UCLA Anderson

Accounting Theory Study Guide

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This study document spans most of the signaling papers covered in the Winter 2019 *Doctoral Seminar in Financial Accounting Theory* (MGMTPHD 236) class thought by Professor Brett Trueman and the Spring 2020 *Theoretical models in accounting* (MGMTPHD 236) class thought by Professor Judson Caskey.

The theory sections of the UCLA Anderson Accounting area comprehensive examination is different from most final exams. It is best approached by a deep understanding of the base material instead of attempts to "study to the exam" (which is sufficient for many classes). While around 50 theoretical papers, many of which use mathematical theorems you might be encountering for the first time, may seem like a lot, it is good to be able to break these down into categories.

Accounting theory (and this exam) focuses on signaling games, which is a subcategory of sequential games with incomplete information, a subfield of game theory. From there it inherits the main equilibrium concept of Perfect Bayesian Equilibrium (PBE). This should be reviewed beforehand to better appreciate the many papers using them as a base.

We can attempt to (imperfectly) break down most of the main signaling papers covered in these classes into the following categories along with their representative papers:

- Persuasion games
    - with Full Revelation, aka "unraveling" (e.g.: Grossman [1981])
    - with Disclosure Costs (e.g.: Verrecchia [1983])
    - with Information Uncertainty (e.g.: Jung & Kwon [1988])
    - with Uncertainty About Sender Incentives (e.g.: Einhorn [2007])

- Costless Signaling Games, aka "Cheap Talk" (e.g.: Crawford & Sobel [1982])

- Costly Reporting Games, aka "Signal Jamming"
    - with Costly Disclosure and Full Revelation (e.g.: Stein [1989])
    - with Costly Disclosure and Reporting Uncertainty (e.g.: Fischer & Verrecchia [2000])

- Rational Expectations
    - Price Determined by Market Clearing (e.g.: Grossman & Stiglitz [1980])
    - Price Determined by Competitive Market Maker (e.g.: Kyle [1985])

Herding could also be used as a useful category, exemplified in papers such as Scharfstein & Stein (1990) and Trueman (1994).

Solving for equilibrium is usually your main concern and challenge in the questions you may face. There are a few types of equilibria to keep in mind:

- Fully Revealing (also known as Fully Separating)
- Partially Revealing (or Partially Separating)

- Pooling
- Partition Equilibrium (as seen in “Cheap Talk” models)
    - where Babbling Equilibrium is a special case

Together these canonical papers span the main equilibrium solution styles you should master and know when to use, these are:

- Solving for the indifference threshold
    - When there is a cost to disclosure
    - When there is uncertain information endowment
    - When there is uncertainty about Sender’s objective function

- Inverting the Sender’s reporting function
    - Which may include solving for the other agent’s linear conjecture coefficients, as often seen in Rational Expectations models

- Use of the Intuitive Criterion refinement, such as for an “unraveling” argument and commonly seen in achieving certain pooling equilibria

Finally, there are papers that escape the above categories or span a number of them. The rest of this document is heavily based on notes handed out in class, with some extra steps added in for clarity. It tends to focus either on the paper’s main equilibrium result of interest or what was focused on in the class it was presented.

One particularly helpful aspect of the process boxes I made use of is the ability to clearly isolate all the “inputs” to a paper’s model, and have them for quick reference. Between 50 papers, it is easy to confuse variables and assumptions (“was *r* risk-aversion or risk-tolerance in this paper? Was x demand or liquidity?”), this document attempts to help with this by trying to have everything on a single (sometimes two-sided) sheet of paper.

I would like to thank the UCLA Anderson Accounting professors for their support and guidance. All typos and mistakes are my own.

The next section lists the papers and problems covered in this document.

Doctoral Seminar in Financial Accounting Theory
(MGMTPHD 236, Winter 2019)
Professor Brett Trueman

Rational Expectations

- S. Grossman and J. Stiglitz, “On the Impossibility of Informationally Efficient Markets,” American Economic Review, June 1980.
- R. Verrecchia, “Information Acquisition in a Noisy Rational Expectations Economy,” Econometrica, November 1982.
- A. Kyle, “Continuous Auctions and Insider Trading,” Econometrica, November 1985.
- O. Kim and R. Verrecchia, “Trading Volume and Price Reactions to Public Announcements,” Journal of Accounting Research, Autumn 1991.

Signaling

- G. Akerlof, “The Market for Lemons,” Quarterly Journal of Economics, August 1970.
- M. Spence, “Job Market Signaling”, Quarterly Journal of Economics, August 1973.
- H. Leland and D. Pyle, “Informational Asymmetries, Financial Structure, and Financial Intermediation,” Journal of Finance, May 1977.
- S. Titman and B. Trueman, “Information Quality and the Valuation of New Issues,” Journal of Accounting and Economics, May 1986.
- S. Ross, “The Determination of Financial Structure: The Incentive Signaling Approach,” Bell Journal of Economics, Spring 1977.
- M. Miller and K. Rock, “Dividend Policy Under Asymmetric Information,” Journal of Finance, September 1985.
- S. Datar, J. Feltham, and J. Hughes, “The Role of Audits and Audit Quality in Valuing New Issues,” Journal of Accounting and Economics, March 1991.

Voluntary Disclosure

- R. Verrecchia, “Discretionary Disclosure,” Journal of Accounting and Economics, December 1983.
- R. Dye, “Disclosure of Nonproprietary Information,” Journal of Accounting Research, Spring 1985.
- W. Jung and Y. Kwon, “Disclosure When the Market is Unsure of Information Endowment of Managers,” Journal of Accounting Research, Spring 1988.
- J. Hughes and S. Pae, “Voluntary Disclosure of Precision Information,” Journal of Accounting and Economics, June 2004.
- M. Darrough and N. Stoughton, “Financial Disclosure Policy in an Entry Game,” Journal of Accounting and Economics, January 1990.
- M. McNichols and B. Trueman, “Public Disclosure, Private Information Collection and Short-term Trading”, Journal of Accounting and Economics, January 1994.
- P. Fischer, and R. Verrecchia, “Reporting Biases,” The Accounting Review, April 2000.
- E. Einhorn, “Voluntary Disclosure Under Uncertainty About the Reporting Objective,” Journal of Accounting and Economics, July 2007.

Asymmetric Information

- S. Myers and N. Majluf, “Stock Issues and Investment Policy When Firms Have Information That Investors Do Not Have,” Journal of Financial Economics, June 1984.
- B. Trueman, “Analyst Forecasts and Herding Behavior,” Review of Financial Studies, Spring 1994.
- A. Beyer, “Financial Analysts’ Forecast Revisions and Managers’ Reporting Behavior,” Journal of Accounting and Economics, December 2008.
- D. Scharfstein and J. Stein, “Herd Behavior and Investment,” American Economic Review, June 1990.

Theoretical models in accounting
(MGMTPHD 236, Spring 2020)

Professor Judson Caskey

Background

- Milgrom, P., and N. Stokey. 1982. Information, trade and common knowledge. Journal of Economic Theory 26(1): 17-27
- Grossman, J. 1981. The informational role of warranties and private disclosure about product quality. Journal of Law and Economics 24(3): 461-484
- Milgrom, P. 1981. Good news and bad news: Representation theorems and applications. Bell Journal of Economics 12(2): 380-391

Ohlsonomics

- Ohlson, J. 1995. Earnings, book values and dividends in equity valuation. Contemporary Accounting Research 11(2): 661-687

Rational Expectations

- Grossman, S., and J. Stiglitz. 1980. On the impossibility of informationally efficient markets. American Economic Review 70(3): 393-408
- Caskey, J. 2009. Information in equity markets with ambiguity-averse investors. Review of Financial Studies 22(9): 3595-
- Guttman, I. 2010. The timing of analysts' earnings forecasts. The Accounting Review
- Kyle, A. 1985. Continuous auctions and insider trading. Econometrica 53(6): 1315-1335
- Kyle, A. 1989. Informed speculation with imperfect competition. Review of Economic Studies 56(3): 317-355

Verifiable disclosure models

- Verrecchia, R. 1983. Discretionary disclosure. Journal of Accounting and Economics 5: 179-194 Accounting Research 26(1): 146-153
- Jung, W., and Y. Kwon. 1988. Disclosure when the market is unsure of information endowment of managers. Journal of Accounting Research 26(1): 146-153

- Einhorn, E., and A. Ziv. 2012. Biased voluntary disclosure. Review of Accounting Studies 17(2): 420-442

Cheap Talk

- Crawford, V., and J. Sobel. 1982. Strategic information transmission. Econometrica 50(6): 1431-1451
- Ottaviani, M., and P. Sorensen. 2006. The strategy of professional forecasting. Journal of Financial Economics 81(2): 441-466
- Bertomeu, J., and I. Marinovic. 2016. A theory of hard and soft information. The Accounting Review 91(1): 1-20

Earnings management/Costly signaling

- Kartik, N. 2009. Strategic communication with lying costs. Review of Economic Studies 4(1): 1359-1395
- Caskey, J. 2014. The pricing effects of securities class action lawsuits and litigation insurance. Journal of Law, Economics and Organization 30(3): 493-532
- Fischer, P., and R. Verrecchia. 2000. Reporting bias. The Accounting Review 75(2): 229-245
- Stein, J. 1989. Efficient capital markets, inefficient firms: A model of myopic corporate behavior. Quarterly Journal of Economics 104(4): 655-669

Voluntary + mandatory disclosure

- Einhorn, E. 2018. Competing information sources. The Accounting Review 93(4): 151-176
- Cheynel, E., and C. Levine. 2020. Public disclosures and information asymmetry: A theory of the mosaic. The Accounting Review 95(1): 79-99

Continuous time dynamic models

- Pastor, L., and P. Veronesi. 2003. Stock valuation and learning about profitability. Journal of Finance 58(5): 1749-1789

Solutions to Homework Assignments and Previous Theory Comps

Prof. Brett Trueman Assignments:

- Assignment 1 (Normal Update of Expected Value)
- Assignment 2 (Rational Expectations)
- Assignment 3 (Costly Signaling)
- Assignment 4 (Inverting Sender’s Function)
- Assignment 5 (Uncertain Information Endowment)
- Assignment 6 (Herding)

Prof. Judson Caskey Assignments:

- Assignment 1 Q1 (Rational Expectations)
- Assignment 1 Q4 (Guttman [2010])
- Assignment 2 Q3 (Signal Jamming and Cheap Talk)

Previous Theory Comp

- Theory 1: Comp 2019
- Theory 1: Comp 2020
- Theory 2: Comp 2019
- Theory 2: Comp 2020

Grossman & Stiglitz (1980)

I. Rational Expectations (Partially Revealing Rational Expectations Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

$t = 0$ — $t = 1$

Trading — Cashflow Realization μ

1 Demand is independent of wealth

The goal of this section is to show that CARA utility makes it so the demand for risky asset does not depend on wealth, explaining why endowments $\overline{M}_i$ & $\overline{X}_i$ and wealth W_{ti} do not enter the systems found in further sections.

CARA coef. of risk-aversion: a $\longrightarrow$

$U_i(W_1 i) = -exp\{-aW_{1i}\}$ $\longrightarrow$

risk-free endowment $\overline{M}_i$ $\longrightarrow$

risky endowment $\overline{X}_i$ $\longrightarrow$

price P $\longrightarrow$

constraint $PX_i + M_i = W_{0i} \equiv \overline{M}_i + \overline{X}_i$ $\longrightarrow$

information I_i $\longrightarrow$

t=1 wealth: $W_{1i} = M_i + \mu X_i$ $\longrightarrow$

knowing that $\mu \sim N(\overline{\mu}, \sigma_x^2)$ $\longrightarrow$

Show demand is independent of wealth

Knowing that $W_{1i} \sim Normal$

Trader's expected utility can be written as:

$$E[U_i(W_{1i})|I_i] = -exp\{-a[E[W_{1i}|I_i] - \frac{a}{2}var[W_{1i}|I_i]]\}$$

maxing utility is thus equiv. to maxing:

$$E[W_{1i}|I_i] - \frac{a}{2}var[W_{1i}|I_i]$$

note that: $E[W_{1i}|I_i] = W_{0i} + (E[\mu|I_i] - P)X_i$

and: $var[W_{1i}|I_i] = X_i^2 var(\mu|I_i)$,

substituting, we get objective function:

$$W_{0i} + (E[\mu|I_i] - P)X_i - \frac{a}{2}X_i^2 var(\mu|I_i)$$

FOC: demand $X_i = \frac{E(\mu|I_i) - P}{a \cdot var(\mu|I_i)}$

$\longrightarrow X_i = \frac{E(\mu|I_i)}{a \cdot var(\mu|I_i)}$

2 Equilibrium when fraction λ are informed

The equilibrium threats λ as given thus cost of information acquisition c is ignored.

share of informed: $\lambda \longrightarrow$

CARA coef. of risk-aversion: $a \longrightarrow$

per-capita liquidity $x \sim N(0, \sigma_x^2) \longrightarrow$

private signal: $\theta = \mu + \epsilon \longrightarrow$

$\mu \sim N(\overline{\mu}, \sigma_\mu^2) \perp \epsilon \sim N(0, \sigma_\epsilon^2) \longrightarrow$

signal conjecture form: $P = \alpha + \beta\theta + \gamma x \longrightarrow$

Show $\exists$ Partially Revealing

Rational Expectations Equilibria

Uninformed Conjecture:

$z \equiv \theta + bx = (\mu + \epsilon) + bx$

where $b = \gamma/\beta$

demand $X_U = \frac{E[\mu|z] - P}{a \cdot var(\mu|z)}$

$E[\mu|z] = \overline{\mu} + \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\epsilon^2 + b^2\sigma_x^2}(z - \overline{\mu})$

$Var(\mu|z) = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\sigma_\mu^2 + \sigma_\epsilon^2 + b^2\sigma_x^2}$

demand $X_I = \frac{E[\mu|\theta] - P}{a \cdot var(\mu|\theta)}$

$E[\mu|\theta] = \overline{\mu} + \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\epsilon^2}(\theta - \overline{\mu})$

$Var(\mu|\theta) = \sigma_\mu^2 - \frac{\sigma_\mu^4}{\sigma_\mu^2 + \sigma_\epsilon^2}$

Thus we have as a function of:

$X_U(\overline{\mu}, a, b, \theta, \sigma_\mu^2, \sigma_\epsilon^2, \sigma_x^2)$

$X_I(\overline{\mu}, a, b, \theta, \sigma_\mu^2, \sigma_\epsilon^2, \sigma_x^2)$

plugging into **Demand = Supply :**

$\lambda X_I + (1 - \lambda) X_U = x$

gives: $P = \alpha + \beta\theta + \gamma x$

So conjecture terms converge to specific, terms that are expressions of setting parameters. This means $\exists$ a partially revealing price equilibrium in this setting.

$\longrightarrow$ informed demand X_I

$\longrightarrow$ uninformed demand X_U

$\longrightarrow$ β as a function of parameters

$\longrightarrow$ γ as a function of parameters

$\longrightarrow$ α as a residual constant

$\longrightarrow$ $P = \alpha + \beta\theta + \gamma x$

$\longrightarrow$ price informativeness proxy: $\frac{\beta^2}{\gamma^2}$

$\longrightarrow$ $\exists$ partially revealing equilibrium

Additional notes:

The process makes use the previous section's optimal demand function and the following properties of normally distributed random variables:

$$E[\mu|z] = E[\mu] + \frac{cov(\mu, z)}{var(z)}(z - E[z])$$

$$var(\mu|z) = var[\mu] - \frac{cov^2(\mu, z)}{var(z)}$$

Law of total variance: $var(X) = \mathbf{E}[var(X \mid Y)] + var(\mathbf{E}[X \mid Y])$

$$q(\mathrm{E}[v] - p) - \frac{\gamma}{2} var(v) q^2 \quad FOC: \; 0 = \mathrm{E}[v] - p - \gamma var(v) q \Rightarrow \begin{cases} p = \mathrm{E}[v] - \gamma var(v) q \\ q = \frac{\mathrm{E}[v]-p}{\gamma var(v)} \end{cases}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}, \begin{pmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \right) \quad where \quad \rho_{xy} = \frac{cov(x,y)}{\sigma_x \sigma_y}$$

$$x \mid y \sim \mathcal{N}\left(\mu_x + \frac{cov(x,y)}{var(y)}(y - \mu_y), \sigma_x^2 - \frac{cov(x,y)^2}{\sigma_y^2} \right) \qquad Note: \quad var(x \mid y) = \sigma_x^2 \left(1 - \rho_{xy}^2\right)$$

Kim & Verrecchia (1991)

I. Rational Expectations (Partially Revealing Rational Expectations Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

1	2	3
Round 1 trading at P_1	**Round 2 trading at P_2**	**Realization u**
public signal y_1	public signal y_2	
private signal z_i		

Kim & Verrecchia examine the price and volume reactions to a public announcement. This paper has individual traders (subindexed i) with their own particular endowments E_i & levels of risk tolerance r_i.

Inputs:

- CARA risk-**tolerance** coef: r_i →
- risky asset $u \sim N(\overline{\mu}, h^{-1})$ →
- random supply $x \sim N(0, t^{-1})$ →
- (t=1) public signal $y_1 = u + \eta$; $\eta \sim N(0, m^{-1})$ →
- (t=1) private signal $z_i = u + \epsilon_i$; $\epsilon_i \sim N(0, s_i^{-1})$ →
- (t=2) public signal $y_2 = u + \nu$; $\nu \sim N(0, n^{-1})$ →
- conjecture form of $P_1 = \alpha_1 + \theta_1 y_1 + \beta_1 u - \gamma_1 x$ →
- conjecture form of $P_2 = \alpha_2 + \theta_{21} y_1 + \beta_2 u - \gamma_2 x$ →

Show ∃ Partially Revealing Equilibria Through Price Amids Random Supply

i's assessment of mean of u at t=2:

$u_{2i} = E[u|Info_{t=2}]$

i's assessment of precision of u at t=2:

$K_{2i} = var^{-1}[u|Info_{t=2}]$

thus trader i's t=2 demand:

$D_{2i} = r_i K_{2i}(u_{21} - P_2)$

(demand D_{1i} is complex & omitted)

Notes omit this explicitly, but as in previous Rational Expecations models, demand is then used to solve for the coefficients on price conjectures, (ie: $\alpha_1, \theta_1, \beta_1, \gamma_1$ for P_1, and $\alpha_2, \theta_{21}, \beta_2, \gamma_2$ for P_2) as functions of economic parameters. This also lets us solve for P_1 & P_2 as functions of economic parameters.

Thus ∃ a Partially Revealing Rational Expectations Equilibriumin here. QED.

Outputs:

- → i's demand at t=2: D_{2i}
- → P_2 & P_1 as functions of params
- → Thus ∃ partial price equilibrium
- → $P_2 - P_1 = \frac{n}{K_2}[Surprise\ +\ Noise]$
- → $Surprise = y_2 - \frac{h + my_1 + su + r^2 s^2 tq}{K_1}$
- → $Noise = \frac{x}{rK_1}$
- → Volume generated by i at t=2 is:
- → $|D_{2i} - D_{1i}| = |r_i(s_i - s)(P_2 - P_1)|$
- → Total Volume of all traders is:
- → $\left(\frac{1}{2}\int r_i|s_i - s|di\right)|P_2 - P_1|$

Notice in the notes that while trader i's riskless endowment E_i is specified and eq.(4) elaborates i's wealth, given CARA utility, they do not play a role above.

Kyle (1985)

I. Rational Expectations (Partially Revealing Rational Expectations Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

$t = 0$ — Trading

$t = 1$ — Cashflow Realization v

Kyle derives the rational expectations equilibrium using a different set of assumptions about how the market operates from those of Grossman & Stiglitz (1980), eg: traders are **risk neutral**.

liquidity trader demand $u \sim N(0, \sigma_u^2)$

signal of informed trader: $\theta = v + \epsilon$

where $v \sim N(\overline{v}, \sigma_v^2) \quad \perp \quad \epsilon \sim N(0, \sigma_\epsilon^2)$

y is the t=0 order flow: $y = x + u$

competitive mkt maker sees order flow as a signal

and he sets $P = E^{mkt}[v|y]$ then clears the market

risk neutral obj. function: $\max_x E^I[(v - P)x|\theta]$

Informed trader can only place market orders and

he considers mkt maker's price setting strategy

to predict the effect his demand will have on price

conjecture form of informed demand: $x = \alpha + \beta\theta$

conjecture form of price: $P = \mu + \lambda y$

show $\exists$ a partial equilibrium in this setting

Show $\exists$ Partially Revealing

Rational Expectations Equilibria

$$\max_x E^I[(v - P)x|\theta] = \max_x E^I[(v - E[v|y])x|\theta]$$

$$= \max_x E^I[(v - \mu + \lambda(x + u))x|\theta]$$

$$= \max_x (E^I[v|\theta]x - (\mu + \lambda x)x)$$

FOC: $x = \frac{E^I[v|\theta] - \mu}{2\lambda}$

where $E[v|\theta] = \overline{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}(\theta - \overline{v})$

from conjecture on x: $\beta = \frac{\sigma_v^2}{2\lambda(\sigma_v^2 + \sigma_\epsilon^2)}$

$$\alpha = \frac{\sigma_\epsilon^2 \overline{v}}{2\lambda(\sigma_v^2 + \sigma_\epsilon^2)} - \frac{\mu}{2\lambda}$$

knowing that: $y = x + u = \alpha + \beta\theta + u$

now: $P = E(v|y) = \overline{v} + \frac{cov(v,y)}{var(y)}(y - E(y))$

$$= \overline{v} + \frac{\beta^2 \sigma_v^2 (y - \alpha - \beta\overline{v})}{\beta^2(\sigma_v^2 + \sigma_\epsilon^2) + \sigma_u^2}$$

from P conjecture: $\mu = \frac{(\beta^2\sigma_\epsilon^2 + \sigma_u^2)\overline{v} - \alpha\beta\sigma_v^2}{\beta^2(\sigma_v^2 + \sigma_\epsilon^2) + \sigma_u^2}$

and y coefficient : $\lambda = \frac{\beta\sigma_v^2}{\beta^2(\sigma_v^2 + \sigma_\epsilon^2) + \sigma_u^2}$

Plug in x above to get rid of P's

conjecture elements in conjecture of x:

$\beta = \left[\frac{\sigma_u^2}{\sigma_v^2 + \sigma_\epsilon^2}\right]^{0.5}$ and $\lambda = \frac{\sigma_v^2}{2\sigma_u^2(\sigma_v^2 + \sigma_\epsilon^2)^{0.5}}$

would then solve for $\alpha = -\beta\overline{v}$ and $\mu = \overline{v}$

So conjecture terms are convergent,

this means $\exists$ partially revealing

price equilibrium in this setting. QED.

Note: Mkt Maker makes 0 expected profits.

x's conjecture coefs

P's conjecture coefs

market depth: λ^{-1}

shows $\exists$ equilibrium

Verrechia (1982)

I. Rational Expectations (Partially Revealing Rational Expectations Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

$t = 0$ ——————————————————————— $t = 1$

Trading ——————————————————————— Cashflow Realization μ

The setting here is more general than in Grossman and Stiglitz (1980).

T Traders ($T \to \infty$) differ in their risk aversion and in the private signals they receive.

The subindex t index that the variable is at the individual trader level.

CARA coef. of risk-**tolerance**: $r_t \longrightarrow$

risky cashflow $\mu \sim N(y_0, h_0^{-1}) \longrightarrow$

private signal of trader t: $y_t = \mu + \epsilon_t \longrightarrow$

where $\epsilon_t \sim N(0, s_t^{-1}) \longrightarrow$

cost of signal with precision $s_t : c(s_t) \longrightarrow$

per capita liquidity $x \sim N(0, V) \longrightarrow$

signal conjecture form: $P = \alpha y_0 + \beta\mu + \gamma x \longrightarrow$

Show $\exists$ Partially Revealing

Rational Expectations Equilibria

$$E_t(\mu_t|y_t, P) = y_0 + \frac{s_t\gamma^2 V(y_t - y_0) + \beta(P - y_0)}{(h_0 + s_t)\gamma^2 V + \beta^2}$$

$$Var(\mu_t|y_t, P) = \frac{1}{h_0 + s_t + \beta^2/\gamma^2 V}$$

Notes skip exact derivation of the above but it's said to be done in the same manner as Grossman & Stiglitz (1980)

note: $\beta^2/\gamma^2 V$ is proxy for price informativeness

$$\longrightarrow D_t(y_t, P) = \frac{r_t(E_t(\mu_t|y_t, P) - P)}{Var(\mu_t|y_t, P)}$$

Akerlof (1978)

II. Signaling (Pooling Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

1

Trading

Akerlof shows that in presence of asymmetric information about quality, markets can break down when private information is unverifiable.

1 Asymmetric information case

Used car quality is unverifiable $\longrightarrow$

Group 1 Utility: $U_{1i} = M + \Sigma_{1i} x_{1i}$ $\longrightarrow$

Group 2 Utility: $U_{2i} = M + \frac{3}{2}\Sigma_{2i} x_{2i}$ $\longrightarrow$

Group 1 endowed with N cars $\longrightarrow$

Group 2 endowed with 0 cars $\longrightarrow$

Priors of used car quality: $x \sim Unif(0,2)$ $\longrightarrow$

income of group j: Y_j $\longrightarrow$

(M is other consumption) $\longrightarrow$

Show this is a Pooling Equilibrium where markets break down (Demand=0)

Supply: $S = S(p)$

$$S_1 = \begin{cases} \frac{p}{2}N & \text{if} \quad p \leq 2 \quad (\text{if quality} \leq \text{price}) \\ N & \text{if} \quad p > 2 \end{cases}$$

$S_2 = 0$

E[quality of used car on sale]: $\mu = p/2$

Demand:

$$D_1 = \begin{cases} \frac{Y_1}{p} & \text{if} \quad \mu \geq p \quad (\text{quality} \geq \text{price}) \\ 0 & \text{if} \quad \mu < p \quad (\text{quality} < \text{price}) \end{cases}$$

$$D_2 = \begin{cases} \frac{Y_2}{p} & \text{if} \quad \frac{3}{2}\mu \geq p \\ 0 & \text{if} \quad \frac{3}{2}\mu < p \end{cases}$$

Total demand:

$$D(p,\mu) = \begin{cases} \frac{Y_1+Y_2}{p} & \text{if} \quad p \leq \mu \\ \frac{Y_2}{p} & \text{if} \quad \mu < p \leq \frac{3}{2}\mu \\ 0 & \text{if} \quad p > \frac{3}{2}\mu \end{cases}$$

but for a given p, μ is $p/2$, plugging $p = 2\mu$ above renders $D(p,\mu) = 0$

$\longrightarrow$ Total Demand = 0

2 Symmetric information case (no one knows a car's quality)

Group 1 Utility: $U_{1i} = M + \Sigma_{1i} x_{1i}$

Group 2 Utility: $U_{2i} = M + \frac{3}{2}\Sigma_{2i} x_{2i}$

Group 1 endowed with N cars

Group 2 endowed with 0 cars

Priors of used car quality: $x \sim Unif(0,2)$

income of group j: Y_j

(M is other consumption)

Find price and demand

in this Pooling Equilibria.

Supply: $S = S(p)$

$$S_1 = \begin{cases} N & \text{if} \quad p \geq 1 \;\; \text{(avg quality} \leq \text{price)} \\ 0 & \text{if} \quad p < 1 \end{cases}$$

$S_2 = 0$

E[quality of used car on sale]: $\mu = 1$

We knew this was a Pooling Equilibrium.

Total demand:

$$D(p,\mu) = \begin{cases} \frac{Y_1+Y_2}{p} & \text{if} \quad p \leq 1 \\ \frac{Y_2}{p} & \text{if} \quad 1 < p \leq \frac{3}{2} \\ 0 & \text{if} \quad p > \frac{3}{2} \end{cases}$$

In equlibrium:

$$\begin{cases} p = 1 & \text{if} \quad Y_2 \leq N \\ p = \frac{Y_2}{N} & \text{if} \quad \frac{2Y_2}{3} < N \leq Y_2 \\ p = \frac{3}{2} & \text{if} \quad N < \frac{2Y_2}{3} \end{cases}$$

Price p

then demand $D(p,\mu)$

Datar, Feltham & Hughes (1991)

II. Signaling (Fully Revealing Signaling Equilibria)

This paper takes Leland & Pyle's setting of a CEO signaling with his level of retained ownership and augments it with an auditor, allowing him to potentially signal firm value without retaining as much.

$t = 0$	$t = 0$	$t = 1$
CEO picks auditor of quality a to release audit y	CEO sells $(1-\alpha)\%$ of firm Total firm valued at $E[m \vert \alpha, y, a]$	Cashflow Realization x

CEO's CARA risk-**aversion** coefficient is b

Investors are risk-**neutral**

Project payoff is $x \sim N(m,\sigma^2)$ & r_f rate =0%

Where CEO privately knows m perfectly

Project requires $\$k$ outlay

CEO sells $(1-\alpha)\%$ of firm at t=0 & keeps α

CEO's end wealth: $W = \alpha x + (1-\alpha)E^{mkt}[x|\alpha] - k$

$\max_{\alpha\in[0,1]} \alpha m + (1-\alpha)E^{mkt}[x|\alpha] - b\sigma^2\alpha^2/2$

$\alpha(m)$ denotes α retained by CEO with signal m

Through α, market infers CEO's info m

$v^{mkt}(\alpha)$ denotes mkt assessed value m given α

CEO conjectures his α will perfectly reveal info

$\exists$ functions $\alpha(m)$ & $v^{mkt}(\alpha)$ that solve † while

fully revealing m through α: $v(\alpha(m)) = m$, thus:

† $\max_{\alpha\in[0,1]} \alpha m + (1-\alpha)v(\alpha(m)) - k - b\sigma^2\alpha^2/2$

CEO can hire an auditor of quality a to report y

Audit report y is random & conditional on m & a

$m \supset y$ has no new info about x for CEO given m

y is released alongside α & changes z to $\underline{m}(y,a)$

such that $-b\sigma^2[\log(1-\alpha)+\alpha] = m - \underline{m}(y,a)$

FOC: $m - v(\alpha) + (1-\alpha)v'(\alpha) - b\sigma^2\alpha = 0$

$v(\alpha) = m \Rightarrow v'(\alpha) = \frac{b\alpha\sigma^2}{(1-\alpha)}$

then integrating: $m = -b\sigma^2[\log(1-\alpha)+\alpha] + z$, z is a constant

Letting $\delta \equiv m - \underline{m}(y,a)$

$v(\alpha) = -b\sigma^2[\log(1-\alpha)+\alpha] = \delta$

Totally differentiating both sides (LHS:dα, RHS:dδ)

$v'(\alpha)\mathrm{d}\alpha = 1\mathrm{d}\delta$

$\frac{b\alpha\sigma^2}{(1-\alpha)}\mathrm{d}\alpha = \mathrm{d}\delta$

$\frac{\mathrm{d}\alpha}{\mathrm{d}\delta} = \frac{1-\alpha}{b\sigma^2\alpha} > 0$

That is, the percentage of retained ownership is an increasing function of the difference between the entrepreneur's type and the worst type from which he must separate himself given the audited report.

Leland & Pyle (1977)

II. Signaling (Fully Revealing Signaling Equilibria)

Based on Winter 2019 Accounting Theory (simplified) class notes on the paper (Prof. Trueman)

$t = 0$ —————————————————————— $t = 1$

CEO sells $(1 - \alpha)\%$ of firm

Total firm valued at $\mu^c(\alpha)$

IPO size: $(1 - \alpha)\mu^c(\alpha)$

Cashflow Realization $\mu + x$

1 Fully Revealing Signaling Equilibrium

Firm value μ is unknown to investors and a financial variable (α: share of ownership retained by the CEO) is used to credibly communicate μ.

CEO is risk-**averse** →

Investors are risk-**neutral** →

$\exists$ risk-free asset yielding 0% →

Public priors of firm value: $E[V] = \mu + x$ →

where $x \sim N(0, \sigma_x^2)$ →

CEO perfectly knows μ privately →

CEO sells $(1 - \alpha)\%$ of firm at t=0 & keeps α →

CEO can only credibly convey μ through α →

At t=0 mkt conjectures μ as: $V(\alpha) = \mu^c(\alpha)$ →

CEO's contraint: $Y = W_0 + (1 - \alpha)\mu^c(\alpha)$ →

where W_0 is CEO's endowment of r_f asset →

and Y is CEO's final holding of r_f asset →

but W_1 is the CEO's end-of-period wealth: →

$W_1 = (\alpha)(\mu + x) + W_0 + (1 - \alpha)\mu^c(\alpha)$ →

We want to show $\exists$ a fully revealing equilibrium →

CEO conjectures his α will perfectly reveal info →

The *consistency condition** applies →

Show $\exists\alpha^*$ s.t. α^* results in a fully revealing equilibrium. So CEO's choice of optimal α^* given his μ private info must perfectly reveal it, i.e.: $\mu^c(\alpha^*(\mu)) = \mu$

To find this $\alpha^*(\mu)$, the FOC of his utility function w.r.t. α:

$$\frac{\partial E[U(W_1)]}{\partial \alpha} = E[U'(W_1)[\mu + x - \mu^c(\alpha^*) + (1 - \alpha^*)\frac{\partial \mu^c(\alpha)}{\partial \alpha}] = 0$$

Note: U(W) is not explicitly specified and this $\frac{\partial E[U(W_1)]}{\partial \alpha^*}$ is given as is in the notes

Imposing the consistency condition ($\mu^c(\alpha*) = \mu$) we get:

$$E[U'(W_1)[x + (1 - \alpha^*)\frac{\partial \mu^c(\alpha)}{\partial \alpha}]] = 0$$

$$E[U'(W_1)x] = -(1 - \alpha^*)\frac{\partial \mu^c(\alpha)}{\partial \alpha}E[U'(W_1)]$$

$$(1 - \alpha^*)\frac{\partial \mu^c(\alpha)}{\partial \alpha} = -\frac{E[U'(W_1)x]}{E[U'(W_1)]}$$

Authors then show that this optimal α^* converges & is > 0. Thus $\exists$ a fully revealing equilibrium. QED.

→ $\exists\alpha* > 0$ in a fully revealing equilibrium

*The *consistency* (or incentive compatibility) *condition* is a necessary condition for equilibrium. Here it relates to the investors. Their conjecture of μ, given that they observe α, is equal to the actual value of μ. That is, in equilibrium their conjecture is correct.

2 Solution to question in Section IV of the notes

Solve CEO's problem and find market's optimal conjecture schedule in a fully revealing signaling equilibrium.

Knowing that CEO is risk averse: $E[U(W_1)] = E[W_1] - \frac{b}{2}\sigma^2(W_1)$
and given that $W_1 = (\alpha)(\mu + x) + W_0 + (1-\alpha)\mu^c(\alpha)$:

$$E[U(W_1) = E[\alpha(\mu + x) + W_0 + (1-\alpha)\mu^c(\alpha)] - \frac{b}{2}\sigma^2(\alpha\mu + \alpha x + W_0 + (1-\alpha)\mu^c(\alpha))$$

Note: $E[x] = 0$ and μ is deterministic. $\mu^c(\alpha)$ is also deterministic because manager knows that market's conjecture function (which has no random component). Thus:

$$\max_\alpha E[U(W_1)] = \max_\alpha E[\alpha\mu + W_0 + (1-\alpha)\mu^c(\alpha)] - \frac{b}{2}\alpha\sigma_x^2$$

$$FOC: \quad \frac{\partial E[U(W_1)]}{\partial\alpha} = \mu - \mu^c(\alpha) + (1-\alpha)\frac{\partial\mu^c(\alpha)}{\partial\alpha} - \alpha b\sigma_x^2 = 0$$

Now that we've derived the FOC, we can impose the consistency condition letting $\mu^c(\alpha) = \mu$:

$$(1-\alpha)\frac{\partial\mu^c(\alpha)}{\partial\alpha} - \alpha b\sigma_x^2 = 0$$

$$\frac{\partial\mu^c(\alpha)}{\partial\alpha} = \frac{\alpha b\sigma_x^2}{(1-\alpha)}$$

Integrating:

$$\mu^c(\alpha) = -\alpha b\sigma_x^2(\alpha + log(|\alpha - 1|)) + constant$$

Now because we know that $0 < \alpha < 1$ (how do we know < 1? not sure), the term inside the RHS parenthesis is always negative, making the whole RHS positive given that $-\alpha b\sigma_x^2$ is also always negative.

From this we can know that: $min(\alpha) = 0$ gives $\mu^c(\alpha = 0) = constant$ and that $\mu^c(\alpha)$, which is the investors' conjecture of the value of the firm given α does indeed rise with with α.

Miller & Rock (1985)

II. Signaling (Fully Revealing Signaling Equilibria)

In this setting the firm uses its dividend to signal its value. Firms can cheat by investing below optimal $I_0 < I_0^*$ and releasing proceeds as dividend D_1 to fool investors into estimating a higher persistent earnings surprise factor ϵ_1. This, along with then private info I_0 & ϵ_1, is only made obvious to investors at t=2.

$t=0$	$t=1$ start	$t=1$ end	$t=2$
CEO invests I_0 to start project with $\tilde{X}_1 = F(I_0)$ cashflow	CEO chooses I_1 & D_1 Investors assess $E_1^{mkt}[V_1]$	k% of shareholders sell their stake for $k \cdot E_1^{mkt}[V_1]$	Remaining $(1-k)$% collect $(1-k)X_2$ cashflow

t=0: Firm invest I_0 in production funtion $F(I_0)$ →

Firm's earnings at t=1: $\tilde{X}_1 = F(I_0) + \tilde{\varepsilon}_1$, and →

t=2: $\tilde{X}_2 = F(I_1) + \tilde{\varepsilon}_2 = F\left(\tilde{X}_1 - D_1\right) + \tilde{\varepsilon}_2$ →

where D_1 are dividends paid →

$\tilde{\varepsilon}$'s are random with mean 0 and $E(\tilde{\varepsilon}_2 \mid \varepsilon_1) = \gamma\varepsilon_1$ →

$V_1 = D_1 + \frac{1}{1+i}E[\tilde{X}_2] = D_1 + \frac{1}{1+i}[F(I_1) + \gamma\varepsilon_1]$ →

s.t. $X_1 = I_1 + D_1 \Leftrightarrow X_1 - I_1 = D_1$ →

t=1 firm value: $V_1 = X_1 - I_1 + \frac{1}{1+i}[F(I_1) + \gamma\varepsilon_1]$ →

F is concave: $F(I) \geq 0, F(0) = 0, F' > 0,\ F'' < 0$ →

I_1^* is optimal investment level s.t. $F'(I_1^*) = 1 + i$ →

or $\frac{F'(I_1^*)}{(1+i)} - 1 = 0$, where i is firm's WACC →

$E_0^{mkt}[V_1] = E_0^{mkt}[\tilde{X}_1 - I_1] + \frac{1}{1+i}E_0^{mkt}[F(I_1)]$ →

at t=0: $E_0^{mkt}[V_1] = F(I_0) - I_1^* + \frac{1}{1+i}[F(I_1^*)]$ →

realizes at t=1: $V_1 = X_1 - I_1^* + \frac{1}{1+i}\left[E_1^{mkt}(X_2)\right]$ →

$= F(I_0) + \varepsilon_1 - I_1^* + \frac{1}{1+i}[F(I_1^*) + \gamma\varepsilon_1]$ →

Earnings announcement effect: $V_1 - E_0^{mkt}[V_1]$ →

$= (X_1 - E_0^{mkt}[\tilde{X}_1])[1 + \frac{\gamma}{1+i}] = E_0^{mkt}[\varepsilon_1][1 + \frac{\gamma}{1+i}]$ →

So market can back out ε_1 from t=1 financial flows: →

$\varepsilon_1 = D_1 - E_0^{mkt}[D_1] = X_1 - E_0^{mkt}[\tilde{X}_1]$ →

CEO can cheat by investing $I_0 < I_0^*$ to increase D_1 →

this inflates $E_1^{mkt}[\varepsilon_1]$, $E_1^{mkt}[\varepsilon_2 = \gamma\varepsilon_1]$ and $E_1^{mkt}[V_2]$ →

$(1-k)$% of shareholders are LT investors (*keepers*) →

k% sell their stock when D_1 is announced (*sellers*) →

sellers get $V_1^m(D_1) = D_1 + \frac{1}{1+i}E_1^{mkt}[F(I_1) + \gamma\tilde{\varepsilon}_1]$ →

keepers get $V_1^d(X_1, D_1) = D_1 + \frac{1}{1+i}[F(I_1) + \gamma\varepsilon_1]$ →

CEO weights both interests in objective function: →

$\max_{D_1, I_1} W_1 = kV_1^m + (1-k)V_1^d$ s.t. $D_1 + I_1 = X_1$ →

Show $\exists D^*$ s.t. $D^* \Rightarrow$ fully revealing equilibrium →

We are looking for a signaling schedule where $\forall X_1\ \exists$ an optimal D_1 which the market uses to conjecture X_1 from the signal D_1 when valuing the firm. We assume this schedule is homogenous among rational shareholders such that:

$$V_1^m(D_1) = V_1^d(X_1(D_1), D_1) = V_1^d(X_1, D_1)$$

Substituting above into objective function:

$$\max_{D_1, I_1} W_1 = kV_1^d(X_1, D_1) + (1-k)V_1^d(X, D_1)^d$$

s.t. $D_1 + I_1 = X_1 \Leftrightarrow I_1 = X_1 - D_1$ (eliminating I_1)

FOC $\frac{\partial W_1}{\partial D_1}$: $k\frac{\partial V^d(X(D_1), D_1)}{\partial X_1}\frac{\partial X_1(D_1)}{\partial D_1} + k\frac{\partial V^d(X_1(D_1), D_1)}{\partial D_1}$ $+(1-k)\frac{\partial V_d(X, D_1)}{\partial D_1} = 0$ → $\exists D^*$ st $D^* \Rightarrow$ fully revealing equilibrium

Changing notation this is equivalent to:

$$kV_x^d(X_1(D_1), D_1)X_1'(D_1) + kV_d^d(X_1(D_1), D_1)$$
$$(1-k)V_d^d(X_1, D_1) = 0$$

And since conjectures are fulfilled in equilibrium, i.e.: $X_1(D_1) = X_1$:

$$kV_x^d(X_1(D_1), D_1)X_1'(D_1) + V_d^d(X_1(D_1), D_1) = 0$$

This is an ODE which uniquely describes the signaling schedule $X_1(D_1)$ which perfectly reveals X_1

Ross (1977)

II. Signaling (Fully Revealing Signaling Equilibria)

This paper uses a level of maximum debt D^* as a threshold for CEOs to be able to perfectly signal whether their firm is good (Type A) or bad (Type B).

$t = 0$	$t = 0$	$t = 1$
CEO picks auditor y	CEO sells $(1 - \alpha)\%$ of firm	Cashflow Realization x

$$M = \gamma_0 V_0 (1 + r) + \gamma_1 \begin{bmatrix} V_1 & if & V_1 \geq D \\ V_1 - C & if & V_1 < D \end{bmatrix}$$

M is CEO compensation, V_1 is future firm value

$V_0 = V_1/(1 + r)$ is current firm value

γ_0 and γ_1 are positive constants

r is the one period interest rate

D is the face value of firm debt

C: penalty paid if bankruptcy occurs ($V_1 <$ D)

Two types of firms: good type A & bad type B

A firms are worth more than Type B: $V_a > V_b$

D^* is max debt B can have without bankruptcy

CEO uses D^* as a threshold to signal firm type

$D > (<) D^*$ means CEO signals it is type $A(B)$

CEO earns more if he accurately signals type:

A truth: $M_a = \gamma_0 V_{1a} + \gamma_1 V_{la}$ if $D^* < D \leq V_{la}$

A lie: $M_a = \gamma_0 V_{1b} + \gamma_1 V_{1a}$ if $D < D^*$

B lie: $M_b = \gamma_0 V_{1a} + \gamma_1 (V_{1b} - C)$ if $D^* < D \leq V_{1a}$

B truth: $M_b = \gamma_0 V_{1b} + \gamma_1 V_{1b}$ if $D < D^*$

Clearly, CEOs of firms of type A will tell the truth by choosing D > D* because they earn more compensation if they do because $V_b < V_a$.

For CEOs of type B firms, compensation must be set s.t. they earn more by correctly choosing $D < D^*$. This is true when: $\gamma_0 \left(V_{1a} - V_{1b}\right) < \gamma_1 C$ or $\gamma_0 V_{1a} + \gamma_1 \left(V_{1b} - C\right) < \gamma_0 V_{1b} + \gamma_1 V_{1b}$

Compensation gain from lying must be less than the compensation loss. The LHS is the B CEO's share of gain from initially lying that firm is a type A. The RHS is the penalty due to bankruptcy.

Unsuccessful firms don't have enough cash to avoid bankruptcy if D > D* and lying CEOs will earn less. **Thus the signaling equilibrium is fully revealing.**

$\exists D^*$ st $D^* \Rightarrow$ fully revealing equilibrium

Spence (1973)

II. Signaling (Fully Separating Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

before date 0	0
Worker Education Decision	Wage offered $w(MPL^c \vert educ)$

Spence (1973) shows how an unknown characteristic can be publicly revealed through the use of a signal. **Note:** The model actually doesn't have multiple dates, but I added the above for clarity.

1 Perfectly Revealing/Separating Signaling Equilibrium

Here we show a set of employer conjectures on education of workers that: (1) is fulfilled by the worker's optimal action, and (2) allows for education level to perfectly reveal information on worker's productivity.

Risk neutral agents

Worker's marginal prod. of labor: $MPL \in \{1, 2\}$

Worker knows his MPL; employer doesn't

Worker population $Prob(MPL = 1) = q$

Worker population $Prob(MPL = 2) = 1 - q$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 1) = y$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 2) = y/2$

Employer conjecture: $\begin{cases} MPL^c = 1 \text{ if } y < y^* \\ MPL^c = 2 \text{ if } y \geq y^* \end{cases}$

Employer offers wage: $w(MPL^c|\mathbf{educ}) = MPL^c$

We want a **fully seperating equilibrium**

Show this setting leads to a Fully Separating Equilibrium.

