

PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 19

Electric Dipole

Electric Fields in Matter

Induced Dipoles

Electric Dipole in an external electric field

Case 1: Uniform electric field case

The dipole experiences a *torque* τ , due to *equal but oppositely directed forces*

τ tries to align \mathbf{p} along \mathbf{E} . Its magnitude is zero for $\theta = 0^\circ$ and also for $\theta = 180^\circ$

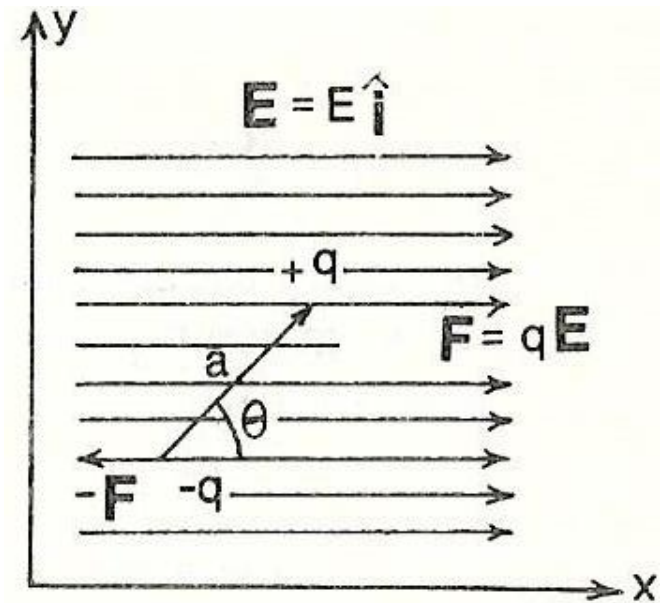
$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} = pE \sin \theta$$

$\xrightarrow{\mathbf{p}} \theta = 0^\circ$, stable equilibrium

\longrightarrow Direction of \mathbf{E}

$\xleftarrow{\mathbf{p}} \theta = 180^\circ$, unstable equilibrium

Uniform electric field: *no translational force* on a dipole, but only a *torque* which tries to make \mathbf{p} parallel to \mathbf{E}



Electric Dipole in an external electric field

Case 2: When \mathbf{E} varies from point-to-point (\mathbf{E} non-uniform)

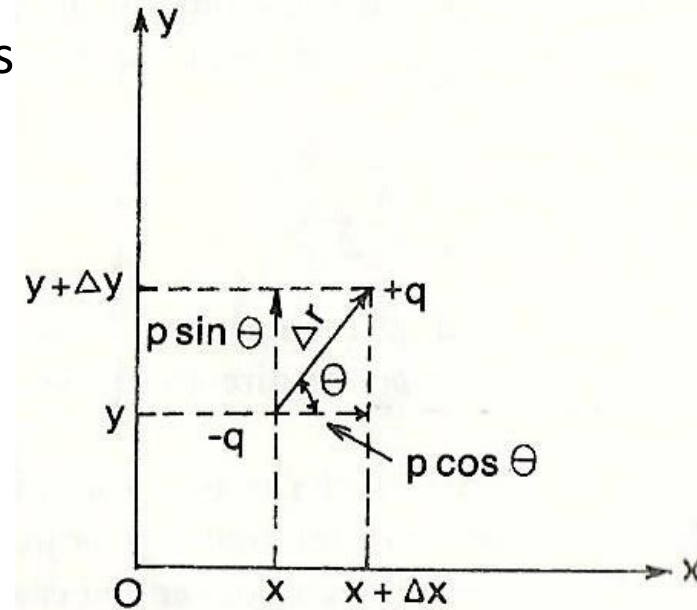
The x-component of force acting on the dipole is

$$F_x = qE_x(x + \Delta x, y + \Delta y) - qE_x(x, y)$$

$$\approx q \left[E_x(x, y) + \Delta x \frac{\partial E_x}{\partial x} + \Delta y \frac{\partial E_x}{\partial y} \right] - qE_x(x, y)$$

