Q2-1

$T(n) = 4T(n/2) + n$

$T(n) = 4\cdot\left(4T(n/4) + \frac{n}{2}\right) + n$

$T(n) = 16\cdot T(n/4) + 3n$

$T(n) = 16\cdot\left(4T(n/8) + \frac{n}{4}\right) + 3n$

$T(n) = 48T(n/8) + 7n$

$T(n) = 4^t\cdot T(n/2^t) + (2^t - 1)n$

$T(n) = n^2\cdot T(1) + (n-1)n$

$= n^2\left(1 + 1 - \frac{1}{n}\right) = 2n^2 - n$

Inductive Hyp: $T(j) = 2j^2 - j$ for all $1 \leq j \leq n$

Base case: $T(1) = 2(1)^2 - 1 = 1$

Inductive step:

$T(k) = 4\cdot\left(\cancel{k}\,2\left(\frac{k}{2}\right)^2 - \frac{k}{2}\right) + k$

$= 2k^2 + k = 2k^2 + k$

$\forall n \geq 1,\ T(n) = 2n^2 + n$

Q2-2

$2^{n+1} = O(2^n)$

$2^{n+1} = 2\cdot 2^n = O(2^n)$

$2^{2n} \neq O(2^n)$

$2^{2n} = 2^n\cdot 2^n = O(2^{2n}) \neq O(2^n)$

Q2-3

$2\sqrt{\log_2 n} < \log_2 n < \sqrt{2n} < n^{2.5} < 10^n < n^n$

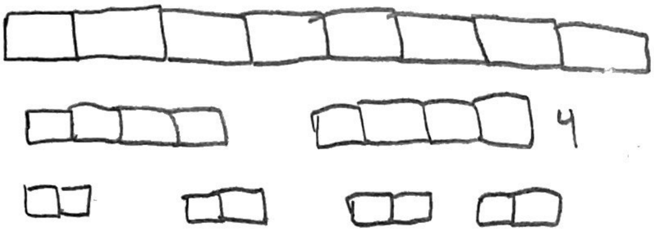
$(\log_2 n)^{\frac{1}{2}} < \log_2 n < (n)^{\frac{1}{2}} < n^{2.5} < 10^n < n^n$

*assuming $n \to \infty$, not really sure how to further explain besides knowing how these functions grow, could plug in an arbitrarily large value to prove further

3-1a

$n/k = 2$

$n/k = 4$



8 $k = n$ $O(n^2) = O(n \cdot n)$

$\boxed{64}$

$O(n \cdot 4) = O(4n)$

$= \boxed{32}$ per list

$* 2 = 64$

$O(n \cdot 2) = O(2n)$

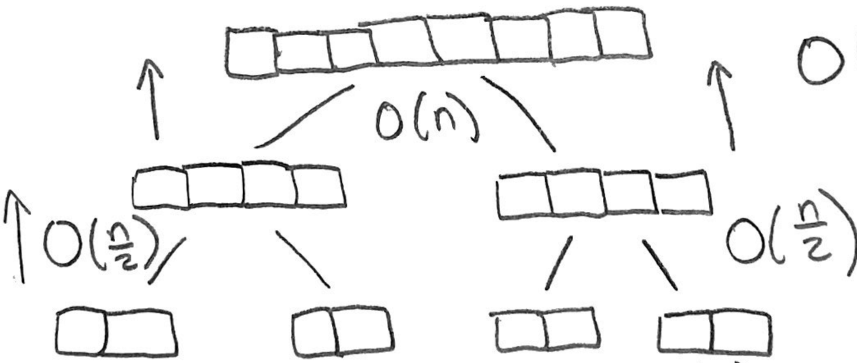
$= \boxed{16}$ per list

$* 4 = 64$

$$\frac{n}{k} \cdot O(n \cdot k) = O(n^2)$$

* Therefore for list of size k it will take $O(n \cdot k)$, but it will be $\frac{n}{k}$ lists, proving sorting all the lists would take $O(n^2)$.

3-1b



$O(n + \frac{n}{2} + \frac{n}{2}) = O(2n)$

* for specific case

$O(n \lg(4))$

$O(n + 2(\frac{n}{2}) + 4(\frac{n}{4}) + \ldots + k(\frac{n}{k}) = O(n \cdot \lg(\frac{n}{k})) = O(2n)$

3-1c

$$\frac{nk + n\log(n/k)}{n} = \frac{n\log n}{n}$$

$k + \log(n/k) = \log(n)$

$k + \cancel{\log(n)} - \log(k) = \cancel{\log(n)}$

$k = \log k$