Steady State AC Circuits
AC Power
and
Complex Impedance
Phasor

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Review-AC Circuits

- Introduction
- Power in Resistive Components
- Power in Capacitors
- Power in Inductors
- Circuits with Resistance and Reactance
- Active and Reactive Power
- Power Factor Correction

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Introduction

- The instantaneous power dissipated in a component is a product of the instantaneous voltage and the instantaneous current

$$p = vi$$

- In a resistive circuit the voltage and current are in phase – calculation of p is straight forward
- In reactive circuits, there will normally be some phase shift between v and i, and calculating the power becomes more complicated

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Power in Resistive Components

- Suppose a voltage $v = V_p \sin \omega t$ is applied across a resistance R. The resultant current i will be

$$i = \frac{v}{R} = \frac{V_P \sin \omega t}{R} = I_P \sin \omega t$$

- The result power p will be

$$p = vi = V_P \sin \omega t \times I_P \sin \omega t = V_P I_P (\sin^2 \omega t) = V_P I_P (\frac{1 - \cos 2\omega t}{2})$$

- The average value of (1 - cos $2\omega t$) is 1, so

$$\text{Average Power } P = \frac{1}{2} V_P I_P = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} = VI$$

where V and I are the **r.m.s. voltage and current**

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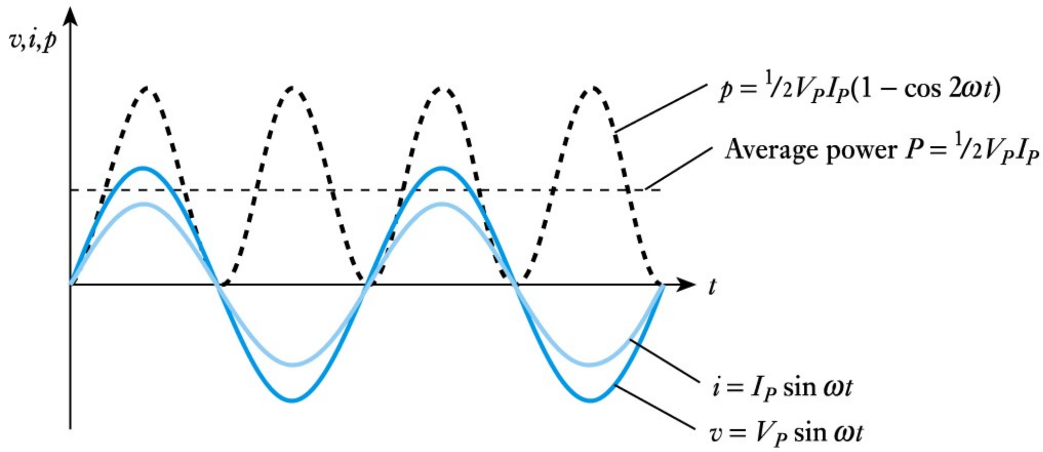
- **Relationship between *v*, *i* and *p* in a Resistive Circuit**



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Power in Capacitive Circuit

- From our discussion of capacitors we know that the current leads the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across a capacitance *C*, the current will be given by $i = I_p \cos \omega t$
- Then

$$
\begin{aligned}
p &= vi \\
&= V_P \sin \omega t \times I_P \cos \omega t \\
&= V_P I_P (\sin \omega t \times \cos \omega t) \\
&= V_P I_P \left(\frac{\sin 2\omega t}{2}\right)
\end{aligned}
$$

- The average power is zero

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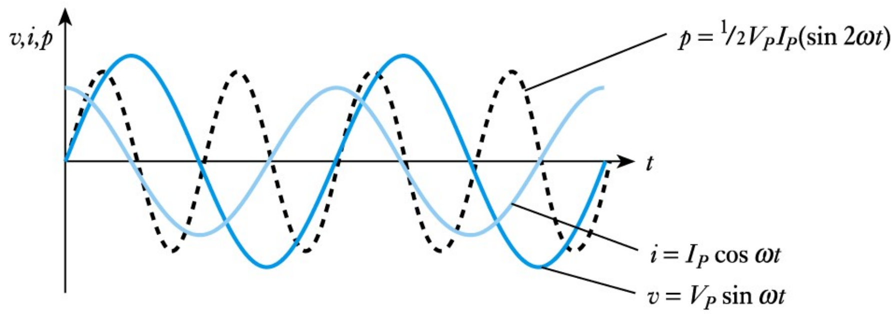
- **Relationship between *v*, *i* and *p* in a capacitor**



