PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 30

Electrodynamics
Motional emf
ELECTROMAGNETIC INDUCTION

To make a current flow, charges need to be pushed. How fast they move, in response to a given push, depends on the nature of the material.

For most materials, current density (***J***) is proportional to the force per unit charge (***f***)

$$\boldsymbol{J} = \sigma \boldsymbol{f}$$

(σ is the conductivity of medium, the inverse of which is resistivity $\rho = \frac{1}{\sigma}$)

If forces that drive the charges are electromagnetic,

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Since velocity of charges in a conductor is very small,

$$\boxed{\mathbf{J} = \sigma \mathbf{E}.}$$

Ohm's law

For perfect conductors, $\boldsymbol{E} = \boldsymbol{J}/\sigma$ is zero ($\sigma \to \infty$) which means for good conductors (metals), electric field **required to drive the current** is negligible.

Material	Resistivity	Material	Resistivity
Conductors:		*Semiconductors:*	
Silver	1.59×10^{-8}	Sea water	0.2
Copper	1.68×10^{-8}	Germanium	0.46
Gold	2.21×10^{-8}	Diamond	2.7
Aluminum	2.65×10^{-8}	Silicon	2500
Iron	9.61×10^{-8}	*Insulators:*	
Mercury	9.61×10^{-7}	Water (pure)	8.3×10^{3}
Nichrome	1.08×10^{-6}	Glass	$10^{9} - 10^{14}$
Manganese	1.44×10^{-6}	Rubber	$10^{13} - 10^{15}$
Graphite	1.6×10^{-5}	Teflon	$10^{22} - 10^{24}$

Resistivities, in ohm-meters (all values are for 1 atm, 20° C).
Source: Handbook of Chemistry and Physics, 78th ed.
(Boca Raton: CRC Press, Inc., 1997).

(a) Find the charge density of mobile charge carriers in a piece of Copper, assuming each atom contributes one free electron

(a) $\rho = \dfrac{\text{charge}}{\text{volume}} = \dfrac{\text{charge}}{\text{atom}} \cdot \dfrac{\text{atoms}}{\text{mole}} \cdot \dfrac{\text{moles}}{\text{gram}} \cdot \dfrac{\text{grams}}{\text{volume}} = (e)(N)\left(\dfrac{1}{M}\right)(d)$, where

$$\begin{aligned} e &= \text{charge of electron} &&= 1.6 \times 10^{-19}\,\text{C}, \\ N &= \text{Avogadro's number} &&= 6.0 \times 10^{23}\,\text{mole}, \\ M &= \text{atomic mass of copper} &&= 64\,\text{gm/mole}, \\ d &= \text{density of copper} &&= 9.0\,\text{gm/cm}^3. \end{aligned}$$

$$\rho = (1.6 \times 10^{-19})(6.0 \times 10^{23})\left(\frac{9.0}{64}\right) = \boxed{1.4 \times 10^4\,\text{C/cm}^3.}$$

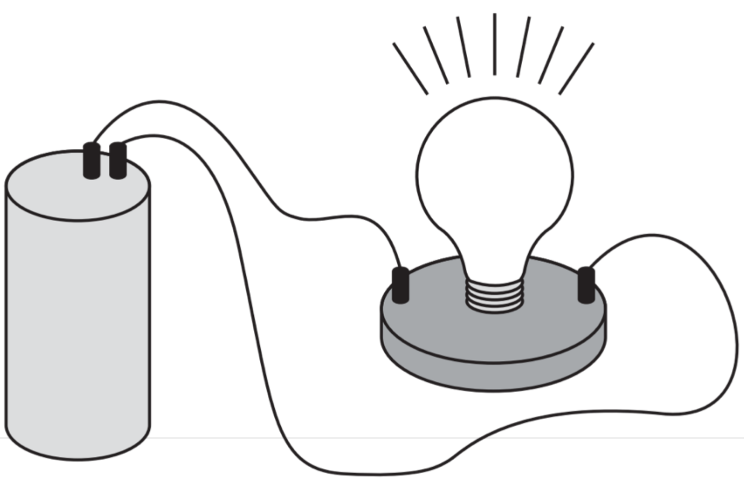
(b) Then calculate the average electron velocity in a Copper wire 1 mm in diameter, carrying current of 1A.

