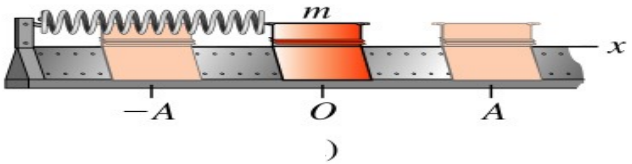
PHY101: Introduction to Physics I

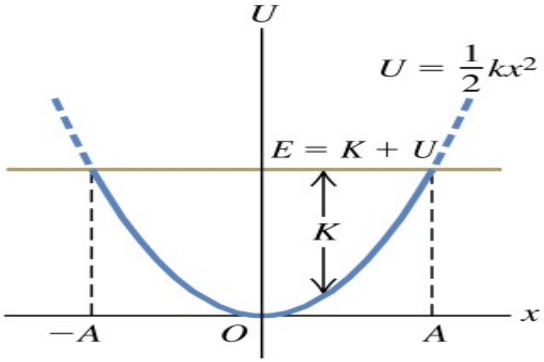
Monsoon Semester 2024
Lecture 16

Department of Physics, School of Natural Sciences,
Shiv Nadar Institution of Eminence, Delhi NCR

Previous Lecture

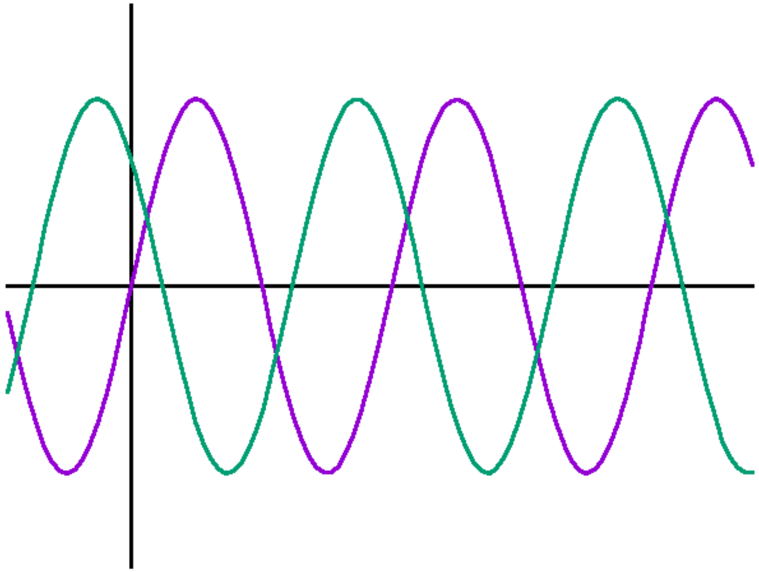


Potential energy
Energy diagram



This Lecture

Oscillation



What is oscillation?

⇒ Displacement from position of stable equilibrium that occurs periodically as a function of time.

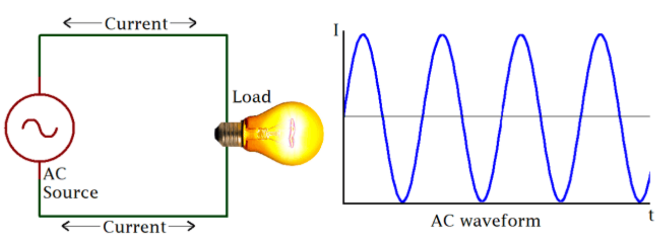
⇒ Occurs when system is disturbed from the position of stable equilibrium.

