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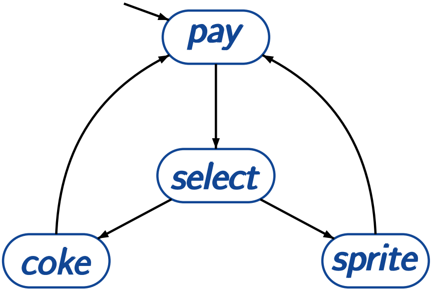
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \to 2^{AP}$ the labeling function

Transition system for beverage machine



Program graph (PG)

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example: if α is the assignment $x:=x+y$ then

$$Effect(\alpha, [x=1, y=7]) = [x=8, y=7]$$

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TS-semantics of a program graph

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$$\boxed{\begin{array}{c}\text{program graph } \mathcal{P} \text{ over } \textit{Var} \\ \Downarrow \\ \text{transition system } \mathcal{T}_{\mathcal{P}}\end{array}}$$

TS-semantics of a program graph

program graph $\mathcal{P}$ over $\boldsymbol{Var}$

$$\Downarrow$$

transition system $\mathcal{T}_{\mathcal{P}}$

states in $\mathcal{T}_{\mathcal{P}}$ have the form

$$\langle \ell, \eta \rangle$$

location   variable evaluation

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Let $\mathcal{P} = (Loc, Act, Effect, \hookrightarrow, Loc_0, g_0)$ be a PG.
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Structured operational semantics (SOS)

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$$\frac{premise}{conclusion}$$

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It means that $\longrightarrow$ is the smallest relation such that:

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Parallelism and communication

"real" parallel system

$$P = P_1 \parallel \ldots \parallel P_n$$

$$\uparrow \quad \downarrow$$

transition system

$$\mathcal{T} = \mathcal{T}_1 \parallel \ldots \parallel \mathcal{T}_n$$

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compositional design $\uparrow$ $\downarrow$ analysis semantics

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goal: define semantic parallel operators on transition systems or program graphs that model "real" parallel operators

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- interleaving of concurrent, independent actions of parallel processes (modelled by TS)
- representation by nondeterministic choice:

  "which subprocess performs the next step?"

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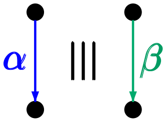

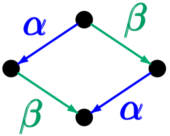

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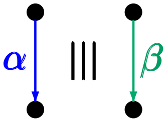

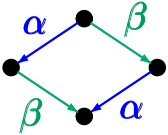

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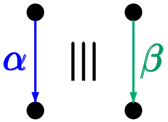

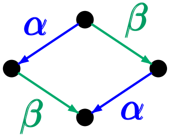

parallel execution of α and β on *two processors* $\widehat{=}$ serial execution on a *single processor* in arbitrary order

Interleaving operator $|||$ for TS

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Interleaving operator ||| for TS

PC2.2-DEF-INTERLEAVING-TS

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The transition system $\mathcal{T}_1 \,|||\, \mathcal{T}_2$ is defined by:

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SOS-notation (structured operational semantics)

just a simple notation for operational semantics

$$\frac{\textbf{premise}}{\textbf{conclusion}}$$

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E.g., *"the relation $\longrightarrow$ is given by ..."*

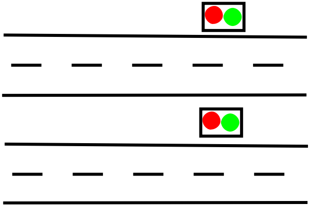
$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \qquad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

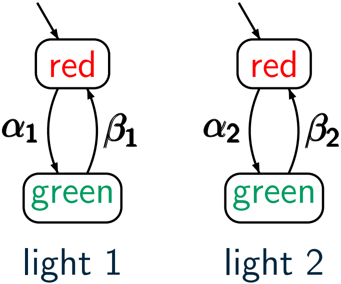
means that $\longrightarrow$ is the smallest relation such that:

(1) If $s_1 \xrightarrow{\alpha}_1 s_1'$, then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle$

(2) If $s_2 \xrightarrow{\alpha}_2 s_2'$, then $\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle$

Useless lights for non-crossing streets





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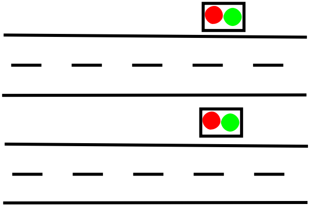
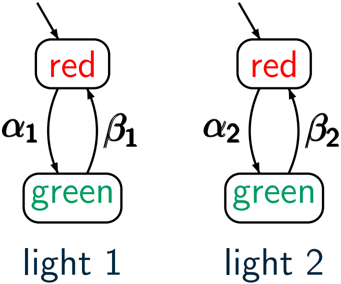
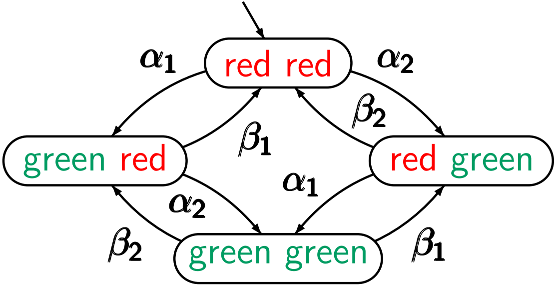




light 1 ||| light 2



Dependent actions

dependent actions $\alpha \mathrel{\widehat{=}} x{:=}2x$ and $\beta \mathrel{\widehat{=}} x{:=}x{+}1$

representations in transition systems

$$\begin{array}{c} x{=}3 \\ \downarrow \alpha \\ x{=}6 \end{array} \qquad\qquad \begin{array}{c} x{=}3 \\ \downarrow \beta \\ x{=}4 \end{array}$$

