

④ r -combination of n -elements set is Permutation without regards of order!

$$\rightarrow P(n, r) = C(n, r) \cdot \underbrace{r!}_{\text{number of same set, different order}}$$

$$\rightarrow \boxed{C(n, r) = \frac{n!}{(n-r)! r!} = \binom{n}{r}}$$

Ex: How many 3-person committee from 4 people:

$$C(4, 3) = \frac{4!}{1! \cdot 3!} = 4$$

Ex: How many permutations of MISSISSIPPI?

Sol:

1. 11 places for 1 M : $\binom{11}{1} = 11$
2. 10 places for 4 S : $\binom{10}{4} = 210$
3. 6 places for 4 I : $\binom{6}{4} = 15$
4. 2 places for 2 P : $\binom{2}{2} = 1$

$$\rightarrow P = 11 \cdot 210 \cdot 15 \cdot 1 = \boxed{3300}$$

⊛ Theorem: Suppose a collection of n objects
 u_1 of type 1 are indistinguishable
 u_2 _____ 2 _____
...
 u_k _____ k _____
and $u_1 + u_2 + \dots + u_k = n$.

→ The number of distinguishable permutation :

$$\frac{n!}{u_1! u_2! \dots u_k!} = \binom{n}{u_1} \binom{n-u_1}{u_2} \binom{n-u_1-u_2}{u_3} \dots$$