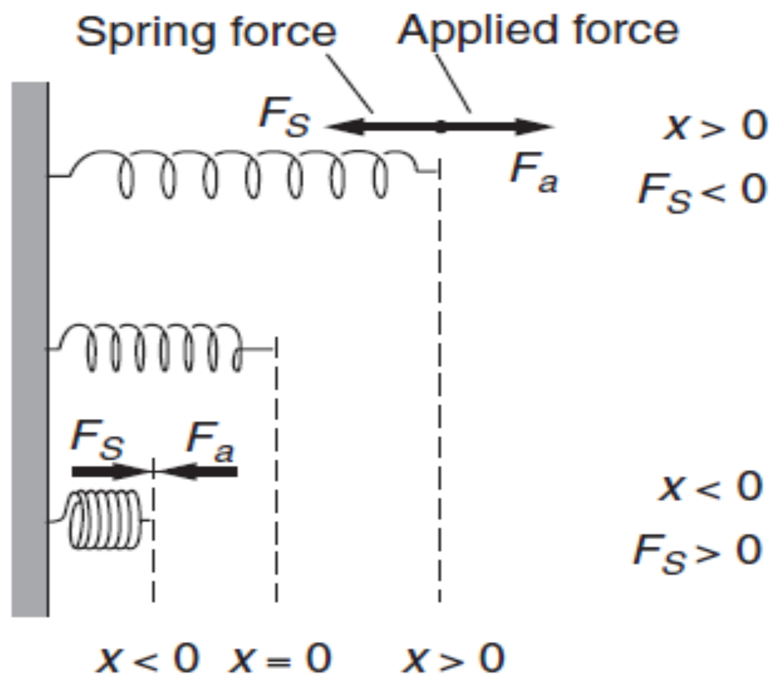
⇒ Examples:

(a) Mass on spring

(b) A swinging pendulum

(c) An AC electric circuit





Why do we study oscillation?

⇒ A common phenomenon in everyday life

(a) Motion of clock pendulum

(b) Vibration of string on musical instrument

(c) Car suspension

(d) Suspension of a bridge (Solder marching on a bridge!!)

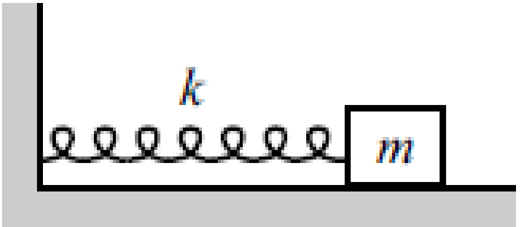
1831: Broughton Bridge, England broke while a brigade of soldiers was marching across the bridge.

1850: Angers bridge, France broke while a brigade of soldiers was marching across the bridge. 200 people died.

1940: Tacoma Narrows Bridge (Washington) broke because of wind (64 km/h).

Simple harmonic motion (SHM)

SHM is an oscillatory motion a system undergoes when subjected to a restoring force, which is directly proportional to the system's displacement from the equilibrium position and acts in the direction opposite to the displacement.



A body that undergoes simple harmonic motion is called a **harmonic oscillator**.

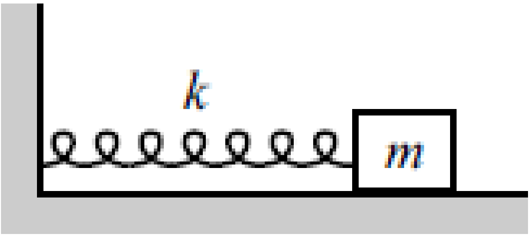
It is called "harmonic" because its oscillation generates a "pure" sinusoidal tone at the frequency ω . Together with its overtones ($n\omega$) it forms a superposition of tones that are felt by human ears as harmony.

If the amplitude of oscillation is small enough, we can use SHM as an approximate model for many different periodic motions, such as

- vibration of a tuning fork
- electric current in an alternating-current circuit
- oscillations of atoms in molecules and solids

Equation of motion

Force on a spring,



$$F = -kx$$

$$\Rightarrow \ddot{x} + \omega^2 x = 0, \qquad \omega = \sqrt{\frac{k}{m}}$$

Hook's law holds good for small stress or compression.

This a homogeneous (RHS = 0), linear (power of $\ddot{x}$ or x is 1) differential equation of 2nd order type (double derivative of x).

Trial solution, $\boldsymbol{x = Ae^{\alpha t}}$

Where A and α are constants to be determined from boundary conditions.

$$\Rightarrow x(t) = Ae^{i\omega t} + Be^{-i\omega t} \qquad (1)$$

General solution is the arbitrary linear combination of the solutions.

(Verify if this is really a solution by substituting (1) in the equation of motion.)

Different form of the solution

Can we write the solution in the form?

$$x(t) = C\cos\omega t + D\sin\omega t \qquad (2)$$

Use $\quad e^{i\theta} = \cos\theta + \sin\theta \quad$ in (1)

Another form of the solution is:

$$x(t) = P\cos(\omega t + \varphi_1) \qquad (3)$$

Can you obtain (3) from (2) ?

