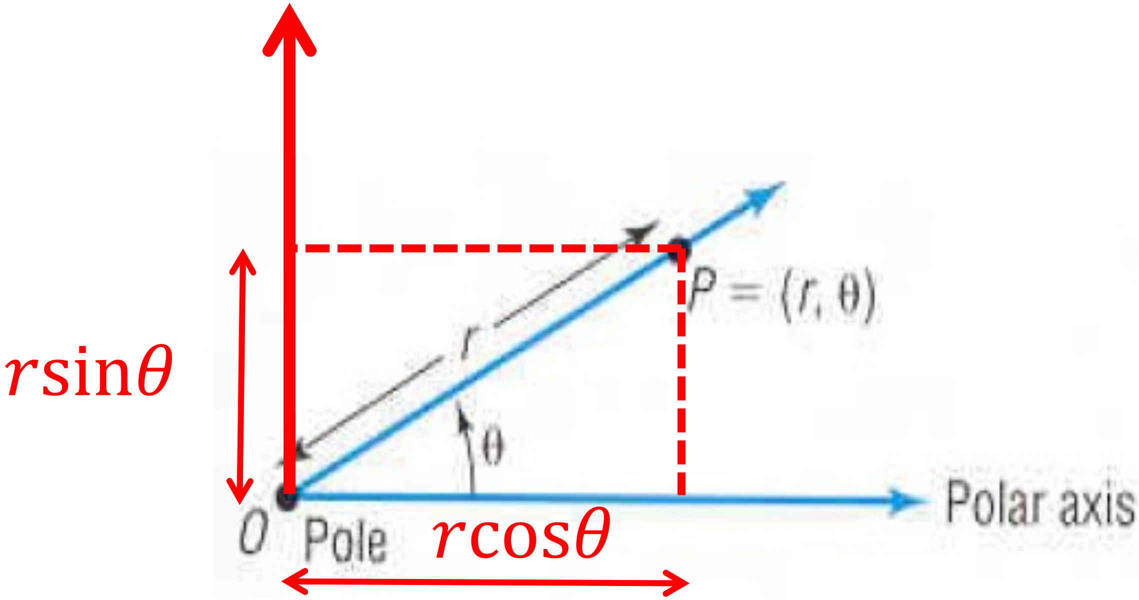
Previous Lecture

Derivative of vectors,
Finding unit vectors etc.

$$\hat{r} = cos\theta\,\hat{\imath} + sin\theta\,\hat{\jmath}$$

$$\hat{\theta} = -sin\theta\,\hat{\imath} + cos\theta\,\hat{\jmath}$$



This Lecture

Acceleration in plane polar coordinate
Newton's laws of motion

Vectors

Acceleration in plane polar coordinate system

$$\mathbf{a} = \frac{d}{dt}\mathbf{v} = \frac{d}{dt}(\dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}) \qquad \boxed{\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$

$$= \ddot{r}\hat{\mathbf{r}} + \dot{r}\frac{d}{dt}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\theta}\frac{d}{dt}\hat{\boldsymbol{\theta}}.$$

$$= \ddot{r}\hat{\mathbf{r}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + \dot{r}\dot{\theta}\hat{\boldsymbol{\theta}} + r\ddot{\theta}\hat{\boldsymbol{\theta}} - r\dot{\theta}^2\hat{\mathbf{r}}$$

$$= (\underbrace{\ddot{r}}_{\text{radial}} - \underbrace{r\dot{\theta}^2}_{\text{centripetal}})\hat{\mathbf{r}} + (\underbrace{r\ddot{\theta}}_{\text{tangential}} + \underbrace{2\dot{r}\dot{\theta}}_{\text{Coriolis}})\hat{\boldsymbol{\theta}}.$$

