

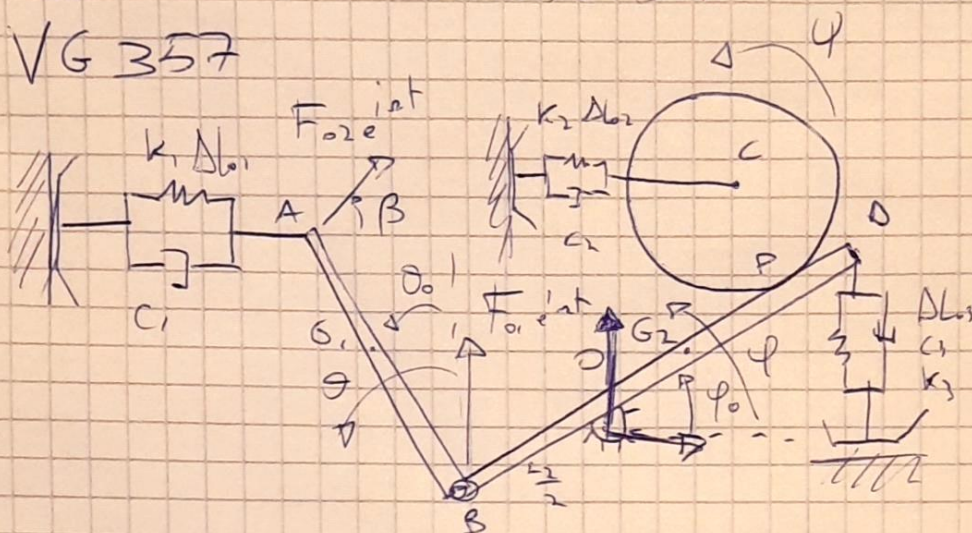
Giulio Vettori 966357  
10582927

11/02/2021

PAGINA 1

(+)  
→

VG 357



$$\underline{x} = \begin{Bmatrix} \theta \\ \varphi \\ \psi \end{Bmatrix}$$

$$\underline{x}_0 = \begin{Bmatrix} \theta_0 \\ \varphi_0 \\ 0 \end{Bmatrix}$$

$$T = \frac{1}{2} m_2 \dot{v}_{G2}^2 + \frac{1}{2} I_2 \dot{\varphi}^2 + \frac{1}{2} m_1 \dot{x}_{G1}^2 + \frac{1}{2} m_1 \dot{y}_{G1}^2 + \frac{1}{2} I_1 \dot{\theta}^2 + \frac{1}{2} \dot{x}_C^2 m_3 + \frac{1}{2} m_3 \dot{y}_C^2 + \frac{1}{2} I_3 \dot{\psi}_{ass}^2$$

$$Q = \frac{1}{2} \sum_{i=1}^3 c_i \Delta L_i^2 \quad V_{el} = \frac{1}{2} \sum_{i=1}^3 k_i \Delta L_i^2$$

$$V_g = m_1 h_{G1} + m_2 h_{G2} + m_3 h_C$$

$$\delta W = F_{02} e^{i\omega t} \cos(\beta) \delta x_A + F_{02} e^{i\omega t} \sin(\beta) \delta y_A + F_{01} e^{i\omega t} \delta y_B$$

$$v_{G2} = \frac{L_2}{2} \dot{\varphi} \quad x_{G1} = -\frac{L_2}{2} \cos(\varphi) - L_1 \sin(\theta)$$

$$y_B = -\frac{L_2}{2} \sin(\varphi) \quad y_{G1} = -\frac{L_2}{2} \sin(\varphi) + L_1 \cos(\theta)$$

$$x_A = -\frac{L_2}{2} \cos(\varphi) - 2L_1 \sin(\theta)$$

$$\psi_{ass} = \varphi + \theta$$

$$y_A = -\frac{L_2}{2} \sin(\varphi) + 2L_1 \cos(\theta)$$

$$y_D = \frac{3}{2} L_2 \sin(\varphi)$$

$$x_C = (2 - \varphi R) \cos(\varphi) - R \sin(\varphi)$$

$$y_C = (2 - \varphi R) \sin(\varphi) + R \cos(\varphi)$$



$$\Delta L_1 = -(\dot{y}_A - \dot{y}_B)$$

$$\Delta L_2 = x_C - x_B$$

$$\Delta L_3 = y_B - y_{B0}$$

$$[-L_m] = \begin{bmatrix} \theta & \varphi & \varphi \\ -L_1 \cos(\theta_0) & \frac{L_2}{2} \sin(\varphi_0) & 0 \\ -L_1 \sin(\theta_0) & -\frac{L_2}{2} \cos(\varphi_0) & 0 \\ 1 & 0 & 0 \\ \cancel{L_1/2} 0 & L_2/2 & 0 \\ 0 & 1 & 0 \\ 0 & -a \sin(\varphi_0) - R \cos(\varphi_0) & -R \cos(\varphi_0) \\ 0 & +a \cos(\varphi_0) - R \sin(\varphi_0) & -R \sin(\varphi_0) \\ 0 & 1 & 1 \end{bmatrix} \begin{matrix} x_{B0} \\ y_{B0} \\ \dot{\theta} \\ v_{B0} \\ \dot{\varphi} \\ x_C \\ y_C \\ \varphi_{ass} \end{matrix}$$