Substitute $C = P\cos\varphi_1, \quad D = P\sin\varphi_1$.

Then $P = \sqrt{C^2 + D^2}$, $\tan\varphi_1 = \frac{D}{C}$

Substitute $D = P\cos\varphi_2, \quad C = P\sin\varphi_2$.

where $P = \sqrt{C^2 + D^2}$, $\tan\varphi_2 = \frac{C}{D}$ to obtain

$$x(t) = P\sin(\omega t + \varphi_2) \qquad (4)$$

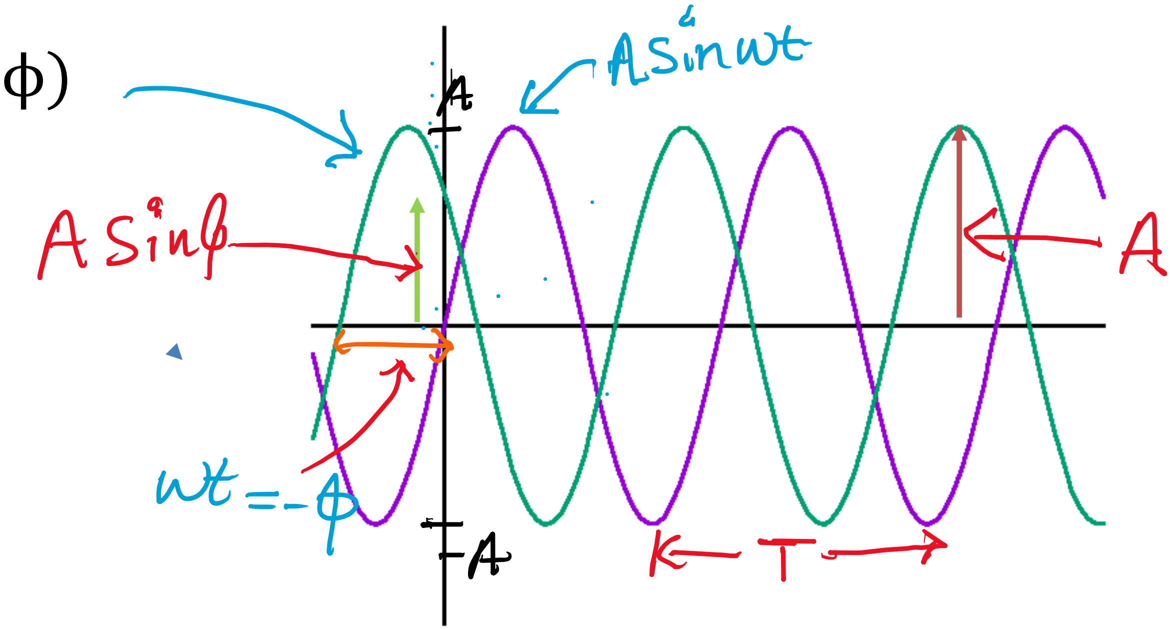
Forms (3) and (4) are most popular.

Depending on the specific system and situation, the forms can be chosen for convenient mathematical calculations.

=> Physically, all forms will describe exactly the same motion as the constants taken in equations will change accordingly.

