

⊛ Pascal triangle :

$$\begin{array}{ccccccc}
 & & & & 1 & & \\
 & & & 1 & & 1 & \\
 & & 1 & & 2 & & 1 \\
 & 1 & & 3 & & 3 & 1 \\
 1 & \swarrow & \searrow & & & & \\
 & 1 & & 4 & & 6 & & 4 & 1
 \end{array}
 \quad (5)$$

Notice: 1. 2 number in row  $N$  make a number in row  $n+1$

2. Number on row  $n+1$ , index  $r$  ( $0 \leq r \leq n$ ) can be expressed as  $\binom{n}{r}$

Consider row  $n+1=5$  :  $\binom{4}{0}=1$ ,  $\binom{4}{1}=4$ ,  $\binom{4}{2}=6$ ,...

→ ⊛ Pascal's identity:  $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$

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⊛ Binomial theorem:

$$(x+y)^n = \sum_{r=0}^n \binom{n}{r} x^r y^{n-r}$$

⊛ Special cases:  $\sum_{r=0}^n \binom{n}{r} = 2^n$  (sum of a row is  $2^n$ )

$\sum_{r=0}^n (-1)^r \binom{n}{r} = 0$  (sum of a row with alternate signs is 0)

⊛ Note: If  $p$  is probability of event happening  
 $q = 1-p$  not happening

→ The probability in  $N$  trials there are  $r$  successes is the  $r^{\text{th}}$  term in binomial expansion of  $(p+q)^N$

$$\text{a.k.a. } \binom{N}{r} p^r q^{n-r}$$

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$$\binom{10}{5} (2x)^8 (3y^2)^5$$