Assignment 4 (for Lecture 4) Solutions

June 29, 2020

A1.

ϕ	ψ	$\phi \implies \psi$	$\psi \implies \phi$	$\boxed{\phi \iff \psi}$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

A2.

ϕ	ψ	$\phi \implies \psi$	$\neg\phi \vee \psi$	$\boxed{(\phi \implies \psi) \iff (\neg\phi \vee \psi)}$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

A3.

ϕ	ψ	$\neg(\phi \implies \psi)$	$\phi \wedge \neg\psi$	$\boxed{\neg(\phi \implies \psi) \iff (\phi \wedge \neg\psi)}$
T	T	F	F	T
T	F	T	T	T
F	T	F	F	T
F	F	F	F	T

A4.

(a)

ϕ	ψ	$\phi \implies \psi$	$\phi \wedge (\phi \implies \psi)$	$\boxed{[\phi \wedge (\phi \implies \psi)] \implies \psi}$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) The truth table obtained demonstrates that *modus ponens* is a valid rule of inference because all truth values are TRUE, and thus, it is a *logical validity* or *tautology*.

A5. Proof that $\neg(\phi \vee \psi)$ is equivalent to $(\neg\phi) \wedge (\neg\psi)$:

1. $\phi \vee \psi$ means that at least one of ϕ, ψ is true.
2. Thus $\neg(\phi \vee \psi)$ means that it is *not* the case that at least one of ϕ, ψ is true.
3. If it is not true that at least one of ϕ, ψ is true, then ϕ and ψ must both be false.

4. This is the same as saying that both $\neg\phi$ and $\neg\psi$ are true. (By the definition of negation).

5. By the meaning of *and*, this can be expressed as $(\neg\phi) \wedge (\neg\psi)$.

A6.

(a) $34,159$ is *not* a prime number.

(b) *Neither* are roses red *nor* violets blue.

(c) There are no hamburgers, and I will *not* have a hot dog.

(d) Fred will *not* go, but he *will* play.

(e) The number x is greater than or equal to 0 and less than or equal to 10.

(f) We will lose the first game and the second.

A7. Proof that $\phi \iff \psi$ is equivalent to $(\neg\phi) \iff (\neg\psi)$:

1. $\phi \iff \psi$ means that both ϕ and ψ are true or both are false.

2. Thus, this is the same as saying $(\phi \wedge \psi) \vee (\neg\phi \wedge \neg\psi)$.

2. Since logical disjunction is commutative, we can write the above as $(\neg\phi \wedge \neg\psi) \vee (\phi \wedge \psi)$.

3. But this is read as both $\neg\phi$ and $\neg\psi$ are true or both are false, or $(\neg\phi) \iff (\neg\psi)$.

A8.

(a)

ϕ	ψ	$\phi \implies \psi$	$\psi \implies \phi$	$\phi \iff \psi$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(b)

ϕ	ψ	θ	$\psi \vee \theta$	$\phi \implies (\psi \vee \theta)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

A9.

ϕ	ψ	θ	$\psi \wedge \theta$	$\phi \Longrightarrow (\psi \wedge \theta)$	$\phi \Longrightarrow \psi$	$\phi \Longrightarrow \theta$	$(\phi \Longrightarrow \psi) \wedge (\phi \Longrightarrow \theta)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

A10. Proof that $\phi \Longrightarrow (\psi \wedge \theta)$ is equivalent to $(\phi \Longrightarrow \psi) \wedge (\phi \Longrightarrow \theta)$:

1. $\phi \Longrightarrow (\psi \wedge \theta)$ means that whenever ϕ is true, $\psi \wedge \theta$ is also true.
2. This is the same as saying that whenever ϕ is true, both ψ, θ are also true.
3. Thus, whenever ϕ is true, $\psi$is also true *and* whenever ϕ is true, ψ is also true.
4. By the meaning of *and*, this can also be expressed as $(\phi \Longrightarrow \psi) \wedge (\phi \Longrightarrow \theta)$.

A11. Proof that $(\phi \Longrightarrow \psi) \Longleftrightarrow (\neg\psi \Longrightarrow \neg\phi)$:

ϕ	ψ	$\neg\psi$	$\neg\phi$	$\phi \Longrightarrow \psi$	$\neg\psi \Longrightarrow \neg\phi$	$(\phi \Longrightarrow \psi) \Longleftrightarrow (\neg\psi \Longrightarrow \neg\phi)$
T	T	F	F	T	T	T
T	F	T	F	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

A12.

