⊛ Let S be a sample space. A probability function P from the set of events in the sample space satisfy:

1. $0 \leq P(E) \leq 1$
2. $P(\emptyset) = 0, \quad P(S) = 1$
3. If $E_1 \cap E_2 = 0, \quad P(E_1 \cup E_2) = P(E_1) + P(E_2)$

⊛
1. $P(E^c) = 1 - P(E)$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

⊛ Ex: A card is drawn from a 52-deck. What's the probability of red OR face card

⊛ $$P(\text{Red} \cup \text{Face}) = P(\text{Red}) + P(\text{Face}) - P(\text{Red} \cap \text{Face})$$
$$= \frac{26}{52} + \frac{12}{52} - \frac{6}{52}$$
$$= \frac{8}{13}$$

⊛ Expected value: Average chance of all outcomes, each outcome weighted by their probability.

⊛ An experiment with n real number outcomes $a_1, a_2, \dots a_n$ each with probability $p_1, p_2, \dots p_n$ has expected value:

$$\sum_{k=1}^{n} a_k p_k = a_1 \cdot p_1 + a_2 \cdot p_2 + \dots$$

⊛ If $E(X)$ is the expected value of, we have:

$$E(X \pm Y) = E(X) \pm E(Y)$$

(*) Ex: A carnival game consist of 10 numbers, 3 out of which the operator picked beforehand.

You get 100 $ if you guess 3 correct

5 $ 2 correct

1 $ 1 correct

What's the expected value of money?

Sol: The probability of getting 3 right: $\frac{3C3}{10C3} = \frac{1}{120}$

2 : $\frac{3C2 \cdot 7C1}{10C3} = \frac{21}{120}$

1 : $\frac{3C1 \cdot 7C2}{10C3} = \frac{63}{120}$

0 : $\frac{7C3}{10C3} = \frac{35}{120}$

$\Rightarrow E = 100 \cdot \frac{1}{120} + 5 \cdot \frac{21}{120} + 1 \cdot \frac{63}{120} + 0 \cdot \frac{35}{120} \approx 2.23\$$