(b) $J = \dfrac{I}{\pi s^2} = \rho v \Rightarrow v = \dfrac{I}{\pi s^2 \rho} = \dfrac{1}{\pi(2.5 \times 10^{-3})(1.4 \times 10^4)} = \boxed{9.1 \times 10^{-3}\,\text{cm/s},}$ or about 33 cm/hr. This is astonishingly small—literally slower than a snail's pace.

Electromotive Force

When you hook up a battery to a light bulb by electric wire, current is same all the way around the loop, although the 'driving force' is inside the battery

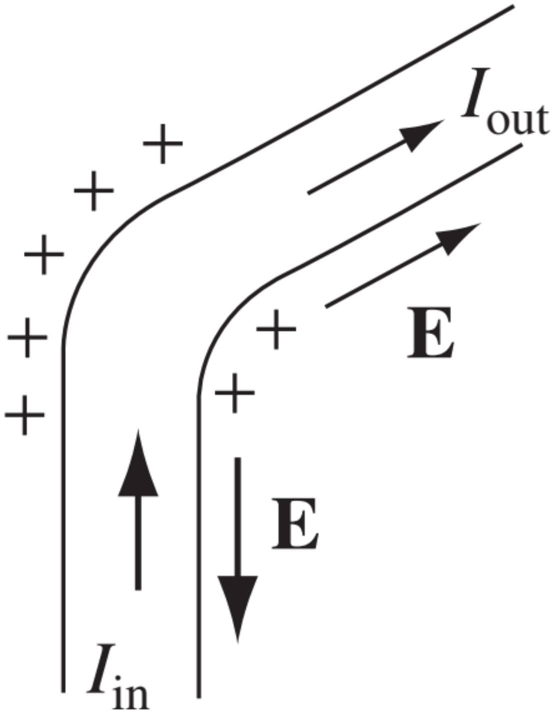
- Who is doing the pushing in the rest of the circuit?
- How is it that the 'push' is exactly right to produce the same current in each segment?
- Why doesn't it take a finite time (say 1 hr) for the current to reach the bulb?



Electromotive Force

If current were not the same all the way around, charge would be piling up- ***E*** *field due to accumulating charge* is in such a direction so as to *even out the flow*

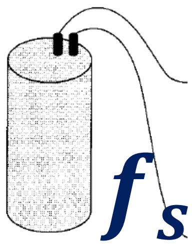
If $I_{in} > I_{out}$, charge accumulates at the 'knee'- it produces a field so as to *promote* I_{out} and *oppose* I_{in} until $I_{out} = I_{in}$ (equilibrium)



This process is automatically self-correcting to keep the current uniform- so one feels the current is uniform all around the circuit

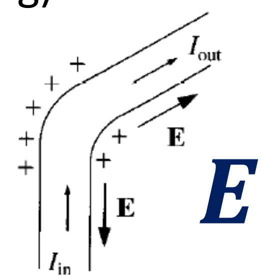
Two forces involved in driving current around a circuit

the *source* f_s confined to one portion of the loop (battery, say)



$$\boldsymbol{f} = \boldsymbol{f_s} + \boldsymbol{E}$$

and an *electrostatic force* ***'E'*** serving to smooth out the flow (field due to charge piling)



The physical agency responsible for $\boldsymbol{f_s}$ can be many different things: in a battery it's a chemical force; in a piezoelectric crystal mechanical pressure is converted into an electrical impulse; in a thermocouple it's a temperature gradient that does the job; in a photoelectric cell it's light; and in a Van de Graaff generator the electrons are literally loaded onto a conveyer belt and swept along.

The net effect is determined by the line integral of $\boldsymbol{f}$ around the circuit

$$\boxed{\mathcal{E} \equiv \oint \mathbf{f} \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l}.}$$

$\oint E.dl = 0$ for electrostatic fields

$\boldsymbol{\varepsilon}$ is called the **electromotive force (emf)** of the circuit- it is the integral of a force per unit charge and is responsible for driving current in a circuit.

