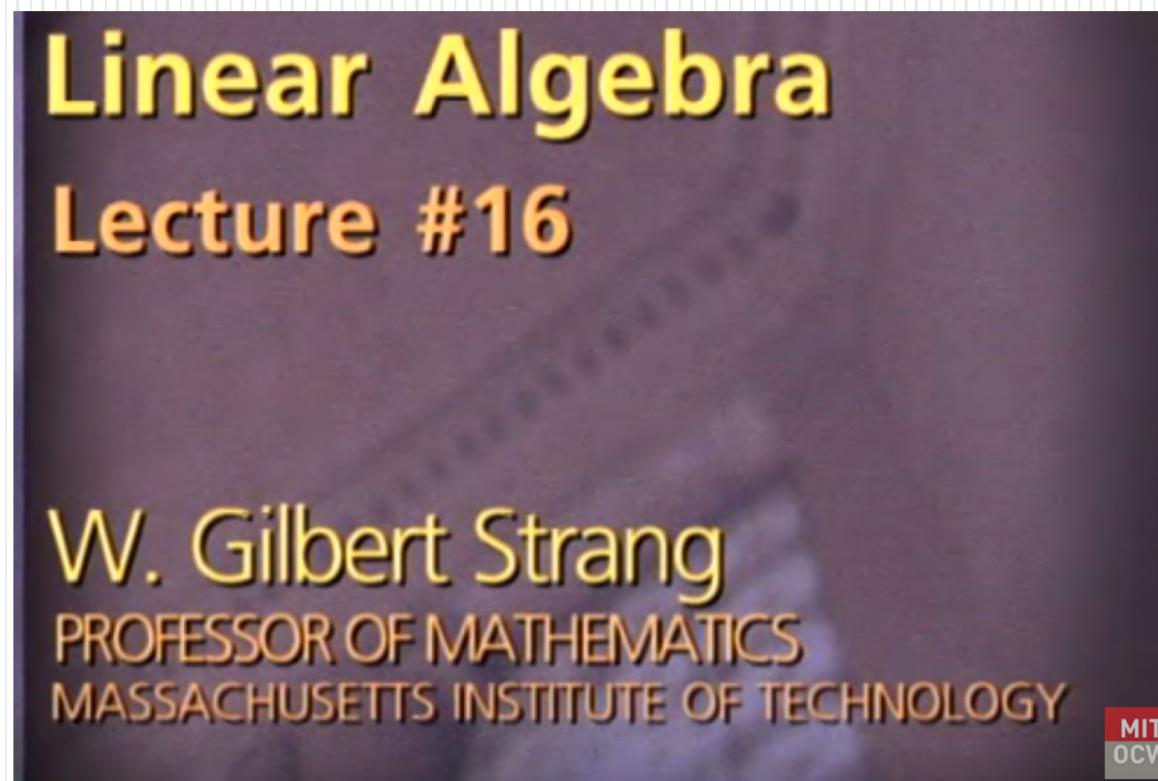


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# Linear Algebra – Lecture #16

## Projections



# Linear Algebra Lecture 16

Linear Algebra Lecture 16 Proj. matrix  
Projections  $P = A(A^T A)^{-1} A^T$   
Least squares and best straight line If  $b$  in column space  $Pb = b$   
If  $b \perp$  column space  $Pb = 0$