A worker, knowing his MPL, will choose education level of:

$$\begin{cases} 0 \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) < w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \\ y^* \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) \geq w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \end{cases}$$

This will be a fully revealing separating equilibrium if:

For $MPL = 1$ worker : $2 - y^* < 1 - 0 \Rightarrow 1 < y^*$

For $MPL = 2$ worker : $2 - y/2^* > 1 - 0 \Rightarrow 2 > y^*$

Thus conjectures will be fulfilled if $1 < y^* < 2$

OUTCOME:

· All workers get either exactly 0 or exactly y^* units of education

· The signaling schedule is thus: $\begin{cases} MPL^c = 1 \text{ if } y < y^* \\ MPL^c = 2 \text{ if } y \geq y^* \end{cases}$

· The signal perfectly reveals a worker's MPL.

Fully Separating Equilibrium will result from value of y^* in $1 < y^* < 2$. QED.

Note for section 1: The inverse relation between productivity and cost of education is a necessary condition for education to serve as a signal for productivity.

2 Pooling Equilibrium

A pooling equilibrium can exist in which it would be suboptimal for any worker to get education:

Risk neutral agents $\longrightarrow$

Worker's marginal prod. of labor: $MPL \in \{1, 2\}$ $\longrightarrow$

Worker knows his MPL; employer doesn't $\longrightarrow$

Worker population $Prob(MPL = 1) = q$ $\longrightarrow$

Worker population $Prob(MPL = 2) = 1 - q$ $\longrightarrow$

Thus $E[MPL] = q(1) + (1 - q)(2) = 2 - q$ $\longrightarrow$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 1) = y$ $\longrightarrow$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 2) = y/2$ $\longrightarrow$

New emp. conj.: $\begin{cases} y < y^* : MPL^c = E[MPL] = 2 - q \\ y \geq y^* \text{: } MPL^c = 2 \end{cases}$ $\longrightarrow$

Employer offers wage: $w(MPL^c|\mathbf{educ}) = MPL^c$ $\longrightarrow$

We want a **pooling equilibrium** $\longrightarrow$

A worker, knowing his MPL, will choose education level of:

$$\begin{cases} 0 \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) < w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \\ y^* \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) \geq w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \end{cases}$$

This will be a pooling equilibrium if:

For $MPL = 1$ worker : $2 - y^* < (2 - q) - 0 \Rightarrow y^* > q$

For $MPL = 2$ worker : $2 - y^*/2 < (2 - q) - 0 \Rightarrow y^* > 2q$

Thus conjectures will be fulfilled if $y^* > 2q$

OUTPUT RESULTS:

· All workers get either exactly 0 education.

· All workers are better off than in the separating equilibrium:

$$\begin{cases} \text{For } MPL = 1 \text{ worker: } 2 - q > 1 \\ \text{For } MPL = 2 \text{ worker: } 2 - q > 2 - y^*/2 \end{cases},$$

Pooling Equilibrium will result from value of y^* in $y^* > 2q$. QED.

Alternative Pooling Equilibrium

Risk neutral agents $\longrightarrow$

Worker's marginal prod. of labor: $MPL \in \{1, 2\}$ $\longrightarrow$

Worker knows his MPL; employer doesn't $\longrightarrow$

Worker population $Prob(MPL = 1) = q$ $\longrightarrow$

Worker population $Prob(MPL = 2) = 1 - q$ $\longrightarrow$

Thus $E[MPL] = q(1) + (1 - q)(2) = 2 - q$ $\longrightarrow$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 1) = y$ $\longrightarrow$

cost of y $\mathbf{educ}^y$ units: $c(y|MPL = 2) = y/2$ $\longrightarrow$

Alt. emp. conj.: $\begin{cases} y < y^* : MPL^c = 1 \\ y \geq y^* : \; MPL^c = E[MPL] = 2 - q \end{cases}$ $\longrightarrow$

Employer offers wage: $w(MPL^c|\mathbf{educ}) = MPL^c$ $\longrightarrow$

We want a **pooling equilibrium** $\longrightarrow$

A worker, knowing his MPL, will choose education level of:

$$\begin{cases} 0 \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) < w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \\ y^* \text{ if } w(MPL^c|\mathbf{educ}^{y^*}) - c(y^*|MPL) \geq w(MPL^c|\mathbf{educ}^0) - c(0|MPL) \end{cases}$$

This will be **another** pooling equilibrium if:

For $MPL = 1$ worker : $(2 - q) - y^* > 1 - 0$

$\Rightarrow 2 - q - y^* > 1$

For $MPL = 2$ worker : $(2 - q) - y^*/2 > 1 - 0$

$\Rightarrow 2 - q - y^*/2 > 1q$

Thus conjectures will be fulfilled if $2 - q - y^* > 1$

OUTPUT RESULTS:

· All workers get either exactly y^* education.

· and are worse off than in previous pooling equilibrium:

$$\begin{cases} \text{For } MPL = 1 \text{ worker: } (2 - q) - y^* < (2 - q) \\ \text{For } MPL = 2 \text{ worker: } (2 - q) - y^* < (2 - q) \end{cases}$$

If however an employer offers $(2 - q)$ regardless of education, all workers would take that option instead, and that employer would get the same expected level of productivity.

Pooling Equilibrium will result
from value of y^* in $2 - q - y^* > 1$. QED.

Titman & Trueman (1986)

II. Signaling (Fully Separating Signaling Equilibria)

Based on Winter 2019 Accounting Theory (simplified) class notes on the paper (Prof. Trueman)

$t = 0$			$t = 1$
CEO private signal x	auditor choice $q(x)$ public signal $\theta = \mu + \epsilon_2$ where $\epsilon_2 \sim N(0,\ q(x)^{-1})$	CEO sells $(1-\alpha)\%$ of firm	Cashflow Realization μ

1 Fully Revealing Signaling Equilibrium

The goal of this paper is to show how the quality of the chosen auditor can signal the firm's value.

CEO has CARA coef. of risk-**aversion** a $\longrightarrow$

Investors are risk-**neutral** $\longrightarrow$

CEO sells $(1-\alpha)\%$ of firm & α is exogenous $\longrightarrow$

$\exists$ risk-free asset yielding 0% $\longrightarrow$

Diffuse priors of firm value μ $\longrightarrow$

CEO gets private signal $x = \mu + \epsilon_1$ $\longrightarrow$

public signal $\theta = \mu + \epsilon_2$ $\longrightarrow$

$\epsilon_1 \sim N(0, h^{-1}) \perp \epsilon_2 \sim N(0, q^{-1})$ $\longrightarrow$

precision q depends on CEO's auditor quality $\longrightarrow$

auditor quality is function of CEO's $x : q(x)$ $\longrightarrow$

mkt posterior of μ given auditor quality: $f(q*)$ $\longrightarrow$

$E[U^{CEO}(W)|x] = \int\int U(W) g(\theta, \mu|x) d\mu d\theta$ $\longrightarrow$

$W = W_0 - I + (1-\alpha)E^{mkt}[\mu|\theta, q] - c(q) + \alpha\mu$ $\longrightarrow$

where: W_0 is CEO's initial wealth $\longrightarrow$

$c(q)$ is cost of auditor of quality q & $c'(q) > 0$ $\longrightarrow$

and I is amount CEO invested in the project $\longrightarrow$

Show $\exists$ a Fully Separating Signaling Equilibria

by proving $\exists$ a signal inference schedule

CEO's objective function is to maximimize:

$E[W|x] - \frac{a}{2} var(W|x)$ given his CARA utility

Given that priors are diffuse: $E^{CEO}[\mu|x] \sim N(x,\ h^{-1})$

and firm value: $E^{mkt}[\mu|\theta, q^*] = \frac{q*}{q*+h}\theta + \frac{h}{q*+h} f(q*)$

Notes simply give **equilibrium** signaling schedule:

$$f(q*) = \frac{\int_{q_{min}}^{q^*} c'(y)(y+h)dy}{(1-\alpha)h} + \frac{a(1+\alpha)[ln(q^*+h) - ln(q_{min}+h)]}{2h} + z$$

from which we can tell that $f(q*)$ is $\nearrow$ in x (in equilib.)

(z is not elaborated on in the notes).

CEO's expectation of market price in equilibrium:

$E^{CEO}[E^{mkt}[\mu|\theta, q^*] = \frac{q*}{q*+h} E^{CEO}[\theta|q^*] + \frac{h}{q*+h} f(q*)]]$

using consistency condition: $E^{CEO}[\theta|q^*] = f(q*)$

gives $E^{CEO}[E^{mkt}[\mu|\theta, q^*]] = f(q*)$

$\longrightarrow$ $f(q*)$ is $\nearrow$ in x in equilibrium.

The equilibrium signaling schedule shows that the CEO with the least favorable info consistent with taking on the project signals with minimum precision q_{min}.

Darrough & Stoughton (1990)

III. Voluntary Disclosure (Partial Signaling Equilibria)

This paper looks at equilibria in the context of an Incumbent who has to trade off between losing profit share to a potential Entrant by revealing info to raise money in the market for a profitable project.

Incumbant decides whether to release info	Incumbant raises $\$K$ from the market by selling share	Entrant decides whether to enter market at cost $\$K_E$	Upon entry, Entrant becomes informed with certainty

If Entrant E enters Monopolist M's industry:
then M becomes Incumbant I in a duopoly
To finance project $M \in$ {type H,L} raises $\$K$
M can credibly (d)isclose its type before
selling fraction $\alpha(d) = \frac{K}{V(d)}$ of the firm where
$V(d)$: market's valuation given available info
Afterwards E decides whether to enter
It costs $\$K_E$ for K to enter M's industry
Upon entry M's revenue falls from Π_M to Π_I
and E learns M's type if previously undisclosed
then E earns $\bar{\Pi}_E$. For $i = M$,I,E profits Π_i are:
$\bar{\Pi}_i$ (if M was type H) $> \underline{\Pi}_i$ (if M was type L)
Assume $\underline{\Pi}_M \geq \bar{\Pi}_I$ and $\bar{\Pi}_I > \underline{\Pi}_I > K > 0$
d_j is the probability M discloses given type
where $d_1 = P(d = H|H)$, $d_2 = P(d = L|L)$
$e = P(entry|ND)$ where $ND =$ No disclosure
E enters if $d = H$ since $\bar{\Pi}_E > K_E > \underline{\Pi}_E > 0$
$p = P(H)$ is prior belief M is H type while
$q = P(H|d = ND)$ is posterior given ND
Sequential equilibrium defined by $\{d_1,d_2,e,q\}$
Even with no disclosure (ND), E would enter
if: $\mathrm{E}(\Pi_E|ND) = q\bar{\Pi}_E + (1-q)\underline{\Pi}_E > K_E$, or if:
$q > \frac{K_E - \underline{\Pi}_E}{\bar{\Pi}_E - \underline{\Pi}_E} \equiv \mu$ can be intepreted as entry cost
$e = 1$ if $q > \mu$, $q = \mu$ if $e \in (0,1)$, $e = 0$ if $q < \mu$
$V(d) = \mathbf{E}(\Pi_I|d,e)$: value of I given E's entry
M optimizes $\max_{d \in \{D,ND\}} \mathrm{E}(\Pi_I \mid d,e) - K$
Note: $V(d) = e(1-\frac{K}{V})\bar{\Pi}_I + (1-e)(1-\frac{K}{V})\bar{\Pi}_M$
is the valuation under ND, where $V =$
$e(\underline{\Pi}_I + q(\bar{\Pi}_I - \underline{\Pi}_I)) + (1-e)(\underline{I}_M + q(\bar{\Pi}_M - \underline{I}_M))$
Show that all 3 types of signaling equilibria exist
depending on whether $q > \mu$, $q = \mu$ or $q < \mu$

$\exists$ **A fully revealing equilibrium if** $q > \mu \Rightarrow e = 1$
defined by disclosure/entry policies $\{d_1 \in [0,1], d_2 = 1, e = 1\}$
It is sufficient to show that both types of M (L & H)
would prefer diclosure & $q = 1$ is a consistent belief:

$$\text{for } L : V(d) = \begin{cases} \underline{I}_M - K & \text{if } d = L \ (E \text{ won't enter}) \\ \left(1 - \frac{K}{V}\right)\underline{\Pi}_I & \text{if } \ d = ND \end{cases}$$

$$V = e(\underline{\Pi}_I + q(\bar{\Pi}_I - \underline{\Pi}_I)) + (1-e)(\underline{I}_M + q(\bar{\Pi}_M - \underline{I}_M)) = \bar{\Pi}_I$$

Thus $K \leq \frac{\bar{\Pi}_I(\underline{I}_M - \underline{I}_I)}{\bar{\Pi}_I - \underline{I}_I}$ knowing that $\frac{(\underline{\Pi}_M - \underline{\Pi}_I)}{(\bar{\Pi}_I - \underline{\Pi}_I)} \geq 1$
And type H cannot prevent E from entering given $e = 1$.:

$\exists$ **A pooling equilibrium if** $q < \mu \Rightarrow e = 0$
defined by disclosure/entry policies $\{d_1 = 0, d_2 = 0, e = 0\}$
Type H will prefer ND if $\left(1 - \frac{K}{V}\right)\bar{\Pi}_M \geq \bar{\Pi}_I - K$
$K \leq \frac{V\left(\bar{\Pi}_M - \bar{\Pi}_I\right)}{\bar{\Pi}_M - V}$ holds since we know that $\bar{\Pi}_I \leq \underline{\Pi}_{\mathbf{M}} < V$
Investor posteriors become: $q = \frac{p(1-d_1)}{p(1-d_1)+(1-p)(1-d_2)} = p$
And since $e = 0$, L types will prefer ND with certainty.:

$\exists$ **A partially revealing equilibrium if** $q = \mu$
defined by policies: $\left\{d_1 = 0, d_2 = \frac{\mu - p}{(1-p)\mu}, e = e^* \in (0,1)\right\}$
This will only occur if posterior given ND leave E indifferent to entering, meaning $q = \mu$. Since here only the type H incumbents always choose ND: $q > p$. This yields the necessary condition that $\mu > p$. Note that when $d_2 = \frac{\mu - p}{(1-p)\mu}$, the posterior is $q = \frac{p}{p + (1 - \frac{\mu - p}{(1-p)\mu})(1-p)} = \frac{p}{p + 1 - p - \frac{\mu - p}{\mu}} = \frac{p}{p/\mu} = \mu$

$$V = e^*\left(\underline{\Pi}_I + \mu\left(\bar{\Pi}_I - \underline{\Pi}_I\right)\right) + (1-e^*)\left(\underline{\Pi}_M + \mu\left(\bar{\Pi}_M - \underline{\Pi}_M\right)\right)$$

Since L is indifferent between $d = L$ & ND, e^* much be s.t.:
$\underline{\Pi}_M - K = \left(1 - \frac{K}{V}\right)\left(e^*\underline{I}_I + (1-e^*)\,\underline{\Pi}_M\right)$. Given $\Pi_M \geq \bar{\Pi}_I$, $\exists$ entry probability e^* s.t. above holds. To verify that at e^*, type H prefers ND: $\bar{\Pi}_I - K < \left(1 - \frac{K}{V}\right)\left(e^*\bar{\Pi}_I + (1-e^*)\,\bar{\Pi}_M\right)$
But: $\bar{\Pi}_I - K \leq \underline{\Pi}_M - K = \left(1 - \frac{K}{V}\right)\left(e^*\underline{\Pi}_I + (1-e^*)\,\underline{\Pi}_M\right)$

$$\left(1 - \frac{K}{V}\right)\left(e^*\underline{I}_I + (1-e^*)\,\underline{\Pi}_M\right) < \left(1 - \frac{K}{V}\right)\left(e^*\bar{\Pi}_I + (1-e^*)\,\bar{\Pi}_M\right)$$

all 3 types of signaling equilibria exist in this setting and depend on whether $q > \mu$, $q = \mu$, or $q < \mu$

Dye (1985) + Jung & Kwon (1988)

III. Voluntary Disclosure (Partial Signaling Equilibria)

Here partial disclosure can arise even when there are no disclosure costs.

0	1	1	2
CEO sees private signal with probability $1-p$	CEO can choose to disclose it it perfectly reveals x	Investors price firm and trade	Cashflow x is realized

Firm has risky cashflow with prior mean μ →
Investors are risk **neutral** →
at t=0, CEO sees private signal with prob. $1-p$ →
If received, it perfectly reveals x →
CEO can choose to credibly disclose it →
Disclosure is costless →
CEO cannot credibly convey he's uninformed →
CEO's goal is to maximize firm valuation →
Show $\exists$ a partially revealing equilibrium →

If CEO discloses x, t=1 price will equal x

otherwise price considering that:

$Prob(\text{CEO uninformed}) = p$

$Prob(\text{CEO witholding}) = (1-p)F(\hat{x})$

$Prob(\text{CEO discloses}) = (1-p)[1-F(\hat{x})]$

where $F(\hat{x}) = Prob(x \leq \hat{x}) = \int_{\underline{x}}^{\overline{x}} dF(x)$

Now given non-disclosure (ND):

$$E[x|ND] = \frac{p}{p+(1-p)P(x<\hat{x})}E[x] + \frac{(1-p)P(x<\hat{x})}{p+(1-p)P(x<\hat{x})}E[x|x<\hat{x}]$$

$$E[x|ND] = \frac{p}{p+(1-p)F(\hat{x})}\mu + \frac{(1-p)F(\hat{x})}{p+(1-p)F(\hat{x})}\int_{\underline{x}}^{\hat{x}} \frac{xdF(x)}{F(\hat{x})}$$

$$= \frac{p}{p+(1-p)F(\hat{x})}\mu + \frac{(1-p)F(\hat{x})}{p+(1-p)F(\hat{x})}\int_{\underline{x}}^{\hat{x}} xdF(x)$$

Authors then show threshold level of $\hat{x}$ is unique.

→ $\exists$ a unique $\hat{x}^*$ s.t. $\hat{x}^*$ $\Rightarrow$ partially revealing equilibrium

Einhorn (2007)

III. Voluntary Disclosure (Partial Signaling Equilibria)

This paper departs from the typical setting of voluntary disclosure in a rational expectations model by the addition of an assumption that investors are uncertain about the manager's reporting objective.

CEO sees private signal $\tilde{s}$ with probability τ	Motivated by his type $\tilde{t}$, CEO decides $D(\tilde{t})$ whether to disclose $\tilde{s}$	Investors price the firm	Firm value $\tilde{v}$ is realized

Investors are risk **neutral**

Uncertain firm value $\tilde{v} \sim N(\mu, \sigma^2)$

CEO sees private signal $\tilde{s}$ with probability τ

where $\tilde{s} \sim N(0,1)$

denote the pdf of $\tilde{s}$ by z and cdf by Φ

$\tilde{s}$ positively correlated with $\tilde{v}$: $\rho(\tilde{v},\tilde{s}) > 0$

CEO can decide (D) to freely & credibly

disclose $\tilde{s}$ $(D = dis)$ or keep quiet $(D = nd)$

His strategy depends on his private type $D(\tilde{t})$

& his conjecture of mkt pricing rule $\hat{P}(D,s)$

determined by mkt's conjecture about his $\hat{D}(\tilde{t})$

where type $\tilde{t} \in \{$***inf****lating*, ***def****lating*$\}$

Investor priors: $Prob(t = inf) = \pi$

and $Prob(t = def) = (1-\pi)$

If $\pi = 0$ or 1: equilibrium is fully revealing,

as CEO will always dis & $P(dis,s) = \mu + \rho\sigma s$

Show $\exists s^*$ s.t. $s^* \Rightarrow$ partial equilibrium

Then show it in the case where $\tau = 0$

and instead the cost to acquire $\tilde{s}$ is $c > 0$

c affects the CEO personally and not the firm

note: $P()$ here refers to pricing rule

$Prob()$ is for probability

We start by case where $\tau = 1$, in equilibrium : $\forall s \in \Re$:

$$D(\text{inf}, s) = \hat{D}(\text{inf}, s) = \begin{cases} dis \text{ if } s \geq s^* \\ nd \text{ if } otherwise \end{cases}$$

$$D(def, s) = \hat{D}(def, s) = \begin{cases} dis \text{ if } s \leq s^* \\ nd \text{ if } otherwise \end{cases}$$

$$P(dis,s) = \hat{P}(dis,s) = \mu + \rho\sigma s$$

$$P(nd,s) = \hat{P}(nd,s) = E[\tilde{v} \mid nd] = \mu + \rho\sigma E[\tilde{s} \mid nd]$$

$$= \mu + \rho\sigma[Prob(inf|nd)E\left[\tilde{s} \mid inf, nd\right] + Prob(def|nd)E\left[\tilde{s} \mid def, nd\right]]$$

$$= \mu + \rho\sigma[Prob(inf|nd)E\left[\tilde{s} \mid \tilde{s} \leq s^*\right] + Prob(def|nd)E\left[\tilde{s} \mid \tilde{s} \geq s^*\right]]$$

$$Prob(inf|nd) = \frac{\pi\Phi(s^*)}{\pi\Phi(s^*)+(1-\pi)(1-\Phi(s^*))} \ ; \ E\left[\tilde{s} \mid \tilde{s} \leq s^*\right] = -\frac{z(s^*)}{\Phi(s^*)}$$

$$Prob(def|nd) = \frac{(1-\pi)(1-\Phi(s^*))}{\pi\Phi(s^*)+(1-\pi)(1-\Phi(s^*))} \ ; \ E\left[\tilde{s} \mid \tilde{s} \geq s^*\right] = \frac{z(s^*)}{1-\Phi(s^*)}$$

$$P(nd,s) = \mu + \rho\sigma\left[\frac{-\pi\Phi(s^*)z(s^*)(1-\Phi(s^*))/\Phi(s^*)+(1-\pi)(1-\Phi(s^*))z(s^*)}{(1-\Phi(s^*))(\pi\Phi(s^*)+(1-\pi)(1-\Phi(s^*)))}\right]$$

$$= \mu + \rho\sigma\frac{-\pi z(s^*)+(1-\pi)z(s^*)}{\pi\Phi(s^*)+(1-\pi)(1-\Phi(s^*))} = \mu - \rho\sigma\frac{(2\pi-1)z(s^*)}{(2\pi-1)\Phi(s^*)+(1-\pi)}$$

With uncertain info endowment $0 < \tau \leq 1$, the following changes:

$$Prob(inf|nd) = \frac{\tau\pi\Phi(s^*)}{(1-\tau)+\tau\pi\Phi(s^*)+\tau(1-\pi)(1-\Phi(s^*))}$$

$$Prob(def|nd) = \frac{\tau(1-\pi)(1-\Phi(s^*))}{(1-\tau)+\tau\pi\Phi(s^*)+\tau(1-\pi)(1-\Phi(s^*))}$$

$$P(nd,s) = \mu - \rho\sigma\frac{\tau(2\pi-1)z(s^*)}{\tau(2\pi-1)\Phi(s^*)+(1-\tau\pi)}$$

Disclosure costs: if $\tau = 0$ & instead cost to acquire $\tilde{s}$ is $c > 0$ then the points of indifference for an inflating and deflating CEO are:

$$P\left(\text{nd}, s^*_{\text{inf}}\right) = P\left(\text{inf}, s^*_{\text{def}}\right) - c; -P\left(\text{nd}, s^*_{\text{def}}\right) = -P\left(\text{dis}, s^*_{\text{def}}\right) - c$$

For nd: $P(\text{nd}, s^*_{\text{inf}}) = P(\text{nd}, s^*_{\text{def}}) \Rightarrow P(dis, s^*_{inf}) - P(dis, s^*_{def}) = 2c$

$$P(nd,s) : (\mu + \rho\sigma s^*_{inf}) - (\mu + \rho\sigma s^*_{def}) = 2c \Leftrightarrow \rho\sigma(s^*_{inf} - s^*_{def}) = 2c$$

$$s^*_{\text{inf}} = s^*_{\text{def}} + \frac{2c}{\rho\sigma} \qquad \text{then:}$$

$$Prob(inf|nd) = \frac{\pi\Phi(s^*_{inf})}{\pi\Phi(s^*_{inf})+(1-\pi)(1-\Phi(s^*_{inf}))} \ ; \ E[\tilde{s} \mid \tilde{s} \leq s^*_{inf}] = -\frac{z\left(s^*_{inf}\right)}{\Phi\left(s^*_{inf}\right)}$$

$$Prob(def|nd) = \frac{(1-\pi)(1-\Phi(s^*_{def}))}{\pi\Phi(s^*_{def})+(1-\pi)(1-\Phi(s^*_{def}))} \ ; \ E[\tilde{s} \mid \tilde{s} \geq s^*_{def}] = \frac{z\left(s^*_{def}\right)}{1-\Phi\left(s^*_{def}\right)}$$

$$P(\text{nd}, s) = \mu - \rho\sigma\frac{\tau\pi z(s^*_{\text{inf}})-\tau(1-\pi)z(s^*_{\text{def}})}{1-\tau+\tau\pi\Phi\left(s^*_{\text{inf}}\right)+\tau(1-\pi)\left(1-\Phi\left(s^*_{\text{def}}\right)\right)}$$

In equilibrium, $-P\left(\text{nd}, s^*_{\text{def}}\right) = -P\left(\text{ dis }, s^*_{\text{def}}\right) - c$:

$$s^*_{def} + \frac{\tau\pi z(s^*_{def}+2c/\rho\sigma)-\tau(1-\pi)z(s^*_{def})}{1-\tau+\tau\pi\Phi(s^*_{def}+2c/\rho\sigma)+\tau(1-\pi)(1-\Phi(s^*_{def}))} + \frac{c}{\rho\sigma} = 0.$$

Solving this gives unique s_{def} yielding a partial equilibrium. QED.

$\exists$ a unique $\hat{s^*}$ s.t. $\hat{s^*}$ $\Rightarrow$ partial reveal equilib.

Fischer & Verrechia (2000)

III. Voluntary Disclosure (Fully Revealing Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

CEO privately sees $\tilde{e}$ — CEO reports r — Mkt assess.: $E^{Mkt}[\tilde{v}|r, \hat{b}(e,x)]$ — Trading — Realization v

This paper shows that uncertainty regarding managerial incentives can result in biasing activity by the managers that affects firm value.

CEO is risk **neutral** →
Public priors on $\tilde{v} \sim N(0, \sigma_v^2)$ →
CEO privately sees signal $\tilde{e} = \tilde{v} + \tilde{n}$ →
where $\tilde{n} \sim N(0, \sigma_n^2)$ →
Commonly known that $\tilde{v} \perp \tilde{n}$ →
After seeing $\tilde{e} = e$: CEO reports r →
$r = e + b$ where b is CEO report bias →
Given x CEO maxes: $x\hat{P} - \frac{cb^2}{2}$ →
where random event $\tilde{x} \sim N(\mu_x, \sigma_x^2)$ →
has outcome only CEO sees: $\tilde{x} = x$ →
$c > 0$ is publicly known cost parameter →
$cb^2/2$ is CEO's known report bias cost →
CEO choice of bias is $b(e,x)$ →
$\hat{b}(e,x)$: Mkt conjecture of CEO's $b(e,x)$ →
has linear form $\hat{b}(e,x) = \hat{\lambda}_e e + \hat{\lambda}_x x + \hat{\delta}$ →
In equilibrium: $\hat{b}(e,x) = b(e,x) \;\; \forall\{e,x\}$ →
Mkt assess: $P(r) = E^{Mkt}[\tilde{v} \mid r, \hat{b}(e,x)]$ →
$\hat{P}$: CEO conject. of market pricing rule →
has linear form $\hat{P}(r) = \hat{\beta}r + \hat{\alpha}$ →
In equilib.: $\hat{P}(r) = P(r) = \beta r + \alpha \; \forall r$ →

Show that the coefficients on the linear conjectures converge to specific economic parameter based terms, thus proving that $\exists$ a Fully Separating Equilibrium in this setting

CEO's conjecture of market price dynamics given his report r : $P = \hat{\beta}r + \hat{\alpha} = \hat{\beta}e + \hat{\beta}b + \hat{\alpha}$

CEO maximizes $x(\hat{\beta}e + \hat{\beta}b + \hat{\alpha}) - \frac{cb^2}{2}$ w.r.t. b

$0 = \hat{\beta}x - cb \Rightarrow b(e,x) = \frac{\hat{\beta}}{c}x$: optimal bias function.

Knowing this, investors estimate the coefficients of their conjecture of CEO's bias function: $b(e,x) = \lambda_e e + \lambda_x x + \delta$

as: $\quad \lambda_e = 0, \quad \lambda_x = \frac{\hat{\beta}}{c}, \quad \delta = 0$

Given this, investors assess value of firm as:

$$E[\tilde{v} \mid r, \hat{b}(e;x)] = E[\tilde{v} \mid r] = E[\tilde{v} \mid e + \hat{\lambda}_x x]$$

$$= E[\tilde{v} \mid \tilde{v} + \tilde{n} + \hat{\lambda}_x x] = E[\tilde{v}] + \frac{cov(\tilde{v}, \tilde{v}+\tilde{n}+\hat{\lambda}_x x)}{var(\tilde{v}+\tilde{n}+\hat{\lambda}_x x)}(r - E[r])$$

$$E[\tilde{v} \mid r, \hat{b}(e,x)] = \frac{\sigma_v^2}{\sigma_v^2+\sigma_n^2+\hat{\lambda}_x^2\sigma_x^2}(r - \hat{\lambda}_x\mu_x)$$

Knowing this, CEO estimates the coefficients in his conjecture of investor's pricing strategy, $\hat{P}(r) = \hat{\beta}r + \hat{\alpha}$,

as $\beta = \frac{\sigma_v^2}{\sigma_v^2+\sigma_n^2+\hat{\lambda}_x^2\sigma_x^2}$ and $\alpha = -\beta\hat{\lambda}_x\mu_x$

Plugging $\lambda_x = \frac{\beta}{c}$, in equilibrium, $\beta = \frac{\sigma_v^2}{\sigma_v^2+\sigma_n^2+\left(\frac{\beta}{c}\right)^2\sigma_x^2}$

Giving: $\beta^3\sigma_x^2 + \beta\left(\sigma_v^2+\sigma_n^2\right)c^2 - \sigma_v^2c^2 = 0$

This gives us a solution that also lets us solve for α as a function of economic parameters, proving that $\exists$ a Fully Separating Equilibrium in this setting. QED.

→ Conjecture coefficients as functions of economic parameters proves that $\exists$ a Fully Separating Equilibrium in this setting

Hughes & Pae (2004)

III. Voluntary Disclosure (Partially Revealing Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

This paper explores the real effects of partial disclosure equilibria on the firm.

Mandatory Disclosure (subscript: $_{man}$)

CEO decides whether to pay c to be CEO_I & privately know precision s perfectly	CEO observes private signal $y = x + \epsilon$ where $\epsilon \sim N(0, s^{-1})$	It's **mandatory** that CEO releases all private info he acquired	To finance project CEO sells firm for $E^{mkt}[x\|info]$	Project realization x

CEO & traders are risk **neutral** →
CEO has project that requires outlay k →
It's worthless without the investment →
Project has positive NPV ($\theta > k$) →
Public priors on PV: $x \sim N(\theta, h^{-1})$ →
CEO sells firm to investors to raise k →
Mandatory discl. mkt posterior: $E^{mkt}_{man}[x|info]$ →
Before sale: CEO sees private signal: $y = x + \epsilon$ →
where $\epsilon \sim N(0, s^{-1})$; no one knows value of s →
Before seeing y: CEO can buy info on s →
This info costs c and perfectly reveals s →
c is random & only known to the CEO →
Denote CEO who got info on s: CEO_I →
Denote CEO who didn't get info on s: CEO_U →
All disclosures are assumed truthful →
$Prob(CEO_I) = \lambda$ is exogenously imposed →
$E[\frac{s}{s+h}|y, ND]$: posterior of $\frac{s}{s+h}$ given y & ND →
it's small s.t.: $E[\frac{s}{s+h}|y, ND] << E[\frac{h}{s+h}|y, ND]$ →
This is a fully separating equilibrium →

Show that this fully revealing equilibrium has real effects on whether the project is undertaken

Let $D=$ disclosure of s, $ND=$ no disclosure of s.

Under the **mandatory** disclosure case we have:

$$Prob(CEO_I \Leftrightarrow D) = \lambda \;\&\; Prob(CEO_U \Leftrightarrow ND) = 1 - \lambda$$

$$E^{CEO}(x|y, D) = \frac{s}{s+h}y + \frac{h}{s+h}\theta$$

$$E^{CEO}(x|y, ND) = E[\frac{s}{s+h}|y, ND]y + (1 - E[\frac{s}{s+h}|y, ND])\theta$$

Then investor expectation of net value of the firm is:

Case 1: Given Disclosure of s (D)

Here $CEO_I \Leftrightarrow D$, so $E^{mkt}_{man}[x - k|y, D] = \frac{s}{s+h}y + \frac{h}{s+h}\theta - k$

$$= \frac{s}{s+h}y + (1 - \frac{s}{s+h})\theta - k = \frac{s}{s+h}(y - \theta) + (\theta - k)$$

(A) Case 1 relations show that project will be taken regardless of the value of $\frac{s}{s+h}$ **if** $y > k$. But **if** $y < k$, project will be taken only if $\frac{s}{s+h}$ is small enough

Case 2: Given No-Disclosure of s (ND)

Here $CEO_U \Leftrightarrow ND$, so $E^{mkt}_{man}[x - k|y, ND]$

$$= E^{mkt}_{man}[\frac{s}{s+h}|y, ND]y + (1 - E^{mkt}_{man}[\frac{s}{s+h}|y, ND])\theta - k$$

$$E^{mkt}_{man}[x - k|y, ND] = E^{mkt}_{man}[\frac{s}{s+h}|y, ND](y - \theta) + (\theta - k)$$

(B) Case 2 relations show that project will be taken regardless of the value of $E^{mkt}_{man}[\frac{s}{s+h}|y, ND]$ **if** $y > k$. But **if** $y < k$, project will be taken only if $E^{mkt}_{man}[\frac{s}{s+h}|y, ND]$ is small enough

→ Real effects **(A)** and **(B)**

Voluntary Disclosure (subscript: $_{vol}$)

Here investors do not know whether no disclosure (ND) means that the CEO did not collect information on s ($CEO_U|ND$) or if CEO is informed but withholding said information on s ($CEO_I|ND$).

CEO decides whether to pay c to be CEO_I & privately know precision s perfectly	CEO observes private signal $y = x + \epsilon$ where $\epsilon \sim N(0, s^{-1})$	CEO discloses level of y and decides whether to **voluntarily** disclose info on s if he acquired it	To finance project CEO sells firm for $E^{mkt}[x\|info]$	Project realization x

CEO & traders are risk **neutral**

CEO has project that requires outlay k

It's worthless without the investment

Project has positive NPV ($\theta > k$)

Public priors on PV: $x \sim N(\theta, h^{-1})$

CEO sells firm to investors to raise k

Voluntary disclosure mkt posterior: $E^{mkt}_{vol}[x|info]$

Before sale: CEO sees private signal: $y = x + \epsilon$

where $\epsilon \sim N(0, s^{-1})$; no one knows value of s

Before seeing y: CEO can buy info on s

This info costs c and perfectly reveals s

c is random & only known to the CEO

Denote CEO who got info on s: CEO_I

Denote CEO who didn't get info on s: CEO_U

CEO cannot credibly convey he is CEO_U

All disclosures are assumed truthful

$E^{mkt}_{vol}[\frac{s}{s+h}|y, ND]$: mkt posterior given y & ND

is small: $E^{mkt}_{vol}[\frac{s}{s+h}|y, ND] << E^{mkt}_{vol}[\frac{h}{s+h}|y, ND]$

This is a partially revealing equilibrium

Show that this partially revealing equilibrium has real effects on whether the project is undertaken

Let D= disclosure of s, ND= no disclosure of s.

$$E^{CEO}(x|y, D) = \tfrac{s}{s+h}y + \tfrac{h}{s+h}\theta$$

$$E^{CEO}(x|y, ND) = E[\tfrac{s}{s+h}|y, ND]y + (1 - E[\tfrac{s}{s+h}|y, ND])\theta$$

Then investor expectation of net value of the firm is:

Case 1: Given Disclosure of s (D)

$$E^{mkt}_{vol}[x - k|y, D] = E^{mkt}_{man}[x - k|y, D] = \tfrac{s}{s+h}y + \tfrac{h}{s+h}\theta - k$$

$$= \tfrac{s}{s+h}y + (1 - \tfrac{s}{s+h})\theta - k = \tfrac{s}{s+h}(y - \theta) + (\theta - k)$$

(A) Case 1 has the same result as in mandatory disclosure: The relations show that project will be taken regardless of the value of $\frac{s}{s+h}$ **if** $y > k$. But **if** $y < k$, project will be taken only if $\frac{s}{s+h}$ is small enough

Case 2: Given No-Disclosure of s (ND)

$$E^{mkt}_{vol}[x - k|y, ND]$$

$$= E^{mkt}_{vol}[\tfrac{s}{s+h}|y, ND]y + (1 - E^{mkt}_{vol}[\tfrac{s}{s+h}|y, ND])\theta - k$$

$$E^{mkt}_{vol}[x - k|y, ND] = E^{mkt}_{vol}[\tfrac{s}{s+h}|y, ND](y - \theta) + (\theta - k)$$

(B) Case 2 relations show that project will be taken regardless of the value of $E^{mkt}_{vol}[\frac{s}{s+h}|y, ND]$ **if** $y > k$. But **if** $y < k$, project will be taken only if $E^{mkt}_{vol}[\frac{s}{s+h}|y, ND]$ is small enough

Now investors aren't sure if CEO is CEO_U or CEO_I so they put some weight on $Prob(CEO_I|ND)$ giving result:

$$E^{mkt}_{man}[\tfrac{s}{s+h}|y, ND] < E^{mkt}_{vol}[\tfrac{s}{s+h}|y, ND] \text{ when } y < \theta$$

$$E^{mkt}_{man}[\tfrac{s}{s+h}|y, ND] > E^{mkt}_{vol}[\tfrac{s}{s+h}|y, ND] \text{ when } y > \theta$$

Real effects **(A)** and **(B)**

McNichols & Trueman (1994)

III. Voluntary Disclosure (Fully Revealing Signaling Equilibria)

This paper shows the effects of information acquisition and trading before public disclosure.

Basic framework follows Kyle (1985) with 2 changes: (1) informed trader now has a short term holding strategy and (2) after he has made his purchase, there is a possible public signal before he closes his position which he now needs to consider.

0	1	2 starts	2 ends	3
Informed trader sees $\theta = v + \epsilon_1$	1st round of trading Informed opens position	Firm discloses z with probability p	2nd round of trading Informed closes position	Cash flow v

risky asset v prior: $v \sim N(\bar{v}, \sigma^2)$ $\longrightarrow$

t=1: informed sees signal $\theta = v + \epsilon_1$ $\longrightarrow$

where $\epsilon_1 \sim N(0, \sigma_1^2)$ $\longrightarrow$

Risk neutral informed t=1 demand: x $\longrightarrow$

Liquidity t=1 demand: $\mu \sim N(0, \sigma_u^2)$ $\longrightarrow$

Market maker sets price $P_1 = E(v|y)$ $\longrightarrow$

where y is order flow $y = x + \mu$ $\longrightarrow$

t=2: with probabiliy p firm discloses: $\longrightarrow$

$z = v + \epsilon_2$ where $\epsilon_2 \sim N(0, \sigma_2^2)$ $\longrightarrow$

and $cov(\epsilon_1, \epsilon_2) = \sigma_{1,2}$ $\longrightarrow$

Then both traders close their positions $\longrightarrow$

Market maker sets price $P_2 = E(v|y, z)$ $\longrightarrow$

We start with Kyle (1985)'s framework: $\longrightarrow$

where informed trader's conjecture of $P_1 = \mu + \lambda y$ $\longrightarrow$

depends on mkt maker's conjecture of $x = \alpha + \beta\theta$ $\longrightarrow$

with both knowing eachother's conjecture forms $\longrightarrow$

have coefs: $\beta = \left[\frac{\sigma_u^2}{\sigma_v^2 + \sigma_\epsilon^2}\right]^{0.5}$; $\lambda = \frac{\sigma_v^2}{2\sigma_u^2(\sigma_v^2+\sigma_\epsilon^2)^{0.5}}$, $\longrightarrow$

and again $x = \alpha + \beta\theta = -\beta\bar{v} + \beta\theta = \beta(\theta - \bar{v})$ $\longrightarrow$

$P_1 = \mathrm{E}(v \mid y) = \bar{v} + \lambda y$ $\longrightarrow$

if z not disclosed: $P_1 = P_2$ $\longrightarrow$

Informed can acquire info on θ's σ_1^2 for $C_1(\frac{1}{\sigma_1^2})$ $\longrightarrow$

Show $\exists$ Partially Revealing Rational Expectations Equilibria

The new price at t=2 conjecture is:

$$P_2 = \mathrm{E}(v \mid y, z) = \bar{v} + b_1 y + b_2(z - \bar{v})$$

which the paper solves (assuming $b_2 > 0$):

$$b_1 = \frac{(b_2)\sigma_1^2\left(\sigma^2+\sigma_1^2\right)^{1/2}\left(\sigma_2^2-\sigma_{12}\right)}{\sigma_u\left[2\left(\sigma^2+\sigma_1^2\right)\left(\sigma^2+\sigma_2^2\right)-\left(\sigma^2+\sigma_{12}\right)^2\right]}$$

$$b_2 = \frac{\sigma^2\left(2\sigma_1^2+\sigma^2-\sigma_{12}\right)}{2\left(\sigma^2+\sigma_1^2\right)\left(\sigma^2+\sigma_2^2\right)-\left(\sigma^2+\sigma_{12}\right)^2}$$

Plugging into informed demand's:

problem: $max_x E[(P_2 - P_1)x|\theta]$ where $x = \alpha + \beta\theta$

Paper integrates over all possible values of θ to get

Profits: $\mathrm{E}(\pi) = p[\lambda - b_1]\sigma_u = p[cov(P_2 - P_1, \theta - \bar{v})\beta]$

Profits occur if z disclosed, otherwise $P_1 = P_2$

With costly info acquisition, informed optimizes for his choice of $\frac{1}{\sigma_1^{2*}}$: $\max_{\frac{1}{\sigma_1^2}} \mathrm{E}(\pi) - C_1(\frac{1}{\sigma_1^2})$

$\longrightarrow$ Conjecture coefficients as functions of economic parameters proves that $\exists$ a Partial Rational Expectations Equilibrium in this setting

Verrechia (1983)

III. Voluntary Disclosure (Partial Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

Relative to Verrechia (1983), traders here are risk neutral as opposed to risk averse.

0	1	2	3
CEO is endowed with signal y	CEO decides whether to release y incurring disclosure cost c	Trading	Cashflow Realization μ

This paper shows that under reasonable conditions, a partial disclosure equilibrium will arise: good news disclosed and bad news withheld.

Traders are risk neutral $\longrightarrow$

Risky cashflow $\mu \sim N(y_0, h_0^{-1})$ $\longrightarrow$

Private signal: $y = \mu + \epsilon$ $\longrightarrow$

where $\epsilon \sim N(0, s^{-1})$ $\longrightarrow$

Disclosure cost: c $\longrightarrow$

Trader's piors Ω are diffuse $\longrightarrow$

We are looking for a partial equilibrium $\longrightarrow$

Show $\exists \hat{x}$ s.t. $\hat{x}$ results in a Partial Equilibrium

Let D= disclosure, ND= no disclosure.

Since we are looking for a partial equilibrium,

we want to show that $\exists \hat{x}$ such that:

(1) $\forall y \geq \hat{x}$: CEO discloses y

(2) $\forall y < \hat{x}$: no disclosure of y

For above cases, value of firm =

(1) $E(\mu|D) = E(\mu - c \mid y)$

(2) $E(\mu|ND) = E(\mu \mid y < \hat{x})$

To solve, we consider that y at threshold $y = \hat{x}$ will render CEO indifferent between disclosing and not:

$$E(\mu - c \mid y = \hat{x}) = E(\mu \mid y < \hat{x})$$

Author shows $\exists$ unique $\hat{x}$ solving above relation, proving that $\exists$ a partial equilibrium in this setting

$\longrightarrow$ $\exists$ unique $\hat{x}$ as a function of params resulting in partial signaling equilibrium

Example p.5 (here we find specific term for threshold $\hat{x}$)

traders are risk neutral $\longrightarrow$

risky cashflow $\mu \sim N(y_0, h_0^{-1})$ $\longrightarrow$

private perfect signal: $y = \mu$ $\longrightarrow$

disclosure cost: c $\longrightarrow$

trader's piors $\sim Unif(0, t)$ $\longrightarrow$

partial equilibrium exists $\longrightarrow$

Find value of threshold $\hat{x}$ that results in a partial signaling equilibrium

where

(1) $\forall y \geq \hat{x}$: CEO discloses y

(2) $\forall y < \hat{x}$: no disclosure of y

For above cases, Value of firm =

(1) $E(\mu | Diclosure) = E(y - c \mid y)$

(2) $E(\mu | No\ Diclosure) = E(y \mid y < \hat{x})$

y at threshold $y = \hat{x}$ renders manager indifferent between disclosing and not:

$$E(y - c \mid y = \hat{x}) = E(y \mid y < \hat{x})$$

$$\hat{x} - c = E[Unif(0, \hat{x})]$$

$$\hat{x} - c = \hat{x}/2$$

$$\hat{x} = 2c$$

$\longrightarrow \hat{x} = 2c$

Beyer (2008)

VI. Asymmetric Information (Fully Separating Signaling Equilibrium)

This paper studies analyst forecasting strategies and CEO's earnings management policy. When reporting earnings, CEO trades off disutility he gets from falling short of the analyst forecast against costs of manipulating earnings.

$t=1$	$t=2$	$t=3$
The analyst observes a private signal ω_1, and issues his first forecast AF_1.	With probability λ the analyst observes a second private signal ω_2, and issues a revised forecast AF_2	The manager privately observes x and issues the earnings report R.