Dependent actions

PC2.2-5

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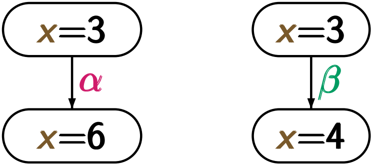
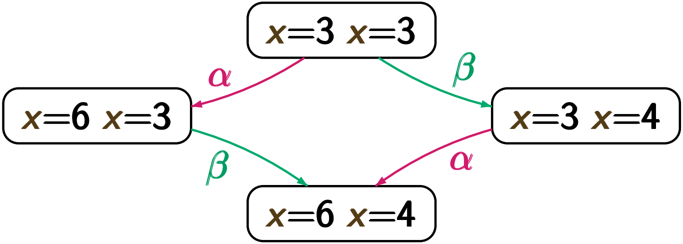


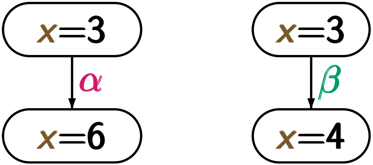
interleaving operator $|||$



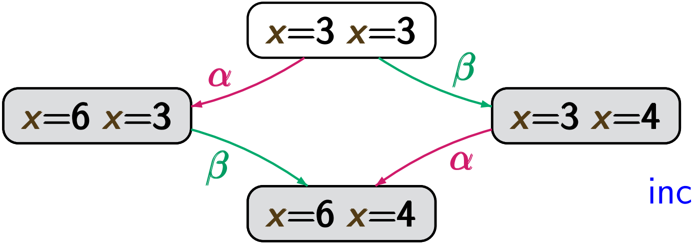
Interleaving fails for dependent actions

dependent actions $\alpha \mathrel{\widehat{=}} x{:=}2x$ and $\beta \mathrel{\widehat{=}} x{:=}x{+}1$

representations in transition systems



interleaving operator $|||$ for transition systems "fails"



inconsistent states

Interleaving for program graphs

. . . for modeling parallel systems with subprocesses communicating via shared variables

Interleaving for program graphs

program graph $\mathcal{P}_1$
$(Loc_1, \ldots, \hookrightarrow_1, \ldots)$

program graph $\mathcal{P}_2$
$(Loc_2, \ldots, \hookrightarrow_2, \ldots)$

interleaving operator

$$\mathcal{P}_1 \,|||\, \mathcal{P}_2 \;=\; (Loc_1 \times Loc_2, \ldots, \hookrightarrow, \ldots)$$

Interleaving for program graphs

program graph $\mathcal{P}_1$
$(Loc_1, \ldots, \hookrightarrow_1, \ldots)$

program graph $\mathcal{P}_2$
$(Loc_2, \ldots, \hookrightarrow_2, \ldots)$

interleaving operator

$$\mathcal{P}_1 \,|||\, \mathcal{P}_2 \;=\; (Loc_1 \times Loc_2, \ldots, \hookrightarrow, \ldots)$$

$$\frac{\ell_1 \overset{g:\alpha}{\hookrightarrow}_1 \ell_1'}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha}{\hookrightarrow} \langle \ell_1', \ell_2 \rangle} \qquad \frac{\ell_2 \overset{g:\alpha}{\hookrightarrow}_2 \ell_2'}{\langle \ell_1, \ell_2 \rangle \overset{g:\alpha}{\hookrightarrow} \langle \ell_1, \ell_2' \rangle}$$

Example: interleaving for PG

$\mathcal{P}_1$

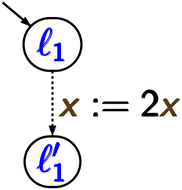
$$\begin{tikzcd} \ell_1 \arrow[d, dotted, "x := 2x"] \\ \ell_1' \end{tikzcd}$$

$\mathcal{P}_2$

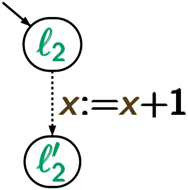
$$\begin{tikzcd} \ell_2 \arrow[d, dotted, "x := x+1"] \\ \ell_2' \end{tikzcd}$$

