Spanning Trees

A connected undirected graph can have n^{n-2} (max.) no. of spanning trees where 'n' is the no. of nodes.

General properties :-

→ All spanning trees have same no. of edges and vertices.
→ It does not have any cycle (loops)

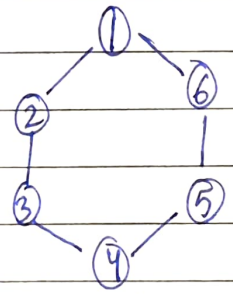
Minm Spanning Trees

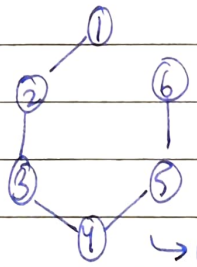
- Connected undirected graph with a weight (or cost) associated with each edge.
- Cost of spanning tree would be sum of costs of its edges
- Minm spanning tree - ST that has lowest cost.
- No. of edges = V-1 (where V is no. of vertices)

Greedy Algos to find MST:-

1) Prim's Algorithm
2) Kruskal's Algorithm

} both work with connected, weighted and undirected graphs.





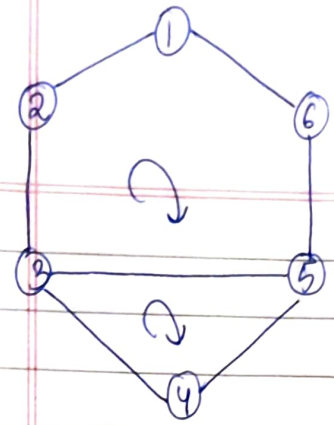
→ can be multiple

$S \subseteq G$
$V' = V$ $\quad |E'| = |V| - 1$

$G = (V, E)$
$V = \{1, 2, 3, 4, 5, 6\}$
$E = \{(1,2), (2,3) \dots\}$

$|V| = n = 6$
$|V| - 1 = 5 = |E'|$

No. of spanning trees = ${}^{6}C_{5} = 6$
(6 → total no. of edges; 5 → edges we want)



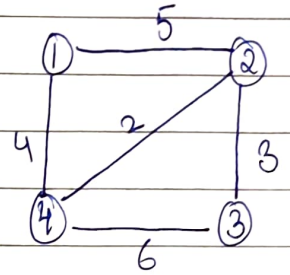
no. of spanning trees :-

$^{7}C_{5} - 2$

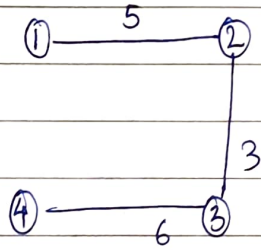
It is forming 2 cycles of no. of vertices < 6

$^{|E|}C_{|V|-1}$ - no. of cycles

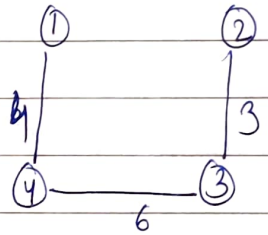
Weighted graph :



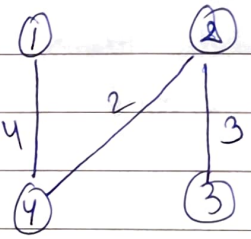
$|V| - 1 = 3$



cost = 14



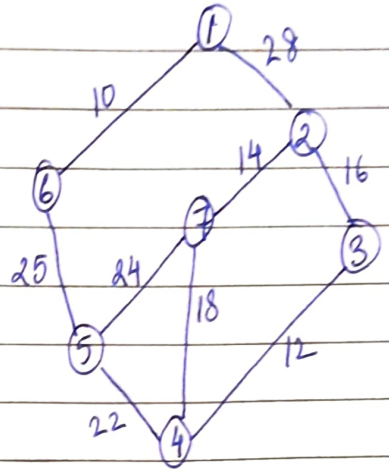
cost = 13

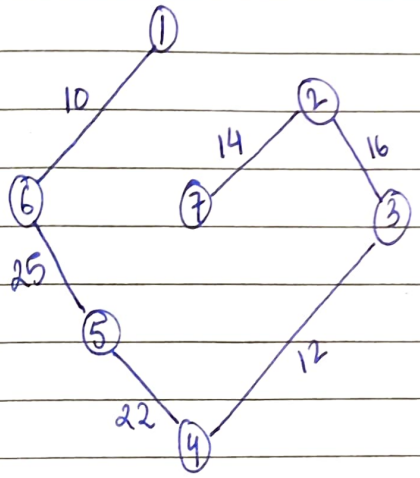


cost = 9

Prim's Algorithm

* Firstly, select the min^m cost edge
* After that, select the min^m cost edge but make sure it is connected to the already selected vertices