Firm has unmanaged earnings $\widetilde{x} \sim N(\mu_x, \tau_x^{-1})$ →

Realizations of $\widetilde{x}$ only observable to CEO →

Analyst's private signal ω_1 is a noisy measure of $\widetilde{x}$ →

$\omega_1 = x + \varepsilon_1 + \varepsilon_2$; $\varepsilon_1 \sim N(0, \tau_1^{-1}) \perp \varepsilon_2 \sim N(0, \tau_2^{-1})$ →

which he bases himself on to release forecast AF_1 →

He has a λ% chance of getting another private signal →

$\omega_2 = x + \varepsilon_2$ leading him to release 2nd forecast AF_2 →

Forecasts minimize error: $\min_{CF} E[|\widetilde{R} - CF||\{\omega\}]$ →

where $CF \in \{AF_1, AF_2\}$ is his *latest* forecast →

CEO sees forecasts and then observes realized x →

Finally, CEO releases a reports R on x →

If R diverges from x, CEO incurs costs $\frac{k_m}{2}(R-x)^2$ →

If R below analyst expectations, cost: $\frac{k_f}{2}(R-CF)^2$ →

$U^{CEO}(R-AF) - \frac{k_m}{2}d^2$ and $U' > 0, U'' < 0$ →

$d = R - x$ is CEO's choice of discretionary accruals →

Equilib. defined by policies: $\{AF_1(), AF_2(), R(), P()\}$ →

$P()$ is market's pricing strategy: $P(\Omega) = E[\tilde{x}|info]$ →

$R()$ is CEO's reporting rule $R(x, AF_1, AF_2)$ which →

minimizes $\frac{k_f}{2}(R-CF)^2 + \frac{k_m}{2}(R-x)^2$ if $R < CF$ →

minimizes $\frac{k_m}{2}(R-x)^2$ if $R \geq CF$ →

Show ∃ a fully revealing equilibrium in this setting →

Proof by backwards induction ∃ fully revealing equilibrium

If the realization of unmanaged earnings exceed the analyst's most current forecast the CEO has no incentives to manipulate earnings

Hence $R(x, CF) = x$ if $x \geq CF$

If $x < CF$ then the manager solves $\min_R \frac{k_f}{2}(R-CF)^2 + \frac{k_m}{2}d^2$

and hence $R(x, CF) = x + \frac{k_f}{k_f + k_m}(CF - x)$

let $g_1(\widetilde{x}) = g(\widetilde{x} \mid \omega_1)$ and $g_2(\widetilde{x}) = g(\widetilde{x} \mid \omega_1, \omega_2)$

Then at $t = 1, 2$ the analyst minimizes

$\int_{-\infty}^{\infty} |R(\widetilde{x}, AF_t) - AF_t|\, g_t(\widetilde{x}) dx$ which is equivalent to

$\frac{k_m}{k_f + k_m} \int_{-\infty}^{AF_t} (AF_t - \widetilde{x})\, g_t(\widetilde{x}) dx + \int_{AF_t}^{\infty} (\widetilde{x} - AF_t)\, g_t(\widetilde{x}) dx$

The FOC to these optimization problem simplifies to

$$G_t(AF_t) = \frac{k_f + k_m}{k_f + 2k_m}$$

Note that the solution to this condition, $AF_t(\omega_t)$, is a function of ω_t because $G_t(\cdot)$ depends on ω_t.

The second order condition to the analyst's optimization problem is $g_t(AF_t) > 0$ and hence $G_t(AF_t)$ characterizes a unique solution. since the RHS of $G_t(AF_t)$ exceeds $\frac{1}{2}$, it follows that $AF_t > E[\widetilde{x} \mid \omega_t]$. Thus we can rewrite $G_t(AF_t)$ as

$$\frac{2}{\sqrt{\pi}} \int_0^{\frac{AF_t - E[\tilde{x}|\omega_t]}{\sqrt{2}\sigma_t}} e^{-y^2} dy = \frac{k_f + k_m}{k_f + 2k_m} - \frac{1}{2}$$

which implies $AF_t(\omega_t, \sigma_t) = E[\widetilde{x} \mid \omega_t] + \sqrt{2}\sigma_t \operatorname{erf}^{-1}\left(\frac{k_f}{k_f + 2k_m}\right)$

since $AF_1(\omega_1, \sigma_1)$ is strictly increasing in ω_1, it is one-to-one and therefore observing AF_1 is informationally equivalent to observing ω_1 (this follows because σ_1 is a known parameter of the model).

Therefore, the capital market price at $t = 1$ is satisfies

$P(AF_1) = E[\tilde{x} \mid AF_1] = E[\tilde{x} \mid \omega_1]$. similarly, $P(AF_1, AF_2) = E[\widetilde{x} \mid \omega_2]$

At $t = 3$ the price can be derived by solving $R(x, CF)$ for x. QED.

Myers & Majluf (1982)

VI. Asymmetric Information (Partial Signaling Equilibrium)

This paper describes investment and financing decisions when managers have private information about firm value which cannot be credibly convey to risk **neutral** investors. Acting in the interests of old shareholders', CEOs may pass up profitable projects as their positive NPV would be outweighed by the dilution. Thus the market interprets the decision of (not) issuing as negative (positive).

-1	0	$+1$	$+1$
CEO & Investors have priors for value of firm 's assets-in-place $E[\tilde{A}]$ and potential project $E[\tilde{B}]$	CEO sees realizations a & b decides whether to issue stock E to raise I to finance project		Investors see realizations a & b

Firm has project requiring \$$I$ outlay →

I can be partly financed by known cash (S) →

$S < I$, so firm would need to issue $E = I - S$ →

t=-1 expected value of assets: $\overline{A} = E_{-1}[\tilde{A}]$ →

at t=0 CEO sees realized value of assets: $a \geq 0$ →

t=-1 expected NPV of investment: $\overline{B} = E_{-1}[\tilde{B}]$ →

at t=0 CEO sees realized value of project: $b \geq 0$ →

CEOs act in interest of *old* (t=-1) shareholders →

Thus CEOs maximize $V_0^{old} = V(a,b,E)$ →

P : market value at t = 0 if stock is not issued →

P′ : mkt value at t = 0 of *old* shares if stock issued →

If firm forfeits investment: $V^{old} = S + a$ →

If it issues E: $V^{old} = \frac{P'}{P'+E}(E + S + a + b)$ then →

old only gain if $S + a < \frac{P'}{P'+E}(E + S + a + b) \Leftrightarrow$ →

$\frac{E}{P'+E}(S + a) < \frac{P'}{P'+E}(E + b) \Leftrightarrow \frac{E}{P'}(S + a) < E + b$ →

if $\frac{E}{P'}(S + a) > E + b$: firm does nothing $\rightarrow M$ →

if $\frac{E}{P'}(S + a) < E + b$: it issues & invests $\rightarrow M'$ →

Firm is indifferent at line where $\frac{E}{P'}(S + a) = E + b$ →

Issue fairly priced if: $P' = S + \bar{A}(M') + \bar{B}(M')$ →

where $\bar{A}(M') \equiv E^{mkt}(\tilde{A} \mid E = I - S)$ →

and $\bar{B}(M') \equiv E^{mkt}(\tilde{B} \mid E = I - S)$ →

Note that under no issue/project: $\mathrm{P} = S + \bar{A}(M)$ →

Likelihood of forfeiting project: $F(M)$ →

Expected loss due to forfeit: $L \equiv F(M)\bar{B}(M)$ →

L increases with $E = I - S$ →

Special Case 1: Pooling equilibrium (a is known)

If a is also known to investors ($\overline{A}(M') = a$) we have:

$P' = S + a + \bar{B}(M')$ and since $\overline{\mathrm{B}}(\mathrm{M'}) \geq 0$ and $\mathrm{P'} \geq \mathrm{S + a}$

The firm will issue if $\mathrm{E}\left(\frac{\mathrm{S+a}}{\mathrm{P'}}\right) < \mathrm{E + b}$

Which is satisfied given $\mathrm{b} \geq 0$, because $\frac{(\mathrm{S+a})}{\mathrm{P'}} \leq 1$

Thus firm will issue whenever $\mathrm{b} > 0$, and $\mathrm{P'} = \mathrm{S + a} + \overline{B}$

Special Case 2: Issues always dilute ($P' \leq P$)

Let a^* be the threshold of assets that renders the firm indifferent between M' and M (issuing and not). Using the indifference line, threshold a^* solves: $a^* + S = P'(1 + b/E)$

Note that $\bar{A}(M) + S > a^* + S$, because any $a < a^*$ would lead the firm to issue ($a < a^*$ implies $a^* + S < P'(1 + b/E)$).

Since $\mathrm{P} = \overline{\mathrm{A}}(M) + \mathrm{S}, \quad \mathrm{P} > \mathrm{P'}(1 + \mathrm{b/E})$.

Since $\mathrm{b} > 0, \quad \mathrm{P'}(1 + \mathrm{b/E}) \geq \mathrm{P'}$ and $P \geq P'$

Scharfstein & Stein (1990)

VI. Asymmetric Information (No Partial Signaling Equilibrium)

This paper shows the dynamics of herding in a situation where a manager is being judged by the accuracy of his private signal which he reports only after another manager has reported his.

CEO A receives signal and decides whether to invest	CEO B receives signal and decides whether to invest	Investment outcome $x \in \{x_H, x_L\}$ occurs	Market assesses $P(B = smart \mid info)$

Game is played 1 more time

Possible investment outcome $x \in \{x_H, x_L\}$ →
$x_H > 0$ occurs with probability $P(x_H) = \alpha$ →
$x_L < 0$ occurs with probability $P(x_L) = 1 - \alpha$ →
CEOs have private signal $s \in \{s_g, s_b\}$ →
smart CEOs get informative signal: →
$P(s_G \mid x_H, smart) = p > P(s_G \mid x_L, smart) = q$ →
dumb CEOs get uninformative diffuse signal: →
$P(s_G \mid x_H, dumb) = P(s_G \mid x_L, dumb) = z$ →
CEOs don't know their type, priors are public: →
Priors: $P(smart) = \theta$ and $P(dumb) = 1 - \theta$ →
There are 2 CEOs: A invests before B →
Before making his investment decision →
B can consider his signal & A's decision →
Outcomes are public even if no one invests →
Market updates its belief $\hat{\theta}$ about CEO B →
where $\hat{\theta}$ is likelihood CEO B is smart →
$\hat{\theta}$(A's signal, B's signal, ex-post $s \in \{s_g, s_b\}$) →
based on investment decision outcome →
and if B followed A's decision (herding) →
Both types equally as likely to see positive s_g →
$P(x_H \mid s_G) = \mu_G$; $P(x_H \mid s_B) = \mu_B$ →
Project is attractive if good signal is received →
$\mu_G x_H + (1 - \mu_G) x_L > 0 > \mu_B x_H + (1 - \mu_B) x_L$ →
Thus if A goes first, B would know A's signal →
Signal draws are independent unless both →
CEOs are *smart* then they're identical →
CEOs are risk neutral & maximize $\hat{\theta}$ →
Assume that $\alpha = 1/2$ and show that there is no →
fully revealing equilibrium for B in this setting →

Proof by contradiction:

Since *smart* & *dumb* are equally as likely to see positive signal s_g:

$$P(s_G \mid smart) = P(s_G \cap x_H \mid smart) + P(s_G \cap x_H^c \mid smart)$$

Knowing that $P(B \mid A) = \frac{P(A \cap B)}{P(A)} \Leftrightarrow P(A \cap B) = P(A) P(B \mid A)$

$$P(s_G \mid smart) = P(x_H) P(s_G \mid x_H, smart) + P(x_L) P(s_G \mid x_L, smart)$$

$$P(s_G \mid smart) = \alpha p + (1 - \alpha) q = P(s_G \mid dumb) = z = P(s_G)$$

$$\mu_G = P(x_H \mid s_G) = \frac{P(s_G \mid x_H) P(x_H)}{P(s_G)} = \frac{\theta p + (1-\theta) z}{z} \alpha$$

$$\mu_B = P(x_H \mid s_B) = \frac{P(s_B \mid x_H) P(x_H)}{P(s_B)} = \frac{\theta (1-p) + (1-\theta)(1-z)}{1-z} \alpha$$

Probability 2 *dumb* managers observe s_G is z^2

$$P(s_G, s_G \mid x_H \text{ or } x_L, dumb, dumb) = z^2$$

Probability 2 *smart* managers observe s_G when state is high is p

$$P(s_G, s_G \mid x_H, smart, smart) = p \quad (\text{not } p^2)$$

Note that $p = 1 - q > q = 1 - p$. If $\alpha = 1/2$ this implies that:

$$\alpha p + (1 - \alpha) q = z \Leftrightarrow p/2 + (1/2)(1 - p) = 1/2 = z$$

What market surmises if following high state x_H,

it knew that A had a bad signal & B had a good signal

(which is what B would let on if he breaks with A):

If $\hat{\theta}$(A's signal, B's signal, $s \in \{s_g, s_b\}$) $= \hat{\theta}(s_B, s_G, x_H)$

$$\hat{\theta}(s_B, s_G, x_H) = \frac{\frac{1}{2}(1-\theta) p \theta}{\frac{1}{2}(1-\theta) p \theta + \frac{1}{2}(1-\theta)(1-p)\theta + \frac{1}{4}(1-\theta)^2} = 2\theta p / (1 + \theta)$$

If instead A saw s_B & didn't invest, B would communicate the following if he followed A

$$\hat{\theta}(s_B, s_B, x_H) = \frac{2\theta(1-p)(1+\theta)}{4\theta(1-p) + (1-\theta)^2} \; ; \; \hat{\theta}(s_B, s_B, x_L) = \frac{2\theta p (1+\theta)}{4\theta p + (1-\theta)^2}$$

Whereas if he broke with A:

$$\hat{\theta}(s_B, s_G, x_H) = 2\theta p / (1 + \theta) \; ; \; \hat{\theta}(s_B, s_G, x_L) = 2\theta (1-p) / (1 + \theta)$$

continued on next page...

→ No fully reveal equilib. in this setting.

Note that $P(s_G|smart) = P(x_H)P(s_G|x_H, smart) + P(x_L)P(s_G|x_H, smart) = \alpha p + (1-\alpha)q$

and $P(s_L|smart) = P(x_L)P(s_L|x_L, smart) + P(x_H)P(s_L|x_H, smart) = (1-\alpha)p + \alpha q = p - \alpha p + \alpha - \alpha p$

$= \alpha + p - 2\alpha p = 1 - \alpha p - 1 - \alpha p + \alpha + p = 1 - \alpha p - (1-\alpha)(1-p) = 1 - (\alpha p + (1-\alpha)(1-p)) = 1 - P(s_G|smart)$

Just to confirm that $P(s_L|x_L, smart) = p$ **and** $P(s_L|x_H, smart) = q = 1-p$

Proof of equation 12 :

I will use this relation: $P(A) = P(A\&B) + P(A\&B^c)$ and $P(B \mid A) = \frac{P(A\&B)}{P(A)} = \frac{P(A\&B)}{P(A\&B)+P(A\&B^c)}$

Thus: $P(B \mid A\&C) = \frac{P((A\&C)\&B)}{P(A\&C)} = \frac{P((A\&C)\&B)}{P((A\&C)\&B)+P((A\&C)\&B^c)}$

So: $P(x_H|s_B, s_G) = \frac{P(s_B,s_G,x_H)}{P(s_B,s_G,x_H)+P(s_B,s_G,x_H^c)} = \frac{P(s_B,s_G,x_H)}{P(s_B,s_G,x_H)+P(s_B,s_G,x_L)}$

$P(s_B, s_G, x_L) = \theta p(1-\theta)(1-z) + (1-\theta)(z)\theta(1-p) + (1-\theta)z(1-\theta)(1-z)$

$P(s_B, s_G, x_L) = \frac{1}{2}\theta(1-\theta)p + \frac{1}{2}\theta(1-\theta)(1-p) + \frac{1}{4}(1-\theta)^2 = \frac{1}{2}\theta(1-\theta) + \frac{1}{4}(1-\theta)^2$ (given that z=1/2)

$P(s_B, s_G, x_H) = \theta(1-p)(1-\theta)\frac{1}{2} + (1-\theta)\frac{1}{2}\theta p + \frac{1}{4}(1-\theta)^2$

$P(s_B, s_G, x_H) = \theta(1-p)(1-\theta)\frac{1}{2} + (1-\theta)\frac{1}{2}\theta p + \frac{1}{4}(1-\theta)^2 = \frac{1}{2}\theta(1-\theta) + \frac{1}{4}(1-\theta)^2$

Thus: $P(x_H|s_B, s_G) = \frac{P(s_B,s_G,x_H)}{P(s_B,s_G,x_H)+P(s_B,s_G,x_L)} = \frac{1}{2}$ and $P(x_L|s_B, s_G) = \frac{1}{2}$.

Proof of equation 13 : Similarly,

$P(s_B, s_B, x_H) = \theta^2(1-p) + \theta(1-p)(1-\theta)(1-\frac{1}{2}) + (1-\theta)(1-\frac{1}{2})\theta(1-p) + \frac{1}{4}(1-\theta)^2$

$P(s_B, s_B, x_H) = \theta^2(1-p) + \theta(1-\theta)(1-p) + \frac{1}{4}(1-\theta)^2 = \theta(1-p) + \frac{1}{4}(1-\theta)^2$

$P(s_B, s_B, x_L) = \theta^2 p + 2\theta p(1-\theta)\frac{1}{2} + \frac{1}{4}(1-\theta)^2 = \theta^2 p + \theta(1-\theta)p + \frac{1}{4}(1-\theta)^2 = \theta p + \frac{1}{4}(1-\theta)^2$

$P(x_H|s_B, s_B) = \frac{P(s_B,s_B,x_H)}{P(s_B,s_B,x_H)+P(s_B,s_B,x_L)} = \frac{\theta(1-p)+\frac{1}{4}(1-\theta)^2}{\theta+\frac{1}{2}(1-\theta)^2} = \frac{4\theta(1-p)+(1-\theta)^2}{4\theta+2(1-\theta)^2}$ and

$P(x_L|s_B, s_B) = \frac{P(s_B,s_B,x_L)}{P(s_B,s_B,x_H)+P(s_B,s_B,x_L)} = \frac{\theta p+\frac{1}{4}(1-\theta)^2}{\theta+\frac{1}{2}(1-\theta)^2} = \frac{4\theta p+(1-\theta)^2}{4\theta+2(1-\theta)^2} = 1 - P(x_H|s_B, s_B)$

Inequality (14) states the requirement that if B receives signal s_G, he prefers to invest (and identify himself as someone who had observed s_G), rather than not invest (and masquerade as someone who had received signal s_B). There for break with A in order to reveal his signal. Note from previous page:

$\hat{\theta}(s_B, s_G, x_H) = \hat{\theta}(s_G, s_B, x_L) = \frac{2\theta p}{1+\theta}$ $\quad$ $\hat{\theta}(s_B, s_G, x_L) = \hat{\theta}(s_G, s_B, x_H) = \frac{2\theta(1-p)}{1+\theta}$

$\hat{\theta}(s_B, s_B, x_H) = \hat{\theta}(s_G, s_G, x_L) = \frac{2\theta(1-p)(1+\theta)}{4\theta(1-p)+(1-\theta)^2}$ $\quad$ $\hat{\theta}(s_B, s_B, x_L) = \hat{\theta}(s_G, s_G, x_H) = \frac{2\theta p(1+\theta)}{4\theta p+(1-\theta)^2}$

$\hat{\theta}(s_B, s_G, x_H)\,\mathrm{P}(x_H|s_B, s_G) + \hat{\theta}(s_B, s_G, x_L)\,\mathrm{P}(x_L|s_B, s_G) \geq \hat{\theta}(s_B, s_B, x_H)\,\mathrm{P}(x_H|s_B, s_G) + \hat{\theta}(s_B, s_B, x_L)\,\mathrm{P}(x_L|s_B, s_G)$

$\frac{1}{2}\hat{\theta}(s_B, s_G, x_H) + \frac{1}{2}\hat{\theta}(s_B, s_G, x_L) \geq \frac{1}{2}\hat{\theta}(s_B, s_B, x_H) + \frac{1}{2}\hat{\theta}(s_B, s_B, x_L)$

$\frac{2\theta p}{1+\theta} + \frac{2\theta(1-p)}{1+\theta} \geq \frac{2\theta(1-p)(1+\theta)}{4\theta(1-p)+(1-\theta)^2} + \frac{2\theta p(1+\theta)}{4\theta p+(1-\theta)^2} \Leftrightarrow \frac{2\theta}{1+\theta} \geq 2\theta(1+\theta)\left[\frac{1-p}{4\theta(1-p)+(1-\theta)^2} + \frac{p}{4\theta p+(1-\theta)^2}\right]$

$1 \geq (1+\theta)^2\left[\frac{(1-p)[4\theta p+(1-\theta)^2]+(p)[4\theta(1-p)+(1-\theta)^2]}{[4\theta p+(1-\theta)^2][4\theta(1-p)+(1-\theta)^2]}\right] = (1+\theta)^2\frac{4\theta p+(1-\theta)^2}{[4\theta p+(1-\theta)^2][4\theta(1-p)+(1-\theta)^2]} = (1+\theta)^2\frac{1}{4\theta(1-p)+(1-\theta)^2}$

$4\theta(1-p) + (1-\theta)^2 \geq (1+\theta)^2 \Leftrightarrow 4\theta - 4\theta p + 1 - 2\theta + \theta^2 \geq 1 + 2\theta + \theta^2 \Leftrightarrow 4\theta - 4\theta p \geq 4\theta \Leftrightarrow -4\theta p \geq 0 \rightarrow\leftarrow$

The inequality in equation 14 is indeed violated. Thus there is no fully revealing equilibrium in which manager B's investment decision depends on the signal he observes. Thus the only possible equilibria are those where manager B mimics manager A regardless of the signal, or where manager B does the opposite of manager A regardless of the signal. This is Proposition 1 in the paper.

Trueman (1994)

IV. Asymmetric Information (Partially Revealing Signaling Equilibria)

Based on Winter 2019 Accounting Theory class notes on the paper (Prof. Trueman)

This paper looks at the incentives driving forecasting analysts to herd on their earnings forecasts and to examine the implications of this herding behavior on forecast accuracy and the return-earnings relation.

Note: While the notes introduce EPS sign $\{-,+\}$, only high/low level $\{h,l\}$ of EPS matters in analysis.

Equilibrium with simultaneous forecast release

... current period starts — current period ends ...

Analyst k observes his signal $y_i^k,\ i \in \{h,l\}$ → Analyst k releases his forecast $f_i^k,\ i \in \{h,l\}$ → Realization of $e_i,\ i \in \{h,l\}$ → Market assesses k's skill: $P^{mkt}(k=g|y_i,f_i,e_i)$

Infinitely repeated game

Analyst knows his skill k, investors don't →

$k \in \{b,g\}$: $k=g$ for good, $k=b$ for bad →

k forecasts $e \in \{e_l,e_h\}$ as $f^k(y_i^k) \in \{f_l,f_h\}$ →

$P(e_l)=2t, P(e_h)=1-2t, 2t>1-2t$ →

k observes his signal for level of e: y_i^k →

$P^k(y_i^k|e_i)=k$; $P^k(y_j^k|e_i)=1-k$ →

where $i \in \{h,l\}$; $k \in \{g,b\}$ with $g>b$: →

e.g.: $P^b(y_i^b|e_i)=b$; $P^b(y_j^b|e_i)=1-b$ →

† k truthfully reports his signal: $f^k(y_i^k)=f_i$ →

† except b can lie at a rate α about his y_h^b →

such that $f^b(y_h^b)=(1-\alpha)f_h+\alpha f_l$ →

Market assesses k's skill: $P_t^{mkt}(g|y_i,f_i,e_i,P_{t-1}^{mkt})$ →

$P^{mkt}(g|f_i,e_i,P_{t-1}^{mkt})$ assessed over ∞ periods →

As $T\to\infty$: $P_t \to P_{t-1} \Rightarrow P^{mkt}(g|f_i,e_i)$ →

$\Rightarrow b$ seeks policy to maximize $E^b[P^{mkt}(g|f_i,e_i)]$ →

Market conjectures that b uses α^* to solve for →

optimal policy $max_\alpha E^b[P^{mkt}(g|f^b(y_h^b),e_i)|y_i^b]$ →

Show $\exists$ a seperating equilibrium in this setting →

Show $\exists\alpha^*$ which optimizes a b analyst's policy and yields a Partially Separating Equilibrium

Market Assessment is $P^{mkt}(g \mid f_l,e_l)=$

$$P(g \mid y_l,f_l,e_l)*P(y_l \mid,f_l,e_l)+P(g \mid y_h,f_l,e_l)*P(y_h \mid,f_l,e_l)$$

where $P(g \mid y_h,f_l,e_l)=0$ (†) and $P(g \mid y_l,f_l,e_l)=\frac{P(g,y_l,f_l,e_l)}{P(y_l,f_l,e_l)}$

$$=\frac{P(y_l,f_l,g|e_l)P(e_l)}{P(y_l,f_l|e_l)P(e_l)}=\frac{P(y_l,f_l|g,e_l)P(g|e_l)}{gP(g)+bP(b)}=\frac{gP(g)}{gP(g)+bP(b)}$$

Noting that: $P(y_l \mid f_l,e_l)=P(y_l,g \mid f_l,e_l)+P(y_l,b \mid f_l,e_l)$

$$P(y_l,g \mid f_l,e_l)=\frac{gP(g)}{gP(g)+(b+(1-b)\alpha)P(b)}=\frac{gP(g)}{gP(g)+bP(b)+\alpha(1-b)P(b)}$$

Similarly, $P(y_l,b \mid f_l,e_l)=\frac{bP(b)}{gP(g)+bP(b)+\alpha(1-b)P(b)}$

Thus, $P(y_l \mid f_l,e_l)=\frac{gP(g)+bP(b)}{gP(g)+bP(b)+\alpha(1-b)P(b)}$

And: $P^{mkt}(g \mid f_l,e_l)=\frac{gP(g)}{gP(g)+bP(b)}\frac{gP(g)+bP(b)}{gP(g)+bP(b)+\alpha(1-b)P(b)}$

So: $P^{mkt}(g \mid f_l,e_l)=\frac{gP(g)}{gP(g)+bP(b)+\alpha(1-b)P(b)}$

Similarly: $P^{mkt}(g \mid f_l,e_h)=\frac{(1-g)P(g)}{(1-g)P(g)+(1-b)P(b)+\alpha bP(b)}$

$$P^{mkt}(g \mid f_h,e_l)=\frac{(1-g)P(g)}{(1-g)P(g)+(1-\alpha)(1-b)P(b)}$$

$$P^{mkt}(g \mid f_h,e_h)=\frac{gP(g)}{gP(g)+(1-\alpha)bP(b)}$$

$$E^b[P^{mkt}(g \mid f_h,e_i)|y_h^b]=\frac{(1-g)P(g)}{(1-g)P(g)+(1-\alpha)(1-b)P(b)}P^b\left(e_l \mid y_h^b\right)+\frac{gP(g)}{gP(g)+(1-\alpha)bP(b)}P^b\left(e_h \mid y_h^b\right)$$

$$E^b[P^{mkt}(g \mid f_l,e_i)|y_h^b]=\frac{gP(g)}{gP(g)+bP(b)+\alpha(1-b)P(b)}P^b\left(e_l \mid y_h^b\right)+\frac{(1-g)P(g)}{(1-g)P(g)+(1-b)P(b)+\alpha bP(b)}P^b\left(e_h \mid y_h^b\right)$$

Given above, if market conjectures that upon seeing y_h^b, analyst b lies with probability α^* then at equilibrium, b is indifferent between the two, thus equating:

$$E^b[P^{mkt}(g \mid f_h,e_i)|y_h^b]=E^b[P^{mkt}(g \mid f_l,e_i)|y_h^b]$$

gives an expression for α^* in a Partial Equilibrium. QED.

→ In this setting $\exists\alpha^*$ st $\alpha^* \Rightarrow$ Partial Signaling Equilibrium

In order to lower the number of variables, I've simplified the possible e levels of e_h^-, e_l^-, e_l^+, e_h^+, where: $e_h^- < e_l^- < 0 < e_l^+ < e_h^+$ to simply e_h and e_l while maintaining the result given that e's pdf is symmetric & E[e]=0 (See Fig.).

herding.png

$P(e^l) = 2P(e_l^-) = 2P(e_l^+) = 2t$, $P(e_l) = 2t > 0.5 > 1 - 2t = P(e_h)$

I also extended the use of the probabilities of accurate symbols (b/g) to serve as indicators for weak/strong analysts (w/s, in the notes) again to lower the number of variables.

† This is the market's set of conjectures (S1)-(S6) in the paper. It stems from the fact that: Since low signals are more common in nature, for $k \in \{g, b\}$:$E^k[P^{mkt}(g \mid E^k[e|y_l^k], f_l)] > E^k[P^{mkt}(g \mid E^k[e|y_l^k], f_h)]$ So neither analyst has incentive to misreport low y_l's

A good g analyst has no incentive to misreport y_h^g : $E^g[P^{mkt}(g \,|\, E^g[e|y_h^g], f_h)] > E^b[P^{mkt}(g \,|\, E^b[e|y_l^b], f_h)]$ Thus g's optimal policy is never lying $\alpha^{g*} = 0$.

Some of the conditional probability relationships used here are listen below:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(B \mid A) = \frac{P(A \,\&\, B)}{P(A)} \Leftrightarrow P(B|A)P(A) = P(A \,\&\, B)$$

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A^c)\,P(A^c)}$$

Mostly not used but listed for reference:

$$P(A \mid B \cap C) = \frac{P(B \mid A \,\&\, C)P(A \mid C)}{P(B \mid C)}$$

$$\frac{P(A \,\&\, B \,\&\, C)}{P(B \,\&\, C)} = \frac{P(A \,\&\, B \,\&\, C)P(A \mid C)}{P(A \,\&\, C)P(B \mid C)}$$

$$\frac{P(A \,\&\, B \,\&\, C)}{P(B \,\&\, C)} = \frac{P(A \,\&\, B \,\&\, C)P(A \,\&\, C)}{P(A \,\&\, C)P(B \mid C)P(C)}$$

Grossman (1981)

1. Background (Unraveling)

Under some circumstances, agents will voluntarily fully disclose credible private information. Though sometimes it may be in their best interest for a seller to for instance keep product quality undisclosed, we consider an equilibrium where the seller has incentives to disclose quality, even when it's low. Here, the way adverse selection works against sellers is by forcing full disclosure, instead of a market breakdown.

ex ante

Seller chooses what set of possible product qualities D to disclose

Those who don't get price associated with min(set of possible qualities)

ex post

Buyer conjectures that quality is $min(D)$

Let I be consumer's income

γ_i is belief about likelihood of quality q_i

Expect. util. $V(\gamma,p) = \sum_i \gamma_i U(q_i, I-p)$

Given willingness to pay price $p(\gamma)$:

price p solves $V(\gamma,p) = \bar{u}$

where $\bar{u}$ is his baseline utility from not consuming the product

Seller can make **credible & true** disclosure of privately known product quality $q \in Q$

This is costlessly veriable ex-post

Let $D(q)$ be disclosure if true quality is q

Two ways consumer can infer quality q:

1) q is fully revealed, and if not ...

2) he conjectures value for unrevealed q

p_i: price buyer's willing to pay for quality q_i

Let $p_1 < p_2 < p_3 < \ldots$ for $q_1 < q_2 < q_3 < \ldots$

If seller says quality q is in $D = \{q_3, q_6, q_7\}$

Buyer will infer that $q = min(D) = q_3$

This is because disclosure is costless and seller can't lie but has max profit incentive

Show $\exists$ a consumer belief function $\gamma(D)$ that results in a fully revealing equilibrium

Here, it is consumers' conjectures about the seller's quality out of equilibrium which forces the seller to choose a particular equilibrium (ie: off-equilibrium beliefs).

The seller is unable to mislead consumers because of the rational conjectures consumers have regarding what his quality must be if he deviates from full-information.

This occurs because buyers are unable to directly conjecture the quality through other means (eg: a costly signal) and because seller's disclosure is credible but costless.

Thus consumer hold the following beliefs γ_i^e about expected quality given disclosed possible quality set D:

$$Prob(q = q_i \in D) : \gamma_i^e(D) = \begin{cases} 0 \text{ if } q_i \text{ is not lowest quality in } D \\ 1 \text{ if } q_i \text{ is the lowest quality in } D \end{cases}$$

Proof: let $\underline{q}_i$ be lowest quality in D

A seller with quality $k > \underline{q}_i$ would get higher price if his disclosed set of possible qualities D^* excluded all qualities below k. Thus a profit maximizing seller would always disclose a set D^* which had as its lowest quality member actual quality k s.t. $min(D) = k$. His profit from signaling D^* would be greater than for signaling D.

Thus this is a fully revealing equilibrium. "Off-equilibrium" beliefs would include that any seller who does not disclose has product with minimal possible product quality $min(Q)$. QED.

$\exists$ a set of conjectures resulting in a Fully Revealing Equilibrium

Milgrom & Stokey (1982), Milgrom (1981a), Hirshleifer (1971)

1. Background (Common knowledge and trade)

Based on Spring 2020 Accounting Theory class notes on the paper (Prof. Caskey)

This paper can be described as the equivalent of M&M's take on how capital structure effects investment but applied to how info affects trade: it should not. If a population of risk-averse traders begins at a Pareto-optimal allocation then new private info alone will cause prices to change but generate no trade.

-1 ———————————————— 0

Agents are at a Pareto optimal allocation. | New information affecting prices is released. No trading occurs.

Pure exchange economy (no production) →

Individuals i trade consumption bundles →

state-space $\Omega = \Theta \times X$ is partitioned into: →

-realization/payoff-relevant states $\theta \in \Theta$ →

-purely informational (re: θ) states $x \in X$ →

giving state of the world $\omega = \{\theta, x\}$ →

i has state-contingent endowment $e_i(\theta)$ →

assume a single consumption good →

utility $U_i(\theta, e_i(\theta))$, prior beliefs $p_i(\omega)$ →

i has particular information partition $\hat{P}_i$ →

$q(\theta)$: ex ante price of consumption in state θ →

$q(\theta)$ was a Pareto-optimal allocation →

In a competitive equilibrium prices satisfy: →

$\frac{q(\theta)}{q(\theta')} = \underbrace{\frac{p_i(\theta)U_i'(\theta, e_i(\theta))}{p_i(\theta')U_i'(\theta', e_i(\theta'))}}_{MRS\ between\ \theta\ \&\ \theta'}$, in other words: →

relative prices of allocations θ & θ' equal relative →

marginal benefits of consuming those bundles →

Payoffs depend on the state $\omega_k \in \Omega$ →

where $\Omega = \{\omega_1, \ldots, \omega_K\}$, $K < \infty$ →

i's information set is defined by a partition P_i →

where if the true state is ω, i knows ω is in $P_i(\omega)$ →

$\wedge$: 'meet' of partitions is finest common *coarsening* →

$\vee$: 'join' of partitions is coarsest common *refinement* →

Beliefs are *concordant* if all agents agree on the →

likelihood of payoff-relevant state θ yielding purely →

informational state x: $p_1(x \mid \theta) = \cdots = p_n(x \mid \theta)\ \forall x, \theta$ →

Example:

$$P_1 : \left[\begin{array}{ccc|cc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}\right]$$

$$P_2 : \left[\begin{array}{cc|cc|c|cc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}\right]$$

$$\text{Meet } P_1 \wedge P_2 : \left[\begin{array}{ccccc|ccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}\right]$$

$$\text{Join } P_1 \vee P_2 : \left[\begin{array}{cc|c|c|c|cc|c} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}\right]$$

Use R to denote the partitions defined by $P_1 \wedge P_2$ where $R(\omega)$ is the element of R that includes ω.

In the above example, $R(\omega_1) = R(\omega_2) = \cdots = R(\omega_5) = \{1, 2, 3, 4, 5\}$ and $R(\omega_6) = R(\omega_7) = R(\omega_8) = \{6, 7, 8\}$ and the trivial event $\omega \in \Omega$.

Milgrom (1981a) shows this is equivalent to an event that satisfies the following axioms where $K_A = \{\omega \in \Omega : A \text{ is common knowledge at } \omega\}$

$\cdot\ K_A \subset A$: A can be common knowledge only if it actually occurs

$\cdot\ \forall \omega \in K_A, \forall i, P_i(\omega) \subset K_A$: If A is common knowledge, then everyone knows it is common knowledge.

$\cdot\ B \subset A \Rightarrow K_B \subset K_A$: When B is common knowledge, any consequence of B is as well. For example, if B is "the S&P 500 decreased by more than 5%" and A is "the S&P 500 decreased", then if it is common knowledge that the S&P decreased by more than 5%, then it is common knowledge that the S&P decreased.

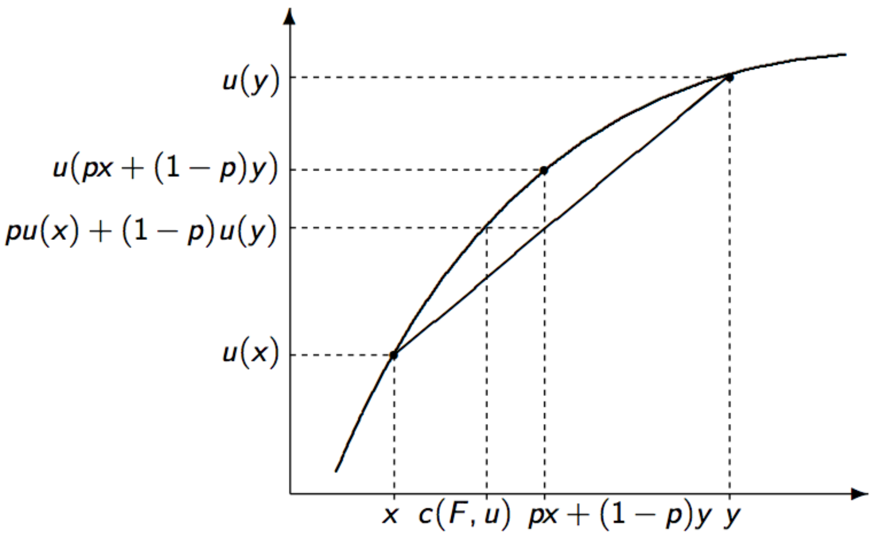
$\cdot\ \{\forall i, \forall \omega \in A, P_i(\omega) \subset A\} \Rightarrow A = K_A$: All public events are common knowledge. When a $\omega \in A$ occurs, all agents know (from $P_i(\omega) \subset A$).

Concordant beliefs are a key condition in the case of private info. For public information, the condition can be weakened to *essentially concordant* beliefs, which restricts only relative likelihoods of payoff relevant states generating signals: $\frac{p_1(x|\theta)}{p_1(x|\theta')} = \cdots = \frac{p_n(x|\theta)}{p_n(x|\theta')}$, $\quad \forall x, \theta, \theta'$

Hirshleifer (1971) shows that information can be detrimental because it eliminates opportunities to insure. For example, assume that there are two types of individuals $i \in \{1, 2\}$ and two states $\omega_i \in \{\omega_1, \omega_2\}$ that occur with equal probability. There is one commodity and individuals have utility $u(c)$. Type i individuals have endowments c_i with payouts:

	ω_1	ω_2
c_1	\$2	\$1
c_2	\$1	\$2

If no information is released prior to trade, individuals will tracle so that each holds $\frac{1}{2}$ unit of c_1 and $\frac{1}{2}$ unit of c_2, giving expected utility of $u(3/2)$[1] If individuals could not trade, the expected utility of type i individuals is $\frac{1}{2}u(2) + \frac{1}{2}u(1) < u\left(\frac{1}{2} \times 2 + \frac{1}{2} \times 1\right) = u(3/2)$ for any concave u. If information is released prior to trade, prices immediately shift and eliminate chances to insure.



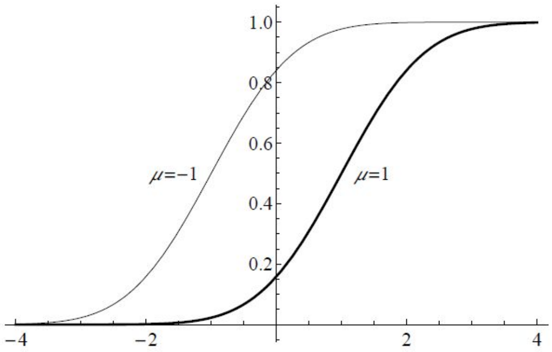
Milgrom (1981b)

1. Background (Notions of 'good news': FOSD/SOSD/MLRP)

Based on Accounting Theory class notes (Prof. Caskey Spring 2020): A general and unambiguous notion of good news is first-order stochastic dominance (FOSD). If distribution $F_1 \succ_{\text{FOSD}} F_2$, then its CDF lies to the right of F_2. Thus, its mass is pushed to higher values. The following are equivalent definitions:

$$F_1 \succeq_{\text{FOSD}} F_2 \Leftrightarrow F_1(\theta) \leq F_2(\theta) \forall \theta \Leftrightarrow \int \underbrace{u(\theta) \mathrm{d} F_1(\theta)}_{\mathrm{E}_1[\theta]} \geq \int \underbrace{u(\theta) \mathrm{d} F_2(\theta)}_{\mathrm{E}_2[\theta]} \quad \forall \text{ increasing } u$$

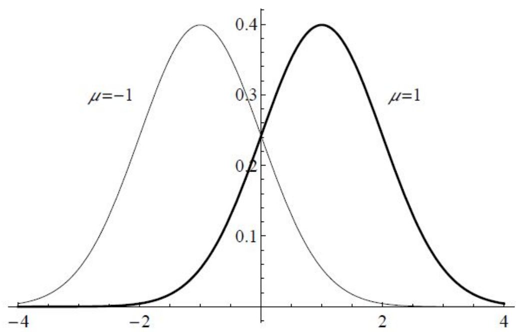
For example, if F_1 and F_2 are both normal distributions with the same variance, the one with the higher mean dominates. If we take an observation x to be a signal about the mean $\theta \in \{-1, 1\}$, a higher x is 'good news' in the sense that $\mathrm{P}(\theta = 1 \mid x)$ is increasing in x



Monotone likelihood ratio property (MLRP) is a less general, but sometimes more useful, definition of good news. MLRP implies FOSD but not vice versa. A distri- bution with signal x has the MLRP if the ratio $\frac{f(x \mid \theta_1)}{f(x \mid \theta_2)}$ is increasing in x for every $\theta_1 > \theta_2$. This is equivalent to:

$$f(x_1 \mid \theta_1) f(x_2 \mid \theta_2) - f(x_1 \mid \theta_2) f(x_2 \mid \theta_1) > 0, \forall x_1 > x_2, \theta_1 > \theta_2$$

MLRP implies FOSD. It also implies that the PDFs cross at a single point. For example, if F_1 and F_2 are both normal distributions with the same variance, the one with the higher mean MLRP dominates and the PDFs cross at a single point.



Ohlson (1995)

2. Ohlsonomics (Asset prices and earnings)

Based on material seen in Accounting Theory (Prof. Caskey Spring 2020)

Ohlson shows how comtemporaneous and future earnings, book values and dividends are price into the firm's market value.

PV(expect. divs): $P_t = y_t + \sum_{s=1}^{\infty} R_f^{-s} E_t\left[d_{t+s}\right]$ $\longrightarrow$

where R_f is $(1+r_f)$ & r_f is the risk-free rate $\longrightarrow$

assumes flat yield curve & risk neutral investor $\longrightarrow$

Clean Surplus Relation: $y_{t-1} = y_t + d_t - x_t$ $\longrightarrow$

y_t is **net book value** & x_t is **earnings** $\longrightarrow$

Earnings process: $x_{t+1}^a = \omega x_t^a + v_t + \epsilon_{1,t+1}$ $\longrightarrow$

where x_t^a is abnormal earnings, v_t is other $\longrightarrow$

relevant info yet to show up in earnings: $\longrightarrow$

$v_{t+1} = \gamma v_t + \epsilon_{2,t+1}$; ϵ_1, ϵ_2 are mean 0 shocks $\longrightarrow$

Restictions: $0 \le \omega \le 1$ and $0 \le \gamma \le 1$ $\longrightarrow$

Define abnormormal earnings x_t^a as $\longrightarrow$

$x_t^a \equiv x_t - (R_f - 1)y_{t-1}$ $\longrightarrow$

$$\boldsymbol{z}_t = \begin{pmatrix} x_t^a \\ \boldsymbol{v_t} \end{pmatrix} = \boldsymbol{\Gamma} \begin{pmatrix} x_{t-1}^a \\ \boldsymbol{v}_{t-1} \end{pmatrix} + \boldsymbol{\epsilon}_t \text{ where } \boldsymbol{\Gamma} = \begin{pmatrix} \omega & 1 \\ 0 & \gamma \end{pmatrix}$$

This implies that $\mathrm{E}_t\left[\boldsymbol{z}_{t+s}\right] = \boldsymbol{\Gamma}^s \boldsymbol{z}_t$

To compute the infinite sum $\sum_{s=1}^{\infty} R^{-s} \mathrm{E}_t\left[\boldsymbol{z}_{t+s}\right]$,

write the eigenvalue decomposition $\boldsymbol{\Gamma} = \boldsymbol{X}^{-1}\boldsymbol{\Lambda}\boldsymbol{X}$

so that $\Gamma^s = \mathbf{X}^{-1}\boldsymbol{\Lambda}^s\mathbf{X}$ and:

$$\sum_{s=1}^{\infty} R^{-s}\mathrm{E}_t\left[\boldsymbol{z}_{t+s}\right] = \left(\sum_{s=1}^{\infty} R^{-s}\boldsymbol{\Gamma}^s\right)\boldsymbol{z}_t = \boldsymbol{X}^{-1}\left(\sum_{s=1}^{\infty} R^{-s}\boldsymbol{\Lambda}^s\right)\boldsymbol{X}\boldsymbol{z}_t$$

Assuming the sume converges: $\sum_{s=1}^{\infty} R^{-s}\boldsymbol{\Lambda}^s = R^{-1}\boldsymbol{\Lambda}(\boldsymbol{I} - R^{-1}\boldsymbol{\Lambda})^{-1}$

$$\sum_{s=1}^{\infty} R^{-s}\mathrm{E}_t\left[\boldsymbol{z}_{t+s}\right] = \boldsymbol{X}^{-1}R^{-1}\boldsymbol{\Lambda}\left(\boldsymbol{I} - R^{-1}\boldsymbol{\Lambda}\right)^{-1}\boldsymbol{X}\boldsymbol{z}_t$$

$$\sum_{s=1}^{\infty} R^{-s}\mathrm{E}_t\left[\boldsymbol{z}_{t+s}\right] = R^{-1}\boldsymbol{\Gamma}\left(\boldsymbol{I} - R^{-1}\boldsymbol{\Gamma}\right)^{-1}\boldsymbol{z}_t$$

This gives: $\Rightarrow P_t = y_t + \begin{pmatrix} 1 & \mathbf{0}' \end{pmatrix} R^{-1}\boldsymbol{\Gamma}\left(\boldsymbol{I} - R^{-1}\boldsymbol{\Gamma}\right)^{-1}\boldsymbol{z}_t$

$$(1\ \mathbf{0}')\, R^{-1}\boldsymbol{\Gamma}\left(\boldsymbol{I} - R^{-1}\boldsymbol{\Gamma}\right)^{-1} = \begin{pmatrix} \frac{\omega}{R-\omega} & \frac{R}{(R-\omega)(R-\gamma)} \end{pmatrix}$$

$$\Rightarrow P_t = y_t + \tfrac{\omega}{R-\omega}x_t^a + \tfrac{R}{(R-\omega)(R-\gamma)}v_t$$

where $\alpha_1 = \frac{\omega}{R-\omega} \ge 0$ is weight on abnormal earnings

and $\alpha_2 = \frac{R}{(R-\omega)(R-\gamma)} \ge 0$ is weight on other info

The $R-\omega$ in the denominators is like the $r-g$ in a growing perpetuity where $R = 1+r$ and $\omega = 1+g$. The ω in the numerator for the abnormal earnings x_t^a coefficient appears because perpetuity begins after one period of growth.

