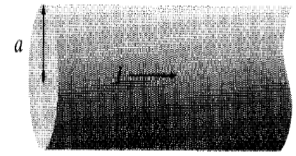
Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 10

(a) A current of magnitude I is uniformly distributed over a wire of circular cross section, with radius a. Find the magnitude, J, of the volume current density.



b) Suppose the magnitude of current density in the wire is proportional to the distance s from the axis, i.e.,

$$J = k\,s\,,$$

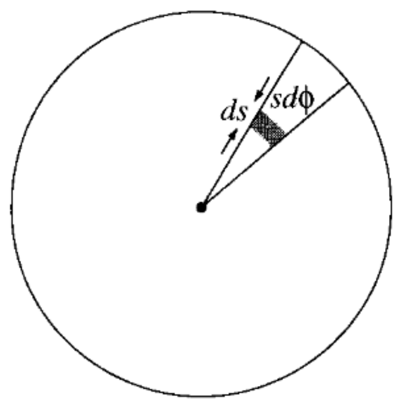
where k is a constant. Find the magnitude, I, of the total current in the wire.

Solution: The area-perpendicular-to-flow is πa^2, so

(a)
$$J = \frac{I}{\pi a^2}.$$

This was trivial because the current density was uniform.

(b) Because J varies with s, we must integrate over the cross-sectional area of the wire.



The magnitude, dI, of the current in the shaded patch is

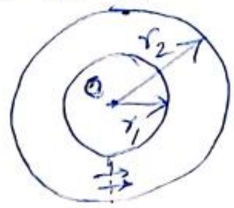
$J\,da_\perp$, and $da_\perp = s\,ds\,d\phi$. So,

$$I = \int (ks)(s\,ds\,d\phi) = 2\pi k \int_0^a s^2\,ds = \frac{2\pi k a^3}{3}.$$

2. Suppose a thin metallic ribbon carrying a steady current I is bent into the form of a circular ring of inner and outer radii r_1 and r_2, respectively. Find the magnetic field **B** at the centre of the ring.

Sol Current through an elementary ring of radius r and thickness dr is

$$dI = \frac{I}{r_2 - r_1}\, dr$$



magnetic field at the centre due to it is

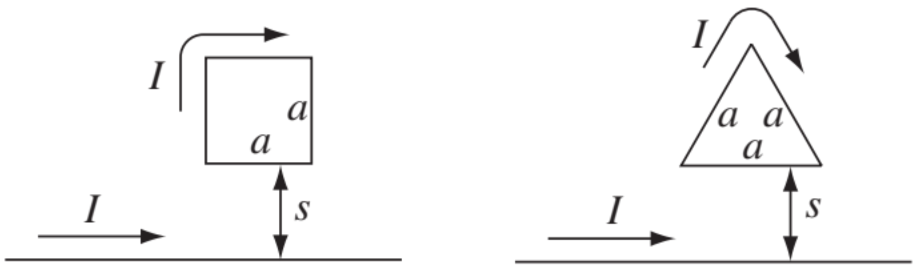
$$dB = \frac{\mu_0\, dI}{2r} = \frac{\mu_0 I}{2(r_2 - r_1)} \frac{dr}{r}$$

$\therefore$ Total field at the centre would be

$$B = \int dB = \frac{\mu_0 I}{2(r_2 - r_1)} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$= \frac{\mu_0 I}{2(r_2 - r_1)} \ln \frac{r_2}{r_1}$$

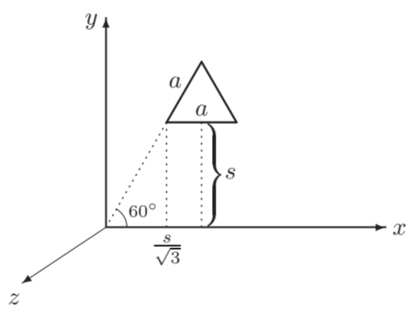
3. Find the force on a square loop and the triangular loop as shown in the figure below, placed near an infinite straight wire. Both the loop and the wire carry a steady current I.