Radial acceleration: Due to change of radial speed

Centripetal acceleration: Due to change of direction of tangential velocity

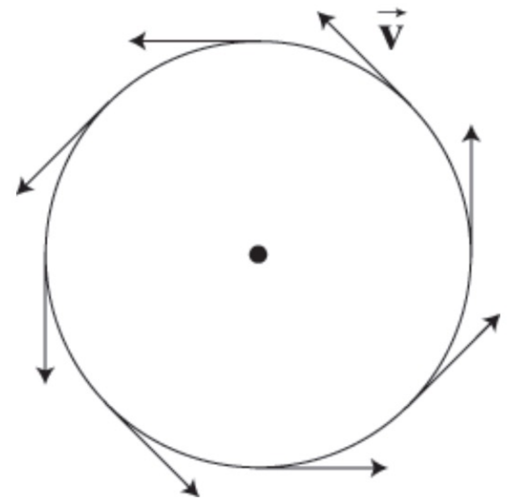
Tangential acceleration: Due to change of tangential speed

Coriolis acceleration: Due to change of radius and angle, both with time

Acceleration in polar coordinate system

Uniform circular Motion

$$\mathbf{a} = \frac{d}{dt}\mathbf{v} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\boldsymbol{\theta}}.$$



$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

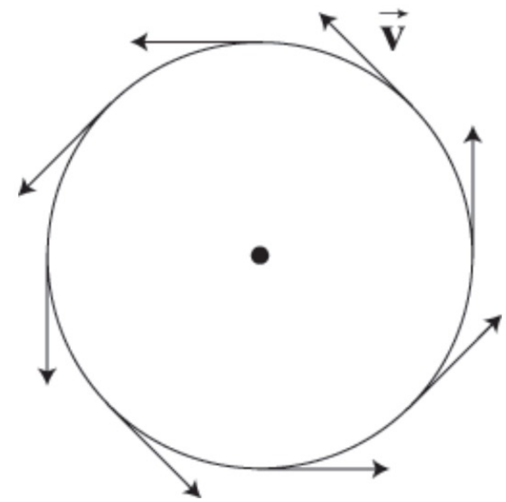
For a circular motion, $r = R$, the radius of the circle.

Hence, $\dot{r} = \ddot{r} = 0$

So, $a_\theta = R\ddot{\theta}$ and $a_r = -R\dot{\theta}^2$

Acceleration in polar coordinate system

Nonuniform circular Motion



For non-uniform circular motion, ω is function of time. Hence, $a_\theta = R\dfrac{d\omega}{dt} = R\alpha$,

where $\alpha = \dfrac{d\omega}{dt}$ is the angular acceleration.

However, the radial acceleration is always $a_r = -R\dot{\theta}^2 = -R\omega^2$

Therefore, an object traveling in a circular orbit with a constant speed is always accelerating towards the center. Though the magnitude of the velocity is a constant, the direction of it is constantly varying. Because the velocity changes direction, the object has a nonzero acceleration.

Frame of reference

A frame of reference is a set of coordinates that can be used to determine positions and velocities of objects in that frame.

Example: Imagine you threw and caught a ball while you were on a train moving at a constant velocity past a station. To you, the ball appears to simply travel vertically up and then down under the influence of gravity. However, to an observer stood on the station platform the ball would appear to travel in a parabola, with a constant horizontal component of velocity equal to the velocity of the train. This is illustrated in **Figure 1** below.

