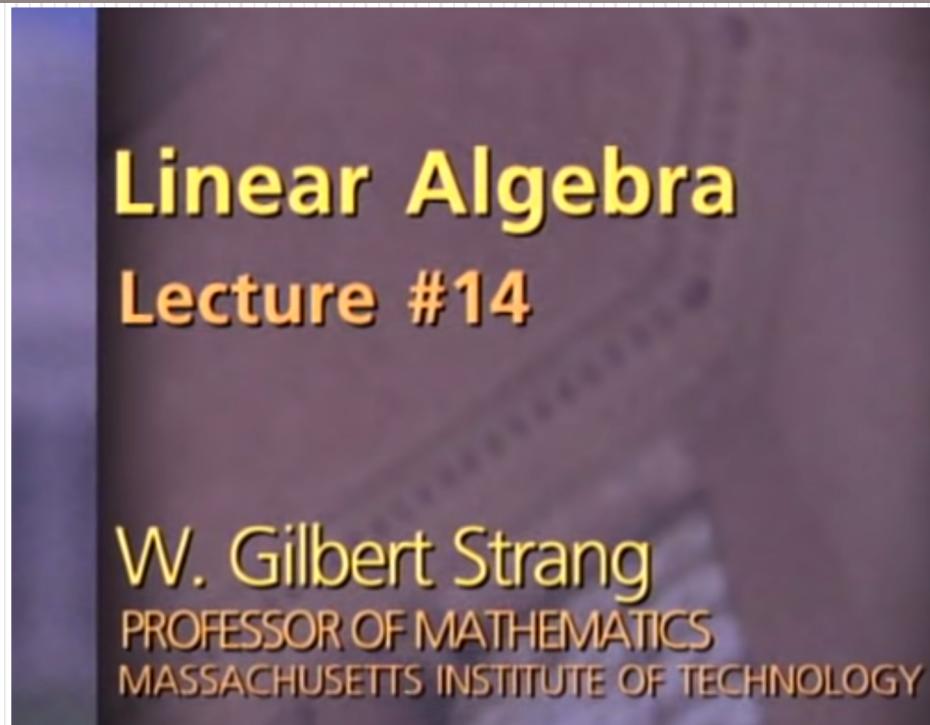


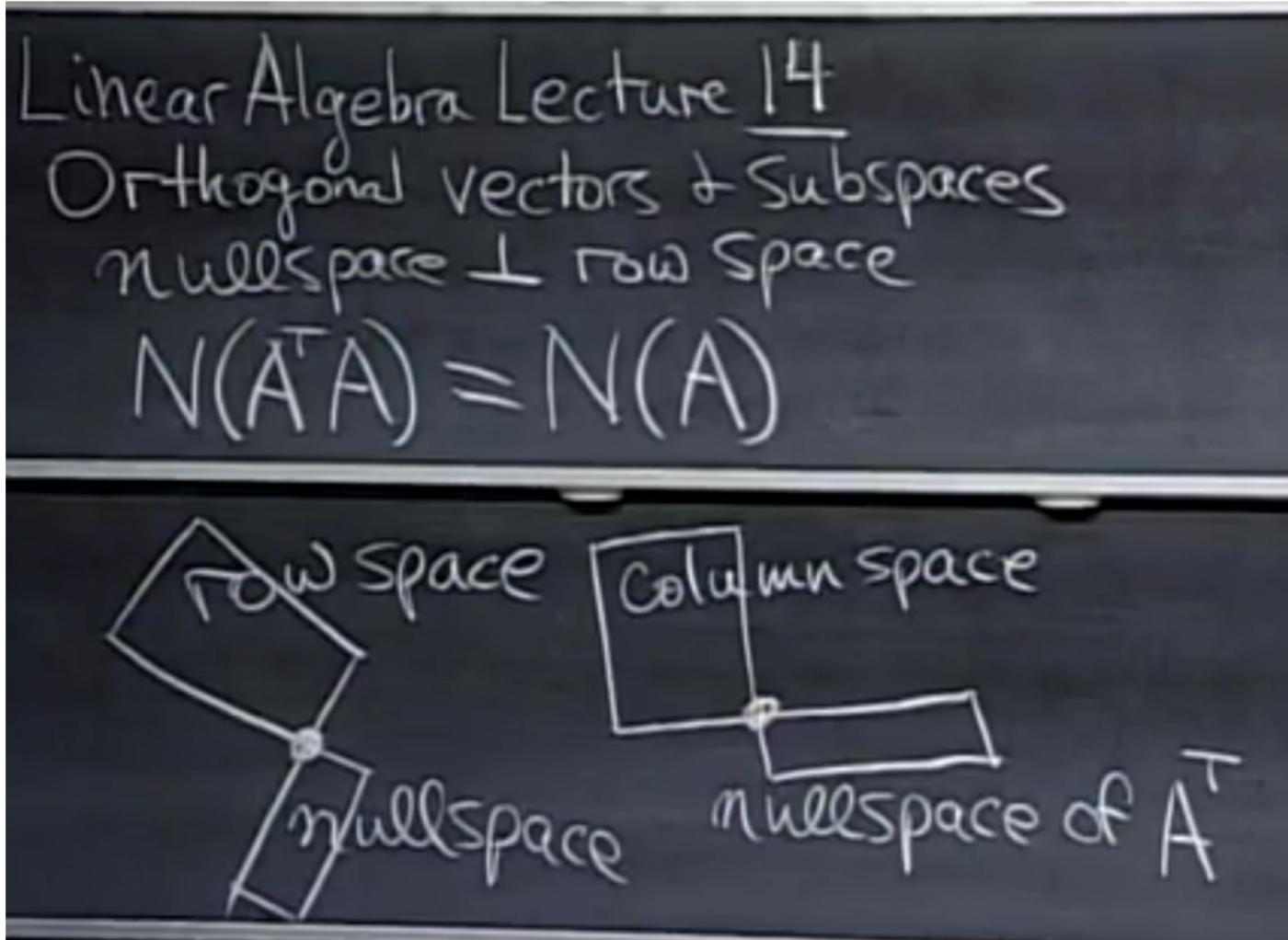
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Linear Algebra – Lecture #14

