PHY 102 Introduction to Physics II

Spring Semester 2025

Lecture 36

Superposition of two sinusoidal waves

$$x_1(t) = a_1 \cos(\omega t + \theta_1)$$
$$x_2(t) = a_2 \cos(\omega t + \theta_2)$$

The two displacements of disturbances have same frequency but different amplitudes and different *initial phases*

$$x(t) = x_1(t) + x_2(t)$$ (Resultant displacement)

$$= \cos\ \omega t(a_1\ cos\theta_1 + a_2 \cos\theta_2) - \sin \omega t(a_1\ sin\theta_1 + a_2 sin\theta_2)$$

$$x(t) = \mathrm{a} \cos(\omega \mathrm{t} + \theta)$$

Resultant disturbance is also simple harmonic with different amplitudes and initial phases

$$\tan\theta = \frac{a_1 \sin\theta_1 + a_2 \sin\theta_2}{a_1 \cos\theta_1 + a_2 \cos\theta_2}$$

$$a\cos\theta = a_1 cos\theta_1 + a_2 cos\theta_2$$
$$a\ sin\theta = a_1 sin\theta_1 + a_2 sin\theta_2$$

$$a = [a_1^2 + a_2^2 + 2a_1a_2 \cos(\theta_1 - \theta_2)]^{1/2}$$

Superposition of two sinusoidal waves

$$\theta_1 \sim \theta_2 = 0, 2\pi, 4\pi, \dots \dots .\,.$$

$$a = a_1 + a_2 \quad \text{(Constructive interference)}$$

$$\theta_1 \sim \theta_2 = \pi, 3\pi, 5\pi \dots \dots .\,.$$

$$a = a_1 - a_2 \quad \text{(Destructive interference)}$$

In general, if we have n displacements

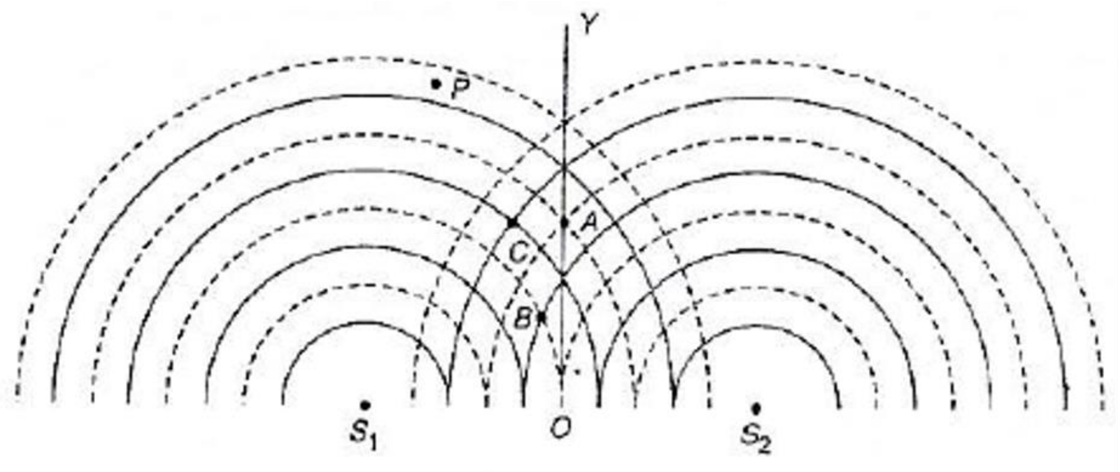
$$\left.\begin{aligned} x_1 &= a_1 \cos(\omega t + \theta_1) \\ x_2 &= a_2 \cos(\omega t + \theta_2) \\ &\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \\ x_n &= a_n \cos(\omega t + \theta_n) \end{aligned}\right\}$$

$$x = x_1 + x_2 + \cdots \ldots . + x_n = \mathrm{a} \cos(\omega t + \theta)$$

$$a \cos\theta = a_1 cos\theta_1 + \cdots \ldots . a_n cos\theta_n$$

$$a\ sin\theta = a_1 sin\theta_1 + \cdots \ldots . a_n sin\theta_n$$

Superposition of two sinusoidal waves



Consider a point B such that $S_2B - S_1B = {}^{\lambda}/_{2}$. At such a point, disturbance reaching form S_1 will always be <u>*out of phase*</u> with disturbance from S_2.

