

No Teachers Left Behind: An Analysis of Teacher Shortages in the Chicago Public School System

TEAM #11685

MODELING MEGAMINDS

MODELING THE FUTURE CHALLENGE 2023

March 3, 2023

Executive Summary

Since 2010, teacher count in Illinois public schools have decreased by roughly 15%, and has shown no signs of stopping[1]. As the number of teachers falls, the effects are reflected in the enrollment and performance of students. Since 2010, the number of students enrolled in CPS has fallen by 14.6%, and since the changing of the SAT grading scale in 2017, the average student score has fallen by 5.5%[2][3]. It has become strikingly clear that this teacher shortage issue has impacted and will continue to impact the future generations of our society. Our model aims to identify the main causes of teacher shortages, predict future trends, and propose viable recommendations to address the problem.

To determine the severity of the teacher shortage problem, we first needed to establish a forecasting model. We used a Fourier transform to find the most appropriate function to represent our data, followed by adding linear component to model the consistent growth in teacher shortages. Then, utilizing gradient boosting, we fitted the curve and got viable coefficients, which revealed an overall periodic growth in teacher vacancies. This indicates that teacher shortages is a problem that is worsening over time. We decided on the 7 main factors that led to teacher shortage: student enrollment, student daily attendance, student misconduct, student ELA performance, student math performance, teacher salary, and the 5Essential leadership index[4][5][6]. Data of these factors for each active school in the CPS was then put into a Random Forest algorithm to determine which factors contributed the most to teacher shortage. We concluded that the main factors contributing towards teacher shortages was too much student enrollment, student misconduct, and poor teacher leadership.

With these predictions, we were able to quantify the risk posed by teacher shortages. We first modeled our factors in a partial dependence plot to determine critical thresholds that would put schools in danger of facing a teacher shortage. We then created our own Risk Index calculation. First, it takes each of the seven values and correlates it to the probability based on our partial dependence plot. We then multiplied it by an independently determined impact index to calculate its Risk Index. By comparing our index with real world data, we determined that our calculation was accurate and could be applied throughout CPS.

Based on our analysis as well as our Random Forest algorithm, we outlined policy recommendations for the Chicago Public Schools System that will mitigate teacher shortage problems. We centered these recommendations upon the three main factors: excessive enrollment, student misconduct, and poor leadership. For enrollment, we found that teachers generally like low enrollment as it supports student-teacher relationships, so we recommend that schools engage in active promotion of a positive learning environment with frequent communication. For misconduct, we found that the easiest way to minimize misconduct was to remove detention and instead implement rehabilitation for students. For leadership, we decided to increase the requirements to become a school administrator in order to ensure quality administration of schools. Overall, for a financial plan to make these changes possible, we suggested that schools lower the allocated budget for these changes during years where teacher shortage dipped below the linear regression, such as from now to 2024, and increase the budget when teacher shortage rates rose above the linear regression, such as from 2024 to 2032.

Contents

1	Background	4
2	Assumptions	5
3	Data Methodology	6
4	Mathematical Methodology	9
4.1	Fourier Transform and Gradient Boosting	9
4.2	Random Forest Feature Importance	10
4.2.1	Variables	10
4.2.2	Results	11
4.2.3	Strengths	12
4.2.4	Weaknesses	12
4.3	Partial Dependence Plot	12
4.3.1	Results	13
4.4	Sensitivity Analysis	14
4.4.1	Permutation Feature Importance	14
4.4.2	Results	14
5	Risk Analysis	16
5.1	Risk Model Development	16
5.2	Results	17
5.3	Analysis	18
6	Recommendations	19
6.1	Enrollment	19
6.2	Misconduct	19
6.3	Leadership	19
6.4	Overall Funding	20
7	Conclusion	21
8	Acknowledgements	21
9	References	22
A	Appendix	25
B	Code Used	29

1 Background

The night of November 20th, 2022 was a joyful one for the students at Dawes Elementary School. Students in Skokie and Evanston, Illinois, were overjoyed to learn that their Thanksgiving break was extended by two days. While this news brought delight to the children, it created a nightmare for the administrators in their schools[7]. The real reason behind a prolonged break was none other than the ongoing teacher shortage crisis — but Dawes Elementary School isn't the only school facing this pressing issue. All throughout the Chicago Public School District (CPS), administrators are forced to cut courses, increase class sizes, and in turn, decrease the quality of education students receive due to a lack of teachers[8].

Many people point to COVID-19 for being the leading cause of teacher shortage, and argue that the crisis will resolve itself as the pandemic dies out. However, evidence of the current shortage in educational staff has been present for over a decade, and the pandemic has only recently brought attention to the matter. From 2010 to 2019, CPS saw its teacher population decline 5.5%[1], a substantial number considering the sheer size of the district. Thus, it becomes clear that this issue is here to stay unless we confront it. As the pandemic recedes and schooling systems return to normalcy, the teacher crisis still holds true.

There are several reasons why teachers are leaving the industry en masse. Some cite low pay: certain schools have such a low minimum wage that the average McDonald's worker is earning more than an educator. Others cite that the students are the problem: student performance is directly correlated to a teacher's willingness to teach. In some cases, the shortage is even attributed to the school administration: poor administration leads to an unwillingness to teach. Regardless of the underlying reason, this issue needs to be addressed.[9][10][11].

There exist many ways to mitigate this issue. Schools have made countless efforts to incentivize teachers to stay by improving the workplace environment. The CPS Board increased teacher salaries drastically by 16% over a five year term[12]. They also provided financial incentives to the highest performing teachers[13]. In fact, some schools in the district are even lowering the bar and dropping qualification requirements to become a teacher[14]. However, these are often only temporary solutions that do not address the main causes of this shortage.

Our paper aims to determine the relative importance of the following factors contributing to the teacher shortage crisis: student enrollment, student daily attendance, student misconduct, student ELA performance, student math performance, teacher salary, and the 5 Essentials leadership index. Furthermore, we will predict the number of teachers that will be in the CPS district in future years if current trends continue. This will help the CPS board better target mitigation efforts in the right places in order to truly solve this problem.

So, while students at Dawes Elementary School may appreciate their extra days off, two days will eventually turn into two weeks, and two weeks into two months if the teacher shortage continues to worsen. Without proper education, the futures of many bright children will be jeopardized, and they will be unable to live a successful and fulfilling life.

2 Assumptions

1. *Teacher vacancies in schools are determined by comparing student-teacher ratios to the ideal student-teacher ratio, 18:1.*

- **Justification:** Schools may have differing policies that have result in different optimal ratios, but looking into each school's curriculum is beyond the scope of our analysis and would prevent us from finding significant results. It is reasonable to assume all schools want to achieve the ideal ratio of 18:1. [15].

2. *We will only analyze data prior to COVID-19 (specifically, 2014-2019).*

- **Justification:** As previously established in our background, teacher shortage issues have existed for years before COVID-19 began. The impacts that COVID-19 has specifically had on teacher attrition rates have begun to recede as schools are returning to normal[16]. Thus, the only major lingering impacts of COVID-19 on the education system existed prior to the pandemic and were simply brought to the spotlight by the pandemic. Due to this, we will neglect data from the COVID-19 period in order to better address and analyze long-term problems present in the education system.

3. *We will only account for schools that have been open every year from 2014-2019.*

- **Justification:** Many schools exit and join CPS each year, so to maintain consistent data and obtain consistent trends, we only account for schools that have been around during our whole analysis time frame. Our data sets begin from 2014 as that is the time the US job market fully recovered from the Great Recession of 2007-2009[17]. This recession severely effected the economy and job market, and we believe it would skew our results if we included this anomalous event in our past data. Furthermore, we ended the data set at 2019, as that was the beginning of COVID-19, which also is an anomalous event which would skew off our data[18].

4. *We did not account for the effects of inflation.*

- **Justification:** The average inflation rate in the United States from 2014-2019 was around 1.8%[19], which is within the typical range of inflation fluctuations that leads to the natural rate of unemployment(around 2% inflation[20]). Therefore the impact of inflation on the data is very small and can be safely ignored as it will not impact teacher employment greatly.

5. *We did not account for people with special needs.*

- **Justification:** People with such special needs also need special services in order to learn - this includes specialized teachers and a smaller class size[21]. As such, we will not account for them because the average student does not require such resources.

3 Data Methodology

We draw data from three primary sources on the seven factors affecting teacher employment. The sources include the Illinois State Board of Education (ISBE)[4], Chicago Public Schools (CPS)[5], and the University of Chicago (UChicago)[6]. These are all reliable sources as they all directly deal with the local area around CPS and are the official education governing bodies dealing with education within CPS. The seven factors we looked at within these sources include student enrollment, student daily attendance, student misconduct, student ELA performance, student math performance, teacher salary, and the 5Essential Leadership Index. All of this data is relevant to determining the extent in which each of the eight factors contribute to teacher shortages, as well as predicting future trends of teacher shortage in CPS as a whole.

It is important to note that no data was found on the number of vacancies nor the types of teacher vacancies per school (i.e. science, math, reading, etc.). Thus, instead of looking at this on an individual level, we choose to address teacher shortages by school and discuss what characteristics of a school prevent a teacher shortage at their school.

We got our data ready for modeling in many ways. We first took the data and removed the data for schools that were not open for the entirety of 2014-2019 per Assumption 2. The main obstacle was the way different data sets classified schools. Some used school ID, others used the school abbreviated name, and others used the full name of the school. To organize all the data sets so that they match in school reference, we found a data set provided by CPS that linked each school's short name, long name, and ID. Then, we imported this into Excel and ran a matching program, which converted each data set with short names and long names into school IDs. Following this, we utilized a secondary matching algorithm to combine the data sets, organizing everything into a final data set which contained each school ID and each corresponding factor values. Finally, we dropped schools with missing data for any of the factors because our random forest it could lead to bias of selection. We opted for this instead of substituting empty values with the average of non-empty values to also void bias. In the end, this turned out to be 476 rows of the 3828 rows we choose to analyze. This allows our data set to be easily and efficiently utilized in our model.