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Power in Inductive Circuit

- From our discussion of inductors we know that the current lags the voltage by 90°. Therefore, if a voltage $v = V_p \sin \omega t$ is applied across an inductance *L*, the current will be given by $i = -I_p \cos \omega t$
- Therefore

$$\begin{aligned} p &= vi \\ &= V_P \sin \omega t \times -I_P \cos \omega t \\ &= -V_P I_P (\sin \omega t \times \cos \omega t) \\ &= -V_P I_P \left(\frac{\sin 2\omega t}{2}\right) \end{aligned}$$

- Again the average power is zero

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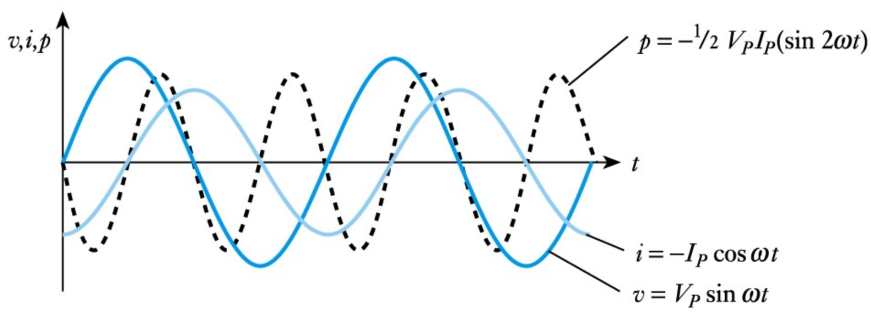
- **Relationship between *v*, *i* and *p* in an inductor**



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Circuit with Resistance and Reactance

- When a sinusoidal voltage $v = V_p \sin \omega t$ is applied across a circuit with resistance *and* reactance, the current will be of the general form $i = I_p \sin(\omega t - \phi)$
- Therefore, the instantaneous power, *p* is given by

$$\begin{aligned} p &= vi \\ &= V_P \sin \omega t \times I_P \sin(\omega t - \phi) \\ &= \frac{1}{2} V_P I_P \{\cos \phi - \cos(2\omega t - \phi)\} \end{aligned}$$

$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

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$$p = \frac{1}{2} V_P I_P \cos \phi - \frac{1}{2} V_P I_P \cos(2\omega t - \phi)$$

- The expression for *p* has two components
- The second part oscillates at 2ω and has an average value of zero over a complete cycle
  - this is the power that is stored in the reactive elements and then returned to the circuit within each cycle.

- The first part represents the power dissipated in resistive components. **Average power dissipation** is

$$P = \frac{1}{2} V_P I_P (\cos \phi) = \frac{V_P}{\sqrt{2}} \times \frac{I_P}{\sqrt{2}} \times (\cos \phi) = VI \cos \phi$$

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- The average power dissipation given by

$$P = \frac{1}{2} V_P I_P (\cos \phi) = VI \cos \phi$$

is termed the **active power** in the circuit and is measured in watts (W)

- The product of the r.m.s. voltage and current i.e. *VxI* is termed as the **apparent power**, *S*. To avoid confusion this is given the units of volt amperes (VA)

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- From the above discussion it is clear that

$$P = VI\cos\phi$$
$$= S\cos\phi$$

- In other words, the active power is the apparent power times the cosine of the phase angle.
- This cosine is referred to as the **power factor**

$$\frac{\text{Active power (in watts)}}{\text{Apparent power (in volt amperes)}} = \text{Power factor}$$

$$\text{Power factor} = \frac{P}{S} = \cos\phi$$

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Active (P) and Reactive Power (Q)