Electrodynamics

For an ideal source of emf (say, resistanceless battery), the net force $\boldsymbol{f}$ on the charges is zero

$\sigma \to \infty$, hence $f = \frac{J}{\sigma} \to 0$ (Quality of an ideal conductor)

$\boldsymbol{E} = -\boldsymbol{f_s}$ **(Since, $\boldsymbol{f} = \boldsymbol{f_s} + \boldsymbol{E}$)**

Potential difference between two terminals of a battery (a and b)

$$V = -\int_a^b \mathbf{E} \cdot d\mathbf{l} = \int_a^b \mathbf{f}_s \cdot d\mathbf{l} = \oint \mathbf{f}_s \cdot d\mathbf{l} = \mathcal{E}$$

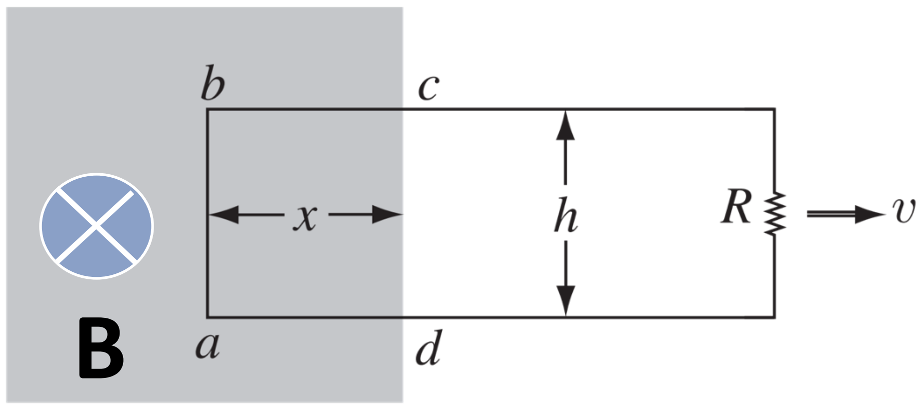
(Extending the integral to the entire loop as $f_s = 0$ outside the source)

The function of the battery is to establish and maintain a *voltage difference equal to the electromotive force*- the resulting electrostatic field due to the potential difference drives current around rest of the circuit.

***emf* can be defined as the work done per unit charge**

Motional emf

A generator based on **motional emf** is the most common source of emf. **Motional emf** is generated when a wire moves through a magnetic field



Principle

If the entire loop is pulled to the right with speed v, charges in segment ab experience a magnetic force, whose vertical component qvB drives current around the loop, in the clockwise direction.

Emf generated,

$$\varepsilon = \oint \boldsymbol{f}_{mag} . \boldsymbol{dl} = vBh$$

(Horizontal components (bc & ad) contribute nothing to emf (integral) as force is ⊥ to the wire)

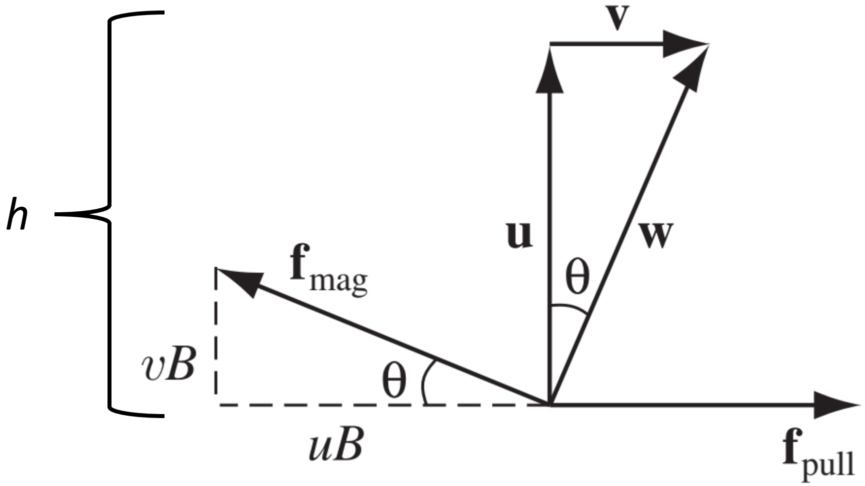
Motional emf

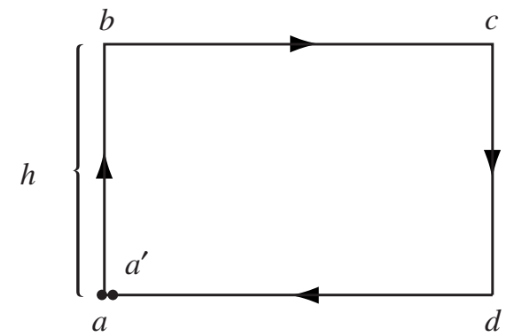
Although the magnetic force is responsible for establishing emf, it is not doing any work (magnetic forces never do any work)- who then is supplying the energy that heats the resistor?

