PHY101: Introduction to Physics I

Monsoon Semester 2024

Lecture 20

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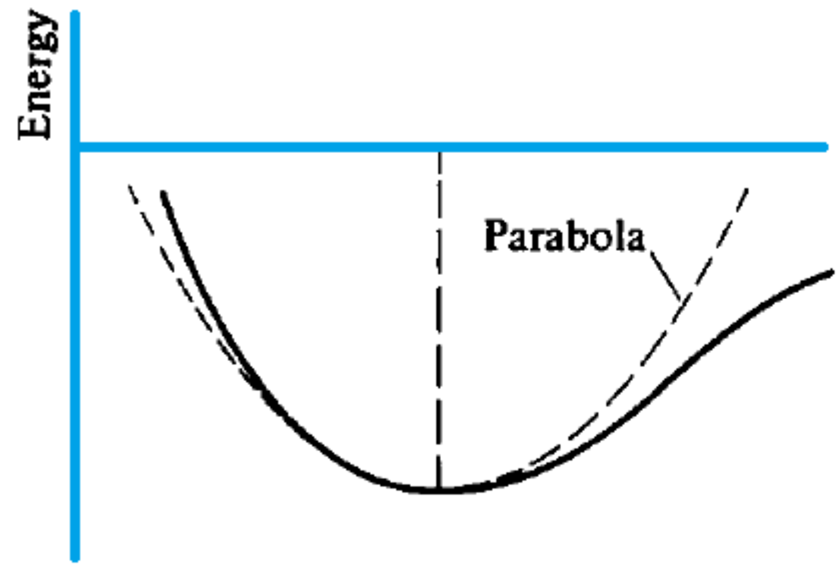
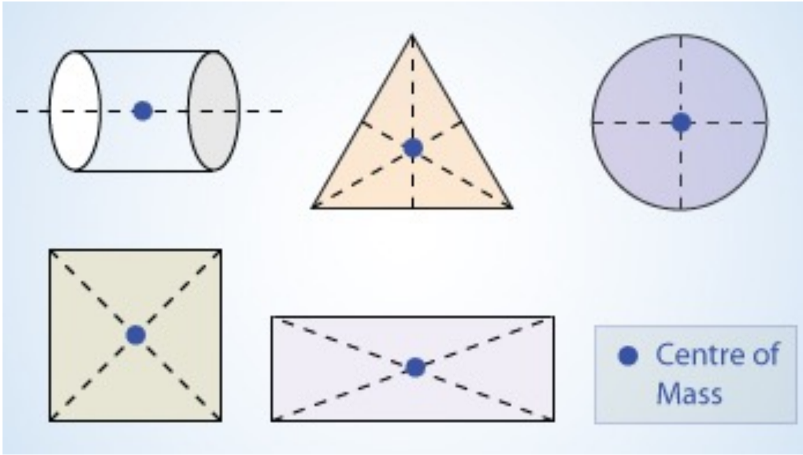
Previous Lecture

Bound potential
Small oscillation



This Lecture

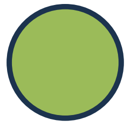
Centre of mass
Momentum
Impulse

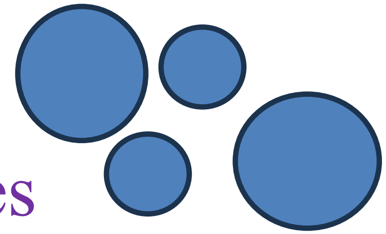


System of particles

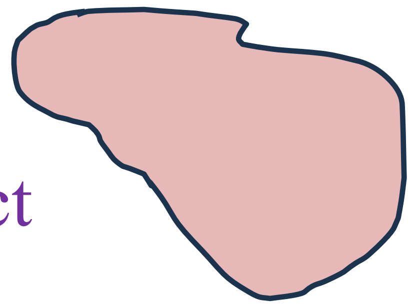
A physical system can consist of the followings,

