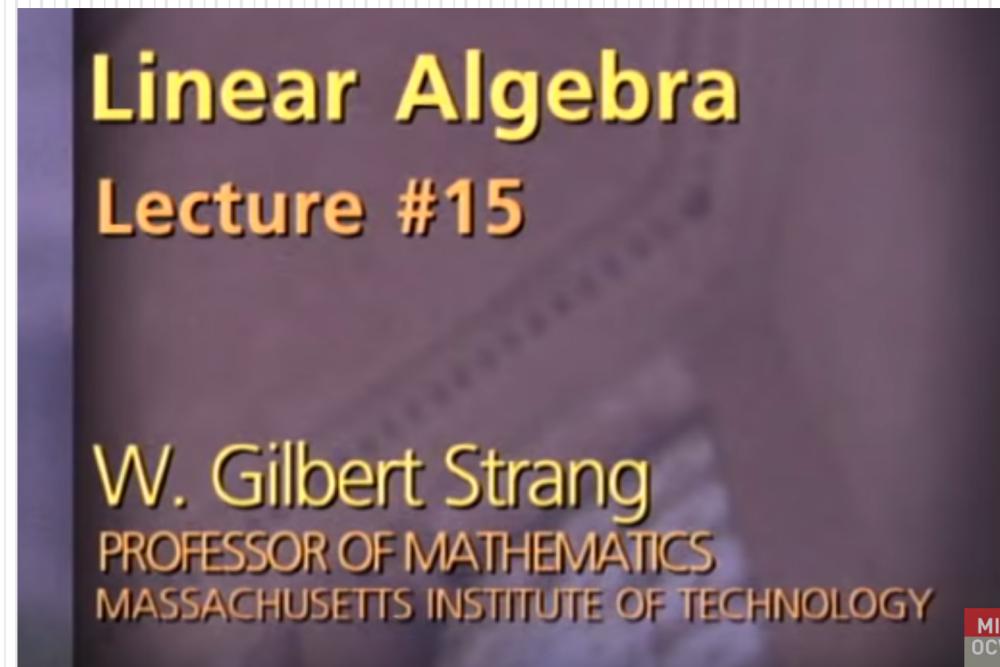


By Ariel Guerrero  
[ariel.guerrero@uc.edu.py](mailto:ariel.guerrero@uc.edu.py)  
+595 (981) 425 040