$y = y_1 + y_2 = a\,cos(\omega t) + a\,cos(\omega t - \pi) = 0$. This is called destructive interference

In a similar manner, we may consider a point C such that $S_2C - S_1C = \lambda$, the phase of vibration are exactly the same as the point A (constructive interference)

In general, if a point P is such that

$$S_2P - S_1P = n\lambda \qquad \text{(intensity maxima)}$$

then the disturbances reaching the point P from two sources will be *in phase*.

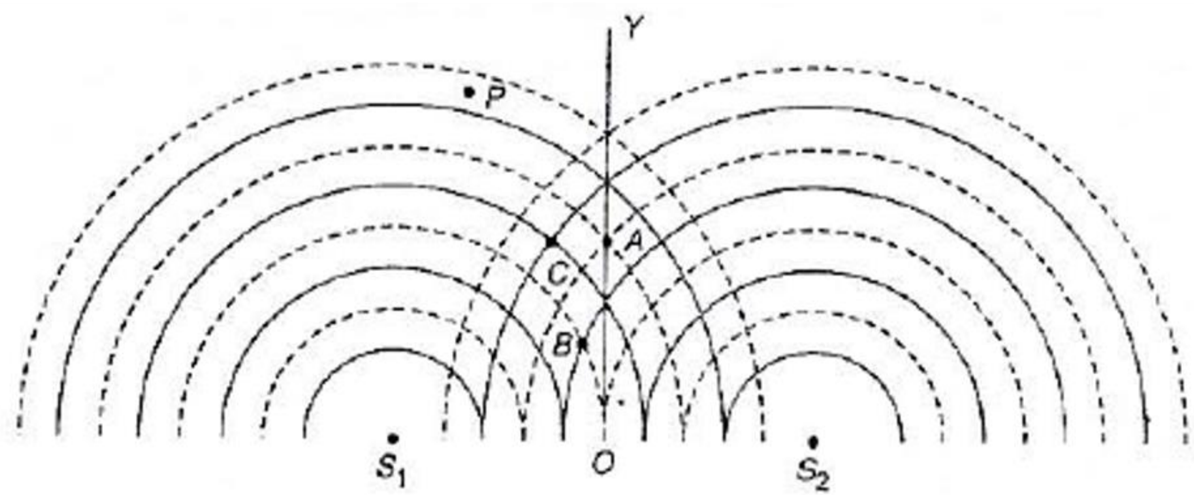
$$n = 0,1,2,3 \ldots \ldots \ldots ..$$

But if the point P is such that

$$S_2P - S_1P = (n + \frac{1}{2})\lambda \qquad \text{(intensity minima)}$$

then the disturbances reaching the point P from two sources will be *out-of- phase*- the interference would be destructive and intensity would be minimum.

Whenever two needles vibrate with a constant phase difference, a stationary interference pattern is produced- *positions of maxima and minima will depend on the phase difference of vibration* . Two sources which vibrate at a *constant phase difference* are said to be coherent.



Whenever two needles vibrate with a constant phase difference, a stationary interference pattern is produced- *positions of maxima and minima will depend on the phase difference of vibration* . Two sources which vibrate at a *constant phase difference* are said to be coherent.