The multiplier on 'other information' v_t has R in the numerator because of the lag for including in price and the denominator is a perpetuity-of-perpetuities: The 'other information' has growth rate $\gamma - 1$

continued on next page...

By moving one period forward, we have an expression for stock return (proof below):
$\frac{P_{t+1}+d_{t+1}}{P_t} = R_f + (1+\alpha_1)\frac{\epsilon_{1,t+1}}{P_t} + \alpha_2 \frac{\epsilon_{2,t+1}}{P_t}$

P_t could also be expressed as a weighted average of earnings-multiplier value and book value:
$P_t = k(\varphi x_t - d_t) + (1-k)y_t + \alpha_2 v_t$ where $\varphi = \frac{R_f}{R_f - 1}$, $k = \frac{(R_f-1)\omega}{R_f-\omega}$

If no other information could be used to predict earnings, i.e., $v_t = 0$, assumed persistent abnormal earnings where $\omega = 1$, stock will be valued based on earnings multiplier. Moreover, assumed random walk of abnormal earnings where $\omega = 0$, stock will be valued based on book value.

Proof of equation (6) in the paper: $\frac{P_{t+1}+d_{t+1}}{P_t} = R_f + (1+\alpha_1)\frac{\epsilon_{1,t+1}}{P_t} + \alpha_2 \frac{\epsilon_{2,t+1}}{P_t}$
Derive the expression for $P_{t+1} + d_{t+1} - R_f P_t$ and subsequently divide by P_t

$$P_{t+1} + d_{t+1} - R_f P_t = y_{t+1} + d_{t+1} + \alpha_1 x_{t+1}^{\boldsymbol{a}} + \alpha_2 v_{t+1} - R_f\left(y_t + \alpha_1 x_t^{\boldsymbol{a}} + \alpha_2 v_t\right)$$

$$= (\alpha_1 + 1)\, x_{t+1}^{\mathrm{a}} + \alpha_2 v_{t+1} - R_f \alpha_1 x_t^{\mathrm{a}} - R_f \alpha_2 v_t$$

Substituting $x_{t+1}^{\mathrm{a}} = \omega x_t^{\mathrm{a}} + v_t + \varepsilon_{1t+1}$ and $v_{t+1} = \gamma v_t + \varepsilon_{2t+1}$

$$P_{t+1} + d_{t+1} - R_f P_t = (\alpha_1 + 1)\, \varepsilon_{1t+1} + \alpha_2 \varepsilon_{2t+1} + \beta_1 x_t^{\mathrm{a}} + \beta_2 v_t$$

where $\beta_1 \equiv (\alpha_1 + 1)\,\omega - \alpha_1 R_f$ and $\beta_2 \equiv (\alpha_1 + 1) + \alpha_2 \gamma - \alpha_2 R_f$
Since $\alpha_1 = \omega / (R_f - \omega)$ and $\alpha_2 = R_f / (R_f - \gamma)(R_f - \omega)$ we thus have $\beta_1 = \beta_2 = 0$ then

$$(P_{t+1} + d_{t+1}) / P_t = R_f + (\alpha_1 + 1)\, \varepsilon_{1t+1} / P_t + \alpha_2 \varepsilon_{2t+1} / P_t$$

$$E[v_{t+\tau}] = \gamma^\tau v_t$$

$$E[x_{t+\tau}^a] = \omega^\tau x_t + \sum_{j=0}^{\tau-1} \omega^j \gamma^j v_{t+\tau-j}$$

Caskey (2009)

3. Rational Expectations Model (with Ambiguity Aversion)

Based on material seen in Accounting Theory (Prof. Caskey Spring 2020)

This paper shows that persistent mispricing is consistent with a market that includes ambiguity-averse investors. In particular, ambiguity-averse investors may prefer to trade based on aggregate signals that reduce ambiguity at the cost of a loss in information.

- Paper uses Smooth ambiguity aversion: $f \succeq g \Leftrightarrow \int_{p\in P} h\left(E_p[u(f)]\right) dQ(p) \geq \int_{p\in P} h\left(E_p[u(f)]]\right) dQ(p)$
- Ambiguity-neutrality can be expressed by $h(u) = u$, giving: $f \succeq g \Leftrightarrow \int_{\omega\in\Omega} u(f)\mathrm{d}p^*(\omega) \geq \mathrm{E}_{p^*}[u(g)]$ where $\mathrm{d}p^*(\omega) = \int_{p\in P} \mathrm{d}p(\omega)\mathrm{d}Q(p)$
- Just as risk-aversion (concave utility) places more weight on adverse *outcomes*, ambiguity-aversion (concave $h(u)$) places more weight on adverse *distributions*.
- Competitive noisy rational expectations model (a la Grossman & Stiglitz, 1980).
- The model has payoff $v = v_1 + v_2$, with corresponding signals $s_i = v_i + e_i$
- All investors have risk-**tolerance** ρ, ie: $u(c) = exp\{-\gamma c\}$ and eg: $\mathrm{E}[u(c)] = -exp\{-\frac{1}{\gamma}\mu_o + \frac{1}{2}\sigma_c^2\frac{1}{\gamma^2}\}$
- Ambiguity-aversion is represented as the concave function $h(u) = -(-u)^a$ where $a = 1$ corresponds to ambiguity-neutrality and $a > 1$ corresponds to ambiguity-aversion.
- Given a payoff v that is joint-normal with some ambiguous parameter b, and conditioning on signal s, the ambiguity-averse optimization problem can be written as:

$$\max_q \int h\underbrace{\left(\int u(q(v-p))\mathrm{d}F(v \mid b, s)\right)}_{\mathrm{E}[u(q(v-p))|b,s]} \mathrm{d}F(b \mid s)$$

- Evaluating interior of expectation: $\mathrm{E}[u(q(v-p)) \mid b, s] = -\exp\left\{-\frac{1}{\rho}\left(q(\mathrm{E}[v \mid b, s] - p) - \frac{1}{2\rho}q^2 var(v \mid b, s)\right)\right\}$
- With risk-ambiguity: $h(\mathrm{E}[u(q(v-p)) \mid \boldsymbol{b}, s]) = -\exp\left\{-\frac{a}{\rho}\left(q(\mathrm{E}[v \mid \boldsymbol{b}, \boldsymbol{s}] - p) - \frac{1}{2\rho}q^2 var(v \mid \boldsymbol{b}, \boldsymbol{s})\right)\right\}$

$$\mathrm{E}[h(\mathrm{E}[u(q(v-p)) \mid b, s]) \mid s] = -\exp\left\{-\frac{a}{\rho}\left(q(\mathrm{E}[v \mid s] - p) - \frac{1}{2\rho}q^2(a\, var(\mathrm{E}[v \mid b, s] \mid s) + var(v \mid b, s))\right)\right\}$$

Using Law of Total Variance: $var(v \mid s) = var(v \mid b, s) + var(\mathrm{E}[v \mid b, s] \mid s)$, we get the following certainty equivalent after substituting $var(v \mid b, s) = var(v \mid s) - var(\mathrm{E}[v \mid b, s] \mid s)$:

$$q(\mathrm{E}[v \mid s]-p)-\frac{1}{2\rho}q^2(var(v \mid s)+(a-1)var(\mathrm{E}[v \mid b, s] \mid s)) = q(\mathrm{E}[v \mid s]-p)-\frac{1}{2\rho}q^2 var(v \mid s)(1+(a-1)R(v, b \mid s))$$

where $R(v, b \mid s) = \frac{var(\mathrm{E}[v|b,s]|\theta)}{var(v|\theta)} = \frac{var(v|\theta)-var(v|b,s)}{var(v|s)}$ reflects the contribution of b to uncertainty about v given s,

- If we were to aggregate signals s_1 and s_2, we could compare $R(v, b \mid s_1, s_2)$ to $R(v, b \mid s^*)$ where $s^* = \mathrm{E}[v \mid s_1, s_2]$.

- Conditional variance is unchanged from this aggregation of signals:
$var(v \mid s_1, s_2) = var(v) - var(\mathrm{E}[v \mid s_1, s_2]) = var(v) - var(\mathrm{E}[v \mid s^*]) = var(v \mid s^*)$

- Thus: $R(v, b \mid s_1, s_2) > R(v, b \mid s^*) \Leftrightarrow var(\mathrm{E}[v \mid b, s_1, s_2] \mid s_1, s_2) > var(\mathrm{E}[v \mid b, s^*] \mid s^*)$

- Using the Kalman filter: $\mathrm{E}[v \mid b, s_1, s_2] = \mathrm{E}[v \mid s_1, s_2] + \frac{cov(v,b|s_1,s_2)}{var(b|s_1,s_2)}(b - \mathrm{E}[b \mid s_1, s_2])$

- so that: $var(\mathrm{E}[v \mid b, s_1, s_2] \mid s_1, s_2) = \frac{cov(v,b|s_1,s_2)^2}{var(b|s_1,s_2)} = \underbrace{\left(\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\right)^2}_{Impact\ of\ s_1\ on\ \mathrm{E}[v|s_1,s_2]} \frac{\sigma_{s1}^2}{var(s_1|b)} \cdot \sigma_b^2$

  where $\frac{\sigma_{s1}^2}{var(s_1|b)}$ is the Variation of s_1 caused by b.

- Similar computations give: $var(\mathrm{E}[v \mid b, s^*] \mid s^*) = \underbrace{\left(\frac{\sigma_{v1}^2}{\sigma_{s1}^2}\right)^2}_{Impact\ of\ s_1\ on\ s^*} \frac{var(s^*)}{var(s^*|b)}\sigma_b^2$

  where again $\frac{var(s^*)}{var(s^*|b)}$ is the Variation of s^* caused by b.

- So that $R(v, b \mid s_1, s_2) > R(v, b \mid s^*) \Leftrightarrow \frac{\sigma_{\rho 1}^2}{var(s_1|b)} = \frac{1}{1-corr(s_1,b)^2} > \frac{var(s^*)}{var(s^*|b)} = \frac{1}{1-corr(s^*,b)^2}$
$\Leftrightarrow corr(s_1, b)^2 > corr(s^*, b)^2$

- Because s^* is an aggregate signal, b causes a lower proportion of its variation, giving $R(v, b \mid s_1, s_2) > R(v, b \mid s^*)$. Formally:

$$corr(s_1, b)^2 = \frac{\sigma_b^2}{\sigma_{s1}^2} > corr(s^*, b)^2 = \frac{\sigma_{v1}^4/\sigma_{s1}^2}{var(s^*)} \frac{\sigma_b^2}{\sigma_{s1}^2}$$

  where the inequality follows because $var(s^*) = \frac{\sigma_{v1}^4}{\sigma_{s1}^2} + \frac{\sigma_{v2}^4}{\sigma_{s2}^2}$

- The commingling of s_2 with s_1 makes ambiguity-averse investors more sure of how to interpret the aggregate signal.

- Because ambiguity-averse investors strictly prefer the aggregate information, they would be willing to pay for an aggregate signal. This holds even if the signal destroys information (Prop. 3 in paper). This can lead to mispricing.

Guttman (2010)

3. Rational expectations (Competitive trader models)

Based on material seen in Accounting Theory (Prof. Caskey Spring 2020)

This study's model endogenizes the timing decision of analysts and analyzes their equilibrium timing strategies. Analysts face a trade-off between the timeliness and the precision of their forecasts.

Investors trade over $t \in [0,T]$ $\longrightarrow$

They have CARA coefficient γ $\longrightarrow$

Asset payoff is $\pi \sim \mathcal{N}\left(\mu_{\pi 0}, \sigma^2_{\pi 0}\right)$ $\longrightarrow$

Public information at time t has precision: $\longrightarrow$

$f(t)$ with $f(0) = \frac{1}{\sigma^2_{\pi 0}} \geq 1$ (i.e. $\sigma^2_{\pi 0} \leq 1$) $\longrightarrow$

$f(t) \to \infty$ as $t \to T$ and $f'(t) > 0$ $\longrightarrow$

Investors can buy signal $\psi_i(t) = \pi + e_i(t)$ $\longrightarrow$

from **analyst** i where $var(e_i(t)) = \frac{1}{f_i(t)}$ $\longrightarrow$

Imputing the cost of information c from $\longrightarrow$

the informed/uninformed indifference†: $\longrightarrow$

† from Grossman & Stiglitz (1985) $\longrightarrow$

$c = \frac{1}{\gamma} \log \sqrt{\frac{var_u(v-p)}{var_i(v-p)}}$ $\longrightarrow$

where $var_u(\pi - p) = \frac{1}{f(t)}$ $\longrightarrow$

and $var_i(\pi - p) = \frac{1}{f(t)+f_i(t)}$ and so: $\longrightarrow$

$U^i(f(t)) = C(t) = \frac{1}{\gamma} \log \sqrt{\frac{var_u(v-p)}{var_{(v-p)}}} =$ $\longrightarrow$

$\frac{1}{2\gamma} \log \frac{f(t)+f_i(t)}{f(t)} \Leftarrow$ analyst i maximizes this $\longrightarrow$

By assumption $f_i(t) = F_i + \alpha_i \log f(t)$ $\longrightarrow$

where α_i is speed of analyst learning $\longrightarrow$

and F_i is his initial info advantage $\longrightarrow$

for some constants $F_i \geq 0$ and $\alpha_i > 0$ $\longrightarrow$

Both $f(t)$ & $f_i(t)$ increase over time $\longrightarrow$

$f_i'(t) = \frac{\alpha_i}{f(t)} f'(t)$: analyst precision grows $\longrightarrow$

faster than public precision if $f(t) < \alpha_i$ $\longrightarrow$

FOC to find optimal t^* at which analyst's payoff is maximized:

$$\frac{\partial U^i}{\partial f} = \frac{1}{2\gamma} \frac{\alpha_i - \overbrace{(F_i + \alpha_i \log f(t))}^{f_i(t)}}{f(t)(f(t)+f_i(t))} = 0 \Rightarrow \alpha_i - (F_i + \alpha_i \log f(t)) = 0$$

$$\Rightarrow \frac{F_i}{\alpha_i} - 1 = -\log f(t) \Rightarrow f(t^*) = \mathrm{e}^{1-F_i/\alpha_i} \Rightarrow t^* = f^{-1}\left(\mathrm{e}^{1-F_i/\alpha_i}\right)$$

If $\alpha_i < f_i(0)$, then $\frac{\partial U^i}{\partial f} < 0 \; \forall f$ and the analyst immediately releases a forecast.

The condition $\alpha_i < f_i(0)$ is equivalent to$f(0) > \mathrm{e}^{1-F_i/\alpha_i}$

Using the **implicit function theorem** for $f(t) - e^{1-F_i/\alpha_i} = 0$

we get marginal effects of initial analyst precision F_i on optimal $f(t^*)$:

$$\frac{\mathrm{d}t^*}{\mathrm{d}F_i} = -\frac{\frac{\partial}{\partial F_i}\left(f(t) - \mathrm{e}^{1-F_i/\alpha_i}\right)}{\frac{\partial}{\partial t}\left(f(t) - \mathrm{e}^{1-F_i/\alpha_i}\right)} = -\frac{1}{\alpha_i}\frac{e^{1-F_i}}{f'(t^*)} = -\frac{1}{\alpha_i} \underbrace{\frac{f(t^*)}{f'(t^*)}}_{>0} < 0$$

and marginal effects of speed of learning α_i on optimal $f(t^*)$:

$$\frac{\mathrm{d}t^*}{\mathrm{d}\alpha_i} = -\frac{\frac{\partial}{\partial \alpha_i}\left(f(t) - \mathrm{e}^{1-F_i/\alpha_i}\right)}{\frac{\partial}{\partial t}\left(f(t) - \mathrm{e}^{1-F_i/\alpha_i}\right)} = \frac{F_i}{\alpha_i^2} \underbrace{\frac{f(t^*)}{f'(t^*)}}_{>0} \Rightarrow \text{where } sign\left(\frac{\mathrm{d}t^*}{\mathrm{d}\alpha_i}\right) = sign(F_i)$$

$\frac{\mathrm{d}t^*}{\mathrm{d}F_i} < 0 \;\Rightarrow$ The more the analyst already knows, the earlier releases his forecast.

Extreme case leads him to release at $t = 0$ to bank his initial info.

$\frac{\mathrm{d}t^*}{\mathrm{d}\alpha_i} > 0 \Rightarrow$ analysts who learn faster wait longer to issue forecasts.

Two analysts i, j: To find earliest time τ_i at which analyst i will release a forecast we use the indifference interval defined by the $t = \tau_i$ that solves:

$$\overbrace{\underbrace{\log \frac{f(t) + f_i(t)}{f(t)}}_{from\ releasing\ first}}^{i's\ payoff\ at\ t} = \underbrace{\log \frac{\overbrace{f(t) + f_j(t)}^{public\ info,\ now\ includes\ j's\ fcast} + \overbrace{F_i + \alpha_i \log\left(f(t) + f_j(t)\right)}^{f_i(t)}}{f(t) + f_j(t)}}_{i's\ payoff\ at\ t\ from\ releasing\ 2nd\ after\ j}$$

Figure 5 shows that the analyst will never unilaterally issue before τ_i since even if analyst j 'jumps ahead' he can get a higher payoff by delaying. The main proposition says that the first analyst to forecast will release at the earlier of when the other analyst is indifferent (τ_2) or his own unconstrained optimum (t_1^*). The other analyst will either cluster with the first or wait until his unconstrained optimum (t_2^*). Fig. 6 & 7 illustrate the proposition.

Implicit Function Theorem: If you have a function (say: $x^2 + y^2 + z^2 = e^{xy}$) and you want to find a partial derivative of one of these variables w.r.t another (eg:$\frac{\partial z}{\partial x}$) put it in the form $F(x, y, z) = 0$ (so: $F(x, y, z) = x^2 + y^2 + z^2 - e^{xy} = 0$) then (note the variable inversion):

$$\frac{\partial z}{\partial x} = -\frac{\frac{\partial F(x,y,z)}{\partial x}}{\frac{\partial F(x,y,z)}{\partial z}} \qquad eg: \frac{\partial z}{\partial x} = -\frac{\frac{\partial F(x,y,z)}{\partial x}}{\frac{\partial F(x,y,z)}{\partial z}} = -\frac{2x - ye^{xy}}{2z}$$

FIGURE 5
Interior Indifference Interval

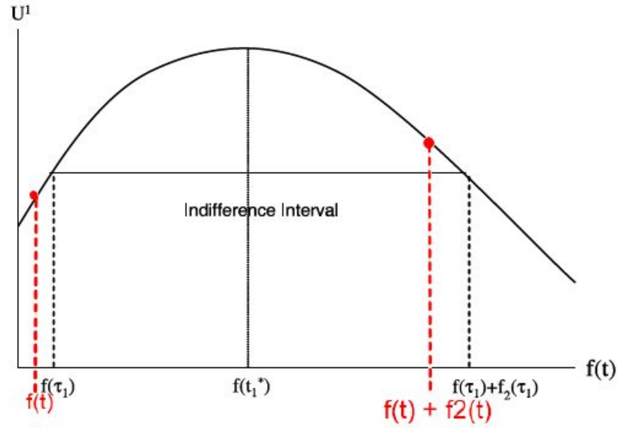
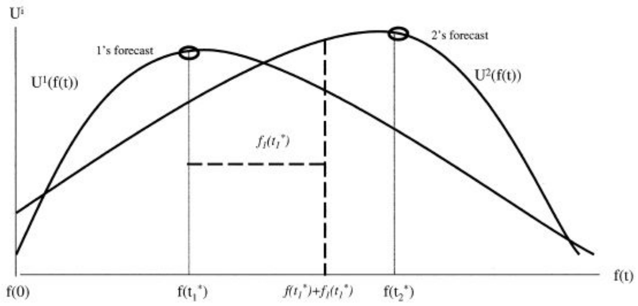
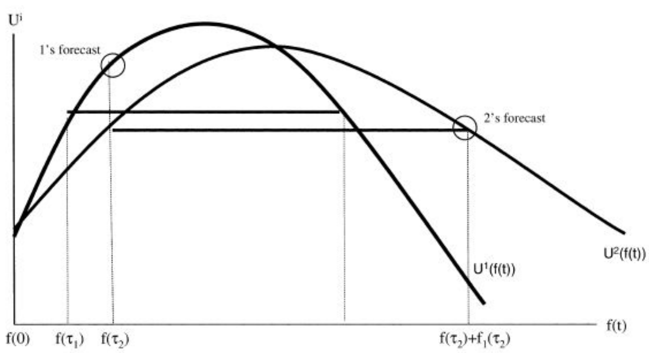


FIGURE 6
Non-Clustering Pattern



Analyst 1 forecasts first at his unconstrained optimum: $f(t_1^*)$. Following his forecast, the precision of the investors' beliefs "jumps" to $f(t_1^*) + f_1(t_1^*)$, which is still lower than the unconstrained optimum of analyst 2. Analyst 2 waits until the precision of the investors' beliefs equals his unconstrained optimum and then publishes his forecast.

FIGURE 7
Clustering Pattern



Since $\tau_1 < \tau_2$, analyst 1 is the first to forecast. He issues his forecast at τ_2. Following his forecast, the precision of the investors' beliefs "jumps" to $f(\tau_2) + f_1(\tau_2)$, which is higher than the unconstrained optimum of analyst 2. Since the payoff of analyst 2 at this region is decreasing in the precision of the investors' beliefs, analyst 2 issues his forecast immediately after the forecast of analyst 1.

Kim & Verrecchia (1994)

4. Rational Expectations (Strategic Trader Models)

This paper suggests that earnings announcements provide information that allows certain traders to make judgments about a firm's performance that are superior to the judgments of other traders.

$s = \tau$	$s = \tau$	$s > \tau$	$s = T$
Earnings announcement $\tilde{Y}$	Trader i can also buy info $\tilde{O}_i$ at cost C	Market maker sees flow $\tilde{w}_s$ sets price & clears the market	Realized Liquidation firm value $= \tilde{U}$

T periods, $s = 1, 2, \cdots, T$

Firm generates $\tilde{u}_s$ in every period s

where $\tilde{u}_s \sim N(0, a_s)$

It has liquidating value $\tilde{U} = \sum_{s=1}^{T} \tilde{u}_s$

Firm discloses EPS $\tilde{Y}$ at time $s = \tau < t$

$\tilde{Y} = \tilde{u}_t + \tilde{\delta}$ where $\tilde{u}_t \sim N(0, a_t)$

$\tilde{\delta} \sim N(0, d)$ is discretionary earnings

At time τ a trader i can pay fixed C to see

$\tilde{O}_i = \tilde{\delta} + \tilde{\epsilon}_i$ where $\tilde{\epsilon}_i \sim N(0, e)\ \forall i$

Thus i gets a better idea of fundamental

performance: $\tilde{Y} - \tilde{O}_i = \tilde{u}_t - \tilde{\epsilon}_i$

Whereas market maker only observes $\tilde{Y}$

Firm announces EPS only at date τ

Between informed traders: $cov(\tilde{\epsilon}_i, \tilde{\epsilon}_i') = \rho e$

Liquidity trader's net demand is $\sim N(0,1)$

The market maker observes $\tilde{w}_s$ where

$\tilde{w}_s \equiv \tilde{x}_s + \tilde{z}_{ns} + \tilde{z}_{ds}$ where $\tilde{x}_s \equiv \sum_{i=1}^{N_s} \tilde{x}_{is}$

and $\tilde{x}_s$ is informed aggregate demand

$\tilde{z}_{ns}$ is nondiscretionary liquidity demand

and $\tilde{z}_{ds}$ is discretionary liquidity demand

Mkt maker conjectures: $\tilde{x}_i = \beta\tilde{Y} + \gamma\tilde{O}_i$

Informed conjectures: $\tilde{P}_z = \tilde{U}_{t-1} + \alpha\tilde{Y} + \lambda\tilde{w}$

where $\tilde{U}_{t-1} \equiv \sum_{s=1}^{t-1} \tilde{u}_s$

Let Π_i^x be the profit of informed trader i

Then informed trader i must optimize:

$\max_{\tilde{x}_i} \Pi_i^x = E\left[\tilde{x}_i\left(\tilde{U} - \tilde{P}_\tau\right) \mid \tilde{U}_{t-1}, \tilde{Y}, \tilde{O}_i\right]$

Show $\exists$ a rationale expectations equilibrium by solving for i's conjecture coefficients in terms of raw parameters

Solving informed trader i's problem:

$$\max_{\tilde{x}_i} \Pi_i^x = E\left[\tilde{x}_i\left(\tilde{U} - \tilde{P}_\tau\right) \mid \tilde{U}_{t-1}, \tilde{Y}, \tilde{O}_i\right]$$

$$\max_{\tilde{x}_i} \Pi_i^x = E\left[\tilde{x}_i\left(\tilde{U} - \tilde{U}_{t-1} - \alpha\tilde{Y} - \lambda\tilde{w}\right) \mid \tilde{U}_{t-1}, \tilde{Y}, \tilde{O}_i\right]$$

$$\max_{\tilde{x}_i} \Pi_i^x = \mathrm{E}\left[\tilde{x}_i\left\{\tilde{u}_t - \alpha\tilde{Y} - \lambda\left(\tilde{x} + \tilde{z}_n + \tilde{z}_d\right)\right\} \mid \tilde{Y}, \tilde{o}_i\right]$$

$$\max_{\tilde{x}_i} \Pi_i^x = \mathrm{E}\left[\tilde{x}_i\left\{\tilde{u}_t - \alpha\tilde{Y} - \lambda\tilde{x}_i - (N-1)\lambda\beta\tilde{Y} - \lambda\gamma\sum_{k\neq i}\tilde{O}_k\right\} \mid \tilde{Y}, \tilde{o}_i\right]$$

$$\max_{\tilde{x}_i} \Pi_i^x = \tilde{x}_i\left[\tfrac{a(d+e)\tilde{Y} - ad\tilde{O}_i}{ad+ae+de} - \{\alpha + (N-1)\lambda\beta\}\tilde{Y} - \lambda\tilde{x}_i\right.$$
$$\left. -(N-1)\lambda\gamma\tfrac{(1-\rho)de\tilde{Y} + \{ad+\rho e(a+d)\}\tilde{O}_i\}}{ad+ae+de}\right]$$

Taking FOC w.r.t $\tilde{x}_i$ we can solve for:

$$\tilde{x}_i = \tfrac{1}{2\lambda}\left\{\left[-\{\alpha + (N-1)\lambda\beta\} + \tfrac{a(d+e) - (N-1)\lambda\gamma(1-\rho)de}{ad+ae+de}\right]\tilde{Y}\right.$$
$$\left. - \tfrac{ad + (N-1)\lambda\gamma\{ad+\rho e(a+d)\}}{ad+ae+de}\tilde{O}_i\right\}$$

Thus:

$$\gamma = -\tfrac{ad + (N-1)\lambda\gamma\{ad+\rho e(a+d)\}}{2\lambda\{ad+ae+de\}} = \tfrac{-ad}{\lambda[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]}$$

$$\text{and } \beta = \tfrac{1}{2\lambda}\left[-\{\alpha + (N-1)\lambda\beta\} + \tfrac{a(d+e) - (N-1)\lambda\gamma(1-\rho)de}{ad+ae+de}\right]$$

$$\Leftrightarrow \beta = \tfrac{1}{(N+1)\lambda}\left[-\alpha + \tfrac{a[(N+1)d + \{2+(N-1)\rho\}e]}{(N+1)ad + \{2+(N-1)\rho\}e(a+d)}\right]$$

Then the market maker sets price equal to his expectation of $\tilde{U}$:

$$\tilde{P}_t = \mathrm{E}\left[\tilde{U} \mid \tilde{U}_{t-1}, \tilde{Y}, \tilde{\omega}\right] = \tilde{U}_{t-1} + \mathrm{E}\left[\tilde{u}_t \mid \tilde{Y}, \tilde{\omega}\right] \text{ where}$$

$$\mathrm{E}\left[\tilde{u}_t \mid \tilde{Y}, \tilde{\omega}\right] = \tfrac{1}{Det}\left\{\left[\tfrac{N^2\alpha a^2 d^2}{(N+1)\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]}\right.\right.$$
$$\left. + av - \tfrac{N(N-1)(1-\rho)a^3d^2e}{(N+1)\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2}\right]\tilde{Y}$$
$$\left. + \tfrac{Na^2d^2}{\lambda[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]}\tilde{\omega}\right\}$$

where the determinant, Det, of the var-covar matrix of $(\tilde{Y}, \tilde{\omega})$ is

$$\text{Det} = v(a+d) + \tfrac{Na^2d^2[Nad + \{1+(N-1)\rho\}e(a+d)]}{\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2} \qquad \text{Therefore:}$$

$$\lambda = \tfrac{1}{Det}\tfrac{Na^2d^2}{\lambda[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]} = \sqrt{\tfrac{Na^2d^2(ad+ae+de)}{v(a+d)[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2}}$$

$$\alpha = \tfrac{1}{Det}\left[\tfrac{N^2\alpha a^2d^2}{(N+1)\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]}\right.$$
$$\left. + av - \tfrac{N(N-1)(1-\rho)a^3d^2e}{(N+1)\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2}\right]$$

$$\text{simplified: } \alpha = \frac{a\left[v - \frac{N(N-1)(1-\rho)a^2d^2e}{(N+1)\hat{\lambda}^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2}\right]}{(a+d)\left[v - \frac{N(N-1)(1-\rho)a^2d^2e}{(N+1)\lambda^2[(N+1)ad + \{2+(N-1)\rho\}e(a+d)]^2}\right]}$$

since the inside of the big square bracket is positive, $\alpha = a/(a+d)$

$$\beta = \sqrt{\tfrac{vd^2}{N(a+d)(ad+ae+de)}} \text{ and } \gamma = -\sqrt{\tfrac{v(a+d)}{N(ad+ae+de)}} \qquad \text{QED.}$$

Kyle (1989)

4. Rational Expectations (Strategic Trader Models)

This papers models investors which consider the price impact of their trades when optimizing their optimal demands.

$t=0$ ——— $t=1$

Trading ——— Cashflow Realization v

Liquidity trader demand $z \sim N(0,\sigma_z^2)$ $\longrightarrow$

n's obj. function: $\max_{q_n} \mathrm{E}_n[u(w_n + q_n(v-p))]$ $\longrightarrow$

places **limit orders** & has risk-aversion: r_n $\longrightarrow$

n conjectures his price impact: $p = p_0 + \lambda_n q_n$ $\longrightarrow$

$\max_{q_n} \mathrm{E}_n[u(w_n + q_n(v-(p_0+\lambda_n q_n)))]$, FOC: $\longrightarrow$

$0 = \mathrm{E}_n[u'(w_n + q_n(v-p))(v-p-\lambda_n q_n)] =$ $\longrightarrow$

$\mathrm{E}_n[u'(w_n + q_n(v-p_0-\lambda_n q_n))(v-p_0-2\lambda_n q_n)]$ $\longrightarrow$

with certainty equivalence and $u(c) = -e^{-r_n c}$: $\longrightarrow$

$0 = r_n \mathrm{e}^{-r_n(w_n + q_n' E_n[v-p] - \frac{p_n}{2} q_n var_n(v-p) q_n)}$ $\longrightarrow$

$\times(E_n[v-p] - \lambda_n q_n - r_n var_n(v-p) q_n)$ $\longrightarrow$

$\Leftrightarrow 0 = E_n[v-p] - \lambda_n q_n - r_n var_n(v-p) q_n$ $\longrightarrow$

$\Rightarrow q_n = \frac{E_n[v-p]}{r_n var_n(v-p) + \lambda_n}$ so λ_n deflates demand $\longrightarrow$

N_I informed & N_U uninformed investors $\longrightarrow$

Informed risk aversion: r_I, uninformed: r_U $\longrightarrow$

Use mkt clearing cond.: $\sum_{n=1}^{N} q_n(p) + x = 0$ $\longrightarrow$

Assume p is known: $q_n = \frac{E_n[v]-p}{r_n var_n(v) + \lambda_n}$ $\longrightarrow$

Informed n trader's signal: $s_n = v + e_n$ $\longrightarrow$

where $v \sim N\left(\mu_v, \tau_v^{-1}\right) \perp$ iid $\epsilon \sim N(0,\tau_e^{-1})$ $\longrightarrow$

Uninformed conjectures informed $q_I(p,s_n)$ $\longrightarrow$

$q_I(p,s_n) = \mu_I + \beta s_n - \gamma_I p$ $\longrightarrow$

Informed conjectures uninformed $q_U(p)$ $\longrightarrow$

$q_U(p) = \mu_U - \gamma_U p$ $\longrightarrow$

Overall market price conjecture form: $\longrightarrow$

$p = p_0 + p_{\bar{s}} \bar{s} + p_x x$ $\longrightarrow$

Thus $E[v|s_n] = \mu_v + \frac{\tau_v^{-1}}{\tau_v^{-1}+\tau_e^{-1}}(s_n - \mu_v)$

$= \mu_v + \frac{1}{1+\frac{\tau_v}{\tau_e}}(s_n-\mu_v) = \mu_v + \frac{\tau_e}{\tau_e+\tau_v}(s_n-\mu_v)$

n also has price p as signal, so 1st consider:

$E[v \mid p] = \mathrm{E}[v] + \frac{cov(v,p)}{var(p)}\,(p - E[p])$

Since s_n affects both p and v we condition above on it:

$\mathrm{E}[v|p,s_n] = \mathrm{E}[v|s_n] + \frac{cov(v,p|s_n)}{var(p|s_n)}\,(p - \mathrm{E}\,[p \mid s_n])$

where $p - \mathrm{E}[p \mid s_n] = p - (\mathrm{E}[p] + \frac{cov(p,s_n)}{var(s_n)}(s_n-\mu_v))$

Informed demand: $q_n = \frac{\mathrm{E}[v|s_n,p]-p}{\rho_I var(v|s_n,p)+\lambda_I}$

$$= \underbrace{\frac{\frac{\tau_v}{\tau_v+\tau_e}\mu_v - \frac{cov(v,p|s_n)}{var(p|s_n)}\left(\mathrm{E}[p] - \frac{cov(p,s_n)}{var(s_n)}\mu_v\right)}{\rho_I \tau_I^{-1} + \lambda_I}}_{\mu_I}$$

$$+ \underbrace{\frac{\frac{\tau_e}{\tau_v+\tau_e} + \frac{cov(v,p|s_n)}{var(p|s_n)}\frac{cov(p,s_n)}{var(s_n)}}{\rho_I \tau_I^{-1} + \lambda_I}}_{\beta} s_n - \underbrace{\frac{1-\frac{cov(v,p|s_n)}{var(p|s_n)}}{\rho_I \tau_I^{-1} + \lambda_I}}_{\gamma_I} p$$

where $\tau_I = var\,(v \mid s_n, p)^{-1}$

Meanwhile uninformed have: $q_U = \frac{\mathrm{E}[v|p]-p}{\rho_U var(v|p)+\lambda_U}$

$$= \underbrace{\frac{\mu_v - \frac{cov(v,p)}{var(p)}\mathrm{E}[p]}{\rho_U \tau_U^{-1} + \lambda_U}}_{\mu_U} - \underbrace{\frac{1 - \frac{cov(v,p)}{var(p)}}{\rho_U \tau_U^{-1} + \lambda_U}}_{\gamma_U} p$$

where $\tau_U = var(v \mid p)^{-1}$

Now the market clearing condition where all flows = 0:

$0 = \sum_{n=1}^{N_I} (\mu_I + \beta s_n - \gamma_I p) + N_U(\mu_U - \gamma_U p) + x$

$= N_I(\mu_I - \gamma_I p) + \beta N_I \bar{s}_I + N_U(\mu_U - \gamma_U p) + x$

$\Rightarrow p = \frac{N_I\mu_I + N_U\mu_U}{N_I\gamma_I + N_U\gamma_U} + \frac{\beta N_I}{N_I\gamma_I + N_U\gamma_U}\bar{s}_I + \frac{1}{N_I\gamma_I + N_U\gamma_U}x$

where $\bar{s}_I$ is the average signal: $\bar{s}_I = \frac{1}{N_I}\sum_{n=1}^{N_I} s_n$

Thus price satisfies the following

$$p = \underbrace{\frac{N_I\mu_I + N_U\mu_U}{N_I\gamma_I + N_U\gamma_U}}_{p_0} + \underbrace{\frac{\beta N_I}{N_I\gamma_I + N_U\gamma_U}}_{p_{\bar{s}}}\bar{s}_I + \underbrace{\frac{1}{N_I\gamma_I + N_U\gamma_U}}_{p_x} x$$

$\longrightarrow$ Informed and

$\longrightarrow$ uninformed

$\longrightarrow$ demands:

$\longrightarrow$ X_I and X_U.

$\longrightarrow$ Price

$\longrightarrow$ function p's

$\longrightarrow$ conjecture

$\longrightarrow$ coefficients:

$\longrightarrow$ p_x, $p_{\bar{s}}$, p_0

$\longrightarrow$ as functions

$\longrightarrow$ of parameters.

In Kyle (1985), the price setting mechanism is that market-makers set price equal to the expected value given order flow $y = x + u$ (equivalent to $\hat{q}_i = q_i + x$ in here). This is equivalent to the market clearing condition with a single informed trader and an unbounded number of uninformed traders. One way to see this is to assume that uninformed trader n conjectures that other uninformed traders have demand of the form $\mu_U - \gamma_U p$ and that the informed trade q_i is joint-normal with v. The market clearing condition ($\sum_{n=1}^{N} q_n(p) + x = 0$) then gives (from the point of view of uninformed trader n):

$$q_n + (N_U - 1)(\mu_U - \gamma_U p) + q_I + x = 0$$

giving the following conjectures for price and informed demand $\hat{q}_i$:

$$p \quad = \frac{(N_u - 1)\mu_u + q + x}{(N_u - 1)\gamma_u} + \underbrace{\frac{1}{(N_u - 1)\gamma_u}}_{\lambda_u} q_n$$

$$\hat{q}_i = (N_u - 1)(\gamma_u p - \mu_u) - q_n = q_i + x$$

$$\mathrm{E}[v \mid p, q_n] = \mathrm{E}[v \mid \hat{q}_i] = \mu_v + \frac{cov(v, q_i)}{var(q_i) + \sigma_x^2}(\hat{q}_i - \mathrm{E}[q_i] - \mu_x)$$

Uninformed trade quantity is then the following where the second line follows after taking into account the q_u embedded in $\hat{q}_i$ and rearranging:

$$q_u = \frac{\mathrm{E}[v \mid \hat{q}_i] - p}{\rho_u var(v \mid \hat{q}_i) + \lambda_u} = \underbrace{\frac{\mu_v - \frac{cov(v,q_i)}{var(q_i)+\sigma_x^2}((N_u - 1)\mu_u + \mathrm{E}[q_i + x])}{\rho_u var(v \mid \hat{q}_i) + \lambda_u + \frac{cov(v,q_i)}{var(q_i)+\sigma_w^2}}}_{\mu_u} - \underbrace{\frac{1 - \frac{cov(v,q_i)}{var(q_i)+\sigma_x^2}(N_u - 1)\gamma_u}{\rho_u var(v \mid \hat{q}_i) + \lambda_u + \frac{cov(v, q_i)}{var(q_i) + \sigma_x^2}}}_{\gamma_u} p$$

This satisfies the linear conjecture and we can get the expression for price by solving the following expression using the μ_u and γ_u expressions above: $N_u(\mu_u - \gamma_u p) + q_i + x = 0 \Rightarrow p = \frac{\mu_u}{\gamma_u} + \frac{1}{N_u \gamma_u}(q_i + x)$
The expression for the γ_u coefficient implies: $\gamma_u = \frac{1}{\rho_u var(v|\hat{q}_i) + \lambda_u + \frac{cov(v,q_i)}{var(q_i)+\sigma_x^2} N_u}$
and the expression for the μ_u coefficient implies: $\mu_u = \gamma_u \left(\mu_v - \frac{cov(v,q_i)}{var(q_i)+\sigma_x^2}\mathrm{E}[q_i + x]\right)$
Using the above conjecture for $\hat{q}_i$, the relation $\lambda_u = \frac{1}{(N_u-1)\gamma_u}$ implies

$$\lambda_u = \frac{\rho_u}{N_u - 2} var(v \mid \hat{q}_i) + \frac{cov(v, q_i)}{var(q_i) + \sigma_x^2}\frac{N_u}{N_u - 2} \Rightarrow \gamma_u = \frac{N_u - 2}{N_u - 1}\frac{1}{\rho_u var(v \mid q_i) + \frac{cov(v,q_i)}{var(q_i)+\sigma_w^2} N_u}$$

$$\Rightarrow p = \underbrace{\mu_v + \frac{cov(v, q_i)}{var(q_i) + \sigma_x^2}(q_i + x - \mathrm{E}[q_i + x])}_{\mathrm{E}[v|q_i+x]} + \frac{1}{N_u - 2}\frac{cov(v, q_i)}{var(q_i) + \sigma_x^2}(q_i + x) + \frac{N_u - 1}{N_u - 2}\frac{\rho_{u_\mu}}{N_u} var(v \mid \hat{q}_i)$$

which approaches the $\mathrm{E}[v \mid q_i + x]$ from Kyle (1985) as $N_u \to \infty$

Einhorn & Ziv (2012)

5. 'Semi' Verifiable Disclosure Models (Signal Jamming)

CEO sees private signal s with probability τ	Motivated by his type t, CEO decides $D(t)$ whether to disclose s	Investors price the firm	Firm value v is realized

Investors are risk **neutral** $\longrightarrow$

Uncertain firm value $v \sim N(\mu, \sigma^2)$ $\longrightarrow$

CEO sees signal s with probability τ $\longrightarrow$

where $s = v + \varepsilon$ and $\varepsilon \sim N(0, \sigma_\epsilon^2)$ $\longrightarrow$

CEO can make report r if informed $\longrightarrow$

or CEO can choose to not disclose (ND) $\longrightarrow$

This report incurs cost $\lambda + c(r-s)^2$ $\longrightarrow$

Let $\hat{v} = \mathrm{E}[v \mid s]$ and $\delta = \frac{\sigma^2}{\sigma^2+\sigma_\ell^2}$ $\longrightarrow$

where $var(\hat{v}) = cov(v,\hat{v}) = \frac{\sigma^4}{\sigma^2+\sigma_\epsilon^2} = \delta\sigma^2$ $\longrightarrow$

$\delta \in (0,1)$ represents extent of CEO info $\longrightarrow$

Φ denotes standard normal distribution $\longrightarrow$

ϕ denotes the standard normal density $\longrightarrow$

with standard normal being: $\sim N(0,1)$ $\longrightarrow$

Case 1: truthful disclosure $(c = 0)$:

Solve by finding threshold $\hat{v}_{b1}$ that renders CEO indifferent between N & ND:

$$\mathrm{E}[v \mid D] - \lambda = \mathrm{E}[v \mid ND] \text{ where}$$

$$\mathrm{E}[v \mid ND] = P(Uninformed \mid ND)E[v] + P(Informed \mid ND)\mathrm{E}[v \mid v < \hat{v}_{b1}]$$

$$\text{Thus: } \hat{v}_{b1} - \lambda = \underbrace{\frac{1-\tau}{1-\tau+\tau F(\hat{v}_{b1})}}_{\mathbf{P}(\text{ Uninformed } | ND)} \mu + \underbrace{\frac{\tau F(\hat{v}_{b1})}{1-\tau+\tau F(\hat{v}_{b1})}}_{\mathbf{P}(\text{ Informed } | ND)} \underbrace{\frac{1}{F(\tilde{v}_{b1})} \int_{-\infty}^{\hat{v}_{b1}} x \mathrm{d}F(x)}_{\mathbf{E}[v | \hat{v} < \hat{v}_{b1}]}$$

$$\hat{v}_{b1} - \lambda = \underbrace{\frac{1-\tau}{1-\tau+\tau\Phi\left(\frac{\hat{v}_{01}-\mu}{\sigma\sqrt{d}}\right)}}_{\mathbf{P}(\text{ Uninformed } | ND)} \mu + \underbrace{\frac{\tau\Phi\left(\frac{\hat{v}_{b1}-\mu}{\sigma\sqrt{d}}\right)}{1-\tau+\tau\Phi\left(\frac{\hat{v}_{b1}-\mu}{\sigma\sqrt{d}}\right)}}_{\mathbf{P}(\text{ Informed } | ND)} \underbrace{\left(\mu - \sigma\sqrt{\delta}\frac{\phi\left(\frac{\hat{v}_{\Delta 1}-\mu}{\sigma\sqrt{d}}\right)}{\Phi\left(\frac{\hat{v}_{b1}-\mu}{\sigma\sqrt{d}}\right)}\right)}_{E[v | \hat{v} < \hat{v}_{b1}]}$$

$$\hat{v}_{b1} - \lambda = \mu - \frac{\tau\phi\left(\frac{\hat{v}_{01}-\mu}{\sigma\sqrt{d}}\right)}{1-\tau+\tau\Phi\left(\frac{\hat{v}_{01}-\mu}{\sigma\sqrt{d}}\right)}\sigma\sqrt{\delta}$$

$$\Rightarrow \frac{\lambda}{\sigma\sqrt{d}} = x + \frac{\tau_O(x)}{1-\tau+\tau\Phi(x)} \text{ where } x = \frac{\hat{v}_{b1}-\mu}{\sigma\sqrt{\delta}}$$

