This is currently a work in progress.
Constructive comments and feedback welcome.

Calculate MPMD nozzle displacement for bed switch activation

Objective is to determine explicit function for the required nozzle z displacement to activate bed switch based on given nozzle x and y location. Goal is to have reliable compact solution to integrate into firmware for accurate onboard bed probe data.

Free body diagram for the bed will be constructed with upward forces located at each bed switch and downward force applied by nozzle. Goal is to have forces placed at bed switch location will be a function of z displacement so that a downward reaction force is very weak and upward motion reaction force is very strong (consistent with bed clip acting as a much much firmer spring than the bed switch). For simplicity bed clip reaction force will be applied at the bed switch location.

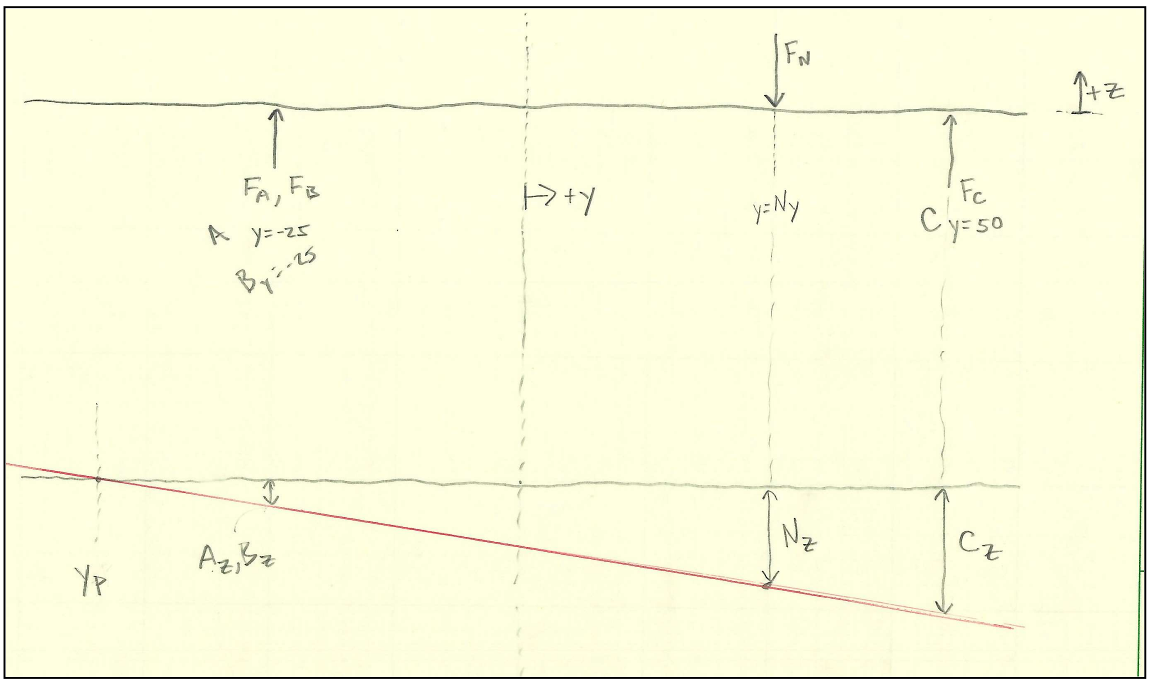
```
set_draw_defaults(
    xaxis = true, xaxis_color = black,
    xaxis_type = solid, xaxis_width = 2,
    yaxis = true, yaxis_color = black,
    yaxis_type = solid, yaxis_width = 2,
    dimensions = [600,600],
    font = Arial,
    font_size = 14,
    line_width = 3
)$;
```

1 Tower Axis Case:

Start analysis by looking at case were nozzle is inline with one of the tower axis. This case has symmetric arrangement of bed switches about the given axis. This allows problem to be evaluated in 2 dimensions. y_p is the y co-ordinate in which the bed pivots about. It is undefined at y=0 as the bed is assumed to depress parallel.