[Taylor series expansion of a function of two variables]

$$q\Delta x \frac{\partial E_x}{\partial x} + q\Delta y \frac{\partial E_x}{\partial y} = p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_x}{\partial y}$$



$$p_x = q\Delta x = p \cos \theta$$

$$p_y = q\Delta y = p \sin \theta$$

In general \mathbf{p} can have a projection along z-axis also which would have a contribution to F_x

$$F_x = p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_x}{\partial y} + p_z \frac{\partial E_x}{\partial z} = (\mathbf{p} \cdot \nabla) E_x$$

Electric Dipole in an external electric field

In a similar manner, the y-component and z-component of the force on dipole would be

$$F_y = p_x \frac{\partial E_y}{\partial x} + p_y \frac{\partial E_y}{\partial y} + p_z \frac{\partial E_y}{\partial z} = (\mathbf{p} \cdot \nabla) E_y$$

$$F_z = p_x \frac{\partial E_z}{\partial x} + p_y \frac{\partial E_z}{\partial y} + p_z \frac{\partial E_z}{\partial z} = (\mathbf{p} \cdot \nabla) E_z$$

Summing up,

$$\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} \quad \text{in case of a } \underline{\text{non-uniform field}}$$

$$\text{For an electrostatic field, } \nabla \times \mathbf{E} = 0 \quad \frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z} \quad \frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x}$$

$$\frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

Substitute these in the expression for F_x

Electric Dipole in an external electric field

$$F_x = p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_x}{\partial y} + p_z \frac{\partial E_x}{\partial z}$$

$$= p_x \frac{\partial E_x}{\partial x} + p_y \frac{\partial E_y}{\partial x} + p_z \frac{\partial E_z}{\partial x} = \frac{\partial}{\partial x} (\mathbf{p} \cdot \mathbf{E})$$

Similarly $F_y = \frac{\partial}{\partial y} (\mathbf{p} \cdot \mathbf{E}) \quad F_z = \frac{\partial}{\partial z} (\mathbf{p} \cdot \mathbf{E})$

$$\mathbf{F} = \nabla (\mathbf{p} \cdot \mathbf{E})$$

Thus the presence of an electric field has two effects on a dipole:

a) Align \mathbf{p} along \mathbf{E} (ensured by the term $\mathbf{p} \cdot \mathbf{E}$)

b) Move \mathbf{p} to regions of stronger field (ensured by the operator ∇)

Potential Energy of a Dipole in an external electric field

$$\mathbf{F} = \nabla(\mathbf{p} \cdot \mathbf{E})$$

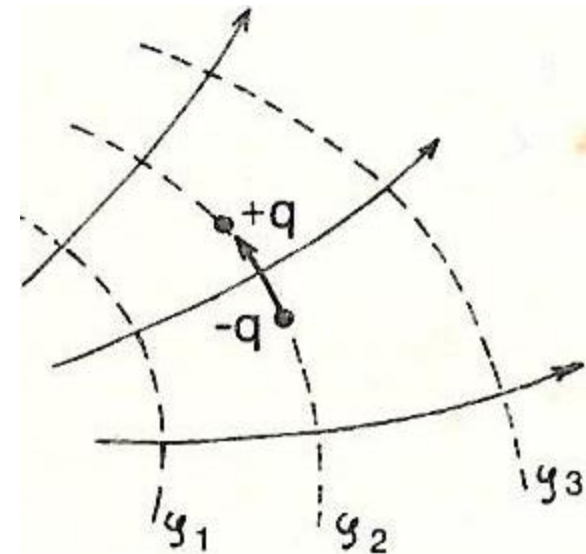
But force is also equal to the negative gradient of a scalar potential, $\mathbf{F} = -\nabla U$

$$U = -\mathbf{p} \cdot \mathbf{E} \quad \text{'U' is the potential energy}$$

Thus if the dipole is free to move, it will tend to travel into regions of **stronger electric field**, so that U takes large and large negative values (more stable)- dipoles tend to attain minimum potential energy in an external electric field.

