Ex. 4.1.1.a

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \qquad A^T A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \Longleftrightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1/7 \\ 10/7 \end{bmatrix}}$$

$$r = b - Ax = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -1/7 \\ 10/7 \end{bmatrix} = \begin{bmatrix} -2/7 \\ -3/7 \\ -1/7 \end{bmatrix}$$

$$\boxed{\|r\|_2 = \sqrt{(14)/(49)}} \approx 0.5345$$

Ex. 4.1.1.c

$$\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \end{bmatrix} \qquad A^T A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 9 \\ 9 & 10 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 16 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} 10 & 9 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 16 \end{bmatrix} \Longleftrightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 16/19 \\ 16/19 \end{bmatrix}}$$

$$r = b - Ax = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 16/19 \\ 16/19 \end{bmatrix} = \begin{bmatrix} 9/19 \\ 25/19 \\ 9/19 \\ -26/19 \end{bmatrix}$$

$$\boxed{\|r\|_2 = \sqrt{(1463)/(361)}} \approx 2.0131$$

EX. 4.1.2.a

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 2 \\ 3 & 6 & 3 \\ 2 & 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 & 2 \\ 3 & 6 & 3 \\ 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \\ 9 \end{bmatrix} \Leftrightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1/3 \\ 2 \end{bmatrix}}$$

$$r \doteq b - Ax = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1/3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ -1/3 \\ 0 \end{bmatrix}$$

$$\boxed{RMSE = \sqrt{(1/3)/4} \approx 0.2887}$$

EX. 4.1.8.b

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 14 \\ 14 & 62 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \end{bmatrix}$$

$$\frac{76}{52} \quad \frac{38}{26} \quad \frac{19}{13}$$

$$\begin{bmatrix} 4 & 14 \\ 14 & 62 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \frac{1}{52} \begin{bmatrix} 62 & -14 \\ -14 & 4 \end{bmatrix} \begin{bmatrix} 8 \\ 30 \end{bmatrix}$$

$$= \frac{1}{52} \begin{bmatrix} 76 \\ 8 \end{bmatrix} \Rightarrow \boxed{y = \frac{19}{13} + \frac{2}{13} x}$$

$$r = b - Ax = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \frac{1}{52} \begin{bmatrix} 76 \\ 8 \end{bmatrix} = \begin{bmatrix} 20/52 \\ 4/52 \\ -56/52 \\ 32/52 \end{bmatrix}$$

$$\boxed{RMSE = \sqrt{(4576)/(10816)} \approx 0.6504}$$

Ex. 4.1.9.b

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

*Using python ↓↓

$$x = \begin{bmatrix} 2.962 \\ -1.013 \\ 0.167 \end{bmatrix} \qquad y = 0.167x^2 - 1.013x + 2.962$$

$RMSE \approx 0.416$

* The RMSE found for polynomial is 0.416 which is less than 0.6504 in the previous problem, which is to be expecting as adding a degree gives at least as good if not better solution than before.

CP. 4.2.6

a) 1D arrays, xx being x-values and yy being y-values

b) This creates the matrix A such that the first column is 1's and the second column is the values from xx where the number of rows corresponds to the number of values in xx, that is each row is an entry from xx.

c) This creates vector b containing ~~[illegible]~~ one column which is ln(yy) - ln(xx); thus same amount of rows as xx/yy as this calculation is made for corresponding indeces.

d) This line calculates the x vector such to minimize the 2-way, using matrix A and vector b.

e) This line is calculating y-values for the graph generated, based on x-range/ ~~[illegible]~~ interval in xxx, using the coefficients from x-vector at certain x-value.

f) c(t) is the concentration of the drug at hour t (ng/ml), where t is the time in hours since being administered.

g) We seem to have a good fit visually, and it seems to be a good model based on low RMSE = 0.0341.

h) The drug is hitting the patient's system (rapidly increasing) and then wears out of their system (slowly decreasing).

i) The drugs stays within therapeutic levels between hours ~0.5-19.