➔

Figure 1:



Similar triangles are used to relate the z displacement of bed switches and nozzle.

(%i55) K=(y_p-A_y)/A_z;K=(y_p-B_y)/B_z;K=(y_p-N_y)/N_z;K=(y_p-C_y)/C_z;

(%o52) $$K=\frac{y_p-A_y}{A_z}$$

(%o53) $$K=\frac{y_p-B_y}{B_z}$$

(%o54) $$K=\frac{y_p-N_y}{N_z}$$

(%o55) $$K=\frac{y_p-C_y}{C_z}$$

Equations from free body diagram.
Eqn1 sum of forces in vertical direction.
Eqn2 sum of moments about N_y.
Eqn3 sum of moments about C_y bed switch.
Eqn4 sum of moments about A_y bed switch. F_a and F_b are equal.

```
(%i5) Eqn1:2·F_a+F_c=F_n;
      Eqn2:2·F_a·(N_y-A_y)=F_c·(C_y-N_y);
      Eqn3:2·F_a·(C_y-A_y)=F_n·(C_y-N_y);
      Eqn4:F_n·(N_y-A_y)=F_c·(C_y-A_y);
```

(%o2) $F_c+2\,F_a=F_n$

(%o3) $2\,F_a\,(N_y-A_y)=F_c\,(C_y-N_y)$

(%o4) $2\,(C_y-A_y)\,F_a=F_n\,(C_y-N_y)$

(%o5) $F_n\,(N_y-A_y)=(C_y-A_y)\,F_c$

Solve Eqn3 for F_n and substitute into Eqn4

```
(%i8) solve(Eqn3,F_n);
      subst(rhs(%[1]), F_n, Eqn4);
      Eqn5:solve(%,F_a)[1];
```

(%o6) $[F_n=-\frac{(2\,C_y-2\,A_y)\,F_a}{N_y-C_y}]$

(%o7) $-\frac{(2\,C_y-2\,A_y)\,F_a\,(N_y-A_y)}{N_y-C_y}=(C_y-A_y)\,F_c$

(%o8) $F_a=-\frac{F_c\,N_y-C_y\,F_c}{2\,N_y-2\,A_y}$

Super simple force model. F(x)=-x is used for now. TODO make this piecewise -x; x<=0 & -100x x>0. Bed clip force much stiffer than bed switch force.

```
(%i57) subst(-C_z,F_c,Eqn5);
       Eqn6:subst(-A_z,F_a,%);
```

(%o56) $F_a=-\frac{C_y\,C_z-C_z\,N_y}{2\,N_y-2\,A_y}$

(%o57) $-A_z=-\frac{C_y\,C_z-C_z\,N_y}{2\,N_y-2\,A_y}$

Relate A_z to C_z with similar triangle. Solve for y_p pivot point.

```
Eqn7:A_z=(y_p-A_y)·C_z/(y_p-C_y);
subst(rhs(Eqn7),A_z,Eqn6);
Eqn8:solve(%,y_p)[1];
```

(%o11) $$A_z=\frac{C_z\,(y_p-A_y)}{y_p-C_y}$$

(%o12) $$-\frac{C_z\,(y_p-A_y)}{y_p-C_y}=-\frac{C_y\,C_z-C_z\,N_y}{2\,N_y-2\,A_y}$$