Class Size from ISBE Illinois Report Card[4]

- **Motivation:** There is no existing teacher vacancy data. The closest comparison to reveal whether a not a shortage exists is average class size.
- **Parameters:** This data set provides us with the average students per class in a given school in the CPS district.
- **Purpose:** We use the average number of students per class to determine the student-teacher ratio. Public schools want the ideal student-teacher ratio of 18:1[22], which is a good balance between cost cutting and student performance. Thus, if the student-teacher ratio is greater than 18:1, teacher vacancies exist in the school. [22]

Student Enrollment from ISBE Illinois Report Card[4]

- **Motivation:** Student enrollment affects the need for a teacher. This, in turn, determines whether there is or is not a teacher shortage. [23]
- **Parameters:** This data set provides us with the number of student enrollments in a given school in the CPS district over the years 2014-2019.
- **Purpose:** We use student enrollment data in to see how an increase/decrease in students impacts the number of teachers required to operate a school. A decrease in teacher shortages can be attributed to a decrease in student enrollment. The opposite holds true as well.

Student Daily Attendance from ISBE Illinois Report Card[4]

- **Motivation:** Student attendance reflects student engagement and commitment, and thus, teacher morale. This directly affects a teacher's willingness to continue working.[24]
- **Parameters:** This data set provides us with the average percentage of students present in a given school throughout a given year.
- **Purpose:** We use student attendance data to see how student commitment and participation impacts teacher shortages. A decrease in teacher shortages can be attributed to increased student commitment and in turn, increased student attendance. The opposite holds true.

Student Misconduct from CPS Metrics[25]

- **Motivation:** Student misconduct affects a teacher's safety and thus, willingness to continue working.
- **Parameters:** This data set provides us with the number of times in a school year that a school reports behaviors that violate the Student Code of Conduct. We took the values given by the data and further divided by the school enrollment in order to account for the fact bigger schools might report more incidences of misconduct.
- **Purpose:** We use student misconduct data to see how student behavior impacts teacher safety. A decrease in teacher shortage can be attributed to increased teacher safety and in turn, a decrease in student misconduct. The opposite holds true as well. We decided not to use suspension and expulsion data as misconduct captures a wider range of behaviors that prove to be disruptive. Expulsion and suspension are both less likely to be given out by schools as well, leading to 0 from the majority of the schools. [26].

Student ELA Performance from ISBE Illinois Report Card[4]

- **Motivation:** Student ELA performance determines a teacher's teaching satisfaction and thus, willingness to continue working.
- **Parameters:** This data set provides us with the percentage of students in a given school and a given year who meet the national average in ELA.
- **Purpose:** We use student ELA performance data to see how student proficiency impacts teacher satisfaction. If student performs extremely poorly, a teacher may feel less motivated to teach them, and this may lead to increased teacher shortage[27].

Student Math Performance from ISBE Illinois Report Card[\[4\]](#)

- **Motivation:** Student math performance determines a teacher's teaching satisfaction and thus, willingness to continue working.
- **Parameters:** This data set provides us with the percentage of students in a given school and a given year who meet the national average in math.
- **Purpose:** We use student math performance data to see how student proficiency impacts teacher satisfaction. If student performs extremely poorly, a teacher may feel less motivated to teach them, and this may lead to increased teacher shortage[\[27\]](#).

Teacher Salary from CPS Employee Position Files[\[28\]](#)

- **Motivation:** The amount of salary directly affects how financially satisfactory the job is and thus, a teacher's willingness to stay.
- **Parameters:** This data set provides us with the average yearly salary of a given teacher in a given school.
- **Purpose:** We use teacher salaries to see how increased salaries increase teacher financial security. A decrease in teacher shortages can be attributed to increased financial security and in turn, increased salary. The opposite holds true as well[\[29\]](#).

5Essential Leadership Index from ISBE 5Essentials[\[30\]](#)

- **Motivation:** The 5Essential Leadership Index is determined by the University of Chicago through the statewide 5Essentials survey given to students, teacher, and parents.
- **Parameters:** This data set provides us with an index that evaluates the performance of a given school's administration (principals, vice principals, etc.).
- **Purpose:** We use this index to see how proficient leadership in the school administration increases teacher stability. A decrease in teacher shortages can be attributed to a higher index score of a given school. The opposite holds true as well[\[31\]](#).

4 Mathematical Methodology

In our model, we seek to predict the future trends of teacher shortages and determine the relative importance of the chosen factors on causing teacher shortages from 2014-2019. We first use a Fourier Transform to determine a regression which is further optimized through gradient boosting. We had considered other regressions such as exponential smoothing and an auto-regressive integrated moving average (ARIMA) model, but were limited by the number of data points and the noisiness of the data. Then, we use the random forest feature importance algorithm to determine the most important factor contributing to teacher shortages in each year from 2014-2019. Next, we combine all the data from each year and run the random forest feature importance algorithm on the large data set. We then use a partial dependence plot to determine how large the value of a factor has to be to have no effect on teacher shortages. The partial dependence plot also determines the interval of values for which the factor impacts teacher shortages. We created partial dependence plots for the most relatively important factor in each year from 2014-2019. We also created partial dependence plots for every factor from the combined data set of all six years. Note that for our data, years such as "2014" refers to the school year "2013-2014".

4.1 Fourier Transform and Gradient Boosting

Our past data for student-teacher ratios varies greatly, constantly alternating from values around 23 to values around 19. Therefore, given this fluctuating data set, we decided upon a Fourier transform to formulate a pattern to generalize and model these trends. Utilizing a Fourier transform, we found that our data could be closely broken down into 4 fundamental frequencies and a linear component, to adjust for the overall slow increase in the ratio over time. Writing out these 4 sine and cosine functions alongside the linear increase, the general equation for our model was found to be

$$a + bx + c \sin(dx + e) + f \cos(dx + e) + g \sin(hx) + h \cos(gx) \quad (1)$$

Following this, to fit our function closely to this model, we utilized a gradient boosted fitting method. For our loss function, we found that square loss functions outperformed logarithmic hyperbolic cosine functions and absolute loss functions to model our code. We suspect this is due to the rapid acceleration of the square loss function as compared to others, such as absolute loss functions. Large outliers and errors in our predictions would significantly skew our final results and predictions, and would be easily recognizable in the face of our smaller oscillating function. Therefore, it makes sense that we found from reasoning and graphical interpretation that the squared loss function provided less error in the 20 year prediction range we performed. By using this gradient boosting, we found that the closest fit can be graphed and set as the following.

Predictions for Future Student to Teacher Ratio for Elementary

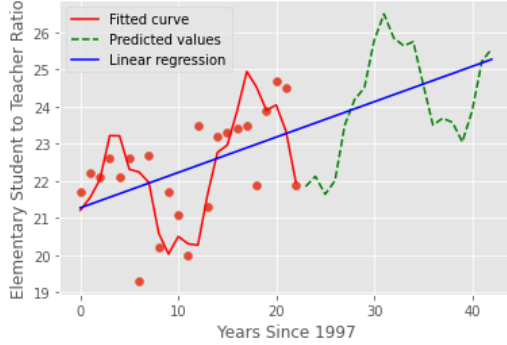


Figure 1: Fourier Transform for CPS elementary schools

Variable	Value
a	100.793183
b	0.115572800
c	54.1830283
d	0.0000239402306
e	5.05552528
f	-86.2378408
g	1.84024119
h	0.450852865

Table 1: Variable Values for Equation 1

Predictions for Future Student to Teacher Ratio for High School

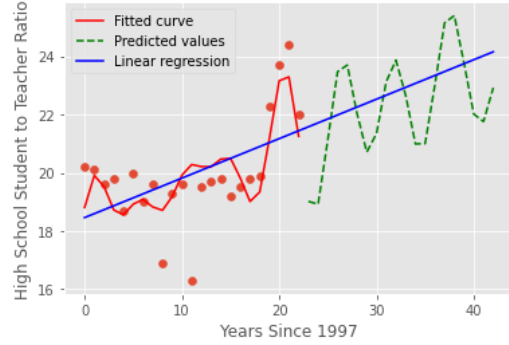


Figure 2: Fourier Transform for CPS elementary schools

Variable	Value
a	18.31325684
b	0.1295682
c	0.54045429
d	0.73775414
e	6.52288523
f	-0.67572896
g	1.1881641
h	1.02244719

Table 2: Variable Values for Equation 1

These results make sense because, as mentioned in the background, the solutions that are generally implemented by CPS currently are band-aid solutions that only solve the problem short term and do nothing to aid the issue long term. However, as seen by the linear regression that acts as the base axis of our Fourier transform, the general trend of the student-teacher ratio is still increasing, showing that this is a problem that will continuously get worse in the future. Thus, this is a problem we need to fix.

4.2 Random Forest Feature Importance

The random forest feature importance algorithm determines the relative importance of each of the seven factors for each year from 2014-2019. We also use this algorithm on the combined data set. Thus, we produce seven such models depicting the importance of each factor. We use an 80-20 train test split to implement the random forest feature importance algorithm. This means we use 80% of the data set to train the decision trees and 20% of the data set to test the model and achieve the relative feature importance. The random forest algorithm will help us determine which factors we need to target our mitigation efforts on based on which factor impacts teacher shortage the most.