Case 2: Mandatory disclosure with only lying costs

write the investors' conjecture as $r = r_0 + r_v v$ to get CEO's payoff:

$$\underbrace{\frac{r-r_0}{r_v}}_{\mathrm{E}[v|r]} - \frac{c}{2}(r-\hat{v})^2 \Rightarrow 0 = \frac{1}{r_v} - c(r-v) \Rightarrow r = v + \frac{1}{r_v c} \Rightarrow r_v = 1, r_0 = \frac{1}{c}$$

Case 3: Voluntary disclosure with manipulation

$$\text{CEO's objective function: } \max_r \mathrm{E}[v \mid r] - \lambda - \tfrac{c}{2}(r-\hat{v})^2$$

$$\text{FOC: } 0 = \frac{\partial \mathrm{E}[v|r]}{\partial r} - c(r-\hat{v}) \Rightarrow r = \hat{v} + \underbrace{\frac{1}{c}\frac{\partial \mathrm{E}[v \mid r]}{\partial r}}_{b(\hat{v})}$$

To prevent a CEO with $\hat{v}$ from mimicking one with $\hat{v} + \Delta\hat{v}$, it must be that:

$$\mathrm{E}[v \mid r = \hat{v} + b(\hat{v})] - \lambda - \tfrac{c}{2}b(\hat{v})^2$$

$$\geq \mathrm{E}[v \mid r = \hat{v} + \Delta\hat{v} + b(\hat{v}+\Delta\hat{v})] - \lambda - \tfrac{c}{2}(\Delta\hat{v} + b(\hat{v}+\Delta\hat{v}))^2$$

$$\text{Then } \mathrm{E}[v \mid r = \hat{v} + b(\hat{v})] = \hat{v} \Rightarrow -b(\hat{v})^2 \geq \tfrac{2}{c}\Delta\hat{v} - (\Delta\hat{v} + \Delta b + b(\hat{v}))^2$$

$$-b(\hat{v})^2 \geq \tfrac{2}{c}\Delta\hat{v} - \left(b(\hat{v})^2 + 2b(\hat{v})(\Delta\hat{v} + \Delta b) + (\Delta\hat{v} + \Delta b)^2\right)$$

$$\Leftrightarrow 0 \leq \left(1 + \tfrac{\Delta b}{\Delta v}\right)^2 + 2\tfrac{b(v)}{\Delta v}\left(1 + \tfrac{\Delta b}{\Delta v}\right) - \tfrac{2}{c}\tfrac{1}{\Delta v}$$

$$\Leftrightarrow 1 + \tfrac{\Delta b}{\Delta v} \notin \left(-\tfrac{1}{\Delta v}\left(\sqrt{\tfrac{2}{c}\Delta\hat{v} + b(\hat{v})^2} + b(\hat{v})\right), \tfrac{1}{\Delta v}\left(\sqrt{\tfrac{2}{c}\Delta\hat{v} + b(\hat{v})^2} - b(\hat{v})\right)\right)$$

where $\Delta b = b(\hat{v} + \Delta\hat{v}) - b(\hat{v})$. Taking the positive root gives:

$$\tfrac{\mathrm{d}b}{\mathrm{d}v} \geq \lim_{\Delta v \to 0} \underbrace{\frac{1}{\Delta v}\left(\sqrt{\frac{2}{c}\Delta\hat{v} + b(\hat{v})^2} - b(\hat{v})\right) - 1}_{\Delta b/\Delta v}$$

$$= \lim_{\Delta v \to 0} \tfrac{1}{c}\left(\tfrac{2}{c}\Delta\hat{v} + b(\hat{v})^2\right)^{-1/2} - 1 = \tfrac{1}{cb(v)} - 1 \quad \text{(using l'Hopital's rule)}$$

Similar to the above, to prevent a manager with $\hat{v}+\Delta\hat{v}$ from mimicking one with $\hat{v}$, it must be the case that:

$$\mathrm{E}[v \mid r=\hat{v}+\Delta\hat{v}+b(\hat{v}+\Delta\hat{v})]-\lambda-\frac{c}{2}b(\hat{v}+\Delta\hat{v})^2 \geq \mathrm{E}[v \mid r=\hat{v}]-\lambda-\frac{c}{2}(b(\hat{v})-\Delta\hat{v})^2$$

$$\Leftrightarrow 0 \geq \left(\frac{\Delta b}{\Delta v}\right)^2+2\frac{b(v)}{\Delta v}\frac{\Delta b}{\Delta v}-\left(\frac{2}{c}\frac{1}{\Delta v}-2\frac{b(v)}{\Delta v}+1\right)$$

$$\Leftrightarrow \frac{\Delta b}{\Delta v} \in \left(-\frac{1}{\Delta v}(\sqrt{(b(\hat{v})-\Delta\hat{v})^2+\frac{2}{c}\Delta\hat{v}}+b(\hat{v})), \frac{1}{\Delta v}(\sqrt{(b(\hat{v})-\Delta\hat{v})^2+\frac{2}{c}\Delta\hat{v}}-b(\hat{v}))\right)$$

Again using the positive root and taking limits gives the following again using l'Hopital's rule:

$$\frac{\mathrm{d}b}{\mathrm{d}v} \leq \lim_{\Delta v\to 0}\frac{1}{\Delta v}(\sqrt{(b(\hat{v})-\Delta\hat{v})^2+\frac{2}{c}\Delta\hat{v}}-b(\hat{v}))=\frac{1}{cb(v)}-1$$

The two inequalities implies that $\frac{\mathrm{d}b}{\mathrm{d}\hat{v}}=\frac{1}{cb(v)}-1$ so that $b(\hat{v})$ solves the differential equation $b(\hat{v})\left(1+b'(\hat{v})\right)=\frac{1}{c}$. The solution is $b(\hat{v})=\frac{1}{c}\left(1+\omega\left(-\mathrm{e}^{-\left(1+c\hat{v}-c^2k\right)}\right)\right)$ where ω is the Lambert-W function and the boundary condition $b\left(\hat{v}_0\right)=0$ implies $k=\frac{\hat{v}_0}{c}$ This gives:

$$b(\hat{v})=\frac{1}{c}\left(1+\omega\left(-\mathrm{e}^{-(1+c(\hat{v}-\hat{v}_0))}\right)\right)$$

Note that we have $b\left(\hat{v}_0\right)=\frac{1}{c}\left(1+\omega\left(-\mathrm{e}^{-1}\right)\right)=0$ and $\lim_{\hat{v}\to\infty} b(\hat{v})=\frac{1}{c}(1+\omega(0))=\frac{1}{c}$

Jung & Kwon (1988)

5. Verifiable Disclosure Models (Partial Signaling Equilibria)

Based on Spring 2020 Accounting Theory class notes on the paper (Prof. Caskey)

Here partial disclosure can arise even when there are no disclosure costs.

0	1	1	2
CEO sees private signal with probability $1-p$	CEO can choose to disclose it it perfectly reveals x	Investors price firm and trade	Cashflow x is realized

- Firm has risky cashflow x with prior mean μ →
- x has support $[\underline{x},\overline{x}]$ and distribution $F(x)$ →
- Investors are risk **neutral** →
- at t=0, CEO sees private signal with probability p →
- If received, it perfectly reveals x →
- CEO can choose to credibly disclose it →
- Disclosure is costless →
- CEO cannot credibly convey he's uninformed →
- CEO's goal is to maximize firm valuation →
- Conjecture that CEO follows a threshold strategy →
- Show $\exists$ a partially revealing equilibrium →

If CEO discloses x, t=1 price will equal x

otherwise price considering that:

$$Prob(\text{CEO uninformed}) = p$$

$$Prob(\text{CEO witholding}) = (1-p)F(\hat{x})$$

$$Prob(\text{CEO discloses}) = (1-p)[1-F(\hat{x})]$$

$$\text{where } F(\hat{x}) = Prob(x \leq \hat{x}) = \int_{\underline{x}}^{\overline{x}} dF(x)$$

Now given non-disclosure (ND):

$$E[x|ND] = \frac{p}{p+(1-p)P(x<\hat{x})}E[x] + \frac{(1-p)P(x<\hat{x})}{p+(1-p)P(x<\hat{x})}E[x|x<\hat{x}]$$

$$E[x|ND] = \frac{p}{p+(1-p)F(\hat{x})}\mu + \frac{(1-p)F(\hat{x})}{p+(1-p)F(\hat{x})}\int_{\underline{x}}^{\hat{x}} \frac{xdF(x)}{F(\hat{x})}$$

$$= \frac{p}{p+(1-p)F(\hat{x})}\mu + \frac{(1-p)}{p+(1-p)F(\hat{x})}\left(\hat{x}F(\hat{x}) - \int_{\underline{x}}^{\hat{x}} F(x)\mathrm{d}x\right)$$

$$E[x|D] = E[x|ND] \Leftrightarrow E[x|x=\hat{x}] = E[x|x<\hat{x}]$$

$$\hat{x} = \frac{p}{p+(1-p)F(\hat{x})}\mu + \frac{(1-p)}{p+(1-p)F(\hat{x})}\left(\hat{x}F(\hat{x}) - \int_{\underline{x}}^{\hat{x}} F(x)\mathrm{d}x\right)$$

$$\hat{x}(p+(1-p)F(\hat{x})) = p\mu + (1-p)\left(\hat{x}F(\hat{x}) - \int_{\underline{x}}^{\hat{x}} F(x)\mathrm{d}x\right)$$

$$0 = p(\mu-\hat{x}) - (1-p)\int_{\underline{x}}^{\hat{x}} F(x)\mathrm{d}x \Rightarrow \text{denote this by } h(\hat{x})$$

→ $\exists$ a unique $\hat{x}^*$ s.t. $\hat{x}^*$ $\Rightarrow$ partially revealing equilibrium

Higher (FOSD) and/or less-risky (SOSD) payoffs lead to higher threshold values of $\hat{x}$, ie: less disclosure.

Proof by contradiction:

Let $F \succeq_{FOSD} G$ and $F \succeq_{SOSD} G$

Then assuming above isn't true, if thresholds $\hat{x}_g \geq \hat{x}_f$:

$$0 = h(\hat{x}_f, F) - h(\hat{x}_g, G)$$

$$0 = p\underbrace{(\hat{x}_g - \hat{x}_f)}_{>0} + p\underbrace{(\mu_f - \mu_9)}_{\geq 0 \text{ if } F\succeq_{FOSD}G}$$

$$+(1-p)\underbrace{\int_{\underline{x}}^{\hat{x}_f}(G(x)-F(x))\mathrm{d}x}_{\geq 0 \text{ if } F\succeq_{SOSD}G} + (1-p)\underbrace{\int_{y_f}^{\hat{x}_g} G(x)\mathrm{d}x}_{>0}$$

$$\rightarrow\leftarrow$$

Verrechia (1983)

5. Verifiable Disclosure Models (Partial Signaling Equilibria)

Based on Spring 2020 Accounting Theory class notes on the paper (Prof. Caskey)

This paper shows that under reasonable conditions, a partial disclosure equilibrium will arise: good news disclosed and bad news withheld.

0	1	2	3
CEO is endowed with signal y	CEO decides whether to release y incurring cost c	Trading	Cashflow Realization μ

- Traders are risk neutral
- Risky cashflow $\mu \sim N(y_0, h_0^{-1})$
- Private signal: $y = \mu + \epsilon$
- where $\epsilon \sim N(0, s^{-1})$
- Disclosure cost: c
- Trader's piors Ω are diffuse
- Let D= disclosure
- ND= no disclosure
- Looking for a partial equilibrium

Show $\exists \hat{x}$ s.t. $\hat{x}$ results in a Partial Equilibrium

we want to show that $\exists \hat{x}$ such that:

(1) $\forall y \geq \hat{x}$: CEO discloses y

(2) $\forall y < \hat{x}$: no disclosure of y

For above cases, value of firm =

(1) $E(\mu|D) = E(\mu - c \mid y)$

(2) $E(\mu|ND) = E(\mu \mid y < \hat{x})$

To solve, we consider that y at threshold $y = \hat{x}$ will render CEO indifferent between disclosing and not:

$$E(\mu - c \mid y = \hat{x}) = E(\mu \mid y < \hat{x})$$

or working with posteriors $\hat{\mu} = E[\mu|y]$ & $\hat{\mu}_x = E[\mu|y = x]$:

$$E(\hat{\mu} - c \mid \hat{\mu} = \hat{\mu}_x) = E(\hat{\mu} \mid \hat{\mu} < \hat{\mu}_x)$$

For normal variables $\mu \sim N\left(y_0, h_0^{-1}\right)$ & $\varepsilon \sim N\left(0, s^{-1}\right)$

This gives $E[u|y] = \hat{u} = y_0 + \frac{\frac{1}{h_0}}{\frac{1}{h_0} + \frac{1}{s}}(y - y_0) = \frac{1}{h_0+s}(h_0 y_0 + s y)$

$E[\hat{u}] = \frac{1}{h_0+s}(h_0 y_0 + s E[y]) = y_0$ and $var(\hat{u}) = \frac{s}{h_0+s}\frac{1}{h_0} = cov(u, \hat{u})$

Let $\phi(\cdot)$ & $\Phi(\cdot)$ denote pdf and cdf of $\hat{u} \sim \mathcal{N}\left(y_0, \frac{s}{h_0+s}\frac{1}{h_0}\right)$

$$\mathrm{E}[u \mid y < \hat{x}] = \mathrm{E}\left[\hat{u} \mid \hat{u} < \hat{u}_x\right] = y_0 - \sigma_u \frac{\phi\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)}{\Phi\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)}$$

The disclosure threshold will satisy:

$$\mathrm{E}\left[\hat{u} \mid \hat{u} < \hat{u}_x\right] = \hat{u}_x - c \Leftrightarrow \frac{c}{\sigma_u} = \frac{\hat{u}_x - y_0}{\sigma_u} + \frac{\phi\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)}{\Phi\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)} = f\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)$$

Let's use the implicit function theorem to get $\frac{\mathrm{d}\hat{u}_x}{\mathrm{d}\sigma_u}$

Let $0 = F(c, \sigma_u, y_0, \hat{u}_x) = f\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right) - \frac{c}{\sigma_u}$

$$\frac{\mathrm{d}\hat{u}_x}{\mathrm{d}\sigma_u} = -\frac{\frac{\partial F}{\partial \sigma_u}}{\frac{\partial F}{\partial \hat{u}_x}} = -\frac{-f'\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)\frac{\hat{u}_x - y_0}{\sigma_u^2} + \frac{c}{\sigma_u^2}}{f'\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)\frac{1}{\sigma_u}} = \frac{\hat{u}_x - y_0}{\sigma_u} - \frac{\frac{c}{\sigma_u}}{f'\left(\frac{\hat{u}_x - y_0}{\sigma_u}\right)}$$

$\exists$ unique $\hat{x}$ as a function of params resulting in partial signaling equilibrium

Ottaviani & Sorensen (2006)

6. Cheap Talk Models (Partition Equilibria)

This paper shows that if analysts have incentives to appear talented, then cheap talk distorts their behavior and limits communication. The paper considers impact of various objective functions on analyst forecasts where all parties have common priors.

Analyst of type t releases his forecast m — *Outcome x is realized* — *Market evaluates analyst's talent t*

n analysts simlutaneously issue forecasts →
Common prior of outcome $x \sim N(\mu, v^{-1})$ →
Each forecaster i sees $s_i = x + \epsilon_i$ →
where $s_i|x \sim N\left(x,\tau_i^{-1}\right)$ and their →
posterior becomes: $x|s_i \sim N\left(\frac{\tau_i s_i + v\mu}{\tau_i + v}, \frac{1}{\tau_i + v}\right)$ →
Denote pdf of this posterior by $q_i\left(x|s_i\right)$ →
Denote an honest forecast from i by h_i: →
$h_i\left(s_i\right) = \mathrm{E}\left(x|s_i\right) = \left(\frac{\tau_i}{\tau_i + v}\right) s_i + \left(\frac{v}{\tau_i + v}\right)\mu$ →
Let $t_i > 0$ be forecast accuracy talent of i →
Common prior: t is distributed by pdf $p(t)$ →
i issues a forecast m for outcome of x →
Ex-post: market uses $\{m,x\}$ to update →
its beliefs about the analyst's talent. →
Let $e = x - m$ be the forecast error →
t dictates error e's distribution by pdf $\tilde{g}(e|t)$ →
$\tilde{g}(e|t) = \frac{1}{2} t\hat{g}(t|e|)$ where $\hat{g}$ is a pdf over $[0,\infty)$ →
To ensure $e \sim N(0,\tau^{-1})$ priors must satisfy: →
(normal pdf) $\sqrt{\frac{\tau}{2\pi}}\exp\{\frac{\tau}{2}e^2\} = \int_0^\infty \tilde{g}(e|t)dP(t)$ →
Likelihood ratio $\frac{\tilde{g}(e|t)}{\tilde{g}(e|t')} \nearrow$ in $|e|$ when $t < t'$ →
meaning higher types have lower errors →
The analyst payoff is defined by $w(m|x)$ →
$w(m|x) = \int_0^\infty u(t)\mathrm{d}P(t|m,x) = \mathrm{E}^{mkt}[u(t)|m,x]$ →
$u(t) \nearrow$ in t & $P(t|m,x)$: mkt's posterior over t →
Analyst maximizes $u(m|s) = \mathrm{E}[w(m|x)|s]$ →

Analyst minimizes his perceived error $\hat{e}|s \sim N\left(\hat{s} - \mathrm{E}[x|s], var(x|s)\right)$

so that $\mathrm{E}[|\hat{e}|]$ is minimized at $\hat{s}^* = \mathrm{E}[x|s] = m_h$. This gives the forecast:

$$\underbrace{\frac{\nu}{\nu+\tau}\mu + \frac{\tau}{\nu+\tau}\hat{s}^*}_{m^*} = \left(1 - \left(\frac{\tau}{\nu+\tau}\right)^2\right)\mu + \left(\frac{\tau}{\nu+\tau}\right)^2 s \neq \underbrace{\left(1 - \frac{\tau}{\nu+\tau}\right)\mu + \frac{\tau}{\nu+\tau}s}_{m_h}$$

In other words, the honest forecast m_h minimizes $\mathrm{E}\left[(m-x)^2|s\right]$

while the analyst's optimal forecast minimizes $\mathrm{E}\left[(\hat{s}-x)^2|s\right]$

To infer types t, market has conjecture schedule with distribution $\varphi(m|x,t)$

Denote by S_m the partition of s that issues m under the conjectured strategy,

and $\varphi(m|x,t) = \int_{S_m} \mathrm{d}\tilde{G}(s|x,t)$. Then posterior distribution over types is:

$$p(t|m,x) = \frac{\varphi(m|x,t)}{\int_0^\infty \varphi(m|x,t)\mathrm{d}P(t|m,x)}$$

Two partition case where $m \in \{m_h, m_l\}$ for $S_h = s > \tilde{s}$, then:

$$\varphi\left(m_h|x,t\right) = \int_{\check{s}-x}^\infty \tilde{g}(e|t)\mathrm{d}e = \frac{1}{2}\int_{\check{s}-x}^\infty t\hat{g}(t|e|)\mathrm{d}e$$
$$= \frac{1}{2}\left(\int_{\min\{0,\check{s}-x\}}^0 t\hat{g}(t|e|)\mathrm{d}e + \int_{\max\{0,\check{s}-x\}}^\infty t\hat{g}(t|e|)\mathrm{d}e\right)$$
$$= \frac{1}{2}\left(\int_0^{\max\{0,x-\check{s}\}} t\hat{g}(te)\mathrm{d}e + \int_{\max\{0,\check{s}-x\}}^\infty t\hat{g}(te)\mathrm{d}e\right)$$
$$= \underbrace{\frac{1}{2}\left(\int_0^{\max\{0,t(x-\check{s})\}} \hat{g}(z)\mathrm{d}z + \int_{\max\{0,t(\check{s}-x)\}}^\infty \hat{g}(z)\mathrm{d}z\right)}_{\textit{Change of variable to z=te}}$$
$$= \frac{1}{2}(1 - \hat{G}(\max\{0, t(\check{s}-x)\}) + \hat{G}(\max\{0, t(x-\check{s})\}))$$

Note that $\varphi\left(m_l|2\check{s}-x,t\right) = 1 - \varphi\left(m_h|2\check{s}-x,t\right)$

$$= 1 - \frac{1}{2}(1 - \hat{G}(\max\{0, t(x-\check{s})\}) + \hat{G}(\max\{0, t(\check{s}-x)\}))$$
$$= \frac{1}{2}(1 - \hat{G}(\max\{0, t(\check{s}-x)\}) + \hat{G}(\max\{0, t(x-\check{s})\})) = \varphi\left(m_h|x,t\right)$$

This implies that the payoff $w\left(m_h|x\right) = w\left(m_l|2\check{s}-x\right)$.

To compute the difference $u\left(m_h|s\right) - u\left(m_l|s\right)$, first compute:

$$u\left(m_l|s\right) = \int_{-\infty}^\infty w\left(m_l|x\right)q(x|s)\mathrm{d}x$$
$$= \int_{-\infty}^{\check{s}} w\left(m_l|x\right)q(x|s)\mathrm{d}x + \int_{\check{s}}^\infty w\left(m_l|x\right)q(x|s)\mathrm{d}x$$
$$= \int_{\check{s}}^\infty \underbrace{w\left(m_l|2\check{s}-z\right)q(2\check{s}-z|s)\mathrm{d}z}_{\textit{Change variable to } z=2\check{s}-x} + \int_{\check{s}}^\infty w\left(m_l|x\right)q(x|s)\mathrm{d}x$$
$$= \int_{\check{s}}^\infty \left(w\left(m_h|x\right)q(2\check{s}-x|s) + w\left(m_l|x\right)q(x|s)\right)\mathrm{d}x$$

Similarly: $u\left(m_h|s\right) = \int_{\check{s}}^\infty \left(w\left(m_l|x\right)q(2\check{s}-x|s) + w\left(m_h|x\right)q(x|s)\right)\mathrm{d}x$

$$u\left(m_h|s\right) - u\left(m_l|s\right) = \int_{\check{s}}^\infty \left(w\left(m_h|x\right) - w\left(m_l x\right)\right)\left(q(x|s) - q(2\check{s}-x|s)\right)\mathrm{d}x$$

Bertomeu & Marinovic (2016)

6. Cheap Talk Model (without Partition Equilibrium)

This paper develops a cheap talk model that has a combination of cheap talk (s for 'soft') and verifiable information (h for 'hard'). Authors show that certain soft disclosures may contain as much information as hard disclosures. The paper develops a way of dealing with cheap talk without having partitions.

CEO decides whether to disclose h — *CEO makes cheap talk report r* — *Market prices payoff π*

CEO has firm with payoff $\pi = h + s$

CEO knows h & s, can convey h at cost c

CEO can make a cheap-talk report r

CEO optimizes $P(1_h, h, r) = E[\pi | 1_h h, r] - 1_h c$

where 1_h is an indicator for disclosing h

$Prob(CEO\ is\ constrained\ [\tau = 1]) = \gamma$

$Prob(CEO\ is\ unconstrained\ [\tau = 0]) = 1 - \gamma$

If CEO is type $\tau = 0$, r can be anything but if $\tau = 1$, he can't overreport: $r \leq s$ or $r \leq h + s$

Common prior of outcome $x \sim N(\mu, v^{-1})$

CEO has disclosure strategy $D(h, s, \tau)$ and reporting strategy $R(h, s, \tau)$

Formally his problem is: $\max_{d,r} P(d, 1_h h, r)$

s.t. $\tau r \leq \tau(1_h s + (1 - 1_h)\pi)$

Market pricing rule is: $P(0, 0, r) = \min(r, \lambda_0)$

and $P(1, h, r) = h - c + \min(r, z)$

where λ_0 is max possible price if $1_h = 0$

($\Rightarrow \lambda_0 = \max_r P(0, 0, r)$) and

z is max valuation of soft report if $1_h = 1$

($\Rightarrow h - c + z = \max_r P(1, h, r)$)

Consider the unconstrained CEO's problem of whether to disclose h:

$\tau = 0$ Unconstrained CEO will disclose h IFF

the resulting payoff is $\geq$ some threshold λ_0:

$$\underline{\textit{Payoff from issuing } h} \rightarrow h - c + z \geq \lambda_0 \leftarrow \underline{\textit{Payoff from issuing only } s}$$

$$\Leftrightarrow h \geq \lambda_0 - z + c \equiv k$$

Meanwhile a $\tau = 1$ type Constrainted CEO will disclose if

$$h - c + \min(s, z) \geq \min(\lambda_0, h + s)$$

Or equivalently,

the reporting strategies for disclosing h are:

$$\textit{Unconstrained}: \quad h - c + z \geq \lambda_0$$

$$\textit{Constrained}: \quad h - c + \min\{s, z\} \geq \lambda_0$$

z solves: $z = \mathrm{E}[s \mid \tau = 0 \cup \{\tau = 1, s \geq z\}]$

The maximum soft-only value λ_0 solves:

$$\lambda_0 = \mathrm{E}[\pi | \{\tau = 0, h - c + z < \lambda_0\} \bigcup$$

$$\{\tau = 1, \{\lambda_0 \leq \pi < \lambda_0 + c \text{ or } \underbrace{\lambda_0 + c \leq \pi < \lambda_0 + c + s - z}_{\textit{Exists only if } s>z}\}\}]$$

Crawford & Sobel (1982)

6. Cheap Talk Models (Partition Equilibria)

This paper explores the extent to which information transmission occurs when a sender's report is **not verifiable** and the sender **does not incur a direct cost** associated with a function of the action the receiver takes in response to the report. Full revelation is possible when sender and receiver incentives are perfectly aligned, whereas partial information transmission is possible when they are not too misaligned. An uninformative "babbling equilibrium" occurs when they become too misaligned.

Sender observes θ perfectly	Sender sends message $m(\theta)$ to Receiver regarding value of θ	Receiver estimates $E_R[\theta\|m]$ and takes action $y(E_R[\theta\|m]) = y^R(m)$

Sender S & Receiver R are risk neutral
and have common priors: $\widetilde{\theta} \sim Unif(0,1)$
S observes θ perfectly
S sends message m to Receiver
R then takes action y given m
S's utility: $U^S(y,\theta,b) \equiv -(y-\theta-b)^2$
which is concave in y, has bias term $b>0$
FOC w.r.t. y: optimum at $y^{S*}(\theta) = \theta + b$
R's utility: $U^R(y,\theta) \equiv -(y-\theta)^2$
FOC w.r.t. y: optimum at $y^{R*}(\theta) = \theta$
thus bias b makes it so $y^{S*}(m) > y^{R*}(m)$
The partitioning equilibrium exists if:
(i) $\exists$ action schedule s.t. $y^R(m) = E_R[\theta|m]$
for all m where $\exists$ a θ s.t. $m(\theta) = m$
(ii) messaging schedule $m(\theta)$ that solves
S's problem: $\max_m E_S[-(y-\theta-b)^2|m]$
Show that any such equilibria that exists
are all partition equilibria of type:

$$m(\theta) = \begin{cases} m_1 & \theta \in [0,a_1) \\ \vdots & \vdots \\ m_i & \theta \in [a_{i-1},a_i) \\ \vdots & \vdots \\ m_N & \theta \in [a_{N-1},1] \end{cases}$$

(**Note:** In the paper θ is m, and m is n)

Two Partitions Case: find threshold a^* that splits them

Sender: $m(\theta) = m_1$ if $\theta \in [0,a)$ and $m(\theta) = m_2$ if $\theta \in [a,1]$

Receiver: $\mathrm{E}_R(\theta \mid m_1) = a/2$ and $\mathrm{E}_R[\theta \mid m_2] = a/2 + 1/2$

At threshold a^*, Sender will be indifferent between both partitions

s.t. $U^S(E_S[y(m_1) \mid \theta \in (0,a^*)]) = U^S(E_S[y(m_2) \mid \theta \in (a^*,1)])$

$$\max_m E_S[-(y-a^*-b)^2|m_1] = \max_m E_S[-(y-a^*-b)^2|m_2]$$

$$-(E_S[y|m_1] - a^* - b)^2 = -(E_S[y|m_2] - a^* - b)^2$$

$$-(\tfrac{a^*}{2} - a^* - b)^2 = -(\tfrac{1}{2} + \tfrac{a^*}{2} - a^* - b)^2 \Leftrightarrow -(-\tfrac{a^*}{2} - b)^2 = -(\tfrac{1}{2} - \tfrac{a^*}{2} - b)^2$$

$$\Leftrightarrow -(\tfrac{(a^*)^2}{4} - a^*b + b^2) = -(\tfrac{1}{4} - \tfrac{a^*}{2} - b + \tfrac{(a^*)^2}{4} + a^*b + b^2)$$

$$\Leftrightarrow \quad 0 = -\tfrac{1}{4} + \tfrac{a^*}{2} + b \quad \Leftrightarrow \quad a^* = \tfrac{1}{2} - 2b \quad \text{QED.}$$

Thus the 2-partition equilibrium exists IFF $b \leq 1/4$

If $b > 1/4$: incentives are so misaligned that Receiver will only work with his unconditional $E[\theta] = \frac{\theta}{2}$ no matter the m that Sender sends.

N Partition Case: find thresholds a_i that splits them, ie:

Again $\forall i$, Sender with $\theta = a_i$ (right between two partitions) should be indifferent between sending signal m_i and m_{i+1}

Sender's optimal $y = a_i + b$ is between $\frac{a_{i-1}+a_i}{2}$ and $\frac{a_i+a_{i+1}}{2}$

$$a_i + b = \frac{\frac{a_{i-1}+a_i}{2} + \frac{a_i+a_{i+1}}{2}}{2} = \frac{a_{i-1}+2a_i+a_{i+1}}{4} \Rightarrow [a_{i+1} - a_i] = [a_i - a_{i-1}] + 4b$$

So since partition intervals are spaced out by distances of $4b$:

$$a_i = a_1 + (a_1 + 4b) + (a_1 + 2(4b)) + \cdots + (a_1 + (i-1)(4b))$$

$$a_i = ia_1 + (1 + 2 + \cdots + (i-1))4b = ia_1 + \frac{i(i-1)}{2}4b$$

$$\Rightarrow a_N = 1 = Na_1 + N(N-1)2b \quad \Rightarrow \quad a_1 = 1/N - 2(N-1)b$$

Thus the partitions' split points are $\Rightarrow \quad a_i = i/N - 2ib(N-i)$

Thus we've defined the N partition's intervals. Using the above to solve for b, the N partition equilibrium exists IFF $b < \frac{1}{2N(N-1)}$

$\Rightarrow N < \frac{1+\sqrt{1+2/b}}{2} \Rightarrow$ we know the lowest number of paritions N is equal to 2 if $b = 1/4$ and $N \to \infty$ as $b \to 0$

Caskey (2014)

7. Earnings management/Costly signaling (Partially Revealing Equilibrium)

This is an example of a **Fischer and Verrecchia (2000)**-type model where litigation is restricted to cases where investors may have actually suffered harm. The paper illustrates the feedback effect of litigation and the effect of unknown manager incentives to bias.

Time 0	Time 1	Time 2	Time 3
Investors observe signal y	CEO with bias b_m observes r reports r_m, Time 1 investors buy at price p_1	Time 2 information s is released, Time 2 investors buy at price p_2	v is realized. If Time 1 investors successfully sue they receive $1-a\times$ the firm's costs

Firm has terminal cashflow $v \sim N(\mu_v,\tau_v^{-1})$ ⟶
CEO observes r and makes report r_m ⟶
where $r = v + e_r$ & $e_r \sim N(0,\tau_e^{-1})$ ⟶
CEO has private information b_m about ⟶
his incentives to bias his report r_m ⟶
CEO optimizes: ⟶
$\max_{r_m} \mathrm{E}\left[bp_1\left(r_m,y\right) - \frac{c}{2}\left(r_m - r\right)^2 |r,b\right]$ ⟶
c is cost of misreporting parameter ⟶
Investors have prior $b \sim N\left(\mu_b, \sigma_b^2\right)$ ⟶
t=1 investors use r_m & y to price v at p_1 ⟶
at t=2, public signal s is released ⟶
where $y = v + e_y$, $s = v + e_s$ ⟶
$e_y \sim N(0,\tau_y^{-1})$ & $e_s \sim N(0,\tau_s^{-1})$ ⟶
t=2 investors price v at p_2 using s, y, r_m ⟶
If $p_2 < p_1$ t=1 investors can choose to sue ⟶
thus p_2 reflects risk of lawsuit costs: ⟶
$p_2 = \mathrm{E}_2[v - \theta\max\{0,p_1 - p_2\}] =$ ⟶
$\mathrm{E}_2[v] - 1_{p_2<p_1}\mathrm{E}_2[\theta](p_1 - p_2)$, where $\theta =$ ⟶
$Prob(\text{suit lost})E[\Delta\%\text{price}\searrow|\text{suit lost}]$ ⟶
ie: θ summarizes expected damages and ⟶
$\mathrm{E}_2[\theta] < 1$ (cannot gain value from suit) ⟶
$1_{p_2<p_1}$ is indicator function for $p_2 < p_1$ ⟶
note: $v_1 = \mathrm{E}_1[v|r_m,y]$ & $v_2 = \mathrm{E}_2[\mathrm{v}|r_m,y,s]$ ⟶
Investors conjecture CEO reporting rule: ⟶
$r_m = \rho_0 + \rho_r r + \rho_b b$ ⟶
CEO conjectures investor t=1 pricing rule: ⟶
$p_1 = \mathrm{E}\left[v \mid r_m,y\right] + \pi_0 + \pi_r r_m + \pi_y y$ ⟶

Solving for p_2 :

$$p_2 = \mathrm{E}_2\left[v - \theta\max\left\{0,p_1-p_2\right\} \mid r_m,y,s\right] = v_2 - \mathrm{E}_2\left[\theta\max\left\{0,p_1-p_2\right\} \mid r_m,y,s\right]$$

$$\Rightarrow \quad p_2 = v_2 - 1_{v_2<p_1}\frac{\mathrm{E}_2[\theta|r_m,y,s]}{1-\mathrm{E}_2[\theta|r_m,y,s]}\left(p_1 - v_2\right)$$

(note that v_2 is like a valuation of the v at $t=2$ before litigation is considered)

The price $p_2 = v_2$ whenever $v_2 \geq p_1$ or $1_{p_2<p_1}\mathrm{E}_2[\theta] = 0$

The $\frac{\mathrm{E}_2[\theta]}{1-\mathrm{E}_2[\theta]}$ term reflects the feedback effect of litigation: expected lawsuits cause price declines, which increase the expected magnitude of lawsuits.

Example:

Denote $v_2 = \mathrm{E}_2[v] = \frac{\tau_v\mu_v + \tau_e(s_1+s_2)}{\tau_v + 2\tau_e}$ and $v_1 = \mathrm{E}_1[v] = \frac{\tau_{\mathrm{v}}\mu_{\mathrm{v}} + \tau_*\partial_1}{\tau_{\mathrm{s}} + \tau_{\mathrm{e}}}$

Assume: $p_2 = v_2 - 1_{v_2<p_1}\theta\left(v_2 - p_1\right)$ where $\theta \in (0,1)$

and: $p_1 = \mathrm{E}_1\left[p_2\right] = v_1 - \theta\mathrm{E}_1\left[1_{v_2<p_1}\left(v_2 - p_1\right)\right]$

$$var_1\left(v_2\right) = \underbrace{\frac{1}{\tau_v+\tau_e}}_{var_1(v)} - \underbrace{\frac{1}{\tau_v+2\tau_e}}_{var_2(v)} = \frac{1}{\tau_v+\tau_e}\frac{\tau_e}{\tau_v+2\tau_e}$$

Using the formulas for expectations of truncated normals, we have:

$$\mathrm{E}_1\left[1_{v_2<p_1}\left(v_2-p_1\right)\right] = \left(v_1-p_1\right)\Phi\left(\frac{p_1-v_1}{std_1(v_2)}\right) - std_1\left(v_2\right)\phi\left(\frac{p_1-v_1}{std_1(v_2)}\right)$$

Thus: $p_1 = v_1 - \theta\left(\left(v_1-p_1\right)\Phi\left(\frac{p_1-v_1}{std1(v_2)}\right) - std_1\left(v_2\right)\phi\left(\frac{p_1-v_1}{std_1(v_2)}\right)\right)$

$$\Rightarrow 0 = \theta\left(\frac{p_1-v_1}{std_1(v_2)}\Phi\left(\frac{p_1-v_1}{std_1(v_2)}\right) - \phi\left(\frac{p_1-v_1}{std_1(v_2)}\right)\right) - \frac{p_1-v_1}{std_1(v_2)} = g\left(\frac{p_1-v_1}{std_1(v_2)}\right)$$

where $g(x) = \theta(x\Phi(x) - \phi(x)) - x, \quad g'(x) = \theta\Phi(x) - 1 < 0$

Because $g(x) \to \infty$ as $x \to -\infty$ and $g(0) = -\theta\phi(0) < 0$, there is a unique $x < 0$ that solves $g(x) = 0$, or, equivalently, a $x > 0$ that solves $g(-x) = 0$, so we can define p_1 as:

$$\frac{p_1-v_1}{std_1(v_2)} = -\hat{x} \quad \Rightarrow \quad p_1 = v_1 - std_1\left(v_2\right)\hat{x}$$

where $\hat{x}$ depends only on θ so that p_1 is linear in s_1.

Insurance: The model includes the possibility of an insurance policy that covers up to X in damages, in exchange for a premium x:

$$x = \theta\mathrm{E}\left[1_{v_2<\underline{v}_2}1_{p_2<p_1}\min\left\{X,p_1-p_2\right\}\right].$$

The ex ante firm value is: $\mathrm{E}[v] - \theta\alpha\times$ Expected damages (D)

Kartik (2009)

7. Earnings management/Costly signaling (Partially Revealing Equilibrium)

This paper introduces lying costs to cheap talk models. Main result is that lying costs make the sender's objective depend directly on his report, which removes some of the cheap talk aspects of the behavior.

Sender has type t | Sender sends message m | Receiver takes action $a_r(m)$

Sender S has type $t \in [0,1]$

S sends message $m \in M$ to R

Receiver R reacts with action a

Bias: S prefers $a_s(t) > a_r(m = t)\ \forall t$

R's utility: $u_r(a,t)$

S's utility: $u_s(a,t) - kC(\nu(\Psi(m)),t)$

where $kC(\nu(\Psi(m)),t)$ is cost of lying

$\Psi(m') = t'$ maps messages m to types t

$\nu(t) = r$: increasing reporting function

s.t. cost to report higher ts is greater

$C(r,t)$ mets out cost for reports $r > t$

k magnifies cost (assume $k > 0$)

FOC satisfies: $\frac{\partial C}{\partial r} = C_1(\nu(r^*), t) = 0$

SOC > 0 for cost <u>minimizing</u> report r^*

SOC satisfies: $C_{11}(\nu(r^*),t)\nu'(r^*)^2 > 0$

Assume $C_{12} < 0 < C_{11}$, so by IFT

on FOC: $\frac{\partial r^*}{\partial t} = -\frac{C_{12}(\nu(r^*),t)\nu'(r)}{C_{11}(\nu(r^*),t)\nu'(r)^2} > 0$

Let $\rho(t)$ be S's messaging rule

Assume $t \in (t_l,t_h)$ results in a fully

revealing equilibrium where R can

invert $\rho^*(t)$ to perfectly infer S's type.

Show $\exists$ Low-types-separate/High-types

-pool (LSHP) equilibrium defined by:

1) $\forall t < \underline{t}$: the claimed type $\rho(t) = \rho^*(t)$

2) $\forall t \geq \underline{t}$: the claimed type $\rho(t) = 1$

(with $\underline{t} \leq \bar{t} < 1$)

Low-types-separate/High-types-pool Equilibrium

Finding $\rho^*(t)$: for t in the fully revealing partition ($t \in (t_l,t_h)$)

S claims type $\tilde{t}$ to optimize: $\max_{\tilde{t}} u_s\left(a_r(\tilde{t}),t\right) - kC(\nu(\rho(\tilde{t})),t)$

$$\text{FOC: } 0 = \frac{\partial u_s\left(a_r(\tilde{t}),t\right)}{\partial a_r} a_r'(\tilde{t}) - kC_1(\nu(\rho(\tilde{t})),t)\nu'(\rho(\tilde{t}))\rho'(\tilde{t})$$
$$\Rightarrow \rho'(t) = \frac{\frac{\partial u_s(a_r(t),t)}{\partial a_r} a_r'(t)}{kC_1(\nu(\rho(t)),t)\nu'(\rho(t))}$$

and integrating $\rho'(t)$ gives us $\rho^*(t)$: the invertable messaging rule that R can use to perfectly infer S's type t (full separation).

Knowing that types below $\underline{t}$ separate and those above $\underline{t}$ all pool at max t (=1), Sender of type $\underline{t}$ will be indifferent between:

$$\underbrace{u_s\left(a_r(\underline{t}),\underline{t}\right) - kC\left(\nu\left(\rho^*(\underline{t})\right),\underline{t}\right)}_{Payoff\ to\ revealing\ type\ \underline{t}} = \underbrace{u_s(\alpha(\mu(\underline{t})),\underline{t}) - kC(\nu(1),\underline{t})}_{Payoff\ to\ type\ \underline{t}\ claiming\ type\ 1}$$

From this we can find a unique $\underline{t}$.

Types $t \geq \underline{t}$ have typical Cheap Talk partition equilibrium

There are J partitions satisfying $t_{j+1} > t_j$, $t_0 = \underline{t}$ & $t_J = 1$ with:

$$1)\ a_r\left(t_{j-1},t_j\right) \quad = \quad \arg\max_a \int_{t_{j-1}}^{t_j} u_r(a,t)\mathrm{d}F(t)$$

$$2) u_s\left(a_r\left(t_{j-1},t_j\right),t_j\right) \quad = \quad u_s\left(a_r\left(t_j,t_{j+1}\right),t_j\right)$$

$$3)\ u_s(a_r(\underline{t},t_1),\underline{t}) - kC(\nu(1),\underline{t}) = u_s(a_r(\underline{t}),\underline{t}) - kC(\rho^*(\underline{t}),\underline{t}) \text{ if } \underline{t} > 0$$

[Kartik (2009, S4)] Example with uniform type distribution: $t \sim Unif(0,1)$

with $F(t) = t$; $u_r(a_r,t) = -(a-t)^2$; $u_s = -(a-t-b)^2$; $\nu(t) = t$; $C(x,t) = -(x-t)^2$

Receiver's Problem: $max_{a_r} u_r(a_r,t) = max_{a_r} - (a-t)^2$

FOC: $\frac{\partial u_r}{\partial a_r} = 0 = -2(a_r - \tilde{t}) \Leftrightarrow a_r = \tilde{t}$

Sender's problem: $max_{\tilde{t}}\ u_s(a_r(\tilde{t}),t) + kC(\nu(\rho(\tilde{t})),t) = max_{\tilde{t}}\ -(\tilde{t}-t-b)^2 - k(\rho(\tilde{t})-t)^2$

FOC (where conjectures are met and $\tilde{t} = t$) $\frac{\partial u_s}{\partial a} = 0 = -2b - 2k(\rho(t)-t)\rho'(t) \Leftrightarrow \rho'(t) = \frac{b}{k(\rho(t)-t)}$

Let $v(x) = \rho(t) - t$:

$$\frac{dv}{dt} = \frac{\frac{b}{k} - v}{v} \Leftrightarrow \frac{v}{\frac{b}{k} - v} dv = dt$$

By integrating we get

$$-v - \frac{b}{k} \ln\left(\frac{b}{k} - v\right) = t + constant$$

Let $v = \mu - x$, then

$$-(\rho(t) - t) - \frac{b}{k} \ln\left(\frac{b}{k} - (\rho(t) - t)\right) = t + constant$$

using $\rho(0) = 0$ we get $constant = -\frac{b}{k} \ln \frac{b}{k}$ and

$$-\rho(t) - \frac{b}{k} \ln\left(\frac{b}{k} - \rho(t) + t\right) = -\frac{b}{k} \ln \frac{b}{k} \Leftrightarrow -\frac{k}{b}\rho(t) - \ln\left(\frac{b}{k} - \rho(t) + t\right) = -\ln \frac{b}{k}$$

$$\Leftrightarrow -\frac{k}{b}\rho(t) + \ln \frac{b}{k} = \ln\left(\frac{b}{k} - \rho(t) + t\right) \Leftrightarrow \exp\left(-\frac{k}{b}\rho(t) + \ln \frac{b}{k}\right) = \left(\frac{b}{k} - \rho(t) + t\right)$$

$$\Leftrightarrow \frac{b}{k} \exp\left(-\frac{k}{b}\rho(t)\right) = \frac{b}{k} - \rho(t) + t \Leftrightarrow \exp\left(-\frac{k}{b}\rho^*(t)\right) = 1 - \frac{k}{b}\left(\rho^*(t) - t\right)$$

Note that this implies inflated language, with $t = 0$ giving $\rho(0) = 0$ and $t > 0$ giving $\rho(t) > t$ because $\omega(x) > 0$ for $x > -e^{-1}$ The ratio $\frac{k}{b}$ of lying costs to bias determines the extent of separation. All types $t < \underline{t} < \bar{t} < 1$ separate, and there is no separation ($\underline{t} = 0$) if $b \geq k + \frac{1}{4}$. In other words, when $b \geq k + \frac{1}{4}$ there is only cheap talk. The cheap-talk partitions look similar to Crawford and Sobel's (1982) uniform/quadratic example (Kartik 2009, p. 1373).

