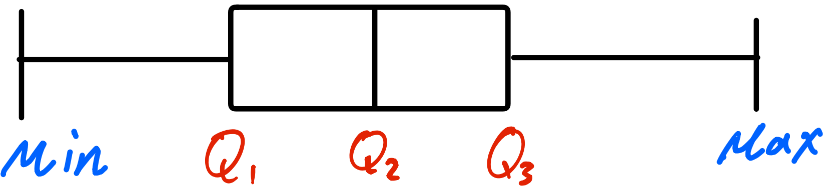


Example: ~~11~~, ~~22~~, ~~20~~, ~~14~~, ~~29~~, ~~8~~, ~~35~~, ~~27~~, ~~13~~, ~~49~~, ~~10~~, ~~24~~, ~~17~~

Min | Max

Low to big → 8, 10, 11, 13, 14, 17, 20, 22, 24, 27, 29, 35, 49

Q_1 | middle ※

13 ※

avg 11 and 13 | $Q_2 = 20$ | avg 27, 29

$Q_1 = 12$ | $Q_3 = 28$

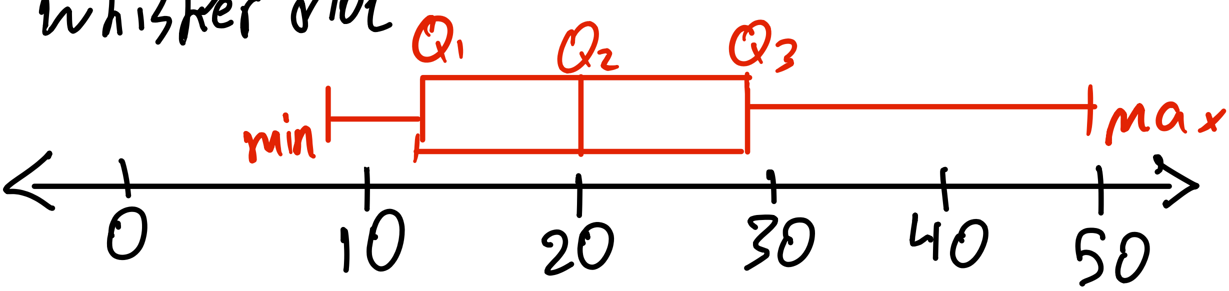
checking for outliers $[Q_1 - 1.5\,IQR,\ Q_3 + 1.5\,IQR]$

$IQR = Q_3 - Q_1 = 28 - 12 = 16$

$[12 - 1.5(16),\ 28 + 1.5(16)]$

$[-12, 52]$ anything outside this rang is an outlier

* Drawing a Box and whisker plot



Example: ~~18~~, ~~34~~, ~~76~~, ~~29~~, ~~15~~, ~~41~~, ~~46~~, ~~25~~, ~~54~~, ~~38~~, ~~20~~, ~~32~~, ~~43~~, 22