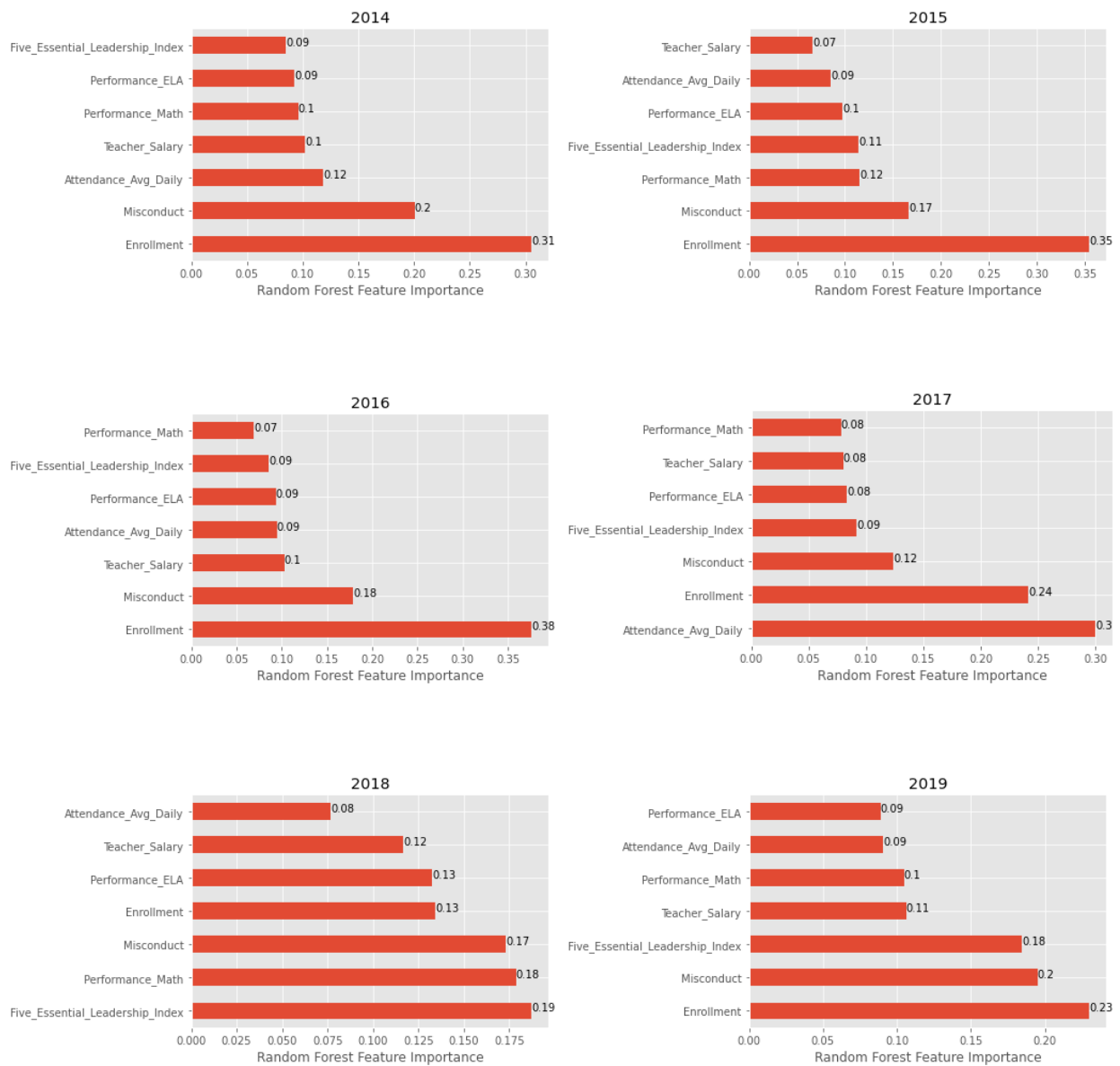
4.2.1 Variables

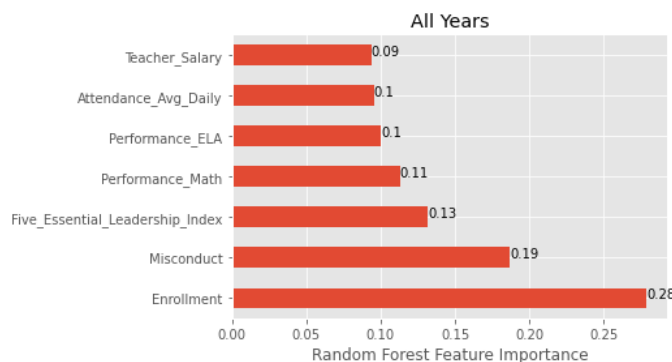
Factor	Units
Class size	Student to teacher ratio
Student enrollment	Students enrolled
Student daily attendance	Average percentage
Student misconduct	Misconducts per student
Student ELA performance	Percentage proficient
Student math performance	Percentage proficient
Teacher salary	Dollars
5Essential Leadership Index	Index value

Table 3: Variables for our mathematical models

4.2.2 Results

The results we return from the algorithm are depicted in the following graphs:





Random Forest Feature Importances for Years 2014-2019

In four of the six years that we ran the random forest feature importance, the most relatively important factor was enrollment (2014, 2015, 2016, 2019). In 2017 and 2018, the most relatively important factor was daily student attendance, and the 5Essential Leadership Index, respectively.

The random forest algorithm was responsible for classifying vacancies based on the factor data. The random forest results were then compared with existing vacancies to determine whether the model correctly predicted the existence of a vacancy. The accuracy score of the combined relative feature importance of our Random Forest algorithm is 0.8987. This means the random forest algorithm correctly predicts a vacancy 89.87% of the time. We find a mean absolute error (MAE) of 0.1013, and a root mean square error (RMSE) of 0.3183, meaning our data is of high quality.

4.2.3 Strengths

A strength of our model is that random forest is highly robust and resistant to over-fitting, which means it can handle noisy data and outliers effectively. As seen with trends in our factors, some years have unexpectedly high or low values that can't be adequately explained so we use multiple. The algorithm is computationally efficient and can handle larger data sets.

4.2.4 Weaknesses

A weakness of our model is its bias towards categorical variables: Random Forest feature importance may be biased towards categorical variables with many levels. This is because each level of a categorical variable is treated as a separate feature, which can lead to an over-representation of categorical variables in the feature importance ranking.

4.3 Partial Dependence Plot

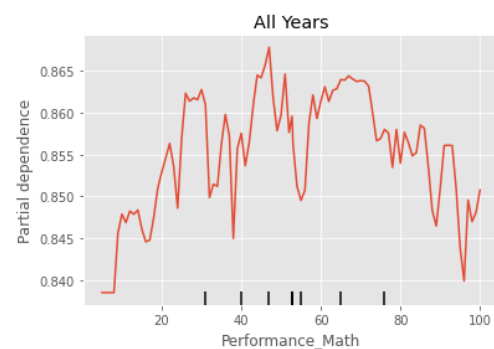
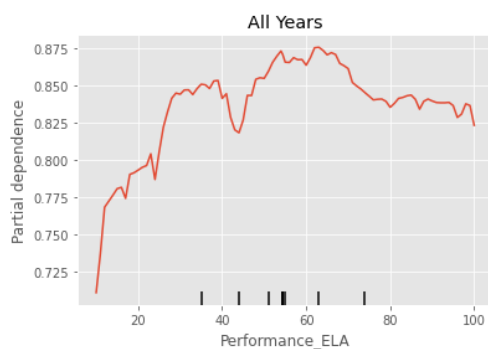
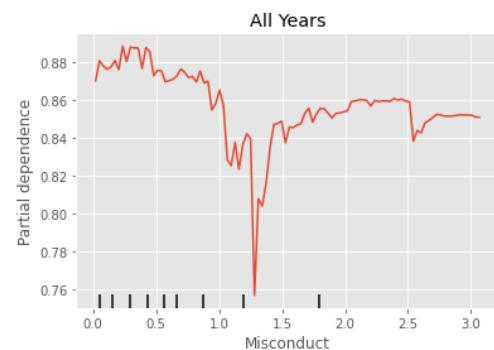
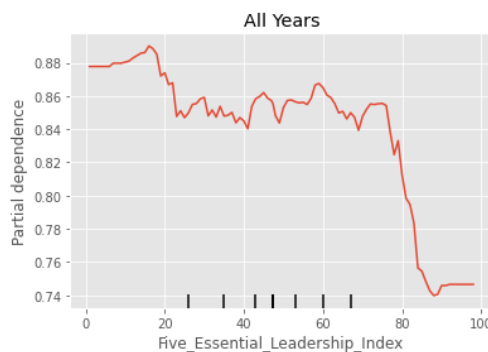
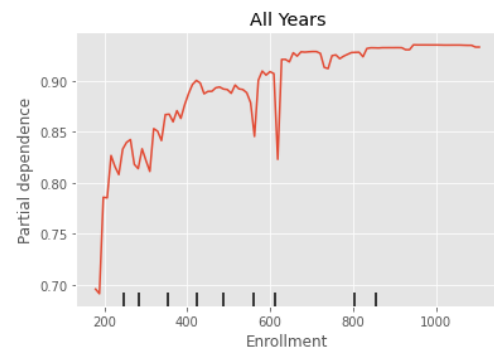
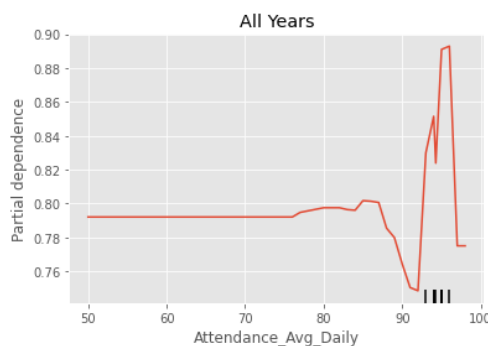
A partial dependence plot is a powerful visualization tool that we use to show the relationship between a factor variable with our dependent student to teacher ratio. In this, we chose to utilize one way partial dependence plots to ensure easy understanding and visual interpretation. We can model this with the equation

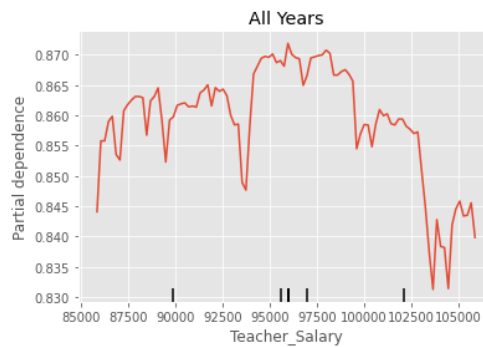
$$\boxed{PD(x) = E[f(X', x)]} \quad (2)$$

where $PD(x)$ is the partial dependence of student-teacher ratio on a single factor variable x , E is the expected value operator, and $f'(X, x)$ is the random forest model's predicted outcome for the set of predictor variables X' , where the value of x has been fixed to a specific value. With our data points regarding partial dependence, we had a discrete set of values. To turn this into a continuous graph which can be visualized, we utilized gradient boosting to fit a curve through the discrete data points. This was done using the sklearn python programmed, which had readily available partial dependence plot gradient boosting.