Example: interleaving for PG

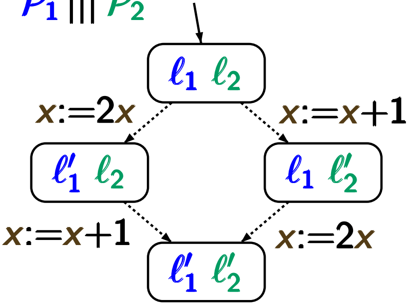
$\mathcal{P}_1$



$\mathcal{P}_2$

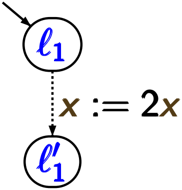


$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

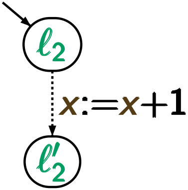


Example: interleaving for PG

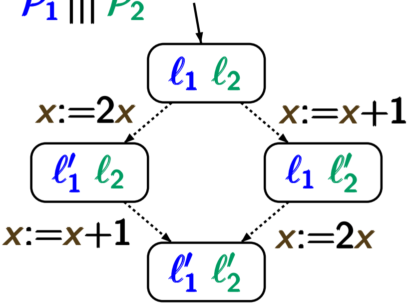
$\mathcal{P}_1$



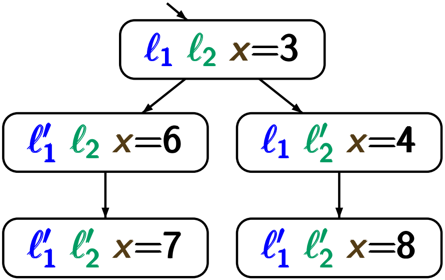
$\mathcal{P}_2$



$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

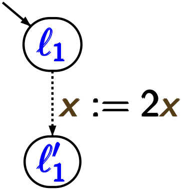


transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$

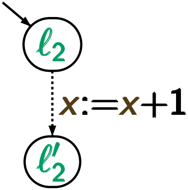


Example: interleaving for PG

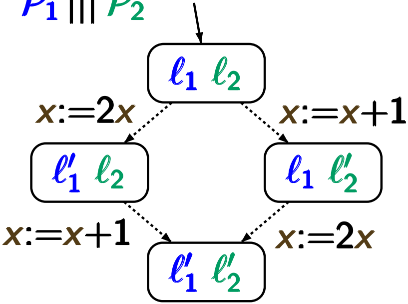
$\mathcal{P}_1$



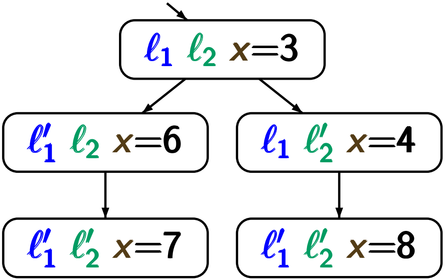
$\mathcal{P}_2$



$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

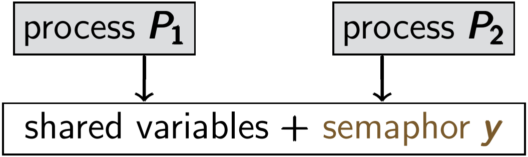


transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



note: $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2} \neq \mathcal{T}_{\mathcal{P}_1} \,|||\, \mathcal{T}_{\mathcal{P}_2}$

Mutual exclusion with semaphore



Mutual exclusion with semaphore

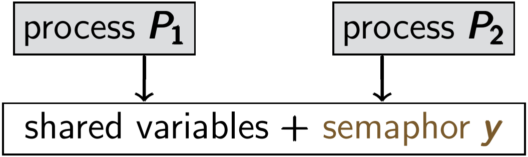
process P_1 process P_2

shared variables + semaphor y

protocol for process P_i

```
LOOP FOREVER
  noncritical actions;
  AWAIT y > 0 DO
          y:=y−1
  OD
  critical actions;
  y:=y+1
END LOOP
```

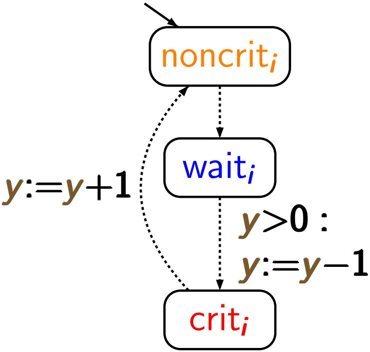
Mutual exclusion with semaphore



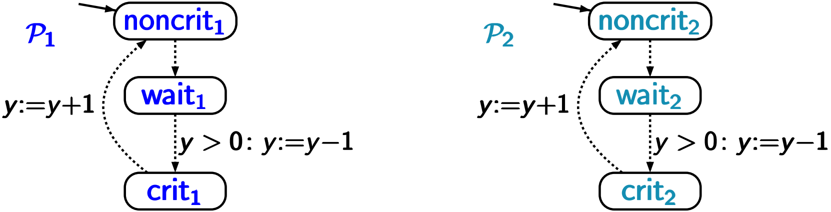
protocol for process P_i

```
LOOP FOREVER
  noncritical actions;
  AWAIT y > 0 DO
          y:=y−1
  OD
  critical actions;
  y:=y+1
END LOOP
```

