Campus Alto Paraná y Sub Campus Santa Rita

Lecture 01
The geometry of Linear Equations

- n linear equations, n unknows
  - Row picture,
  - Column picture
  - Matrix form
  - Matrix multiplications

2 Equations, 2 Unknowns

$$\begin{cases} +2x - 1y = +0 \\ -1x + 2y = +3 \end{cases}$$

$$\mathbf{Ax=b}$$

$$\begin{bmatrix} +2 & -1 \\ -1 & +2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} +0 \\ +3 \end{bmatrix}$$

$$\mathrm{A}=\begin{bmatrix} +2 & -1 \\ -1 & +2 \end{bmatrix} \quad \mathbf{b}=\begin{bmatrix} +0 \\ +3 \end{bmatrix}$$

ROW PICTURE

$$\begin{cases} +2x - 1y = +0 \cdots Recta_1 \\ -1x + 2y = +3 \cdots Recta_2 \end{cases}$$

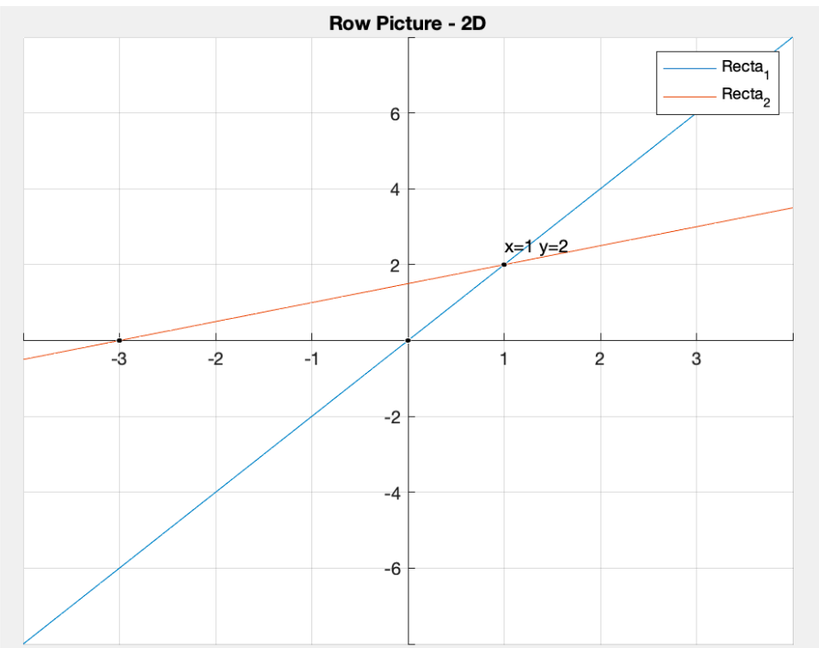
x	y
0	0
1	2

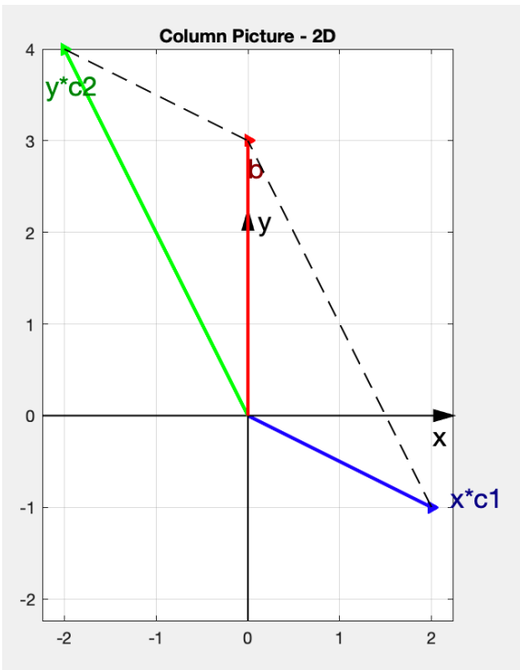
x	y
-3	0
1	2

COLUMN PICTURE

$$x \begin{bmatrix} +2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ +2 \end{bmatrix} = \begin{bmatrix} +0 \\ +3 \end{bmatrix}$$