A single particle



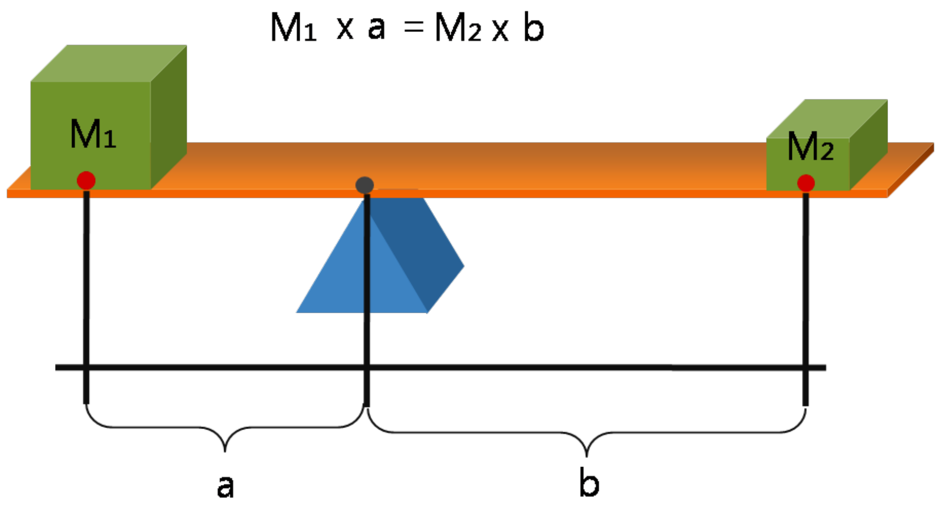


Or, a small number of particles



Or, an extended object

System of particles



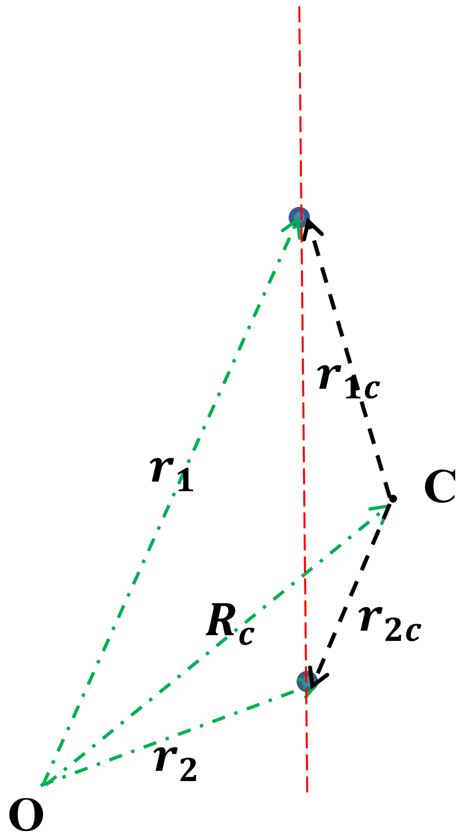
- For a **system of particles** in motion, if the entire system moves as if the net external force were applied to a single particle located at a special point within the system, the point is called **'center of mass'** of that system of particles.

- This particle model is independent of the nature of motion, for example, translation, rotation, vibration, and deformation etc.

Center of mass

Two particle case (bold letter representing a vector quantity here)

Here we are going to prove that 'the center of mass of two particles lie on the line joining the two particles'



Two particle, at some position:

Let the origin of my reference is at 'O'

Let the position vector of first particle is $\boldsymbol{r_1}$

Let the position vector of second particle is $\boldsymbol{r_2}$

We define center of mass for two particle at $\boldsymbol{R_c}$, Let it be at C

$$\boldsymbol{R_c} = \frac{(m_1\boldsymbol{r_1} + m_2\boldsymbol{r_2})}{(m_1 + m_2)}$$

