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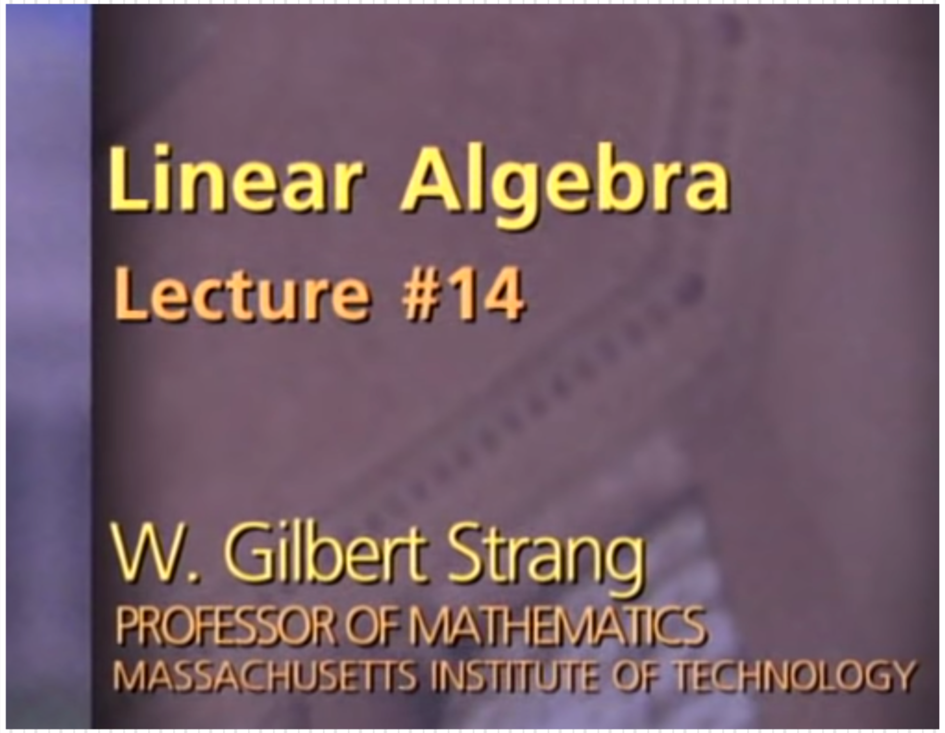
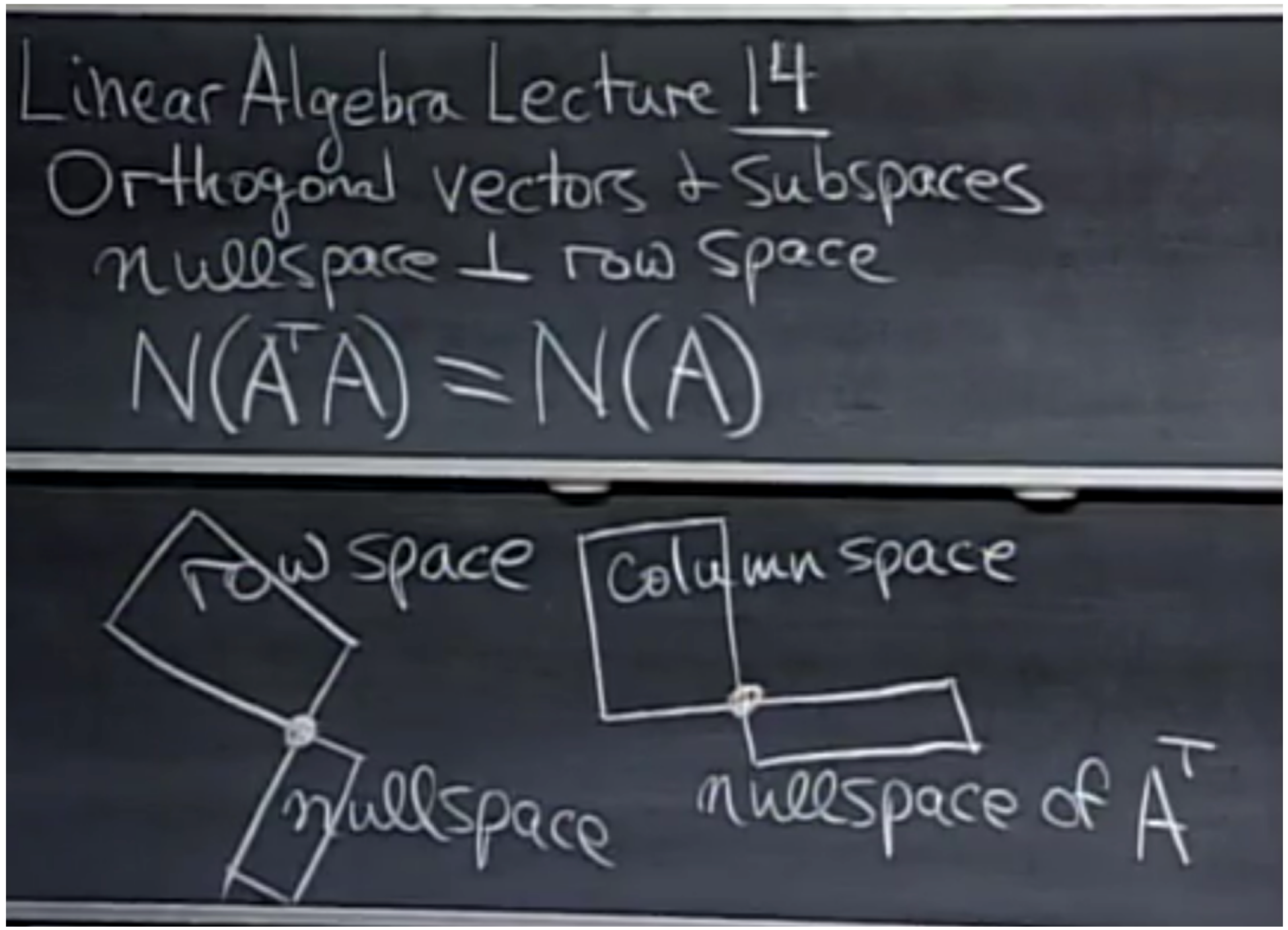
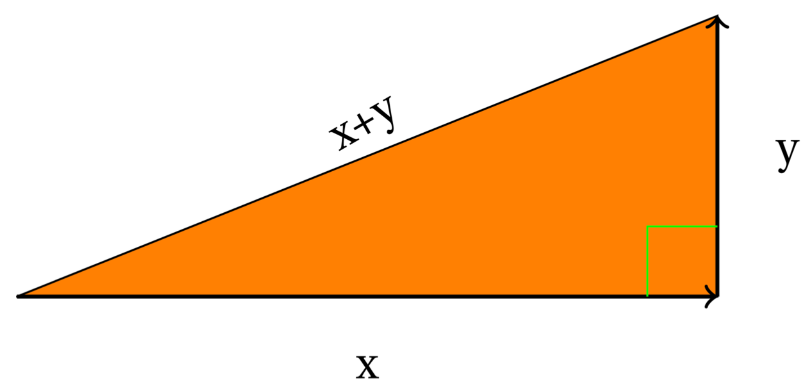
Linear Algebra – Lecture #14
Orthogonal vectors and subspaces