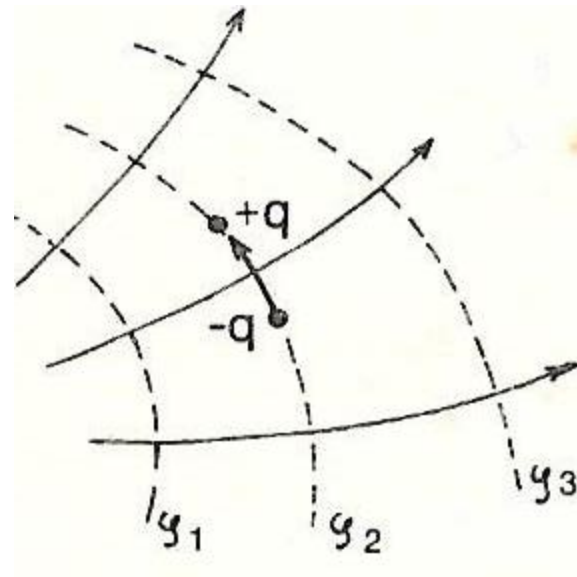
U is zero when \mathbf{p} is perpendicular to \mathbf{E} . This means it is possible to move a dipole anywhere in a field \mathbf{E} without doing any work, provided the orientation of \mathbf{p} is kept at right angles to the local direction of \mathbf{E}

Dotted lines are ***equipotentials***- no work is done if you move a dipole along an equipotential (why?)



Potential Energy of a Dipole in an external electric field

One can always bring a dipole, \mathbf{p} , from infinity to any point in a electric field without doing any work, provided final orientation of \mathbf{p} is perpendicular to local \mathbf{E} .



After doing this, if the direction of \mathbf{p} has to be changed until it makes an angle θ with \mathbf{E} , it is necessary to do work against the torque, $dW = \tau d\theta$

$$\text{Total work done, } W = \int_{\theta=\pi/2}^{\theta} pE \sin\theta d\theta = -\mathbf{p} \cdot \mathbf{E}$$

This work done, by definition is the potential energy of the dipole in electric field.

Mutual Potential energy of two dipoles

For two point charges q_1 and q_2 , separated by distance ' r ', potential energy is

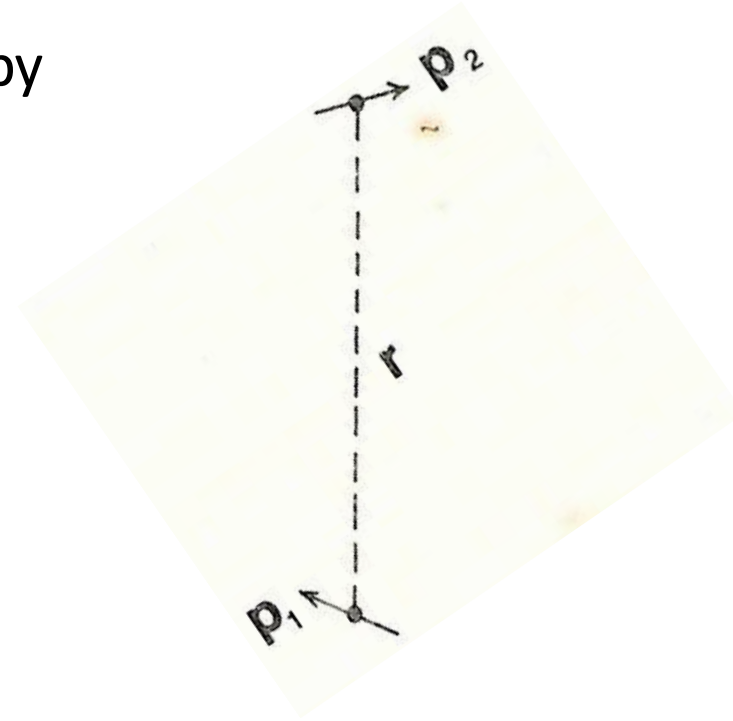
$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

What is the mutual potential energy of two dipoles \mathbf{p}_1 and \mathbf{p}_2 separated by distance ' r '?