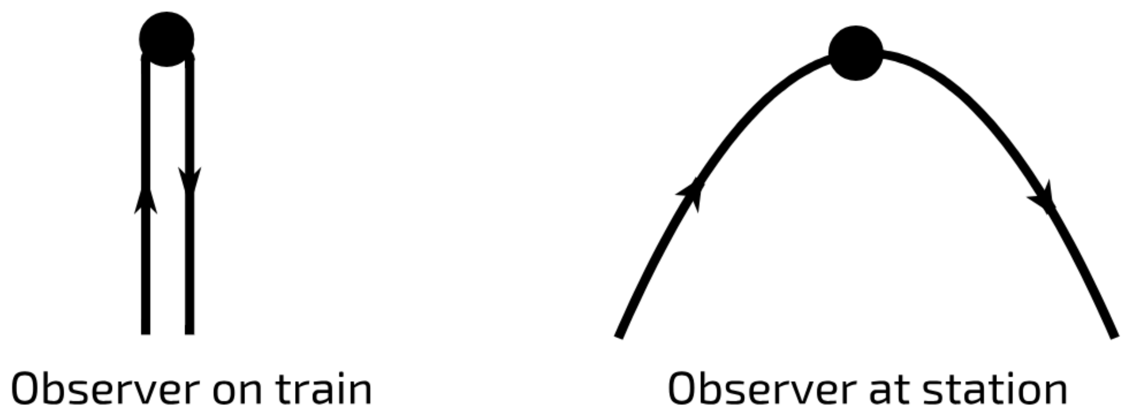


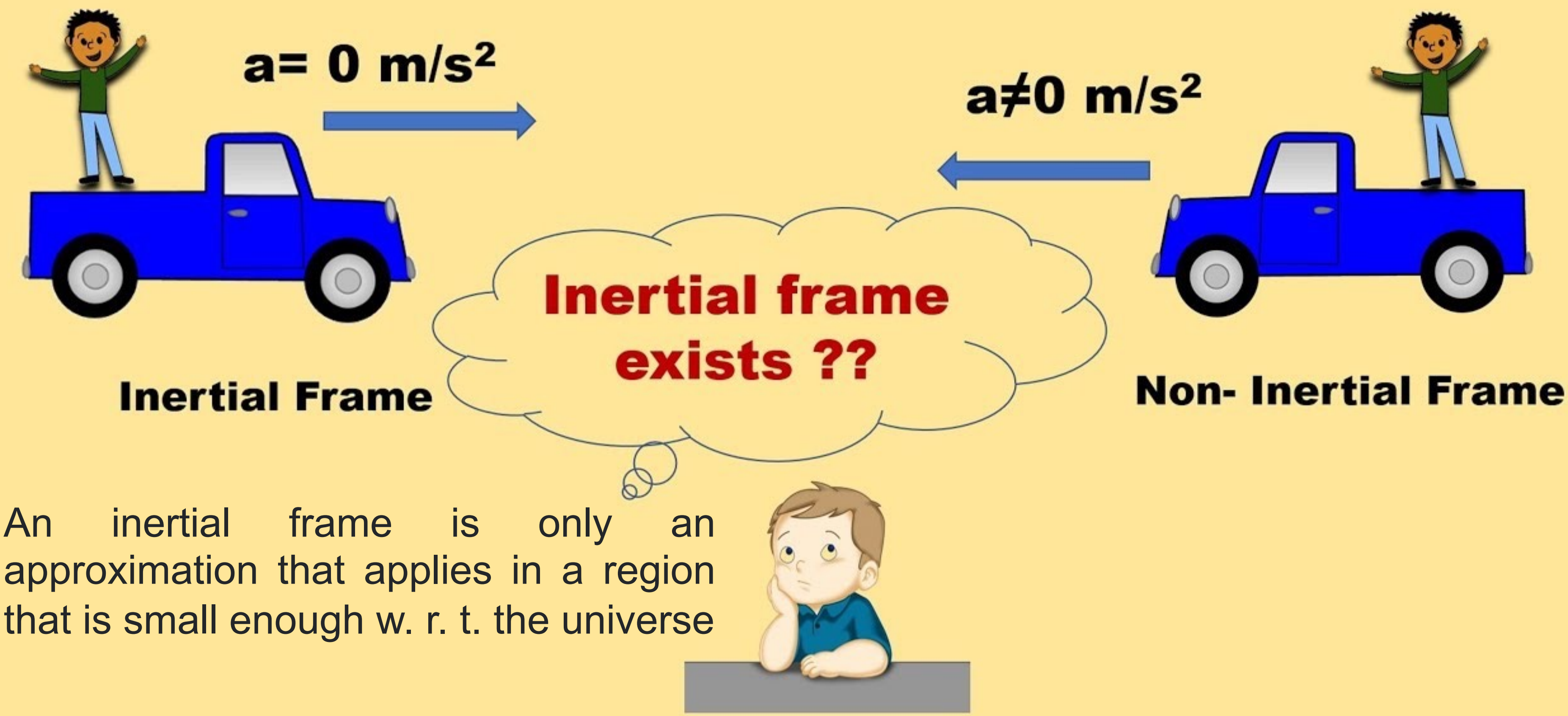
Figure 1: Path of the ball as seen by an observer on the train and an observer at the station.