If the phase difference varies with great rapidity, interference pattern would change rapidly, and no stationary interference would occur

Let the displacement produced by the sources at S_1 and S_2 be

$$y_1 = a\, cos\omega t \qquad y_2 = a\cos(\omega t + \varphi)$$

$$y = y_1 + y_2 = 2a\cos\left(\frac{\varphi}{2}\right) cos\left(\omega t + \frac{\varphi}{2}\right)$$

Intensity is proportional to square of amplitude

$$I = 4I_0 cos^2\frac{\varphi}{2}$$

[I_0 is the intensity produced by each of the sources individually]

For coherent sources

$$I = 4I_0 cos^2 \frac{\varphi}{2}$$ (Stationary interference occurs => Oscillatory intensity)

$\varphi = \pm\pi, \pm3\pi, \pm5\pi, \dots$ Intensity is zero (minima)

$\varphi = 0, \pm2\pi, \pm4\pi, \dots$ Intensity is $4I_0$ (maxima)

Incoherent sources

If the phase difference between S1 and S2 is changing with time

$$I = 4I_0 \left\langle cos^2 \frac{\varphi}{2} \right\rangle = 2I_0$$ => Uniform Intensity

(No stationary interference occurs)

Time-averaged intensity

$$\langle f(t) \rangle = \frac{1}{\tau} \int_{-\tau/2}^{+\tau/2} f(t)\, dt.$$

The interval τ over which the averaging is done will depend on the time-resolving power of the observer/instrument.

For example, if the interference pattern is viewed by a normal eye, this averaging will be over about $\tau = 0.1$ s; for a camera with exposure time 0.001 s, $\tau = 0.001$ s.

Clearly, if ϕ varies in a random manner in time scale which is small compared to τ, then $\cos^2(\phi/2)$ will randomly vary between 0 and 1, and what the observer will interpret will be the average of this, $\langle \cos^2(\phi/2) \rangle = 1/2$.

Therefore, for such a case

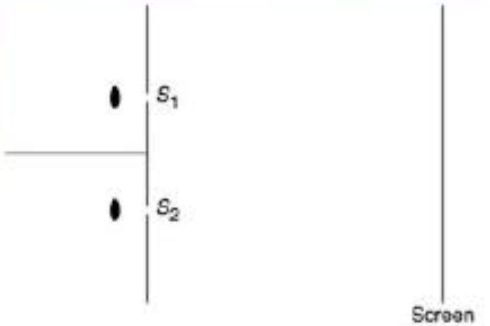
$$\langle I \rangle = 2I_0.$$

This signifies that if the sources are incoherent, then the resultant (average) intensity is the sum of two intensities. The observer will constantly observe an intensity of $2I_0$.

Interference of light waves

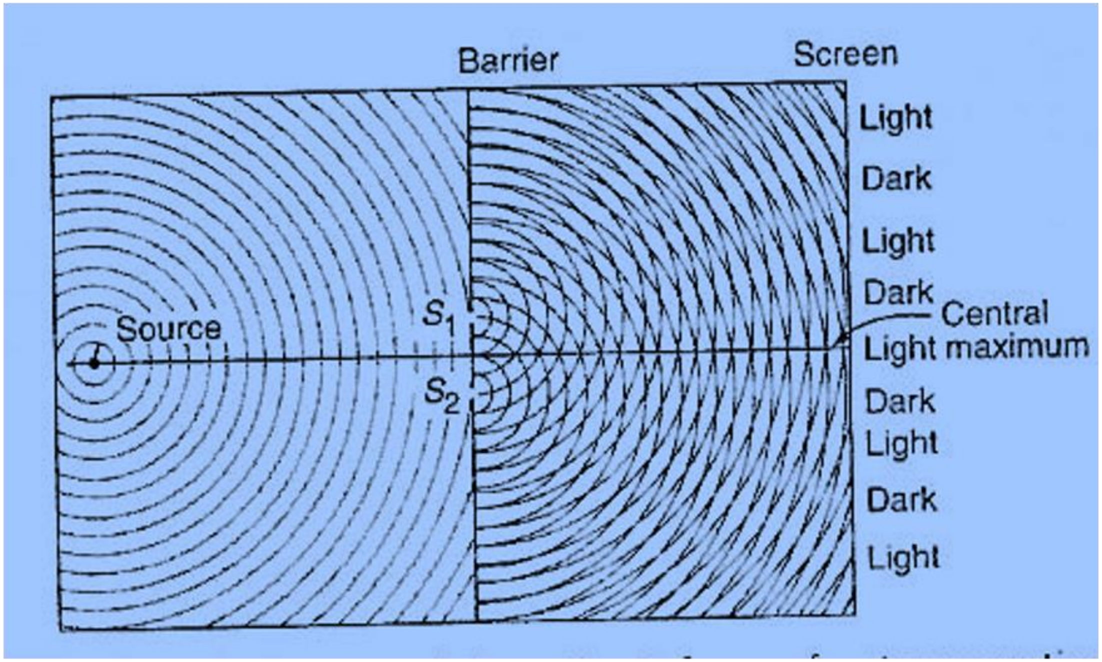
We will now discuss the interference produced by light waves. However, for light it is difficult to observe a stationary interference pattern. If we use conventional light sources (such as two sodium lamps) illuminating two pin holes, we will not observe any interference pattern on the screen. This can be understood from the following reasoning:

