EX. 0.2.4. Sauer

a) $(11)_{10} = (8+3)_{10} = (1011)_2$

$(.25)_{10} = (.01)_2 \rightarrow (11.25)_{10} = \boxed{(1011.01)_2}$

$0.25 * 2 = 0.5 + 0$

$0.5 * 2 = 0 + 1$

b) $(2/3)_{10} = \boxed{(0.\overline{10})_2}$

$2/3 * 2 = 1/3 +$

$1/3 * 2 = 2/3 +$

$2/3 * 2 = 1/3 -$

c) $(3/5)_{10} = (0.\overline{1001})_2$

$3/5 * 2 = 1/5 + 1$

$1/5 * 2 = 2/5 + 0$

$2/5 * 2 = 4/5 + 0$

$4/5 * 2 = 3/5 + 1$

$3/5 * 2 =$

d) $(3)_{10} = (2+1)_{10} = (11)_2$

$(0.2)_{10} = (.\overline{0011})_2$

$\rightarrow (3.2)_{10} = \boxed{(11.\overline{0011})_2}$

$1/5 * 2 = 2/5 + 0$

$2/5 * 2 = 4/5 + 0$

$4/5 * 2 = 3/5 + 1$

$3/5 * 2 = 1/5 + 1$

$1/5 * 2 = 2/5 +$

e) $(30)_{10} = (16+8+4+2)_{10} = (11110)_2$

$(0.6)_{10} = (0.\overline{1001})_2 \rightarrow (30.6)_{10} = \boxed{(11110.\overline{1001})_2}$

f) $(99)_{10} = (64+32+2+1)_{10} = (1100011)_2$

$(0.9)_{10} = (0.\overline{11100})_2$

$\downarrow$

$(99.9)_{10} = \boxed{(1100011.\overline{11100})_2}$

$9/10 * 2 = 8/10 + 1$

$8/10 * 2 = 6/10 + 1$

$6/10 * 2 = 2/10 + 1$

$2/10 * 2 = 4/10 + 0$

$4/10 * 2 = 8/10 + 0$

$8/10 * 2 = 6/10 + 1$

EX. 0.2.8. Sauer

a) $1+2+8+16 = (27)_{10}$

b) $1+2+4+16+32 = (55)_{10}$
$0.125 \rightarrow (55.125)_{10}$

c) $1+2+4 = (7)_{10}$
$(0.\overline{001})_2 = (1/7)_{10} \rightarrow (7.1/7)_{10}$

d) $2+8 = (10)_{10}$
$(0.\overline{01})_2 = (1/3)_{10} \rightarrow (10\ 1/3)_{10}$

e) $1+2+4+16 = (23)_{10}$
$(0.\overline{10101})_2 = (2/3)_{10} \rightarrow (23\ 2/3)_{10}$

f) $1+2+4+8 = (15)_{10}$
$(0.010\overline{001}) = (0.268)_{10} \rightarrow (15.268)_{10}$

* got fractions / decimals from colab

EX. 0.3.6. Sauer

a) $(1+(2^{-51}+2^{-52}+2^{-54}))-1$

$1+2^{-51} \rightarrow 1.0000\ 0000\ 0000\ 0000\ 0000\ \ldots\ 0010 \quad * 2^0$

$1+2^{-51}+2^{-52} \rightarrow 1.0000\ 0000 \longrightarrow 0011 \quad * 2^0$

$1+2^{-51}+2^{-52}+2^{-54} \rightarrow 1.000 \longrightarrow 0011 \mid 01 \quad * 2^0$

don't round ↑

0011

$1+2^{-51}+2^{-52}+2^{-54}-1 \rightarrow 0.0000\ 0000\ 0000\ 0000\ 0000\ 0000\ 0000$

$0000\ 0000\ 0000\ 0000\ 0011\ 01 * 2^0$

chop ↑

sign exponent

$= 2^{-51} + 2^{-52}$

$= 1.1 * 2^{-51}$

b) * same result as part a as 2^{-54} gets chopped in a; likewise 2^{-60} would be chopped here. rest is the same