Answer: the person who is pulling the loop.

Motional emf

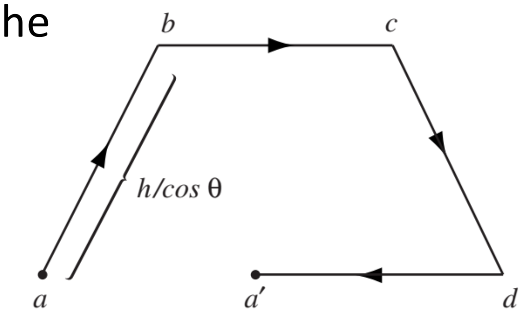
With current flowing, free charges in segment ab have a vertical velocity ($\boldsymbol{u}$) in addition to horizontal velocity ($\boldsymbol{v}$, due to pull)- accordingly, magnetic force has a component quB to the left- to counteract $f_{pull} = uB$ (per unit charge) is necessary to maintain current in the circuit. This force is transmitted to the charge by the structure of the wire.





(a) Integration path for computing $\mathcal{E}$ (follow the wire at one instant of time).

Meanwhile, the particle is actually moving in the direction of the resultant velocity **w**, and the distance it goes is **(h/ cos θ).**



(b) Integration path for calculating work done (follow the charge around the loop).

The work done per unit charge is therefore

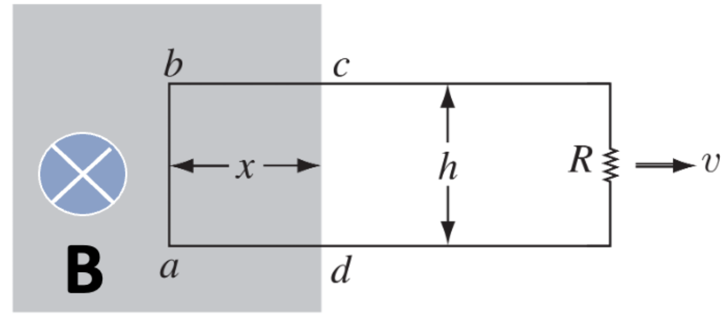
$$\int \mathbf{f}_{\text{pull}} \cdot d\mathbf{l} = (uB)\left(\frac{h}{\cos\theta}\right)\sin\theta = vBh = \mathcal{E}$$

(sin θ coming from the dot product).