Calculate \mathbf{E}_1 at the position of \mathbf{p}_2

$$\mathbf{E}_1 = \frac{1}{4\pi\epsilon_0 r^3} [3(\mathbf{p}_1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}_1]$$

$$\text{Calculate } U = -\mathbf{p}_2 \cdot \mathbf{E}_1 = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} [\mathbf{p}_1 \cdot \mathbf{p}_2 - 3(\mathbf{p}_1 \cdot \hat{\mathbf{r}})(\mathbf{p}_2 \cdot \hat{\mathbf{r}})]$$

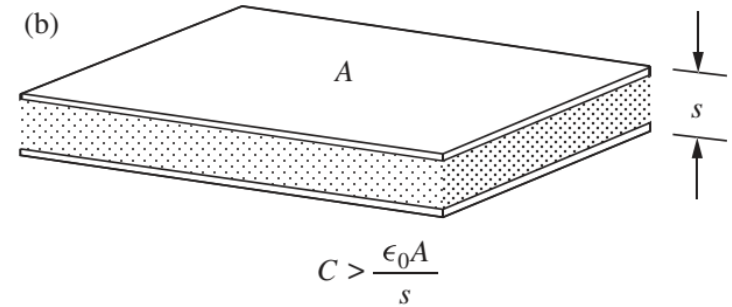
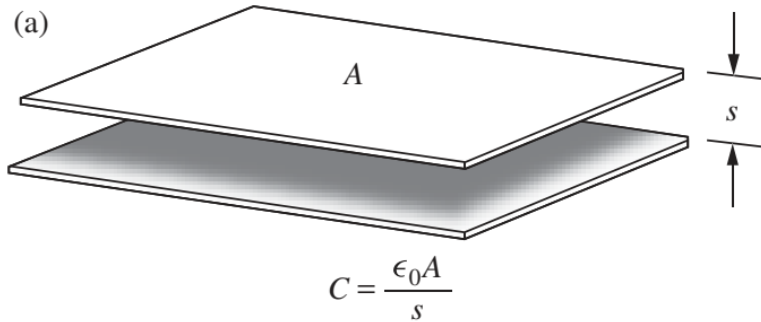




Electric Fields in Matter

Dielectrics

Faraday discovered that the capacitance of such a capacitor is *increased* when an insulator is put between the plates.



$$C = \frac{Q}{\phi_{12}}$$

If the insulator completely fills the space between the plates, the capacitance is increased by a factor κ which depends only on the nature of the insulating material.

Insulating materials are also called **dielectrics**; the factor κ is then a property of the dielectric and is called the **dielectric constant**.

The dielectric constant of a vacuum is, of course, **unity**.