4.3.1 Results

The values of local extrema in each of the factors represents critical values which are thresholds of putting schools at risk. The results we return from the algorithm are depicted in the following graphs:





Partial Dependency Plots for Years 2014-2019

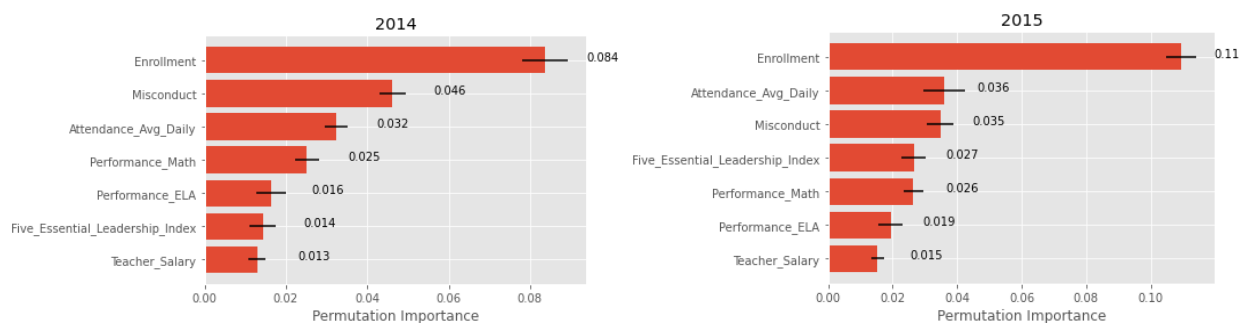
4.4 Sensitivity Analysis

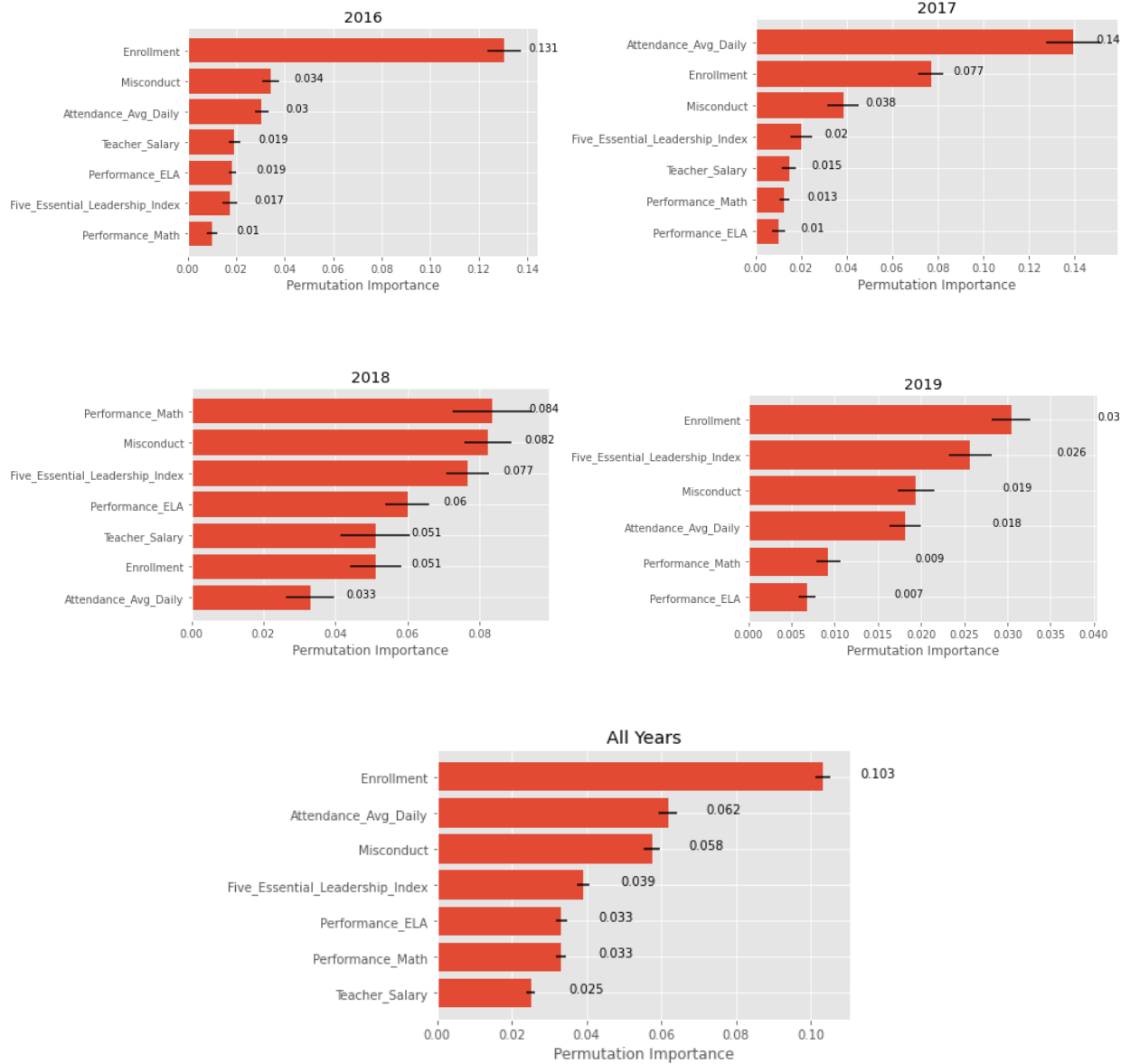
4.4.1 Permutation Feature Importance

We analyzed the sensitivity of our variables using a permutation feature importance algorithm. This algorithm is another method of determining the relative feature importance. It works by randomly permuting the values of each factor in the test data set and measuring how much the permutation reduces the model's accuracy. If a feature is important to the model, then permuting its values should lead to a significant decrease in the model's performance. On the other hand, if a feature is not important, permuting its values should not have much effect on the model's performance. Model performance is determined by computing the difference between the original accuracy and the accuracy on the permuted data set.

4.4.2 Results

We conduct a sensitivity analysis on every factor and added an error bar that represents one standard deviation above and below the calculated value. Since in every plot except 2018 the error bar of the leading cause does not overlap with the error bars of the other factors, we find that our results are statistically significant and that those specific factors are indeed the most significant cause of teacher shortage that year. While 2018 does not follow the above, the top 3 factors do as their error bars do not overlap with those of the other factors. Thus, we decided to analyze only the top 3 factors as those factors are statistically significant for all years.





Permutation Feature Importances for Years 2014-2019

5 Risk Analysis

5.1 Risk Model Development

Teacher shortages are devastating to both the schools and the communities it impacts. The potential losses that people will experience from teacher vacancies can range greatly. As such, we aim our risk analysis at determining which schools and which communities are most at risk from the ongoing teacher shortage. The main risk present is the development and future of the children attending school. Therefore, we define a risk index that evaluates the total risk present to the future of a student attending a school. This risk index can be calculated using the following formula

$$\text{Risk Index} = P_a * I_a + P_e * I_e + P_l * I_l + P_m * I_m + P_{el} * I_{el} + P_{ma} * I_{ma} \quad (3)$$

where the variables mentioned are the ones present in the following table.

Variable	Meaning
P_a	Probability of Teacher Shortage Impacting Attendance
P_e	Probability of Teacher Shortage Impacting Enrollment
P_l	Probability of Teacher Shortage Impacting Leadership
P_m	Probability of Teacher Shortage Impacting Misconduct
P_{el}	Probability of Teacher Shortage Impacting ELA Performance
P_{ma}	Probability of Teacher Shortage Impacting Math Performance
I_a	Impact of Attendance on the Future and Development of Students
I_e	Impact of Enrollment on the Future and Development of Students
I_l	Impact of Leadership on the Future and Development of Students
I_m	Impact of Misconduct on the Future and Development of Students
I_{el}	Impact of ELA Performance on the Future and Development of Students
I_{ma}	Impact of Math Performance on the Future and Development of Students

Table 4: Variable Values for Equation 3

Variable	Value	Reasoning
I_a	5	We determined this value because although personal attendance matters greatly towards a student's individual performance, the attendance of other people should not impact another student's development greatly.[32]
I_e	10	Enrollment matters greatly because the more students there are, the less time a teacher can spend on each individual student, greatly impacting a student's development.[33]
I_l	5	Leadership decisions often do not have a direct impact on student performance, though some prominent ones may arise through actions such as policies changing a student's environment.[34]
I_m	10	Student misconduct can bring about a negative school environment for other students, greatly impacting the way in which they develop. [35]
I_{el}	15	English performance scores directly correlate to and measure how much students have developed.
I_{ma}	15	Math performance scores directly correlate to and measure how much students have developed.

Table 5: Variable Values for Equation 3

To determine the probability values, we can utilize the partial dependence plots found in our earlier model. The partial dependence plot gives us how each factor is connected to teacher shortages when the factor is at a certain value. This relationship can be reversed, and we see the influence of teacher shortages on each factor is related by the same partial dependence. Therefore, a value of a factor with a lower partial dependence on teacher shortages implies that the probability of teacher shortage impacting that factor is lower. This justifies us to set each probability per factor per school to be the partial dependence value corresponding to that factor.