- When a circuit has resistive and reactive parts, the resultant power has 2 parts:
  - The first is *dissipated* in the resistive element. This is the **active power, *P***
  - The second is *stored* and *returned* by the reactive element. This is the **reactive power**, ***Q*** , which has units of **volt amperes reactive** or **VAR**
- While reactive power is not dissipated it does have an effect on the system
  - for example, it increases the current that must be supplied and increases losses with cables

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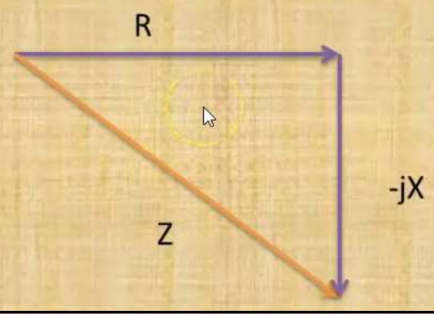
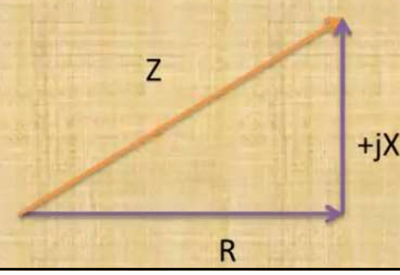
What is impedance?

- Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied.
- Impedance is defined as the ratio of RMS value of Voltage to RMS value of current Z=V/I or V=IZ
- Z = R + jX (for inductive circuit) or Z = R – jX (for capacitive circuit)





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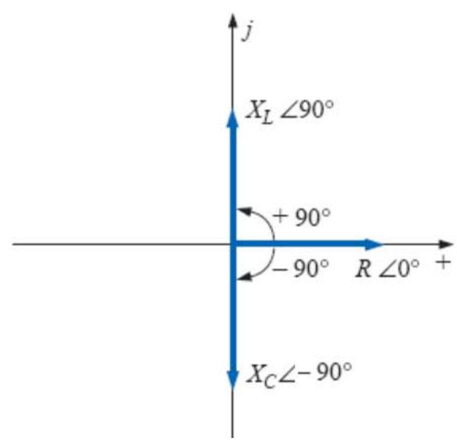
Impedance Diagram

For any configuration (series, parallel, series-parallel, etc.), the angle associated with the total impedance is the angle by which the ***applied voltage leads the source current.***

For Resistive network , θ_T will be 0

For inductive networks, θ_T will be positive,

For capacitive networks, θ_T will be negative.



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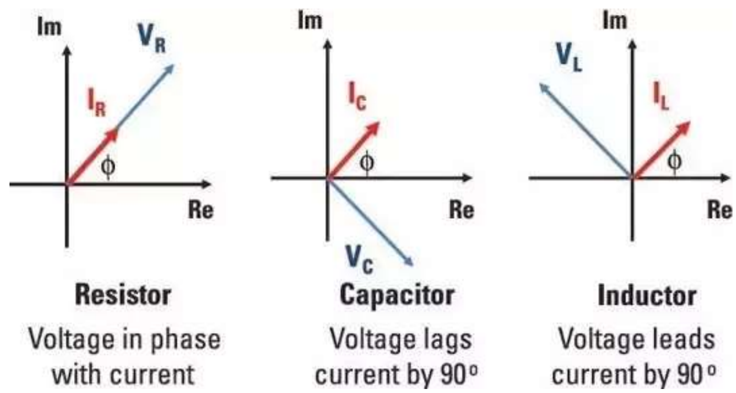
Phasor Diagram of individual Element



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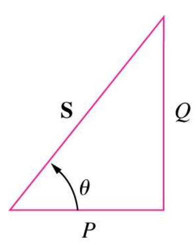
Power Triangle

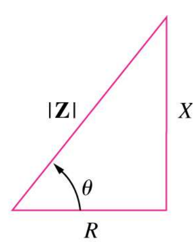
S= P+ jQ

Z= R+ j(X$_L$-X$_C$)