In a conventional light source, light comes from a large number of independent atoms, each atom emitting light for about 10^{-10} s, i.e., light emitted by an atom is essentially a pulse lasting for only 10^{-10} s. Consequently, light coming out from holes S1 and S2 will have a fixed phase relationship for about 10^{-10} s, hence the interference pattern will keep on changing every billionth of a second. The eye can notice intensity changes which last at least for 0.1s, and hence we will observe a uniform intensity over the screen.



If two sodium lamps illuminate two pinholes S_1 and S_2, no interference pattern will be observed on the screen.

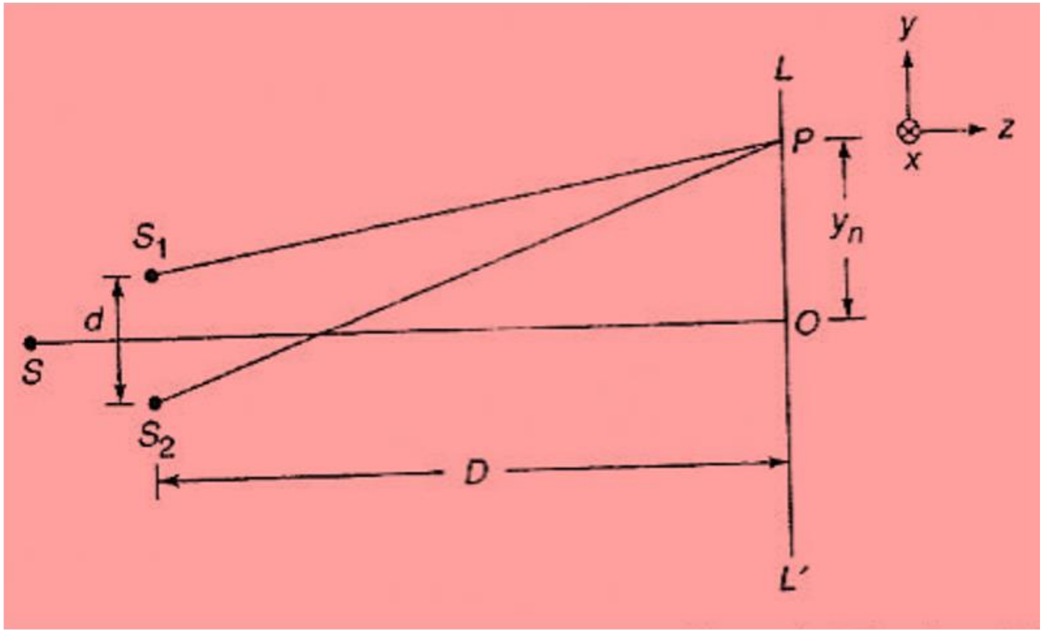
Thomas Young in 1801 devised an ingenious but simple method to lock the phase relationship between two sources- the trick is "division of a single wavefront into two": these two split wavefronts act as if they emanated from two sources having a fixed phase relationship. Therefore, when these two waves were allowed to interfere, it produced stationary interference pattern.