5.2 Results

Utilizing values from our data sets as well as Equation 3, we found the highest and lowest risk schools in CPS. The following tables represent the values

Factor	Value	Partial Dependence	Risk Index
Attendance	93.0	0.82396	4.1198
Enrollment	314	0.78826	7.8826
Leadership	82.0	0.77340	3.867
Misconduct	1.038	0.83284	8.3284
ELA Performance	23.951	0.79469	11.92035
Math Performance	57.259	0.85498	12.8247
Total			48.94285

Table 6: Chicago Math and Science Academy Charter School (**Low** Risk)

Factor	Value	Partial Dependence	Risk Index
Attendance	93.0	0.82251	4.11255
Enrollment	275	0.78826	7.8826
Leadership	86.0	0.75027	3.75135
Misconduct	1.185	0.82325	8.2325
ELA Performance	22.169	0.77503	11.62545
Math Performance	56.533	0.82391	12.35865
Total			47.9631

Table 7: Theodore Roosevelt High School (**Low** Risk)

Factor	Value	Partial Dependence	Risk Index
Attendance	95.9	0.92121	4.60605
Enrollment	685	0.89183	8.9183
Leadership	15.0	0.88756	4.4378
Misconduct	0.476	0.89021	8.9021
ELA Performance	59.388	0.86634	12.9951
Math Performance	53.804	0.85783	12.86745
Total			52.7268

Table 8: Academy for Global Citizenship Charter School (**High** Risk)

Factor	Value	Partial Dependence	Risk Index
Attendance	95.9	0.92121	4.60605
Enrollment	685	0.89183	8.9183
Leadership	15.0	0.88756	4.4378
Misconduct	0.524	0.86937	8.6937
ELA Performance	60.236	0.87129	13.06935
Math Performance	51.124	0.85872	12.8808
Total			52.606

Table 9: Catalyst Elementary Charter School - Circle Rock (**High Risk**)

5.3 Analysis

From our risk index, we found that the Chicago Math and Science Academy Charter School and the Theodore Roosevelt High School have the lowest risk index in CPS, and the Academy for Global Citizenship Charter School and the Catalyst Elementary Charter School - Circle Rock had the highest risk index in CPS. This aligns closely with what is true, as the Chicago Math and Science Academy Charter School is ranked as the 4th best charter school in Illinois, and the Theodore Roosevelt High School is ranked 78 out of all CPS schools. This confirms our index model, as they clearly are very good schools, likely due to the low-risk factor we determined during this teacher shortage crisis.[36][37] Furthermore, the Catalyst Elementary Charter School - Circle Rock is ranked 2038 out of all elementary schools in Illinois, and the Academy for Global Citizenship Charter School is ranked in the bottom 50% of all schools in Illinois.[38][39] This aligns with what our model expects, as these schools are expected to be suffering due to the high risk index in these times of teacher shortage crisis.

This risk index is extremely versatile, as it can be applied to any school regardless of time or location, as it only requires data regarding the factors. Furthermore, by finding the index of staple good schools, one can compare indices between schools to see if their school is of higher or lower risk.

6 Recommendations

We now propose recommendations for action that can be taken by the Chicago Public School District to help reduce homicide rates. These recommendations will focus on three most pressing factors of teacher shortages based upon our Random Forest algorithm: excessive enrollment, student misconduct, and poor leadership. Additionally, we outlined a possible budgeting plan for CPS to follow as these changes are bound to cost a lot of money.

6.1 Enrollment

As our random forest depicts, enrollment is consistently significant factor towards teacher shortages. Therefore, it is clearly a pressing issue which, once solved, could greatly reduce teacher vacancies in Chicago. One way to make sure teachers are content with school size is the addition of more schools. As mentioned, most teachers report higher levels of happiness and morale when teaching at smaller schools, usually attributing it towards closer student-teacher relationships and more administrative attention. The only exception to this is large magnet-type schools in which there was a high student population. This is because the population is due to the number of applicants wanting to enroll and the school choosing its students versus a regular large public high school where students need to attend and a moral obligation to fulfill. Therefore, we recommend large public schools to implement often faculty meetings, in which administrative officers can listen to teacher ideas and complaints to form a closer and more attentive relationship with teachers. Furthermore, we recommend schools to promote positive learning environments, utilizing systems of school wide core values and accessible communication lines to teachers, as it has been shown to foster better teacher student relationships.[40]

6.2 Misconduct

Alongside enrollment, student misconduct and behavior was shown to be a significant factor as well in teacher vacancies. One way to minimize misconduct would be to implement rehabilitation in schools, as opposed to traditional solutions such as detention or suspension. Things such as educational courses and community service following a misconduct can help students grow to learn how to be better people and improve themselves, rather than simply punishing them and allowing the misconduct to occur again. As shown by the following study, rehabilitation can reduce further offences by around 62%. Thus, while traditional punishments may improve student behavior on a short term, on a long term scale, in order to truly solve the problem we must replace such traditional punishments with rehabilitation efforts. [41]

6.3 Leadership

As shown in our random forest model, leadership is a significant factor as well in causing teacher shortages. To eliminate this problem, we suggest that CPS creates mandatory leadership sessions and classes, where school administrators such as principals can work on improving their leadership continuously. This helps as school administrators often went to school a long time ago, and consistently taking classes can ensure that they stay qualified to administrate a school. Additionally, another solution is to raise the necessary qualifications to become an administrator unlike what

schools are currently doing. While this may seem counter intuitive and against the actions that are done right now - lowering requirements to get more personnel, in the long run, this will be more beneficial as it will raise teacher retention rates and lower teacher attrition rates, better solving the problem on a long term scale. [42]

6.4 Overall Funding

In reality, funding is heavily limited for public schools, as cannot be solely spent on solving the issue of teacher shortages. For all schools, resources, maintenance, and other bills all require portions of the overall funds. Therefore, we suggest the following strategy to maintain a healthy spending plan while still regulating the teacher shortage crisis. As shown from our model for student-teacher ratios, it fluctuates around our base linear regression. When the model falls under this trend line, we suggest lowering the expenditure on the solutions mentioned above. This is because during these times, teacher shortage is relatively moderate and needs more minimal regulation. Then, during the times when student-teacher ratios peak past our linear regression, we recommend significantly increasing the budget spent on the solutions we mentioned, as these are times when teacher shortages are high enough to significantly impact the education and growth of students. Over time, this strategy should slowly flatten the peaks of our function, resulting in an altered model which will oscillate around a slowly decreasing sloped line.

For example, in our 20 year estimate, we recommend decreasing funding from 2019 to 2024, as our model predicts student-teacher ratios to be relatively lower during these times, needing less attention and direct countermeasures. Then, from 2024 to 2032, we recommend increasing the budget spent on combating teacher shortages. For this, as shown by our curve, the amount spent should increase alongside the increase in the student-teacher ratio. Therefore, we recommend slowly scaling up the budget from 2024 to 2028, as that prevents ever going over the budget as the school can slowly make plans and gradually cut down on other costs. Then, as our model predicts teacher shortages going down, we recommend slowly cutting back on this budget, creating plans to reallocate the funds as the school gradually returns to regular spending.

7 Conclusion

Overall, our model successfully determined the main factors contributing to the current teacher shortage crisis, predicted general trend of student teacher ratios into the future, and analyzed the risk that teacher shortages bring to the future generations of Chicagoans. We analyzed the risks of teacher shortages in specific schools within CPS, and provided recommendations for CPS to implement in order to mitigate teacher shortages. We were able to determine that the main factors that we needed to mitigate were excessive enrollment, student misconduct, and poor leadership. As such, we outlined the following recommendations: the addition of more schools, the transition from traditional punishment methods to rehabilitation methods, and increasing the requirements to become a school administrator. We then outlined a financial plan that would target funding at specific years that our Fourier transform determined to be years with high teacher vacancies that would help with budgeting and mitigate the problem further.

8 Acknowledgements

There were many significant parties who provided us with the resources and knowledge necessary to have this wonderful experience. We sincerely thank the following:

1. Our team coach: Paul Kim
2. Our mentors: Elizabeth Cvelbar and Tony Pistilli
3. Members of The Actuarial Foundation for organizing and running this competition

9 References

References

- [1] CPS teacher statistics. [Online]. Available: <https://www.isbe.net/Pages/Annual-Statistical-Report.aspx>
- [2] SAT Scores Illinois. [Online]. Available: <https://www.illinoisreportcard.com/state.aspx?source=trends&source2=sat.average&Stateid=IL>
- [3] Enrollment in CPS. [Online]. Available: https://www.cps.edu/globalassets/cps-global-media/ara/ara-1718-documents/ara_districtoverview_1718.pdf
- [4] Illinois school report card. [Online]. Available: <https://www.isbe.net/Pages/Illinois-State-Report-Card-Data.aspx>
- [5] CPS district data. [Online]. Available: <https://www.cps.edu/about/district-data/>
- [6] 5 essentials data. [Online]. Available: <https://uchicagoimpact.org/our-offerings/5essentials>
- [7] Dawes Elementary Days Off. [Online]. Available: <https://chicago.suntimes.com/education/2022/3/21/22989205/chicago-public-schools-cps-calendar-2022-23-academic-year-august-22-first-day-earliest-start>
- [8] Effects of teacher shortage. [Online]. Available: <https://learningpolicyinstitute.org/blog/teacher-shortages-take-center-stage#:~:text=The%20Impact%20of%20Teacher%20Shortages,to%20teach%20their%20subject%20matter.>
- [9] Reasons for teacher shortage. [Online]. Available: <https://news.illinois.edu/view/6367/678256548>
- [10] More reasons for teacher shortage. [Online]. Available: <https://www.nytimes.com/2022/12/14/opinion/teacher-shortage-education.html>
- [11] 1 in 3 schools went without teachers. [Online]. Available: <https://www.npr.org/local/309/2019/08/05/748114048/1-in-3-chicago-public-schools-went-without-a-teacher-for-a-year>
- [12] Benefits for CPS teachers. [Online]. Available: <https://www.cps.edu/globalassets/cps-pages/staff/hr4u/cps-benefits-handbook-2022-v3.pdf>
- [13] Financial incentives CPS. [Online]. Available: <https://columbiachronicle.com/98f6c230-5b09-588b-9966-f720bbb393a6>
- [14] Dropping qualifications for teachers. [Online]. Available: <https://abc7chicago.com/teacher-shortage-teachers-pay-back-to-school-salary/12112027/>
- [15] Student teacher ratio. [Online]. Available: [https://www.learner.com/blog/student-to-teacher-ratio-in-high-schools#:~:text=Numerous%20experts%20have%20landed%20on,engaged%20\(Public%20School%20Review\)..](https://www.learner.com/blog/student-to-teacher-ratio-in-high-schools#:~:text=Numerous%20experts%20have%20landed%20on,engaged%20(Public%20School%20Review)..)

