Q1-2

$\forall n \in \mathbb{N}, \sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

Base case

$n=1$

$\sum_{k=1}^{1} 1(1+1) = 2 \qquad \frac{1(1+1)(1+2)}{3} = 2$

Thus, the statements are equal for our base case.

Inductive step

Assume that the statement holds for an arbitrary $n \in \mathbb{Z}$ \$ $n \geq 1$.

$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

Now suppose we have $n+1$:

$\sum_{k=1}^{n+1} k(k+1) = \frac{(n+1)(n+2)(n+3)}{3}$

Suppose we had a m such that $m=n+1$

$\sum_{k=1}^{m} k(k+1) = \frac{m(m+1)(m+2)}{3}$

Conclusion

Therefore, the base case holds, and the inductive step holds proving our statement for any $n \geq 1, n \in \mathbb{Z}$.

Q1-3 Counterfeit coin among 3^n coins takes n weighings.

Base case

$n = 1$

- Two coins can be placed on the scale, if the coins balance then it's the third coin.
- Here, we actually would need to know if the counterfeit was lighter/heavier because if the two coins didn't balance, we wouldn't know which coin it was off one weighing. (*** assumption)

Inductive step

$n+1$ for some arbitrary $n \in \mathbb{Z}, n > 0$.

- Suppose n holds; that is, we can find the counterfeit among 3^n coins in n weighings. Now, we have $n+1$ where 3^{n+1} must hold for $n+1$ weighings. ~~That is~~ Therefore, $3^{n+1} = 3^n \cdot 3$; now, we must prove we can find the coin in our 3 larger groups.
- The same assumption holds as above, we'd need to know lighter/heavier. To find the coin in the 3 larger groups we can do the same thing as above; weigh two groups, if they are equal weight then the coin is in the third group, otherwise it's in one of the two groups.

Conclusion

Our base case holds, and our inductive step holds such that 3^n coins takes n weighings - assuming we know if the counterfeit coin is lighter/heavier.