The position vector of first particle with respect to C: $\boldsymbol{r_{1c}}$

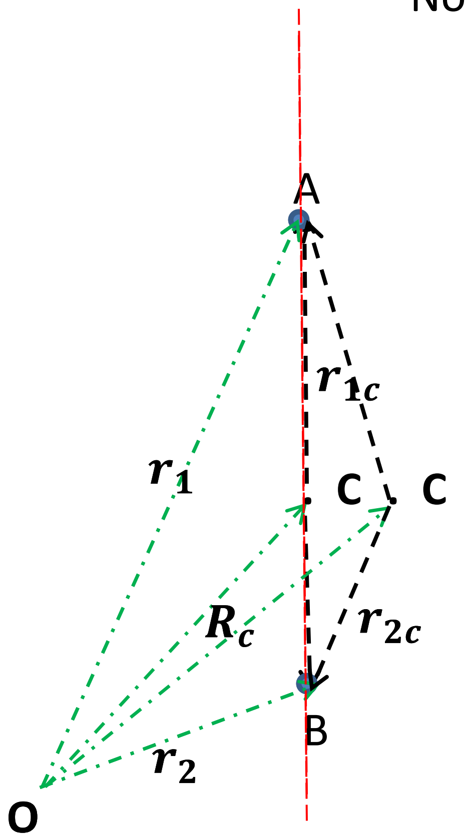
The position vector of second particle with respect to C: $\boldsymbol{r_{2c}}$

$$\boldsymbol{R_c} = \frac{(m_1\boldsymbol{r_1}+m_2\boldsymbol{r_2})}{(m_1+m_2)}$$

We are dealing with vectors, and bold letter are representing vector quantity.

Note from Fig. $\boldsymbol{r_1} = \boldsymbol{R_c} + \boldsymbol{r_{1c}}$ or $\boldsymbol{r_{1c}} = -\boldsymbol{R_c} + \boldsymbol{r_1}$

$\boldsymbol{r_2} = \boldsymbol{R_c} + \boldsymbol{r_{2c}}$ or $\boldsymbol{r_{2c}} = -\boldsymbol{R_c} + \boldsymbol{r_2}$



$$m_1\boldsymbol{r_{1c}} = m_1(-\boldsymbol{R_c} + \boldsymbol{r_1}) \quad , \quad m_2\boldsymbol{r_{2c}} = m_2(-\boldsymbol{R_c} + \boldsymbol{r_2})$$

$$m_1\boldsymbol{r_{1c}} + m_2\boldsymbol{r_{2c}} = -(m_1 + m_2)\boldsymbol{R_c} + (m_1\boldsymbol{r_1} + m_2\boldsymbol{r_2})$$

$$m_1\boldsymbol{r_{1c}} + m_2\boldsymbol{r_{2c}} = -(m_1 + m_2)\boldsymbol{R_c} + (m_1+m_2)\boldsymbol{R_c} = \boldsymbol{0}$$

$$m_1\boldsymbol{r_{1c}} + m_2\boldsymbol{r_{2c}} = \boldsymbol{0}$$

$\Rightarrow \quad m_1\boldsymbol{r_{1c}} = -m_2\boldsymbol{r_{2c}},$ Direction of $\boldsymbol{r_{1c}}$ is opposite to $\boldsymbol{r_{2c}}$

The center of mass of two particles lie on the line joining the two particles

$$\Rightarrow \quad \frac{d(m_1\boldsymbol{r_{1c}} + m_2\boldsymbol{r_{2c}})}{dt} = 0 \quad \Rightarrow \quad m_1\boldsymbol{v_{1c}} + m_2\boldsymbol{v_{2c}} = \boldsymbol{0}$$

