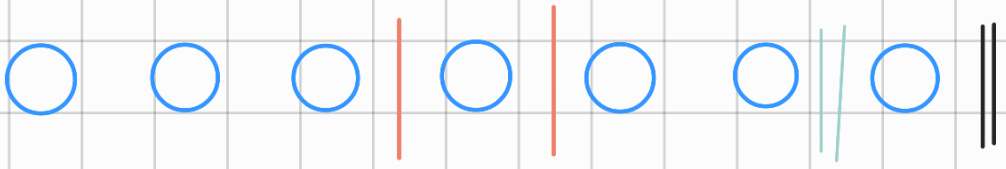
⊛ A r-combination with repetition allowed, is unordered selection of r elements from n-set where elements of X are allowed to repeat.

⊛ Consider placing r identical balls into u identical urns.

⊛ Ex: $r = 7$, $u = 3$



→ There are u dividers space
r-1 dividers

Dividers can be in same spot (empty urn)

→ Cut = $\binom{r+u-1}{u-1}$

⊛ Note: problem is called stars and bars

⊛ Ex: How many non-negative integer solution to

$x + y + z + a + b + c = 35$ (1)

. Aus: $\binom{35+6-1}{6-1}$

How many positive integer solution?

. Aus: Set $x' = x+1$, $y' = y+1$, ...

→ (1) ⇔ $x' + y' + z' + a' + b' + c' = 29$

→ Cut = $\binom{29+6-1}{6-1}$

⊛ Ex:

```
for  k = 1 -> n
    for  j = 1 -> k
        for  i = 1 -> j
            // code
```

How many times do inner loop runs?

- Ans:

There're r stars, 3 bars

Since $i \le j \le k$, num of stars before each bars indicate the values.

→ No bar appears before first star

→ r-1 stars, 3 bars

→ Cnt = $\binom{r-1+3}{3}$

Summary

For counting:

⊛ If repetition is not allowed:

- If order matter: $P(n,r)$
- If order doesn't matter: $C(n,r)$

⊛ If repetition is allowed:

- If order matter: n^r
- If order doesn't matter: $\binom{r+n-1}{n-1}$