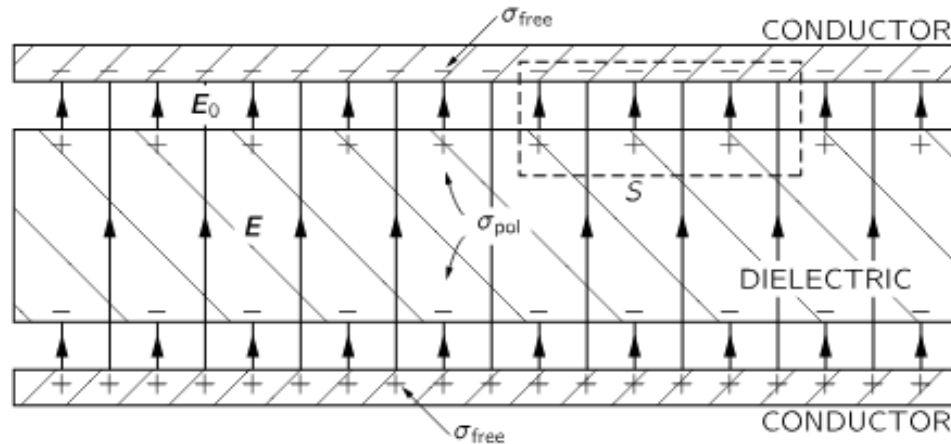
Dielectrics

Dielectric constants of various substances

Substance	Conditions	Dielectric constant (κ)
Air	gas, 0 °C, 1 atm	1.00059
Methane, CH ₄	gas, 0 °C, 1 atm	1.00088
Hydrogen chloride, HCl	gas, 0 °C, 1 atm	1.0046
Water, H ₂ O	gas, 110 °C, 1 atm	1.0126
	liquid, 20 °C	80.4
Benzene, C ₆ H ₆	liquid, 20 °C	2.28
Methanol, CH ₃ OH	liquid, 20 °C	33.6
Ammonia, NH ₃	liquid, −34 °C	22.6
Mineral oil	liquid, 20 °C	2.24
Sodium chloride, NaCl	solid, 20 °C	6.12
Sulfur, S	solid, 20 °C	4.0
Silicon, Si	solid, 20 °C	11.7
Polyethylene	solid, 20 °C	2.25–2.3
Porcelain	solid, 20 °C	6.0–8.0
Paraffin wax	solid, 20 °C	2.1–2.5
Pyrex glass 7070	solid, 20 °C	4.00

Dielectrics

Parallel-plate capacitor with some charges on the surfaces of the conductors



$$C = \frac{\epsilon_0 A}{d}$$

$$Q = CV$$

If we put a piece of insulating material like lucite or glass between the plates, we find that the capacitance is larger.

That means, of course, that the voltage is lower for the same charge.

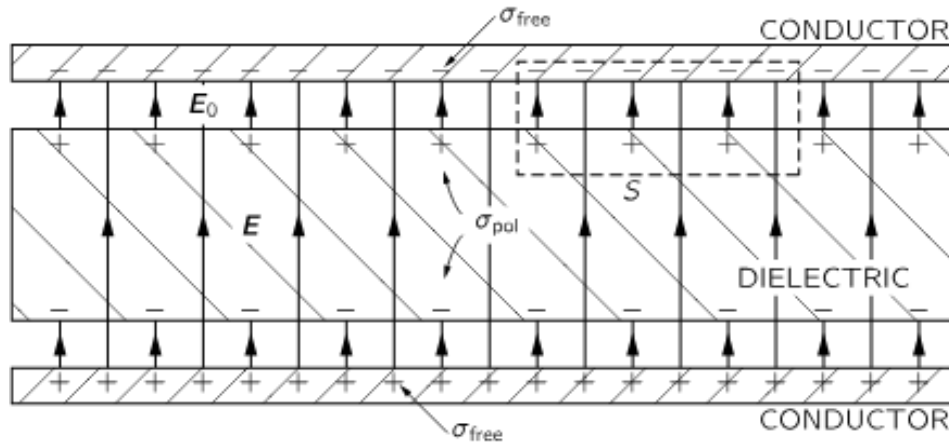
But the voltage difference is the integral of the electric field across the capacitor; so we must conclude that inside the capacitor, the electric field is reduced even though the charges on the plates remain unchanged.

Consider the Gaussian surface S shown by broken lines.

Since the electric field is reduced with the dielectric present, we conclude that the net charge inside the surface must be lower than it would be without the material. There is only one possible conclusion, and that is that there must be positive charges on the surface of the dielectric. Since the field is reduced but is not zero, we would expect this positive charge to be smaller than the negative charge on the conductor.

