Ex. 3.1.1. b2c

b) $(-1,0),\ (2,1),\ (3,1),\ (5,2)$

*mixed up l's, sorry if it's hard to follow

$$l_1(x) = \frac{(x+1)(x-2)(x-3)}{(5+1)(5-2)(5-3)} \qquad l_2(x) = \frac{(x+1)(x-2)(x-5)}{(3+1)(3-2)(3-5)}$$

$$l_3(x) = \frac{(x+1)(x-3)(x-5)}{(2+1)(2-3)(2-5)} \qquad l_4(x) = \frac{(x-2)(x-3)(x-5)}{(-1-2)(-1-3)(-1-5)}$$

$$p(x) = 2 \cdot l_1 + 1 \cdot l_2 + 1 \cdot l_3 + 0 \cdot l_4$$

$(x^2 - x - 2)(x-3)$

$x^3 - x^2 - 2x - 3x^2 + 3x + 6$

$(x^2 - x - 2)(x-5)$

$x^3 - x^2 - 2x - 5x^2 + 5x + 10$

$(x^2 - 2x - 3)(x-5)$

$x^3 - 2x^2 - 3x - 5x^2 + 10x + 15$

$$\boxed{p(x) = \frac{2}{36}(x^3 - 4x^2 + x + 6) - \frac{1}{8}(x^3 - 6x^2 + 3x + 10) + \frac{1}{9}(x^3 - 7x^2 + 7x + 15)}$$

*checked with desmos

c) $$l_1(x) = \frac{(x-2)(x-4)}{(0-2)(0-4)} \qquad l_2 = \frac{(x-0)(x-4)}{(2-0)(2-4)} \qquad l_3 = \frac{(x-0)(x-2)}{(4-0)(4-2)}$$

$$P(x) = -2 \cdot l_1 + 1 \cdot l_2 + 4 \cdot l_3$$

$$\boxed{P(x) = -\frac{1}{4}(x^2 - 6x + 8) - \frac{1}{4}(x^2 - 4x) + \frac{1}{2}(x^2 - 2x)}$$

*checked with desmos

Ex. 3.1.2. b2c

$$\frac{1-0}{2-(-1)} \qquad \frac{2-1}{5-3} \qquad \frac{0-\frac{1}{3}}{3-(-1)} \qquad \frac{-\frac{1}{3}}{4} \qquad \frac{\frac{1}{2}}{3}$$

b)

-1	0			
2	1	$\frac{1}{3}$		
3	1	0	$-\frac{1}{12}$	
5	2	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{3}{72}$

$$p(x) = 0 + \frac{1}{3}(x+1) - \frac{1}{12}(x+1)(x-2) + \frac{3}{72}(x+1)(x-2)(x-3)$$

c)

0	-2		
2	1	$\frac{3}{2}$	
4	4	$\frac{3}{2}$	0

$\dfrac{1-(-2)}{2-0}$ $\qquad \dfrac{4-1}{4-2}$ $\qquad \dfrac{\frac{3}{2}-\frac{3}{2}}{}$

$P(x) = -2 + \frac{3}{2}(x-0)$

EX. 3.1.4

a)

0	0			
1	1	1		
2	2	1	0	
3	7	5	2	$\frac{2}{3}$

$x^2 - 3x + 2$

$x^3 - 3x^2 + 2x$

$P_3(x) = 0 + 1(x-0) + 0(x-0)(x-1) + \frac{2}{3}(x-0)(x-1)(x-2)$

$\boxed{= \frac{2}{3}x^3 - 2x^2 + \frac{7}{3}x}$

b) $P_4(x) = \frac{2}{3}x^3 - 2x^2 + \frac{7}{3}x + 1(x-0)(x-1)(x-2)(x-3)$

$P_4(x) = \frac{2}{3}x^3 - 2x^2 + \frac{7}{3}x + 2(x-0)(x-1)(x-2)(x-3)$

c) There doesn't exist a polynomial with degree 3 or less containing the additional point as part a gives a unique solution with degree 3, not containing (4, 2), and adding this point would result in degree 4.