The parameters in the equation

$$x = A\sin(\omega t + \phi)$$



1. Angular velocity:

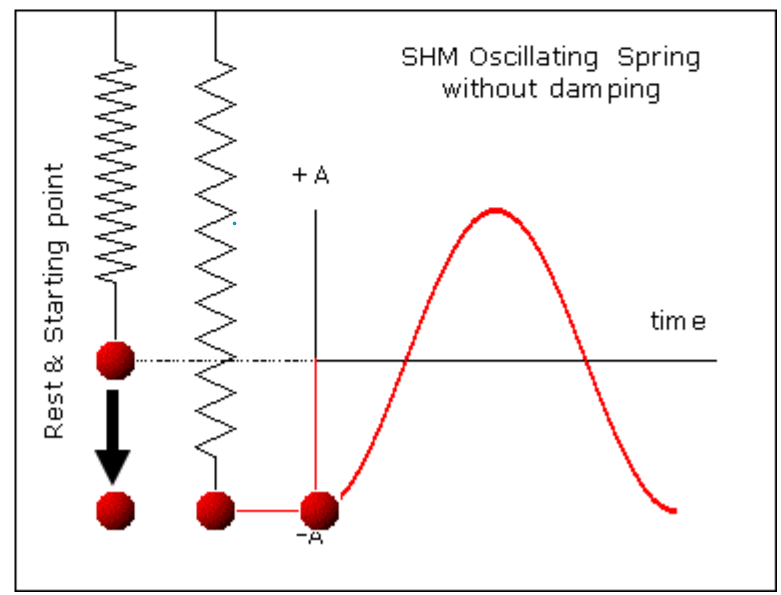
$$\omega = \sqrt{\frac{k}{m}}$$

2. Phase = φ

3. Period, $T = \frac{2\pi}{\omega}$, after T, the position and velocity are back to initial condition

4. Frequency : $\nu = \frac{1}{T}$

5. Amplitude (A): Maximum displacement from equilibrium position



Displacement

$$x = A\omega\cos(\omega t + \varphi)$$

Velocity

$$v = \frac{dx}{dt} = -A\omega\sin(\omega t + \varphi)$$

Acceleration

$$\mathrm{a} = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \varphi)$$

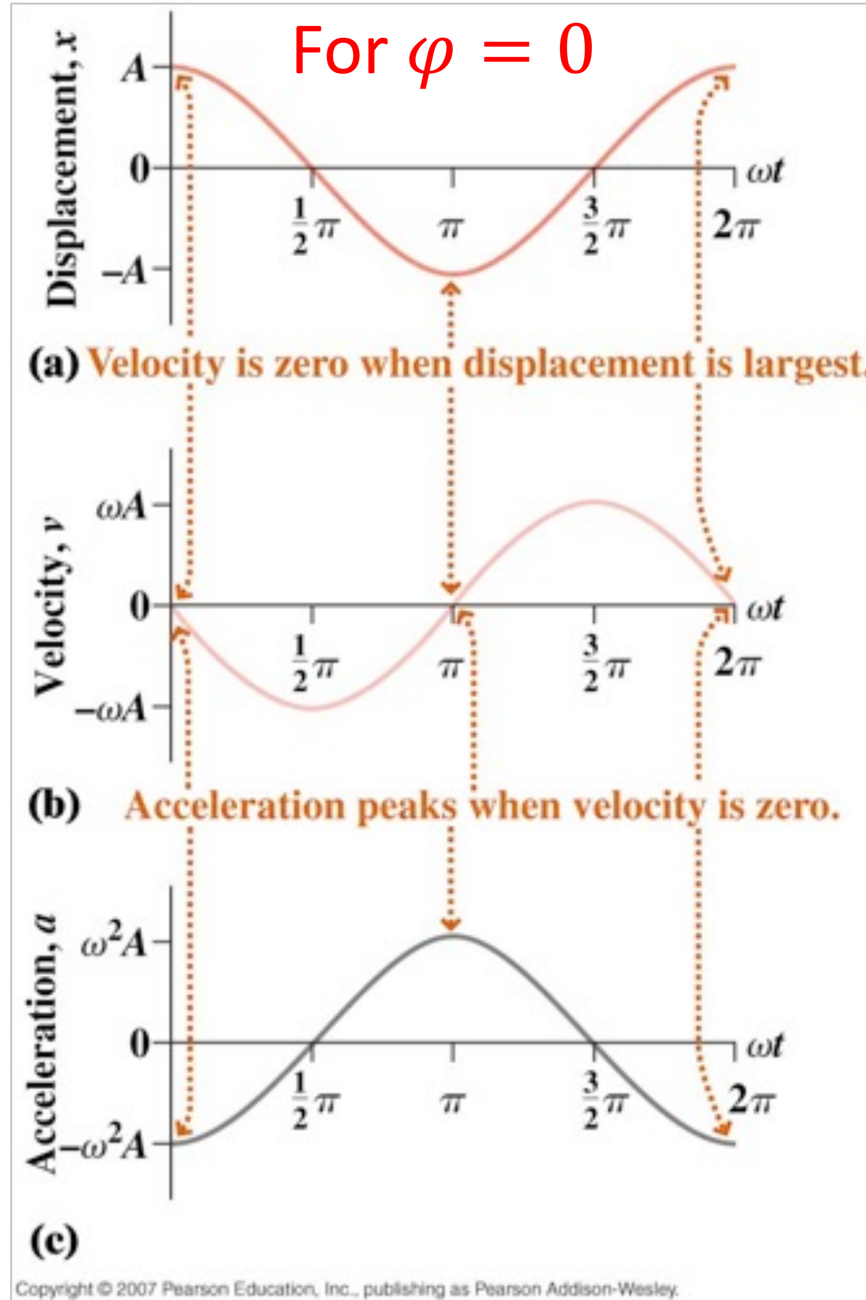
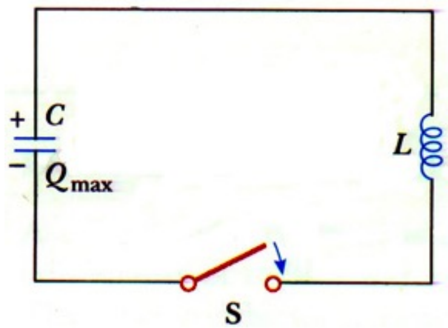


