PHY101: Introduction to Physics I

Monsoon Semester 2024

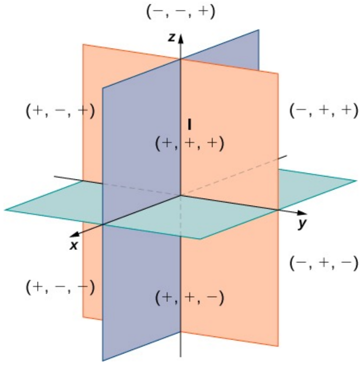
Lecture 4

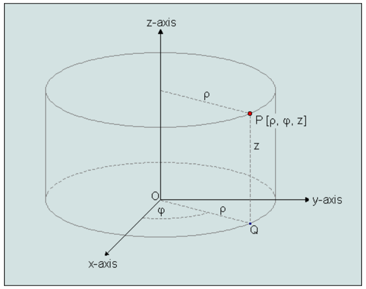
Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

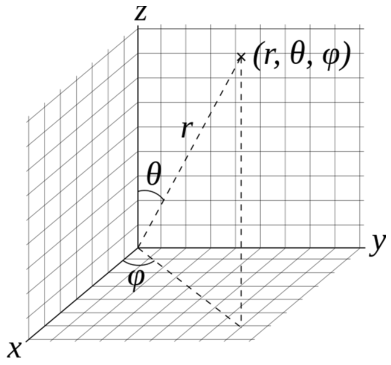
Coordinate system in 3 Dimension (3D)

- **Cartesian coordinates system**





- **Cylindrical polar coordinates system**



- **Spherical coordinates system**

This Lecture

Incremental length, surface, and volume element

Scalars and Vectors

3D Cartesian coordinates system

(a) Incremental length

$$\Delta \boldsymbol{l_x} = \Delta x\, \overrightarrow{a_x}$$

$$\Delta \boldsymbol{l_y} = \Delta y\, \overrightarrow{a_y}$$

$$\Delta \boldsymbol{l_z} = \Delta z\, \overrightarrow{a_z}$$

(b) Incremental surface

$$\Delta \boldsymbol{A_x} = \Delta y\, \Delta z\, \overrightarrow{a_x}$$

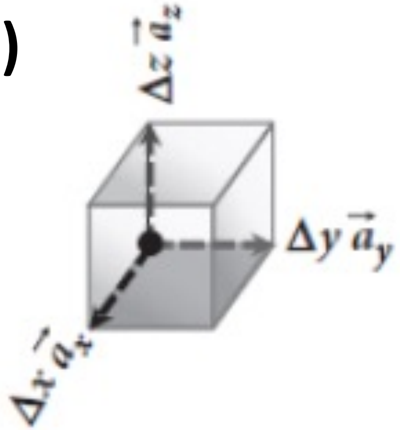
$$\Delta \boldsymbol{A_y} = \Delta z\, \Delta x\, \overrightarrow{a_y}$$

$$\Delta \boldsymbol{A_z} = \Delta x\, \Delta y\, \overrightarrow{a_z}$$

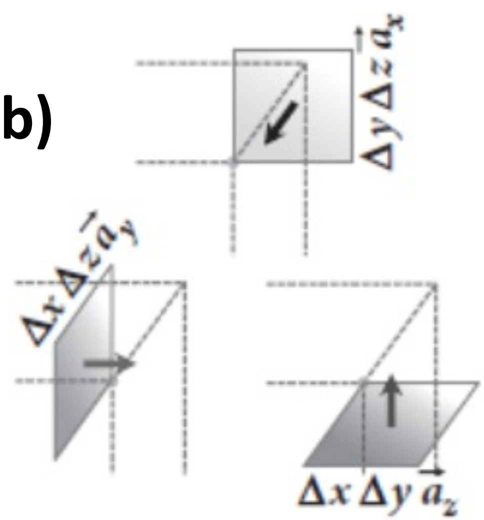
(c) Incremental volume

$$\Delta V = \Delta x\, \Delta y\, \Delta z$$

(a)



(b)