Linear Algebra Lecture 14



Orthogonal Vectors



$$x^T y = 0$$

$$\|x\|^2 + \|y\|^2 = \|x+y\|^2 \qquad \text{Pythagoras}$$

$$x^T x + y^T y = (x+y)^T (x+y)$$

$$x^T x + y^T y = x^T x + x^T y + y^T x + y^T y$$

$$\cancel{x^T x} + \cancel{y^T y} = \cancel{x^T x} + x^T y + y^T x + \cancel{y^T y}$$

$$0 = x^T y + y^T x = 2x^T y$$

Then $x^T y = 0$ ■ Q.E.D.

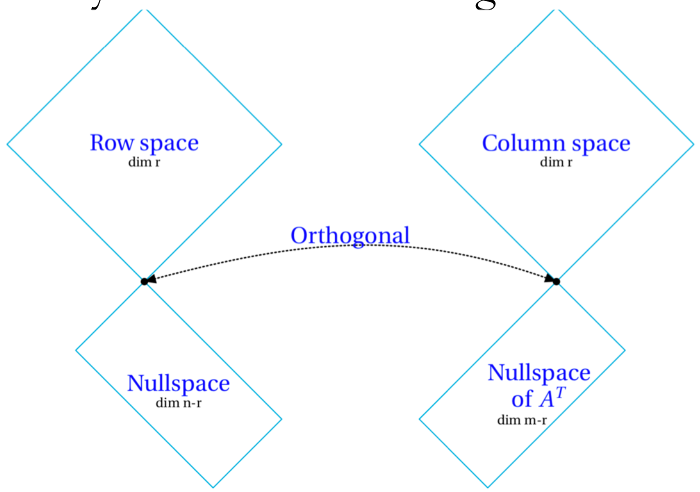
$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad x + y = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

$\|x\|^2 = 14$ $\|y\|^2 = 5$ $\|x+y\|^2 = 19$

Subpace **S** is orthogonal to Subspace **T**

Means:

Every vector in **S** is orthogonal to every vector in **T**



$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix}$$

n=3, r=1

Dim N(A) = 2

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [0]$$

Row space is orthogonal to nullspace

Why? $Ax = 0$

$$\begin{bmatrix} row_1\ of\ A \\ row_2\ of\ A \\ \vdots \\ row_m\ of\ A \end{bmatrix} [x] = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

In the row space also it's the linear combination of row_i

$$(c_1 row_1 + c_2 row_2 + \cdots)^T x = 0$$

Nullspace and rowspace are ***orthogonal complements*** **in $\mathbb{R}^n$**

Nullspace contains ALL vector $\perp$ row space

Coming: Solve $Ax = b$

when there is not solution m>n

$$A^T{}_{n\times m} A_{m\times n}$$

$n\times n$

symmetric

$(A^TA)^T = A^T A^{T^T} = A^T A$

$$\boxed{\begin{array}{c} Ax = b \\ \downarrow \\ A^T A\hat{x} = A^T b \end{array}}$$

$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}$ r=2 $\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 8 & 30 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 3 \\ 1 & 3 \\ 1 & 3 \end{bmatrix}$ r=1 $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 3 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 9 \\ 9 & 27 \end{bmatrix}$

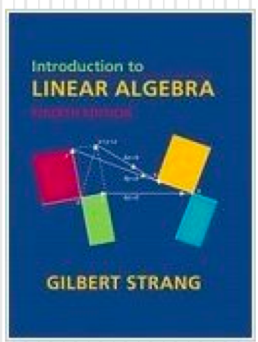
$A^T A$ is invertible exactly if A has independents columns

$$N(A^TA) = N(A)$$

Credits

https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/index.htm

[Gilbert Strang Web Site](http://www-math.mit.edu/~gs/) http://www-math.mit.edu/~gs/



http://math.mit.edu/~gs/linearalgebra/