(Spherical waves emanating from S1 and S2 were coherent)

Interference pattern of Young's double slit experiment

The aim is to determine the positions of maxima and minima on the line LL' which is parallel to y-axis and lies in plane containing S, S1 and S2.



$$S_2P - S_1P = n\lambda;\ n = 0, 1, 2, \ldots$$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y_n + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y_n - \frac{d}{2}\right)^2\right] = 2y_n d$$

$$S_2P - S_1P = \frac{2y_n d}{S_2P + S_1P}$$

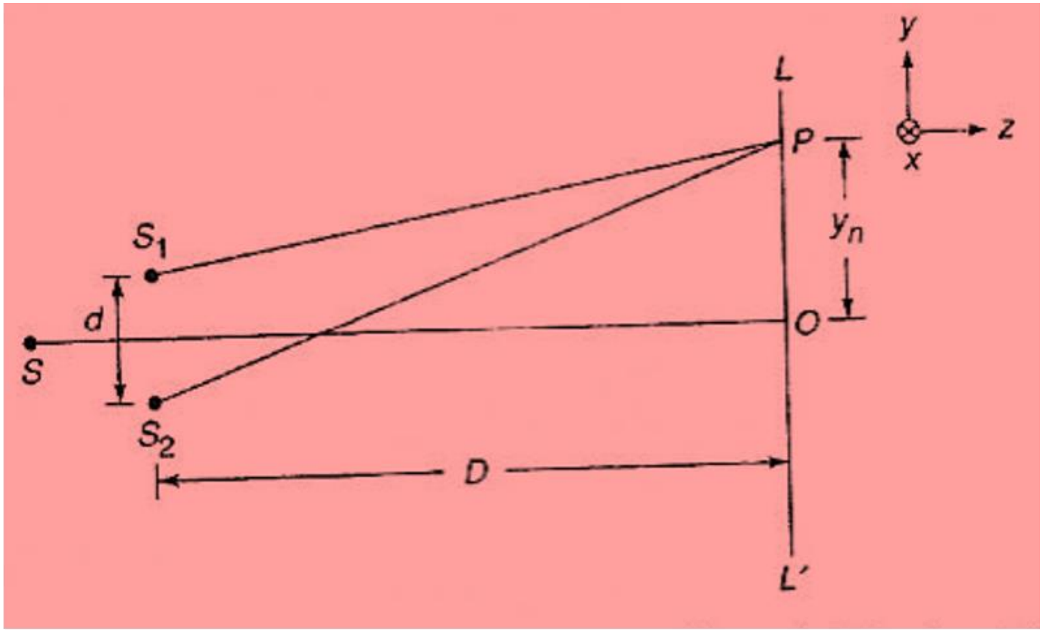
$$S_2P - S_1P \approx \frac{y_n d}{D}$$

$y_n, d \ll D \qquad S_1P + S_2P \sim 2D$

Interference pattern of Young's double slit experiment

$$y_n = \frac{n\lambda D}{d}$$

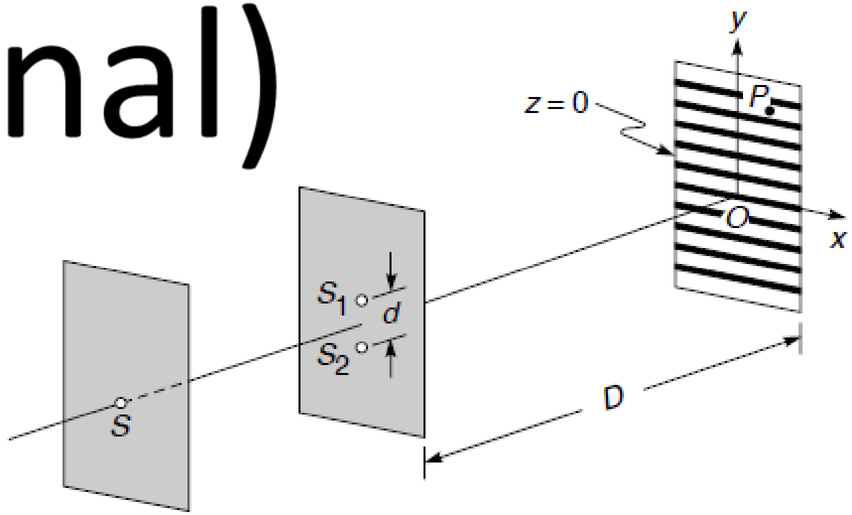
Dark and bright fringes are equally spaced; distance between consecutive dark or bright fringes is



$$\beta = y_{n+1} - y_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\beta = \frac{\lambda D}{d}$$

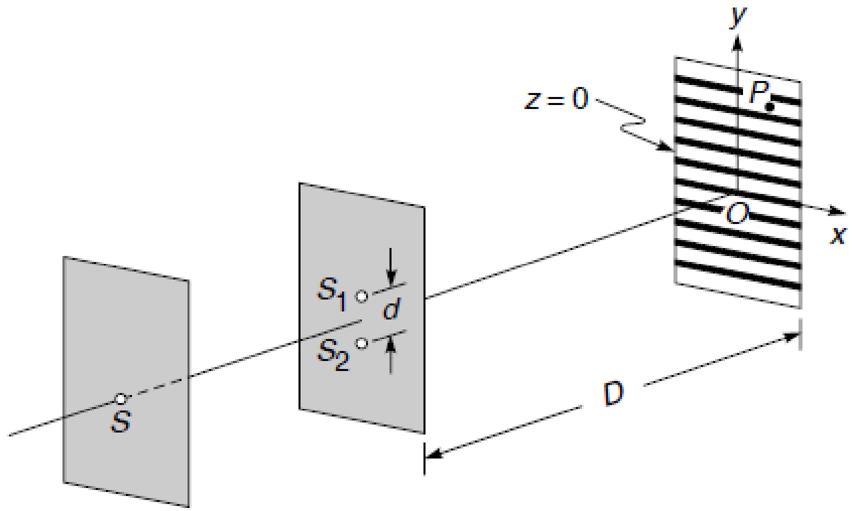
(Optional)