# Projections

16 Proj. matrix

$$P = P_F = A (A^T A)^{-1} A^T b$$

If  $b$  in column space  $Pb = b$

If  $b \perp$  column space  $Pb = 0$

The results is zero

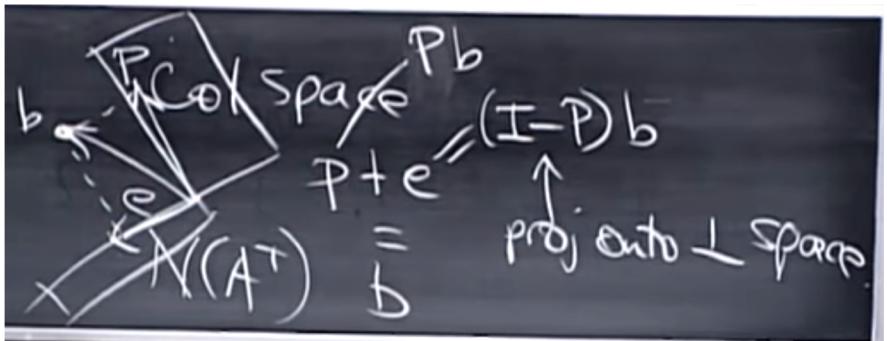
16 Proj. matrix

$$P = P_F = A (A^T A)^{-1} A^T Ax$$

If  $b$  in column space  $Pb = b$

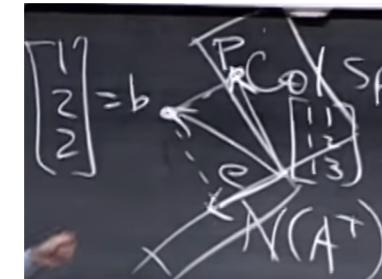
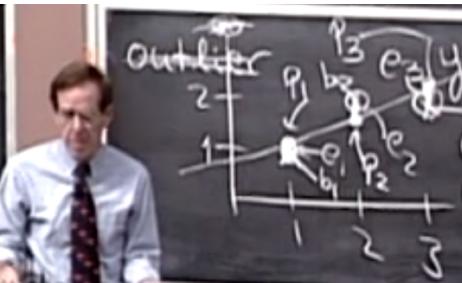
If  $b \perp$  column space  $Pb = 0$

The results is  $b$



# Least squares and best straight line

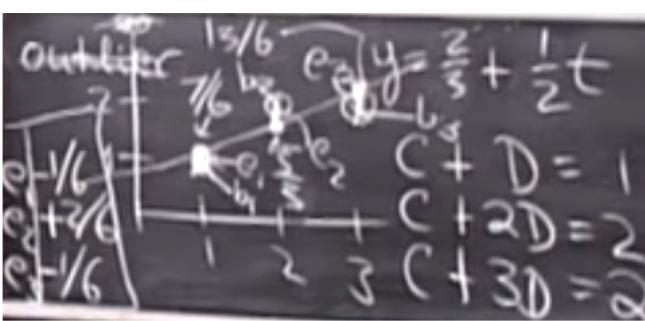
$$\begin{aligned}
 & \text{Graph: } y = C + Dt \\
 & \text{Equations: } \\
 & \begin{aligned} C + D = 1 \\ C + 2D = 2 \\ C + 3D = 2 \end{aligned} \\
 & \text{Matrix form: } \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}
 \end{aligned}$$



$$\begin{aligned}
 & \text{Find } \hat{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 6 & 5 \\ 6 & 14 & 11 \end{bmatrix} \\
 & A^T A \hat{x} = A^T b \text{ normal eqns}
 \end{aligned}$$

$$\begin{aligned}
 3C + 6D &= 5 \\
 6C + 14D &= 11 \\
 \hline
 2D &= 1
 \end{aligned}
 \quad \begin{cases} 3C + 3 = 5 \\ D = \frac{1}{2} \\ C = \frac{2}{3} \end{cases}$$

$$\begin{aligned}
 & \text{Minimize } \|Ax - b\|^2 = \|e\|^2 \\
 & = e_1^2 + e_2^2 + e_3^2 \\
 & = (C + D - 1)^2 + (C + 2D - 2)^2 \\
 & \quad + (C + 3D - 2)^2
 \end{aligned}$$

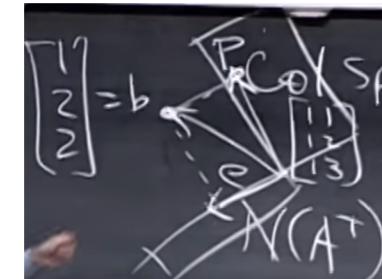
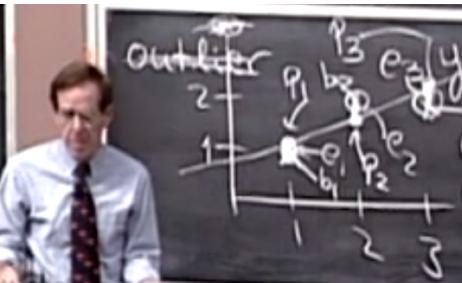


$$\text{Best line } \frac{2}{3} + \frac{1}{2}t$$

$$\begin{aligned}
 b &= p + e \\
 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} &= \begin{bmatrix} 7/6 \\ 5/3 \\ 13/6 \end{bmatrix} + \begin{bmatrix} 1/6 \\ 2/6 \\ -1/6 \end{bmatrix}
 \end{aligned}$$