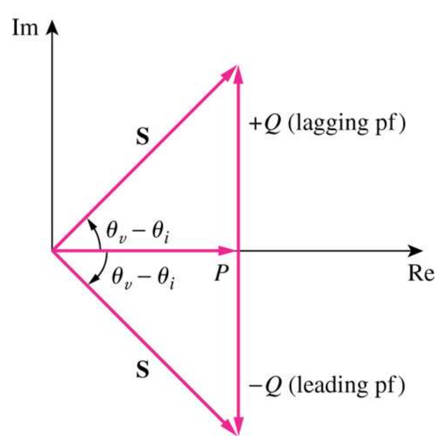
(a)

Power triangle



(b)

Impedance triangle



Power triangle

Power factor $= \frac{P}{S} = \cos\varphi$ = R / Z

For inductive (lagging): Power factor $= \frac{P}{S} = \cos\varphi$ = R / (R + jX_L)

For capacitive (leading): Power factor $= \frac{P}{S} = \cos\varphi$ = R / (R - jX_C)

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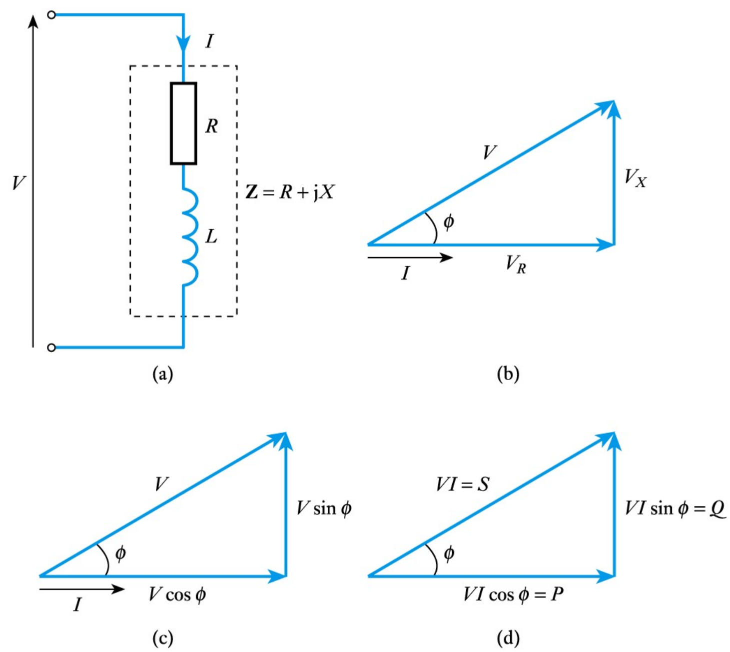
- Consider an RL circuit
  - the relationship between the various forms of power can be illustrated using a power triangle



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- Therefore, *V* and *I being rms values*

Active Power (Average Power)

$P = VI \cos\phi$ watts or $P = S\cos\phi$

Reactive Power (Quadrature Power)

$Q = VI \sin\phi$ *VAR* or $Q = S\sin\phi$

Apparent Power $S = VI$ VA

Complex Power $\boldsymbol{S} = P + jQ = VI\cos\phi + j\,VI\sin\phi = \boldsymbol{VI^*}$

with $S^2 = P^2 + Q^2$

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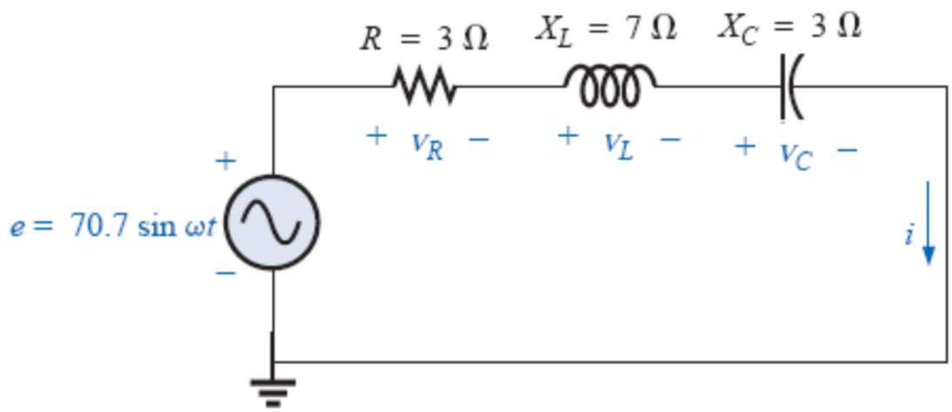
Find total Impedance Z_T ,
Draw Impedance diagram (Triangle),
Draw Phasor Diagram (Current and Voltage Phasor) ,
Power Factor