Dielectrics

Parallel-plate capacitor with some charges on the surfaces of the conductors



$$C = \frac{\epsilon_0 A}{d}$$

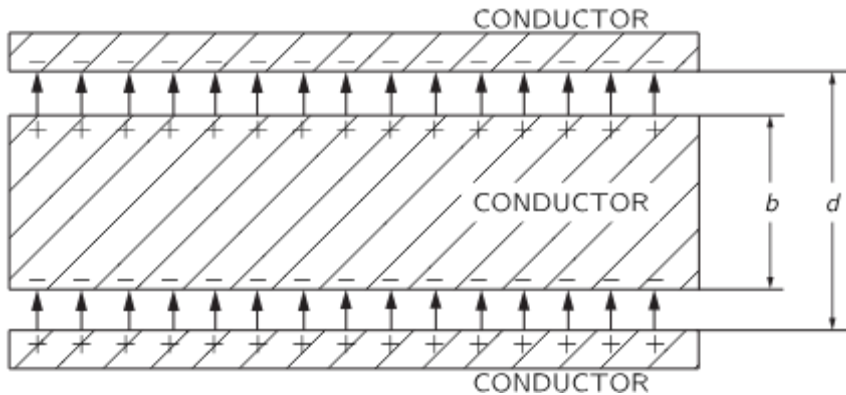
$$Q = CV$$

Consider the Gaussian surface S shown by broken lines.

Since the electric field is reduced with the dielectric present, we conclude that the net charge inside the surface must be lower than it would be without the material. There is only one possible conclusion, and that is that there must be positive charges on the surface of the dielectric. Since the field is reduced but is not zero, we would expect this positive charge to be smaller than the negative charge on the conductor. So the phenomena can be explained if we could understand in some way that when a dielectric material is placed in an electric field there is positive charge induced on one surface and negative charge induced on the other.

Dielectrics

Parallel-plate capacitor with conducting plate



The induced charges reduce the field in the conductor to zero.

The field in the rest of the space is the same as it was without the conductor, because it is the surface density of charge divided by ϵ_0 ; but the distance over which we have to integrate to get the voltage (the potential difference) is reduced.

$$V = \frac{\sigma}{\epsilon_0} (d - b)$$

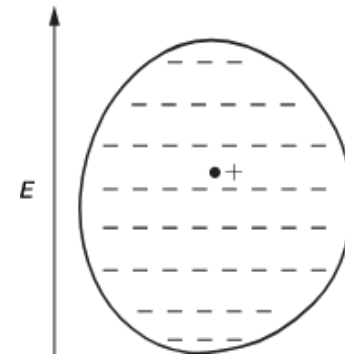
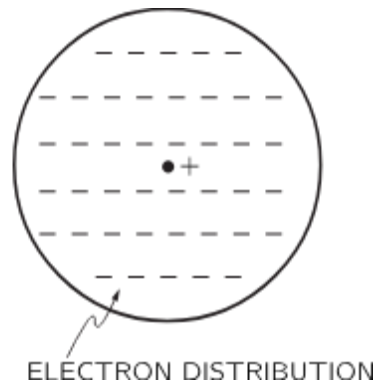
$$C = \frac{\epsilon_0 A}{d[1 - (b/d)]}$$

The capacitance is increased by a factor that depends upon **(b/d)**, the proportion of the volume which is occupied by the conductor.

Induced Dipoles

In dielectrics, by contrast, all charges are attached to specific atoms or molecules—they're on a tight leash, and all they can do is move a bit within the atom or molecule.

There are actually two principal mechanisms by which electric fields can distort the charge distribution of a dielectric atom or molecule: stretching and rotating.



In each atom there are charges q separated by a distance δ , so that $q\delta$ is the dipole moment per atom.

$$p = q\delta$$

If there are N atoms per unit volume, there will be a *dipole moment per unit volume* equal to $Nq\delta$. This dipole moment per unit volume will be represented by a vector, \mathbf{P} .

$$\mathbf{P} = Nq\delta$$

Induced Dipoles

In general, \mathbf{p} will vary from place to place in the dielectric. However, at any point in the material, \mathbf{p} is proportional to the electric field \mathbf{E} .