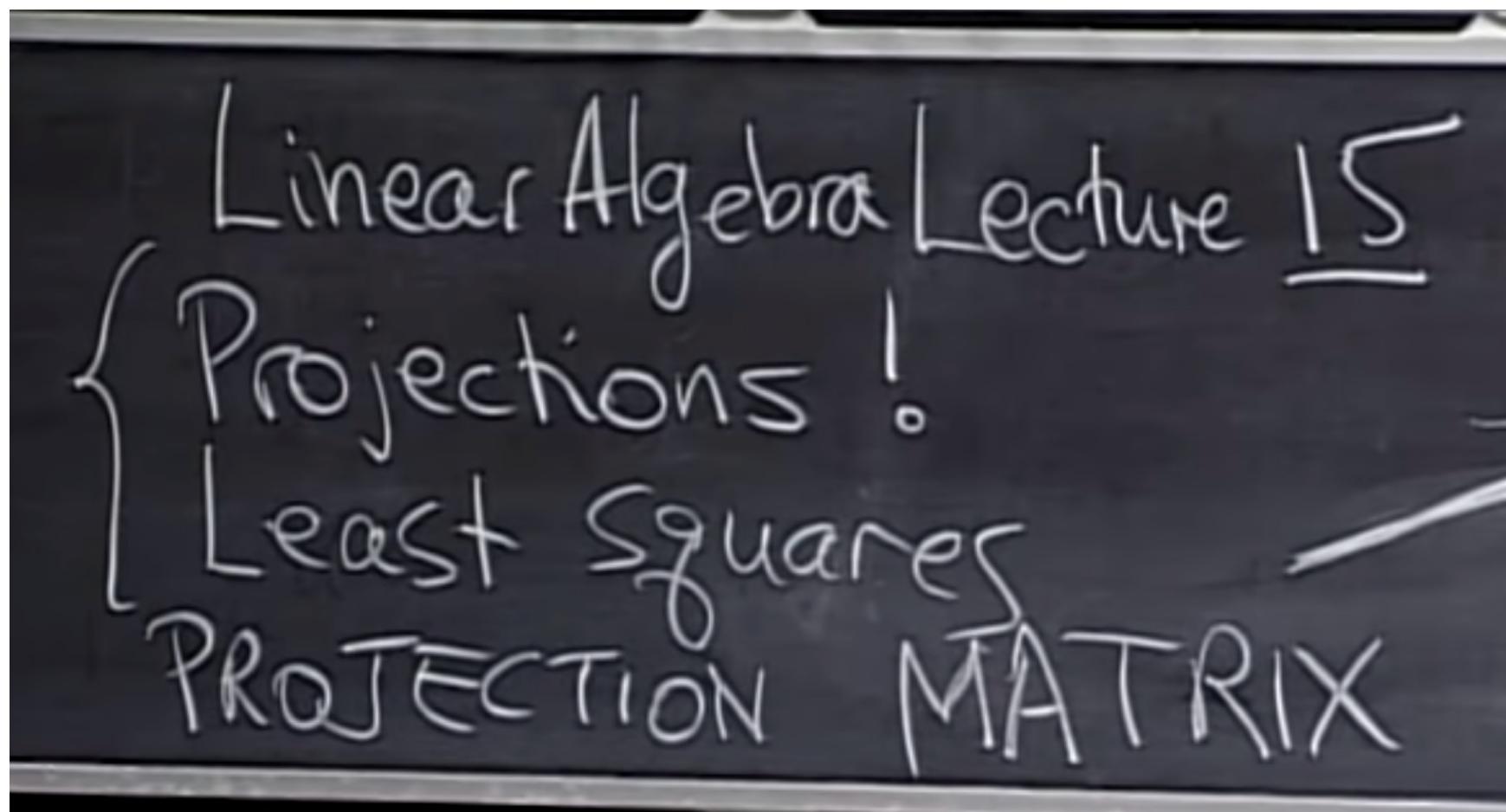


# Linear Algebra – Lecture #15

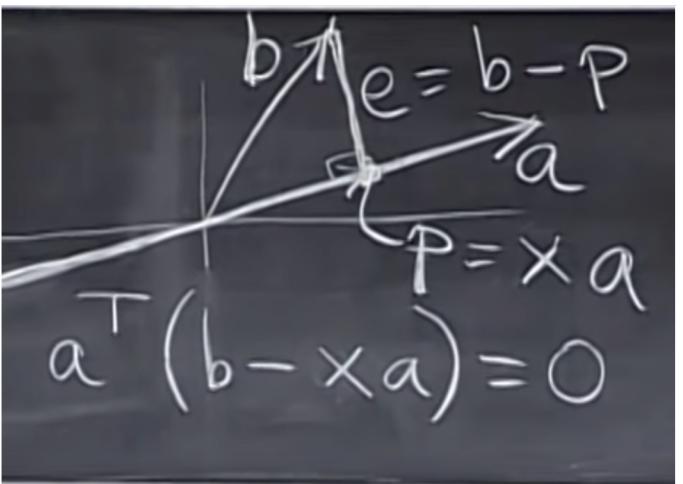
## Projections onto subspaces



# Linear Algebra Lecture 15



# Projections into a line



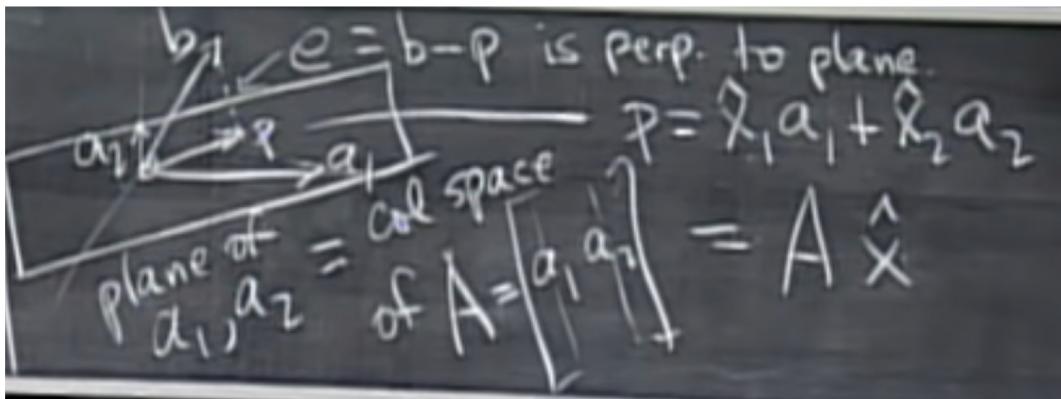
$$\begin{aligned} x a^T a &= a^T b \\ x = \frac{a^T b}{a^T a} &\quad P = a x \end{aligned}$$

$$\begin{aligned} P &= a \frac{a^T b}{a^T a} && \times \\ \text{Proj } P &= P b & \text{MATRIX } & \times \\ P &= \frac{a a^T}{a^T a} \end{aligned}$$

$$\begin{aligned} C(P) &= \text{line through } a \\ * \text{rank}(P) &= 1 \\ \boxed{P^T = P} & \quad \boxed{P^2 = P} \end{aligned}$$

# Projections into a subspace

Why project?  
Because  $Ax = b$  may have no solution  
Solve  $A\hat{x} = P$  instead  
↓ proj of  $b$  onto col space



$P = A\hat{x}$  Find  $\hat{x}$   
Key:  $b - A\hat{x}$  is perp. to plane  
 $a_1^T(b - A\hat{x}) = 0$   $a_2^T(b - A\hat{x}) = 0$

$$\begin{bmatrix} a_1^T \\ a_2^T \end{bmatrix} (b - A\hat{x}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$A^T(b - A\hat{x}) = 0.$$

$e \in N(A^T)$   
 $e \perp C(A)$  YES!  
 $\boxed{A^T A \hat{x} = A^T b}$

# Projections into a subspace

$$\hat{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$