# Least squares and best straight line

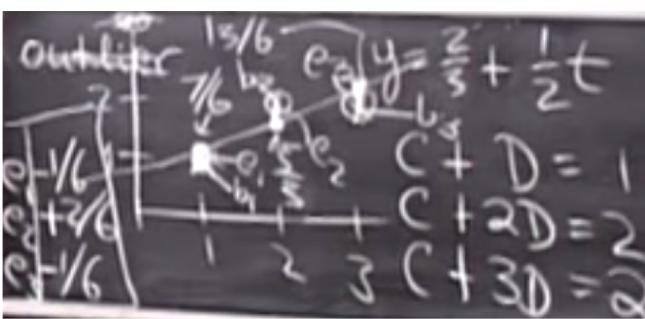
$$\begin{aligned}
 & \text{Graph: } y = C + Dt \\
 & \text{Equations: } \\
 & \begin{aligned} C + D = 1 \\ C + 2D = 2 \\ C + 3D = 2 \end{aligned} \\
 & \text{Matrix form: } \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}
 \end{aligned}$$



$$\begin{aligned}
 & \text{Function: } \hat{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, P = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & 6 & 5 \\ 6 & 14 & 11 \end{bmatrix} \\
 & A^T A \hat{x} = A^T b \text{ normal eqns}
 \end{aligned}$$

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 & \quad + (C + 3D - 2)^2
 \end{aligned}$$



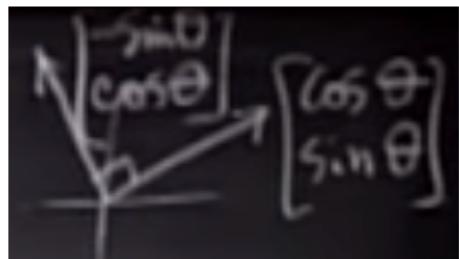
$$\text{Best line } \frac{2}{3} + \frac{1}{2}t$$

$$\begin{aligned}
 b &= p + e \\
 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} &= \begin{bmatrix} 7/6 \\ 5/3 \\ 13/6 \end{bmatrix} + \begin{bmatrix} 1/6 \\ 2/6 \\ -1/6 \end{bmatrix}
 \end{aligned}$$

# Least squares and best straight line

$$\begin{aligned} \text{Find } \hat{\mathbf{x}} &= \begin{bmatrix} \hat{c} \\ \hat{d} \end{bmatrix} \\ \left\{ \begin{array}{l} \mathbf{A}^T \mathbf{A} \hat{\mathbf{x}} = \mathbf{A}^T \mathbf{b} \\ \mathbf{P} = \mathbf{A} \hat{\mathbf{x}} \end{array} \right. \end{aligned}$$

IF  $\mathbf{A}$  has indep columns  $\leftarrow$   
then  $\mathbf{A}^T \mathbf{A}$  is invertible <sup>TO PROVE</sup>  
Suppose  $\mathbf{A}^T \mathbf{A} \mathbf{x} = \mathbf{0}$ .  $\mathbf{x}$  must be  $\mathbf{0}$  <sup>(square)</sup>  $\xrightarrow{\mathbf{A} \text{ has indep cols}}$   $\mathbf{A} \mathbf{x} = \mathbf{0} \Rightarrow \mathbf{x} = \mathbf{0}$



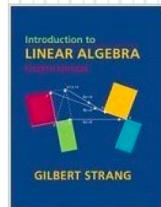


<https://github.com/aegiloru/linearAlgebra>

# Credits



[Gilbert Strang Web Site](#)



<https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/index.htm>

<http://www-math.mit.edu/~gs/>

<http://math.mit.edu/~gs/linearalgebra/>