EX. 0.3.7

a) $16 = (10000)_2 = 1.0000 * 10^4$ $\quad 4 + 1023 = 1027$

0100 0000 0011

4 0 3

$0x(4030\ 0000\ 0000\ 0000)$

b) $130 = 10000010 = 1.0000010 * 10^7$ $\quad 7 + 1023 = 1030$

0100 0000 0110

$0x(4060\ 4000\ 0000\ 0000)$

c) $1/4 = 0.0100\ldots = 1.0 * 10^{-2}$ $\quad -2 + 1023 = 1021$

0011 1111 1101

$0x(3FD0\ 0000\ 0000\ 0000)$

d) $fl(1/7) = 0.\overline{001}$ $\quad -3 + 1023 = 1020$ 0011 1111 1100

$\frac{1}{7} * 2 = \frac{2}{7} + 0$

$\frac{2}{7} * 2 = \frac{4}{7} * 0$

$\frac{4}{7} * 2 = \frac{1}{7} + 1$

$\frac{1}{7} * 2 = \frac{2}{7} + 0$

0010 0100 1001 0010 0100

1001 0010 0100 1001 0010

0100 1001 0010|0

$0x(3FC2\ 4924\ 9249\ 2492)$

e) $fl(4/7) = 0.\overline{100}$

$1023 + -1 = 1022_{10}^{2}$ 0011 1111 1110

$4/7 * 2 = \frac{1}{7} + 1$
$\frac{1}{7} * 2 = \frac{2}{7} + 0$
$\frac{2}{7} * 2 = \frac{4}{7} + 0$
$\frac{4}{7} * 2 = \frac{1}{7} + 1$

1001 0010 0100 1001 0010
0100 1001 0010 0100 1001
0010 0100 1001 | 0

0x(3FE2 4924 9249 2492)

f) $fl(0.01)$

$1023 - 7 = 1016$ 0011 1111 1100

.01 * 2 = 0.02 + 0
.02 * 2 = 0.04 + 0
.04 * 2 = .08 + 0
.08 * 2 = .16 + 0
.16 * 2 = .32 + 0
.32 * 2 = .64 + 0
.64 * 2 = .28 + 1
.28 * 2 = .56 + 0
.56 * 2 = .12 + 1
.12 * 2 = .24 + 0
.24 * 2 = .48 + 0
.48 * 2 = .96 + 0
.96 * 0 = .92 + 1
.92 * 2 = .84 + 1
.84 * 2 = .68 + 1
.68 * 2 = .36 + 1
.36 * 2 = .72 + 0
.72 * 2 = .44 + 0
.44 * 2 = .88 + 0
.88 * 2 = .76 + 0
.76 * 2 = .52 + 1
.52 * 2 = .04 + 1
.04 * 2 = .08 + 0

1011 round up

0x(3F84 7AE1 47AE 147B)

g) $fl(-0.01)$

1023 - 7 + 1024 = 2040 1011 1111 1000

0x(BF84 7AE1 47AE 147B)

h) *using f as reference
$fl(-0.02)$

1023 - 6 + 1024 = 2041 1011 1111 1001

0x(BF94 7AE1 47AE 147B)

EX. 0.4.2. Sauer

$x^2 + 3x - 8^{-14} = 0$ $\quad x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ $\quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

$x_1 = \frac{-3 + \sqrt{9 - 4(1)(-8^{-14})^2}}{2} \approx$

$x_2 = \frac{-3 - \sqrt{9 - 4(1)(-8^{-14})^2}}{2} \approx -3.00$

$x_1 = -\frac{b + \sqrt{b^2 - 4ac}}{2a}$ $\quad x_2 = \frac{2c}{\left(b + \sqrt{b^2 - 4ac}\right)}$

