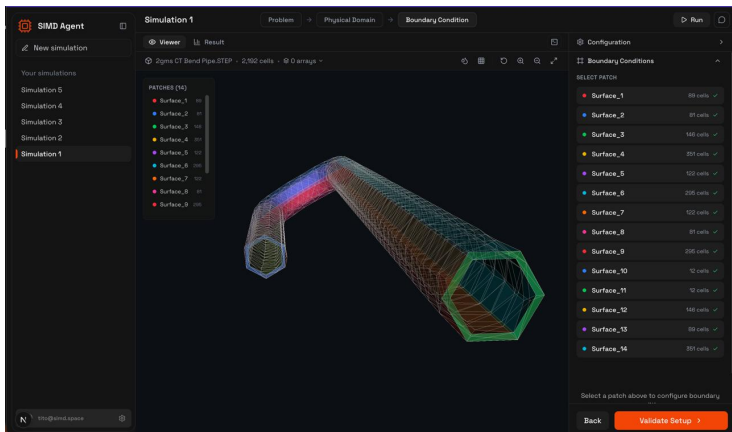


physics-AI For CFD Simulations: SciML

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- 2 Modeling of Cryogenic Systems-H₂, N₂
- 3 Dyad Agent- Jüiahüb
 - Cryogenic H₂/N₂
 - Dyad agent
 - Czochralski Crystal Growth Process
 - Modeling Assumptions and Domain
 - Governing Equations
 - Dimensionless Analysis
 - Improved PINN Methodology
 - Equation-to-Loss Function Mapping
 - Training and Validation



Dyad

Hardware Engineering at the Speed of Software

[Get Started](#)[Get Dyad Studio](#)[Get Help](#)

Dyad is the modeling system for next-generation engineers.

Dyad combines the latest methods for differentiable programming, scientific machine learning, and generative AI with the techniques of traditional modeling environments, e.g. acausal physical models, controls analysis, and embedded code generation.

Using a single-artifact philosophy ensures that the same file can be used for both developing models in a GUI and within textual environments, such as VS Code. Modern design principles of Dyad include a package manager for sharing composable models, test-driven development CI/CD integration, and deployment to real-world hardware.

What is the Czochralski (CZ) Process?

- Dominant method for single-crystal silicon growth
- Used in semiconductor and photovoltaic industries

CZ Furnace Schematic

CZ Furnace: Crucible, Melt, Heater, Crystal Pulling

- Solidification at melt–crystal interface
- Strong heat extraction through crystal
- Melt flow affects dopant and defect transport

Why Modeling is Difficult

- Strong thermal–fluid coupling
- High Rayleigh number convection
- Thin boundary layers
- Multi-physics and multi-scale nature

Axisymmetric Approximation

- Nearly rotationally symmetric geometry
- Reduces 3D problem to 2D (r, z)

Computational Domain

Axisymmetric Melt Domain in (r, z)

Boundary Identification

Crucible Wall, Free Surface, Crystal Interface

Physical Assumptions

- Incompressible Newtonian melt
- Laminar, steady-state flow
- Constant material properties
- Boussinesq approximation

Continuity Equation

$$\frac{1}{r} \frac{\partial}{\partial r}(ru_r) + \frac{\partial u_z}{\partial z} = 0$$

Radial Momentum Equation

$$u_r \partial_r u_r + u_z \partial_z u_r = -\frac{1}{\rho} \partial_r p + \nu \left(\nabla^2 u_r - \frac{u_r}{r^2} \right)$$

Axial Momentum Equation

$$u_r \partial_r u_z + u_z \partial_z u_z = -\frac{1}{\rho} \partial_z p + \nu \nabla^2 u_z + g \beta (T - T_0)$$

Energy Equation

$$u_r \partial_r T + u_z \partial_z T = \alpha \nabla^2 T$$

Thermal–Fluid Coupling

- Temperature \rightarrow buoyancy force
- Velocity \rightarrow heat advection
- Strong nonlinear feedback loop

Rayleigh Number

$$\text{Ra} = \frac{g\beta\Delta TL^3}{\nu\alpha}$$

- Measures strength of natural convection

High Rayleigh Number Regime

- Thin thermal and velocity boundary layers
- Strong nonlinear advection
- Major source of instability for PINNs

Standard PINN Concept

- Neural network approximates PDE solution
- Physics enforced via residual minimization

PINN Inputs and Outputs

- Inputs: (r, z)
- Outputs: u_r, u_z, p, T

Spatial Information (SI) Embedding

- Coordinates injected into hidden layers
- Preserves geometric sensitivity
- Improves boundary-layer resolution

Adaptive Loss Balancing

- Separate losses for each equation
- Trainable weights
- Prevents gradient domination

PINN Loss Function Structure

$$\mathcal{L} = \lambda_c \mathcal{L}_c + \lambda_m \mathcal{L}_m + \lambda_e \mathcal{L}_e + \lambda_b \mathcal{L}_b$$

Continuity Loss

$$\mathcal{L}_c = \frac{1}{N} \sum_i \left| \frac{1}{r} \partial_r (ru_r) + \partial_z u_z \right|^2$$

$$\mathcal{L}_m = \|\mathcal{R}_{u_r}\|^2 + \|\mathcal{R}_{u_z}\|^2$$

- Enforces Navier–Stokes equations

$$\mathcal{L}_e = \frac{1}{N} \sum_i |u_r \partial_r T + u_z \partial_z T - \alpha \nabla^2 T|^2$$

Boundary Condition Loss

$$\mathcal{L}_b = \|u - u_{BC}\|^2 + \|T - T_{BC}\|^2$$

Why Adaptive Weighting Matters

- Different PDEs have different stiffness
- High-Ra momentum residuals dominate gradients
- Learned weights improve conditioning

- No experimental or CFD data
- Physics-only collocation points

Reference Solution

- COMSOL Multiphysics
- Weak-form finite element solver

COMSOL vs PINN Velocity Contours

Temperature Comparison

COMSOL vs PINN Temperature Contours

- Relative L_2 norms
- Improved PINN outperforms standard PINN

- Transient growth
- Rotation and Marangoni effects
- Real-time process optimization

Thank You