530.612
Midterm

Write a finite element program in Matlab to solve a 2D problem. Your program will read in input files provided for you. A different input file will be used to evaluate your program. The input file specifies the number of nodes, number of elements and number of integration points, coordinates of the nodes, the element connectivity, displacement boundary conditions, the body force, and material properties. Your program must do the following:

1. Calculate the element stiffness matrix and force vector and assemble into the global stiffness matrix and force vectors.
2. Provide an option for 4 node bilinear, 8 node serendipity and 9 node biquadratic elements.
3. Provide an option for plane stress and plane strain.
4. Integrate using Gaussian integration and provide an option for 4 point, 8 point, and 9 point Guassian integration.
5. Calculate nodal stress field using L2 projection scheme.
6. Create output files for nodal coordinates, displacements, and stresses. Also create output files for integration point stresses.
7. Calculate nodal reaction forces and total reaction forces at the fixed boundary.

To turn in the midterm, please email a tar zipped file of all the files needed to run your FEA code as well as instructions on how to run your code. Include the instructions in a README.txt. Also email me a report of the studies requested below.

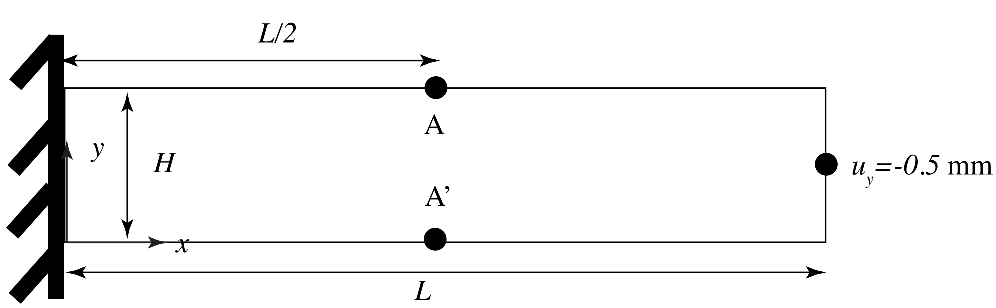

Figure 1: Cantilever beam

Apply the program to solve for the displacements of a cantilever beam bending problem loaded under gravity and a fixed displacement applied at $x = L$ and $y = H/2$ (Fig. 1) for different element sizes h and element types, bilinear vs. biquadratic (8 node and 9 node). The dimensions of the beam are $L = 12$mm $H = 1$mm, $B = 1$mm. You will be analyzing beams of 2 different materials: 1) aluminum (Al) and polyurethane (PU). Write a report that includes the following items.

1. Consider first an aluminum (Al) beam bending under plane stress. For Al, the Young's modulus is $E = 70$ GPa, Poisson's ratio is $\nu = 0.35$, and density is $\rho = 2000$ kg/m^3.

   (a) For the Al beams discretized by a 16x4 (4 node) bilinear mesh, plot the deformed shape and contours of the nodal stresses.

   (b) Calculate the error in the total reaction force compared to the analytical results from Euler-Bernoulli Beam theory. For the analytical solution, approximate the effect of gravity as a uniform distributed force $(q = -\rho g H B)$, where $\rho = 2000 kg/m^3$ and $g = 9.82$m/s^2, $B = 1$ mm, $H = 1$mm. Plot the log error vs the log of a characteristic element size. Compare between the 3 different element types. For the higher order elements, compare the results of under integration, using 4 instead of 9 integration points. You will have to edit the input files to specify the number of integration points. Comment on your results.

   (c) Calculate the error in the nodal displacement and nodal stresses at point A (Fig. 1) compared to the analytical results from Euler-Bernoulli Beam theory. Similarly pick an integration point closest to node A and calculate the error in the integration point stresses. Plot the log error vs the log of a characteristic element size. Compare between the two different element types and the effects of under integration for the higher order elements. Comment on your results.

   (d) For all meshes, plot the vertical displacement, $u_y(x, H/2)$ at the neutral axis and compare to beam theory. Plot the beam bending stress $\sigma_{xx}(y)$ and transverse shear stress $\sigma_{xy}(y)$ of line AA' at $x = L/2$ of the beam. If possible plot the results for the different meshes on the same plot for comparison. Also plot for the cases of under integration Comment on your results.

2. Consider next the PU beam bending under plane strain. The PU is a nearly incompressible material with a Young's modulus of $E = 100$ MPa and density of $\rho = 1500$ kg/m^3.

   (a) For the PU beam discretized by a 16x4 mesh, plot nodal displacement of the neutral axis and the nodal bending stress $\sigma_{xx}(y)$ for the line AA' for Poisson's ratio $\nu = 0.4$, $\nu = 0.49$ and 0.4999. Comment on the results. You will have to edit the input file to change the Poisson's ratio.

   (b) For the case with Poisson's ratio $\nu = 0.4999$, compare the displacement of the neutral axis and the bending stress of line AA' for the 16x4 and 16x8 (4 node) bilinear elements and the 16x4 (9 node) biquadratic element. Comment on the effect of decreasing mesh size and increasing the polynomial order of the shape function.