Min | Max

15, 18, 20, 22, 25, 29, 32 | 34, 38, 41, 43, 46, 54, 76

14 #

$Q_1 = 22$

split #s to 2 equal parts

$Q_3 = 43$

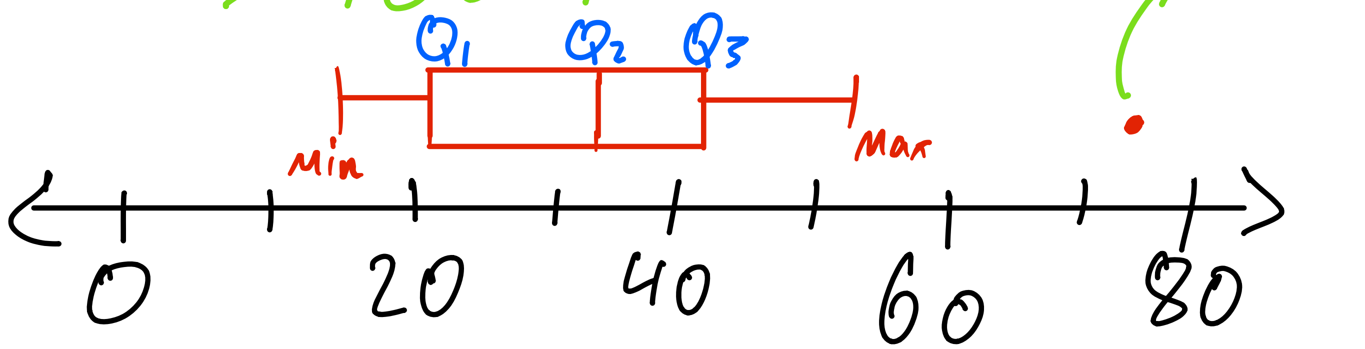
$Q_2 = 3$

$IQR = Q_3 - Q_1 = 43 - 22 = 21$

Outliers $[22 - 1.5(21), 43 + 1.5(21)]$

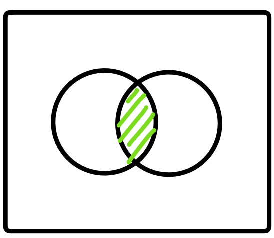
$[-9.5, 74.5]$

→ 76 outlier



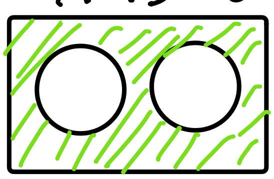
Intersection of A and B → Both A and B occur

$A \cap B$ A and B AB



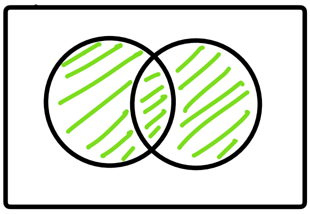
if A and B are mutually exclusive

$A \cap B = 0$



Union of A and B $\longrightarrow$ either A or B occurs

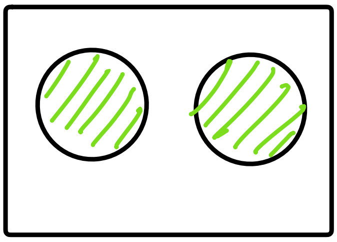
$A \cup B$ $\qquad$ A or B



Addition rule

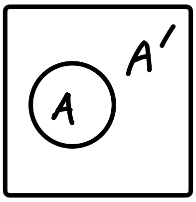
$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Mutually exclusive



$P(A \cup B) = P(A) + P(B)$

A' or $\overline{A}$ or A^c all mean A prime



<u>Example:</u> rolling a six-sided die

$S = [1, 2, \dots, 6]$

we define: $E[1,3,5]$ $\qquad$ $F[4,5,6]$

$G[2,6]$

$P(E) = \frac{3}{6} = \frac{1}{2}$ $\qquad$ $P(G) = \frac{2}{6} = \frac{1}{3}$

$P(F) = \frac{3}{6} = \frac{1}{2}$

$E' = \{2, 4, 6\}$ $\quad P(E') = \frac{3}{6} = 1 - P(E)$

$E \cap F = 5$ $\quad P(E \cap F) = \frac{1}{6}$

$E \cap G = \emptyset$ mutually exclusive

$E \cup F = \{1, 3, 4, 5, 6\}$ $\quad P(E \cup F) = \frac{5}{6}$

or using addition rule

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = 5/6$$

$\overline{E \cup F} = \{2\}$

Example: in a certain population:

10 % have diabetes

30 % have hypertension

7 % have both

A person is randomly selected.

D = The person has diabetes

H = The person has hypertension

$P(D) = 0.10$

$P(H) = 0.30$

$P(D \cap H) = 0.07$

$P(\bar{D}) = 0.90$

$P(\bar{H}) = 0.70$

$P(\overline{D \cap H}) = 0.93$

$$P(D \cup H) = P(D) + P(H) - P(D \cap H)$$
$$= 0.10 + 0.30 - 0.07$$
$$= 0.33$$

---

conditional probability

$P(A/B) \rightarrow$ probability if event A will occure given that event B already occurd

Example:

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 3, 5\}$

$B = \{3, 4, 5\}$

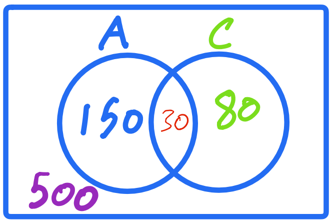
$P(A/B) = ?$

$$= \frac{\frac{2}{6} \,(P(A \cap B))}{\frac{3}{6} \,(P(B))}$$

$$= \boxed{2/3}$$

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$$

1. There are 500 students in a certain school. 150 students are enrolled in an Algebra course and 80 students are enrolled in a Chemistry course. There are 30 students who are taking both Algebra and Chemistry. If a student is chosen at random, (a) What is the probability that the student is taking Algebra? (b) What is the probability that the student is taking Chemistry given that the student is also taking Algebra? (c) What is the probability that the student is taking Algebra given that the student is also taking Chemistry?



a) $P(A) = \frac{150}{500} = \frac{15}{50} = \frac{3}{10} = 0.3$

b) $P(C/A) = \frac{P(C \cap A)}{P(A)} = \frac{30/\cancel{500}}{150/\cancel{500}}$

$= \frac{30}{150} = \frac{1}{5} = 0.2$

c) $P(A/C) = \frac{P(C \cap A)}{P(C)} = \frac{30/\cancel{500}}{80/\cancel{500}}$

$= \frac{30}{80} = \frac{3}{8} = 0.375$

2. There are 200 birds in a zoo. 70 birds are male with brown eyes and 100 birds are female with brown eyes. 20 of the birds are male with blue eyes and 10 birds are female with blue eyes. Construct a contigency table. If a bird is selected at random, what is the probability that the bird is (A) a female? (B) a male with brown eyes? (C) a female given that it has brown eyes? (D) a male given that it has blue eyes? (E) a creature with blue eyes given that it's a female?

G	BR	BL	
M	70	20	90
F	100	10	110
T	170	30	200

a) $P(F) = \frac{110}{200} = \frac{11}{20} = 0.55$

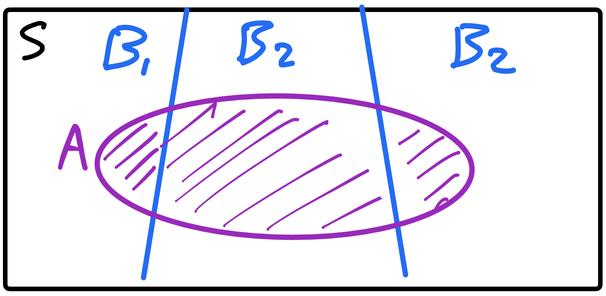
b) $P(M \,\&\, Br) = \frac{70}{200} = \frac{7}{20} = 0.35$

c) $P(F/Br) = \frac{100}{170} = \frac{10}{17} = 0.588$

d) $P(M/BL) = \frac{20}{30} = \frac{2}{3}$

E) $P(BL/F) = \frac{10}{110} = \frac{1}{11}$

Total probability rule



$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3)$

$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3)$$

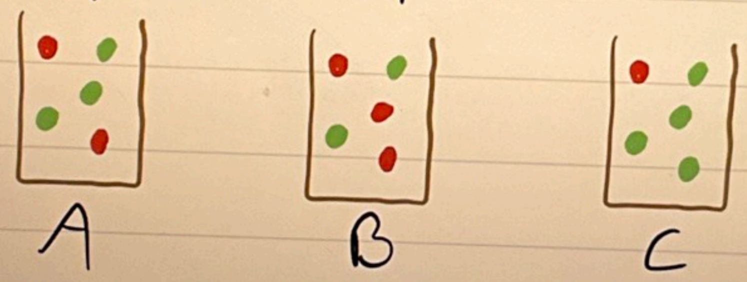
$$P(A \cap B) = P(A/B)\,P(B)$$

so

$$P(A) = P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + P(A/B_3)P(B_3)$$

Example:

Bag A has 2 red balls and 3 green balls.
Bag B has 3 red balls and 2 green balls.
Bag C has 1 red ball and 4 green balls.
A ball is randomly selected from a random bag; what is the probability that the ball is red?