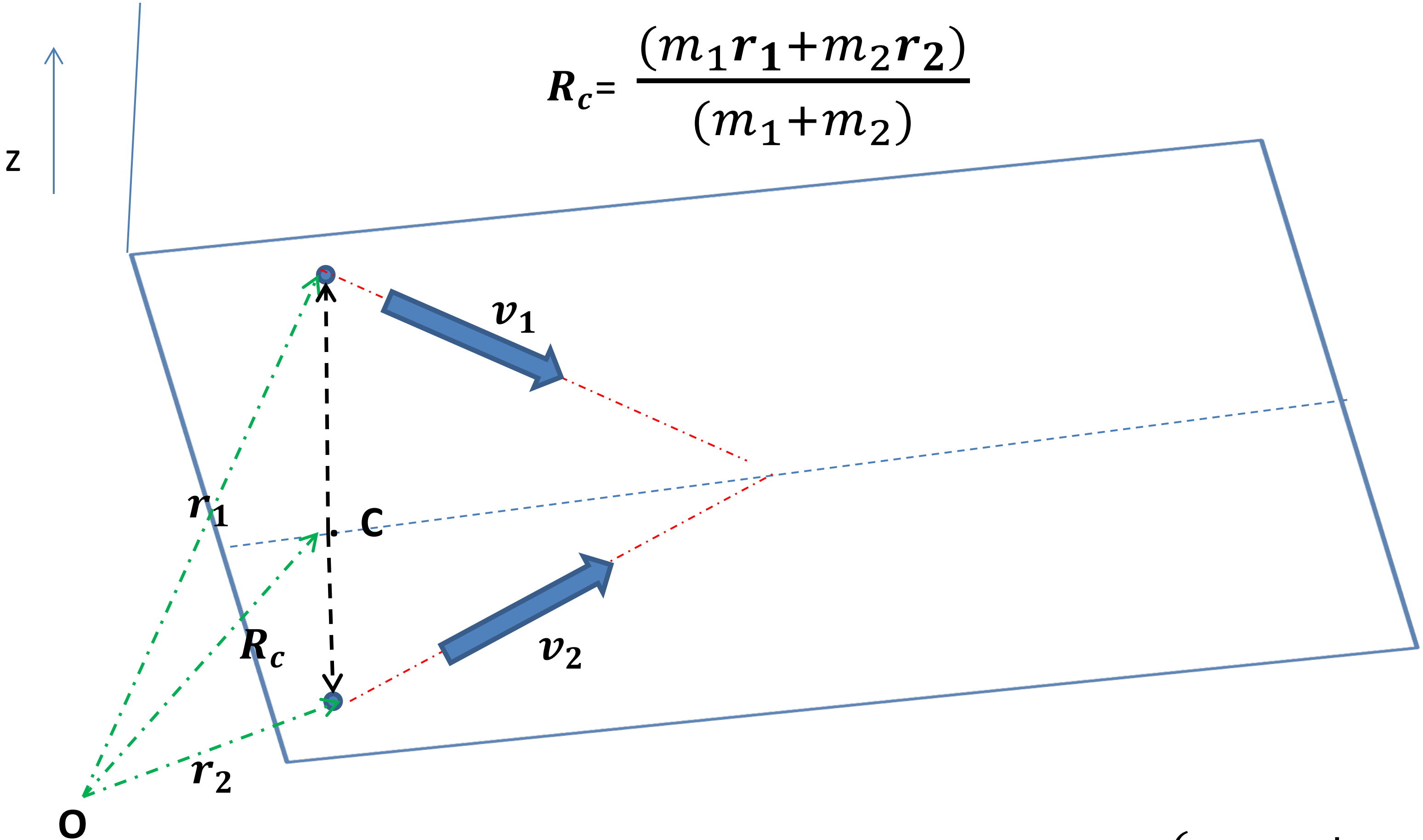
These are the momentum measured with respect to the centre of mass.

Note that the momentum in the Center of Mass reference frame is zero

If the motion of one particle is known, the motion of the other particle follows directly.