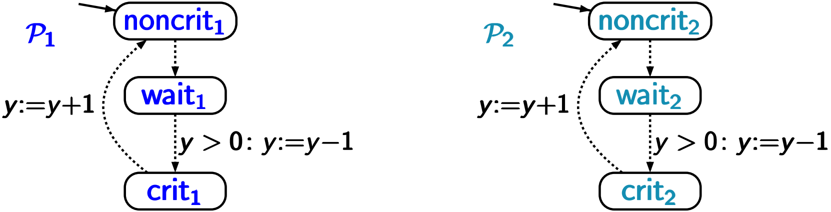
program graph $\mathcal{P}_i$



Mutual exclusion with semaphore

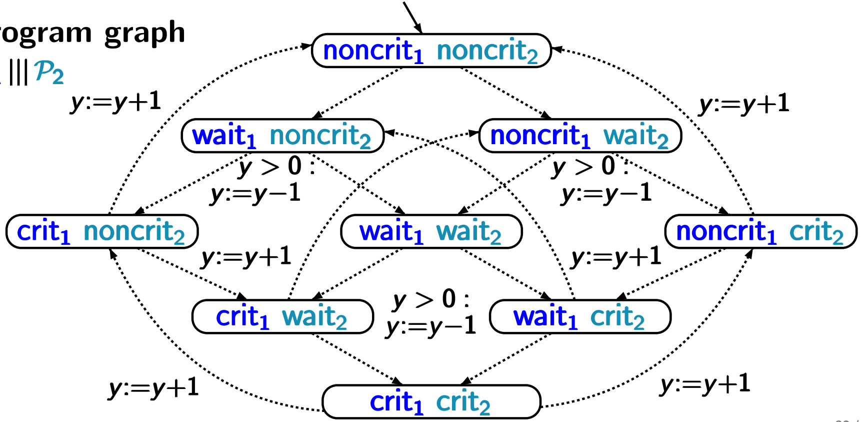


Mutual exclusion with semaphore

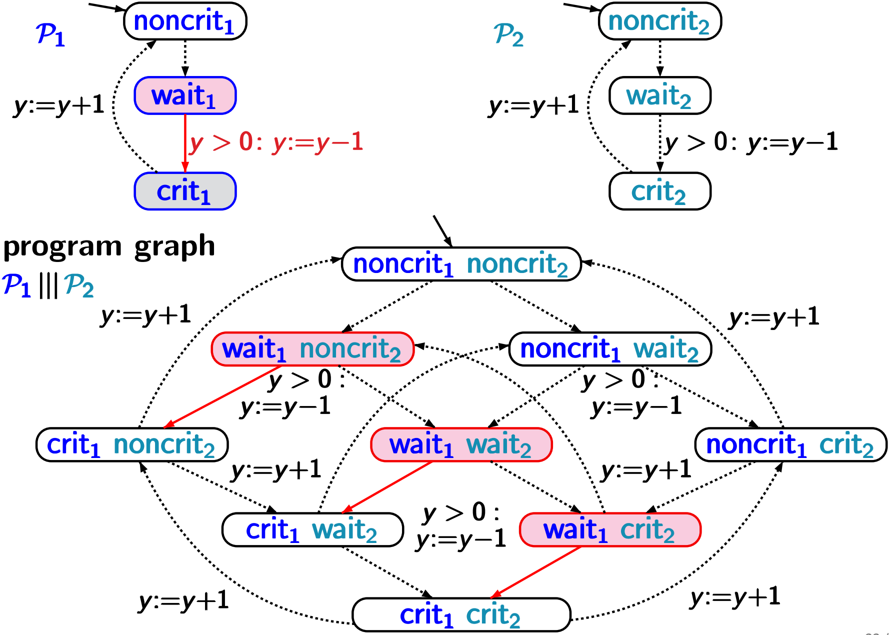


program graph

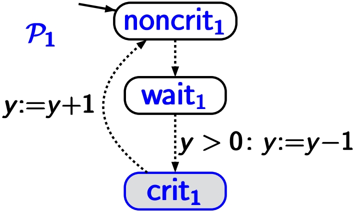
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