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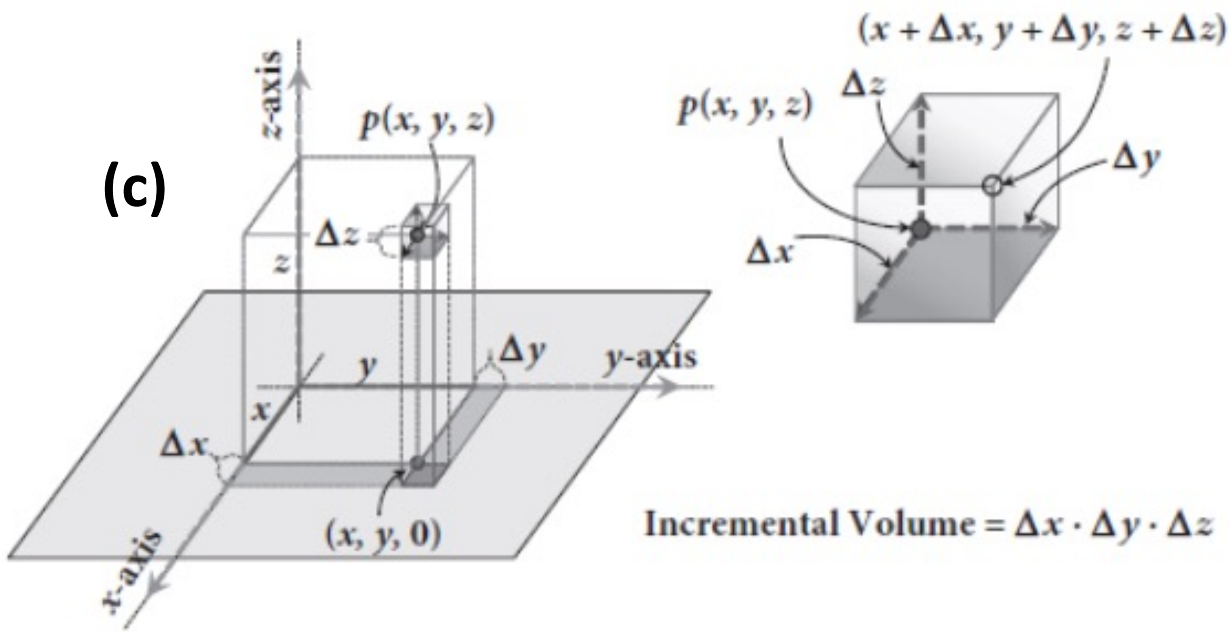

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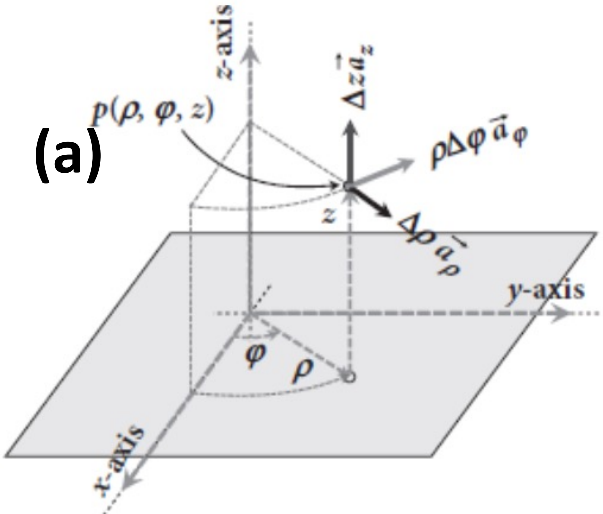
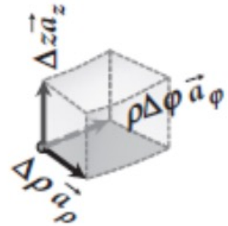
3D cylindrical coordinate system

(a)





Incremental Length Vectors

(a) Incremental length

$$\boldsymbol{\Delta l_\rho} = \Delta\rho \, \overrightarrow{a_\rho}$$

$$\boldsymbol{\Delta l_\varphi} = \rho\Delta\varphi \, \overrightarrow{a_\varphi}$$

$$\boldsymbol{\Delta l_z} = \Delta z \, \overrightarrow{a_z}$$

(b) Incremental surface

$$\boldsymbol{\Delta A_\rho} = (\rho\Delta\varphi)\Delta z \, \overrightarrow{a_\rho}$$

