From Dice Rolls to Dollar Bills: Filtering Financial Markets with Random Matrices

Anupam Bhakta

Columbia University

March 8, 2024

Table of Contents

1. Introduction
2. Theoretical Foundations
3. Applications in Finance
4. Case study
5. Conclusion

Introduction

- Discover how Random Matrix Theory (RMT) unravels the complexity of financial markets.
- Explore the intersection of advanced mathematical concepts and their practical applications in finance.
- See how RMT is applied in portfolio optimization, correlation analysis, and noise reduction in financial data.

Wishart Matrices Overview

Suppose G is a $p \times n$ matrix with each column independently drawn from a p-variate standard normal distribution

$$G = \left(g_i^1, \ldots, g_i^n\right) \sim \mathcal{N}_p(0, V).$$

Wishart Matrices Overview

Suppose G is a $p \times n$ matrix with each column independently drawn from a p-variate standard normal distribution

$$G = \left(g_i^1, \ldots, g_i^n\right) \sim \mathcal{N}_p(0, V).$$

Definition

The **Wishart distribution** is the probability distribution of the $p \times p$ random matrix

$$S = GG^T = \sum_{i=1}^{n} g_i g_i^T$$

denoted by

$$S \sim W_p(V, n).$$

Significance of Wishart Matrices

Example

Let

$$G = \begin{bmatrix} 0.76 & 0.44 & 1.49 \\ 0.12 & 0.33 & -0.21 \end{bmatrix}.$$

Then,

$$S = \begin{bmatrix} 0.76 & 0.44 & 1.49 \\ 0.12 & 0.33 & -0.21 \end{bmatrix} \begin{bmatrix} 0.76 & 0.12 \\ 0.44 & 0.33 \\ 1.49 & -0.21 \end{bmatrix} = \begin{bmatrix} 3.01 & -0.07 \\ -0.07 & 0.17 \end{bmatrix}.$$

Significance of Wishart Matrices

Example

Let

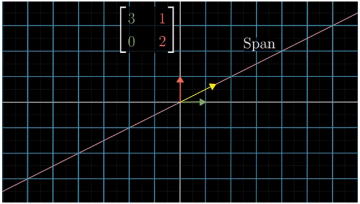
$$G = \begin{bmatrix} 0.76 & 0.44 & 1.49 \\ 0.12 & 0.33 & -0.21 \end{bmatrix}.$$

Then,

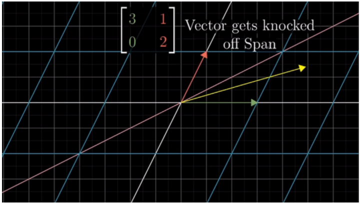
$$S = \begin{bmatrix} 0.76 & 0.44 & 1.49 \\ 0.12 & 0.33 & -0.21 \end{bmatrix} \begin{bmatrix} 0.76 & 0.12 \\ 0.44 & 0.33 \\ 1.49 & -0.21 \end{bmatrix} = \begin{bmatrix} 3.01 & -0.07 \\ -0.07 & 0.17 \end{bmatrix}.$$

Key Property: Ability to model covariance matrices from financial returns.

Eigenvalues - A Visual Introduction



(a) Before



(b) After

Figure 1: Linear transformation showing effect on eigenvalues/eigenvectors.

Eigenvectors - Why They Matter

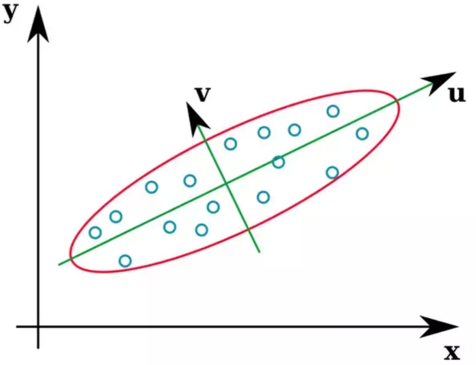


Figure 2: Eigenvectors pointing in the direction of maximum variance.