$$x = +1$$
$$y = +2$$





3 Equations, 3 Unknowns

$$\mathbf{Ax}=\mathbf{b}$$

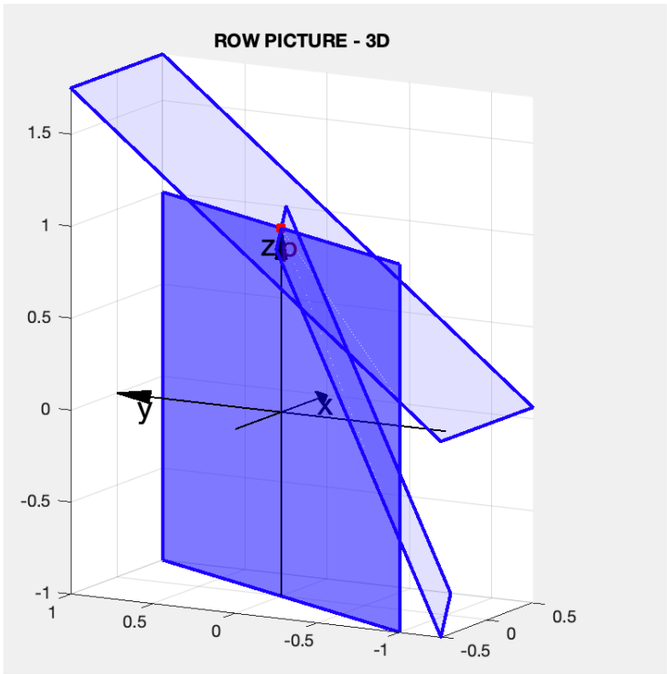
$$\begin{cases} +2x - 1y + 0z = +0 \\ -1x + 2y - 1z = -1 \\ +0x - 3y + 4z = +4 \end{cases} \quad \begin{bmatrix} +2 & -1 & +0 \\ -1 & +2 & -1 \\ +0 & -3 & +4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} +0 \\ -1 \\ +4 \end{bmatrix} \quad \mathrm{A}=\begin{bmatrix} +2 & -1 & +0 \\ -1 & +2 & -1 \\ +0 & -3 & +4 \end{bmatrix} \quad \mathbf{b}=\begin{bmatrix} +0 \\ -1 \\ +4 \end{bmatrix}$$

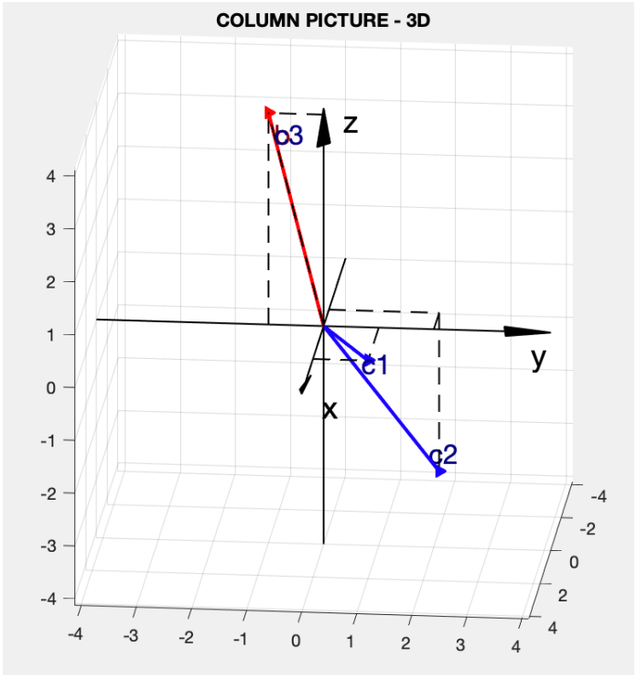
ROW PICTURE

$$\pi_i: ax + by + cz = d \qquad \vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