Motional emf

Let Φ be the flux of B through the loop

$$\Phi = \int \boldsymbol{B.da}$$



$\Phi = Bhx$ for the rectangular loop

As loop moves, flux decreases

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv$$

But this is precisely the emf

$$\boxed{\varepsilon = -\frac{d\Phi}{dt}}$$

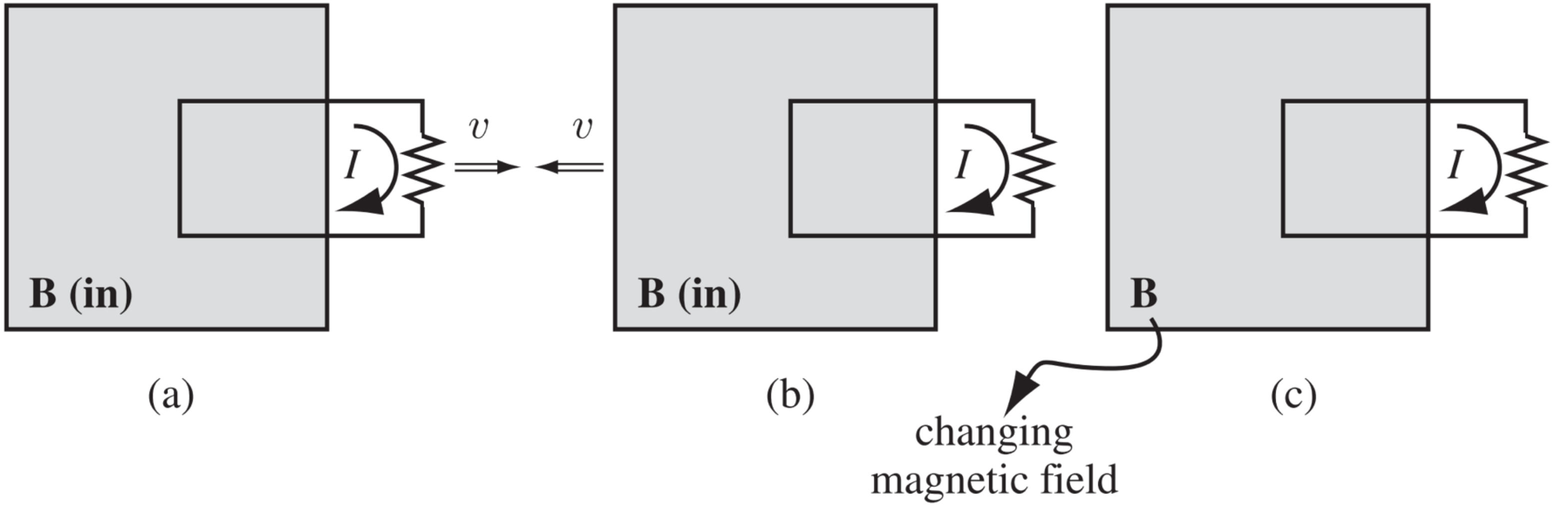
This is the **flux rule** for motional emf.

Emf generated in the loop is minus the change of flux through the loop

Faraday's Law

In 1831 Michael Faraday reported on a series of experiments,

Exp.1: Pulled a loop of wire to the right through a magnetic field.
Exp.2: Moved the magnet to the left, holding the loop still.
Exp.3: Changed the strength of the field, with both the loop and the magnet at rest.



$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Whenever (and for whatever reason) the magnetic flux through a loop changes, an emf will appear in the loop

Faraday's Law

A changing magnetic field induces an electric field.

Induced EMF

Empirical result

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi}{dt}$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\int \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{a}$$

This is **Faraday's law**

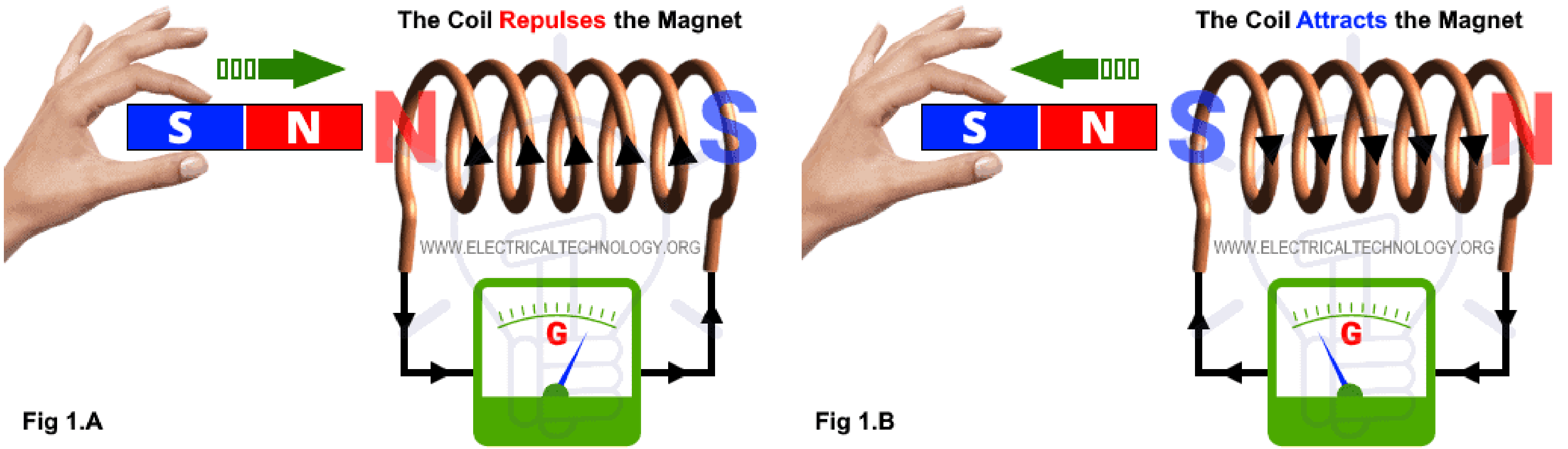
$$\boxed{\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.}$$

Differential form of **Faraday's law**

ELECTROMAGNETIC INDUCTION

Lenz’s Law

An induced Current always flows in a direction such that it opposes the change which produced it.