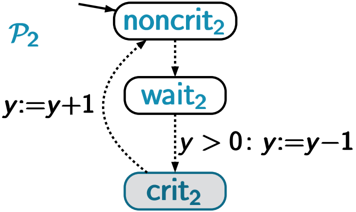


Mutual exclusion with semaphore



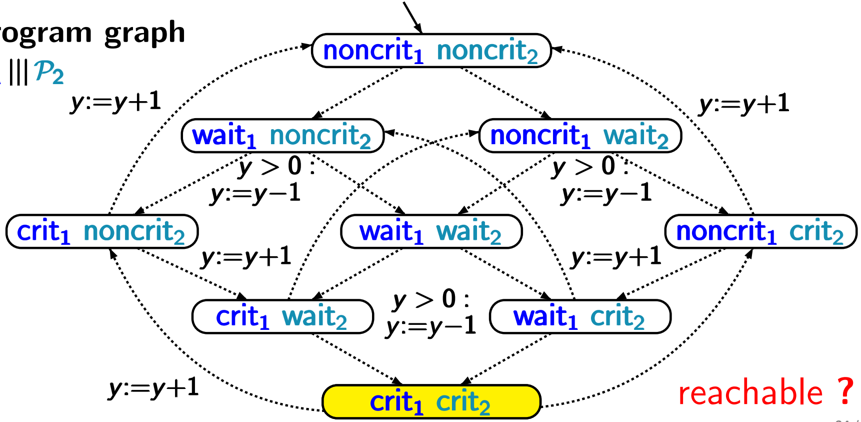
Mutual exclusion with semaphore





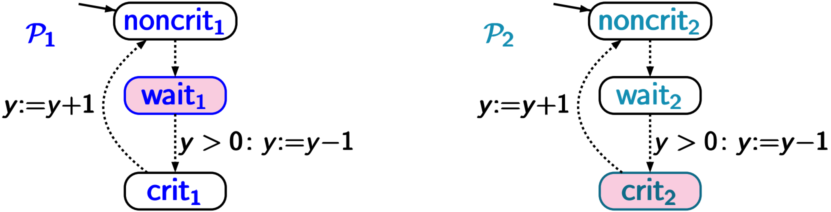
program graph

$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

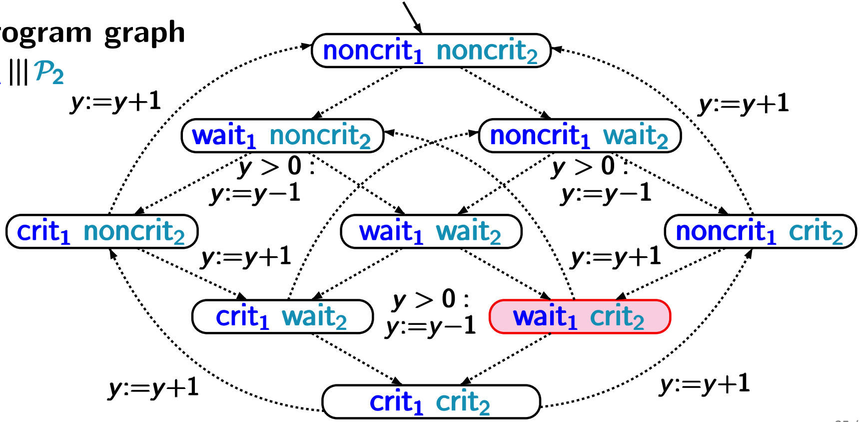


reachable **?**

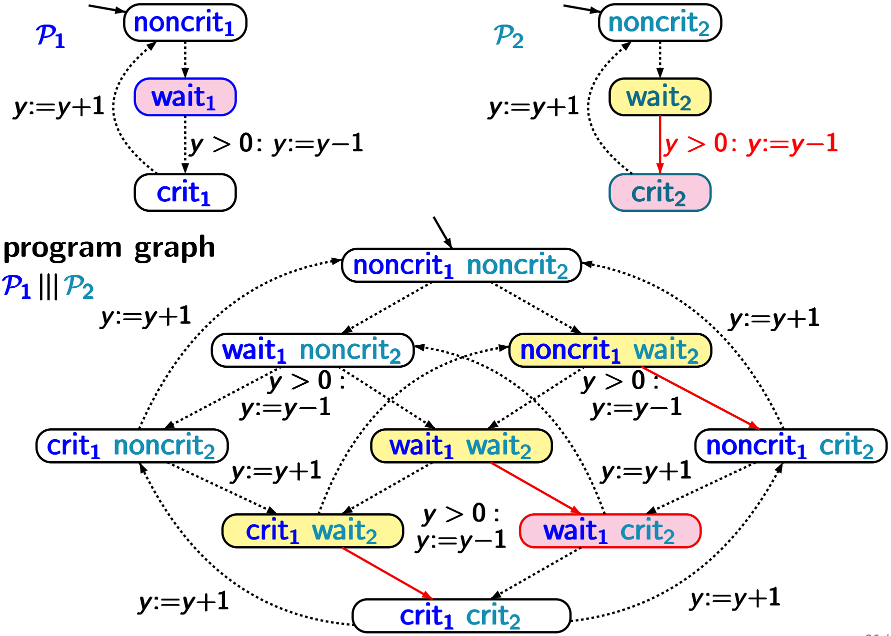
Mutual exclusion with semaphore



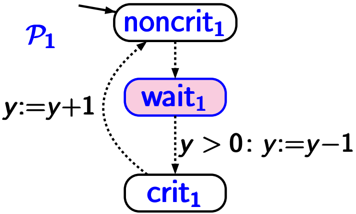
program graph
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