COLUMN PICTURE

$$x\begin{bmatrix} +2 \\ -1 \\ +0 \end{bmatrix} + y\begin{bmatrix} -1 \\ +2 \\ -3 \end{bmatrix} + z\begin{bmatrix} +0 \\ -1 \\ +4 \end{bmatrix} = \begin{bmatrix} +0 \\ -1 \\ +4 \end{bmatrix} \qquad \begin{aligned} x &= +0 \\ y &= +0 \\ z &= +1 \end{aligned}$$





3 Equations, 3 Unknowns

$$\mathbf{Ax}=\mathbf{b}$$

$$\begin{cases} +2x - 1y + 0z = +1 \\ -1x + 2y - 1z = +1 \\ +0x - 3y + 4z = -3 \end{cases} \qquad \begin{bmatrix} +2 & -1 & +0 \\ -1 & +2 & -1 \\ +0 & -3 & +4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} +1 \\ +1 \\ -3 \end{bmatrix} \qquad A = \begin{bmatrix} +2 & -1 & +0 \\ -1 & +2 & -1 \\ +0 & -3 & +4 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} +1 \\ +1 \\ -3 \end{bmatrix}$$

ROW PICTURE ?

Can I solve $\mathbf{Ax}=\mathbf{b}$ for every b?

Do the linear combs of the columns fill 3-D Space?

For This **A**, answer is Yes

COLUMN PICTURE ?

$$x \begin{bmatrix} +2 \\ -1 \\ +0 \end{bmatrix} + y \begin{bmatrix} -1 \\ +2 \\ -3 \end{bmatrix} + z \begin{bmatrix} +0 \\ -1 \\ +4 \end{bmatrix} = \begin{bmatrix} +1 \\ +1 \\ -3 \end{bmatrix}$$

$$x = +1$$
$$y = +1$$
$$z = +0$$

Matrix Mutliplications

$$\mathbf{Ax}=\mathbf{b}$$

$$\begin{bmatrix}+2 & +5\\ +1 & +3\end{bmatrix}\begin{bmatrix}+1\\ +2\end{bmatrix} = 1 * \begin{bmatrix}+2\\ +1\end{bmatrix} + 2 * \begin{bmatrix}+5\\ +3\end{bmatrix} = \begin{bmatrix}+2\\ +1\end{bmatrix} + \begin{bmatrix}+10\\ +06\end{bmatrix} = \begin{bmatrix}+12\\ +07\end{bmatrix}$$

$$\begin{bmatrix}+2 & +5\\ +1 & +3\end{bmatrix}\begin{bmatrix}+1\\ +2\end{bmatrix} = \begin{bmatrix}\left(\begin{bmatrix}+2\\ +5\end{bmatrix}^{T}\begin{bmatrix}+1\\ +2\end{bmatrix}\right)\\ \left(\begin{bmatrix}+1\\ +3\end{bmatrix}^{T}\begin{bmatrix}+1\\ +2\end{bmatrix}\right)\end{bmatrix} = \begin{bmatrix}(2 * 1 + 5 * 2)\\ (1 * 1 + 3 * 2)\end{bmatrix} = \begin{bmatrix}+12\\ +07\end{bmatrix}$$

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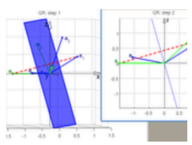
Referencias

GitHub	https://github.com/aegiloru/linearAlgebra
Lec 01 –MIT 18.06	https://www.youtube.com/watch?v=ZK3O402wf1c
MIT 18.06	https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/
Prof. Gilbert Strang	http://www-math.mit.edu/~gs/
DrawLA	https://la.mathworks.com/matlabcentral/fileexchange/23608-drawla-draw-toolbox-for-linear-algebra



drawLA - Draw Toolbox for Linear Algebra.

version 1.0.0.0 (415 KB) by Vladimir Bondarenko

Plot 2D/3D of vectors, planes, lines, spheres, and... display matrix equations.