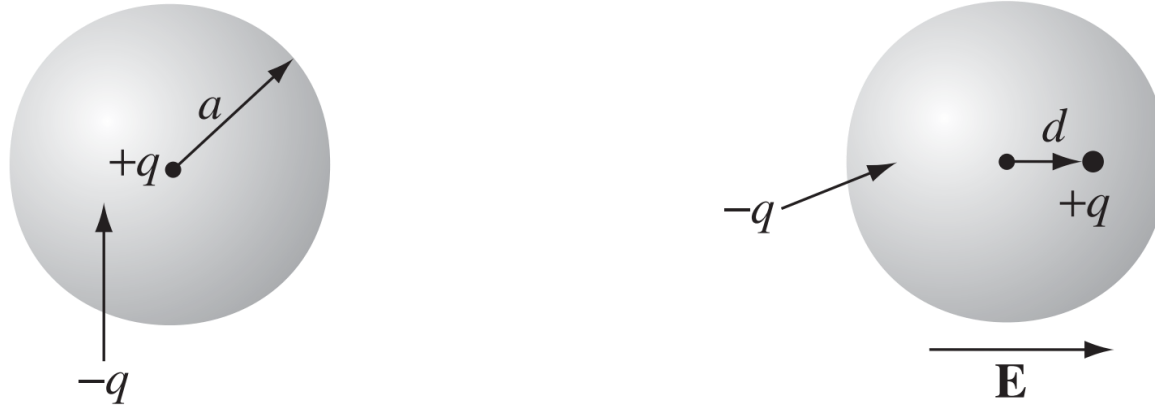
$$\mathbf{p} = \alpha \mathbf{E}$$

The constant of proportionality α is called *atomic polarizability*. The constant of proportionality, which depends on the ease with which the electrons are displaced, will depend on the kinds of atoms in the material. Its value depends on the detailed structure of the atom in question.

H	He	Li	Be	C	Ne	Na	Ar	K	Cs
0.667	0.205	24.3	5.60	1.67	0.396	24.1	1.64	43.4	59.4

Induced Dipoles

A primitive model for an atom consists of a point nucleus (+q) surrounded by a uniformly charged spherical cloud (−q) of radius a. Calculate the atomic polarizability of such an atom.



At the point where the external field pushing the nucleus to the right exactly balances the internal field pulling it to the left: $\mathbf{E} = \mathbf{E}_e$, where \mathbf{E}_e is the field produced by the electron cloud.

The field at a distance d from the center of a uniformly charged sphere is $E_e = \frac{1}{4\pi\epsilon_0} \frac{qd}{a^3}$

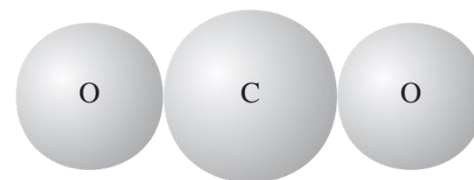
At equilibrium, $E = \frac{1}{4\pi\epsilon_0} \frac{qd}{a^3}$, or $p = qd = (4\pi\epsilon_0 a^3) E$

The atomic polarizability is therefore

$$\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 v, \quad \text{where } v \text{ is the volume of the atom.}$$

Induced Dipoles

For molecules such as carbon dioxide:



$$\mathbf{p} = \alpha_{\perp} \mathbf{E}_{\perp} + \alpha_{\parallel} \mathbf{E}_{\parallel}$$

The induced dipole moment may not even be in the same direction as \mathbf{E} .

For a completely asymmetrical molecule, the most general linear relation between \mathbf{E} and \mathbf{p}

$$p_x = \alpha_{xx} E_x + \alpha_{xy} E_y + \alpha_{xz} E_z$$

$$p_y = \alpha_{yx} E_x + \alpha_{yy} E_y + \alpha_{yz} E_z$$

$$p_z = \alpha_{zx} E_x + \alpha_{zy} E_y + \alpha_{zz} E_z$$

The set of nine constants α_{ij} constitute the **polarizability tensor** for the molecule. Their values depend on the orientation of the axes you use, though it is always possible to choose “principal” axes such that all the off-diagonal terms (α_{xy} , α_{zx} , etc.) vanish, leaving just three nonzero polarizabilities: α_{xx} , α_{yy} , and α_{zz} .