APPROACH:

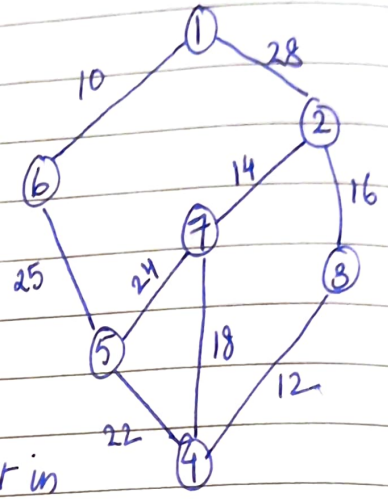
→ Always select the smallest

→ Then, select the connected smallest one

Cost = 10+25+22+12+16+14 = 99

NOTE: We cannot find spanning-trees for non-connected graphs

Kruskal's Algorithm

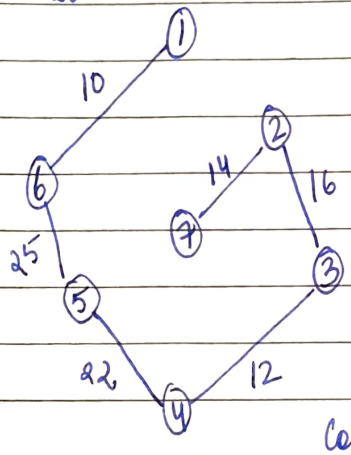


* Always select the min^m cost edge
  - 1-6 (10)
  - 3-4 (12)
  - 2-7 (14)
  - 2-3 (16)

Next smallest is 7-4 (18), but...

* If it forms a cycle, don't include it in the solution.

Then...
- 5-4 (22)
- 5-7 (24) Rejected
- 5-6 (25)



Cost = 99

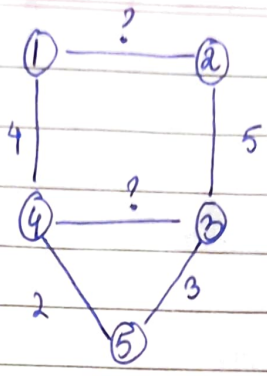
$S.C. = O(E+V)$

$T.C = O(E \log E + E)$

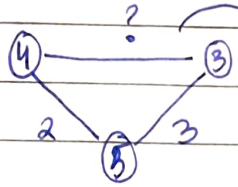
↓

Time to sort edges in Non-dec. order

Q: Missing edges



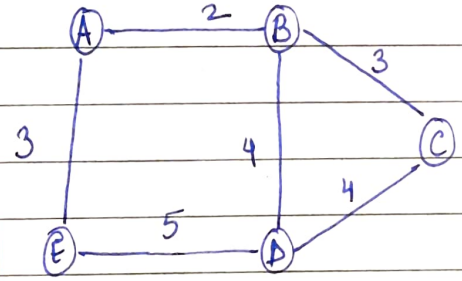
what is the min^{m} possible weight of these edges?



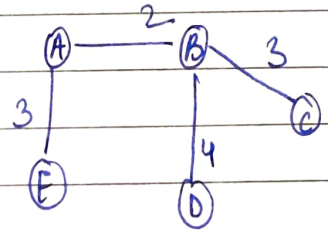
~~2 and~~ 2 and 3 are smaller than this so it could be 4

Top one could be 6.

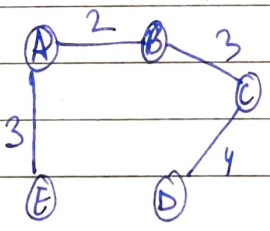
Q: Find MST



No. of MSTs.

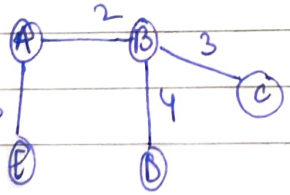


OR



Kruskal's

Prim's



OR