# Calculus 1 Review:

## Limits, Continuity:

$$\lim_{x\to2} \frac{x^2+7x+6}{x+2} = \frac{24}{4} = 6$$

$$\lim_{x\to3} \frac{x^2+2x-15}{x^2-9} = \frac{(x+5)\cancel{(x-3)}}{(x+3)\cancel{(x-3)}} = \frac{8}{6} = \frac{4}{3}$$

$$\lim_{x\to4} \frac{\left(\frac{1}{x}-\frac{1}{4}\right)}{(x-4)} = \frac{\left(\frac{1}{x}-\frac{1}{4}\right)4x}{(x-4)4x} = \lim_{x\to4} \frac{(\cancel{4-x})}{-(\cancel{4-x})(4x)} = \frac{1}{-4x} = -\frac{1}{16}$$

$$\lim_{x\to16} \frac{(\sqrt{x}-4)}{(x-16)} = \frac{(\sqrt{x}-4)}{(x-16)}\frac{(\sqrt{x}+4)}{(\sqrt{x}+4)} = \frac{x+\cancel{4\sqrt{x}-4\sqrt{x}}-16}{\cancel{(x-16)}(\sqrt{x}+4)} = \frac{1}{\sqrt{x}+4} = \frac{1}{8}$$

$$\lim_{x\to7} \frac{|x-7|}{x-7} \quad \lim_{x\to7^-} \frac{|x-7|}{x-7} = -1 \quad \lim_{x\to7^+} \frac{|x-7|}{x-7} = 1 \quad \text{DNE}$$

$$\lim_{x\to0} \frac{\tan(3x)}{5x} = \lim_{x\to0} \frac{\sin(3x)}{\cos(3x)} \cdot \frac{1}{5x} \cdot \frac{3}{3} = \lim_{x\to0} \frac{\sin(3x)}{3x} \cdot \lim_{x\to0} \frac{3}{5} \cdot \frac{1}{\cos(3x)}$$

↳ $y=3x$ $\lim_{y\to0} \frac{\sin y}{y} \cdot \lim_{x\to0} \frac{1}{\cos(3x)} \cdot \frac{3}{5}$

↳ $1 \cdot 1 \cdot \frac{3}{5} = \frac{3}{5}$

## Horizontal asymptote:

$f(x) = \frac{5x-8x^2}{3+2x^2} + 5$ $\qquad \lim_{x\to0} x\sin\left(\frac{1}{x}\right) = 0$

$$\lim_{x\to\infty} \frac{5x+8x^2}{3+2x^2} + 5 = 9$$

## IVT:

$f(x) = x^2+4x-5$ $\quad [a,b]$ $\quad f(a) \neq f(b)$ $\quad f(a) < k < f(b)$

$f(c) = 0$ $\quad [0,2]$ $\quad f(c) = k$

$f(a) = -5 < 0 < 7$

$0 = x^2+4x-5$

$(x+5)(x-1)$

$x = -5, \boxed{1}$

Value of c that will make the function continuous at $x=2$:

$$f(x) = \begin{cases} 7x^2+cx & x<2 \\ 2x^3+5c+3 & x\geq2 \end{cases}$$

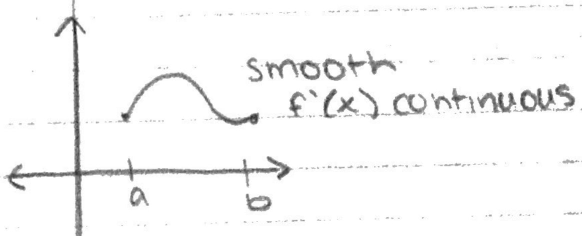
$7x^2+cx = 2x^3+5c+3$

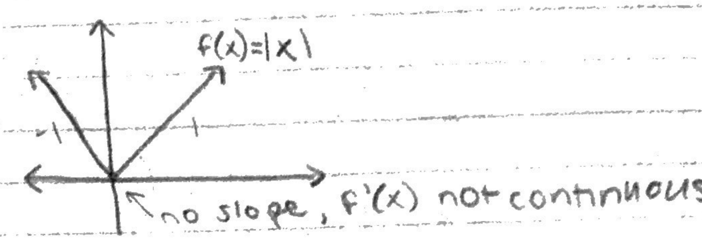
$28+2c = 16+5c+3$ $\quad c=3$

## Calculus I Review:

**Continuity:** f(x) continuous

**Differentiability:** f'(x) continuous




**Derivatives:**

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \qquad f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$$f(x)=x^3 \quad f'(x)=3x^2 \quad f'(x) = \lim_{h\to 0}\frac{(x+h)^3-x^3}{h} = \frac{(\cancel{x+h-x})[(x+h)^2+x(x+h)+x^2]}{\cancel{h}}$$

$$\hookrightarrow \lim_{h\to 0} (x+h)^2 + x(x+h) + x^2 = (x+0)^2 + x(x+0) + x^2 = 3x^2$$

$$f'(4) = \lim_{x\to 4}\frac{x^3-4^3}{x-4} = \frac{(\cancel{x-4})(x^2+4x+4^2)}{\cancel{x-4}} = 16+16+16 = 48$$

**Power Rule:**

$$\frac{d}{dx}(x^n) = nx^{n-1} \qquad \frac{d}{dx}(x^3) = 3x^2 \qquad \frac{d}{dx}(5x^2) = 5\cdot\frac{d}{dx}(x^2) = 10x$$

**Trigonometric**

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x \qquad \frac{d}{dx}[\sec x] = \cancel{\sec x \tan x}$$

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{d}{dx}[\cot x] = -\csc^2 x \qquad \frac{d}{dx}[\csc x] = -\csc x \cot x$$

**Product Rule**

$$[f\cdot g]' = f'\cdot g + f\cdot g' \qquad \frac{d}{dx}[x^2 \sin x] = x^2\cos x + 2x\sin x$$

**Quotient Rule**

$$\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf'-fg'}{g^2} \qquad \frac{d}{dx}\left[\frac{5x+6}{3x-7}\right] = \frac{(3x-7)(5)-(5x+6)(3)}{(3x-7)^2}$$

## Calculus I Review:

### Chain Rule:

$$\frac{d}{dx}\left(f[g(x)]\right) = f'[g(x)] \cdot g'(x)$$

$\frac{d}{dx}[u^n] = n[u]^{n-1} \cdot u'$ $\quad \frac{d}{dx}[5x+3]^4 = 4(5x+3)^3 \cdot 5$

### Exponential/Log/Inverse Trig:

$\frac{d}{dx}[e^u] = e^u \cdot u'$ $\quad \frac{d}{dx}[a^u] = a^u \cdot u' \cdot \ln a$

$\frac{d}{dx}[\ln x] = \frac{1}{x}$ $\quad \frac{d}{dx}[\ln x^2] = \frac{d}{dx}[2\ln x]$

$\frac{d}{dx}[\log_a u] = \frac{u'}{u \ln a}$ $\quad \frac{d}{dx}[\sin^{-1}(u)] = \frac{u'}{\sqrt{1-u^2}}$ $\quad \frac{d}{dx}[\cos^{-1}(u)] = \frac{-u'}{\sqrt{1-u^2}}$

$\frac{d}{dx}[\tan^{-1} u] = \frac{u'}{1+u^2}$ $\quad \frac{d}{dx}[\sec^{-1}(u)] = \frac{u'}{|u|\sqrt{u^2-1}}$ $\quad \frac{d}{dx}[\csc^{-1}(u)] = \frac{-u'}{|u|\sqrt{u^2-1}}$

$\frac{d}{dx}[\cot^{-1} u] = \frac{-u'}{1+u^2}$

### Application of Derivatives:

$m_{sec} = \frac{f(b)-f(a)}{b-a}$ $\quad m_{tan} = f'(c)$

MVT  
Mean Value — $f(x)$ continuous & differentiable on $(a,b)$, $\exists c$ s.t. $f'(c) = \frac{f(b)-f(a)}{b-a}$

