

Applied Linear Algebra (MAT161)

Mid Term Exam 2025

SET - BDate: 3rd March, 2025.

Total Marks: 60

Total Time: 1.5 hrs

Do all the **seven** questions in serial order.**Note:** Use only the elementary row operations to find the inverse of a matrix or to solve any linear system of equations.Q1) Find all the vectors that are orthogonal to the vector $v = (2, 7)$. (5 marks)Q2) Use elementary matrices to find the inverse of the matrix A , where

$$A = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & c \end{bmatrix}, c \neq 0. \quad (10 \text{ marks})$$

Q3) Solve the given linear system by finding the LU -factorization of the coefficient matrix of the system: (10 marks)

$$\begin{aligned} x + y &= 3 \\ x - y &= -1 \end{aligned}$$

Q4) Find (i) a basis for and (ii) the dimension of the subspace W of \mathbb{R}^4
 $W = \{(s + 4t, t, s, 2s - t) : s \text{ and } t \text{ are real numbers}\}$. (8 + 2 = 10 marks)Q5) Find x such that the matrix A is equal to its own inverse, where $A = \begin{bmatrix} 2 & x \\ -1 & -2 \end{bmatrix}$. (10 m)Q6) Test whether the set $\{\sin x, \cos x\}$ of solutions for $y'' + y = 0$ is linearly independent? (8 marks)Q7) Let the column vectors of the matrix be denoted by a_1, a_2, a_3, a_4 , and a_5 (7 marks)

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & -1 & -3 & 1 & 3 \\ -2 & -1 & 1 & -1 & 3 \\ 0 & 3 & 9 & 0 & -12 \end{bmatrix}$$

(a) Find the rank and nullity of A (2 marks)(b) Find a subset of column vectors of A that forms a basis for column space of A . (3 marks)(c) Write third column of A as a linear combination of first two columns of A . (2 marks)(Hint: You can use the reduced row-echelon form R of A given below:

$$R = \begin{bmatrix} 1 & 0 & -2 & 0 & 1 \\ 0 & 1 & 3 & 0 & -4 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$