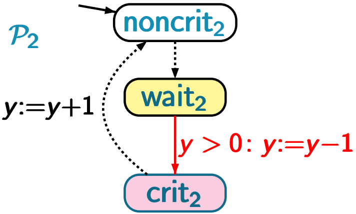


Mutual exclusion with semaphore

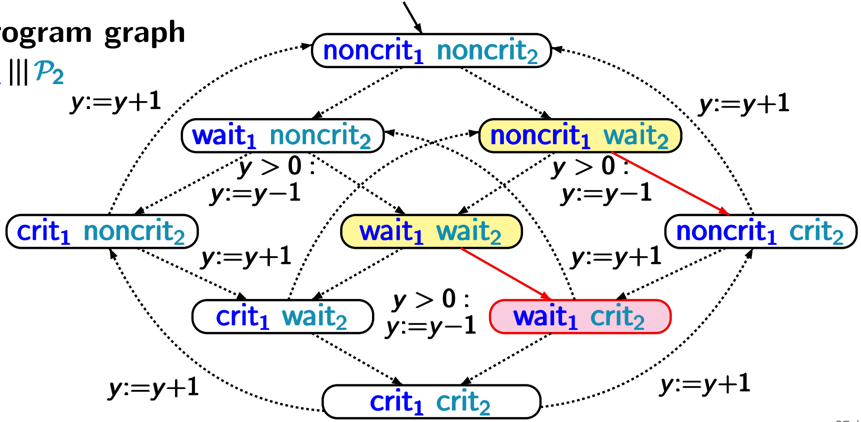


Mutual exclusion with semaphore

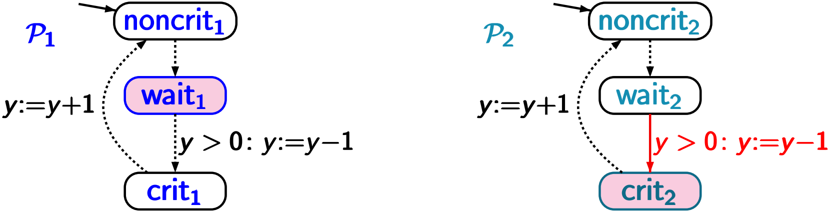




program graph
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$

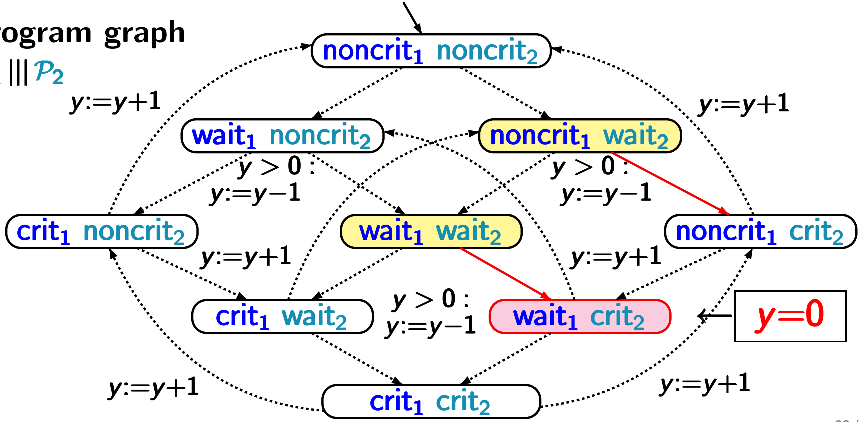


Mutual exclusion with semaphore

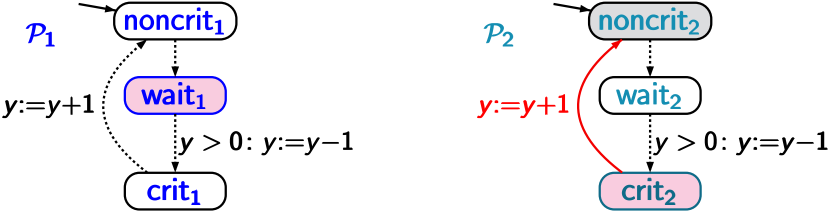


program graph

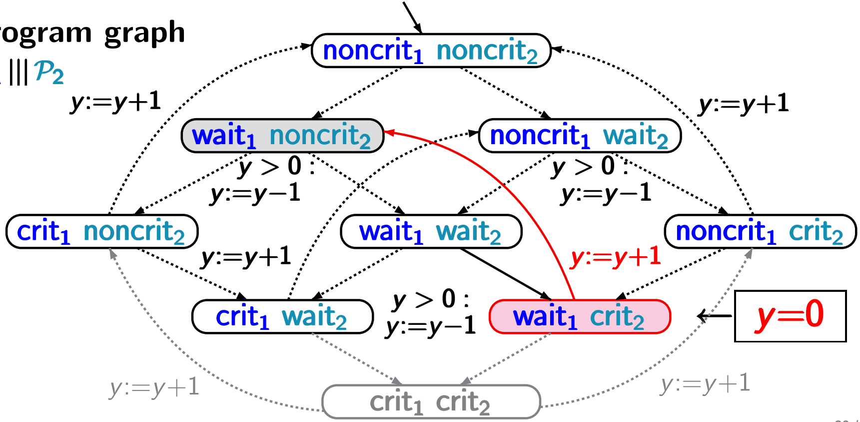
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