Eigenvectors - Why They Matter

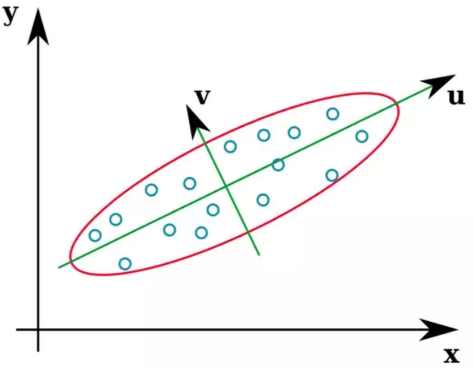


Figure 2: Eigenvectors pointing in the direction of maximum variance.

Significance: In finance, eigenvectors help identify the main factors affecting asset returns.

Introduction to Marchenko-Pastur Law

Definition

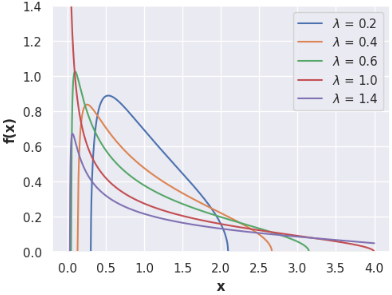
Let S be a matrix with i.i.d. entries. The **Marchenko-Pastur Law** says that the distribution of the eigenvalues (when rows and columns both approach infinity at a **fixed ratio**) is given by the formula:

$$f_\lambda(x) = \frac{1}{2\pi\sigma^2}\frac{\sqrt{(\lambda_{\max} - x)(x - \lambda_{\min})}}{\lambda x}$$

where

- $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum possible eigenvalues defined by $\lambda_{\max/\min} = \sigma^2(1 \pm \sqrt{\lambda})^2$.
- λ is the ratio of the number of rows to the number of columns in the matrix.

Visualizing Marchenko-Pastur Distribution



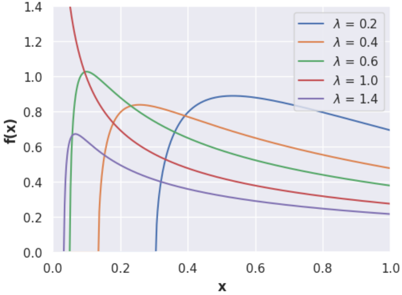


Figure 3: Marchenko-Pastur distribution for different parameters.

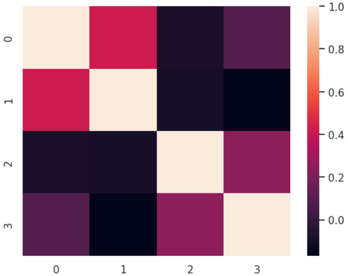
Why Marchenko-Pastur Matters in Finance

- Used differentiate between meaningful ('signal') and random ('noise') eigenvalues in large financial datasets.
- Identifies stable correlations within the market, allowing for the construction of more resilient and efficient investment portfolios.
- Improves risk assessment by distinguishing genuine market trends from random fluctuations, enhancing predictive accuracy.

Correlation Matrices in Finance

$$\begin{bmatrix} 1.00 & 0.42 & -0.09 & 0.08 \\ 0.42 & 1.00 & -0.10 & -0.17 \\ -0.09 & -0.10 & 1.00 & 0.25 \\ 0.08 & -0.17 & 0.25 & 1.00 \end{bmatrix}$$

(a) Random Correlation Matrix



(b) Matrix Heatmap

Figure 4: Visualizing correlation in a matrix.

The Problem of Noisy Data

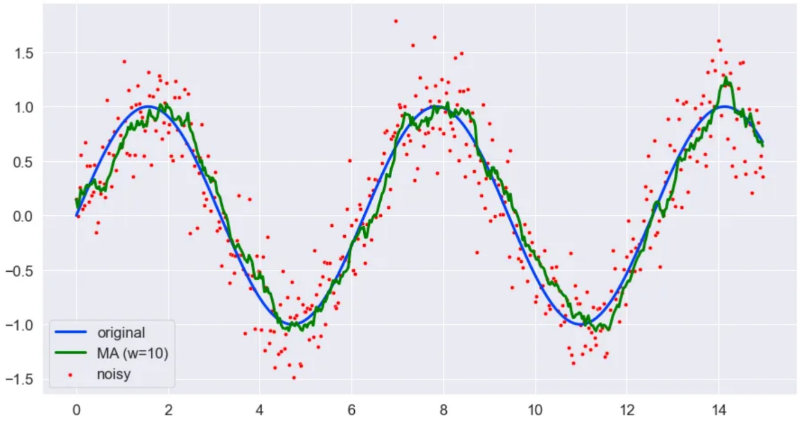


Figure 5: Plot with 10-day moving average.