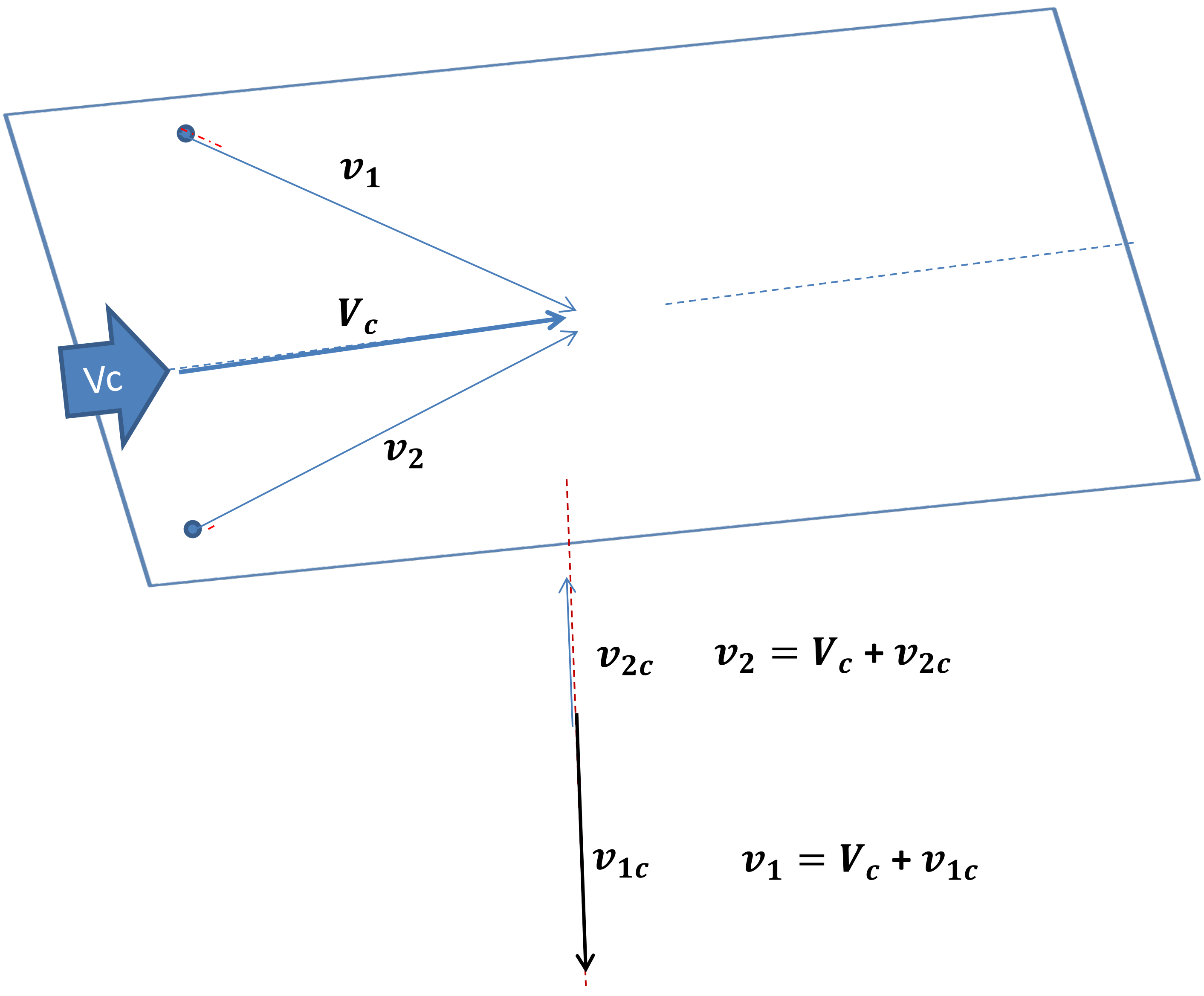
Now consider the particle is moving in lab frame with velocity $\boldsymbol{v_1}$, $\boldsymbol{v_2}$

$$\boldsymbol{R_c} = \frac{(m_1\boldsymbol{r_1}+m_2\boldsymbol{r_2})}{(m_1+m_2)}$$



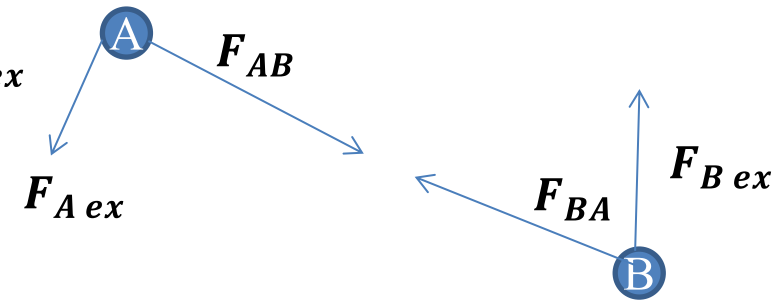
What is the velocity of Center of Mass ?