CP. 4. 2. 7

a) The array t represents the months, that is index i is a certain x (mod 12) to represent the month relative to 12 months in a year for the 5 years.

b) This creates matrix A where the first column is 1's, the second column is cos(2π · t) for each value in t, same for third column but sin(2π · t), same for fourth column but cos(4π · t), where each row is an entry of the columns for each time in t.

c) $f(t)$ is megawatts per hour at relative year t where t is years beginning at 2005 $(t=0)$. $t=1$ represents January 2006, April 2007 is represented by $t=2.25$

d) It doesn't seem like the best model based on points that are far off our model, however this could also be the nature of the data where outliers will make the model look worse than it ~~[illegible]~~ actually is.

CP.4.2.8

$$f(t) = 279 + e^{3.7167 + 0.0212(t-1961)}$$

CP.4.2.9

RMSE for $c_0 + c_1 t$: 2.2333

RMSE for $c_0 + c_1 t + c_2\cos(2\pi t) + c_3\sin(2\pi t)$: 0.8015

The RMSE for the second model is significantly lower $(0.8015 < 2.2333)$ which indicates a better model. Likewise, looking at the graphs it's pretty clear to see the second model is a better representation of the data.

Ex. 4.3.2.b

$$A = \begin{bmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{bmatrix} \qquad r_{00} = \sqrt{16+4+16} = 6 \qquad q_0 = \frac{1}{6}\begin{bmatrix} -4 \\ -2 \\ 4 \end{bmatrix}$$

$$r_{01} = \frac{1}{6}(16-14-20) = -3$$

$$w_1 = \begin{bmatrix} -4 \\ 7 \\ -5 \end{bmatrix} + 3\left(\frac{1}{6}\right)\begin{bmatrix} -4 \\ -2 \\ 4 \end{bmatrix}$$

$$r_{11} = \sqrt{(-6)^2 + (6)^2 + (3)^2} = 9$$

$$q_1 = \frac{1}{9}\begin{bmatrix} -6 \\ 6 \\ 3 \end{bmatrix}$$

$$Q = \begin{bmatrix} -2/3 & -2/3 \\ -1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix} \qquad R = \begin{bmatrix} 6 & -3 \\ 0 & 9 \end{bmatrix}$$

Ex. 4.3.7.b

$$\begin{bmatrix} -4 & -4 \\ -2 & 7 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 0 \end{bmatrix}$$

$$Q^T b = \begin{bmatrix} -2/3 & -1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 3 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -3 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \end{bmatrix} \qquad \boxed{x = \begin{bmatrix} -11/18 \\ 4/9 \end{bmatrix}}$$

Ex. 4.3.8.b

$$r_{00} = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$q_0 = \frac{1}{3} \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} \qquad r_{01} = \frac{1}{3} \begin{bmatrix} 2 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ -1 \\ 3 \end{bmatrix} = 3$$

$$W_1 = \begin{bmatrix} 4 \\ -1 \\ -1 \\ 3 \end{bmatrix} - 3\left(\frac{1}{3}\right) \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -3 \\ 2 \end{bmatrix}$$

$$q_1 = \frac{1}{\sqrt{18}} \begin{bmatrix} 2 \\ -1 \\ -3 \\ 2 \end{bmatrix}$$

$$\boxed{Q = \begin{bmatrix} 2/3 & 2/\sqrt{18} \\ 0 & -1/\sqrt{18} \\ 2/3 & -3/\sqrt{18} \\ 1/3 & 2/\sqrt{18} \end{bmatrix} \qquad R = \begin{bmatrix} 3 & 3 \\ 0 & \sqrt{18} \end{bmatrix}}$$

$$Q^T b = \begin{bmatrix} 2/3 & 0 & 2/3 & 1/3 \\ 2/\sqrt{18} & -1/\sqrt{18} & -3/\sqrt{18} & 2/\sqrt{18} \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -9/\sqrt{18} \end{bmatrix}$$

$$\begin{bmatrix} 3 & 3 \\ 0 & \sqrt{18} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ -9/\sqrt{18} \end{bmatrix}$$

$$\boxed{x = \begin{bmatrix} 5/6 \\ -1/2 \end{bmatrix}}$$