EX. 2.1.2. b2c

b) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ $R3 + 2R1$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & -5 & 3 & -2 \end{array}\right]$ $R3 + \frac{5}{3} R2$ $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 1 & 4 \\ 0 & 0 & 14/3 & 14/3 \end{array}\right]$ $R_2 - \frac{3}{14} R_3$

$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 14/3 & 14/3 \end{array}\right]$ $R_2 / 3$ $R_1 - 2R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 14/3 & 14/3 \end{array}\right]$ $R_1 + 3/14 R_3$ $R_3 / (14/3)$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x = 1$
$y = 1$
$z = 1$

c) $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right]$ $R_3 + R_2$ $R_2 - \frac{1}{2} R_1$ $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3/2 & 3 & 1/2 \\ 0 & 2 & -1 & 4 \end{array}\right]$ $R_3 + 2R_2$

$\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3/2 & 3 & 1/2 \\ 0 & 0 & 5 & 5 \end{array}\right]$ $R_3 / 5$ $R_2 - 3R_3$ $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3/2 & 0 & -29/2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $R_2 / (-3/2)$

$\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & 1 & 0 & 58/6 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $R_1 - R_2$ $\left[\begin{array}{ccc|c} 2 & 0 & -4 & -100/6 \\ 0 & 1 & 0 & 58/6 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $R_1 + 4R_3$

$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -76/6 \\ 0 & 1 & 0 & 58/6 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $R_1 / 2$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -76/12 \\ 0 & 1 & 0 & 58/6 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x = -76/12$
$y = 58/6$
$z = 1$

c) $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ -1 & 3 & -2 & 6 \end{array}\right]$ $R_3 + R_2$ $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 1 & -1 & 1 & -2 \\ 0 & 2 & -1 & 4 \end{array}\right]$ $2R_2 - R_1$

$\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3 & 6 & 3 \\ 0 & 2 & -1 & 4 \end{array}\right]$ $\frac{3}{2}R_3 + R_2$ $\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3 & 6 & 3 \\ 0 & 0 & 9/2 & 9 \end{array}\right]$ $R_3/(9/2)$ $R_2 - 6R_3$

$\left[\begin{array}{ccc|c} 2 & 1 & -4 & -7 \\ 0 & -3 & 0 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_2/-3$ $R_1 - R_2$ $\left[\begin{array}{ccc|c} 2 & 0 & -4 & -10 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_1 + 4R_3$ $R_1/2$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$x = -1$
$y = 3$
$z = 2$

EX. 2.1.6

~~$\frac{.005}{5000} = \sqrt{\frac{5000\,n^2}{5000}}$~~ $n = .001$

~~$2(.001)^2/3 * 5{,}000 \approx 0.0033$ sec~~

$\frac{2(5000)^3/3}{(5000)^2} = \frac{10000}{3} \rightarrow \frac{10{,}000}{3}(.005) \approx$ 17 sec

EX. 2.1.8

~~$2(3000)^3/3 \cdot (3{,}600{,}000{,}000) = 5$~~

$\frac{\pi(3000)^2}{2(3000)^3/3} = \frac{1}{2{,}000}$

$\frac{1}{2{,}000} * 5 = \frac{1}{400} \rightarrow$ 400 ft/sec

EX. 2.2.2.b2c

b) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} R_2 - R_1 \quad \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 2 & 2 & 3 \end{bmatrix} R_3 - \frac{1}{2}R_1 \quad \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix} R_3 - \frac{1}{2}R_2$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \quad \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1\cdot 4 + 0\cdot 0 + 0\cdot 0 & 1\cdot 2 + 0\cdot 2 + 0\cdot 0 & 1\cdot 0 + 0\cdot 2 + 0\cdot 2 \\ 1\cdot 4 + 1\cdot 0 + 0\cdot 0 & 1\cdot 2 + 1\cdot 2 + 0\cdot 0 & 1\cdot 0 + 1\cdot 2 + 0\cdot 2 \\ \frac{1}{2}\cdot 4 + \frac{1}{2}\cdot 0 + 1\cdot 0 & \frac{1}{2}\cdot 2 + \frac{1}{2}\cdot 2 + 1\cdot 0 & \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 2 + 1\cdot 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \checkmark$$

c) $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow[R_3 - R_1]{0*} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 4 & 3 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix} \xrightarrow[R_3 - 2R_2]{0*} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 2 & 1 & -1 \end{bmatrix}$ flip

$R_4 - R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix}$$

* didn't write everything out

✓ but I checked ↩

EX. 2.2.4.b

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

$Lc = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad \begin{aligned} x_0 &= 2 \\ x_1 &= 2 \\ x_2 &= 4 \end{aligned}$$