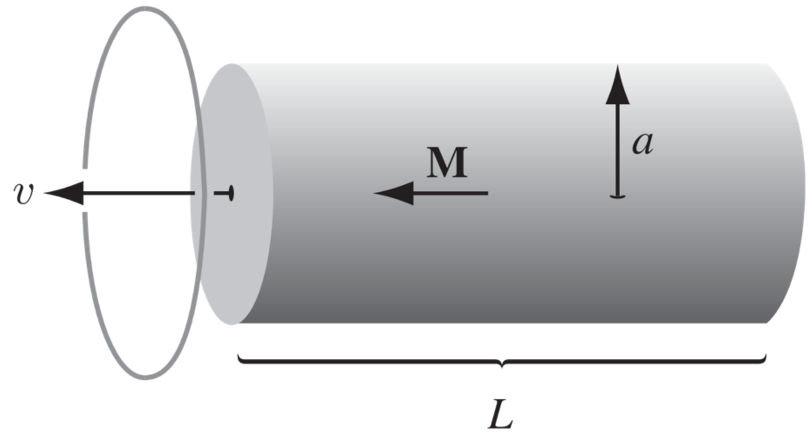


When the "N" Pole of the magnet is moved towards the coil, end of the coil becomes "N" Pole.

When the "N" Poles of the magnet is moved away from the coil, end of the coil becomes "S" Pole.

Lenz's Law

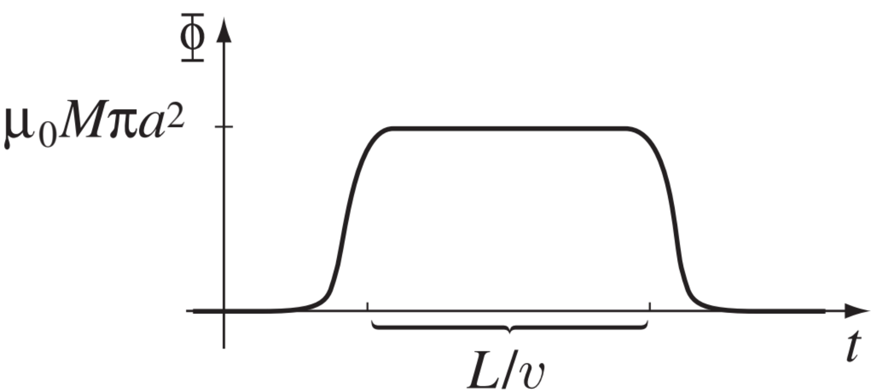
Eg. A long cylindrical magnet of length L and radius a carries a uniform magnetization $\mathbf{M}$ parallel to its axis. It passes at constant velocity v through a circular wire ring of slightly larger diameter. Graph the emf induced in the ring, as a function of time.

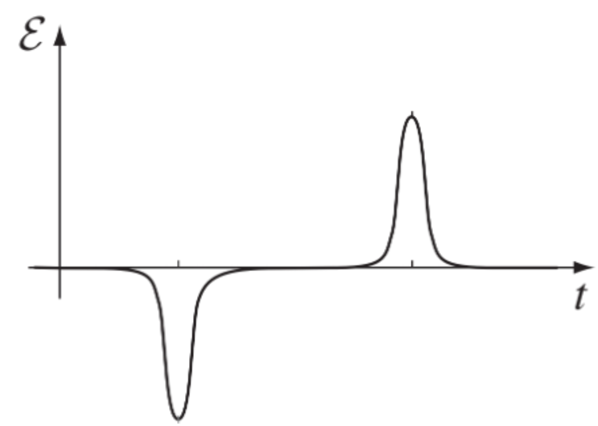


Solution

The magnetic field is the same as that of a long solenoid with surface current $\mathbf{K}_b = M\,\hat{\boldsymbol{\phi}}$

$$\mathbf{B} = \mu_0 \mathbf{M}, \qquad \emptyset = \mu_0 M \pi a^2$$





Nature abhors a change in flux.