$$\boldsymbol{V_c} = \frac{(m_1\boldsymbol{v_1}+m_2\boldsymbol{v_2})}{(m_1+m_2)}$$



Vc
v_1
V_c
v_2
v_{2c}
$v_2 = V_c + v_{2c}$
v_{1c}
$v_1 = V_c + v_{1c}$

Conservation of linear momentum for a two-particle system



External force acting on the particles be $F_{A\,ex}$ *and* $F_{B\,ex}$

force applied by the particle A on particle B be F_{BA}

force applied by the particle B on particle A be F_{AB}

$$F_{A\,ex} + F_{AB} = m_A \boldsymbol{a}_A = \frac{d(m_A \boldsymbol{v}_A)}{dt} \qquad F_{B\,ex} + F_{BA} = m_B \boldsymbol{a}_B = \frac{d(m_B \boldsymbol{v}_B)}{dt}\ ,$$

Total force on the two particle $F_{A\,ex} + F_{AB} + F_{B\,ex} + F_{BA}$,

From Newton's Law we know $F_{AB} + F_{BA} = 0$,

$$\Rightarrow \quad F_{A\,ex} + F_{B\,ex} = \frac{d(m_A \boldsymbol{v}_A)}{dt} + \frac{d(m_B \boldsymbol{v}_B)}{dt} = \frac{d(m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B)}{dt}$$

If external force $F_{A\,ex} + F_{B\,ex} = 0$ *for all times* , $\frac{d(m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B)}{dt} = 0$

$\Rightarrow m_A \boldsymbol{v}_A + m_B \boldsymbol{v}_B =$ **Constant in time or indepedent of time**

Note this statement does not depend on the nature of the internal force. Can be generalized for any number of particle

Impulse

Let a force F(t), time t acts on a particle for some duration

Impulse is defined as follows : $$\mathbf{I = \int_{t_i}^{t_f} F(t)\, dt}$$

We can define an average force $\bar{F}$ over this duration such that :

$$\bar{F}\Delta t = \bar{F} \times (t_f - t_i) = I$$

From Newton's Laws we know that $F = \dfrac{dP}{dt}$

$$I = \int_{t_i}^{t_f} F(t)\, dt = \int_{t_i}^{t_f} \frac{dP}{dt}\, dt = \Delta P$$

$\Rightarrow$ **Impulse = change in momentum**

Note $I = \Delta P = \bar{F}\Delta t$

$$\boldsymbol{F_{ex} = F_{A\,ex} + F_{B\,ex}} = \frac{d(m_A v_A + m_B v_B)}{dt} = \frac{d(\boldsymbol{P_A + P_B})}{dt}$$

$$\boldsymbol{I = \int_{t_i}^{t_f} F_{ex}(t)\,dt = \Delta P = \bar{F}_{ex}\Delta t}$$

$$\boldsymbol{P_f - P_i = \bar{F}_{ex}\Delta t \quad \Rightarrow P_f = P_i + \bar{F}_{ex}\Delta t}$$

Here $\boldsymbol{P_f = P_{Af} + P_{Bf}}$ and $\boldsymbol{P_i = P_{Ai} + P_{iB}}$

$$\boldsymbol{\Rightarrow P_{Af} + P_{Bf} = P_{Ai} + P_{Bi} + \bar{F}_{ex}\Delta t}$$

⇒ **If the impulse of the external force is zero then the total momentum at t_i is equal to the total momentum at t_f .**

- If $I = 0$, total momentum at t_i is equal to the total momentum at t_f .
- If the total external force is zero at every instant between t_i and t_f , total momentum retain the same value at every instant. This is the **Law of Conservation of Linear Momentum**.
- But even in presence of external force, in some cases like collision , we can have an approximate law of conservation of linear momentum . **This is because two object come in contact for a very short duration**.