- [16] Illinois schools return to normal. [Online]. Available: <https://dph.illinois.gov/resource-center/news/2022/august/state-of-illinois-adopts-new-cdc-covid-19-guidelines-for-k-12-sc.html>
- [17] Tracking the post great recession economy. [Online]. Available: <https://www.cbpp.org/research/economy/tracking-the-post-great-recession-economy#:~:text=While%20the%20recovery%20from%20the,same%20quarter%20a%20year%20earlier.>
- [18] COVID19 timeline. [Online]. Available: [https://www.cdc.gov/museum/timeline/covid19.html#:~:text=January%2010%2C%202020,-nCoV\)%20on%20its%20website.](https://www.cdc.gov/museum/timeline/covid19.html#:~:text=January%2010%2C%202020,-nCoV)%20on%20its%20website.)
- [19] Inflation data. [Online]. Available: <https://fred.stlouisfed.org/series/FPCPITOTLZGUSA>
- [20] The curious history of the federal reserve's 2% inflation targeting, explained. [Online]. Available: <https://www.cnbc.com/2023/02/20/the-federal-reserves-2percent-inflation-targeting-policy-explained.html#:~:text=The%202%25%20inflation%20target%20is,to%20the%20International%20Monetary%20Fund.>
- [21] A guide to the individualized education program. [Online]. Available: <https://www2.ed.gov/parents/needs/spced/iepguide/index.html>
- [22] Student teacher ratio effects. [Online]. Available: <https://www.weforum.org/agenda/2022/09/student-teacher-ratios-vary-across-the-globe/>
- [23] Teachers like small schools more. [Online]. Available: https://www.educationworld.com/a_issues/issues108.shtml#:~:text=%22%5BTeachers%5D%20thought%20teaching%20was%20more%20fun%2C%20satisfying%2C%20and,became%20teachers%20in%20the%20first%20place.%20GREATER%20EXPECTATIONS
- [24] Student engagement on teacher happiness. [Online]. Available: <https://www.brookings.edu/research/development-of-student-and-teacher-measures-of-happiness-curriculum-factors/>
- [25] CPS district data metrics. [Online]. Available: <https://www.cps.edu/about/district-data/metrics/>
- [26] Student behavior is the leading cause for teachers leaving. [Online]. Available: <https://districtadministration.com/student-behavior-is-the-leading-cause-for-teachers-leaving/>
- [27] 'there's only so far i can take them' – why teachers give up on struggling students who don't do their homework. [Online]. Available: <https://theconversation.com/theres-only-so-far-i-can-take-them-why-teachers-give-up-on-struggling-students-who-dont-do-their-homework>
- [28] CPS teacher files. [Online]. Available: <https://www.cps.edu/about/finance/employee-position-files/>
- [29] Do teachers make enough money to live comfortably? [Online]. Available: <https://elementaryassessments.com/do-teachers-make-enough-money-to-live-comfortably/#:~:text=Yes%2C%20educators%20absolutely%20do%20and%20can%20live%20well,%20the%20best%20career%20choice%20for%20you%2C%20thats%20fine>

- [30] 5 essentials survey answers. [Online]. Available:
<https://www.isbe.net/Pages/5Essentials-Survey.aspx>
- [31] The teacher shortage is about leadership, not just pay. [Online]. Available:
<https://scholars.org/contribution/teacher-shortage-about-leadership-not-just-pay>
- [32] 10 facts about school attendance. [Online]. Available: <https://www.attendanceworks.org/chronic-absence/the-problem/10-facts-about-school-attendance/>
- [33] Does class size really matter? a chalkbeat look at the research. [Online]. Available:
<https://www.chalkbeat.org/2022/6/10/23162544/class-size-research#:~:text=Research%20suggests%20that%20students%20perform%20better%20in%20smaller%20classes.&text=There%27s%20little%20debate%20among%20teachers,would%20strongly%20boost%20student%20learning.>
- [34] The association between teacher leadership and student achievement: A meta-analysis. [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S1747938X19305317>
- [35] The relationship between student's perceptions of their school environment and academic achievement. [Online]. Available:
<https://www.frontiersin.org/articles/10.3389/fpsyg.2022.959259/full>
- [36] Chicago math and science academy ranking. [Online]. Available:
<https://www.niche.com/k12/chicago-math--and--science-academy-high-school-chicago-il/>
- [37] Roosevelt high school ranking. [Online]. Available: <https://www.usnews.com/education/best-high-schools/illinois/districts/chicago-public-schools/roosevelt-high-school-6582#:~:text=Roosevelt%20High%20School%20is%20ranked%20438%2D673rd%20within%20Illinois.>
- [38] Catalyst charter circle rock ranking. [Online]. Available:
<https://www.usnews.com/education/k12/illinois/catalyst-charter-circle-rock-es-231633#:~:text=Catalyst%20Charter%2DCircle%20Rock%20ES%202021%20Rankings,the%20students%20for%20high%20school.>
- [39] Academy for global citizenship ranking. [Online]. Available:
<https://www.publicschoolreview.com/academy-for-global-citizenship-profile#:~:text=31%20teachers-,School%20Rankings,the%202020%2D21%20school%20year.>
- [40] How to improve student teacher relationships in your school. [Online]. Available:
<https://blog.schoolcues.com/how-to-improve-student-teacher-relationships-in-your-school/#:~:text=5%20Ways%20School%20Admins%20Can%20Push%20for%20Better,5%20%235%3A%20Foster%20a%20Positive%20Learning%20Environment%20>
- [41] Youth rehab better than detention. [Online]. Available: <https://www.abc.net.au/news/2021-05-13/qld-youth-rehab-better-than-detention-says-ted-noffs-foundation/100122222#:~:text=Spokesperson%20Matt%20Noffs%20said%20the%20foundation%20had%20costed,cent%2C%20compared%20with%2026%20per%20cent%20from%20detention.>
- [42] Teaching teachers leadership is extremely important. [Online]. Available:
<https://today.troy.edu/news/teacher-leadership-why-its-more-important-than-ever/>

A Appendix

Below is our whole random forest and with its main branches broken down.