The Problem of Noisy Data

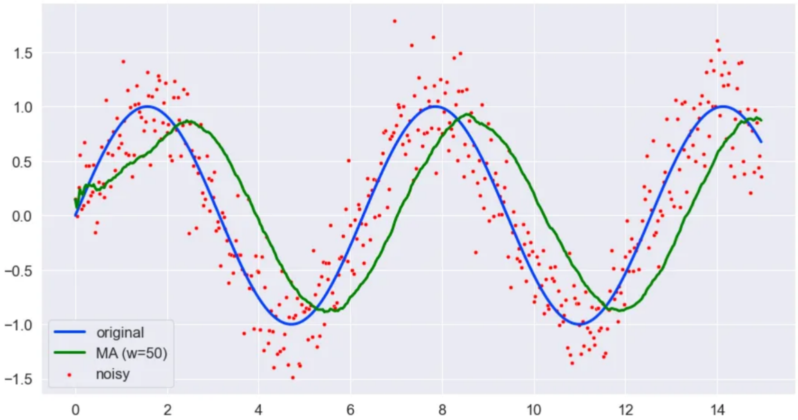


Figure 6: Plot with 50-day moving average.

Filtering with Random Matrix Theory

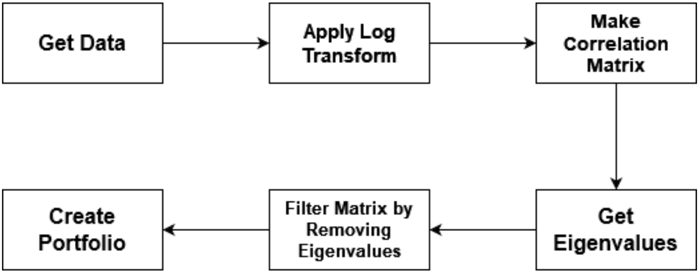


Figure 7: Flowchart for process of applying RMT to filter noise.

QR Code

Case Study Introduction

- Data consists of stock prices from assets listed on the NASDAQ.
- Take a random set of 75 stocks, and put their prices into a dataframe (if possible).
- Do some exploratory data analysis to understand the structure of the data.
- Apply RMT to filter out noise and construct the portfolio.

Visualization Before Filtering

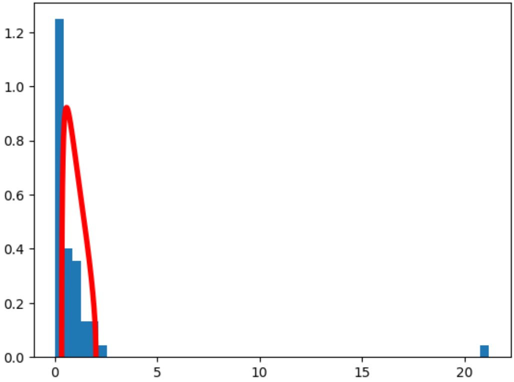


Figure 8: Eigenvalue distribution of correlation matrix.

Applying Wishart Matrix Techniques

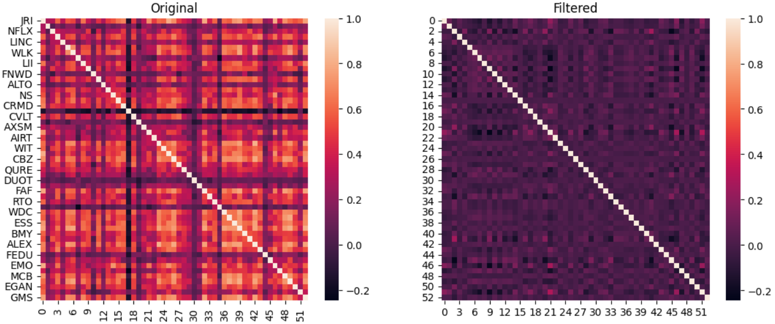


Figure 9: Original and Filtered matrix comparison.