Example: A bat hitting a ball in air under the gravitational force (external force). The ball touches and rebounds from the bat, $\Delta t \approx 0$, and external force gravity is finite such that $P_{Ai} + P_{Bi} \gg \bar{F}_{ex}\Delta t \approx 0$ (here the duration of the hit is very small)

$$\Rightarrow P_{Af} + P_{Bf} = P_{Ai} + P_{Bi} + \bar{F}_{ex}\Delta t \approx P_{Ai} + P_{Bi}$$

Summary

As we have defined Center of Mass position $\quad R_c = \dfrac{(m_1 \boldsymbol{r_1} + m_2 \boldsymbol{r_2})}{(m_1 + m_2)}$

We can define velocity of Center of Mass $\quad \dfrac{d\boldsymbol{R_c}}{dt} = \boldsymbol{V_c} = \dfrac{(m_1 \boldsymbol{v_1} + m_2 \boldsymbol{v_2})}{(m_1 + m_2)}$

And we can define acceleration of Center of Mass $\quad \dfrac{d\boldsymbol{V_c}}{dt} = \boldsymbol{A_c} = \dfrac{(m_1 \boldsymbol{a_1} + m_2 \boldsymbol{a_2})}{(m_1 + m_2)}$

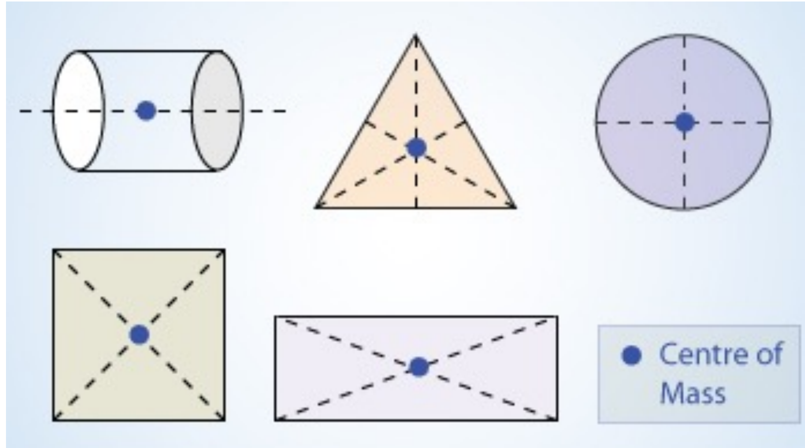
$$\textbf{\textit{If}}\ \ \boldsymbol{F_{A\,ex}} + \boldsymbol{F_{B\,ex}} = \frac{d(m_A \boldsymbol{v_A} + m_B \boldsymbol{v_B})}{dt} = 0 \Rightarrow m_A \boldsymbol{v_A} + m_B \boldsymbol{v_B} = \boldsymbol{Constant}$$

Note If $m_1 \boldsymbol{v_1} + m_2 \boldsymbol{v_2} = \boldsymbol{constant} \Rightarrow \boldsymbol{V_c} = \boldsymbol{constant}$

Point to be noted In the absence of external forces $\quad \boldsymbol{P_{Af}} + \boldsymbol{P_{Bf}} = \boldsymbol{P_{Ai}} + \boldsymbol{P_{Bi}}$

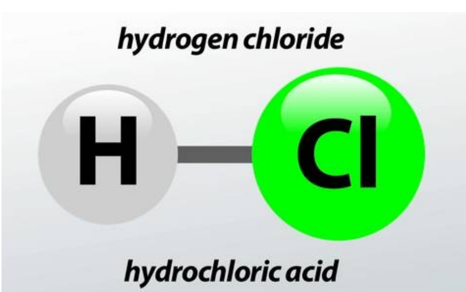
$$\boldsymbol{V_c} = \boldsymbol{constant}$$

Center of mass for objects with standard geometry



Problem and solution

In a HCL molecule, the separation between the nuclei of the two atoms is 0.127 nm. Find the location of the center of mass of the molecule. [A chlorine atom is approximately 35 times heavier than a hydrogen atom].



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Here, $m_2 = 35\ m_1$, $x_1 = 0$ and $x_2 = 0.127$ nm

Ans: 0.123 nm