(%o13) $$y_p=\frac{(C_y+2\,A_y)\,N_y-C_y^2-2\,A_y^2}{3\,N_y-C_y-2\,A_y}$$

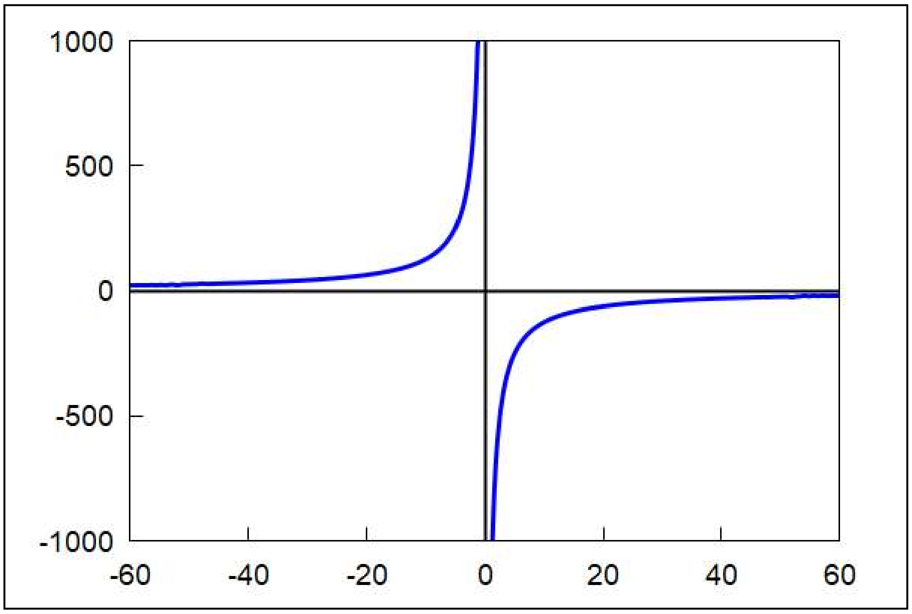
(%i15)
```
subst(50,C_y,Eqn8);
Eqn9:subst(-25,A_y,%);
```

(%o14) $$y_p=\frac{(2\,A_y+50)\,N_y-2\,A_y^2-2500}{3\,N_y-2\,A_y-50}$$

(%o15) $$y_p=-\frac{1250}{N_y}$$

(%i16)
```
wxdraw2d(
    explicit(
        rhs(Eqn9),
        N_y,-60,60
        ),
    yrange=[-1000,1000]
)$
```

(%t16)



Above graph of y_p is as expected. Nozzle in center is undefined. Nozzle near zero moves bed nearly flat representing y_p way off in distance. This is not dependent on switch travel and valid for positive and negative N_y.

TODO need to revalidate this conclusion with more complex F(x) for spring forces.

Substitute force F(x) into Eqn2 (moments around N_y)

(%i59)
```
subst(-C_z,F_c,Eqn2);
Eqn11:subst(-A_z,F_a,%);
```

(%o58) $2\, F_a\, (N_y - A_y) = -C_z\, (C_y - N_y)$

(%o59) $-2\, A_z\, (N_y - A_y) = -C_z\, (C_y - N_y)$

Relate A_z to N_z with similar triangles and subsite equation for y_p(N_y)

```
(%i63) Eqn12:A_z=(y_p-A_y)·N_z/(y_p-N_y);
       subst(rhs(Eqn12),A_z,Eqn11);
       subst(-1250/N_y,y_p,%);
```

(%o61) $$A_z=\frac{N_z\,(y_p-A_y)}{y_p-N_y}$$

(%o62) $$-\frac{2\,(N_y-A_y)\,N_z\,(y_p-A_y)}{y_p-N_y}=-C_z\,(C_y-N_y)$$

(%o63) $$-\frac{2\left(-\frac{1250}{N_y}-A_y\right)(N_y-A_y)\,N_z}{-N_y-\frac{1250}{N_y}}=-C_z\,(C_y-N_y)$$

Solve for N_z and then substitute in C_y, A_y & C_z=0.5 (valid for N_y >0)

```
(%i64) Eqn13:solve(%,N_z)[1];
```