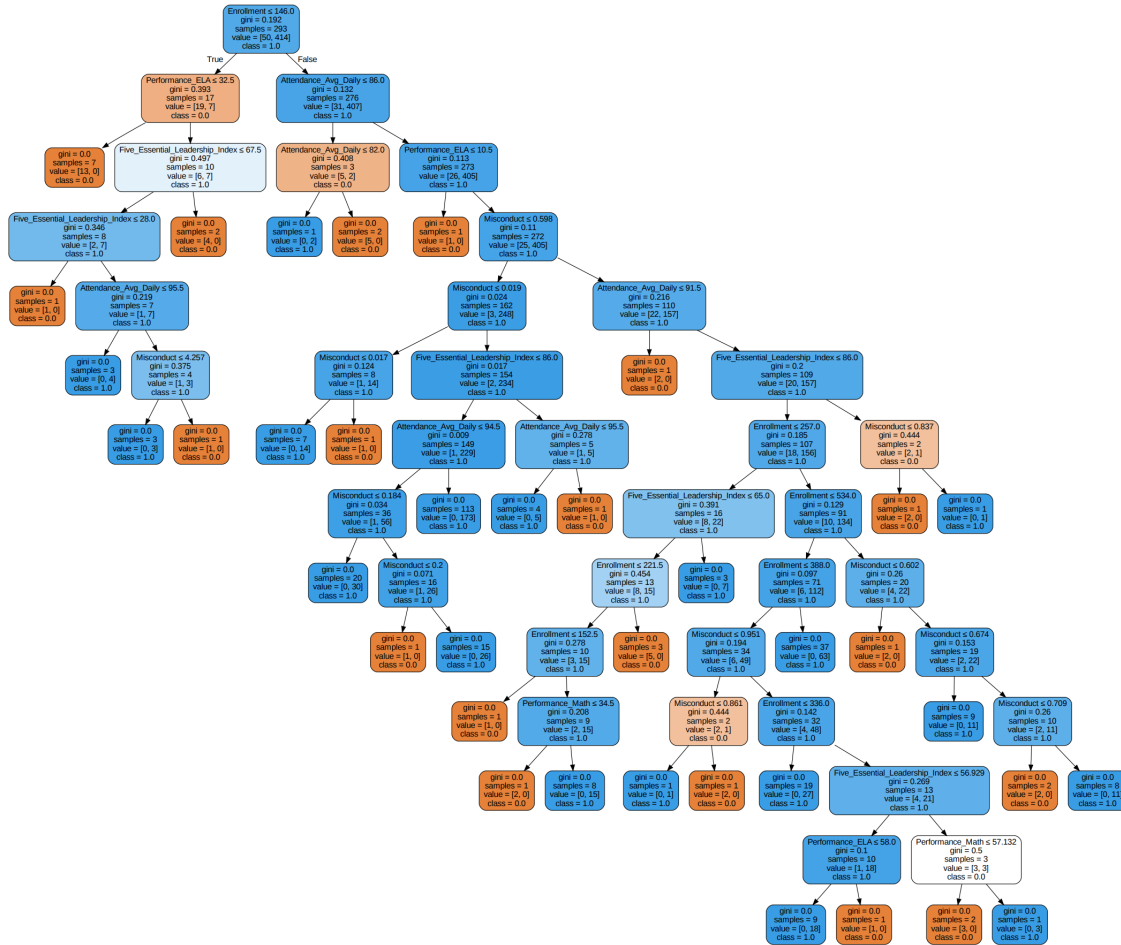


Figure 3: Random Forest Model

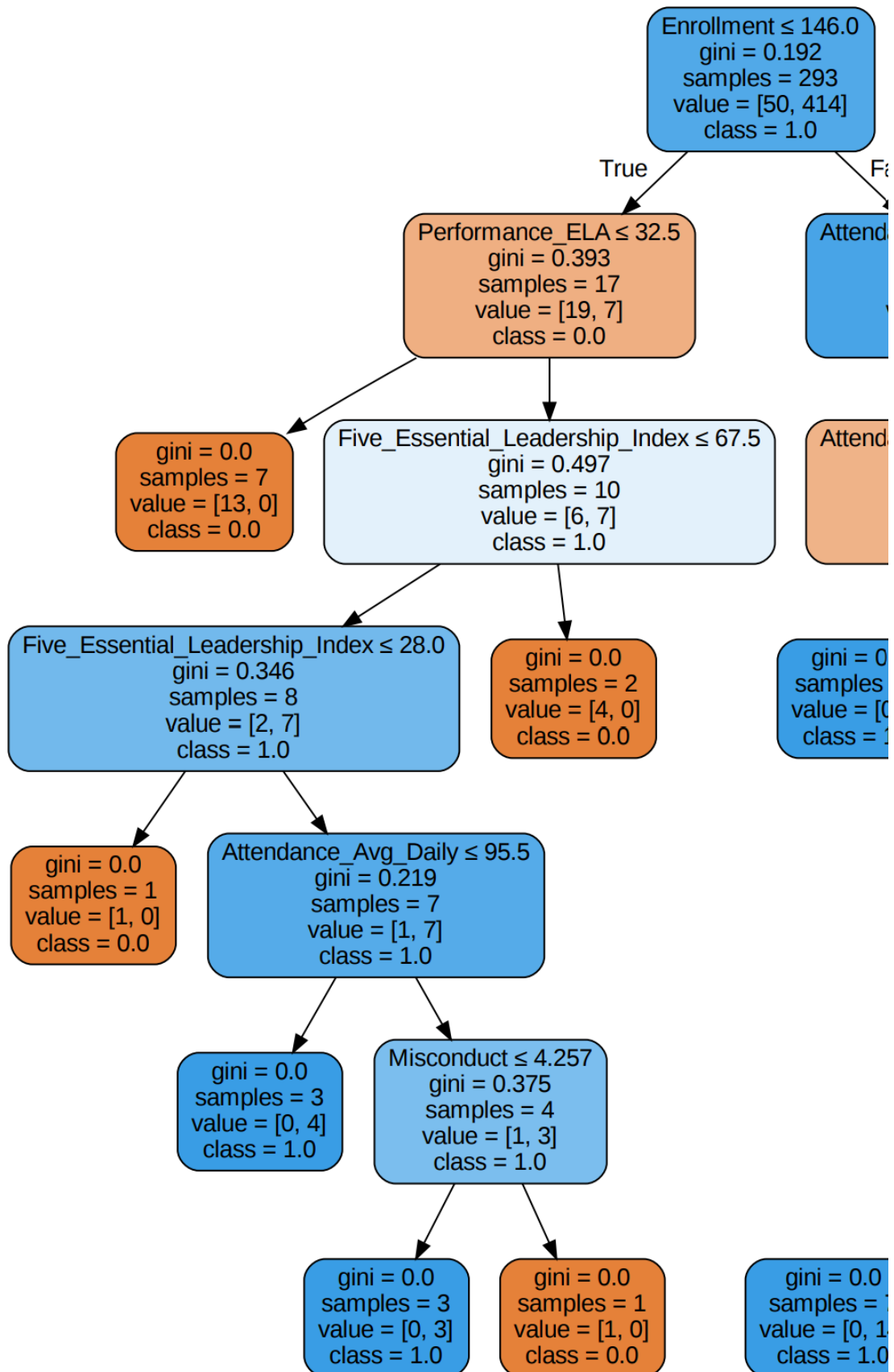


Figure 4: Leftmost Branch of Random Forest

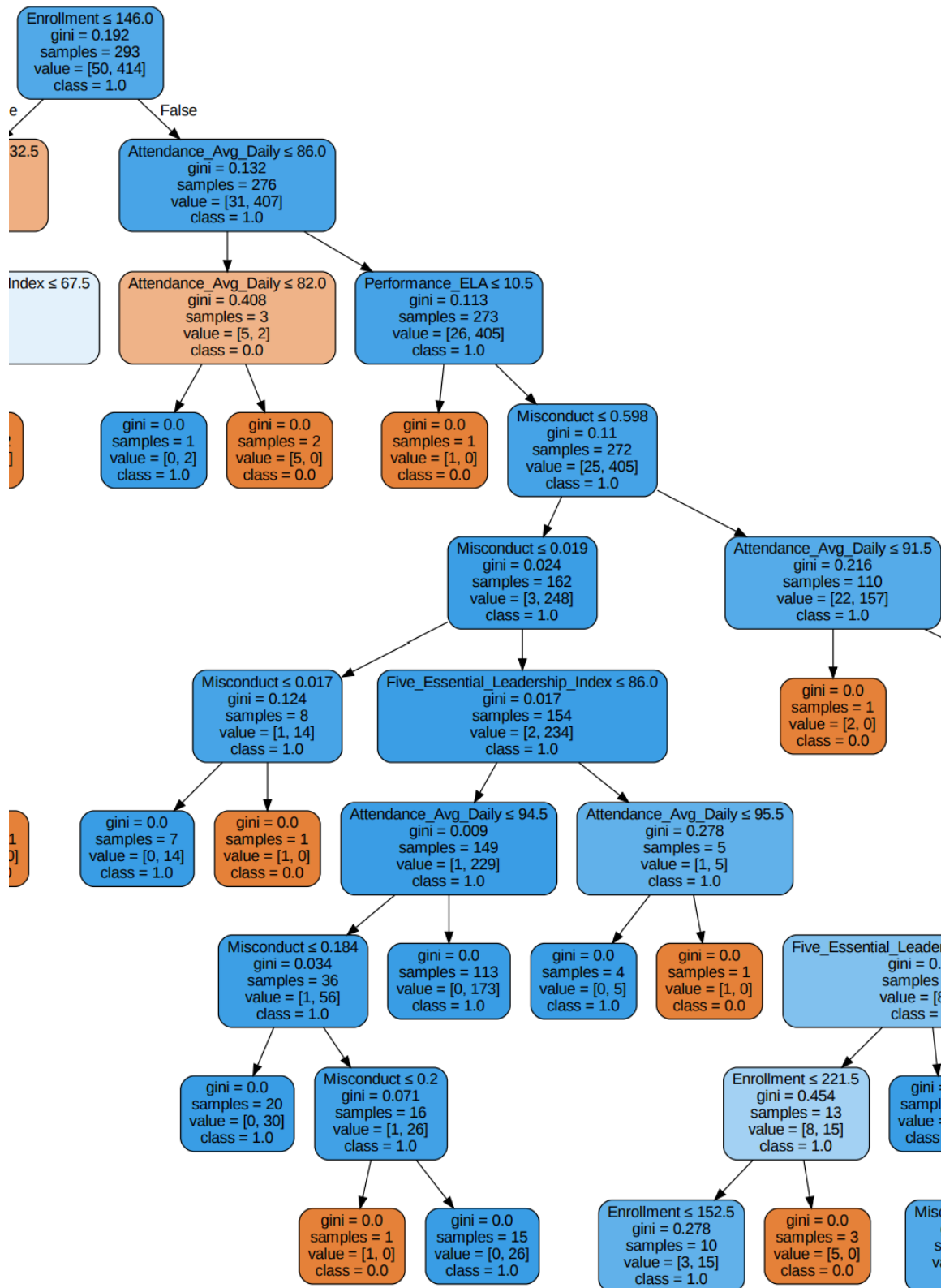




Figure 6: Rightmost Branch of Random Forest

B Code Used

```
1 # Initialize Google Colab Drive
2 from google.colab import drive
3 drive.mount('/content/drive')
4
5 # The path where all data is found
6 path = 'drive/MyDrive/MTFCFiles/'
7
8
9
10 #Import Libraries
11 import graphviz
12 import math
13 import matplotlib.pyplot as plt
14 import numpy as np
15 import pandas as pd
16 from scipy.optimize import curve_fit, fsolve
17 from scipy.signal import find_peaks
18 from sklearn.ensemble import RandomForestClassifier, RandomForestRegressor
19 from sklearn.feature_selection import SelectFromModel
20 from sklearn.inspection import partial_dependence, permutation_importance,
    PartialDependenceDisplay
21 from sklearn.linear_model import LinearRegression
22 from sklearn.metrics import accuracy_score, r2_score
23 from sklearn.model_selection import train_test_split
24 from sklearn.tree import export_graphviz
25
26 plt.style.use('ggplot')
27
28
29
30 # Our dependent variables
31 target_list = ['Enrollment', 'Attendance_Avg_Daily', '
    Five_Essential_Leadership_Index', 'Misconduct', 'Performance_ELA', '
    Performance_Math']
32
33
34 # Clean the data
35 def clean(year):
36
37
38     #File path by year
39     file = path+year+".csv"
40     df = pd.read_csv(file)
41     keep_rows = []
42
43
44     # Fill in holes with average of the column and convert all the strings to floats
45     for col in target_list:
46         count = 0
47         tot = 0
48         for idx, value in df[col].items():
```

```

49     if isinstance(value, str):
50         if value == '--' or math.isnan(int(value)):
51             df[col][idx] = float("nan")
52             continue
53         value = int(value)
54         df[col][idx] = value
55     if(math.isnan(value)):
56         continue
57     tot+=int(value)
58     df[col][idx] = int(value)
59     count+=1
60     keep_rows.append(idx)
61 for idx, value in df[col].items():
62     if math.isnan(value):
63         df[col][idx]=tot/count
64
65
66 # However, we use the drop row strategy, because it is more dependable
67 # Drop rows strategy instead of avg strategy
68 df = df.loc[df.index.isin(keep_rows)]
69
70
71 # Just in case AVG Class Size is hole
72 for idx, value in df["Avg_Class_Size"].items():
73     df = df.dropna(subset=["Avg_Class_Size"])
74
75
76 # Divide Misconduct by enrollment because bigger schools have a higher
77 # misconduct
78 df['Misconduct'] = df['Misconduct']/df["Enrollment"]
79
80 # Create a new vacant col based on the Avg_Class_Size
81 Vacant_Col = []
82 for index, row in df.iterrows():
83
84
85     # Only if class size is > 18, there is a teacher vacancy
86     if row["Avg_Class_Size"] > 18:
87         Vacant_Col.append(1)
88     else:
89         Vacant_Col.append(0)
90 df["Is_Vacant"] = Vacant_Col
91 return df
92
93
94 # Display the Random Forest Feature Importance and Permutation Feature Importance
95 # for each year. Then display the Partial Dependency.
96 def dispYearRandom(year):
97
98     # Get a clean dataset first
99     df = clean(year)

```

```
100
101
102 # Dependent factors and independent variable
103 X = df[target_list]
104 y = df["Is_Vacant"]
105 X = X.astype('float')
106 y = y.astype('float')
107
108
109 # Split the data into train and test
110 X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,
    random_state=0)
111
112
113 # Train the random forest model
114 clf = RandomForestClassifier(n_estimators=100, random_state=0)
115 clf.fit(X_train, y_train)
116
117 # Visualize the forest
118 dot_data = export_graphviz(clf.estimators_[0], out_file=None,
    feature_names=target_list,
119                             class_names=[str(i) for i in clf.classes_],
120                             filled=True, rounded=True,
121                             special_characters=True)
122
123
124 graph = graphviz.Source(dot_data)
125 graph.render('Decision_Tree')
126
127 # Predict the class for each X value
128 y_pred = clf.predict(X_test)
129
130
131 # Print the accuracy of the model
132 accuracy = accuracy_score(y_test, y_pred)
133 print("Accuracy:", accuracy)
134
135
136 # Determine the feature importances and display them
137 importance = clf.feature_importances_
138 feat_importances = pd.Series(clf.feature_importances_, index=X.columns)
139 feat_importances.nlargest(20).plot(kind='barh')
140 plt.xlabel("Random Forest Feature Importance")
141 feature_importances = [(feature, round(importance, 2)) for feature, importance
    in zip(target_list, importance)]
142
143 # Print the error values
144 print('Mean Absolute Error:', metrics.mean_absolute_error(y_test, y_pred))
145 print('Mean Squared Error:', metrics.mean_squared_error(y_test, y_pred))
146 print('Root Mean Squared Error:', np.sqrt(metrics.mean_squared_error(y_test,
    y_pred)))
147 print("Data Size:", df.shape)
148 plt.title(year)
149 for index, value in enumerate(feat_importances.nlargest(20)):
```

```

150     plt.text(value, index, str(round(value, 2)))
151 plt.show()
152
153
154 # Calculate permutation feature importance
155 result = permutation_importance(clf, X, y, n_repeats=10, random_state=42)
156
157
158 # Plot the feature importance scores and the error bars
159 sorted_idx = result.importances_mean.argsort()