$Ux = c$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$\boxed{\begin{aligned} x_2 &= 2 \\ x_1 &= -1 \\ x_0 &= 1 \end{aligned}}$$

EX. 2.2.6

$$\frac{2(1000^3)}{3} = 666,666,666.67$$

$$1000^2 = 1,000,000$$

$$\frac{666,666,666.67}{500(1,000,000) + 666,666,666.67} = 0.5714$$

$$0.5714 \cdot 60 \approx \boxed{34 \text{ sec}}$$

EX. 2.2.8

$$\frac{2n^3/3 + 2kn^2}{2kn^3} = \frac{2(8000)^3/3 + 2(200)(8000)^2}{2(2000)^3/3 + 2(2000)^2} * 0.1 = \boxed{6.63 \text{ sec}}$$

EX. 2.3.2.b2c

b) $\|A\|_\infty = \max(|1|+|2.01|, |3|+|6|) = \max(3.01, 9) = 9$

$$A^{-1} = \begin{pmatrix} 1 & 2.01 \\ 3 & 6 \end{pmatrix}^{-1} = \frac{1}{(1)(6) - 3(2.01)} \begin{pmatrix} 6 & -2.01 \\ -3 & 1 \end{pmatrix} = 33.\overline{3} \begin{pmatrix} -6 & 2.01 \\ 3 & -1 \end{pmatrix}$$

$$\|A^{-1}\|_\infty = 33.\overline{3} \max(|-6| + |2.01|, |3| + |-1|) = 33.\overline{3}(\max(8.01, 4)) = 267$$

$$K_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty = 9 \cdot 267 = \boxed{2403}$$

c) $\|A\|_\infty = \max(|6|+|3|), |4|+|2|) = \max(9, 6) = 9$

$$A^{-1} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}^{-1} = \frac{1}{6 \cdot 2 - 4 \cdot 3} = \frac{1}{0} \rightarrow \boxed{\text{div by 0}}$$

EX. 2.4.2.c2d

c) $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{pmatrix}$ exchange 1 & 3 2

$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & -3 \\ -1 & 0 & 3 \end{bmatrix}$ $R_2 - \frac{1}{2}R_1$, $R_3 + \frac{1}{2}R_1$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 2 & 4 & 2 \\ \frac{1}{2} & 0 & -4 \\ -\frac{1}{2} & 2 & 4 \end{bmatrix}$ exchange 2 & 3 $\begin{bmatrix} 2 & 4 & 2 \\ -\frac{1}{2} & 2 & 4 \\ \frac{1}{2} & 0 & -4 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 2 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & -4 \end{bmatrix}$$

P A L U

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ -2 & 4 & 0 \end{bmatrix}$ swap 1 & 3 $\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ $R_2 + \frac{1}{2}R_1$ $\begin{bmatrix} -2 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 2 \\ 0 & 1 & 0 \end{bmatrix}$ swap 2 & 3

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 2 \end{bmatrix}$ $R_3 - \frac{1}{2}R_2$ $\begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 2 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} -2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{U}$$

EX. 2.4.4

a) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

$Lc = Pb$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$c_1 = 2,\ c_2 = 2,\ c_3 = 4$

$Ux = c$

$$\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$x_3 = 2,\ x_2 = -1,\ x_1 = 1$

b) $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix}$

$Lc = Pb$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/5 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 17 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1/5 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 3 \\ -2 \end{bmatrix}$$

$c_1 = 17,\ c_2 = 11.5,\ c_3 = 4.2$

$Ux = c$

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 5/2 & 1/2 \\ 0 & 0 & 9/5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ 11.5 \\ 4.2 \end{bmatrix}$$

$x_3 = 3,\ x_2 = 4,\ x_1 = 5$

EX. 2.7.2.b

$$L(x_1) = F(x_0) + DF(x_0)(x - x_0)$$

$$F(x_0) = F\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1+e^0\\3\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$$

$$DF\begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix}\partial u(u+e^{v-v}) & \partial v(u+e^{v-v})\\ \partial u(2u+v) & \partial v(2u+v)\end{pmatrix} = \begin{pmatrix}1+e^{v-v} & -e\\ 2 & 1\end{pmatrix}$$

$$DF(x_0) = DF\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1+e^0 & -e^0\\ 2 & 1\end{pmatrix} = \begin{pmatrix}2 & -1\\ 2 & 1\end{pmatrix}$$

$$L\begin{pmatrix}u\\v\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix} + \begin{pmatrix}2 & -1\\ 2 & 1\end{pmatrix}\begin{pmatrix}u-1\\v-1\end{pmatrix} = \begin{pmatrix}2+(2u-2)+(-u+1)\\ 3+(2u-2)+(v-1)\end{pmatrix}$$

$$= \boxed{\begin{pmatrix}2u-v+1\\ 2u+v\end{pmatrix}}$$