R = ball is red

$$P(R) = P(A \cap R) + P(B \cap R) + P(C \cap R)$$

$$= P(R/A)\,P(A) + P(R/B)\,P(B) + P(R/C)\,P(C)$$

P of selecting red ball given bag A

P of bag A being chosen

$$= \frac{2}{5} \cdot \frac{1}{3} + \frac{3}{5} \cdot \frac{1}{3} + \frac{1}{5} \cdot \frac{1}{3}$$

$$= \frac{6}{15}$$

The Law of Total Probability

The Law of Total Probability

If $\{B_1, B_2, B_3, \ldots\}$ is a finite or countably infinite partition of a sample space, and A is an event in the same sample space, then

$$P(A) = \sum_i P(A \cap B_i) = \sum_i P(A|B_i)\,P(B_i).$$

Example:

In a class, 40% of students are male and 60% are female. Sixty percent of the males are taller than 6 feet, and 10% of the females are taller than 6 feet. What percent of the class is shorter than 6 feet?

		Taller	shorter
M	40%	60%	40%
f	60%	10%	90%

$$P(S/M)\,P(M) + P(S/f)\,P(F)$$

$$0.4 \times 0.4 + 0.9 \cdot 0.6$$

$$= 0.7 \text{ or } 70\%$$

Bayes's theorem

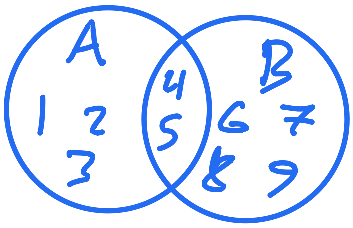
$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$\boxed{P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)}}$$

Example:

$A = \{1,2,3,4,5\}$ $B\ \{4,5,6,7,8,9\}$

$P(A/B) = ?$ $S = \{1,2,3,4,5,6,7,8,9\}$



$P(A) = 5/9$

$P(B) = 6/9$

$P(B/A) = \frac{2}{5}$

$$P(A/B) = \frac{P(B/A)\cdot P(A)}{P(B)}$$

$$= \frac{\frac{2}{\cancel{5}}\cdot\frac{\cancel{5}}{\cancel{9}}}{\frac{6}{\cancel{9}}}\cdot\frac{9}{9} = \frac{2}{6}$$

$$= \boxed{\frac{1}{3}}$$

Example:

1. A particular study showed that 12% of men will likely develop prostate cancer at some point in their lives. A man with prostate cancer has a 95% chance of a positive test result from a medical screening exam. A man without prostate cancer has a 6% chance of getting a false positive test result. What is the probability that a man has cancer given that he has a positive test result?

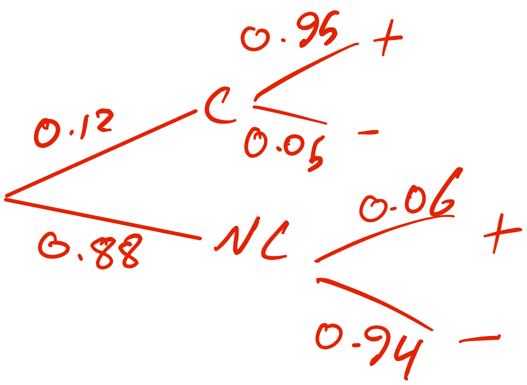
$P(C/P) = ?$

$P(C) = 12\%$

$P(P/C) = 95\%$

$P(P/NC) = 6\%$

$$P(C/P) = \frac{P(P/C) \cdot P(C)}{P(P)} = \frac{0.95 \times 0.12}{0.1168}$$



$P(P) = P(C \cap P) + P(NC \cap P)$

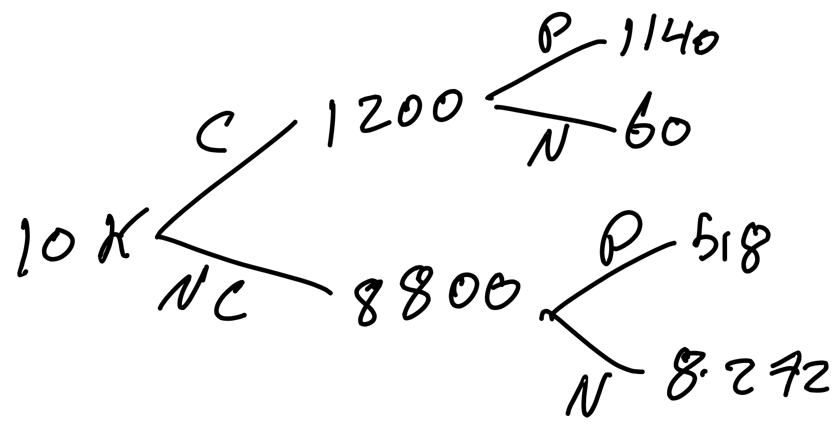
$P(P) = 0.12 \times 0.95 + 0.88 \times 0.06$