$$S_2P - S_1P = \left[x^2 + \left(y + \frac{d}{2}\right)^2 + D^2\right]^{1/2}$$

$$- \left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]^{1/2}$$

$$= \Delta \qquad \text{(say)}$$



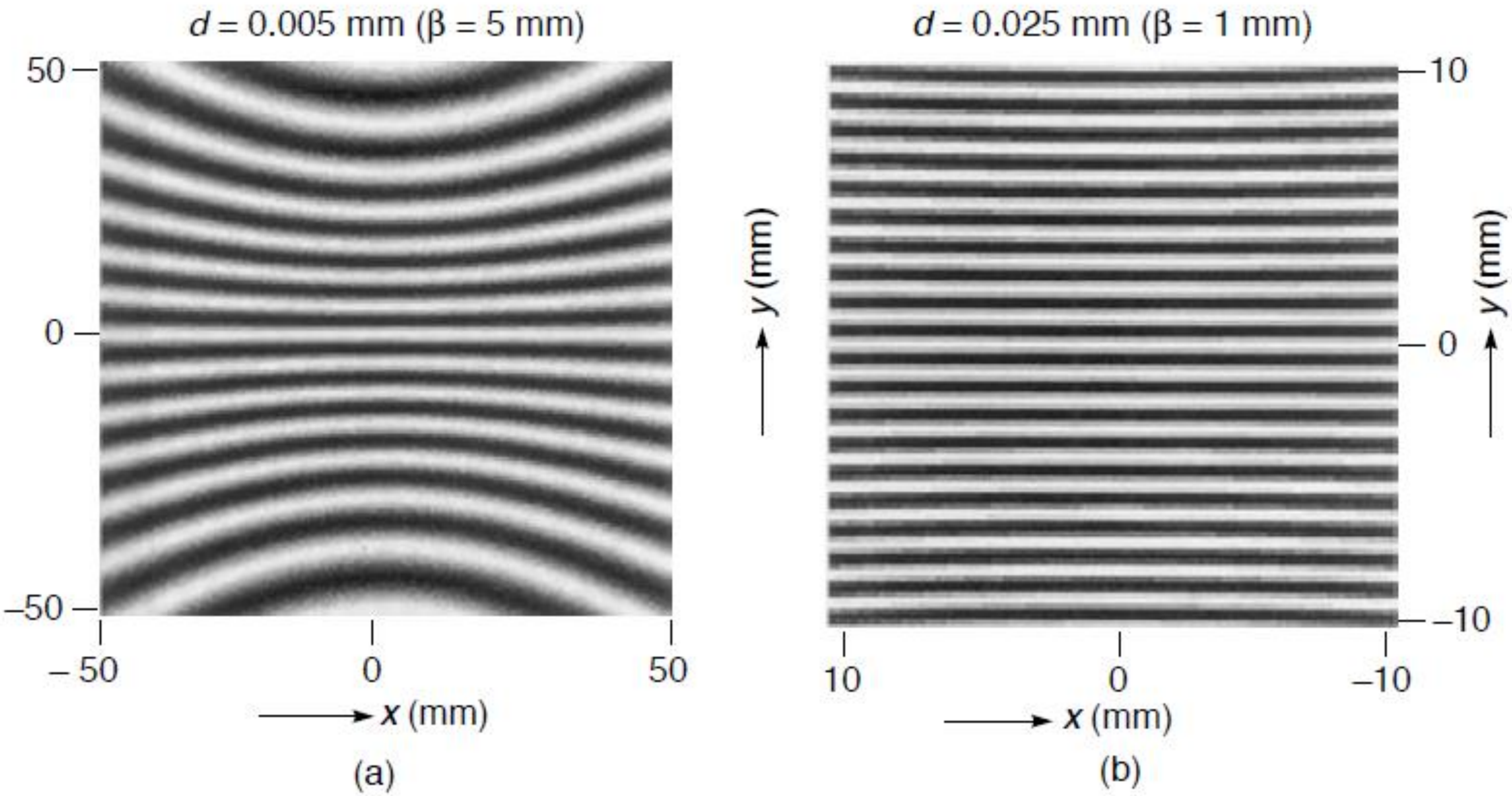
$$\left[x^2 + \left(y + \frac{d}{2}\right)^2 + D^2\right]$$

$$= \left\{\Delta + \left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]^{1/2}\right\}^2$$

or $$(2yd - \Delta^2)^2 = (2\Delta)^2\left[x^2 + \left(y - \frac{d}{2}\right)^2 + D^2\right]$$

$$(d^2 - \Delta^2)y^2 - \Delta^2 x^2 = \Delta^2\left[D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]$$

$$y = \pm\left(\frac{\Delta^2}{d^2 - \Delta^2}\right)^{1/2}\left[x^2 + D^2 + \frac{1}{4}(d^2 - \Delta^2)\right]^{1/2}$$



Computer-generated fringe pattern produced by two point sources S_1 and S_2 on the screen LL' (see Fig. 14.8); (a) and (b) correspond to $d = 0.005$ and 0.025 mm, respectively (both figures correspond to $D = 5$ cm and $\lambda = 5 \times 10^{-5}$ cm).