160 plt.barh(range(X.shape[1]), result.importances_mean[sorted_idx], xerr=result.
    importances_std[sorted_idx])
161 plt.yticks(range(X.shape[1]), [target_list[i] for i in sorted_idx])
162 plt.xlabel('Permutation Importance')
163 plt.title(year)
164
165
166 # Make sure the text fits
167 plt.xlim(right=max(result.importances_mean[sorted_idx])+0.01)
168
169 # Annotate the bar chart with correct values
170 for i, v in enumerate(result.importances_mean[sorted_idx]):
171     plt.text(v + 0.01, i, str(round(v, 3)))
172 plt.show()
173
174
175 # Plot the partial dependence graphs
176 # We want the partial dependency for all factors in our cumulative graph, but
    only the most important one for the other years
177 if year == "All Years":
178
179
180     # Partial Dependence Display
181     for i in range(sorted_idx.shape[0]-1,-1,-1):
182         fig, ax = plt.subplots()
183         disp = PartialDependenceDisplay.from_estimator(clf, X, features=[sorted_idx[
    i]], feature_names=target_list, ax=ax)
184         ax.set_xlabel(target_list[sorted_idx[i]])
185         plt.title(year)
186         plt.show()
187     else:
188
189
190     # Same here, Partial Dependence Display
191     fig, ax = plt.subplots()
192     disp = PartialDependenceDisplay.from_estimator(clf, X, features=[sorted_idx
    [-1]], feature_names=target_list, ax=ax)
193     ax.set_xlabel(target_list[sorted_idx[-1]])
194     plt.title(year)
195     plt.show()
196
197 # Print the scores and errors of each partial dependency
198 for i in result.importances_mean.argsort()[::-1]:

```



```

199     print(f'{target_list[i]:<8}: {result.importances_mean[i]:.3f} +/- {result.
200           importances_std[i]:.3f} (test score: {clf.score(X, y):.3f})')
201
202
203 # Too many warnings, let's get rid of them
204 import warnings
205 warnings.filterwarnings('ignore')
206
207 # Display for each year
208 dispYearRandom("All Years")
209 dispYearRandom("2014")
210 dispYearRandom("2015")
211 dispYearRandom("2016")
212 dispYearRandom("2017")
213 dispYearRandom("2018")
214 dispYearRandom("2019")
215
216
217
218 # Our equation
219 def Fourier(x, a, b, c, d, e, f, g, h):
220     return a + b*x + c*np.sin(d*x + e) + f*np.cos(d*x + e) + g*np.sin(h*x) + h*np.
        cos(g*x)
221
222
223 # Linear Regression and Fourier Transform
224 def LinearAndFourier(school, remove, data):
225     # Read Data
226     data = pd.read_csv(path+data)
227     # Remove the last n years
228     if remove > 0:
229         data = data[:-remove]
230     X = data['Years Since'].values.astype(float)
231     y = data[school].values.astype(float)
232
233     # Fit the linear regression
234     lr_model = LinearRegression().fit(X.reshape(-1, 1), y.reshape(-1, 1))
235     lr_score = lr_model.score(X.reshape(-1, 1), y.reshape(-1, 1))
236     print('Linear regression R-squared:', lr_score)
237
238     # Fit our Fourier Transform model
239     popt, pcov = curve_fit(Fourier, X, y)
240
241     X_future = np.concatenate((X, np.arange(X.max()+1, X.max()+21)))
242
243     # Generate predictions using the fitted function for both past and future
    years
244     y_pred1 = Fourier(X,*popt)
245     r2 = r2_score(y, y_pred1)
246     print("R-squared value:", r2)
247     y_pred2 = Fourier(X_future, *popt)
248

```

```

249 # Plot the data, the fitted curve, and the predicted values
250 plt.scatter(X, y)
251 plt.plot(X, y_pred2[:len(X)], color='red', label='Fitted curve')
252 plt.plot(X_future[len(X):], y_pred2[len(X):], color='green', linestyle='--',
253 label='Predicted values')
254 plt.plot(X_future.reshape(-1, 1), lr_model.predict(X_future.reshape(-1, 1)),
255 color='blue', label='Linear regression')
256 plt.xlabel('Years Since 1997')
257 plt.ylabel(school+ ' Student to Teacher Ratio')
258 plt.legend()
259 plt.title("Predictions for Future Student to Teacher Ratio for " + school)
260 plt.show()
261 m = lr_model.coef_[0][0]
262 b = lr_model.intercept_[0]
263
264 # Print the equation of the line
265 print("Equation of the line: y = {:.2f}x + {:.2f}".format(m, b))
266
267
268 # Remove 3 years due to COVID
269 # Generate Elementary and High School Graphs
270 LinearAndFourier('Elementary',3,'97-22.csv')
271 LinearAndFourier('High School',3,'97-22.csv')
272
273 # Determine the High Risk and Low Risk school
274 df = clean('2019')
275
276 # The ranges of peaks and valleys in our partial dependence plots
277 minHigh = [[650],[93],[0],[0],[50],[45,60],[95000]]
278 maxHigh = [[1200],[97],[20],[1],[70],[55,70],[98000]]
279 minLow = [[200],[89,98],[80],[1],[0],[0,95,55],[104000]]
280 maxLow = [[320],[93,100],[100],[1.30],[25],[20,100,58],[107000]]
281
282 for index, row in df.iterrows():
283     inFullRange = True
284     for i in range(6):
285         inRange = False
286         for x in range(len(minHigh[i])):
287
288             # Check if it is in range for high risk
289             inRange |= row[target_list[i]] >= minHigh[i][x] and row[target_list[i]] <=
maxHigh[i][x]
290         inFullRange &= inRange
291
292 # If the school satisfies all ranges, print it out as high risk
293 if inFullRange:
294     print("High Risk: ",row)
295
296 # Same but for low risk
297 inFullRange = True
298 for i in range(6):

```

```
299     tot = 0
300     inRange = False
301     for x in range(len(minLow[i])):
302         inRange |= row[target_list[i]] >= minLow[i][x] and row[target_list[i]] <=
maxLow[i][x]
303     inFullRange &= inRange
304     if inFullRange:
305         print("Low Risk: ", row)
```