Stein (1989)

7. Costly Signaling (Signal Jamming Equilibrium)

This paper examines the effects of costly lying on manager reporting behavior. It introduces the "myopic" behavior: CEOs always exaggerating their earnings in spite of the fact that investors adjusts for this fact, knowing management always inflates their reports. **Note**: This is a steady-state learning model.

$t-1$ | t | t | t | $t+1$

Investors conjecture earnings management level of $\bar{b}$

CEO picks optimal earnings management level knowing this

Conjectures are met in equilibrium ($b_t = \bar{b}$) and Investors price firm

Let $e_t^n = z_t + v_t$ be natural earnings

Structural earnings z_t follow the random walk:

$z_t = z_{t-1} + u_t$ with $u_t \sim N(0,\sigma_u^2)$

Thus z_t has persistant components over time

vs $v_t \sim N(0,\sigma_v^2)$ which are time independent

z_t & v_t are not observed by CEO or market

Observable earnings are $e_t = e_t^n + b_t - c(b_{t-1})$

by manipulating e_t^n, CEO can "borrow" b_t

and pay it back as $c(b_t)$ next period where

$c()$ is a convex (eg: exponential) cost function

$c'(0) = (1+r)$ and r is firm's discount rate

Firm pays out all earnings e_t as dividends

Its market price is $P_t = E_t^{mkt} \sum_{j=1}^{\infty} \frac{e_{t+j}}{(1+r)} j$

Every period CEOs sell π of their shares

CEO utility: $U_t = e_t + \pi P_t + \frac{(1-\pi)e_{t+1}}{(1+r)}$

In steady state, equilibrium borrowing is $\bar{b}$

Investors have a fixed conjecture on value of $\bar{b}$

So market can reconstruct $\hat{e}_t^n = e_t + c(\bar{b}) - \bar{b}$

So given past $\hat{e}_t^n$, expected future natural

earnings are $E_t(e_{t+k}^n) = \sum_{j=0}^{\infty} \alpha_j \hat{e}_{t-j}^n \quad \forall k > 0$

where α_j sum to one and depend on σ_u^2 and σ_v^2

So coefficient α_0: impact of e_t on expectations

can be derived as $\alpha_0 = \left(k^2/4 + k\right)^{1/2} - k/2$

where $k = \frac{\sigma_u^2}{\sigma_v^2} = \frac{permanent\ earnings\ drift}{same{-}period\ earnings\ shock}$

$\frac{\sigma_u^2}{\sigma_v^2}$ plays following roles in forming expectations:

as $\frac{\sigma_u^2}{\sigma_v^2} = \alpha_0 \to 0$: past values more important

as $\frac{\sigma_u^2}{\sigma_v^2} = \alpha_1 \to 0$: past values less important

Looking at CEO's problem at time t, taking $\bar{b}$ as being fixed by market's conjecture:

$$\max_{b_t} U_t = e_t + \pi P_t + \frac{(1-\pi)e_{t+1}}{(1+r)}$$

$$\text{FOC: } \frac{de_t}{db_t} + \pi \frac{dP_t}{db_t} + \left(\frac{1-\pi}{1+r}\right) \frac{de_{t+1}}{db_t} = 0$$

$$1 + \pi \frac{\alpha_0}{r} + \frac{1-\pi}{1+r}(-c'(b_t))$$

$$c'(b_t) = \left(1 + \pi \frac{\alpha_0}{r}\right) \frac{1+r}{1-\pi}$$

In equilibrium conjectures are fulfilled, thus $b_t = \bar{b}$:

$$c'(\bar{b}) = \left(1 + \pi \frac{\alpha_0}{r}\right) \frac{1+r}{1-\pi}$$

Thus in a steady state signal-jamming equilibrium, managers will borrow a constant amount $\bar{b}$ each period from next period's earnings $e_t + 1$ and the market will correctly anticipate this borrowing.

This setting leads to a steady-state signal jamming equilibrium

Cheynel & Levine(2020)

8. Voluntary + mandatory disclosure (Rational Expectations Model)

This paper examines the interaction between voluntary public disclosure and private information in a setting where the two types of signals interact. The firm's payoff $v = \theta\omega$ where an investor has private information about θ and the firm can release public information about ω.

$t=1$	$t=2$	$t=3$	$t=4$
Informed trader gathers signal about θ	CEO observes s with probability $(1-q)$ then CEO decides whether to disclose $(d=s)$ or not $(d=ND)$	Market maker observes total demand $y = x+u$, then sets price P to clear market.	Value $\theta\omega$ is revealed.

Risky asset $v = \theta\omega$ where $\longrightarrow$

$\theta \sim N\left(m_\theta, \sigma_\theta^2\right)$, $\omega \sim N\left(m_\omega, \sigma_\omega^2\right)$ $\longrightarrow$

$m_\theta > 0$, $m_\omega > 0$ $\longrightarrow$

Noise trader demand $u \sim N(0, \sigma_u^2)$ $\longrightarrow$

Informed trader I knows θ perfectly $\longrightarrow$

I optimizes $\max_x E^I[x(v-p)|d,\theta]$ $\longrightarrow$

CEO sees s with probability $(1-q)$ at $\longrightarrow$

t=2, where $s = \omega + e$ & $e \sim N(0, \sigma_e^2)$ $\longrightarrow$

$E^{CEO}[\omega|s] = \frac{\sigma_\omega^2}{\sigma_\omega^2+\sigma_e^2}s + \frac{\sigma_e^2}{\sigma_\omega^2+\sigma_e^2}m_\omega$ $\longrightarrow$

CEO can then disclose s s.t. $d = s$ $\longrightarrow$

or make no disclosure s.t. $d = ND$ $\longrightarrow$

Mkt maker observes flow $y = x + u$ $\longrightarrow$

then sets price $p = E^{mkt}(\theta\omega|y,d)$ $\longrightarrow$

Conjecture of I's $x = \gamma + \beta(\theta\omega)$ $\longrightarrow$

which equals: $x = \gamma + \beta\theta E^I[\omega|d]$ $\longrightarrow$

Conjecture of pricing rule: $p = \mu + \lambda y$ $\longrightarrow$

Proposition 1

Informed trader solves his problem: $\max_x E^I[x(v-p) \mid d, \theta]$

Knowing market pricing rule $p = \mu + \lambda y$

FOC: $0 = E^I[v|d,\theta] - \mu - 2\lambda x$ (SOC: $\lambda > 0$). Thus $x = \frac{\theta E^I[\omega|d] - \mu}{2\lambda}$

Now knowing mkt maker's conjecture of his demand $x = \gamma + \beta\theta E^I[\omega|d]$

Would mean that $x = \gamma + \beta\theta E^I[\omega|d] = \frac{\theta E^I[\omega|d]}{2\lambda} - \frac{\mu}{2\lambda}$

Which implies that $\beta = \frac{1}{2\lambda}$ and $\gamma = \frac{-\mu}{2\lambda}$

To solve for mkt maker's optimal price, use law of iterated expectations

$$E[v|y,d] = E[E(v|y,d,\theta)|y,d] = E[\theta E(\omega|y,d)|y,d] = E[\theta|y,d]E[\omega|y,d]$$

$$E(\theta \mid Y, d) = E\left(\theta | \beta\theta E(\omega \mid d) + \gamma + u, d\right) = E(\theta \mid k, d) \text{ where } k = \frac{y}{\beta E(\omega|d)}$$

which yields $E(\theta \mid k, d) = m_\theta + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \frac{\sigma_u^2}{\beta E(\omega|d)^2}}(k - E(k))$

$$\lambda = \frac{2\sigma_\theta^2 \lambda E(\omega|d)^2}{4\sigma_u^2\lambda^2 + \sigma_\theta^2 E(\omega|d)^2} \quad \text{and} \quad \mu = \frac{E(\omega|d)\left(\sigma_\theta^2 \mu E(\omega|d) + 4m_\theta\sigma_u^2\lambda^2\right)}{4\sigma_u^2\lambda^2 + \sigma_\theta^2 E(\omega|d)^2}$$

Solving gives: $\lambda_d = \frac{\sigma_\theta}{2\sigma_u}|E(\omega \mid d)|$ and $\mu_d = m_\theta E(\omega \mid d)$

Law of Iterated Expectations (also known as Law of Total Expectation):

For a bivariate random variables, X and Y :

$$\mathrm{E}[Y] = \mathrm{E}[\mathrm{E}[Y \mid X]]$$

For a trivariate random variables, X, Y and Z where E[Y | X, Z] = conditional mean given X and Z :

$$\begin{aligned}
&\mathrm{E[Y] = E[E(Y \mid X, Z)]} \\
&\mathrm{E[Y \mid Z] = E[E(Y \mid X, Z) \mid Z]} \\
&\mathrm{E[Y \mid X] = E[E(Y \mid X, Z) \mid X]} \\
&\mathrm{E[Y] = E[E[E(Y \mid X, Z) \mid Z]]}
\end{aligned}$$

Einhorn (2018)

8. Voluntary + mandatory disclosure (Rational Expectations Model)

The author's Kyle (1985) based model uses competing public & private signals and discretionary manager disclosure as a reason for why firms withhold verifiable information to favor private information collection.

CEO perfectly knows but cannot credibly disclose $\boldsymbol{v}$. But he decides whether to disclose noisy signal $\widetilde{s}_1$.	Public signal $\widetilde{s}_2$. Then Speculator decides whether to additionally buy private signal $\widetilde{s}_3$ for cost c.	Trading. Market maker observes demand flow $y = x + z$ and sets price to clear market.	Value $\tilde{\boldsymbol{v}}$ is revealed.

Prior for risky firm value $\tilde{\boldsymbol{v}}$ is diffuse →
Liquidity trader demand $\tilde{z} \sim N(0, \sigma_z^2)$ →
Risk **neutral** Speculator has demand x →
Risk averse CEO has CARA coefficient a →
CEO perfectly knows but can't disclose $\boldsymbol{v}$ →
CEO has noisy private signal $\tilde{s}_1 = \tilde{v} + \tilde{\epsilon}_1$ →
CEO can choose whether to disclose $\tilde{s}_1$ →
If so then mkt's posterior $\tilde{\boldsymbol{v}}|s_1 \sim N(s_1, \frac{1}{h_1})$ →
Afterwards public signal s_2 is released →
Then Speculator decides whether to →
additionally buy private signal $\widetilde{s}_3$ for cost c →
$\tilde{s}_i = \tilde{v} + \tilde{\epsilon}_i$ and $\tilde{\epsilon}_i \sim N(0, \frac{1}{h_i})$ for $i = 1,2,3$ →
Mkt maker linear price conjecture $p = \alpha + \beta y$ →
clears mkt after seeing total demand $y = x + z$ →
CEO maxes mkt price p: $E[p] - \frac{a}{2} var(p)$ →
CEO doesn't know $\tilde{s}_2$ & $\tilde{s}_3$ beforehand →
We conjecture that Speculator will <u>only</u> find →
s_3 to be worth buying if s_1 was not disclosed →
& his demand $x > 0$ IFF he buys s_3, else $x = 0$ →
This implies that $c_L < c < c_H$, where c_L bound →
prevents him from buying when s_1 is disclosed →
and c_H is the max he'll be willing to pay for s_3 →
He maximized $E^S[x(\widetilde{v} - (\alpha + \beta(x + \tilde{z})))|s_2,s_3]$ →

Additional Definitions & Notes: →
Let $F(x) = \frac{\phi(x)}{\Phi(x)}$ for std normal pdf ϕ & cdf Φ →
Note on FOC/SOC: Given $\max_x\ U(x)$ →
(<u>all w.r.t. x</u>) $\quad FOC: \ \frac{dU}{dx} = U'(x) = 0$ →
$SOC: \ \frac{d^2U}{dx^2} = U''(x) < 0$ for maximization →
$SOC: \ \frac{d^2U}{dx^2} = U''(x) > 0$ for minimization →

Case 1: No competing information benchmark $(h_2 = h_3 = 0)$

Here signals s_2 and s_3 are uninformative. Speculator never buys $\tilde{s}_3$. "Unraveling": CEO discloses s_1 and price $p = E^{mkt}\left[\widetilde{v} \mid \widetilde{s}_1 = s_1\right] = s_1$

By Grossman (1981) if he did not disclose, off-equilibrium $p = -\infty$.

Case 2: Competing public but not private info $(h_2 > 0; h_3 = 0)$

Here signal s_3 is uninformative, so Speculator never buys $\tilde{s}_3$. If CEO discloses s_1, the market conjectures that it must have exceeded some threshold $v - \lambda^0$ for some scalar $\lambda^0 \in \Re$ s.t. $v \leq s_1 + \lambda^0$. Thus:

$$E^{mkt}\left[\hat{v} \mid \tilde{s}_1 = s_1, \tilde{s}_2 = s_2, \tilde{v} \leq s_1 + \lambda^0\right]$$

using Inverse Mills Ratio: $\mathrm{E}[X \mid X < \alpha] = \mu - \sigma \frac{\phi\left(\frac{\alpha-\mu}{\sigma}\right)}{\Phi\left(\frac{\alpha-\mu}{\sigma}\right)} = \mu - \sigma F(\frac{\alpha-\mu}{\sigma})$

$$\mathrm{E}[X \mid X < \alpha] = \mathrm{E}\left[E[\hat{v}|s_1, s_2] \mid \tilde{v} \leq s_1 + \lambda^0\right]$$

$$= E[\hat{v}|s_1, s_2] - \sqrt{var(\hat{v}|s_1, s_2)} \cdot F\left(\frac{(s_1+\lambda^0) - E[\hat{v}|s_1,s_2]}{\sqrt{var(\hat{v}|s_1,s_2)}}\right)$$

Mkt price $p = \frac{h_1 s_1 + h_2 s_2}{h_1 + h_2} - \frac{1}{\sqrt{h_1 + h_2}} F\left(\left[s_1 + \lambda^0 - \frac{h_1 s_1 + h_2 s_2}{h_1 + h_2}\right]\sqrt{h_1 + h_2}\right)$ under disclosure, and under non-disclosure: $p = E^{mkt}[\tilde{v}|s_2] = s_2$. Note that CEO doesn't know the realization of $\tilde{s}_2$ when deciding whether to disclose s_1. By working with CEO's expectations of $\tilde{s}_2$ and equating his expected risk adjusted utility, $\exists$ a unique λ_0 yielding a partial signaling equilibrium.

Case 3: Competing public and private info $(h_2 > 0; h_3 > 0)$

Speculator maxes $E^S[x(\widetilde{v} - (\alpha + \beta(x + \tilde{z})))|s_2,s_3] = xE^S[\widetilde{v}|s_2, s_3] - \alpha x - \beta x^2$ $= x\frac{h_2 s_2 + h_3 s_3}{h_2 + h_3} - \alpha x - \beta x^2 \Rightarrow$ FOC: $\frac{h_2 s_2 + h_3 s_3}{h_2 + h_3} - \alpha - 2\beta x = 0$, SOC: $-2\beta < 0$

This yields optimal $x^* = (2\beta)^{-1}\left(\frac{h_2 s_2 + h_3 s_3}{h_2 + h_3} - \alpha\right)$. Plugging this into his pricing rule, market maker infers $(2\beta)^{-1}\left(\frac{h_2 s_2 + h_3 s_3}{h_2 + h_3} - \alpha\right) + \widetilde{z} = y$

$$\Leftrightarrow \widetilde{s}_3 + 2\beta\frac{h_2 + h_3}{h_3}\widetilde{z} = -\frac{h_2}{h_3}s_2 + \alpha\frac{h_2 + h_3}{h_3} + 2\beta\frac{h_2 + h_3}{h_3}y$$

He sets $p = E^{mkt}\left[\hat{v} \mid \widetilde{s}_3 + 2\beta\frac{h_2 + h_3}{h_3}\widetilde{z} = -\frac{h_2}{h_3}s_2 + \alpha\frac{h_2 + h_3}{h_3} + 2\beta\frac{h_2 + h_3}{h_3}y, \widetilde{s}_2 = s_2\right]$

$$\text{price } p = \frac{\left(\frac{1}{h_3} + \left(2\beta\frac{h_2 + h_3}{h_3}\sigma_z\right)^2\right)^{-1}\left(-\frac{h_2}{h_3}s_2 + \alpha\frac{h_2 + h_3}{h_3} + 2\beta\frac{h_2 + h_3}{h_3}y\right) + h_2 s_2}{h_2 + \left(\frac{1}{h_3} + \left(2\beta\frac{h_2 + h_3}{h_3}\sigma_z\right)^2\right)^{-1}}$$

continued on next page.

$$\alpha=\frac{\left(\frac{1}{h_3}+\left(2\beta\frac{h_2+h_3}{h_3}\sigma_z\right)^2\right)^{-1}\left(-\frac{h_2}{h_3}s_2+\alpha\frac{h_2+h_3}{h_3}\right)+h_2s_2}{h_2+\left(\frac{1}{h_3}+\left(2\beta\frac{h_2+h_3}{h_3}\sigma_z\right)^2\right)^{-1}} \quad and \quad \beta=\frac{\left(\frac{1}{h_3}+\left(2\beta\frac{h_2+h_3}{h_3}\sigma_z\right)^2\right)^{-1}2\beta\frac{h_2+h_3}{h_3}}{h_2+\left(\frac{1}{h_3}+\left(2\beta\frac{h_2+h_3}{h_3}\sigma_z\right)^2\right)^{-1}}$$

$$\alpha=s_2 \quad and \quad \beta=\frac{1}{2\sigma_z}\cdot\sqrt{\frac{h_3}{h_2\left(h_2+h_3\right)}}$$

Plugging these into aggregate demand $y=(2\beta)^{-1}\left(\frac{h_2s_2+h_3s_3}{h_2+h_3}-\alpha\right)+\widetilde{z}$ and pricing coefficient $\alpha=s_2$ into the stock pricing rule $p=\alpha+\beta y$ implies $p=\frac{1}{2}s_2+\frac{1}{2}\frac{h_2s_2+h_3s_3}{h_2+h_3}+\beta z$

$$\Leftrightarrow p=\frac{\left(h_2+1/2h_3\right)s_2+1/2h_3\left(s_3+\sigma_z^{-1}\sqrt{h_2^{-1}+h_3^{-1}}\cdot z\right)}{h_2+h_3}$$

Also Speculator's optimal $x^*=(2\beta)^{-1}\left(\frac{h_2s_2+h_3s_3}{h_2+h_3}-\alpha\right)=\sigma_z\cdot\left(\frac{h_2s_2+h_3s_3}{h_2+h_3}-s_2\right)\cdot\sqrt{\frac{h_2(h_2+h_3)}{h_3}}$

Find upper bound c_H knowing Speculator will acquire s_3 IFF s_1 is not disclosed, ie: $c<c_H$

The max cost c_H he is willing to pay is his maximum expected profit (given he's risk neutral), which is: Given his demand $x=\frac{E^S[\tilde{v}|s_2,s_3]-\alpha}{2\beta}$, his expected utility is $E^S[U^S]=E^S\left[x(\widetilde{v}-(\alpha+\beta(x+\widetilde{z})))|\,s_2,s_3\right]$

$$E^S[U^S|D]=\frac{E[\tilde{v}|s_2,s_3]-\alpha}{2\beta}\left(E[\tilde{v}|s_2,s_3]-\left(\alpha+\beta(\frac{E\left[\tilde{v}\mid s_2,s_3\right]-\alpha}{2\beta}+\underbrace{E^S[\widetilde{z}]}_{=0})\right)\right)$$

$$E^S[U^S|D]=\frac{E[\tilde{v}|s_2,s_3]-\alpha}{2\beta}\left(E[\tilde{v}|s_2,s_3]-\alpha-\frac{E\left[\tilde{v}\mid s_2,s_3\right]-\alpha}{2}\right)=\frac{\left(E\left[\tilde{v}\mid s_2,s_3\right]-\alpha\right)^2}{4\beta}=\frac{\left(\frac{h_2s_2+h_3\tilde{s}_3}{h_2+h_3}-\alpha\right)^2}{4\beta}$$

or $U^S=\frac{\sigma_z}{2}\cdot\frac{h_3}{h_2+h_3}\sqrt{\frac{h_2h_3}{h_2+h_3}}\left(\tilde{s}_3-s_2\right)^2$ after plugging in α & β. However since he needs to make his decision of whether to buy s_3 before observing it, his <u>expected</u> utility is $E^S[U^S|D]$ before cost c.

$$E^S[U^S|D]=\frac{\sigma_z}{2}\cdot\frac{h_3}{h_2+h_3}\sqrt{\frac{h_2h_3}{h_2+h_3}}E\left[\left(\tilde{s_3}-s_2\right)^2\mid s_2\right]=\frac{\sigma_z}{2}\cdot\frac{h_3}{h_2+h_3}\sqrt{\frac{h_2h_3}{h_2+h_3}}var(\tilde{s}_3|s_2)=\frac{\sigma_z}{2}\cdot\sqrt{\frac{h_3}{h_2\left(h_2+h_3\right)}}$$

This implies that the max cost c_H he's be willing to pay is equal to $\frac{\sigma_z}{2}\cdot\sqrt{\frac{h_3}{h_2(h_2+h_3)}}$.

Find lower bound c_L knowing Speculator won't acquire s_3 when s_1 is disclosed, ie: $c_L<c$

This implies that his expected profit under disclosure of s_1 is at least 0 if he buys s_3:

Starting again with his expected profit: $E[x(\widetilde{v}-(\alpha+\beta(x+\widetilde{z})))|s_1,s_2,s_3]$ where $x=\frac{E[\widetilde{v}|s_1,s_2,s_3]-\alpha}{2\beta}$

Linear pricing rule conjecture coefficients become: $\alpha=\frac{h_1s_1+h_2s_2}{h_1+h_2}$ and $\beta=\frac{1}{2\sigma_z}\cdot\sqrt{\frac{h_3}{(h_1+h_2)(h_1+h_2+h_3)}})$

$$E^S[U^S|ND\ with\ s_3]=\frac{\left(\frac{h_1s_1+h_2s_2+h_3s_3}{h_1+h_2+h_3}-\alpha\right)^2}{4\beta}\ \text{or}\ \frac{\sigma_z}{2}\cdot\frac{h_3}{h_1+h_2+h_3}\sqrt{\frac{h_3\left(h_1+h_2\right)}{h_1+h_2+h_3}}\cdot\left(s_3-\frac{h_1s_1-h_2s_2}{h_1+h_2}\right)^2=$$

$$\frac{\sigma_z}{2}\cdot\frac{h_3}{h_1+h_2+h_3}\sqrt{\frac{h_3\left(h_1+h_2\right)}{h_1+h_2+h_3}}\cdot E\left[\left(\tilde{s_3}-\frac{h_1s_1+h_2s_2}{h_1+h_2}\right)^2\mid s_1,s_2\right]=\frac{\sigma_z}{2}\cdot\frac{h_3}{h_1+h_2+h_3}\sqrt{\frac{h_3\left(h_1+h_2\right)}{h_1+h_2+h_3}}\cdot var\left(\tilde{s_3}\mid s_1,s_2\right)$$

$=\frac{\sigma_z}{2}\cdot\sqrt{\frac{h_3}{(h_1+h_2)(h_1+h_2+h_3)}}\Rightarrow$ his expected profit when he has access to s_1 & s_3. Thus to respect the conjecture, c_L must be equivalent to this such that paying c_L for s_3 is unprofitable when s_1 is disclosed.

Pastor & Veronesi (2003)

10. Continuous time dynamic models

The basic idea of this paper is that firm value is convex in growth, so that uncertainty about growth leads to higher multiples.

- The book value of equity b_t follows the process: $\mathrm{d}b_t = \left(\underbrace{y_t}_{Earnings} - \underbrace{d_t}_{dividends} \right) \mathrm{d}t = (\rho_t - c)\, b_t \mathrm{d}t$ where ρ_t and c are the instantaneous earnings and dividend rates.
- Profits follow the mean-reverting process: $\mathrm{d}\rho_t = \phi\,(\bar{\rho} - \rho_t)\,\mathrm{d}t + \boldsymbol{\sigma}_\rho' \mathrm{d}\boldsymbol{w}_t$
- where $\boldsymbol{\sigma}_\rho = \{\sigma_{\rho 1}, \sigma_{\rho 2}\}$ is the two-dimensional vector of volatilities
- $\boldsymbol{w}_t$ is a two-dimensional standard Brownian motion.
- The first component represents systematic risks that are correlated with the later-specified, exogenous stochastic discount factor, and the second component represents idiosyncratic risks. Note that the drift is zero when $\rho_t = \bar{\rho}$, is positive when $\rho_t < \bar{\rho}$, and is negative when $\rho_t > \bar{\rho}$.
- Solving: $\mathrm{d}\rho_t = \phi\,(\bar{\rho} - \rho_t)\,\mathrm{d}t + \boldsymbol{\sigma}_\rho' \mathrm{d}\boldsymbol{w}_t \Leftrightarrow \mathrm{d}e^{\phi t}\rho_t = e^{\phi t}\phi\,(\bar{\rho} - \rho_t)\,\mathrm{d}t + \boldsymbol{\sigma}_\rho' e^{\phi t}\mathrm{d}\boldsymbol{w}_t$
  - **Applying Ito's Lemma to** $dX_t = \mu_t dt + \sigma_t dB_t$
    $\Rightarrow df(t, X_t) = (\frac{\partial f}{\partial t} + \mu_t \frac{\partial f}{\partial x} + \frac{\sigma_t^2}{2}\frac{\partial^2 f}{\partial x^2})dt + \sigma_t \frac{\partial f}{\partial x} dB_t$
  - Here: $X_t = \rho_t$; $\mu_t = \phi(\overline{\rho} - \rho_t)$; $B_t = \boldsymbol{w}_t$; $f(t, X_t) = f(t, \rho_t) = e^{\phi t}\rho_t$

$$Thus: \quad \mathrm{d}f(t, \rho_t) = \left(\frac{\partial f}{\partial t} + \phi(\overline{\rho} - \rho_t)\frac{\partial f}{\partial \rho} + \frac{\sigma_t' \sigma_t}{2}\frac{\partial^2 f}{\partial \rho^2} \right) dt + \sigma_p' \frac{\partial f}{\partial p} dw_t$$

$$\mathrm{d}e^{\phi t}\rho_t = \left(\phi e^{\phi t}\rho_t + \phi(\overline{\rho} - \rho_t)e^{\phi t} + \frac{\sigma_t'\sigma_t}{2}\underbrace{\frac{\partial^2 f}{\partial \rho^2}}_{=0} \right) dt + \sigma_p' e^{\phi t} dw_t = \overline{\rho}\phi e^{\phi t} dt + \sigma_p' e^{\phi t} dw_t$$

$$Integrating\ over\ (0, t): e^{\phi t}\rho_t - e^{\phi 0}\rho_0 = \phi\overline{\rho}\int_0^t e^{\phi s} ds + \sigma_p' \int_0^t e^{\phi s} dw_s$$

$$e^{\phi t}\rho_t - \rho_0 = \phi\overline{\rho}\left(\frac{[e^{\phi s}]_0^t}{\phi} \right) + \sigma_p' \int_0^t e^{\phi s} dw_s = \overline{\rho}[e^{\phi t} - 1] + \sigma_p' \int_0^t e^{\phi s} dw_s$$

$$\frac{e^{\phi t}}{e^{\phi t}}\rho_t = \frac{\rho_0}{e^{\phi t}} + \overline{\rho}\frac{[e^{\phi t} - 1]}{e^{\phi t}} + \sigma_p' \int_0^t e^{\phi s} dw_s \Leftrightarrow \rho_t = \mathrm{e}^{-\phi t}\rho_0 + \left(1 - \mathrm{e}^{-\phi t}\right)\bar{\rho} + \int_0^t \mathrm{e}^{-\phi(t-s)}\boldsymbol{\sigma}_\rho' \mathrm{d}\boldsymbol{w}_s$$

- or, equivalently, for $T > t : \rho_T = \mathrm{e}^{-\phi(T-t)}\rho_t + \left(1 - \mathrm{e}^{-\phi(T-t)}\right)\bar{\rho} + \int_t^T \mathrm{e}^{-\phi(T-s)}\boldsymbol{\sigma}_\rho' \mathrm{d}\boldsymbol{w}_s$
- Note that the expected value drifts towards the long-run mean of $\bar{\rho}$, with ϕ denoting the speed of convergence.
- Firm value m_t is the present value of future expected dividends that, given the exogenous stochastic discount factor π_t, equals: $m_t = \mathrm{E}_t\left[\int_t^T \frac{\pi_s}{\pi_t} c b_s \mathrm{d}s\right] + \mathrm{E}_t\left[\frac{\pi_T}{\pi_t} b_T\right]$ where the second term reflects that the market value m_T equals book value b_T at some known time T.

- The stochastic discount factor follows a geometric Brownian motion, where $\boldsymbol{\sigma}_\pi = \{\sigma_{\pi 1}, 0\}$ so that the first component of w_t represents systematic risk. The solution to $\mathrm{d}\pi_t = -r\pi_t \mathrm{d}t - \pi_t \boldsymbol{\sigma}_\pi' \mathrm{d}\boldsymbol{w}_t$ is: $\pi_t = \mathrm{e}^{-rt}\pi_0 \exp\left\{-\left(\frac{1}{2}\sigma_{\pi 1}^2 t + \int_0^t \boldsymbol{\sigma}_\pi' \mathrm{d}\boldsymbol{w}_s\right)\right\}$
- or, equivalently, for $T > t : \pi_T = \mathrm{e}^{-r(T-t)}\pi_t \exp\left\{-\left(\frac{1}{2}\sigma_{\pi 1}^2 (T-t) + \int_t^T \boldsymbol{\sigma}_\pi' \mathrm{d}\boldsymbol{w}_s\right)\right\}$
- Note that $\mathrm{E}_t\left[\frac{\pi_T}{\pi_t}\right] = \mathrm{e}^{-r(T-t)}$ reflects only the risk-free rate r

Additional notes:

1. $\frac{M_t}{B_t} = c\int_0^\tau z(\overline{\rho}, \rho_t, s)ds + z(\overline{\rho}, \rho_t, \tau)$ $\rightarrow$ This expression is just a statement that the market value equals the risk- adjusted present value of dividends. The 'proposition' part is obtaining the explicit expression z in terms of primitives.

2. The volatilities $\boldsymbol{\sigma}_\rho' \boldsymbol{\sigma}_\rho = \sigma_{\rho 1}^2 + \sigma_{\rho 2}^2$ and $\boldsymbol{\sigma}_\pi' \boldsymbol{\sigma}_\rho = \sigma_{\pi 1}\sigma_{\rho 1}$ affect z only via the q term:
$$q(s) = \frac{\sigma_\rho' \sigma_\rho}{2\phi^3} \underbrace{\left(\frac{1-\mathrm{e}^{-2\phi_s}}{2} + \phi s - 2\left(1-\mathrm{e}^{-\phi s}\right)\right)}_{>0} + \frac{\sigma_\pi' \sigma_\rho}{\phi^2} \underbrace{\left(1-\mathrm{e}^{-\phi s} - \phi s\right)}_{<0}$$
so that $q(s)$, and therefore $z\left(\bar{\rho}, \rho_t, s\right)$, is increasing in the volatility of profitability $\boldsymbol{\sigma}_\rho' \boldsymbol{\sigma}_\rho$ and decreasing in systematic volatility $\boldsymbol{\sigma}_\pi' \boldsymbol{\sigma}_\rho$

3. Current profitability ρ_t clearly increases z, while average proftability $\bar{\rho}$ impacts z by a factor proportional to $1 - \frac{1-e^{-\phi s}}{\phi s} > 0$

4. The case with unknown mean profitability $\bar{\rho}$ is a straightforward application of the Kalman filter. In this particular case, the uncertainty is deterministic.

5. Unknown mean profitability amplifies volatility. When $\bar{\rho}$ is known, a high profit reflects ρ_t that will increase prices by a small amount because the effect dissipates over time as can be seen in expression: $\rho_T = \mathrm{e}^{-\phi(T-t)}\rho_t + \left(1-\mathrm{e}^{-\phi(T-t)}\right)\bar{\rho} + \int_t^T \mathrm{e}^{-\phi(T-s)}\boldsymbol{\sigma}_\rho' \mathrm{d}\boldsymbol{w}_s$. When $\bar{\rho}$ is unknown, a high profit also increases beliefs about $\bar{\rho}$, which has persistent effects.

6. Learning has no effect on expected returns, consistent with the notion that expected returns do not depend on firm-specific information (e.g., Caskey et al. 2015)

Homework 1

Accounting Theory with Brett Trueman

$t=1$	$t=2$	$t=3$
Traders observe signal $y = v + \epsilon$	Investors price firm and trade	Cashflow v is realized

Risky asset prior $v \sim N(5, \sigma^2)$ →

Riskless asset yields zero return →

Traders are all risk **neutral** →

At t=1 →

all traders see $y = v + \epsilon$ →

where $\epsilon \sim N(0,\sigma_\epsilon^2)$ and $\epsilon \perp v$ →

Assume traders saw $y = 7$ →

What will be the market price at date 2?

By using properties of normal distributions:

$$E[v|y=7] = 5 + \tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}(7-5)$$

Or by scaling signals by their weighted average precisions:

$$E[v|y=7] = \tfrac{1/\sigma^2}{1/\sigma^2+1/\sigma_\epsilon^2}5 + \tfrac{1/\sigma_\epsilon^2}{1/\sigma^2+1/\sigma_\epsilon^2}7$$

Which we show to be equivalent here:

$$E[v|y=7] = \tfrac{1/\sigma^2}{1/\sigma^2+1/\sigma_\epsilon^2}5 + \tfrac{1/\sigma_\epsilon^2}{1/\sigma^2+1/\sigma_\epsilon^2}7 = \tfrac{1}{1+\sigma^2/\sigma_\epsilon^2}5 + \tfrac{1}{\sigma_\epsilon^2/\sigma^2+1}7$$
$$\tfrac{\sigma_\epsilon^2}{\sigma^2+\sigma_\epsilon^2}5 + \tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}7 = (1-\tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2})5 + \tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}7 = 5 + \tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}(7-5)$$

Effect on market price from increase in σ_2:

With $P = E[v|y=7] = 5 + \tfrac{\sigma^2}{\sigma^2+\sigma_\epsilon^2}(2)$

$$\tfrac{\partial P}{\partial \sigma_2} = \tfrac{2}{\sigma^2+\sigma_\epsilon^2} + \tfrac{-2\sigma^2}{(\sigma^2+\sigma_\epsilon^2)^2} = \tfrac{2\sigma_\epsilon^2}{(\sigma^2+\sigma_\epsilon^2)^2} > 0$$

Price goes up as investors put more weight on the now relatively more precise higher signal ($y = 7$) rather than the now relatively more imprecise lower prior (5).

Homework 2: Kyle (1985)

Accounting Theory with Brett Trueman

$t=1$	$t=2$	$t=3$
Informed observes	*Informed* places demand	Cashflow v
signal $\theta = v + \epsilon$,	Market Maker sets price to clear market	is realized

Kyle (1985) model →
Risky asset $v \sim N(\bar{v}, \sigma_v^2)$ →
Liquidity demand $u \sim N(0, \sigma_u^2)$ →
Informed trader is now **risk averse**: →
He picks demand x to max his end of →
period wealth: $max_x E[W_3] - \frac{a}{2} var(W_3)$ →
Informed trader initial wealth: W_0 →
W_0 is entirely r_f asset yielding 0% →
All stock purchases are financed →
through sales of this r_f asset →
at $t=1$ *Informed* observes $\theta = v + \epsilon$ →
where $\epsilon \sim N(0, \sigma_\epsilon^2)$ →
Market maker observes flow $y = x + u$ →
Informed's conjecture of Market →
Maker's pricing strategy: $P = \mu + \lambda y$ →
Market Maker's conjecture of private →
information in flow: $x = \alpha + \beta\theta$ →
Define W_1: leftover r_f asset after trading →

Starting with the *Informed* trader's optimization problem:

$$\max_x E[W_3] - \frac{a}{2} var(W_3)$$

$$\max_x E[W_1 + x(v-P)|\theta] - \frac{a}{2} var(W_1 + x(v-P)|\theta)$$

$$\max_x W_1 + xE[v-P|\theta] - \frac{a}{2} var(x(v-P)|\theta)$$

$$\max_x W_1 + xE[v-(\mu+\lambda y)|\theta] - \frac{a}{2} x^2 var(v-(\mu+\lambda y)|\theta)$$

$$\max_x W_1 + xE[v-(\mu+\lambda(x+u))|\theta] - \frac{a}{2} x^2 var(v-\mu-\lambda(x+u)|\theta)$$

$$\max_x W_1 + xE[v|\theta] - xE[\mu+\lambda x|\theta] - \frac{a}{2} x^2 var(v|\theta) - \frac{a}{2} x^2 var(\lambda(x+u)|\theta)$$

$$\max_x W_1 + xE[v|\theta] - x\mu - \lambda x E[x|\theta] - \frac{a}{2} x^2 var(v|\theta) - \frac{a}{2} x^2 var(\lambda u|\theta)$$

$$\max_x W_1 + xE[v|\theta] - x\mu - \lambda x^2 - \frac{a}{2} x^2 var(v|\theta) - \frac{a}{2} x^2 \lambda^2 \sigma_u^2$$

$$\max_x W_1 + x\left(E[v|\theta] - \mu\right) - \lambda x^2 - \frac{a}{2} x^2 \left(var(v|\theta) + \lambda^2\sigma_u^2\right)$$

$$\text{FOC: } 0 = E[v|\theta] - \mu - 2\lambda x - ax\left(var(v|\theta) + \lambda^2\sigma_u^2\right)$$

$$0 = E[v|\theta] - \mu - x\left(2\lambda + a\left(var(v|\theta) + \lambda^2\sigma_u^2\right)\right)$$

$$x = \frac{E[v|\theta]-\mu}{2\lambda + a(var(v|\theta)+\lambda^2\sigma_u^2)}$$

$$E[v|\theta] = E[v] + \frac{cov(v,\theta)}{var(\theta)}(\theta - E[\theta]) = \bar{v} + \frac{cov(v,v+\epsilon)}{var(v+\epsilon)}(\theta - E[v+\epsilon])$$

$$E[v|\theta] = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}(\theta - \bar{v})$$

$$var(v|\theta) = var(v) - \frac{cov^2(v,\theta)}{var(\theta)} = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2+\sigma_\epsilon^2} = \frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2}$$

$$x = \frac{\bar{v} + \frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}(\theta-\bar{v}) - \mu}{2\lambda + a\left(\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2} + \lambda^2\sigma_u^2\right)} = \frac{\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}\theta + \frac{\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2}\bar{v} - \mu}{2\lambda + a\left(\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2} + \lambda^2\sigma_u^2\right)}$$

$$\beta = \frac{\frac{\sigma_v^2}{\sigma_v^2+\sigma_\epsilon^2}}{2\lambda + a\left(\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2} + \lambda^2\sigma_u^2\right)} \qquad \alpha = \frac{\frac{\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2}\bar{v} - \mu}{2\lambda + a\left(\frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_v^2+\sigma_\epsilon^2} + \lambda^2\sigma_u^2\right)}$$

Now working with the pricing rule:

$$P = E[v|y] = E[v] + \frac{cov(v,y)}{var(y)} = \bar{v} + \frac{cov(v,x+u)}{var(x+u)}(y - E[y])$$

$$P = \bar{v} + \frac{cov(v,x+u)}{var(x+u)}(y - E[x+u]) = \bar{v} + \frac{cov(v,x+u)}{var(x+u)}$$

$$P = \bar{v} + \frac{cov(v,\alpha+\beta\theta+u)}{var(\alpha+\beta\theta+u)}(y - E[\alpha+\beta\theta])$$

$$P = \bar{v} + \frac{cov(v,\beta\theta)}{var(\beta\theta)+var(u)}(y - \alpha - \beta E[\theta])$$

$$P = \bar{v} + \frac{\beta cov(v,v+\epsilon)}{\beta^2 var(v+\epsilon)+\sigma_u^2}(y - \alpha - \beta E[v+\epsilon]) = \bar{v} + \frac{\beta\sigma_v^2}{\beta^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma_u^2}(y - \alpha - \beta\bar{v})$$

$$P = \bar{v} + \frac{\beta\sigma_v^2}{\beta^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma_u^2}(y - \alpha - \beta\bar{v})$$

$$\text{Thus: } \mu = \bar{v} - \frac{\beta\sigma_v^2}{\beta^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma_u^2}(\alpha+\beta\bar{v}) \quad \lambda = \frac{\beta\sigma_v^2}{\beta^2(\sigma_v^2+\sigma_\epsilon^2)+\sigma_u^2}$$

Plugging these coefficients into *Informed*'s α & β lets us solve for his demand in terms of raw economic parameters.

Homework 3 $\simeq$ Scharfstein & Stein (1990)

Accounting Theory with Brett Trueman

CEO sees signal y_i — CEO decideds whether to hire auditor — CEO sells $\alpha\%$ of the firm

Costly Signaling

- Firm cashflow $v \in \{v_L, v_H\}$
- $Prob(v_L) = Prob(v_H) = 0.5$
- CEO sees signal $y \in \{y_L, y_H\}$
- where $Prob(y_i|v_i) = p$ for $i \in \{L, H\}$
- CEO cannot credibly disclose y
- CEO sells a fixed $a\%$ of his shares
- He can choose to hire auditor for $\$C$
- who can provide report $a \in \{a_L, a_H\}$
- where $Prob(a_i|v_i) = q$ for $i \in \{L, H\}$
- CEO maximizes stock issue proceeds

Question:

For what range of C will the equilibrium be such that a CEO that sees y_H hires auditor but one who sees y_L doesn't?

Just confirming: $P(v_L|y_L) = \frac{P(y_L|v_L)P(v_L)}{P(y_L)} = \frac{p/2}{p/2+(1-p)/2} = \frac{p/2}{1/2} = p$

Thus $P(v_L|y_L) = P(y_L|v_L)$ and $P(v_H|y_L) = P(y_L|v_H)$ and so on.

An equilibrium in which a CEO hires the auditor only upon seeing y_H means the market can infer that the CEO saw y_H if he hires the auditor, and saw y_L if he did not hire her. **Define** $\hat{y}$ as the signal market infers given hiring decision such that $\hat{y} = y_L$ if no audit, $\hat{y} = y_H$ if auditor hired.