$P(P) = 0.1668$

$$\boxed{P(C/P) = 0.683 = 68.3\%}$$

$P(NC/P) = 1 - 0.683 = 31.7\%$

Same example → let's say we have 10K. population



$$P(C/P) = \frac{1140}{1140+528} = \boxed{0.683}$$

Independence of events

1. A bag consist of 8 red marbles, 7 blue marbles, 6 green marbles, and 4 yellow marbles. What is the probability of selecting (A) a red marble? (B) a blue marble on the first try and then a green marble on the second try with replacement? (C) a yellow marble on the first try and then a red marble on the second try without replacement? (D) two blue marbles with replacement? (E) two green marbles without replacement?

R = 8

B = 7

G = 6

Y = 4

T = 25

a) $P(R) = \frac{8}{25} = 0.32 \rightarrow \boxed{32\%}$

(independent)

b) $P(BG) = \frac{7}{25} \cdot \frac{6}{25} = \frac{42}{625} = \boxed{6.72\%}$

with replacement means marbles are put back after being picked

(dependent)

c) $P(YR) = \frac{4}{25} \cdot \frac{8}{24} = 0.05\bar{3} \rightarrow 5.3\%$

without replacement → not going back to the bag

Rule for independent P

$P(A) = ?$ $\quad P(A|B) = P(A) \rightarrow$ A and B are independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A) \rightarrow \boxed{P(A \cap B) = P(A) \cdot P(B)}$$

permutation and combinations

permutation → order matters

combinations → order doesn't matter

Ex: A B C $\quad$ CAB

2 permutations

1 combinations

Ex: A B C D

permutation

A B	~~B A~~	~~C A~~	~~D A~~
A C	B C	~~C B~~	~~D B~~
A D	B D	C D	~~D C~~

$P = 12$

$C = 6$

$$\boxed{\text{Permutation} \rightarrow P_{k,n} = \frac{n!}{(n-k)!}}$$

$$P_{2\,4} = \frac{4!}{(4-2)!} = 12$$

$$\boxed{\text{Combination} \rightarrow \binom{n}{k} = \frac{n!}{(n-k)!\,k!}}$$

$$\binom{4}{2} = \frac{4!}{(4-2)!\,2!} = 6$$

Example:

In how many different ways can you arrange 3 books on a shelf from a group of 7?

$$P = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{5,040}{24} = \boxed{210}$$

$$\cancel{C = \binom{7}{3} = \frac{7!}{(7-3)!\,3!} = \frac{5040}{144} = \boxed{35}}$$

arrange means order matter!

Example:

How many different ways can we arrange 5 books.

$$P = \frac{5!}{(5-5)!} = \frac{5!}{0!} = \frac{120}{1} = \boxed{120}$$

Example

How many teams of 4 can be produced from a pool of 12 eng?

$$\binom{12}{4} = \frac{12!}{(12-4!)\,4!} = \frac{12!}{8!\,4!} = 495$$

Recall:

- Conditional probability: $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$$

- Total probability rule: $P(B) = \sum_{i=1}^{k} P(B/A_i) \cdot P(A_i)$

$A_1, \ldots, A_k$ forms a partition of Ω, and are mutually exclusive

- Bayes' theorem: $P(A_j/B) = \dfrac{P(A_j \cap B)}{\sum_{i=1}^{k} P(B/A_i) \cdot P(A_i)}$

- Independence of events: $P(A \cap B) = P(A) \cdot P(B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = P(A)$$

Recall:

Topic 2 $\longrightarrow$ Permutations: is an ordered subset

$$P_{k,n} = \frac{n!}{(n-k)!}$$

Combinations: is an unordered subset

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$