Ref: https://isaacphysics.org/concepts/cp_frame_reference?stage=all

Types of frame of reference

- As long as the frame of reference is not moving or moving with a constant velocity it is termed as an **inertial frame of reference**.

- If the frame is accelerating or moving in a circular path with constant speed, it is termed as a **non-inertial frame of reference.**



An inertial frame is only an approximation that applies in a region that is small enough w. r. t. the universe

Ref.: https://www.vedantu.com/ ; https://physics.stackexchange.com/questions/17459

Newton's laws of motion

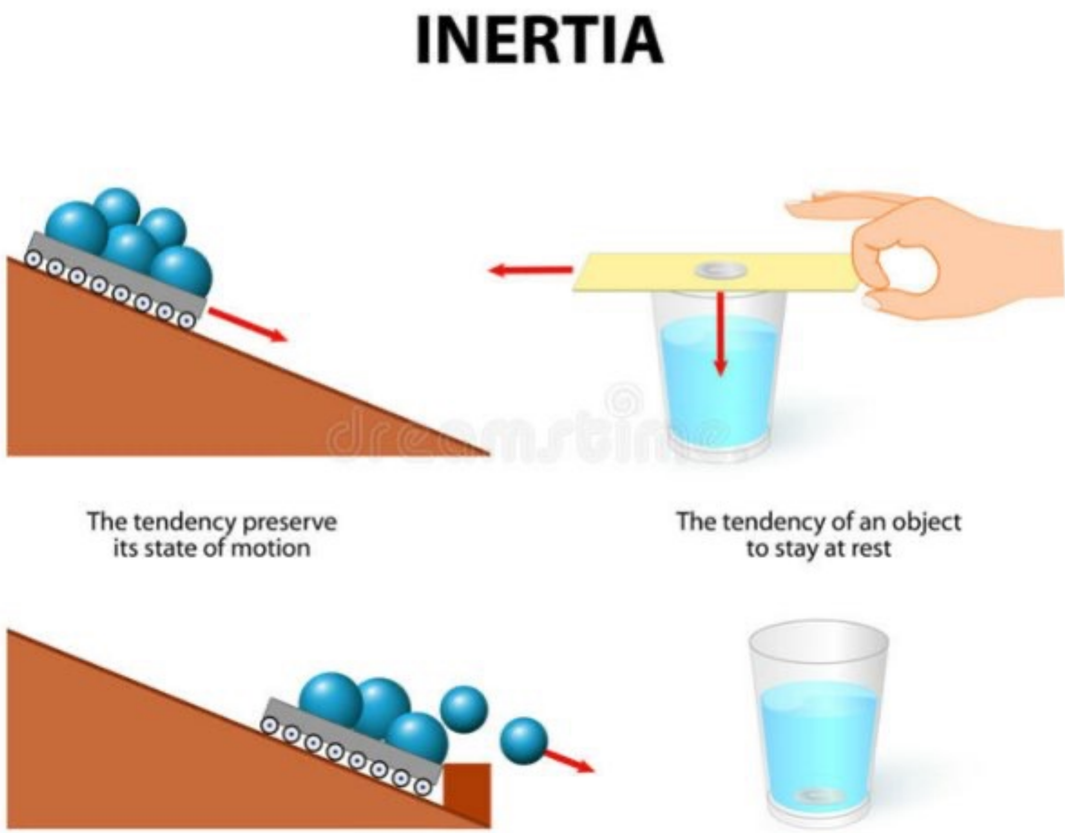
Sir Isaac Newton's laws of motion explain the relationship between a physical object and the forces acting upon it