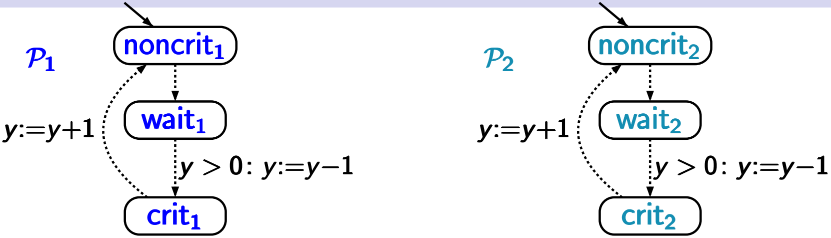
Mutual exclusion with semaphore



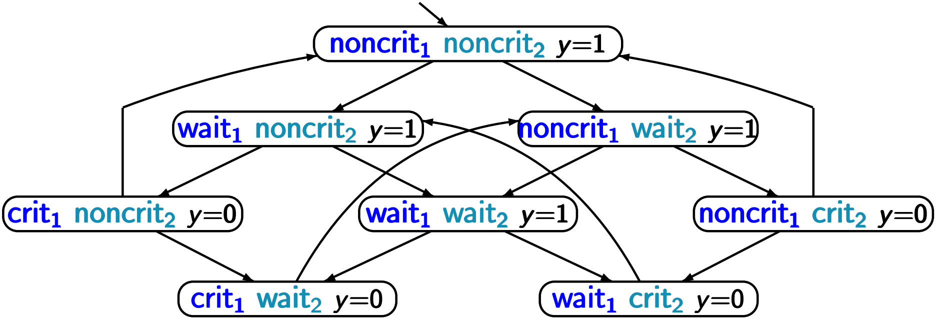
program graph
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



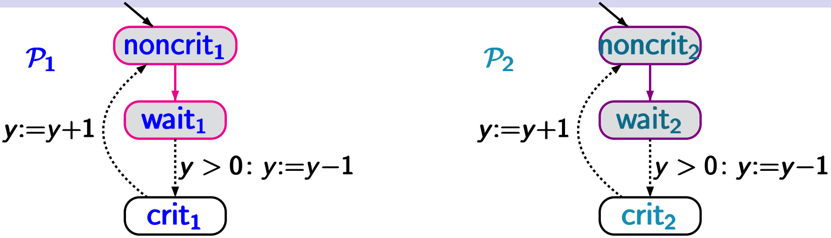
TS for mutual exclusion with semaphore



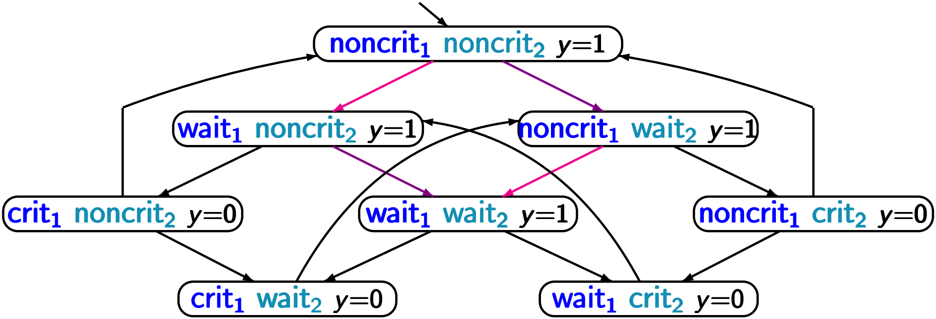
reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