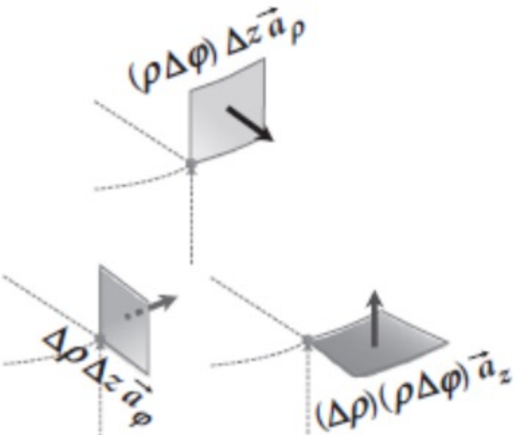
$$\boldsymbol{\Delta A_\varphi} = \Delta\rho \, \Delta z \, \overrightarrow{a_\varphi}$$

$$\boldsymbol{\Delta A_z} = \Delta\rho(\rho\Delta\varphi)\overrightarrow{a_z}$$

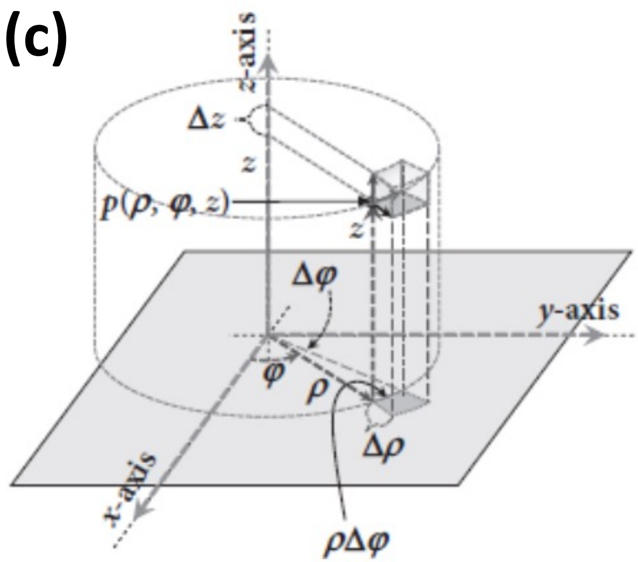
(c) Incremental volume

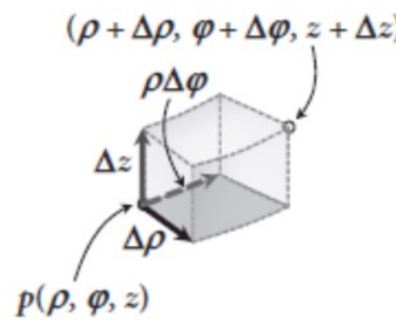
$$\Delta V = \Delta\rho \, (\rho\Delta\varphi) \, \Delta z$$

(b)



(c)

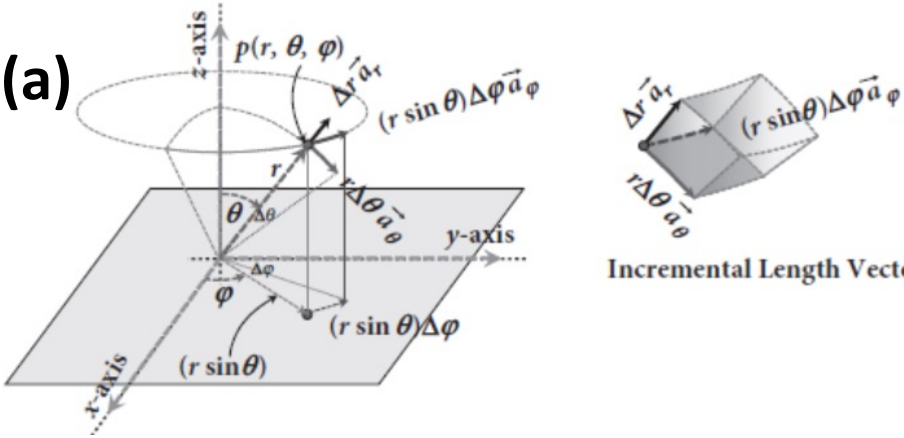




Incremental Volume = $\Delta\rho \cdot \rho\Delta\varphi \cdot \Delta z$

3D spherical coordinate system

(a)