1. **An object at rest remains at rest, and an object in motion remains in motion at constant speed and in a straight line unless acted on by an unbalanced force.**
2. **The acceleration of an object depends on the mass of the object and the amount of force applied.**
3. **Whenever one object exerts a force on another object, the second object exerts an equal and opposite on the first.**

Ref.: https://www1.grc.nasa.gov/beginners-guide-to-aeronautics/newtons-laws-of-motion

Newton’s 1st law of motion

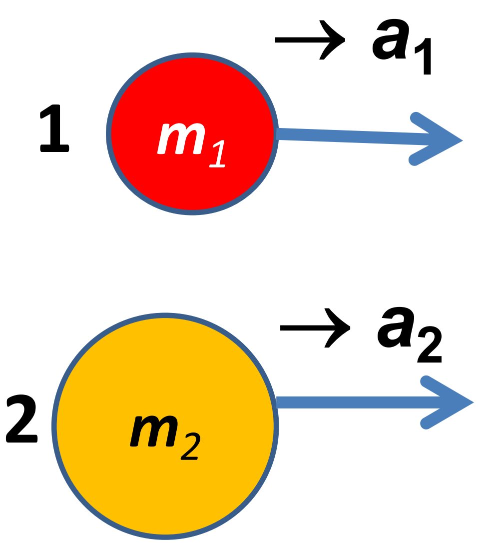
For a given free particle (no interaction with any external agent) it is possible to identify a reference frame (inertial frame) in which it continues its state of rest or a constant-velocity motion (zero acceleration).



Few examples of inertia

Concept of mass

Mass is that intrinsic property of an object that specifies how much resistance an object exhibits to changes in its velocity.



Identical force on 1 and 2

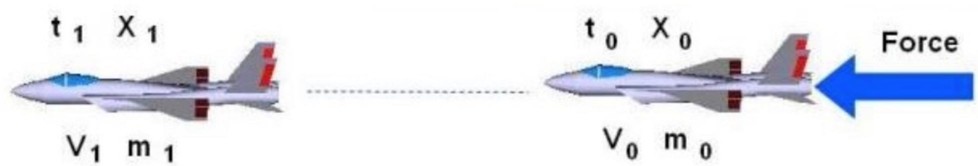
In general the accelerations are unequal, $a_1 \neq a_2$, i.e. the rates of change of velocity of the two objects are different even if they are under the influence of identical external agents.

$$\frac{a_1}{a_2} \equiv \frac{m_2}{m_1}$$

An object having more mass offers more resistance in changing its velocity.

Newton's 2^{nd} law of motion

The rate of change (that is, the time-derivative) of linear momentum *p* of a particle in an inertial reference frame is equal to the net force acting on it.