$= \boxed{-3.00 * 10^0}$ $\quad = \boxed{7.58 * 10^{-14}}$

EX. 0.5.2. Sauer

a) $f(x) = e^x$ on $[0, 1]$

$f(0) = e^0 = 1$ $\quad f(1) = e^1 = e$ $\quad f'(c) = \frac{e - 1}{1}$

$f'(x) = e^x = e - 1$ $\quad \ln e^x = \ln e - 1 \Rightarrow \boxed{c = \ln(e - 1)}$

b) $f(x) = x^2$ on $[0, 1]$

$f(0) = 0^2 = 0$ $\quad f(1) = 1^2 = 1$ $\quad f'(c) = \frac{1 - 0}{1 - 0} = 1$

$f'(x) = 2x$ $\quad \boxed{c = 1/2}$

c) $f(x) = \frac{1}{x + 1}$ on $[0, 1]$ $\quad f(x) = (x + 1)^{-1}$

$f(0) = 1$ $\quad f(1) = \frac{1}{2}$ $\quad f'(c) = \frac{\frac{1}{2} - 1}{1 - 0} = -\frac{1}{2}$

$f'(x) = \frac{-1}{(x + 1)^2}$ $\quad \frac{-1}{(x+1)^2} = -\frac{1}{2} \Rightarrow (x + 1)^2 = 2$

$\Rightarrow x + 1 = \pm\sqrt{2}$

$\Rightarrow x = \pm\sqrt{2} - 1$ $\quad$ * not in interval

$\Rightarrow \boxed{c = \sqrt{2} - 1}$ $\quad -\sqrt{2} - 1$

EX. 0.5.6. Sauer

a) $x^{-2} = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2} + \frac{f'''(c)(x-c)^3}{6} + \frac{f''''(c)(x-c)^4}{24}$

$f(x) = x^{-2}$ $\quad f(1) = 1$

$f'(x) = -2x^{-3}$ $\quad f'(1) = -2$

$f''(x) = 6x^{-4}$ $\quad f''(1) = 6$

$f'''(x) = -24x^{-5}$ $\quad f'''(1) = -24$

$f''''(x) = 120x^{-6}$ $\quad f''''(1) = 120$

$$P_4(x) = 1 - 2(x-1) + \frac{6(x-1)^2}{2} - \frac{24(x-1)^3}{6} + \frac{120(x-1)^4}{24}$$

$$= \boxed{1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + 5(x-1)^4}$$

b) $P_4(1.1) = 1 - 2(0.1) + 3(0.1)^2 - 4(0.1)^3 + 5(0.1)^4 = \boxed{0.8265}$

$P_4(0.9) = 1 - 2(-0.1) + 3(-0.1)^2 - 4(-0.1)^3 + 5(-0.1)^4 = \boxed{1.2345}$

c) ~~$f^{(5)}(x) = -720x^{-7}$~~ $\quad f(x) = P_4(x) + \frac{-720(x-1)^5}{120}$

$R_n(x) = \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!}$ $\quad = P_4(x) - 6(x-1)^5$

$R_4(x) = \frac{f^5(z)(x-1)^5}{120}$ $\quad f^5(z) = -720z^{-7}$ $\quad 0.9 \le z \le 1$

$1 \le z \le 1.1$

$f^5(1) = -720$ $\quad f^5(0.9) = -720(0.9)^{-7}$

$R_4(1.1) = \frac{-720(0.1)^5}{720}$ $\quad R_4(0.9) = \frac{-720(-0.1)^5}{720(0.9)^7}$

$$\frac{|f(x) - P_4(x)|}{f(x)} \le \begin{cases} 1.25 * 10^{-4} & \text{for } x = 0.9 \\ 6.00 * 10^{-5}, & \text{for } x = 1.1 \end{cases}$$

* expect 1.1 to be more accurate as the error is smaller

d) $|f(1.1) - P_4(1.1)| = \boxed{5.37 * 10^{-5}} < 6.00 * 10^{-5}$ ✓

$|f(0.9) - P_4(0.9)| = \boxed{6.79 * 10^{-5}} < 1.25 * 10^{-5}$ ✓