Image source: https://www.physics.louisville.edu/cldavis/phys298/notes/shm_files/image019.png

Oscillations in an LC Circuit



no resistance

Very similar to a mass-and-spring simple harmonic oscillator with **no fricition**

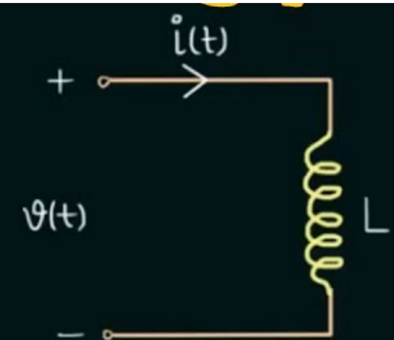
Energy stored in a capacitor

$$\boxed{U = \frac{1}{2} C V^2} \qquad U = \frac{1}{2} Q V$$

$$Q = C V \qquad P = W / t$$

$$\boxed{U = \frac{1}{2} Q^2 / C}$$

Energy stored in an inductor



$$J = \sigma E \qquad \longrightarrow P_d$$

$$V = IR \qquad \longrightarrow E$$

$$I = GV$$

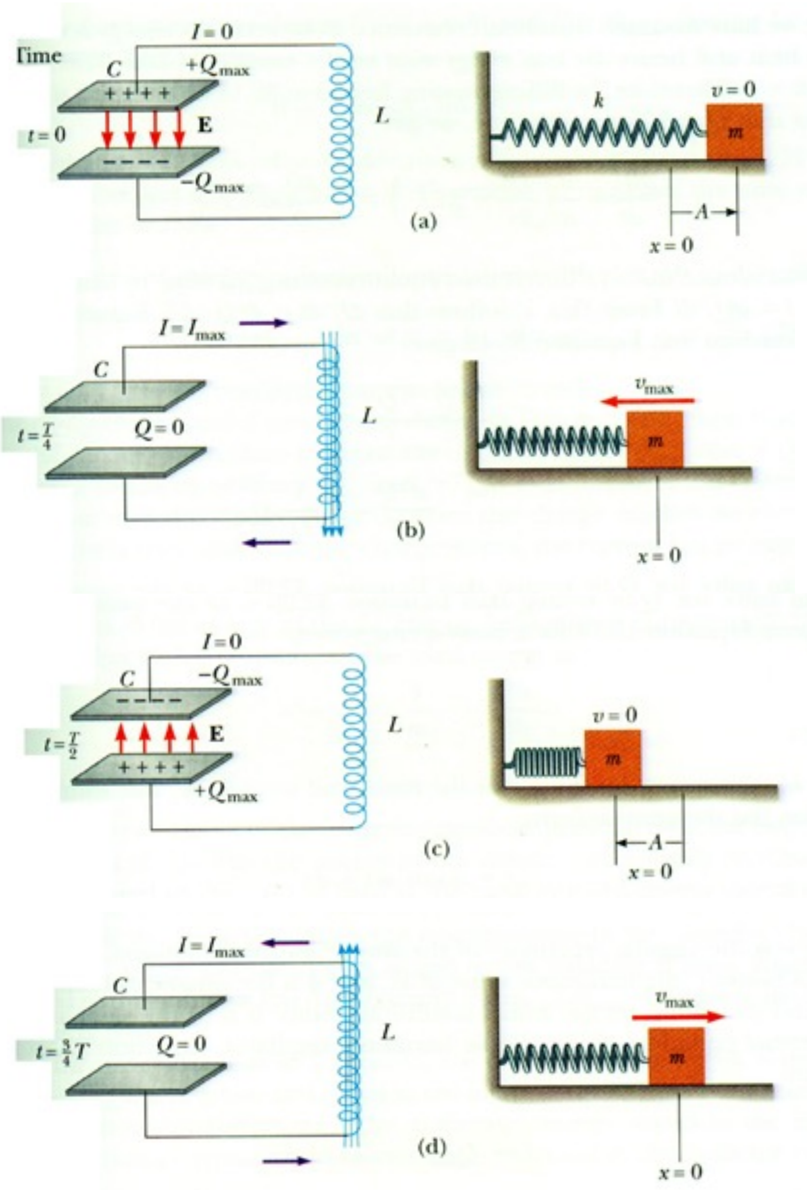