(%o64) $$N_z=-\frac{C_z\,N_y^3-C_y\,C_z\,N_y^2+1250\,C_z\,N_y-1250\,C_y\,C_z}{2\,A_y\,N_y^2+\left(2500-2\,A_y^2\right)N_y-2500\,A_y}$$

```
(%i67) subst(50,C_y,Eqn13);
       subst(-25,A_y,%);
       Eqn14:subst(-.5,C_z,%);
```

(%o65) $$N_z=-\frac{C_z\,N_y^3-50\,C_z\,N_y^2+1250\,C_z\,N_y-62500\,C_z}{2\,A_y\,N_y^2+\left(2500-2\,A_y^2\right)N_y-2500\,A_y}$$

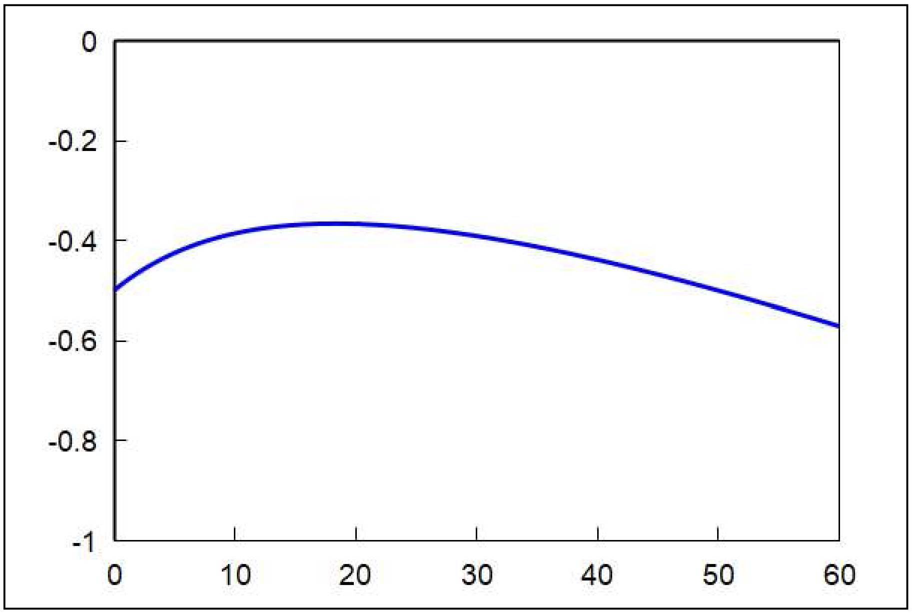
(%o66) $$N_z=-\frac{C_z\,N_y^3-50\,C_z\,N_y^2+1250\,C_z\,N_y-62500\,C_z}{-50\,N_y^2+1250\,N_y+62500}$$

(%o67) $$N_z=-\frac{-0.5\,N_y^3+25.0\,N_y^2-625.0\,N_y+31250.0}{-50\,N_y^2+1250\,N_y+62500}$$

Note that C_z could be factored out so profile is scaled relative to the SWITCH_TRAVLE adjustment.

(%i68)
```
wxdraw2d(
    explicit(
        rhs(Eqn14),
        N_y,0,60
        ),
    yrange=[-1,0]
    );
```

(%t68)



(%o68)

→

Need to repeat for N_y<0

Relate A_c to N_z with similar triangles and substitute equation for y_p(N_y)

(%i71)
```
Eqn15:C_z=(y_p-C_y)·N_z/(y_p-N_y);
subst(rhs(Eqn15),C_z,Eqn11);
subst(-1250/N_y,y_p,%);
```

(%o69) $C_z=\frac{N_z\,(y_p-C_y)}{y_p-N_y}$

(%o70) $-2\,A_z\,(N_y-A_y)=-\frac{(C_y-N_y)\,N_z\,(y_p-C_y)}{y_p-N_y}$

(%o71) $-2\,A_z\,(N_y-A_y)=-\frac{\left(-\frac{1250}{N_y}-C_y\right)(C_y-N_y)\,N_z}{-N_y-\frac{1250}{N_y}}$

