Optimization Review:
Linear Programming (LP) Formulations

Maximize/Minimize: $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to: $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \geq b_2$

$x_1, x_2, \dots, x_n \geq 0$

Objective Function: min $z = 2x_1 + 3x_2$

Constraints: $x_1 + 2x_2 \geq 8$

$2x_1 + x_2 \geq 6$ * Graph

$x_1, x_2 \geq 0$

Find feasible vertices and plug in

Simplex Method

Convert all constraints to equations by introducing slack variables ($\leq$, +)($\geq$, -)

Maximize: $7x_1 + 6x_2$ $\quad 7x_1 + 6x_2 + 0s_1 + 0s_2$

subject to: $2x_1 + 4x_2 \leq 16$ $\quad 2x_1 + 4x_2 + s_1 = 16$

$3x_1 + 2x_2 \leq 12$ $\quad 3x_1 + 2x_2 + s_2 = 12$

$x_1, x_2 \geq 0$ $\quad x_1, x_2, s_1, s_2 \geq 0$

	x_1	x_2	x_3	x_4		
Basis	7	6	0	0		
s_1 0	2	4	1	0	16	$16/2 = 8$
s_2 0	3	2	0	1	12	$12/3 = \boxed{4}$
z	0	0	0	0	0	
$c_j - z_j$	[7]	6	0	0		

R2/3
R1 − 2R2

	x_1	x_2	x_3	x_4		
Basis	7	6	0	0		
s_1 0	0	8/3	1	−2/3	8	[3]
x_1 7	1	2/3	0	1/3	4	6
z_j	7	14/3	0	7/3	28	
$c_j - z_j$	0	[4/3]	0	−7/3		

$R_1 \cdot 3/8$

$R_2 - \frac{2}{3}R_1$

	x_1	x_2	x_3	x_4	
Basis	7	6	0	0	
x_2 6	0	1	3/8	−1/4	3
x_1 7	1	0	−1/4	1/2	2
z	7	6	1/2	2	32
$c_j - z_j$	0	0	−1/2	−2	≤ 0 ✓

$x_1 = 2$

$x_2 = 3$

$s_1 = s_2 = 0$

Optimization Review:
Sensitivity Analysis

$\max \; z = 300x_1 + 200x_2$

s.t. $2x_1 + x_2 \leq 8$ (m_1)

$x_1 + 2x_2 \leq 8$ (m_2)

$x_i \geq 0 \; (i = 1, 2)$

slope $= -\frac{c_1}{c_2}$ $\quad \frac{1}{2} \leq \frac{c_1}{c_2} \leq 2$

* Suppose unit profit of Prod 1 and 2 are changed to 350 and 250. Will optimal solution remain the same?

$\frac{1}{2} \leq \frac{c_1}{c_2} = \frac{350}{250} = \frac{7}{5} \leq 2$ (YES)

* Suppose Prod 2 is 200, what is the range of Prod 2 to keep optimal solution unchanged?

$\frac{1}{2} \leq \frac{x}{200} \leq 2$

$100 \leq x \leq 400$

* Shadow price m_1

$= \frac{z_B - z_A}{(m_1 \text{ capacity})} = \frac{4400/3 - 4000/3}{9 - 8 \; (\text{change of 1 unit})} = \$133/\text{hr}$

* Shadow price m_2

$= \frac{4100/3 - 4000/3}{9-8} = \$33/\text{hr}$

* If we increase capacity of both machines, which should receive priority? (m_1)

* Should we increase availability of m_1 & m_2 for additional cost of \$50/hr? (m_1 yes, m_2 no)

Optimization Review:

Shadow price

Change in optimal objective value per unit increase in RHS constraint

Duality:

$$\min \; 5y_1 + 35y_2 + 20y_3$$
$$\text{s.t.} \; -y_1 + y_2 + y_3 \geq 2$$
$$y_1 + 3y_2 + 0y_3 \geq 3$$

$$y_1, y_2, y_3 \geq 0$$

Dual

$$\max \; 2z_1 + 3z_2$$
$$\text{s.t.} \; -z_1 + z_2 \leq 5$$
$$z_1 + 3z_2 \leq 35$$
$$z_1 \leq 20$$

$$z_1, z_2 \geq 0$$