Incremental Length Vectors

(a) Incremental length

$$\Delta \boldsymbol{l_r} = \Delta r\,\overrightarrow{a_r}$$

$$\Delta \boldsymbol{l_\theta} = r\Delta\theta\,\overrightarrow{a_\theta}$$

$$\Delta \boldsymbol{l_\varphi} = r\sin\theta\,\Delta\varphi\,\overrightarrow{a_\varphi}$$

(b) Incremental surface

$$\Delta \boldsymbol{A_r} = (r\Delta\theta)(r\sin\theta\,\Delta\varphi)\overrightarrow{a_r}$$

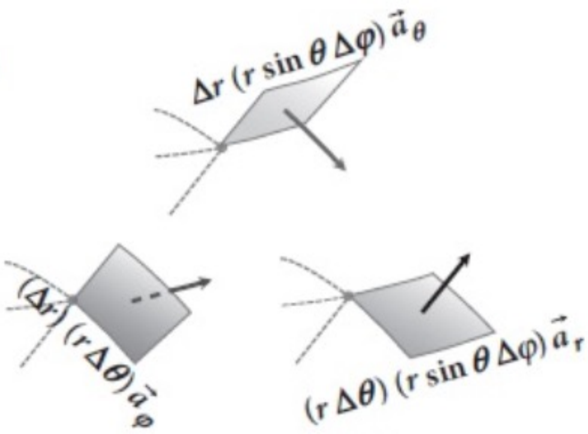
$$\Delta \boldsymbol{A_\theta} = \Delta r(r\sin\theta\,\Delta\varphi)\,\overrightarrow{a_\theta}$$

$$\Delta \boldsymbol{A_\varphi} = \Delta r(r\Delta\theta)\,\overrightarrow{a_\varphi}$$

(c) Incremental volume

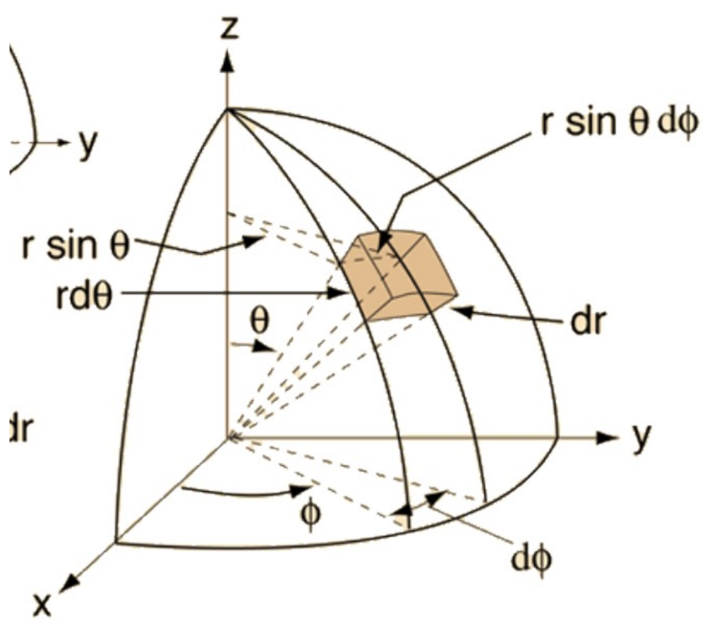
$$\Delta V = r^2\sin\theta\,\Delta r\,\Delta\theta\,\Delta\varphi$$

(b)



Incremental Surface Area Vectors

(c)



Scalar and Vectors

Scalar and Vectors

Scalar: A physical quantity which does not depend on the coordinate system, is called scalar.

Example:

- Mass of a stone:- Its value does not change whether you are carrying it in the East direction or North.
- Temperature, pressure, energy, time, etc.