Solve for N_z and stubstitute constants, this time A_z = SWITCH_TRAVEL

```
(%i72) Eqn16:solve(%,N_z)[1];
```

$$(\%o72)\quad N_z=-\frac{2 A_z N_y^3-2 A_y A_z N_y^2+2500 A_z N_y-2500 A_y A_z}{C_y N_y^2+\left(1250-C_y^2\right) N_y-1250 C_y}$$

```
(%i75) subst(50,C_y,Eqn16);
       subst(-25,A_y,%);
       Eqn17:subst(-.5,A_z,%);
```

$$(\%o73)\quad N_z=-\frac{2 A_z N_y^3-2 A_y A_z N_y^2+2500 A_z N_y-2500 A_y A_z}{50 N_y^2-1250 N_y-62500}$$

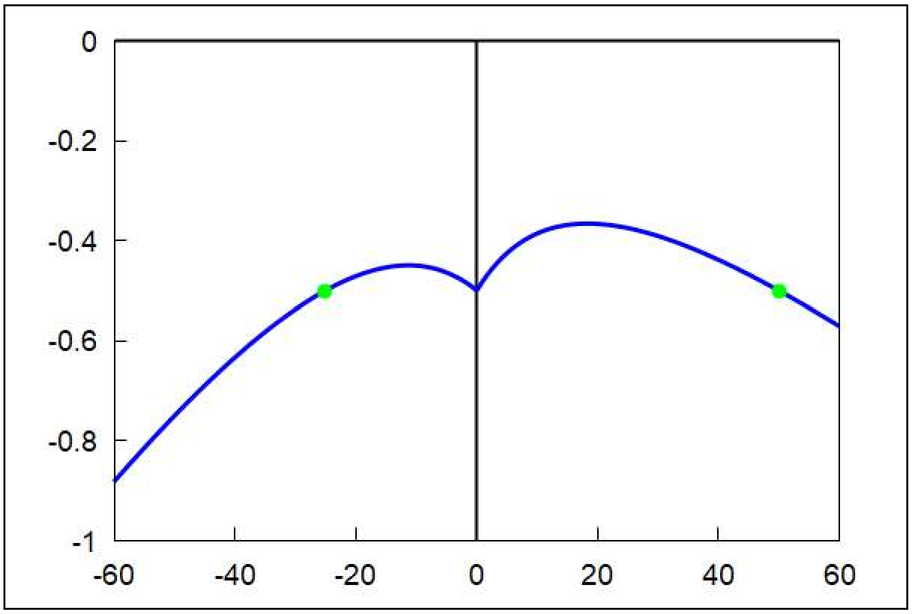
$$(\%o74)\quad N_z=-\frac{2 A_z N_y^3+50 A_z N_y^2+2500 A_z N_y+62500 A_z}{50 N_y^2-1250 N_y-62500}$$

$$(\%o75)\quad N_z=-\frac{-1.0 N_y^3-25.0 N_y^2-1250.0 N_y-31250.0}{50 N_y^2-1250 N_y-62500}$$

Plot both solutions on same graph

```
(%i83) wxdraw2d(
        explicit(
            rhs(Eqn17),
            N_y,-60,0
            ),explicit(
            rhs(Eqn14),
            N_y,0,60
            ),
        yrange=[-1,0],
        dimensions = [1200,600],
        color = green,
        point_size = 1.5,
        point_type = filled_circle,
        points([ [-25,-.5], [50,-.5] ])
        );
```

(%t83)



(%o83)

Review of the different cases seems consistent with expected behavior. N_y<-25 & <50 is beyond the bed switches so it needs to move lower to account for the tilt of bed. N_y=-25, +50 is equal to SWITCH_TRAVEL. This makes sense as nozzle is in line with bed switches. N_y=0, both equations meet at the expected SWITCH_TRAVLE location. Between the center and the bed switches the nozzle has to travel less to get the swtich to actuate.