Department of Physics, Shiv Nadar Institution of Eminence

Spring 2025

PHY102: Introduction to Physics-II

Tutorial – 2

1. Find the directional derivative of $f(x,y) = -2xy - \frac{x^2}{2} - \frac{y^2}{2}$ at (-2,2) in the direction of $\frac{3\pi}{4}$.

2. Find the directional derivative of $f(x,y) = x^3 e^{-y}$ at (3,2) in the direction of $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

3. Compute the gradient of $f(x, y, z) = (x^2 + y^2 + z^2)^{-1}$.

4. The height of a certain hill (in feet) is given by

$$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12),$$

where y is the distance (in miles) north, x the distance east, of South Hadley.

(a) Where is the top of the hill located?

(b) How high is the hill?

(c) How steep is the slope (in feet per mile) at a point 1 mile north and 1 mile east of South Hadley? In what direction is the slope steepest, at that point?

5. Compute the divergence and curl of the following fields

$$\vec{\boldsymbol{E}}(x,y,z) = \frac{x}{(x^2+y^2+z^2)^{3/2}}\hat{\boldsymbol{\imath}} + \frac{y}{(x^2+y^2+z^2)^{3/2}}\hat{\boldsymbol{\jmath}} + \frac{z}{(x^2+y^2+z^2)^{3/2}}\hat{\boldsymbol{k}}$$

for $x^2 + y^2 + z^2 \neq 0$

(Coulomb electric field for a point charge)

$$\vec{\boldsymbol{B}}(x,y,z) = -\frac{y}{x^2+y^2}\hat{\boldsymbol{\imath}} + -\frac{x}{x^2+y^2}\hat{\boldsymbol{\jmath}}$$

for $x^2 + y^2 \neq 0$

(Magnetic field outside an infinite current-carrying wire)

$$\vec{\boldsymbol{A}}(x,y,z) = -y\hat{\boldsymbol{\imath}} + x\hat{\jmath}$$

(Vector potential for a uniform magnetic field)