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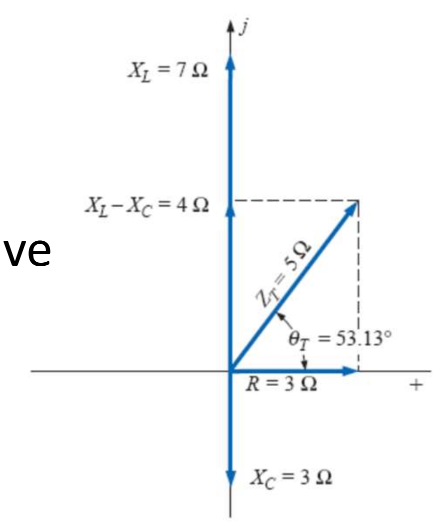
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 = R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= 3\ \Omega + j\,7\ \Omega - j\,3\ \Omega = 3\ \Omega + j\,4\ \Omega$$
$$\mathbf{Z}_T = \mathbf{5\ \Omega\ \angle 53.13^\circ}$$

Inductive



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$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{50\text{ V}\angle 0°}{5\ \Omega\angle 53.13°} = \mathbf{10\ A\angle -53.13°}$$

V_R, V_L, and V_C

$$\mathbf{V}_R = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0°) = (10\text{ A}\angle -53.13°)(3\ \Omega\angle 0°)$$
$$= \mathbf{30\ V\angle -53.13°}$$

$$\mathbf{V}_L = \mathbf{I}\mathbf{Z}_L = (I\angle\theta)(X_L\angle 90°) = (10\text{ A}\angle -53.13°)(7\ \Omega\angle 90°)$$
$$= \mathbf{70\ V\angle 36.87°}$$

$$\mathbf{V}_C = \mathbf{I}\mathbf{Z}_C = (I\angle\theta)(X_C\angle -90°) = (10\text{ A}\angle -53.13°)(3\ \Omega\angle -90°)$$
$$= \mathbf{30\ V\angle -143.13°}$$

Kirchhoff's voltage law:

$$\Sigma_{C}\,\mathbf{V} = \mathbf{E} - \mathbf{V}_R - \mathbf{V}_L - \mathbf{V}_C = 0$$

$$\mathbf{E} = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

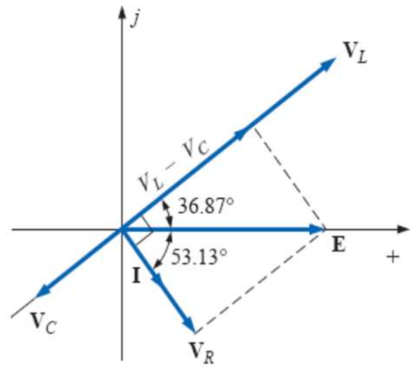


FIG. 15.38
Phasor diagram for the series R-L-C circuit of

For Practicing Complex Algebra calculation: Use Voltage Divider to get Voltage Phasors across R, C, and L

Power Factor:

The total **power factor (Cos θ_T),** determined by the angle between the applied voltage **E** and the resulting current **I**.