Conditional on not hiring auditor:

$$E^{mkt}[v \mid no\ audit] = E^{mkt}[v \mid \hat{y} = y_L] = P(v_H|y_L)v_H + P(v_L|y_L)v_L$$

$$= (1-p)v_H + pv_L = (1-p)v_H + pv_L + (1-p)v_L - (1-p)v_L$$

And so: $E^{mkt}[v \mid no\ audit] = E^{mkt}[v \mid \hat{y} = y_L] = v_L + (1-p)(v_H - v_L)$

Conditional only on hiring auditor (before seeing audit result):

$$E^{mkt}[v \mid audit] = E^{mkt}[v \mid \hat{y} = y_H] = P(v_H|y_H)v_H + P(v_L|y_H)v_L$$

$$E^{mkt}[v \mid audit] = pv_H + (1-p)v_L = v_L + p(v_H - v_L)$$

Conditional on audit (already knowing that $\hat{y} = y_H$)

and a_H : $E^{mkt}[v \mid a_H, y_H] = P(v_H|a_H, y_H)v_H + P(v_L|a_H, y_H)v_L$

and a_L: $E^{mkt}[v \mid a_L, y_H] = P(v_H|a_L, y_H)v_H + P(v_L|a_L, y_H)v_L$

Knowing that:

$$E^{mkt}[v \mid \hat{y} = y_H, y_H] = P(a_H|y_H)E^{mkt}[v \mid a_H, y_H] + P(a_L|y_H)E^{mkt}[v \mid a_L, y_H]$$

$$= [P(v_H|a_H, y_H)P(a_H|y_H) + P(v_H|a_L, y_H)P(a_L|y_H)]\, v_H$$

$$+ [P(v_L|a_H, y_H)P(a_H|y_H) + P(v_L|a_L, y_H)P(a_L|y_H)]\, v_L$$

$$= \left[P(v_H|a_H, y_H)\frac{P(a_H,y_H)}{P(y_H)} + P(v_H|a_L, y_H)\frac{P(a_L,y_H)}{P(y_H)}\right] v_H$$

$$+ \left[P(v_L|a_H, y_H)\frac{P(a_H,y_H)}{P(y_H)} + P(v_L|a_L, y_H)\frac{P(a_L,y_H)}{P(y_H)}\right] v_L$$

$$= \left[\frac{P(v_H,a_H,y_H)}{P(y_H)} + \frac{P(v_H,a_L,y_H)}{P(y_H)}\right] v_H + \left[\frac{P(v_L,a_H,y_H)}{P(y_H)} + \frac{P(v_L,a_L,y_H)}{P(y_H)}\right] v_L$$

$$= \frac{P(v_H,y_H)}{P(y_H)}v_H + \frac{P(v_L,y_H)}{P(y_H)}v_L = P(v_H|y_H)v_H + P(v_L|y_H)v_L$$

$$E^{mkt}[v|audit, y_H] = E^{mkt}[v|\hat{y} = y_H, y_H] = pv_H + (1-p)v_L = v_L + p(v_H - v_L)$$

$$\boldsymbol{aE^{mkt}[v \mid audit, \hat{y} = y_H, y_H] - C > aE^{mkt}[v \mid no\ audit, \hat{y} = y_L, y_H]}$$

$$aE^{mkt}[v \mid \hat{y} = y_H, y_H] - C > aE^{mkt}[v \mid \hat{y} = y_L, y_H]$$

$$a[v_L + p(v_H - v_L)] - C > a[v_L + (1-p)(v_H - v_L)] \Leftrightarrow a(2p+1)(v_H - v_L) > C$$

Thus C has to be smaller than $a(2p-1)(v_H - v_L)$

We now have the upper bound. Now to find the lower bound:

$$\boldsymbol{aE^{mkt}[v \mid audit, y_L] - C < aE^{mkt}[v \mid no\ audit, y_L]}$$

$$aE^{mkt}[v \mid \hat{y} = y_H, y_L] - C < aE^{mkt}[v \mid \hat{y} = y_L, y_L]$$

To calculate this other part, we start with similar structure:

$$E[v|audit, y_L] = E[v|\hat{y} = y_H, y_L] = v_L + P(v_H|a, \hat{y} = y_H, y_L)(v_H - v_L)$$

$$P(v_H|a, \hat{y} = y_H, y_L) = P(a_H|y_L)P(v_H|a_H, y_H) + P(a_L|y_L)P(v_H|a_L, y_H)$$

$$\boldsymbol{P(v_H|a, \hat{y} = y_H, y_L)} = [p(1-q)+(1-p)q]\frac{pq}{pq+(1-p)(1-q)} + [(1-p)(1-q)+pq]\frac{p(1-q)}{p(1-q)+(1-p)q}$$

$$a\left([p(1-q)+(1-p)q]\frac{pq}{pq+(1-p)(1-q)} + [(1-p)(1-q)+pq]\frac{p(1-q)}{p(1-q)+(1-p)q}\right) - C$$
$$< aE^{mkt}[v \mid \hat{y} = y_L, y_H] = a[v_L + (1-p)(v_H - v_L)]$$

Thus the lower bound is:

$$\boldsymbol{a\left([p(1-q)+(1-p)q]\frac{pq}{pq+(1-p)(1-q)} + [(1-p)(1-q)+pq]\frac{p(1-q)}{p(1-q)+(1-p)q}\right)}$$
$$\boldsymbol{-a[v_L + (1-p)(v_H - v_L)] < C}$$

and the upper bound is:

$$\boldsymbol{a(2p+1)(v_H - v_L) > C}$$

Homework 4 $\simeq$ Leland & Pyle (1977)

Accounting Theory with Brett Trueman

$t=0$	$t=0$	$t=1$
CEO picks firm's investment level q_0, sells off some r_0 to finance it	CEO sells off $(1-\alpha)\%$ of his shares after announcing q_0	This investment generates $kf(q_0)$ which is paid out

Denote t=0 investment level q_0 →
at t=0 CEO picks firm's q_0 →
at t=1, outlay generates $kf(q_0)$ →
Function $f(q_0)$ is publicly known →
$f'(q_0) > 0 \qquad f''(q_0) < 0$ →
CEO perfectly knows k while market →
only knows it distribution $k \sim N(\overline{k}, \sigma^2)$ →
Firm has endowment r_0 of riskfree asset →
Riskfree asset has zero return →
CEO finances q_0 by selling some r_0 at t=0 →
Still at t=0, but after announcing q_0: →
CEO sells a fixed $(1-\alpha)\%$ of his shares →
CEO is risk-**averse**: →
His $E[U^{CEO}] = E[W_1] - \frac{a}{2}var(W_1)$ →
While other investors are risk **neutral** →
$\exists$ a signaling equilibrium where →
q_0 perfectly reveals k →

We know that in equilibrium, market infers $\hat{k} = k(q_0) = k$ perfectly

Solving CEO's problem: $E^{CEO}[U] = E^{CEO}[W_1] - \frac{a}{2}var(W_1)$

$$E^{CEO}[W_1] = (1-\alpha)E^{mkt}[kf(q_0)|q_0] + \alpha kf(q_0) + r_0 - q_0$$

$$var^{CEO}(W_1) = var^{CEO}\left((1-\alpha)E^{mkt}[kf(q_0)|q_0] + \alpha kf(q_0) + r_0 - q_0\right)$$
$$var^{CEO}(W_1) = var^{CEO}((1-\alpha)f(q_0)E^{mkt}[k|q_0])$$
$$\boldsymbol{var^{CEO}(W_1) = (1-\alpha)^2 f(q_0)^2 var^{CEO}(E^{mkt}[k|q_0])}$$

because CEO knows:

$\boldsymbol{r_0 - q_0}$: His leftover riskfree asset holding after investing

$\boldsymbol{kf(q_0)}$: Outcome of production since he knows q_0 and k perfectly

$\boldsymbol{\alpha}$ & $\boldsymbol{1-\alpha}$: are exogenously fixed thus deterministic

Thus formaly:

$$max_{q_0} E^{CEO}[W_1] - \frac{a}{2}var(W_1)$$
$$= max_{q_0}(1-\alpha)f(q_0)E^{mkt}[k|q_0] + \alpha kf(q_0) + r_0 - q_0$$
$$-\frac{a}{2}var(1-\alpha)^2 f(q_0)^2 var^{CEO}(E^{mkt}[k|q_0])$$

FOC (conjectures are fulfilled in equilibrium so $E^{mkt}[k|q_0] = k$):

$$0 = (1-\alpha)f'(q_0)k + \alpha kf'(q_0) - 1 - \frac{a}{2}var(1-\alpha)^2 f(q_0)^2 var^{CEO}(k)$$
$$1 = (1-\alpha)f'(q_0)k + \alpha kf'(q_0) \Leftrightarrow \boldsymbol{f'(q_0)k = 1}$$

This is the differential equation that solves k as a function of q_0.

$\boldsymbol{E^{mkt}[kf(q_0)|q_0]}$: equals k since in this fully revealing signaling equilibrium market, market perfectly infers CEO's knowledge of k through his investment level q_0. The CEO, knowing this, can perfectly predict the market price the shares will sell for, eliminating any risk or volatiltity.

This equilibrium would continue to exist if the CEO was risk neutral since his aversion played no role in the currenmt equilibrium solution given that there was no riskiness in his wealth outcome.

Homework 5: Dye (1985) and Jung & Kwon (1988)

Accounting Theory with Brett Trueman

CEO has perfect info about firm value y with probability p	CEO decides whether to release y should he have it	Market Prices y	Cashflow y is realized

Firm has risky cashflow $y \sim Unif(0,t)$ →
With probability p CEO knows y perfectly →
Releasing y is costless if he has it →
He'll only do it if it increases price →
Find cutoff $\hat{y}$ above which he will release y and below which he will not. →

At the cutoff, a CEO with signal $\hat{y}$ will be indifferent between (D) disclosing $\hat{y}$ and (ND) not disclosing it:

$$E^{mkt}[y|\hat{y}, D] = E^{mkt}[y|\hat{y}, ND]$$
$$\hat{y} = P(no\ info)E^{mkt}[y] + P(info)P(withholding)E^{mkt}[y|witholding]$$
$$\hat{y} = \frac{1-p}{(1-p)+pP(y<\hat{y})}\frac{t}{2} + \frac{pP(y<\hat{y})}{(1-p)+pP(y<\hat{y})}E^{mkt}[y|y<\hat{y}]$$
$$\hat{y} = \frac{1-p}{(1-p)+p\frac{\hat{y}}{t}}\frac{t}{2} + \frac{p\frac{\hat{y}}{t}}{(1-p)+p\frac{\hat{y}}{t}}\frac{\hat{y}}{2}$$
$$\hat{y}[(1-p)+p\tfrac{\hat{y}}{t}] = (1-p)\tfrac{t}{2} + p\tfrac{\hat{y}}{t}\tfrac{\hat{y}}{2}$$
$$\hat{y}[(1-p)+p\tfrac{\hat{y}}{t}] = (1-p)\tfrac{t}{2} + p\tfrac{\hat{y}^2}{2t}$$
$$2(1-p)\hat{y} + 2p\tfrac{\hat{y}^2}{t} = (1-p)t + \tfrac{p}{t}\hat{y}^2$$
$$2(1-p)\hat{y} + 2\tfrac{p}{t}\hat{y}^2 - (1-p)t - \tfrac{p}{t}\hat{y}^2 = 0$$
$$\tfrac{p}{t}\hat{y}^2 + 2(1-p)\hat{y} - (1-p)t = 0$$

Using quadratic formula: $\hat{y} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

(keeping only postive term):

$$\hat{y} = \frac{-2(1-p)+\sqrt{4(1-p)^2+4\frac{p}{t}(1-p)t}}{2\frac{p}{t}}$$
$$\hat{y} = \frac{-2(1-p)+2\sqrt{1-2p+p^2+(p-p^2)}}{2\frac{p}{t}}$$
$$\hat{y} = \frac{t}{p}\left[-(1-p)+\sqrt{1-p}\,\right]$$

Homework 6: Trueman (1994)

Accounting Theory with Brett Trueman

CEO sees signal y and forecasts earnings	Earnings are released	Market assesses CEO's ability $P(CEO\ is\ high\ ability)$

Earnings x can be high (2) or low (1)

Ex-ante probability $P(x=1)=0.7$

Ex-ante probability $P(x=2)=0.3$

CEO gets signal y about level of x

CEO can be high ability (CEO_H)

or low ability (CEO_L)

Ex-ante $P(CEO_H)=P(CEO_L)=\frac{1}{2}$

$P(y=2|x=2, CEO_H)=g$

$P(y=1|x=2, CEO_H)=1-g$

$P(y=2|x=2, CEO_L)=b$

$P(y=1|x=2, CEO_L)=1-b$

CEO forecasts level of x: $f\in\{1,2\}$

CEO receives wage equal to

$C*P^{mkt}(CEO_H|f,x)\ -1_{x\neq f}\ \$K$

If realized $x\neq f$, he gets -$\$K$ penalty

Market conjectures:

CEO_H always reports his y truthfully

CEO_L reports a $y=1$ truthfully

But CEO_L will lie at a rate α if $y=2$

ie: α% of the time CEO_L

reports $f=1$ after seeing $y=2$

Find the equilibrium level of α

Denote $y_L=1\,;y_H=2\,;f_L=1\,;f_H=2\,;x_L=1\,;x_H=2$

Denote $W(f,x)$ the compensation a CEO can expect by investors after seeing his forecast f and the realization of earnings x, such that $W(f,x)=C\cdot P(CEO_H|f,x)-\mathbf{1}_{\boldsymbol{f\neq x}}K$

We know CEO_H will always give his private signal as his forecast.

We know CEO_L will as well if his signal is $y=1$.

CEO_L needs to optimize his lying rate α for when he sees $y=2$.

Note: $P(CEO_L)=P(CEO_H)=\frac{1}{2}$ so they're omitted in calculations below.

$\boldsymbol{CEO_L}$'s expected pay when seeing $\boldsymbol{y_H}$ and lying ($\boldsymbol{f=f_L\neq y_H}$):

$$E[W(f_L,x)|y_H]=P(x_H|y_H)W(f_L,x_H)+P(x_L|y_H)W(f_L,x_L)$$

$$=P(x_H|y_H)[CP(CEO_H|f_L,x_H)-K]+P(x_L|y_H)[CP(CEO_H|f_L,x_L)]$$

$$E[W(f_L,x)|y_H]=P(x_H|y_H)[\tfrac{C\cdot(1-g)}{(1-g)+b\alpha+(1-b)}-K]+P(x_L|y_H)[\tfrac{C\cdot g}{g+b+(1-b)\alpha}]$$

$\boldsymbol{CEO_L}$'s expected pay when seeing $\boldsymbol{y_H}$ & not lying ($\boldsymbol{f=f_H=y_H}$):

$$E[W(f_H,x)|y_H]=P(x_H|y_H)W(f_H,x_H)+P(x_L|y_H)W(f_H,x_L)$$

$$=P(x_H|y_H)[CP(CEO_H|f_H,x_H)]+P(x_L|y_H)[CP(CEO_H|f_H,x_L)-K]$$

$$E[W(f_H,x)|y_H]=P(x_H|y_H)[\tfrac{C\cdot g}{g+b(1-\alpha)}]+P(x_L|y_H)[\tfrac{C\cdot(1-g)}{(1-g)+(1-b)(1-\alpha)}-K]$$

Get $P(x_H|y_H)$ & $P(x_L|y_H)$ using Bayes' Theorem (CEO_L's perspective):

$$P(x_H|y_H)=\frac{P(y_H|x_H)P(x_H)}{P(y_H)}=\frac{b(0.3)}{b(0.3)+(1-b)(0.7)}$$

$$P(x_L|y_H)=\frac{P(y_H|x_L)P(x_L)}{P(y_H)}=\frac{(1-b)(0.7)}{b(0.3)+(1-b)(0.7)}$$

At equilibrium α^* will be such that CEO_L will be indifferent between the above two, thus equating $E[W(f_L,x)|y_H]=E[W(f_H,x)|y_H]\Leftrightarrow$

$$\frac{b(0.3)}{b(0.3)+(1-b)(0.7)}[\frac{C\cdot(1-g)}{(1-g)+b\alpha+(1-b)}-K]+\frac{(1-b)(0.7)}{b(0.3)+(1-b)(0.7)}[\frac{C\cdot g}{g+b+(1-b)\alpha}]$$

$$=$$

$$\frac{b(0.3)}{b(0.3)+(1-b)(0.7)}[\frac{C\cdot g}{g+b(1-\alpha)}]+\frac{(1-b)(0.7)}{b(0.3)+(1-b)(0.7)}[\frac{C\cdot(1-g)}{(1-g)+(1-b)(1-\alpha)}-K]$$

will give an expression for a^* in equilibrium.

Assignment 1: Problem 1: Grossman & Stiglitz (1980)

Accounting Theory with Judson Caskey

Public signal $r = v + e_r$	Traders place demands	Cashflow v
Informed signal $s = v + e_s$	Market maker clears market	is realized

Model is based on
Grossman & Stiglitz (1980)
Continuum of **risk averse** traders
With risk **tolerance** coefficient ρ
Thus CARA coefficient $1/\rho$
Prior of payoff: $v \sim N\left(\mu_v, \tau_v^{-1}\right)$
All investors see $r = v + e_r$
$\lambda\%$ of traders are informed so
they see private signal $s = v + e_s$
$e_r \sim N\left(0, \tau_r^{-1}\right) \perp e_s \sim N\left(0, \tau_s^{-1}\right)$
Noisy supply $x \sim N\left(\mu_x, \sigma_x^2\right)$
Prices determined by mkt clearing
Assume linear strategies all around
Uninformed's Conjectures:
$p = p_0 + p_r r + p_s s - p_x x$
$\hat{s} = s + k \times (x - \mu_x)$ & constant k

Question:
1) *What is the equilibrium price?*
2) *Compute ex-ante utility if* $\lambda = 0$
3) *How does it vary with* τ_r?

1) Starting with the *Informed* trader's optimization problem:

$$\max_{q_I} q_I(E^I[v|r,s] - p) - \frac{1}{2\rho} q_I^2 var(v|s,r) \Rightarrow \text{FOC: } q_I = \frac{E^I[v|r,s]-p}{\frac{1}{\rho}\cdot var(v|r,s)}$$

Note: $var\left(v|r\right) = \tau_v^{-1} - \frac{\tau_v^{-2}}{\tau_v^{-1}+\tau_r^{-1}} = \frac{\tau_v^{-1}\tau_r^{-1}}{\tau_v^{-1}+\tau_r^{-1}} = \frac{1}{\frac{\tau_v^{-1}}{\tau_v^{-1}\tau_r^{-1}} + \frac{\tau_r^{-1}}{\tau_v^{-1}\tau_r^{-1}}} = \frac{1}{\tau_v+\tau_r}$

And similarly $var\left(v|s,r\right) = \frac{1}{\tau_v+\tau_r+\tau_s}$

Whereas we know that $E^I[v|r,s] = \frac{\tau_v\mu_v+\tau_r r+\tau_s s}{\tau_v+\tau_r+\tau_s}$

Thus: $q_I = \rho(\tau_v+\tau_r+\tau_s)(\frac{\tau_v\mu_v+\tau_r r+\tau_s s}{\tau_v+\tau_r+\tau_s} - p)$

Then the *Uninformed* trader's optimization problem:

$$\max_{q_U} q_U(E^U[v|r,\hat{s}] - p) - \frac{1}{2\rho} q_U^2 var(v|\hat{s},r) \Rightarrow \text{FOC: } q_U = \frac{E^U[v|r,\hat{s}]-p}{\frac{1}{\rho}\cdot var(v|r,\hat{s})}$$

Note that *Uninformed* doesn't observe s and estimates *Informed*'s private signal s from the price signal ($p = p_0 + p_r r + p_s s - p_x x$) as

$$\hat{s} = E^U[\frac{1}{p_s}\left(p - p_0 - p_r r + p_x x\right)|p,r] = \frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}\mu_x$$

He knows real s would be equal to $s = \frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}x$

$$\hat{s} = \underbrace{\frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \left[\frac{p_x}{p_s}x\right]}_{=s} + \frac{p_x}{p_s}\mu_x - \left[\frac{p_x}{p_s}x\right] = s + \frac{p_x}{p_s}(x-\mu_x)$$

He considers his estimation error $\frac{p_x}{p_s}(x-\mu_x)$ to calculate **error variance**

$$\boldsymbol{var(\hat{s})} = \tau_v^{-1} + \tau_s^{-1} + k^2\sigma_x^2 = \tau_v^{-1} + \hat{\tau}_s^{-1} \Leftrightarrow \hat{\tau}_s^{-1} = \tau_s^{-1} + k^2\sigma_x^2 = \frac{1+(\frac{p_x}{p_s})^2\tau_s\sigma_x^2}{\tau_s}$$

But his expected value $\boldsymbol{E[\hat{s}]} = E[\frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}x] = \frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}\mu_x$

<u>This is because he knows τ_s but not the *realization* of s</u>. Also note: $k = \frac{p_x}{p_s}$.

$$E[v|r,\hat{s}] = \frac{\tau_v\mu_v+\tau_r r+\hat{\tau}_s\hat{s}}{\tau_v+\tau_r+\hat{\tau}_s} = \frac{\tau_v\mu_v+\tau_r r+\hat{\tau}_s(\frac{p}{p_s}-\frac{p_0}{p_s}-\frac{p_r}{p_s}r+\frac{p_x}{p_s}\mu_x)}{\tau_v+\tau_r+\hat{\tau}_s}$$

and $var\left(v|\hat{s}\right) = \tau_v^{-1} - \frac{\tau_v^{-2}}{\tau_v^{-1}+\tau_s^{-1}+k^2\sigma_x^2} = \frac{\tau_v^{-1}\hat{\tau}_s^{-1}}{\tau_v^{-1}+\hat{\tau}_s^{-1}} = \frac{1}{\tau_v+\hat{\tau}_s}$

similarly: $var\left(v|r,\hat{s}\right) = \frac{1}{\tau_v+\tau_r+\hat{\tau}_s}$ where $\hat{\tau}_s = \frac{\tau_s}{1+(\frac{p_x}{p_s})^2\tau_s\sigma_x^2}$

Thus: $q_U = \frac{E^U[v|r,\hat{s}]-p}{\frac{1}{\rho}\cdot var(v|r,\hat{s})} = \rho(\tau_v+\tau_r+\hat{\tau}_s)(\frac{\tau_v\mu_v+\tau_r r+\hat{\tau}_s(\frac{p}{p_s}-\frac{p_0}{p_s}-\frac{p_r}{p_s}r+\frac{p_x}{p_s}\mu_x)}{\tau_v+\tau_r+\hat{\tau}_s} - p)$

We can now calculate aggregate demand using $\lambda q_I + (1-\lambda)q_U$

$$= \lambda\rho\left(\tau_v\mu_v + \tau_r r + \tau_s s - p(\tau_v+\tau_r+\tau_s)\right) +$$

$$(1-\lambda)\rho\left(\tau_v\mu_v + \tau_r r + \hat{\tau}_s(\frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}\mu_x) - p(\tau_v+\tau_r+\hat{\tau}_s)\right)$$

continued on next page

Now Plugging into Supply = Demand: $x = \lambda q_I + (1-\lambda)q_U$

$$x = \lambda\rho(\tau_v\mu_v + \tau_r r + \tau_s s - p(\tau_v+\tau_r+\tau_s)) + (1-\lambda)\rho(\tau_v\mu_v + \tau_r r + \hat{\tau}_s(\frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}\mu_x) - p(\tau_v+\tau_r+\hat{\tau}_s))$$

$$\frac{x}{\rho} = \tau_v\mu_v + \tau_r r + \lambda\tau_s s - p(\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s) + (1-\lambda)\hat{\tau}_s(\frac{p}{p_s} - \frac{p_0}{p_s} - \frac{p_r}{p_s}r + \frac{p_x}{p_s}\mu_x)$$

$$p(\tau_v+\tau_r+\lambda\tau_s+(1-\lambda)\hat{\tau}_s) = -\frac{x}{\rho} + \left[\tau_v\mu_v + (1-\lambda)\hat{\tau}_s(\frac{p_x}{p_s}\mu_x - \frac{p_0}{p_s})\right] + \tau_r r + \lambda\tau_s s + (1-\lambda)(\hat{\tau}_s\frac{p}{p_s} - \hat{\tau}_s\frac{p_r}{p_s}r)$$

$$p(\tau_v+\tau_r+\lambda\tau_s+(1-\lambda)\hat{\tau}_s) - (1-\lambda)\hat{\tau}_s\frac{p}{p_s} = -\frac{1}{\rho}x + \left[\tau_v\mu_v + (1-\lambda)\hat{\tau}_s(\frac{p_x}{p_s}\mu_x - \frac{p_0}{p_s})\right] + \left[\tau_r - (1-\lambda)\hat{\tau}_s\frac{p_r}{p_s}\right]r + \lambda\tau_s s$$

$$p\left(\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(1 - \frac{1}{p_s})\right) = \left[\tau_v\mu_v + (1-\lambda)\hat{\tau}_s(\frac{p_x\mu_x - p_0}{p_s})\right] + \left[\tau_r - (1-\lambda)\hat{\tau}_s\frac{p_r}{p_s}\right]r + \lambda\tau_s s - \frac{1}{\rho}x$$

$$p = \underbrace{\frac{\tau_v\mu_v + (1-\lambda)\hat{\tau}_s(\frac{p_x\mu_x - p_0}{p_s})}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(\frac{p_s-1}{p_s})}}_{p_0} + \underbrace{\frac{\tau_r - (1-\lambda)\hat{\tau}_s\frac{p_r}{p_s}}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(\frac{p_s-1}{p_s})}}_{p_r} r$$

$$+ \underbrace{\frac{\lambda\tau_s}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(\frac{p_s-1}{p_s})}}_{p_s} s - \underbrace{\frac{\frac{1}{\rho}}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(\frac{p_s-1}{p_s})}}_{p_x} x$$

Thus $\boldsymbol{\frac{p_x}{p_s} = \frac{1}{\rho\lambda\tau_s}}$ so now we know that $\boldsymbol{\hat{\tau}_s} = \frac{\tau_s}{1 + (\frac{p_x}{p_s})^2\tau_s\sigma_x^2} = \frac{\tau_s}{1 + (\rho\lambda\tau_s)^{-2}\tau_s\sigma_x^2} = \boldsymbol{\frac{\tau_s}{1 + \frac{\sigma_x^2}{\rho^2\lambda^2\tau_s}}}$

$$p_s = \frac{\lambda\tau_s}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(\frac{p_s-1}{p_s})} \Leftrightarrow \left(\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s(1 - \frac{1}{p_s})\right)p_s = \lambda\tau_s$$

$$\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s - \frac{(1-\lambda)\hat{\tau}_s}{p_s} = \frac{\lambda\tau_s}{p_s} \Leftrightarrow \tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s = \frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}{p_s}$$

$$\Rightarrow \boldsymbol{p_s} = \frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s} \qquad \Rightarrow \boldsymbol{p_x} = \frac{p_x}{p_s}p_s = \frac{p_s}{\rho\lambda\tau_s} = \frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}{(\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s)\rho\lambda\tau_s}$$

$$\Rightarrow \boldsymbol{\frac{p_s - 1}{p_s}} = 1 - \frac{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s} = \frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s - \tau_v - \tau_r - \lambda\tau_s - (1-\lambda)\hat{\tau}_s}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s} = \frac{-\tau_v - \tau_r}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}$$

$$\boldsymbol{\frac{p_r}{p_s}} = \frac{\tau_r - (1-\lambda)\hat{\tau}_s\frac{p_r}{p_s}}{\lambda\tau_s} \Leftrightarrow \frac{p_r}{p_s}\lambda\tau_s = \tau_r - (1-\lambda)\hat{\tau}_s\frac{p_r}{p_s} \Leftrightarrow \frac{p_r}{p_s}[\lambda\tau_s + (1-\lambda)\hat{\tau}_s] = \tau_r \Leftrightarrow \boldsymbol{\frac{p_r}{p_s}} = \frac{\tau_r}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}$$

$$\boldsymbol{p_r} = \frac{\tau_r}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}p_s = \frac{\tau_r}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s} \times \frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s} = \frac{\tau_r}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s}$$

$$\boldsymbol{\frac{p_0}{p_s}} = \frac{\tau_v\mu_v + (1-\lambda)\hat{\tau}_s(\frac{p_x\mu_x - p_0}{p_s})}{\lambda\tau_s} \Leftrightarrow \frac{p_0}{p_s}\lambda\tau_s = \tau_v\mu_v + (1-\lambda)\hat{\tau}_s\left(\frac{p_x}{p_s}\mu_x - \frac{p_0}{p_s}\right) = \tau_v\mu_v + (1-\lambda)\hat{\tau}_s\left(\frac{\mu_x}{\rho\lambda\tau_s} - \frac{p_0}{p_s}\right)$$

$$\Leftrightarrow \frac{p_0}{p_s}(\lambda\tau_s + (1-\lambda)\hat{\tau}_s) = \tau_v\mu_v + \frac{(1-\lambda)\hat{\tau}_s\mu_x}{\rho\lambda\tau_s} \Leftrightarrow \boldsymbol{\frac{p_0}{p_s}} = \frac{\tau_v\mu_v + \frac{(1-\lambda)\hat{\tau}_s\mu_x}{\rho\lambda\tau_s}}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}$$

$$\Rightarrow \boldsymbol{p_0} = \frac{p_0}{p_s}p_s = \frac{\tau_v\mu_v + \frac{(1-\lambda)\hat{\tau}_s\mu_x}{\rho\lambda\tau_s}}{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}\frac{\lambda\tau_s + (1-\lambda)\hat{\tau}_s}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s} = \frac{\tau_v\mu_v + \frac{(1-\lambda)\hat{\tau}_s\mu_x}{\rho\lambda\tau_s}}{\tau_v + \tau_r + \lambda\tau_s + (1-\lambda)\hat{\tau}_s}$$

Thus we now have the equilibrium price in terms of raw parameters: $\boldsymbol{p = p_0 + p_r r + p_s s - p_x x}$

2) [incorrect] ***Compute ex-ante utility if*** $\boldsymbol{\lambda = 0}$ *(the answer on this page is 'wrong', he meant ex-ante before seeing* r*, but here's the 'ex-ante' after seeing* r *but before trading actually takes place and* x *&* P *are realized)* : this would mean that all traders are uninformed and do not see signal s nor can they infer it from the price signal (ie: no $\hat{s}$). The uninformed trader's conjecture then becomes: $p = p_0 + p_r r - p_x x$

$$max_{q_U} E^U[U^U] = max_{q_U}\ q_U(E^U[v|r] - p) - \frac{1}{2\rho} q_U^2 var(v|r) \ \Rightarrow \quad \text{FOC:} \quad q_U = \rho \frac{E^U[v|r] - p}{var(v|r)}$$

$$E^U[v|r] = E^U[v] + \frac{cov(v,r)}{var(r)}(r - E[r]) = \mu_v + \frac{\tau_v^{-1}}{\tau_v^{-1} + \tau_r^{-1}}(r - \mu_v) = \frac{\tau_v \mu_v + \tau_r r}{\tau_v + \tau_r} \leftarrow \text{or just begin with this}$$

$$var\,(v|r) = \tau_v^{-1} - \frac{\tau_v^{-2}}{\tau_v^{-1} + \tau_r^{-1}} = \frac{\tau_v^{-1}\tau_r^{-1}}{\tau_v^{-1} + \tau_r^{-1}} = \frac{1}{\frac{\tau_v^{-1}}{\tau_v^{-1}\tau_r^{-1}} + \frac{\tau_r^{-1}}{\tau_v^{-1}\tau_r^{-1}}} = \frac{1}{\tau_v + \tau_r}$$

$$q_U = \rho \frac{E^U[v|r] - p}{var(v|r)} = \rho(\tau_v + \tau_r)\left(\frac{\tau_v \mu_v + \tau_r r}{\tau_v + \tau_r} - p\right) = \rho(\tau_v \mu_v + \tau_r r - p(\tau_v + \tau_r))$$

In equilibrium: **Demand = Supply**

$$q_U = x \quad \Leftrightarrow \quad \tau_v \mu_v + \tau_r r - p(\tau_v + \tau_r) = x/\rho \quad \Leftrightarrow \quad p(\tau_v + \tau_r) = \tau_v \mu_v + \tau_r r - x/\rho$$

$$p = \frac{\tau_v \mu_v + \tau_r r - \frac{x}{\rho}}{\tau_v + \tau_r} = \underbrace{\frac{\tau_v \mu_v}{\tau_v + \tau_r}}_{p_0} + \underbrace{\frac{\tau_r}{\tau_v + \tau_r}}_{p_r} r - \underbrace{\frac{1/\rho}{\tau_v + \tau_r}}_{p_x} x$$

$$q_U = \rho(\tau_v + \tau_r)\left(\frac{\tau_v \mu_v + \tau_r r}{\tau_v + \tau_r} - \left[\underbrace{\frac{\tau_v \mu_v}{\tau_v + \tau_r} + \frac{\tau_r}{\tau_v + \tau_r} r - \frac{1/\rho}{\tau_v + \tau_r} x}_{p = p_0 + p_r r + p_s s - p_x x}\right]\right) = \rho(\tau_v \mu_v + \tau_r r - \tau_v - \tau_r r + \frac{x}{\rho})$$

$$q_U = \rho(\tau_v \mu_v - \tau_v) + x = \rho\tau_v(\mu_v - 1) + x$$

$$E^U[U^U] = q_U(E^U[v|r] - p) - \frac{1}{2\rho} q_U^2 var(v|r)$$

2) [correct] ***Compute ex-ante utility if*** $\boldsymbol{\lambda = 0}$:

Notes beforehand: Because competitive traders in a CARA/Normal setting with risk **aversion** $\boldsymbol{\gamma}$ have ex-ante quantities $q = \frac{\mathrm{E}[V]-p}{\gamma var(V)}$ their certainty equivalent of buying q units is:

C.E. of $\boldsymbol{q}$ **units:** $q(\mathrm{E}[V]-p) - \frac{\gamma}{2}\,\mathrm{var}(V)q^2 \quad \Rightarrow$ from FOC: $\quad 0 = \mathrm{E}[V]-p-\gamma\,\mathrm{var}(V)q \Leftrightarrow p = \mathrm{E}[V]-\gamma\,\mathrm{var}(V)q$

Plugging this $\boldsymbol{p}$ **into C.E.:** $\Rightarrow \quad q(\mathrm{E}[V]-p) - \frac{\gamma}{2}\,\mathrm{var}(V)q^2 = q(\mathrm{E}[V]-(\mathrm{E}[V]-\gamma\,\mathrm{var}(V)q)) - \frac{\gamma}{2}\,\mathrm{var}(V)q^2$

$= \gamma\,\mathrm{var}(V)q^2 - \frac{\gamma}{2}\,\mathrm{var}(V)q^2 = \boldsymbol{\frac{\gamma}{2} var(V) q^2} \quad \rightarrow$ this is the certainty equivalent of q risky units

In this setting we have a risk **tolerance** parameter so $\gamma = \rho^{-1}$

ALSO NOTE:

Here ex-ante price is uncertain so add it to the mean and variance operators as well: $\boldsymbol{E[V] = E[v-p]}$ **&** $\boldsymbol{var(V) = var(v-p)}$

$$\frac{\gamma}{2} var(V)q^2 = \frac{1}{2\rho} var(v-p) = q^2 \frac{1}{2\rho} var(v-p) \left(\frac{\rho \mathrm{E}[v-p]}{var(v-p)}\right)^2 = \frac{\rho}{2} \frac{\mathrm{E}[v-p]^2}{var(v-p)}$$

However this does not take into consideration the fact that while price is unknown *ex-ante*, it will be known at the time of trade. In calculating the *ex-ante* expectation at time 0, an adjustment has to be made for the fact that the trader will know certain information (signal r here) which will lower his risk level when trading at time 1:

Using the law of total variance: $\quad \boldsymbol{var_0(V-P) = var(V-P) + var_0(\mathrm{E}[V-P])}$

In this setting it would be: $var(v-p) = var(v-p \mid r) + var(\mathrm{E}[v-p \mid r])$

Note: $var(v-p \mid r)$ is the expectation once r is received and p is then also in the information set at that point (deterministic), so in this setting $var(v-p \mid r) = var(v \mid r) = \frac{1}{\tau_v \tau_r}$.

$$\mathrm{E}[u(c)] = -\mathrm{e}^{-\gamma\mu_c + \frac{1}{2}\sigma_c^2\gamma^2} = -\mathrm{e}^{-\gamma\left(\mu_c - \frac{\gamma}{2}\sigma_c^2\right)} = u\left(\mu_c - \frac{\gamma}{2}\sigma_c^2\right)$$

$$\text{We set } \gamma = \frac{1}{\rho} \quad \& \quad X = \frac{\mathrm{E}[v-p]}{\sqrt{\frac{2}{\rho} var(v-p)}} \quad \Rightarrow \quad \mathrm{E}\left[\mathrm{e}^{-\gamma X^2}\right] = \mathrm{E}\left[exp\left\{-\frac{1}{\rho}\frac{\rho}{2}\frac{\mathrm{E}[v-p]^2}{var(v-p)}\right\}\right]$$

Then we would get the following ex-ante certainty equivalent payment where the 0 subscripts denote moments with respect to ex ante information:

$$\frac{1}{2\gamma}\frac{\mathrm{E}_0[V-P]^2}{\mathrm{var}_0(V-P)} + \frac{1}{\gamma}\log\sqrt{\frac{\mathrm{var}_0(V-P)}{\mathrm{var}(V-P)}} = \frac{\rho}{2}\frac{\mathrm{E}[v-p]^2}{\mathrm{var}(v-p)} + \rho\log\sqrt{\frac{\mathrm{var}(v-p)}{\mathrm{var}(v-p \mid r)}}$$

$$= \frac{\rho}{2}\frac{\mathrm{E}[v-p]^2}{\mathrm{var}(v-p)} + \rho\log\sqrt{\frac{\mathrm{var}(v-p)}{\frac{1}{\tau_v+\tau_r}}}$$

$$E[v-p]=\mu_v-\frac{\tau_v\mu_v+\tau_r E[r]-\frac{E[x]}{\rho}}{\tau_v+\tau_r}=\frac{\tau_v\mu_v+\tau_r\mu_v-\tau_v\mu_v-\tau_r\mu_v+\frac{\mu_x}{\rho}}{\tau_v+\tau_r}=\frac{1}{\rho}\frac{1}{\tau_v+\tau_r}\mu_x$$

$$var[v-p]=var\left(v-\frac{\tau_v\mu_v+\tau_r r-\frac{x}{\rho}}{\tau_v+\tau_r}\right)=var\left(\frac{v(\tau_v+\tau_r)-\tau_v\mu_v-\tau_r(v+e_r)+\frac{x}{\rho}}{\tau_v+\tau_r}\right)$$

$$var[v-p]=\frac{var\left(v\tau_v-\tau_r e_r+\frac{x}{\rho}\right)}{(\tau_v+\tau_r)^2}=\frac{\frac{\tau_v^2}{\tau_v}+\frac{\tau_r^2}{\tau_r}+\frac{\sigma_x^2}{\rho^2}}{(\tau_v+\tau_r)^2}=\frac{\tau_v+\tau_r+\frac{\sigma_x^2}{\rho^2}}{(\tau_v+\tau_r)^2}$$

Then:

$$\frac{\rho}{2}\frac{\mathrm{E}[v-p]^2}{\mathrm{var}(v-p)}+\rho\log\sqrt{\frac{\mathrm{var}(v-p)}{\frac{1}{\tau_v+\tau_r}}}=\frac{\rho}{2}\frac{\left[\frac{1}{\rho}\frac{1}{\tau_v+\tau_r}\mu_x\right]^2}{\frac{\tau_v+\tau_r+\frac{\sigma_x^2}{\rho^2}}{(\tau_v+\tau_r)^2}}+\rho\log\sqrt{\frac{\tau_v+\tau_r+\sigma_x^2/\rho^2}{(\tau_v+\tau_r)^2}(\tau_v+\tau_r)}$$

$$E_0[U]=\frac{1}{2\rho}\frac{\mu_x^2}{\tau_v+\tau_r+\frac{\sigma_x^2}{\rho^2}}+\rho\log\sqrt{1+\frac{\sigma_x^2/\rho^2}{\tau_v+\tau_r}}$$

3) Ex-ante utility falls as the precision of public information increases ($\tau_r\uparrow$). Investors earn profit from risk premium, and information lowers that.

Assignment 1: Problem 4

Accounting Theory with Judson Caskey

Based on material seen in Accounting Theory (Prof. Caskey Spring 2020)

Grossman & Stiglitz (1980) →
and **Guttman (2010)** elements →
Investors trade over $t \in [0,T]$ →
They have CARA coefficient γ →
Asset payoff is $\pi \sim \mathcal{N}\left(\mu_{\pi 0}, \sigma_{\pi 0}^2\right)$ →
Public information at time t has precision: →
$f(t)$ with $f(0) = \frac{1}{\sigma_{\pi 0}^2} \geq 1$ (i.e. $\sigma_{\pi 0}^2 \leq 1$) →
$f(t) \to \infty$ as $t \to T$ and $f'(t) > 0$ →
Investors can buy signal $\psi_i(t) = \pi + e_i(t)$ →
from **analyst** i where $var(e_i(t)) = \frac{1}{f_i(t)}$ →
Imputing the cost of information c from →
the informed/uninformed indifference†: →
† from Grossman & Stiglitz (1985) →
$c = \frac{1}{\gamma} \log \sqrt{\frac{var_u(v-p)}{var_i(v-p)}}$ →
where $var_u(\pi - p) = \frac{1}{f(t)}$ →
and $var_i(\pi - p) = \frac{1}{f(t)+f_i(t)}$ and so: →
$U^i(f(t)) = C(t) = \frac{1}{\gamma} \log \sqrt{\frac{var_u(v-p)}{var_{(v-p)}}} =$ →
$\frac{1}{2\gamma} \log \frac{f(t)+f_i(t)}{f(t)} \Leftarrow$ analyst i maximizes this →
By assumption $f_i(t) = F_i + \alpha_i \log f(t)$ →
where α_i is speed of analyst learning →
and F_i is his initial info advantage →
for some constants $F_i \geq 0$ and $\alpha_i > 0$ →
Both $f(t)$ & $f_i(t)$ increase over time →
$f_i'(t) = \frac{\alpha_i}{f(t)} f'(t)$: analyst precision grows →
faster than public precision if $f(t) < \alpha_i$ →
Question's additional assumptions: →
· Noisy supply $x \sim \mathcal{N}\left(\mu_x, \sigma_x^2\right)$ →
· λ% of investors receive the forecast →
All investors know the forecast timing →
parameter $F_{0i} < \alpha_i$ and slope $\alpha_i < \mathrm{e}$ in f_i →
Derive optimal forecast timing →

From *uninformed* in Problem 1 we had that:

$$var(v \mid \hat{s}) = \tau_v^{-1} - \frac{\tau_v^{-2}}{\tau_v^{-1}+\tau_s^{-1}+k^2\sigma_x^2} = \frac{\tau_v^{-1}\hat{\tau}_s^{-1}}{\tau_v^{-1}+\hat{\tau}_s^{-1}} = \frac{1}{\tau_v+\hat{\tau}_s}$$

$$\text{and } \hat{\tau}_s = \frac{\tau_s}{1+\left(\frac{p_x}{p_s}\right)^2\tau_s\sigma_x^2} = \frac{\tau_s}{1+(\rho\lambda\tau_s)^{-2}\tau_s\sigma_x^2} = \frac{\tau_s}{1+\frac{\sigma_x^2}{\rho^2\lambda^2\tau_s}}$$

For *uninformed* in this setting simply replace

$\tau_s = f_i(t)$, $\hat{\tau}_s = \hat{f}_i(t)$ and $\tau_v = f(t)$:

$$var(\pi|\hat{f}_i(t)) = \frac{1}{f(t)+\hat{f}_i(t)} = \left(f(t) + \frac{f_i(t)}{1+\frac{\gamma^2\sigma_x^2}{\lambda^2 f_i(t)}}\right)^{-1}$$

Now precision: $var(\pi|\hat{f}_i(t))^{-1} = f(t) + \frac{f_i(t)}{1+\frac{\gamma^2\sigma_x^2}{\lambda^2 f_i(t)}}$

$$= f(t) + \frac{f_i(t)}{\frac{\lambda^2 f_i(t)+\gamma^2\sigma_x^2}{\lambda^2 f_i(t)}} = f(t) + \frac{f_i(t)^2\lambda^2}{\rho^2\lambda^2 f_i(t)+\gamma^2\sigma_x^2} = f(t) + \frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i(t)$$

Then plug this into analyst's optimization problem

$$\max_f \frac{1}{2\gamma} \log \frac{var^U(\pi|\hat{f}_i(t))}{var^I(\pi|f_i(t))} = \max_f \frac{1}{2\gamma} \log \frac{var^I(\pi|f_i(t))^{-1}}{var^U(\pi|\hat{f}_i(t))^{-1}}$$

$$= \max_f \frac{1}{2\gamma} \log \frac{f(t)+f_i(t)}{f(t)+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i(t)}$$

Analyst revenue is maximized when

the term inside the log is maximized:

$$0 = \frac{\mathrm{d}}{\mathrm{d}f}\left(\frac{f+f_i}{f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_z^2}{f_i(t)}} f_i}\right) = \frac{\partial}{\partial f}\left(\frac{f+f_i}{f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i}\right) + \frac{\partial}{\partial f_i}\left(\frac{f+f_i}{f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i}\right)\frac{\mathrm{d}f_i}{\mathrm{d}f}$$

$$0 = -\frac{\frac{\gamma^2\sigma_x^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i}}}{\left(f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i\right)^2} + \frac{\gamma^2\frac{\sigma_x^2}{f_i}}{\left(f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i(t)}} f_i\right)^2} \frac{\gamma^2\frac{\sigma_x^2}{f_i} f - \lambda^2 f_i}{\left(\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i}\right)^2} \frac{\alpha_i}{f}$$

$$0 = \frac{\gamma^4\sigma_x^2\frac{\sigma_x^2}{f_i}}{\left(f+\frac{\lambda^2}{\lambda^2+\gamma^2\frac{\sigma_i^2}{f_i(t)}} f_i\right)^2} \frac{\frac{\alpha_i}{f_i} - 1 - \lambda^2\frac{f_i}{\gamma^2\sigma_x^2}\left(\frac{\alpha_i}{f}+1\right)}{\left(\lambda^2+\gamma^2\frac{\sigma_x^2}{f_i}\right)^2}$$

Note that putting $\lambda = 0$ implies that the expression is maximized at

$\frac{a_i}{f_i} = 1$, giving $f\left(t^*\right) = \mathrm{e}^{1-F_0/\alpha_i}$ as in Guttman (2010).

Assignment 2: Problem 3: Part 1

Stein (1989): Signal Jamming

Accounting Theory with Judson Caskey

CEO knows $\bar{v}_m$ — CEO reports r — Mkt assess.: $E^{Mkt}[v|r]$ — Trading — Realization v

CEO is risk **neutral** $\longrightarrow$

Investors believe $\tilde{v} \sim N(\mu_v, \sigma_v^2)$ $\longrightarrow$

CEO privately believes $\tilde{v} \sim N(\overline{v}_m, (1-\delta)\sigma_v^2)$ $\longrightarrow$

where $cov(v, \overline{v}_m) = var(\overline{v}_m) = \delta\sigma_v^2$ $\longrightarrow$

CEO reports r to investors to optimize $\longrightarrow$

$max_r \; bE^{mkt}[v|r] - \frac{c}{2}(r - \overline{v}_m)^2$ $\longrightarrow$

b & c are publicly known parameters $\longrightarrow$

Mkt assess: $P(r) = E^{Mkt}[v \mid r]$ $\longrightarrow$

CEO choice of report is $r(\overline{v}_m)$ $\longrightarrow$

Mkt's conjecture of CEO's reporting $\longrightarrow$

strategy: $\hat{r}(\overline{v}_m) = r_0 + r_v\overline{v}_m$ $\longrightarrow$

$\hat{p}$: CEO's conject. of mkt price assessment $\longrightarrow$

has linear form $\hat{E}^{mkt}[v|r,v] = \hat{p}(r) = p_0 + p_r r$ $\longrightarrow$

$\longrightarrow$

In equilibrium: $\hat{r}(\overline{v}_m) = r(\overline{v}_m)$ $\longrightarrow$

In equilib.: $\hat{p}(r) = p(r) = p_0 + p_r r$ $\longrightarrow$

$\longrightarrow$

Note that given this setup Investors' $\longrightarrow$

previous prior is completely dominated $\longrightarrow$

by $\bar{v}_m$ as though they were diffuse $\longrightarrow$

More specifically, to get $E[v|r]$ investors $\longrightarrow$

invert their conjectured $r^{-1}(\overline{v}_m)$ to get v_m $\longrightarrow$

1) Solve for equilibrium policies $r(\bar{v}_m)$ & $p(r)$

Starting with CEO's problem: $max_r \; bE^{mkt}[v|r] - \frac{c}{2}(r - \overline{v}_m)^2$

Plugging mkt's pricing rule: $max_r \; b(p_0 + p_r r) - \frac{c}{2}(r - \overline{v}_m)^2$

FOC: $0 = bp_r - c(r - \overline{v}_m) \Leftrightarrow r = \frac{b}{c}p_r + \overline{v}_m$

Thus market infers that $r_0 = \frac{b}{c}p_r$ and $r_v = 1$

To get the expected value of v it simply inverts

their conjectured reporting function: $E[v|r] = r - \frac{b}{c}p_r = \overline{v}_m$

The CEO can estimate the coefficients in his conjecture

of the market's pricing strategy: $p_r = 1$ and then $p_0 = -\frac{b}{c}$.