Force = Change of Momentum with Change of Time

Difference form: $F = \dfrac{m_1 V_1 - m_0 V_0}{t_1 - t_0}$

With constant mass: $F = m\,\dfrac{V_1 - V_0}{t_1 - t_0}$

t = time
X = location
m = mass
V = Velocity

$F = m\,a$

Force = mass x acceleration

Velocity, acceleration, momentum and force are vector quantities

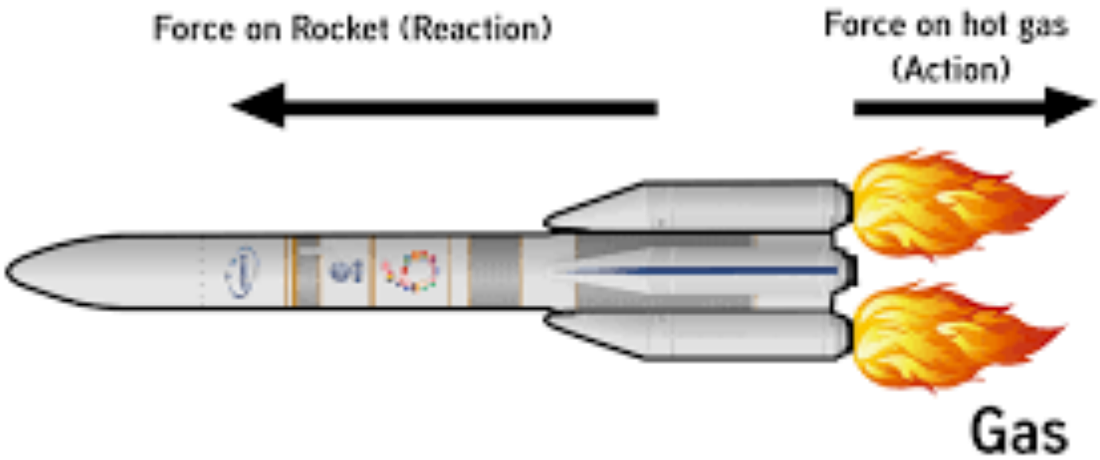
Differential form: $\vec{F} = \dfrac{d\vec{p}}{dt} = m\dfrac{d\vec{v}}{dt} = m\dfrac{d^2\vec{r}}{dt^2} = m\vec{a}$

SI unit of force

$$1\ \mathrm{N} \equiv 1\ \mathrm{kg}\cdot\mathrm{m/s^2}$$

Newton's 3rd law of motion

To every action there is always an equal and opposite reaction: The forces of two bodies on each other are always equal in magnitude and are directed in opposite directions.

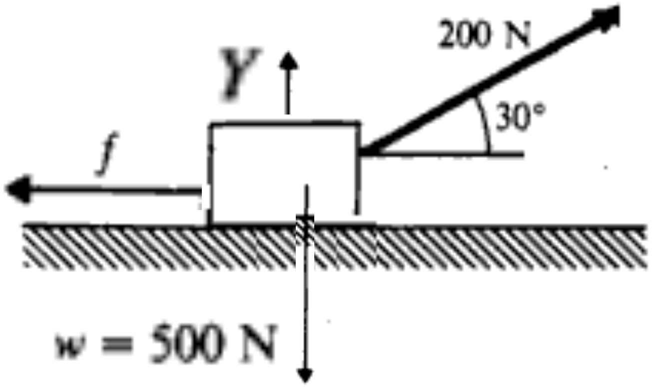


- Example:

The motion of a **rocket** produces thrust and hot exhaust gases flow out the back of the engine, and a thrusting force is produced in the opposite direction.

Problem solving

Free body diagram



- What are the forces acting on the block shown in figure?
- What is the value of **normal reaction** of the floor ?

$$\boxed{Y + 200 \sin 30^\circ - 500 = 0}$$

- A car having mass of 1500 kg achieve the velocity of 5 m/s in 10 second. Calculate the required force to attain required speed by car.

Initial velocity (u) = 0, final velocity (v) = 5m/s, time (t) = 10 second, Mass (m) = 1500 kg, Therefore, force (F)=?

We know that, Force (F) $= m\frac{v-u}{t}$

$$\therefore F = 1500\ kg\ \frac{5\ m/s - 0}{10\ s}$$

$$\Rightarrow F = 1500\ kg \times \frac{1}{2}\ ms^{-2}$$

$$\Rightarrow 750\ kg\ ms^{-2} \Rightarrow 750N$$

Thus required force = 750 N

<u>Next lecture</u>

Contact force