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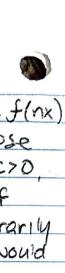
Homework 8:
Suppose ZCIR.
(=>) Suppose Z is a zero set. Then by definition, for all
2>0 there is a countable covering of 2 by open intervals
(ai, bi) such that 5 bi-ai = Ext Equivalentally, we can have
[a: 48 b: 48] where & is ting such that the sum across all
intervals is less than 2/2. So then because adding enapoints
won't change the length of the interval, and so 26-a; ce
as desired.
(E) Suppose for all 270 there is a countable covering of 2 by
closed intervals [a. bi] with total length & bi-ai < 2. Following
the same approach we can have a countable covering of E
by open intervals (a; bi) without changing the interval length,
and thus 2 b-a; ce,
(=) Suppose for all 270 there is a countable covering of Z
by closed intervals [a; bi] with total length & bi-ai < E. Then,
choose sets Si= [a:, bi]. It follows, & bi-a: = 2 diam (Si) ce.
(4) Suppose for all 270, there is a countable covering of 2
by sets Si with total diameter Ediam(si) < E. Let [ai, bi]
represent the endpoints of S: and thus Zaliam(Si)=
Sbi-a: 42 This doesn't have a well-defined meaning,
(=) Because 1-2 are equivalent and 2-3 are equivalent,
1-3 are equivalent.

3.40 Let g(x)= 31, x>0. Then it follows, G(x) = 3x+C2, x>0. Because g(0)=0, it must follows G is continuous at 0, so $\lim_{x\to 0^+} G(x) = \lim_{x\to 0^-} G(x)$ and thus G(0)=0, thousever, $\lim_{x\to 0^+} G(0)=0$, thus G doesn't have a derivative at 0.

Let $f(x) = \frac{50}{5}$ or $\frac{50}{10}$ or Also, suppose $F(x) = \frac{50}{5}$ or $\frac{1}{5}$ or $\frac{1}{5}$ Much more and more near o while bounded between -1 and I prease it follows the areas cancel out closer and closer to 0. Therefore x > 0+ So sin () dx = 0, and is thus continuous everywhere, where F(x) is antidering helder 3.47 Let & be continuous and BWOC suppose there exists point to such that to is contained in the discontinuity set of Ø of, and x isn't contained in the discontinuity set of f. However, it must follow f(xo) is continuous, and because Ø is continuous, Ø (f(xo)) must be continuous. But, this contradicts the assumption o(f(xo)) is discontinuous and contained in the discontinuity set.

2) Suppose D(x)=0 and f(x)= 30, x=0. Then, f(x)'s discontinuity set would be \$03. However, Ø(F(O)) = Ø(O) = O. Moreover of 15 continuous everywhere because it's constant. Thus, the discontinuity set for p(f(x))= Ø and f(x)=303, proving non-equality 3) A sufficient condition would be for \$ to be injective. Suppose & is injective. Then, if for some xo, f(xo) is ascontinuous, O(f(xo)) is also d'Eontinuous from above. Now, if some to is execontinuous for f(xo), where Ø(f(xo)) is injective, it must be mapped to unique off(xo)) where or is continuous everywhere 4) Not necessary Consider F(x= x. Thus, the discontinuity set is 303. Now consider Ø(P(x))=(f(x))2. Thus the discontinuity set is also 303. However, & isn't injective because tx=0 $(-f(x))^2 = (f(x))^2$

Andrew Ebert ** Original HW was 2.7/3 Homework 10: Suppose for all neN, In: 18-> 18 is uniformly continuous. Then for all 270, there exists 8>0 such that for all x, y & IR, with 1x-41 < 8, Ifn(x)-fn(4) < 213. Also, fn= f, so for all &>0, there exists N such that for all n=N and for all XEIR 15n(x)-f(x) | < ≥13, and the same for yETR. 30 then for all x, yETR with 1x-y1<6, |f(x)-f(y) = 15n(x)-f(x)+15n(x)-f(y) + 15n(y) - f(y) | < ≥. b) The same proof above would hold, but instead of having fn: R > R we can let fn: M > N and follow the same logic Cuniform continuty Pollows same rules in different metricapid 4.7 Consider a sequence of functions In in Co. The graph an of In 1s a compact subset of 12? a) (=) Suppose the sequence (Gn) in JE(TR2) converges to the graph of a function JECO. So then, Gn > G in X(R2). Because the metric is measuring maximum vertical distance between In and I and because (Gn) -> G, the maximum distance vertically is LE, and so using the same E, N from that convergence, we can say I fo(x)-F(x) for all x (4) Suppose (fn) converges uniformly as n->0. Also, consider the sequence (Gn) in X(IR). Then it follows, for all 2>0, there exists N such that for all n=N, for all x, Ifn(x)-f(x)(< E. So then, it directly follows that sup! In(x)-f(x) ! E because the largest distance is still <2. So then fn -> f and so it follows the corresponding graphs $G_n = 3(f_n(x);x):x3 \rightarrow G = 3(f(x)x):x3$ b) Let \$>0. Let S be chosen from uniform convergence above. Then from above, there exists some N such that for all n=N, for all x, y, I fn(x)-f(x)|<\(\exists|\) and I fn(y)-f(y)|=\(\exists|\). Also, because f is uniformly convergent If(x)-f(y) 1= £13 and so Ifn(x)-fn(y) 1= £ by triangle inequality.



4.9 If F: R>R is continuous and the sequence fn(x)=f(nx) is equicontinuous, then f must be constant. Bwoc suppose the above is true and fisht constant. Then for all E>D there exists 8>0 such that for all n, and all x, y, if 1x-y1 <8 then If(nx)-f(ny) < E. But, consider arbitrarily large n. Then, let x-y < \frac{1}{2}, But, n If(x)-f(y) < E would not hold, so f must be constant.