Thus the strategies are $r(\overline{v}_m) = r_0 + r_v\overline{v}_m = \frac{b}{c} + \overline{v}_m$

$\hat{p}(r) = p(r) = p_0 + p_r r = -\frac{b}{c} + r = \overline{v}_m$

Assignment 2: Problem 3: Part 2a

Crawford & Sobel (1982): Cheap Talk

Accounting Theory with Judson Caskey

CEO knows $\bar{v}_m$ — CEO reports r — Mkt assess.: $E^{Mkt}[v|r]$ — Trading — Realization v

CEO is risk **neutral**

Investors believe $\tilde{v} \sim N(\mu_v, \sigma_v^2)$

CEO privately believes $\tilde{v} \sim N(\overline{v}_m, (1-\delta)\sigma_v^2)$

where $cov(v, \overline{v}_m) = var(\overline{v}_m) = \delta\sigma_v^2$

CEO reports r to investors to optimize

$\boldsymbol{max_r \; bE^{mkt}[v|r] - \frac{c}{2}(E^{mkt}[v|r] - \overline{v}_m)^2}$

b & c are publicly known parameters

Mkt assess: $P(r) = E^{Mkt}[v \mid r]$

CEO choice of report is $r(\overline{v}_m)$

Mkt's conjecture of CEO's reporting

strategy: $\hat{r}(\overline{v}_m) = r_0 + r_v\overline{v}_m$

$\hat{p}$: CEO's conject. of mkt price assessment

has linear form $\hat{E}^{mkt}[v|r,v] = \hat{p}(r) = p_0 + p_r r$

In equilibrium: $\hat{r}(\overline{v}_m) = r(\overline{v}_m)$

In equilib.: $\hat{p}(r) = p(r) = p_0 + p_r r$

Questions:

Show that this is a cheap talk equilibrium

and that sender's and receiver's implied utility u_r

satisfy conditions in Crawford and Sobel (1982)

where $u_r = (p,\bar{v}_m) = -(p - \bar{v}_m)^2$

and that there is no separating region (v_1,v_2)such

that truthful reporting $\hat{v} = \bar{v}_m$ optimizes:

$\max_{\hat{v}\in(v_1,v_2)} b\hat{v} - \frac{c}{2}(\hat{v} - \bar{v}_m)^2$

Note that this form is the same

as in p.228 of Stocken (2012)

1) For market's utility $u_r(p, \bar{v}_m) = -(p - \bar{v}_m)^2$

and CEO's utility $u_s(p, \bar{v}_m, b) = bp - \frac{c}{2}(p - \bar{v}_m)^2$

We can check the following conditions to see

if this is a Cheap Talk equilibrium:

$$\frac{\partial u_r}{\partial p} = -(p - \bar{v}_m) \quad \frac{\partial u_s}{\partial p} = b - c(p - \bar{v}_m)$$

(↑ This means each agent's optimal action depends on the other's,

which is a requirement in Cheap Talk models

$$\frac{\partial^2 u_r}{\partial p^2} = -1 < 0 \quad \frac{\partial^2 u_s}{\partial p^2} = -c < 0$$

$$\frac{\partial^2 u_r}{\partial p \partial \bar{v}_m} = 1 > 0 \quad \frac{\partial^2 u_s}{\partial p \partial \bar{v}_m} = c > 0$$

Note that part 1 did not satisfy these as

$bp(r) - \frac{c}{2}(r - \bar{v}_m)^2$ so that $\frac{\partial u_s}{\partial p} = b$ and $\frac{\partial^2 u_s}{\partial p^2} = \frac{\partial^2 u_s}{\partial p \partial \bar{v}_m} = 0$

To prove there isn't a separating equilibrium that fully reveals $\hat{v} = \bar{v}_m$

for a range of types $\bar{v}_m \in (v_1, v_2)$ by contradiction let's assume there is.

Then the equilibrium report is equivalent to claiming a type $\hat{v}$.

This gives the objective: $\max_{\hat{v}\in(v_1,v_2)} b\hat{v} - \frac{c}{2}(\hat{v} - \bar{v}_m)^2$

But this has a unique optimum at $\hat{v} = \bar{v}_m + \frac{b}{c}$, implying that

there cannot be any set of types that separate in equilibrium.

Caskey HW2 Q3 2b) The partitioning equilibrium we are looking for exists if:

i) $\exists$ a price schedule such that $p(r) = E^{mkt}[v|r]$

ii) $\exists$ a reporting schedule $r(\overline{v}_m)$ that solves: $max_r \; bE^{mkt}[v|r] - \frac{c}{2}(E^{mkt}[v|r] - \overline{v}_m)^2$

Ex-ante, market's optimal price from FOC would be $p = \overline{v}_m$

For a claimed separating region (v_1, v_2) the equilibrium would be:

CEO reports $r(\overline{v}_m) = v_1$ if $\overline{v}_m < \hat{v}$ and CEO reports $r(\overline{v}_m) = v_2$ if $\overline{v}_m > \hat{v}$

At threshold $\hat{v}$ CEO would be indifferent between both partitions:

$$E^{CEO}[U^{CEO}(p(r = v_1))|\overline{v}_m < \hat{v}]) = E^{CEO}[U^{CEO}(p(r = v_2))|\overline{v}_m > \hat{v}])$$

$$E^{CEO}[bE^{mkt}[v|r = v_1] - \frac{c}{2}(E^{mkt}[v|r = v_1] - \hat{v})^2) = E^{CEO}[bE^{mkt}[v|r = v_2] - \frac{c}{2}(E^{mkt}[v|r = v_2] - \hat{v})^2)$$

$$bE^{CEO}[p|v_1] - \frac{c}{2}(E^{CEO}[p|v_1] - \hat{v})^2) = bE^{CEO}[p|v_2] - \frac{c}{2}(E^{CEO}[p|v_2] - \hat{v})^2)$$

$$bE[p|\overline{v}_m < \hat{v}] - (E[p|\overline{v}_m < \hat{v}] - \hat{v})^2 = bE[p|\overline{v}_m > \hat{v}] - (\frac{c}{2}E[p|\overline{v}_m > \hat{v}] - \hat{v})^2$$

Inverse Mills Ratio: $\mathrm{E}\left[\overline{v}_m \mid \overline{v}_m \geq \hat{v}\right] = \mu_v + \sigma_v\sqrt{\delta}\frac{\phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}{1-\Phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}$ **&** $\mathrm{E}\left[\overline{v}_m \mid \overline{v}_m < \hat{v}\right] = \mu_v - \sigma_v\sqrt{\delta}\frac{\phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}{\Phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}$

$$b\left(\mu_v - \frac{\sqrt{\delta}\sigma_v\phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}{\Phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}\right) - \frac{c}{2}\left(\mu_v - \frac{\sqrt{\delta}\sigma_v\phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}{\Phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})} - \hat{v}\right)^2 = b\left(\mu_v + \frac{\sqrt{\delta}\sigma_v\phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}{1-\Phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}\right) - \frac{c}{2}\left(\mu_v + \frac{\sqrt{\delta}\sigma_v\phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})}{1-\Phi(\frac{\hat{v}-\mu_v}{\sqrt{\delta}\sigma_v})} - \hat{v}\right)^2$$

Then by completing the square: $b\mathrm{E}\left[v \mid v_h\right] - \frac{c}{2}\left(\mathrm{E}\left[v \mid v_h\right] - \hat{v}\right)^2 = b\mathrm{E}\left[v \mid v_\ell\right] - \frac{c}{2}\left(\mathrm{E}\left[v \mid v_\ell\right] - \hat{v}\right)^2$

$\Leftrightarrow \quad 0 = (b + c\hat{v})\left(\mathrm{E}\left[v \mid v_h\right] - \mathrm{E}\left[v \mid v_\ell\right]\right) - \frac{c}{2}\underbrace{\left(\mathrm{E}\left[v \mid v_h\right]^2 - \mathrm{E}\left[v \mid v_\ell\right]^2\right)}_{=(\mathrm{E}[v|v_h]-\mathrm{E}[v|v_\ell])(\mathrm{E}[v|v_h]+\mathrm{E}[v|v_\ell])} \quad \Leftrightarrow \quad 0 = b + c\hat{v} - \frac{c}{2}\left(\mathrm{E}\left[v \mid v_h\right] + \mathrm{E}\left[v \mid v_\ell\right]\right)$

$$\hat{v} = \frac{\mathrm{E}\left[v \mid v > \hat{v}\right] + \mathrm{E}\left[v \mid v < \hat{v}\right]}{2} - \frac{b}{c}$$

$$\hat{v} = \frac{\mu_v + \sigma_v\sqrt{\delta}\frac{\phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}{1-\Phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)} + \mu_v - \sigma_v\sqrt{\delta}\frac{\phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}{\Phi\left(\frac{\hat{v}-\mu_v}{\sigma_v\sqrt{\delta}}\right)}}{2} - \frac{b}{c} = f(\xi)$$

Theory 1 - Comp 2019: Myers & Majluf (1984)

Firm has project requiring funding of I that →

would need an equity issue $E = I$ to go through →

Firm assets $\tilde{A} \sim Unif(0,A)$ with $E_0^{mkt}[\tilde{A}] = \frac{A}{2}$ →

Project NPV $\tilde{B} \sim Unif(0,B)$ with $E[\tilde{B}] = \frac{B}{2}$ →

CEO has private info on realized value of a but →

CEO has no private info on realization b →

CEO acts in interest of *old* (t=-1) shareholders →

Thus CEO maximizes $V_0^{old} = V(a,E[\tilde{B}],E)$ →

P $= E_0^{mkt}[\tilde{A}]$ market value if stock is not issued →

P′ : mkt value of *old* shares if stock issued →

$P' = E_0^{mkt}[\tilde{A}|E] + E_0^{mkt}[\tilde{B}]$ →

If firm forfeits investment: $V^{old} = a$ →

If it issues: $V^{old} = \frac{P'}{P'+I} V = \frac{P'}{P'+I}(I + a + E[\tilde{B}])$ →

old SHs stand to gain if $a < \frac{P'}{P'+I}(I + a + E[\tilde{B}])$ →

$\Leftrightarrow \frac{I}{P'+I}(a) < \frac{P'}{P'+I}(I + E[\tilde{B}]) \Leftrightarrow \frac{I}{P'} a < I + E[\tilde{B}]$ →

if $\frac{I}{P'} a > I + E[\tilde{B}]$: firm does nothing $\rightarrow E = 0$ →

if $\frac{I}{P'} a < I + E[\tilde{B}]$: it issues & invests $\rightarrow E = I$ →

Firm is indifferent at $\frac{I}{P'}(a) = I + E[\tilde{B}]$ →

Assume all agents are risk **neutral** →

What is the range of a for which CEO will issue? →

Problem 1)

CEO's problem: $\max_{E \in \{0,I\}} V_0^{old} = V(a,E[\tilde{B}],E)$

If CEO forfeits investment: $V^{old} = a$ with certainty

If CEO issues: $V^{old} = \frac{P'}{P'+I}(I + a + E[\tilde{B}])$

CEO issues if $a < \frac{P'}{P'+I}(I + a + E[\tilde{B}]) \Leftrightarrow \frac{I}{P'+I} a < \frac{P'}{P'+I}(I + E[\tilde{B}])$

Logic: *old* stockholderss only stand to benefit if the expected incremental value they gain (taking into consideration dilution) $\frac{P'}{P'+I}(I + E[\tilde{B}])$ is greater than the share of existing assets going to *new* stockholders: $\frac{I}{P'+I} a$

Thus if: $\frac{I}{P'+I} a < \frac{P'}{P'+I}(I + E[\tilde{B}]) \Leftrightarrow a < \frac{P'}{I}(I + E[\tilde{B}]) = P'(1 + \frac{E[\tilde{B}]}{I})$

$\Leftrightarrow a < (E^{mkt}[\tilde{A}|E] + E[\tilde{B}])(1 + \frac{E[\tilde{B}]}{I}) \Leftrightarrow a < (E^{mkt}[\tilde{A}|E] + \frac{B}{2})(1 + \frac{B/2}{I})$

Thus market knows that if firm is issuing, it must be because a is below a certain level where the CEO is willing to dilute its ownership for the sake of the investment. To figure out the value of $E^{mkt}[\tilde{A}|E]$ we consider the region of $E^{mkt}[\tilde{A}|E]$ for which the CEO will refuse to dilute the old shareholders because the market valuation of firm assets is too low. I consider a value of a^* which will render the CEO indifferent between issuing E=I and not, which would leave old shareholders equally well off:

$$E[V^{old}|E = 0] = E[V^{old}|E = I]$$

$$E[\tilde{A}|E = 0] = \frac{P'}{P'+I}(I + a^* + E[\tilde{B}])$$

$$E[a|a = a^*] = \frac{E[\tilde{A}|E] + E[\tilde{B}]}{E[\tilde{A}|E] + E[\tilde{B}] + I}(I + a^* + E[\tilde{B}])$$

$$a^* = \frac{E[a|a<a^*] + \frac{B}{2}}{E[a|a<a^*] + \frac{B}{2} + I}(I + a^* + \frac{B}{2})$$

$$a^* = \frac{\frac{a^*}{2} + \frac{B}{2}}{\frac{a^*}{2} + \frac{B}{2} + I}(a^* + \frac{B}{2} + I)$$

$$(\frac{a^*}{2} + \frac{B}{2} + I)a^* = (\frac{a^*}{2} + \frac{B}{2})(a^* + \frac{B}{2} + I)$$

$$\frac{(a^*)^2}{2} + \frac{B}{2}a^* + Ia^* = (\frac{(a^*)^2}{2} + \frac{B}{4}a^* + \frac{a^*}{2}I) + (\frac{B}{2}a^* + \frac{B^2}{4} + \frac{B}{2}I)$$

$$\frac{a^*}{2}I = \frac{B}{4}a^* + \frac{B^2}{4} + \frac{B}{2}I \Leftrightarrow a^*I = \frac{B}{2}a^* + \frac{B^2}{2} + BI$$

$$\Leftrightarrow (I - \frac{B}{2})a^* = (I + \frac{B}{2})B \Leftrightarrow a^* = \frac{I + \frac{B}{2}}{I - \frac{B}{2}}B$$

$$a^* = \frac{2I+B}{2I-B}B$$

Thus CEO will issue & take on investment for $a < \frac{2I+B}{2I-B}B$.

Firm has project requiring funding of I that would need an equity issue $E = I$ to go through

Firm assets $\tilde{A} \sim Unif(0,A)$ with $E_0^{mkt}[\tilde{A}] = \frac{A}{2}$

Project NPV $\tilde{B} \sim Unif(0,B)$ with $E[\tilde{B}] = \frac{B}{2}$

CEO has private info on realized value of a but **CEO has no private info on realization** b

CEO acts in interest of *old* (t=-1) shareholders

Thus CEO maximizes $V_0^{old} = V(a,E[\tilde{B}],E)$

P $= E_0^{mkt}[\tilde{A}]$ market value if stock is not issued

P$'$: mkt value of *old* shares if stock issued

$P' = E_0^{mkt}[\tilde{A}|E] + E_0^{mkt}[\tilde{B}]$

If firm forfeits investment: $V^{old} = a$

If it issues: $V^{old} = \frac{P'}{P'+I}V = \frac{P'}{P'+I}(I + a + E[\tilde{B}])$

old SHs stand to gain if $a < \frac{P'}{P'+I}(I + a + E[\tilde{B}])$

$\Leftrightarrow \frac{I}{P'+I}(a) < \frac{P'}{P'+I}(I + E[\tilde{B}]) \Leftrightarrow \frac{I}{P'}a < I + E[\tilde{B}]$

if $\frac{I}{P'}a > I + E[\tilde{B}]$: firm does nothing $\rightarrow E = 0$

if $\frac{I}{P'}a < I + E[\tilde{B}]$: it issues & invests $\rightarrow E = I$

Firm is indifferent at $\frac{I}{P'}(a) = I + E[\tilde{B}]$

Assume all agents are risk **neutral**

Assume CEO can pay auditor $\$F$ to reveal a, but must decide to do so before seeing a

What's the max F he would be willing to pay?

Problem 2)

Case 1: Expected V^{old} when <u>not</u> paying fee F ex-ante

Let us consider the case where CEO did not pay F.

Since his decision to pay F is made at date -1 before seeing the realization of a, the market cannot infer anything about a from this decision as both the investors and CEO have the same priors for it.

So in the case of forgoing the audit ex-ante, knowing that $a^* = \frac{2I+B}{2I-B}B$ the expected outcome is: $E_{-1}[V^{old}|F \text{ not paid}] =$

$$P(0|\text{ no } F)E_{-1}[V^{old}|E=0, \text{ no } F] + P(I|\text{ no } F)E_{-1}[V^{old}|E=I, \text{ no } F]$$

Taking it one step at a time:

$$P(E=0|\text{ no } F) = P(a > a^*) = \frac{A-a^*}{A} = 1 - \frac{a^*}{A}$$

$$E_{-1}[V^{old}|E=0, \text{ no } F] = E_{-1}[a|a > a^*] = \frac{A}{2} + \frac{a^*}{2}$$

$$P(I|\text{ no } F) = P(a < a^*) = \frac{a^*}{A}$$

$$E_{-1}[V^{old}|E=I, \text{ no } F] = \frac{E_{-1}[P'|E]}{E_{-1}[P'|E]+I}E_{-1}[V]$$
$$= \frac{E[\tilde{A}|E]+E[\tilde{B}]}{E[\tilde{A}|E]+E[\tilde{B}]+I}(I + E[\tilde{A}|E] + E[\tilde{B}])$$
$$= \frac{E[\tilde{A}|a<a^*]+\frac{B}{2}}{E[\tilde{A}|a<a^*]+\frac{B}{2}+I}(I + E[\tilde{A}|a < a^*] + \frac{B}{2})$$
$$= \frac{\frac{a^*}{2}+\frac{B}{2}}{\frac{a^*}{2}+\frac{B}{2}+I}(I + \frac{A}{2} + \frac{B}{2}) = \frac{\frac{a^*}{2}+\frac{B}{2}}{\frac{a^*}{2}+\frac{B}{2}+I}(I + \frac{a^*}{2} + \frac{B}{2})$$
$$E_{-1}[V^{old}|E=I, \text{ no } F] = \frac{a^*}{2} + \frac{B}{2}$$

$$\text{Thus } E_{-1}[V^{old}|\text{ no } F] = \left(1 - \frac{a^*}{A}\right)\left[\frac{A}{2} + \frac{a^*}{2}\right] + \frac{a^*}{A}\left[\frac{a^*}{2} + \frac{B}{2}\right]$$
$$E_{-1}[V^{old}|\text{ no } F] = \left(\frac{A}{2} + \frac{a^*}{2} - \frac{a^*A}{2A} - \frac{(a^*)^2}{2A}\right) + \left[\frac{(a^*)^2}{2A} + \frac{a^*B}{2A}\right]$$
$$E_{-1}[V^{old}|\text{ no } F] = \frac{A}{2} + \frac{a^*B}{2A}$$

Case 2: Expected V^{old} when paying fee F ex-ante

Again since the decision to pay F is made at date -1, the audit

would reveal the value of the value of $\tilde{A}$ which would result in

the following expected value for old shareholders in case of investment:

$E_{-1}[V^{old}|E = I, F \text{ paid}] = E_{-1}[\frac{P'}{P'+I}(I + E_{-1}[\tilde{A}] + E[\tilde{B}])] - F$

$= E_{-1}\left[\frac{E_{-1}[\tilde{A}|E,F]+E_{-1}[\tilde{B}]}{E_{-1}[\tilde{A}|E,F]+E_{-1}[\tilde{B}]+I}(I + E_{-1}[\tilde{A}] + E[\tilde{B}])\right] - F$

$E_{-1}[V^{old}|E = I, F \text{ paid}] = \frac{\frac{A}{2}+\frac{B}{2}}{\frac{A}{2}+\frac{B}{2}+I}(I + \frac{A}{2} + \frac{B}{2}) - F$

$E_{-1}[V^{old}|E = I, F \text{ paid}] = \frac{A}{2} + \frac{B}{2} - F$

We also know that in case CEO paid the fee but forgoes the investment:

$E_{-1}[V^{old}|E = 0, F \text{ paid}] = E[\tilde{A}] - F = \frac{A}{2} - F$

Since this is always strictly inferior to the case in which

CEO issues after paying the fee it's not considered. Thus:

$E_{-1}[V^{old}|F \text{ paid}] = E_{-1}[V^{old}|E = I, F \text{ paid}] = \frac{A}{2} + \frac{B}{2} - F$

And so comparing

Case 1: $E_{-1}[V^{old}| \text{ no } F] = \left(1 - \frac{a^*}{A}\right)\left[\frac{A}{2} + \frac{a^*}{2}\right] + \frac{a^*}{A}\left[\frac{a^*}{2} + \frac{B}{2}\right] = \frac{A}{2} + \frac{a^*B}{2A}$

to

Case 2: $E_{-1}[V^{old}|F \text{ paid}] = \frac{A}{2} + \frac{B}{2} - F$

Implies that the maximum fee CEO is will to pay is

$$F = \left(\frac{A}{2} + \frac{B}{2}\right) - \left(\frac{A}{2} + \frac{a^*B}{2A}\right) = \frac{B}{2} - \frac{a^*B}{2A} = \frac{B}{2}\left(1 - \frac{a^*}{A}\right) = \frac{B}{2} \cdot Prob(a > a^*)$$

which (for clarity) is :

$F = \frac{A}{2} + \frac{B}{2} - E_{-1}[V^{old}| \text{ no } F \text{ paid}]$

$F = \frac{A}{2} + \frac{B}{2} - [P(E = 0| \text{ no } F)E_{-1}[V^{old}|E = 0, \text{ no } F] + P(E = I| \text{ no } F)E_{-1}[V^{old}|E = I, \text{ no } F]\ ]$

$$F = \frac{A}{2} + \frac{B}{2} - \left[P(a > a^*)E_{-1}[a|a > a^*] + P(a < a^*)\frac{E[\tilde{A}|a < a^*] + E[\tilde{B}]}{E[\tilde{A}|a < a^*] + E[\tilde{B}] + I}(E[\tilde{A}|a < a^*] + E[\tilde{B}] + I)\right]$$

$F = \frac{A}{2} + \frac{B}{2} - \left[P(a > a^*)E_{-1}[a|a > a^*] + P(a < a^*)\frac{\frac{a^*}{2}+\frac{B}{2}}{\frac{a^*}{2}+\frac{B}{2}+I}(I + \frac{a^*}{2} + \frac{B}{2}))\right]$

$F = \frac{A}{2} + \frac{B}{2} - \left[\left(1 - \frac{a^*}{A}\right)\left[\frac{A}{2} + \frac{a^*}{2}\right] + \frac{a^*}{A}\left[\frac{a^*}{2} + \frac{B}{2}\right]\right]$

Theory 1 - 2020

by Dimitri Zafirov

Economic Setting

- Risk neutral investors
- Manager knows the value of x precisely but cannot credibly convey it without an auditor charging fee F.
- $x \sim Unif(0,1)$

(a) If manager pays auditor fee to certify the value of x, his payoff is $x - F$. If he opts not to hire the auditor, investors' expectations will place the value of x below that of a certain threshold x^*. To determine this threshold, x^*, the manager is indifferent between disclosing and not disclosing. Thus his payoffs would be equal at x^*:

$$x^* - F = E_{mkt}[x|x < x^*]$$

$$x^* - F = E_{mkt}[Unif(0, x^*)]$$

$$x^* - F = x^*/2$$

$$x^* = 2F$$

Thus $F \geq 1/2$ would result in the threshold being $x^* \geq 1$. Given that the upper bound of x is 1, this would mean that managers would never higher an auditor charging a fee $F \geq 1/2$. Conversely, given the extent of the values x can take ([0,1]), for $F < 1/2$, there exists a manager with cashflow x greater or equal to $2F$ that would pay the fee.

(b) Auditor's expected profits are: *Fee* $\cdot$ *Prob(Fee)*

Auditor's objective function: $\max_F F \cdot P(x > x^*) = \max_F F \cdot [1 - F(x^*)] = \max_F F \cdot (1 - x^*)$

Knowing the entrepreneur's above optimization strategy, auditor conjectures that entrepreneur's optimal $x^* = 2F$ and plugs this into this own objective function: $\max_F F \cdot (1 - 2F)$

FOC w.r.t. F: $0 = 1 - 4F \Leftrightarrow F = 1/4$

(c) The equilibrium where all CEOs hire the auditor when $F < 1/2$ can exist under the following circumstances:

- Since everyone pays the fee, its signal is uninformative, therefore $E^{mkt}[x|\text{F paid}] = E^{mkt}[x] = 1/2$
- The off-equilibrium belief $\mu(F)$ that supports this is $E^{mkt}[x|\text{ F not paid }] = 0$.
- These beliefs are realized in this pooling equilibrium since no CEO has any benefit in deviating from hiring the auditor, given that $E^{mkt}[x] = 1/2 - F > 0$ for $F < 1/2$.

(d) The equilibrium where no CEOs hire the auditor when $F > 1/2$ can exist under the following circumstances:

- Since no one pays the fee, its signal is uninformative, therefore $E^{mkt}[x|\text{F not paid}] = E^{mkt}[x] = 1/2$
- The off-equilibrium belief $\mu(F)$ that supports this is $E^{mkt}[x|\text{ F paid }] = 1$.
- These beliefs are realized in this pooling equilibrium since no CEO has any benefit in deviating to hire the auditor, given that $E^{mkt}[x] = 1/2 > 1 - F$ for $F > 1/2$.

Theory 3 Comp 2019:

Grossman & Stiglitz (1980)

Informed see private signal — Trading — Realization v

- Firm has value $v \sim N(\bar{v},\sigma_v^2)$
- Investors have CARA risk-**aversion** r
- α% of investors are *Informed*
- *Informed* posterior is the following:
- $\boldsymbol{v|Info \sim N(\hat{v},(1-\delta)\sigma_v^2)}$ with $\delta \in (0,1)$
- Noisy supply of shares $x \sim N\left(\bar{x},\sigma_x^2\right)$
- x is correlated with v
- **<u>But x is not correlated with private info</u>**
- Distributional assumptions are above
- Note that $\rho_{vx} \in (0,\sqrt{1-\delta})$
- **Q1) Sketch equilibrium solution**

$$\begin{pmatrix} v \\ \hat{v} \\ x \end{pmatrix} \sim N\left(\begin{pmatrix} \bar{v} \\ \bar{v} \\ \bar{x} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \delta\sigma_v^2 & \rho_{vx}\sigma_v\sigma_x \\ \delta\sigma_v^2 & \delta\sigma_v^2 & 0 \\ \rho_{vx}\sigma_y\sigma_x & 0 & \sigma_x^2 \end{pmatrix}\right)$$

Demands: $\quad q_i = \frac{\mathrm{E}_i[v|\hat{v},x]-p}{r\cdot var_i(v|\hat{v},x)}, \quad q_u = \frac{\mathrm{E}_u[v|s_u]-p}{r\cdot var_u(v|s_u,x)}$

<u>Note</u>: $\boldsymbol{x}$ is in the conditional because in this setting it contains information since it's correlated with $\boldsymbol{v}$.

<u>What follows uses these Week 3 relations for 3 joint-normals</u> $\boldsymbol{X},\boldsymbol{y_1},\boldsymbol{y_2}$

$$\mathrm{E}\left[\boldsymbol{X} \mid \boldsymbol{y}_1,\boldsymbol{y}_2\right] = \mathrm{E}\left[\boldsymbol{X} \mid \boldsymbol{y}_1\right] + cov\left(\boldsymbol{X},\boldsymbol{y}_2 \mid \boldsymbol{y}_1\right) var\left(\boldsymbol{y}_2 \mid \boldsymbol{y}_1\right)^{-1}\left(\boldsymbol{y}_2 - \mathrm{E}\left[\boldsymbol{y}_2 \mid \boldsymbol{y}_1\right]\right)$$

$$var\left(\boldsymbol{x} \mid \boldsymbol{y}_1,\boldsymbol{y}_2\right) = var\left(\boldsymbol{x} \mid \boldsymbol{y}_1\right) - \underbrace{cov\left(\boldsymbol{x},\boldsymbol{y}_2 \mid \boldsymbol{y}_1\right) var\left(\boldsymbol{y}_2 \mid \boldsymbol{y}_1\right)^{-1} cov\left(\boldsymbol{y}_2,\boldsymbol{x} \mid \boldsymbol{y}_1\right)}_{var(\mathrm{E}[\boldsymbol{x}|\boldsymbol{y}_1,\boldsymbol{y}_2]|\boldsymbol{y}_1)}$$

$$\mathrm{E}_i[v] = \mathrm{E}[v|\hat{v},x] = \mathrm{E}[(v|\hat{v})|x] = \hat{v} + \frac{\overbrace{\rho_{vx}\sigma_v\sigma_x}^{cov(v,x|\hat{v})=cov(v,x)}}{\underbrace{\sigma_x^2}_{var(x|\hat{v})=var(x)}}\underbrace{(x-\bar{x})}_{\mathrm{E}[x|\hat{v}]=[[x]} = \hat{v} + \frac{\rho_{vx}\sigma_v}{\sigma_x}(x-\bar{x})$$

$$var_i(v) = var(v \mid \hat{v},x) = (1-\delta)\sigma_v^2 - \frac{\rho_{vx}^2\sigma_v^2\sigma_x^2}{\sigma_x^2} = \left(1-\delta-\rho_{vx}^2\right)\sigma_v^2$$

Conjecture of market pricing rule: $\boldsymbol{p = \pi_0 + \pi_v\left(\mathrm{E}[v|\hat{v},x]-\bar{v}\right) + \pi_x(x-\bar{x})}$

Uninformed infer $\hat{v}$ from the price signal:

$$s_u = \bar{v} + \frac{1}{\pi_v}\left(p-\pi_0\right) = \mathrm{E}_i[v] + \frac{\pi_x}{\pi_v}(x-\bar{x}) = \mathrm{E}[v|\hat{v},x] + b(x-\bar{x})$$

$$\mathrm{E}\left[v|s_u\right] = E[v] + \frac{cov(v,s_u)}{var(s_u)}(s_u - E[s_u]) = \bar{v} + \frac{\left(\delta+\rho_{vx}^2\right)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+b^2\sigma_x^2}\left(s_u-\bar{v}\right)$$

$$\mathrm{E}\left[v|s_u\right] = \bar{v} + \frac{\left(\delta+\rho_{vx}^2\right)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\frac{1}{\pi_v}\left(p-\pi_0\right)$$

$$var(v|s_u) = \sigma_v^2 - \frac{\left(\delta+\rho_{vx}^2\right)^2\sigma_v^4}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2} = \sigma_v^2\left(1 - \frac{\left(\delta+\rho_{vx}^2\right)^2\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\right)$$

By Market Clearing: Supply = Demand $\rightarrow \boldsymbol{x = \alpha q_i + (1-\alpha)q_u}$

$$x = \alpha\frac{\left[\hat{v}+\frac{\rho_{vx}\sigma_v}{\sigma_x}(x-\bar{x})\right]-p}{r\cdot(1-\delta-\rho_{vx}^2)\sigma_v^2} + (1-\alpha)\frac{\bar{v}+\frac{\left(\delta+\rho_{vx}^2\right)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\frac{1}{\pi_v}(p-\pi_0)-p}{r\cdot\sigma_v^2\left(1-\frac{(\delta+\rho_{vx}^2)^2\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\right)}$$

continued on next page

By Market Clearing: Supply = Demand $\rightarrow \boldsymbol{x = \alpha q_i + (1-\alpha) q_u}$

$$x = \alpha \frac{\left[\hat{v} + \frac{\rho_{vx}\sigma_v}{\sigma_x}(x-\bar{x})\right] - p}{r \cdot (1-\delta-\rho_{vx}^2)\,\sigma_v^2} + (1-\alpha)\frac{\bar{v} + \frac{(\delta+\rho_{vx}^2)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2 + \left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\frac{1}{\pi_v}(p-\pi_0) - p}{r\cdot\sigma_v^2\left(1 - \frac{(\delta+\rho_{vx}^2)^2\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\right)}$$

$$p = \underbrace{\frac{\frac{\alpha}{r\cdot var_i(v)}\bar{v} + (1-\alpha)\frac{\bar{v} - \frac{\left(\delta+\rho_{vx}^2\right)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi x}{\pi v}\right)^2\sigma_x^2}\frac{1}{\pi v}\pi_0}{r\cdot var_u(v)} - \bar{x}}{\frac{\alpha}{r\cdot var_i(v)} + (1-\alpha)\frac{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi x}{\pi v}\right)^2\sigma_x^2\pi_v}{r\cdot var_u(v)}}}_{\pi_0} + \underbrace{\frac{\frac{\alpha}{r\cdot var_i(v)}}{\frac{\alpha}{r\cdot var_i(v)} + (1-\alpha)\frac{1-\frac{(\delta+\rho_{vx}^2)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\frac{1}{\pi_v}}{r\cdot var_u(v)}}}_{\pi_v}(\mathrm{E}_i[v]-\bar{v})$$

$$- \underbrace{\frac{1}{\frac{\alpha}{r\cdot var_i(v)} + (1-\alpha)\frac{1-\frac{(\delta+\rho_{vx}^2)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{\pi_x}{\pi_v}\right)^2\sigma_x^2}\frac{1}{\pi_v}}{r\cdot var_u(v)}}}_{\pi_x}(x-\bar{x})$$

1. To solve coefficients, first use:
$$\frac{\pi_x}{\pi_v} = -\frac{r\cdot var_i(v)}{\alpha}$$
which gives $\frac{\pi_x}{\pi_v}$ and, hence, var $_u(v)$, in terms of model parameters.

2. Next solve for π_v :
$$\pi_V = \frac{\frac{\alpha}{rvar_i(v)}}{\frac{1-\frac{\left(\delta+\rho_{v\times}^2\right)\sigma_v^2}{(\delta+\rho_{vx}^2)\sigma_v^2+\left(\frac{rvar_i(v)}{\alpha}\right)^2\sigma_x^2}\pi v}{rvar_u(v)}}$$
which is in terms of model parameters.

3. Multiply π_v by $\frac{\pi_x}{\pi_v}$ to get π_x

4. Lastly, solve for π_0 :
$$\pi_0 = \bar{v} - \frac{1}{\frac{\alpha}{rvar_i(v)} + \frac{1-\alpha}{rvar_u(v)}}\bar{x}$$

Q2) ***Why do uninformed traders participate in the market?***

- They can still some of the informed traders' information from the price signal. Though it will be made noisier from the random supply x, uninformed traders can still make a profit from noise traders given their conjecture.

Q3) ***Without necessarily solving the model, which parameters would you expect to increase the expected value of being privately informed?***

  - $\uparrow \sigma_x^2$ would make it easier for informed traders to camouflage their flow.
  - $\uparrow (1-\alpha)$ the smaller the share of informed traders, the easier it will be for them to camouflage their flow and benefit from their private information.
  - $\uparrow r^{-1}$ higher risk tolerance means informed traders will trade more aggressively on their information.
  - $\uparrow \delta$ the more precise the private information, the more useful it is in forming accurate expectations of the realized value of the asset. A more precise forecast will make knowing which trades will be profitable easier.
  - $\uparrow \hat{v}$ (holding $\overline{v}$ constant) higher posterior average means higher prices thus more profit from trading with uninformed investors forced to work with the prior mean.

The value of being informed will be higher when there are fewer informed (low α). The effects of precision (δ), risk-aversion (r) and noise trade (σ_x) are tricky. For example, high precision increases both the informed and uninformed value functions. When information gets extremely precise, informed traders face such low risk that they reveal most of their information and earn little compared to uninformed traders. Risk aversion may seem to diminish the value of being informed because it leads to smaller positions, but with price-taking traders it also limits aggressive trade and can lead to higher expected utility.

Q4) ***Suppose that instead of α being given exogenously, there was an initial round in which investors choose simultaneously to acquire the private information at some cost. What condition would have to hold in any equilibrium with an interior fraction of informed investors?***

- For an internal equilibrium where $0 < \alpha < 1$, the value of α^e will be such that the net benefit of acquiring information will equal the benefit of remaining uninformed:

$$EV\left(W_I^{\alpha}\right) = EV\left(W_U^{\alpha}\right)$$

If the cost was, say, c, the equilibrium would satisfy:

$$\underbrace{E\left[\frac{1}{2}\frac{\left(E_i[\mathrm{v}]-p\right)^2}{r\cdot var_i(v)}\right]-C}_{Net\ \ value\ of\ being\ informed} = \underbrace{E\left[\frac{1}{2}\frac{\left(E_u[v]-p\right)^2}{r\cdot var_u(v)}\right]}_{Value\ of\ being\ uninformed}$$

Theory 3 Comp 2020 Q1:

Fischer & Verrecchia (2000) and Grossman & Stiglitz (1980)

CEO sees private signal — CEO reports r — Mkt assesses $E^{mkt}[v|r]$ — Trading — Realization v

- Public priors of asset: $v \sim N\left(\bar{v}, \sigma_v^2\right)$
- Traders have CARA risk **aversion** ρ
- CEO sees private signal which gives
- posterior belief $\bar{v}_m \sim N\left(\bar{v}, \boldsymbol{\delta\sigma_v^2}\right)$
- where $cov(v, \bar{v}_m) = \boldsymbol{\delta\sigma_v^2}$ with $\delta \in (0,1)$
- CEO produces report r before trade
- CEO maximizes $\max_r bp - \frac{c}{2}\left(r - \bar{v}_m\right)^2$
- ***b* is a parameter known only to CEO**
- Investors have prior $b \sim N\left(\bar{b}, \sigma_b^2\right)$
- Equilibrium price sets investor demand
- equal to the **known** supply x
- **Add *x* to the pricing rule conjecture**
- CEO's report function is $r(\bar{v}_m, b)$
- $\hat{r}(\bar{v}_m, b)$: Mkt conjecture of CEO's $r(\bar{v}_m, b)$
- has linear form $\hat{r}(\bar{v}_m, b) = \hat{r}_0 + \hat{r}_v \bar{v}_m + \hat{r}_b b$
- In equilibrium: $\hat{r}(\bar{v}_m, b) = r(\bar{v}_m, b)$
- Market asseses $P(r) = E^{mkt}[v \mid r]$
- $\hat{P}$: CEO's conjecture of mkt pricing rule
- has linear form $\hat{p}(r) = \hat{p}_0 + \hat{p}_r r - \hat{p}_x x$
- In equilib.: $\hat{P}(r) = P(r) = p_0 + p_r r - p_x x$

Q1) Solve for the equilibrium price and report

Starting with the CEO's problem given his conjecture of the pricing rule

$$\max_r bp - \frac{c}{2}\left(r - \bar{v}_m\right)^2 = \max_r b(p_0 + p_r r - p_x x) - \frac{c}{2}\left(r - \bar{v}_m\right)^2$$

$$\text{FOC: } 0 = bp_r - c\left(r - \bar{v}_m\right) \quad \Rightarrow \quad \boldsymbol{r = \bar{v}_m + \frac{p_r}{c} b}$$

Then the market's expected value of v given its conjecture $\hat{r}(\bar{v}_m,\, b)$

and taking into consideration from above that $r_0 = 0; \quad r_v = 1; \quad r_b = \frac{p_r}{c}$

$$E[v|r] = E[v] + \frac{cov(v, r_0 + r_v \bar{v}_m + r_b b)}{var(r_0 + r_v \bar{v}_m + r_b b)}(r - E[r]) = \bar{v} + \frac{cov(v, \bar{v}_m + \frac{p_r}{c} b)}{var(\bar{v}_m + \frac{p_r}{c} b)}\left(r - E[\bar{v}_m + \frac{p_r}{c} b]\right)$$

$$\boldsymbol{E^{mkt}[v|r]} = \bar{v} + \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}\left(r - \bar{v} - \frac{p_r}{c}\bar{b}\right) \quad \text{and} \quad \boldsymbol{var(v|r)} = \sigma_v^2 - \frac{\delta^2\sigma_v^4}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}$$

$$\text{Thus demand } q = \frac{E^{mkt}[v|r] - p}{\rho \cdot var(v|r)} = \frac{\bar{v} + \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}\left(r - \bar{v} - \frac{p_r}{c}\bar{b}\right) - p}{\rho\left[\sigma_v^2 - \frac{\delta^2\sigma_v^4}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}\right]} = x \text{ supply}$$

$$\text{Isolate } p\text{: } \rho var(v|r)x = E^{mkt}[v|r] - p \Leftrightarrow p = E^{mkt}[v|r] - \rho var(v|r)x$$

$$p = \underbrace{\bar{v} - \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}\left(\bar{v} + \frac{p_r}{c}\bar{b}\right)}_{p_0} + \underbrace{\frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}}_{p_r} r - \underbrace{\rho\left[\sigma_v^2 - \frac{\delta^2\sigma_v^4}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}\right]}_{p_x} x$$

$$p_r = \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2} = \frac{1}{1 + \left(\frac{p_r}{c}\right)^2 \frac{\sigma_b^2}{\delta\sigma_v^2}} \Rightarrow \quad 0 = p_r^3 + c^2\frac{\delta\sigma_v^2}{\sigma_b^2} p_r - c^2\frac{\delta\sigma_v^2}{\sigma_b^2}$$

This has a unique solution $p_r \in (0,1)$ per Descartes's rule of signs and that the right-hand-side of the inequality is negative (positive) at $p_r = 0\,(p_r = 1)$.

Since $p_r = \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2\sigma_b^2}$, price rule is $\boldsymbol{p = \bar{v} + p_r\left(r - \bar{v} - \frac{p_r}{c}\bar{b}\right) - \rho\sigma_v^2\left(1 - p_r\delta\right)x}$

Theory 3 Comp 2020 Q2:

CEO sees private signal for v — CEO reports r — Mkt assesses $E^{mkt}[v|r, E[b|\hat{b}]\,]$ — Trading — Realization v

Investors sees private signal $\hat{b}$

Public priors of asset: $v \sim N\left(\bar{v},\sigma_v^2\right)$

Traders have CARA risk **aversion** ρ

CEO sees private signal which gives

posterior belief $\bar{v}_m \sim N\left(\bar{v}, \boldsymbol{\delta\sigma_v^2}\right)$

where $cov(v,\bar{v}_m) = \boldsymbol{\delta\sigma_v^2}$ with $\delta \in (0,1)$

CEO produces report r before trade

CEO maximizes $\max_r bp - \frac{c}{2}\left(r - \bar{v}_m\right)^2$

***b* is a parameter known only to CEO**

Investors have prior $b \sim N\left(\bar{b}, \sigma_b^2\right)$

Equilibrium price sets investor demand

equal to the **known** supply x

Add x to the pricing rule conjecture

CEO's report function is $r(\bar{v}_m, b)$

$\hat{r}(\bar{v}_m, b)$: Mkt conjecture of CEO's $r(\bar{v}_m, b)$

has linear form $\hat{r}(\bar{v}_m, b) = \hat{r}_0 + \hat{r}_v\bar{v}_m + \hat{r}_b b$

In equilibrium: $\hat{r}(\bar{v}_m, b) = r(\bar{v}_m, b)$

Market asseses $P(r) = E^{mkt}[v \mid r]$

$\hat{P}$: CEO's conjecture of mkt pricing rule

has linear form $\hat{p}(r) = \hat{p}_0 + \hat{p}_r r - \hat{p}_x x$

In equilib.: $\hat{P}(r) = P(r) = p_0 + p_r r - p_x x$

Question 2 additions:

Prior to trade, investors receive a signal $\hat{b}$

where $cov(b,\hat{b}) = var(\hat{b}) = \gamma\sigma_b^2$ & $\gamma \in (0,1)$

$E[b \mid \hat{b}] = \hat{b}$ and $cov(\hat{b}, v) = cov(\hat{b}, x) = 0$

Q2) how does $\mathrm{E}\left[(r - \bar{v}_m)^2\right]$ vary with γ?

$E[b \mid \hat{b}] = \hat{b}$ and $cov(b,\hat{b}) = var(\hat{b}) = \gamma\sigma_b^2$ with $\gamma \in (0,1)$

$$var(b|\hat{b}) = var(b) - \frac{cov^2(b,\hat{b})}{var(\hat{b})} = \sigma_b^2 - \frac{\gamma^2\sigma_b^4}{\gamma\sigma_b^2} = \sigma_b^2 - \gamma\sigma_b^2 = (1-\gamma)\sigma_b^2$$

$$E^{mkt}[v|r] = \bar{v} + \frac{cov(v,\bar{v}_m + \frac{p_r}{c}\hat{b})}{var(\bar{v}_m + \frac{p_r}{c}\hat{b})}(r - E[\bar{v}_m + \tfrac{p_r}{c}\hat{b}]) = \bar{v} + \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2(1-\gamma)\sigma_b^2}(r - \bar{v} - \tfrac{p_r}{c}\hat{b})$$

$$var(v|r) = \sigma_v^2 - \frac{\delta^2\sigma_v^4}{\delta\sigma_v^2 + (\frac{p_r}{c})^2(1-\gamma)\sigma_b^2}$$

In a similar way to before we can find $p_r = \frac{\delta\sigma_v^2}{\delta\sigma_v^2 + (\frac{p_r}{c})^2(1-\gamma)\sigma_b^2}$

Then $\mathrm{E}\left[(r - \bar{v}_m)^2\right] = \left(\frac{p_r}{c}\right)^2 \mathrm{E}\left[b^2\right] = \left(\frac{p_r}{c}\right)^2\left(\sigma_b^2 + \bar{b}^2\right)$

$$\frac{\mathrm{dE}\left[(r-\bar{v}_m)^2\right]}{\mathrm{d}\sigma_b^2} = \underbrace{\left(\frac{p_r}{c}\right)^2}_{\frac{\partial \mathrm{E}\left[(r-\bar{v}_m)^2\right]}{\partial\sigma_b^2}} + \underbrace{2\frac{p_r}{c^2}\left(\sigma_b^2 + \bar{b}^2\right)}_{\frac{\partial \mathrm{E}\left[(r-\bar{v}_m)^2\right]}{\partial p_r}} \underbrace{\left(-\frac{c^2\frac{\delta\sigma_v^2}{\sigma_b^4}(1-p_r)}{3p_r^2 + c^2\frac{\delta\sigma_v^2}{\sigma_b^2}}\right)}_{\frac{\mathrm{d}p_r}{\mathrm{d}\sigma_b^2}}$$