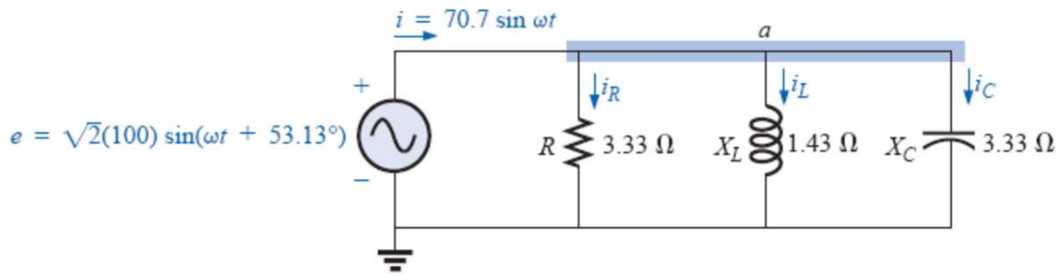
$$F_p = \cos\theta_T = \cos 53.13° = \mathbf{0.6\ lagging}$$

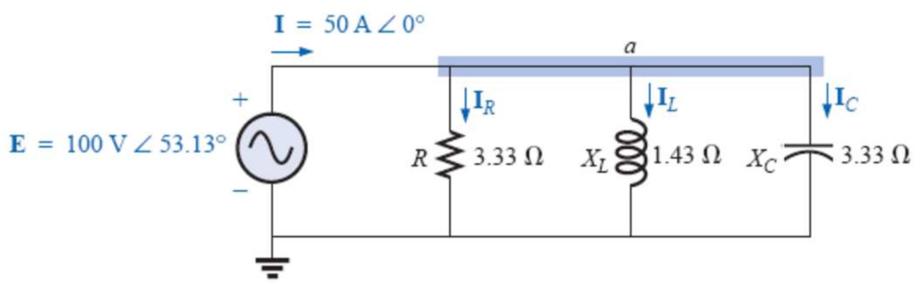
$$F_p = \cos\theta = \frac{R}{Z_T} = \frac{3\ \Omega}{5\ \Omega} = \mathbf{0.6\ lagging}$$

Leading means ***I*** leads ***E: for Capacitive circuit= leading power factor***

Lagging means ***I*** Lags ***E: for inductive circuit= lagging power factor***

Calculate total impedance, currents and draw phasor digram





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$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.5 \text{ S} \angle -53.13°} = \mathbf{2\ \Omega \angle 53.13°}$$

I

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T} = \mathbf{E}\mathbf{Y}_T = (100 \text{ V} \angle 53.13°)(0.5 \text{ S} \angle -53.13°) = \mathbf{50\ A \angle 0°}$$

I_R, I_L, and I_C

$$\begin{aligned}\mathbf{I}_R &= (E \angle \theta)(G \angle 0°)\\ &= (100 \text{ V} \angle 53.13°)(0.3 \text{ S} \angle 0°) = \mathbf{30\ A \angle 53.13°}\\ \mathbf{I}_L &= (E \angle \theta)(B_L \angle -90°)\\ &= (100 \text{ V} \angle 53.13°)(0.7 \text{ S} \angle -90°) = \mathbf{70\ A \angle -36.87°}\\ \mathbf{I}_C &= (E \angle \theta)(B_C \angle 90°)\\ &= (100 \text{ V} \angle 53.13°)(0.3 \text{ S} \angle +90°) = \mathbf{30\ A \angle 143.13°}\end{aligned}$$

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Kirchhoff's current law: At node a,

$$\mathbf{I} - \mathbf{I}_R - \mathbf{I}_L - \mathbf{I}_C = 0$$

$$\mathbf{I} = \mathbf{I}_R + \mathbf{I}_L + \mathbf{I}_C$$

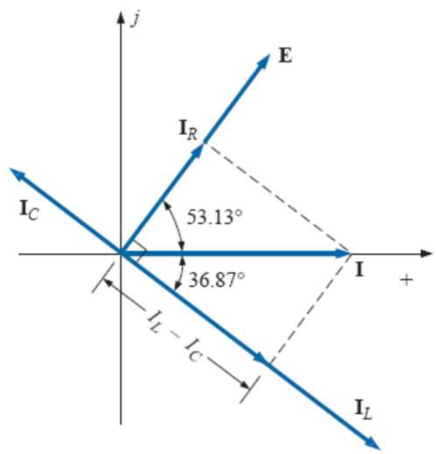

FIG. 15.74
Phasor diagram for the parallel R-L-C

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