$$V = R \frac{dq}{dt}$$

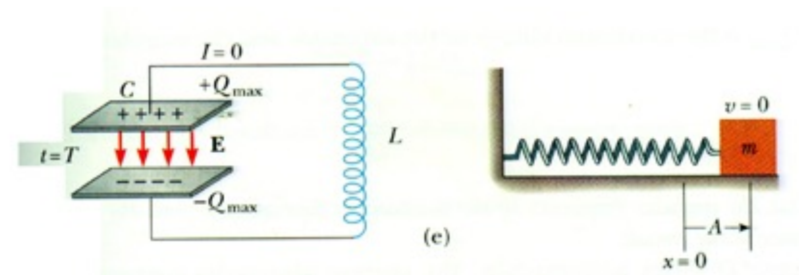
$$\vartheta(t) \propto \frac{di(t)}{dt} \longrightarrow \vartheta \propto \frac{di}{dt} \qquad \boxed{E = \frac{1}{2} L i^2}$$

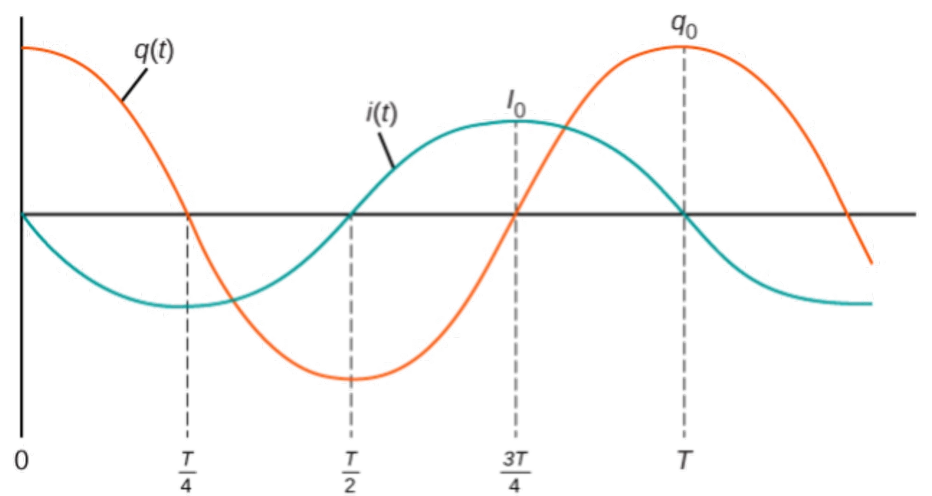
$$\vartheta(t) = L \frac{di(t)}{dt} \longrightarrow$$ 5th form of ohm's law

Oscillations in an LC Circuit

Very similar to a mass-and-spring simple harmonic oscillator with **no fricition**







(e)

angular frequency of the oscillations $\omega = \sqrt{\dfrac{L}{C}}$

"natural frequency" or the "resonant frequency"