$$[m] = \text{diag}([m_1, m_1, J_1, m_2, J_2, m_3, m_3, J_3])$$

$$[-L_c] = [-L_k] = \begin{bmatrix} \theta & \varphi & \varphi \\ +2L_1 \cos(\theta_0) & -\frac{L_2}{2} \sin(\varphi_0) & 0 \\ 0 & -a \sin(\varphi_0) - R \cos(\varphi_0) & -R \cos(\varphi_0) \\ 0 & \frac{3}{2} L_2 \cos(\varphi_0) & 0 \end{bmatrix} \begin{matrix} \Delta L_1 \\ \Delta L_2 \\ \Delta L_3 \end{matrix}$$

$$[-L_a] = \begin{bmatrix} \theta & \varphi & \varphi \\ 0 & -\frac{L_2}{2} \cos(\varphi_0) & 0 \\ -2L_1 \cos(\theta_0) & \frac{L_2}{2} \sin(\varphi_0) & 0 \\ -2L_1 \sin(\theta_0) & -\frac{L_2}{2} \cos(\varphi_0) & 0 \end{bmatrix} \begin{matrix} F_{01} \\ F_{02} \cos(\beta) \\ F_{02} \sin(\beta) \end{matrix} \begin{matrix} y_B \\ x_A \\ y_A \end{matrix}$$



$$[K_{el I}] = K_1 \Delta L_{01}$$

$$\begin{bmatrix} -2L_1 \sin(\theta_0) & 0 & 0 \\ 0 & -\frac{L_2}{2} \cos(\varphi_0) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $\theta$   
 $\varphi$   
 $\psi$ 

$$[K_{el II}] = K_2 \Delta L_{02}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -2\cos(\varphi_0) + R \sin(\varphi_0) & R \sin(\varphi_0) \\ 0 & R \sin(\varphi_0) & 0 \end{bmatrix}$$

 $\theta$   
 $\varphi$   
 $\psi$ 

$$[K_{el III}] = K_3 \Delta L_{03}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{3}{2} L_2 \sin(\varphi_0) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $\theta$   
 $\varphi$   
 $\psi$ 

$$h_{G1} = Y_{G1}$$

$$[K_{G1}] = m_1 g$$

$$\begin{bmatrix} -L_1 \cos(\theta_0) & 0 & 0 \\ 0 & \frac{L_2}{2} \sin(\varphi_0) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $\theta$   
 $\varphi$   
 $\psi$ 

$$h_{G2} = Y_{G2} = \frac{L_2}{2} \sin(\varphi)$$

$$[K_{G2}] = m_2 g$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{L_2}{2} \sin(\varphi_0) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

 $\theta$   
 $\varphi$   
 $\psi$



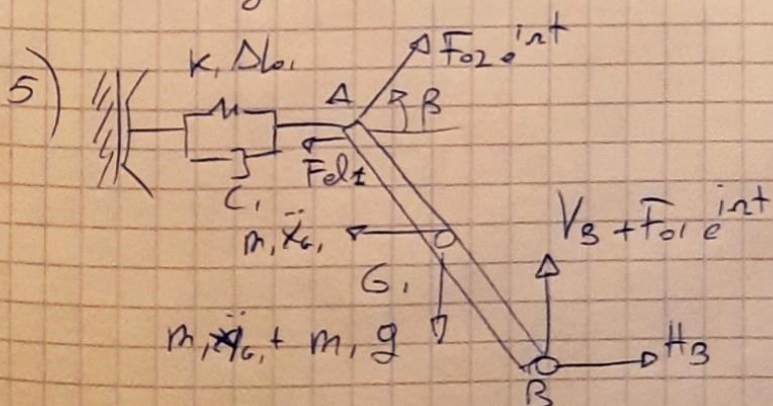
$$h_c = Y_c$$

$$[K_{G3}] = m_3 g \begin{bmatrix} 0 & \varphi & \varphi \\ 0 & 0 & 0 \\ 0 & -\cos(\varphi_0) R \cos(\varphi_0) & -R \cos(\varphi_0) \\ 0 & -R \cos(\varphi_0) & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \varphi \\ \varphi \end{bmatrix}$$

I didn't write any evidence on the procedure, I just differentiated by what I wrote on the matrices' sides.

3) Absolute rotation:  $\varphi_{\text{ass}} = \varphi + \varphi$   
Vertical disk displacement:  $Y_c$

4) Horizontal AB displ:  $X_{G1}$   
Elastic force  $\Delta L_3$ :  $F_{el3} = (k_3 + i \Omega c_3) \Delta L_3$



$$\sum F_{x_{AB}} = 0 \quad H_B + F_{o2}^{int} \cos(\beta) = F_{el1} + m_1 \ddot{X}_{G1} + F_{el01}$$

$$H_B = F_{el1} + m_1 \ddot{X}_{G1} - F_{o2}^{int} \cos(\beta) + F_{el01}$$

$$\sum F_{y_{AB}} = 0 \quad V_B + F_{o1}^{int} + F_{o2}^{int} \sin(\beta) = m_1 g + m_1 \ddot{Y}_{G1}$$

$$V_B = m_1 g + m_1 \ddot{Y}_{G1} - F_{o1}^{int} - F_{o2}^{int} \sin(\beta)$$