⊗  $\frac{\mathbf{a}\mathbf{a}^T}{\mathbf{a}^T \mathbf{a}}$

$$P = \mathbf{A} \hat{x} = \underbrace{\mathbf{A} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T}_{\text{matrix } P} \mathbf{b}$$

$$\mathbf{A}^T \mathbf{b}$$

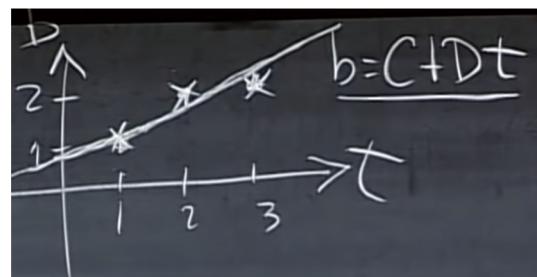
⊗  $\frac{\mathbf{a}\mathbf{a}^T}{\mathbf{a}^T \mathbf{a}}$

$$= \mathbf{A} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}$$
$$= \mathbf{A} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \cancel{\mathbf{A} \mathbf{A}^T \mathbf{A}} \mathbf{A}^T$$

$P^T = P$   
 $P^2 = P$

# Least square fitting by a line

Least squares  
Fitting by a line  
 $(1,1), (2,2), (3,2)$



$$\begin{aligned}C + D &= 1 \\C + 2D &= 2 \\C + 3D &= 2\end{aligned}\quad \begin{bmatrix}1 & 1 \\1 & 2 \\1 & 3\end{bmatrix} \begin{bmatrix}C \\D\end{bmatrix} = \begin{bmatrix}1 \\2 \\2\end{bmatrix}$$

$A \times b$

The video ends with the problem statement

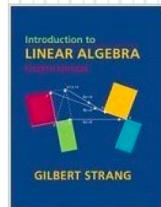


<https://github.com/aegiloru/linearAlgebra>

# Credits



[Gilbert Strang Web Site](#)



<https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/index.htm>

<http://www-math.mit.edu/~gs/>

<http://math.mit.edu/~gs/linearalgebra/>