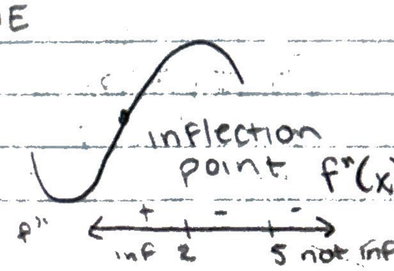
Rolle's — $\hookrightarrow$ … $f(a) = f(b)$ $\quad f'(c) = 0$

$\bar{v} = \frac{s(b)-s(a)}{b-a}$ $\quad \bar{a} = \frac{v(b)-v(a)}{b-a}$ $\quad v(t) = s'(t)$ $\quad a(t) = v'(t)$

crit point: $f'(c) = 0$ or $f'(c)$ DNE

concave up  
$f''(x) > 0$

concave down  
$f''(x) < 0$




inflection point $f''(x) = 0$

# Calculus I Review:

## Integrals:

$$\int [x^n]\,dx = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{x}{4}\,dx = \frac{1}{4}\int x\,dx = \frac{x^2}{8} + C$$

<u>Indefinite</u> $\int 6x^2\,dx$ <u>Definite</u> $\int_1^2 6x^2\,dx$ $\int_1^2 6x^2\,dx = 2x^3\Big|_1^2 = 16-2 = 14$

$$\int_a^b f(x)\,dx = F(b) - F(a) \qquad \int f(x)\,dx = F(x) + C$$

$[a,b]$ FTC

<u>U substitution</u>

$\int 4xe^{x^2}\,dx$ $u = x^2$

$du = 2x\,dx$ $dx = \frac{du}{2x}$

$$= \int 4xe^u \frac{du}{2x} = 2\int e^u\,du = 2e^u + C = 2e^{x^2} + C$$

<u>Fundamental Theorem of Calculus</u>

If $g(x) = \int_a^x f(t)\,dt$, $g'(x) = f(x)$

$\hookrightarrow \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

<u>Implicit Differentiation</u>

$\left[x^3 + y^3 = 8\right] \frac{d}{dx}\ \frac{dy}{dx}$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$

$3y^2 \cdot \frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = \frac{-x^2}{y^2}$

$\left[x^2 + 2xy + y^2 = 5\right] \frac{d}{dx}\ \frac{dy}{dx}$

$2x + 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 2y$

$\frac{dy}{dx} = -1$

# Calculus I Review:

## L'Hopital's Rule

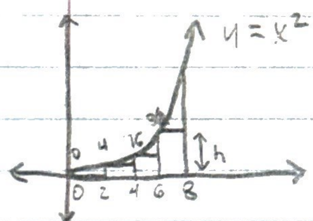
$$\lim_{x\to\infty} \frac{x^2}{e^x} = \frac{(\infty)^2}{e^\infty} = \frac{\infty}{\infty} \quad \to \quad \lim_{x\to\infty} \frac{2x}{e^x} = \frac{\infty}{e^\infty} = \frac{\infty}{\infty} \quad \to \quad \lim_{x\to\infty} \frac{2}{e^x} = \frac{2}{\infty} = 0$$

$$\to \lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

$$\lim_{x\to\infty} \frac{\sin(7x)}{\sin(4x)} = \lim_{x\to\infty} \frac{7\cos(7x)}{4\cos(4x)} = \frac{7}{4}$$

## Riemann Sum




$n=4$

$$\text{Area} = \sum_{i=1}^{n} \Delta x \, f(x_i) \qquad \Delta x = \frac{b-a}{n} = 2$$

Area = 112

$$A_L = \Delta x \left[ f(x_1) + f(x_2) + f(x_3) + f(x_4) \right] = 112 \qquad \frac{A_L + A_R}{2} = 176$$

$$A_R = \Delta x \left[ f(x_2) + f(x_3) + f(x_4) + f(x_5) \right] = 240$$

$$A = \int_a^b f(x)\,dx = \frac{x^3}{3}\Big|_0^8 = \frac{512}{3} = 170.\overline{66}$$