Vector: A quantity which requires both the magnitude & direction for its complete specification.

$$\vec{A} = \boldsymbol{A} = \sum_{i=1}^{n} \overbrace{A_i . \boldsymbol{e}_i}^{\text{Vector}}$$

Scalar component

Notation: A or $\vec{A}$

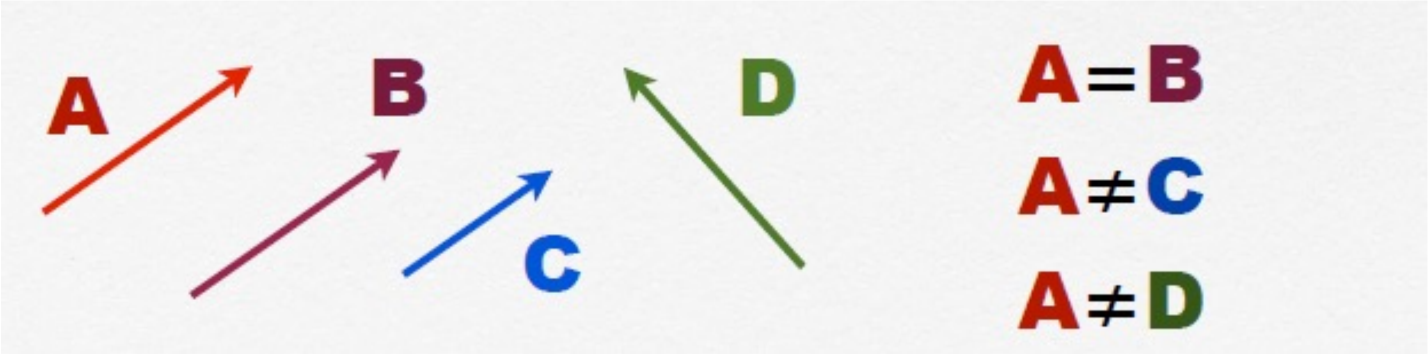
- **Magnitude:** |A| or A
- **Unit vector along A:** Â= **A**/|A|

Vectors

Properties

- Vector quantities have both magnitude and direction.
- The magnitude of a vector represents its size or length. It is always a non-negative value and is denoted by ||**A**|| or |**A**|, where "**A**" is the vector.
- Vectors have a specific direction in space.

- If two vectors have the same length (representing the same physical quantity) and direction, they are equal. A vector can be shifted without changing its value if its length and direction are not changed

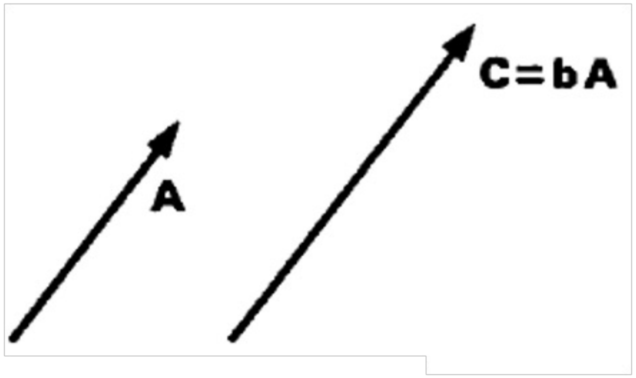


Vectors

Properties

Scalar Multiplication:

- Vectors can be multiplied by scalars (real numbers). Scalar multiplication changes the magnitude of the vector while preserving its direction.



$$|\mathbf{C}| = b|\mathbf{A}|$$

- Multiplying a vector by a negative scalar reverses its direction.

Next lecture

Dot product, cross product

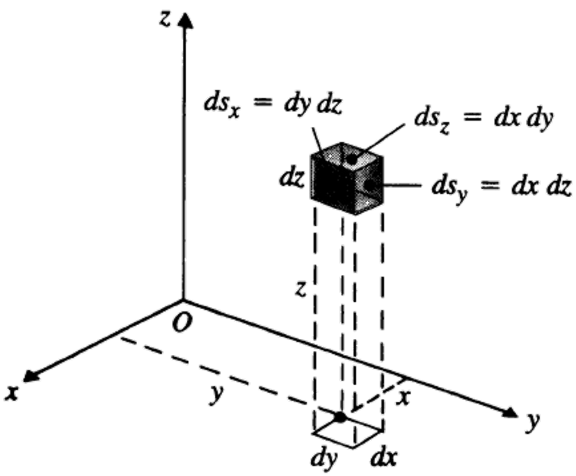
Reserved slides

Coordinate system in 3 Dimension (3D)