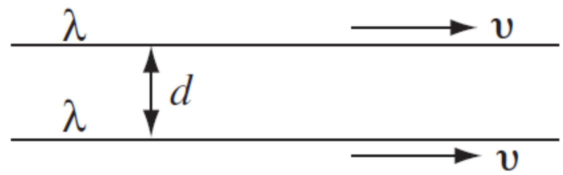
(a) (b)

(a) The forces on the two sides cancel. At the bottom, $B = \frac{\mu_0 I}{2\pi s} \Rightarrow F = \left(\frac{\mu_0 I}{2\pi s}\right) Ia = \frac{\mu_0 I^2 a}{2\pi s}$ (up). At the top, $B = \frac{\mu_0 I}{2\pi(s+a)} \Rightarrow F = \frac{\mu_0 I^2 a}{2\pi(s+a)}$ (down). The net force is $\boxed{\frac{\mu_0 I^2 a^2}{2\pi s(s+a)}}$ (up).

(b) The force on the bottom is the same as before, $\mu_0 I^2 a/2\pi s$ (up). On the left side, $\mathbf{B} = \frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}$; $d\mathbf{F} = I(d\mathbf{l} \times \mathbf{B}) = I(dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \times \left(\frac{\mu_0 I}{2\pi y}\,\hat{\mathbf{z}}\right) = \frac{\mu_0 I^2}{2\pi y}(-dx\,\hat{\mathbf{y}} + dy\,\hat{\mathbf{x}})$. But the x component cancels the corresponding term from the right side, and $F_y = -\frac{\mu_0 I^2}{2\pi}\int_{s/\sqrt{3}}^{(s/\sqrt{3}+a/2)} \frac{1}{y}\,dx$. Here $y = \sqrt{3}x$, so $F_y = -\frac{\mu_0 I^2}{2\sqrt{3}\pi} \ln\left(\frac{s/\sqrt{3}+a/2}{s/\sqrt{3}}\right) = -\frac{\mu_0 I^2}{2\sqrt{3}\pi} \ln\left(1 + \frac{\sqrt{3}a}{2s}\right)$. The force on the right side is the same, so the net force on the triangle is $\boxed{\frac{\mu_0 I^2}{2\pi}\left[\frac{a}{s} - \frac{2}{\sqrt{3}} \ln\left(1 + \frac{\sqrt{3}a}{2s}\right)\right]}$.



4. Suppose you have two infinite straight-line charges λ, a distance *d* apart, moving along at a constant speed *v* (see figure below). How great would *v* have to be for the magnetic attraction to balance the electrical repulsion? Work out the actual number. Is this a reasonable sort of speed?



Using the ***equations discussed/derived in the class*** :

$$f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d},$$

which is the magnetic force per unit length between two wires carrying currents I_1 and I_2, and separated by a distance d, and, the current expressed in terms of the line-charge density and the velocity of electrons :

$$I = \lambda v,$$

we obtain :

Magnetic attraction per unit length: $f_m = \frac{\mu_0}{2\pi}\frac{\lambda^2 v^2}{d}$.

Electric field of one wire (Eq. 2.9): $E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{s}$. Electric repulsion per unit length on the other wire: $f_e = \frac{1}{2\pi\epsilon_0}\frac{\lambda^2}{d}$. They balance when $\mu_0 v^2 = \frac{1}{\epsilon_0}$, or $\boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}}.}$ Putting in the numbers,

$v = \frac{1}{\sqrt{(8.85\times 10^{-12})(4\pi\times 10^{-7})}} = \boxed{3.00\times 10^8\ \mathrm{m/s}.}$ This is precisely the *speed of light(!)*, so in fact you could *never* get the wires going fast enough; the electric force always dominates.