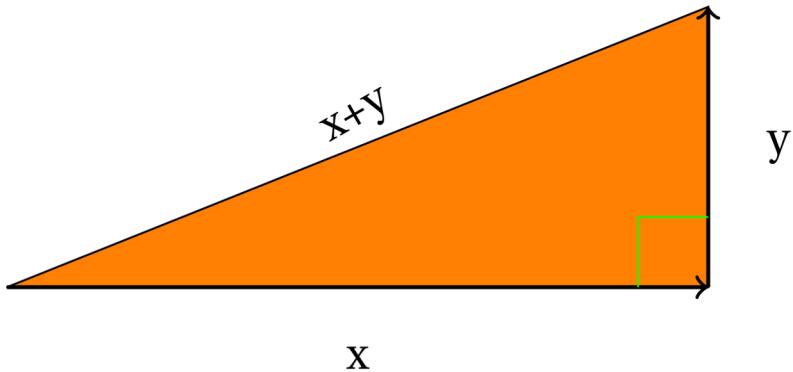
Orthogonal vectors and subspaces



Linear Algebra Lecture 14



Orthogonal Vectors



$$x^T y = 0$$

$$\|x\|^2 + \|y\|^2 = \|x + y\|^2 \quad \text{Pythagoras}$$

$$x^T x + y^T y = (x + y)^T (x + y)$$

$$x^T x + y^T y = x^T x + x^T y + y^T x + y^T y$$

~~$$x^T x + y^T y = x^T x + x^T y + y^T x + y^T y$$~~

$$0 = x^T y + y^T x = 2x^T y$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad x + y = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

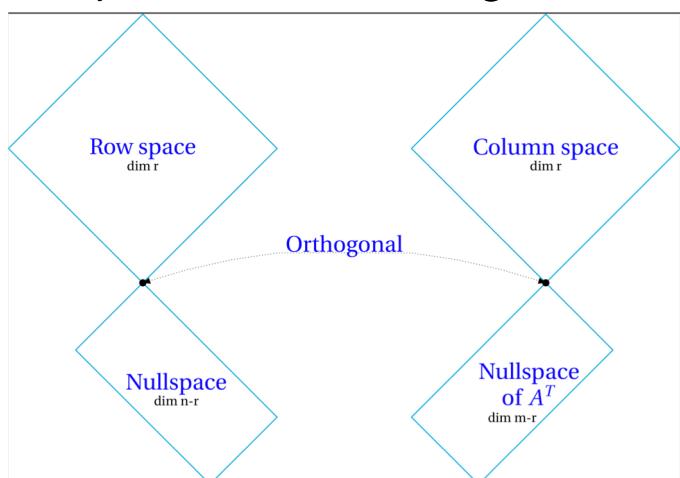
$$\|x\|^2 = 14 \quad \|y\|^2 = 5 \quad \|x + y\|^2 = 19$$

Then $x^T y = 0$ ■ Q.E.D.

Subspace S is orthogonal to Subspace T

Means:

Every vector in S is orthogonal to every vector in T



$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix} \quad n=3, r=1 \quad \text{Dim } N(A) = 2$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [0]$$

Row space is orthogonal to nullspace

Why? $Ax = 0$

$$\begin{bmatrix} \text{row}_1 \text{ of } A \\ \text{row}_2 \text{ of } A \\ \vdots \\ \text{row}_m \text{ of } A \end{bmatrix} [x] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

In the row space also it's the linear combination of row_i

$$(c_1 \text{row}_1 + c_2 \text{row}_2 + \dots)^T x = 0$$

Nullspace and rowspace are orthogonal complements in \mathbb{R}^n

Nullspace contains ALL vector \perp row space

Coming: Solve $Ax = b$

when there is not solution $m > n$

$$A^T \text{ } n \times m \text{ } A \text{ } m \times n$$

$n \times n$

symmetric

$$(A^T A)^T = A^T A^T^T = A^T A$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix} \quad r=2 \quad \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 30 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 3 \\ 1 & 3 \end{bmatrix} \quad r=1 \quad \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 27 \end{bmatrix}$$

$$\boxed{\begin{aligned} Ax &= b \\ \downarrow \\ A^T A \hat{x} &= A^T b \end{aligned}}$$

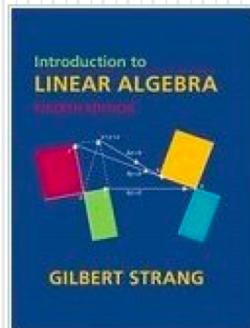
$A^T A$ is invertible exactly if A has independent columns

$$N(A^T A) = N(A)$$

Credits



[Gilbert Strang Web Site](#)



<https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/index.htm>

<http://www-math.mit.edu/~gs/>

<http://math.mit.edu/~gs/linearalgebra/>