EX. 3.1.6

1	1			
2	3	2		
3	3	0	-1	
4	4	1	$\frac{1}{2}$	$\frac{1}{2}$

$P_3(x) = 1 + 2(x-1) - 1(x-1)(x-2) + \frac{1}{2}(x-1)(x-2)(x-3)$

$\boxed{P_5(x) = 1 + 2(x-1) - 1(x-1)(x-2) + \frac{1}{2}(x-1)(x-2)(x-3) + x(x-1)(x-2)(x-3)(x-4)}$

Ex. 3.1.8

$$P(x) = 112 \cdot l_1 + 0 \cdot l_2 + 0 \cdot l_3 + 0 \cdot l_4 + \dots + 0 \cdot l_9 + 2 \cdot l_{10}$$

$$l_1 = \frac{(x-2)(x-3)\dots(x-10)}{(1-2)(1-3)\dots(1-10)}$$

$$l_{10} = \frac{(x-1)(x-2)\dots(x-9)}{(10-1)(10-2)\dots(10-9)}$$

$$P(0) = 112 \cdot \frac{-10!}{-9!} + 2\left(\frac{-9!}{9!}\right) = 1120 - 2 = \boxed{1118}$$

Ex. 3.1.10

$$P(x) = 10 \cdot l_1 + 10 \cdot l_2 + 10 \cdot l_3 + 10 \cdot l_4 + 10 \cdot l_5 + 15 \cdot l_6$$

$$l_1 = \frac{(x-2)(x-3)(x-4)(x-5)(x-6)}{(1-2)(1-3)(1-4)(1-5)(1-6)} \quad \text{at } p(\emptyset): \frac{5!}{-5!} = -1$$

$$l_2 = \frac{(x-1)(x-3)(x-4)(x-5)(x-6)}{(2-1)(2-3)(2-4)(2-5)(2-6)} \quad \text{at } P(\emptyset): \frac{-6!}{5 \cdot 4!} = 6$$

$$l_3 = \frac{(x-1)(x-2)(x-4)(x-5)(x-6)}{(3-1)(3-2)(3-4)(3-5)(3-6)} \quad \text{at } P(\emptyset): \frac{6!}{4 \cdot 2 \cdot -3!} = -15$$

$$l_4 = \frac{(x-1)(x-2)(x-3)(x-5)(x-6)}{(4-1)(4-2)(4-3)(4-5)(4-6)} \quad \text{at } P(\emptyset): \frac{-6!}{3 \cdot 12} = 20$$

$$l_5 = \frac{(x-1)(x-2)(x-3)(x-4)(x-6)}{(5-1)(5-2)(5-3)(5-4)(5-6)} \quad \text{at } P(\emptyset): \frac{6!}{2 \cdot -4!} = -15$$

$$l_6 = \frac{(x-1)(x-2)(x-3)(x-4)(x-5)}{(6-1)(6-2)(6-3)(6-4)(6-5)} \quad \text{at } P(\emptyset): \frac{6!}{5!} = 6$$

$$P(7) = 10(-1 + 6 - 15 + 20 - 15) + 15(6) = \boxed{40}$$

SW

0	1
$\pi/6$	$\sqrt{3}/2$ -0.255873
$\pi/3$	$1/2$ -0.699057 -0.423209
$\pi/2$	0 $-3/\pi$ -0.244340 0.113871

EX. 3.2.2

a)

1	0		
2	$\ln(2)$	$\ln(2)$	$\dfrac{\frac{\ln(4)-\ln(2)}{2} - \ln(2)}{3}$
4	$\ln(4)$	$\dfrac{\ln(4)-\ln(2)}{2}$	

$$\boxed{p(x) = \ln(2)(x-1) + \dfrac{\frac{\ln(4)-\ln(2)}{2} - \ln(2)}{3}(x-1)(x-2)}$$

b) $\ln(2)(2) + \dfrac{\frac{\ln(2)}{2} - \ln(2)}{3}(2)(1) \approx \boxed{1.1552}$

c) $f(x) - P(x) = \dfrac{(x-x_1)(x-x_2)\dots(x-x_n)}{n!} f^{(n)}(c)$

$|f(x) - P(x)| \le \frac{2}{6} |(x-1)(x-2)(x-4)| \rightarrow$ at $x=3$: $\boxed{|f(x)-P(x) \le \frac{2}{3}}$

$f(x) = \ln(x)$ $\quad f'''(x) = 2x^{-3}$: maximized at $c=1$

$f'(x) = \frac{1}{x}$ $\quad$ (assuming interval $1 \le c < 4$)

$f''(x) = -x^{-2}$ $\quad 2(1)^{-3} = 2$

d) actual error: 0.0566

actual error < error bound

$\boxed{0.0566 < 2/3}$