After Clustering

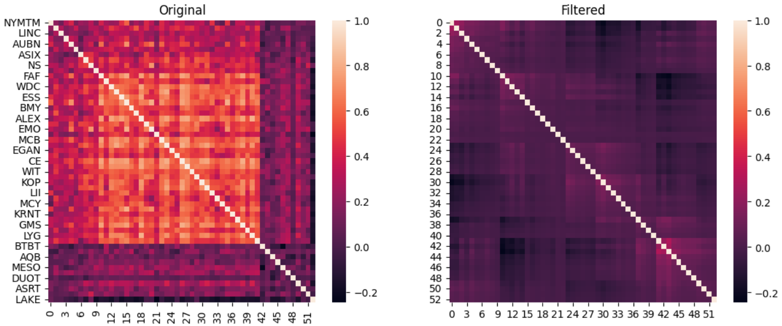


Figure 10: Clustered Original and Filtered matrix comparison.

Achieving the Minimum Variance Portfolio

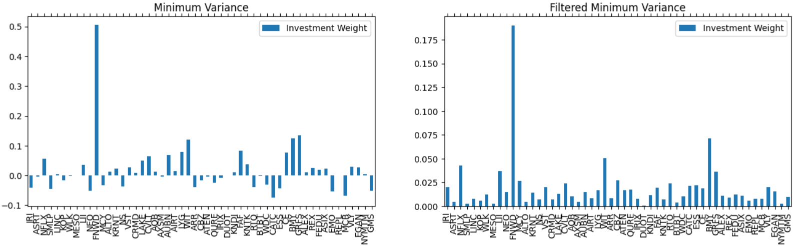


Figure 11: Original and Filtered portfolio comparison.

Achieving the Minimum Variance Portfolio

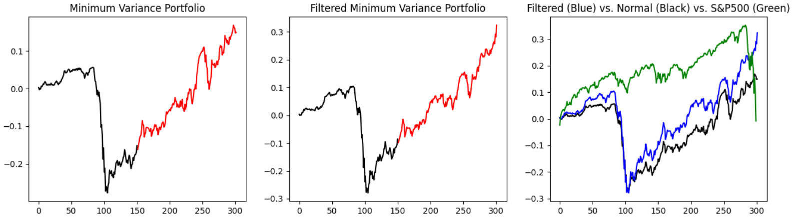


Figure 12: Returns comparison.

Limitations of Random Matrix Theory

- RMT relies on specific statistical assumptions that may not hold in all financial markets, affecting its applicability and accuracy.
- Simplifying complex market behaviors into eigenvalue distributions might overlook crucial dynamics, leading to potential misinterpretations.
- While powerful for certain analyses, RMT is not a one-size-fits-all solution and must be used judiciously within a broader toolkit of financial models.

Future Outlook

- Leverage AI to refine RMT models for predictive financial analysis.
- Develop tools for dynamic market analysis and risk assessment.
- Create systems that adapt to new market conditions using RMT and machine learning, ensuring strategies remain effective over time.

Conclusion

1. RMT provides a mathematical framework for understanding complex financial systems.

Conclusion

1. RMT provides a mathematical framework for understanding complex financial systems.
2. RMT's applications, from correlation matrix analysis to noise filtering, significantly improve portfolio optimization and risk assessment.

Conclusion

1. RMT provides a mathematical framework for understanding complex financial systems.
2. RMT's applications, from correlation matrix analysis to noise filtering, significantly improve portfolio optimization and risk assessment.
3. If you want to make money without doing any of the work, just diversify your portfolio.

Q & A

Questions?

References

- Gupta, A.K., & Nagar, D.K. (1999). Matrix Variate Distributions (1st ed.). Chapman and Hall/CRC.
- Bryson, Jennifer, Roman Vershynin, and Hongkai Zhao. "Marchenko-Pastur law with relaxed independence conditions." Random Matrices: Theory and Applications 10.04 (2021): 2150040.
- Potters M, Bouchaud J-P. Applications to Finance. In: A First Course in Random Matrix Theory: For Physicists, Engineers and Data Scientists. Cambridge University Press; 2020:321-338.
- Laloux, Laurent, et al. "Random matrix theory and financial correlations." International Journal of Theoretical and Applied Finance 3.03 (2000): 391-397.