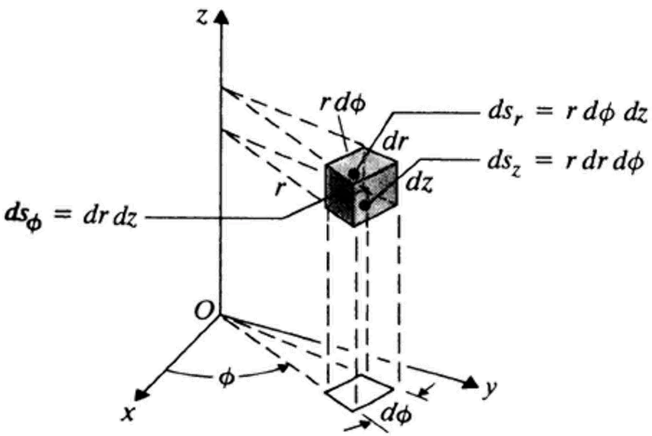
Infinitesimal incremental Length, Surface, and Volume elements

Comparison

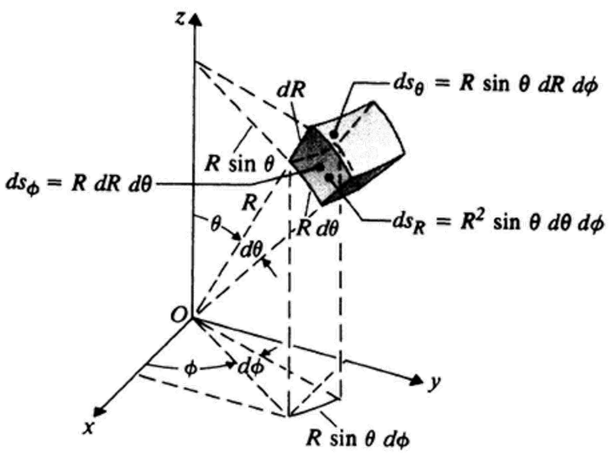
Cartesian



Cylindrical



Spherical



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Coordinate system in 3 Dimension (3D)

Infinitesimal incremental Length, Surface, and Volume elements

Comparison

		Cartesian	Cylindrical	Spherical
Differential Length	dl_1	dx	dr	dR
	dl_2	dy	r dϕ	R dθ
	dl_3	dz	dz	R sinθ dϕ
Differential Area	ds_1	dy dz	r dϕ dz	R^2 sinθ dθ dϕ
	ds_2	dx dz	dr dz	R sinθ dR dϕ
	ds_3	dx dy	r dr dϕ	R dR dθ
Differential Volume	dv	dx dy dz	r dr dϕ dz	R^2 sinθ dR dθ dϕ

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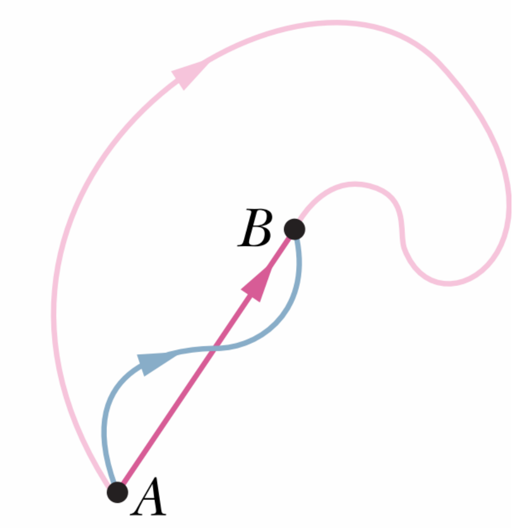
Vectors

Representation of a vector

Example:

The simplest vector quantity is a displacement or change of position. A vector that represents a displacement is called, reasonably, a **displacement vector**.



The displacement vector tells us nothing about the actual path that the particle takes.

For example, all three paths connecting points A and B correspond to the same displacement vector.

Displacement vectors represent only the overall effect of the motion, not the motion itself.