(a) If two rectangles *do not* have the same area, they *are not* congruent.

(b) If in a triangle with sides a, b, c (c largest) it is *not* the case that $a^2 + b^2 = c^2$, then it is *not* right-angled.

(c) If n is *not* prime, then $2^n - 1$ is *not* prime.

(d) If the Dollar does *not* fall, then the Yuan does *not* rise.

A13. Proof that $\phi \Longrightarrow \psi$ is *not* equivalent to $\psi \Longrightarrow \phi$:

ϕ	ψ	$\phi \Longrightarrow \psi$	$\psi \Longrightarrow \phi$	$(\phi \Longrightarrow \psi) \Longleftrightarrow (\psi \Longrightarrow \phi)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

A14.

(a) If two rectangles have the same area, they are congruent.

(b) If in a triangle with sides a, b, c (c largest), $a^2+b^2=c^2$, it is right-angled.

(c) If n is prime, 2^n-1 is prime.

(d) If the Dollar falls, the Yuan rises.

OPTIONAL PROBLEMS

A1. $\phi \Longrightarrow \psi$

A2.

ϕ	ψ	$\phi\dot{\vee}\psi$
T	T	F
T	F	T
F	T	T
F	F	F

A3. $(\phi \wedge \neg\psi) \vee (\neg\phi \wedge \psi)$

A4.

(a) If the number of primes is unbounded, there are infinitely many primes.

(b) If it is raining, the grass is wet.

(c) If $a=1$, $b=2$, then $a+b=3$.

(d) Not possible.

A5.

M	N	$M \times N$	$M + N$
1	1	1	0
1	0	0	1
0	1	0	1
0	0	0	0

A6.

M	N	$M \times N$	$M + N$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	F

(a) $M \times N$ corresponds to $M \wedge N$.

(b) $M + N$ corresponds to $M\dot{\vee}N$.

(c) $\begin{array}{cc} M & -M \\ \hline 1 & -1 \equiv 1 \\ 0 & -0 \equiv 0 \end{array}$, and thus $\neg$ does *not* correspond to $-$.

A7.

M	N	$M \times N$	$M + N$
F	F	F	T
F	T	T	F
T	F	T	F
T	T	T	T

(a) $M \times N$ corresponds to $M \vee N$.

(b) $M + N$ corresponds to $(\neg M \wedge \neg N) \vee (M \wedge N)$, which is equivalent to $M \iff N$.

(c) Identical to 6(c), and thus $\neg$ does *not* correspond to $-$.

It's a strange world if we interpreted 0 as T and 1 as F.

A8. You *need* to turn over 2 cards (the E and the 7). The rule can be (simply) equivalently stated like so: There isn't a vowel on one side OR there is an odd number on one side. Thus, if you see a letter and it isn't a vowel, you don't need to verify the rule; likewise, if you see a number and it's odd, you don't need to verify the rule.

A9. Proof that mn is odd $\iff$ m and n are odd:

Suppose that mn is odd. Assume the contrary, i.e. it is not the case that m and n are odd. This must mean that at least one of m, n is even. Suppose m is even (the argument is identical if we choose n to be even instead). Then m can be expressed as $2k$ for some $k \in \{0, 1, 2, \ldots\}$. But then $mn = 2kn$ is even. Thus, our assumption must be wrong, and m and n are odd.

Suppose now that m and n are odd. Then m and n can be expressed as $2k + 1$ and $2l + 1$ respectively for some $k, l \in \{0, 1, 2, \ldots\}$. Thus $mn = (2k+1)(2l+1) = 4kl + 2k + 2l + 1 = 2(2kl + k + l) + 1$, which is odd. $\square$

A10. mn is even $\iff$ m and n are even is false. Take for example $m = 1$ and $n = 2$. mn is even, but m is odd.

A11. You need to check the ID of the person with the beer, and check the drink of the person under the drinking age. The rule can stated as the following: If you have an alcoholic drink, you must be of drinking age. This can be simplified as: You must *not* have an alcoholic drink OR be of drinking age. Thus, you have to check the ID of anyone with an alcoholic drink or check the drink of anyone not of drinking age.

A12. They are very similar in structure, but Q12 was easier because it was more intuitive.