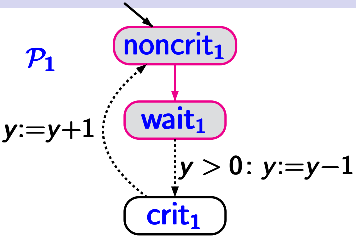
Concurrency of the request actions

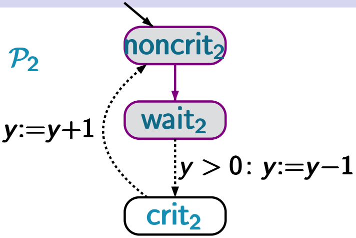


reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$

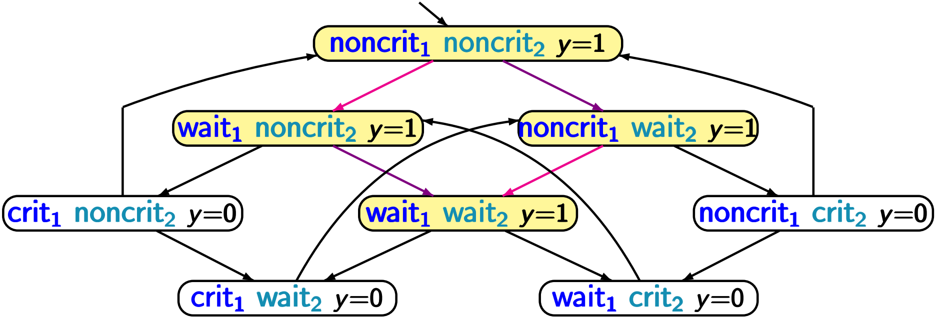


Concurrency of the request actions

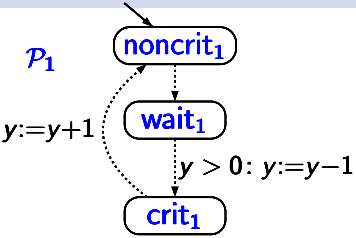


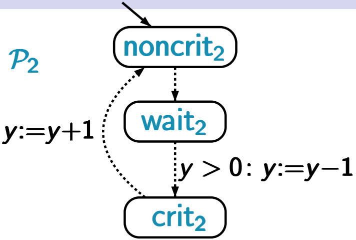


interleaving of the independent request actions

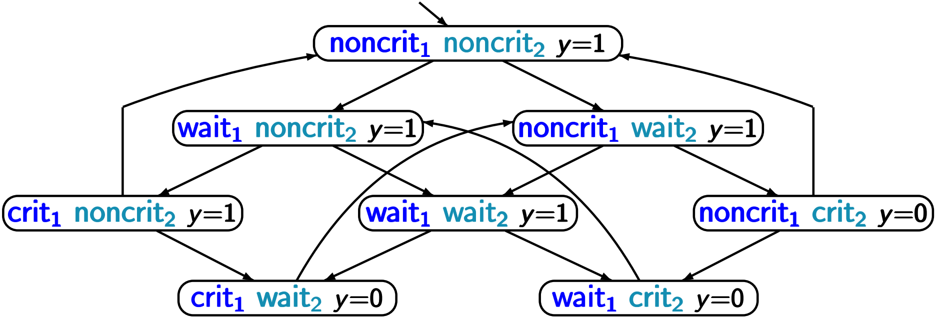


Competition

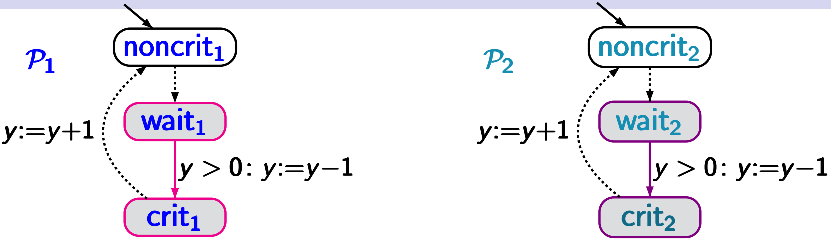




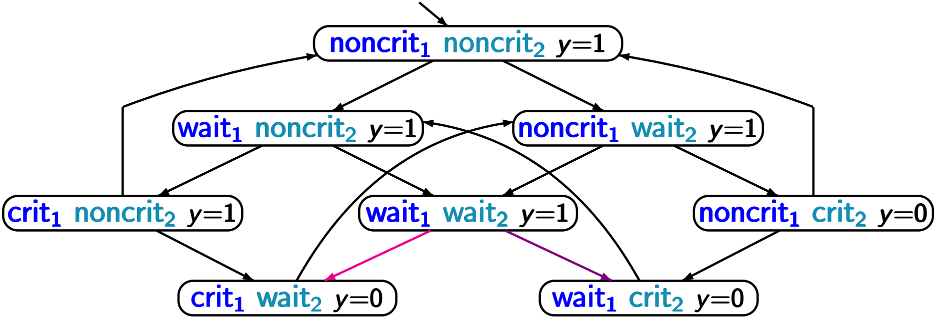
reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



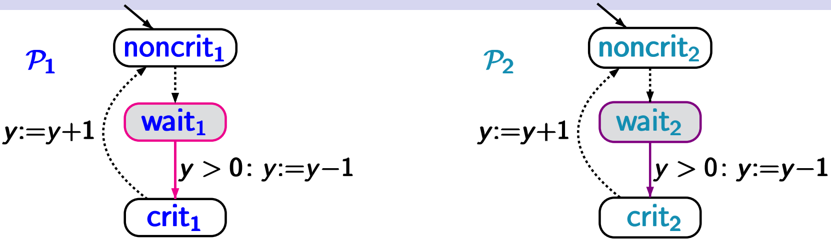
Competition



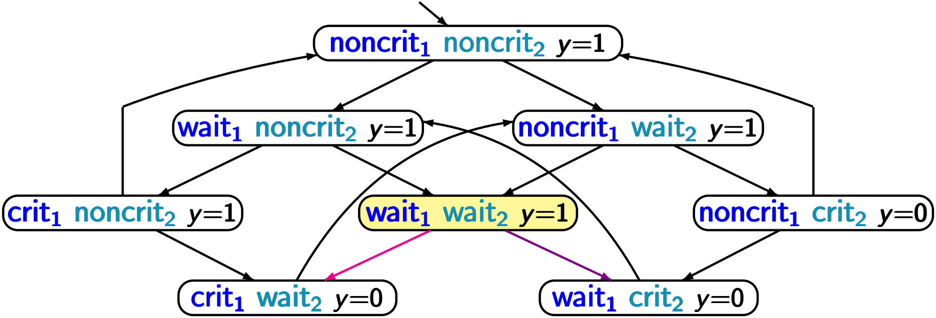
reachable fragment of the transition system $\mathcal{T}_{\mathcal{P}_1 \,|||\, \mathcal{P}_2}$



Competition

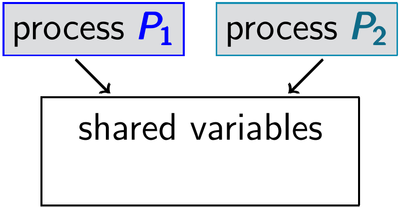


. . . competition between the waiting processes . . .

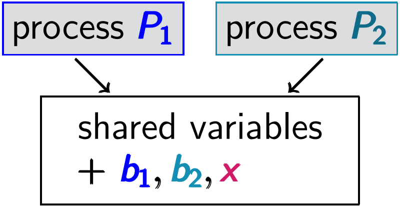


Peterson algorithm for mutual exclusion

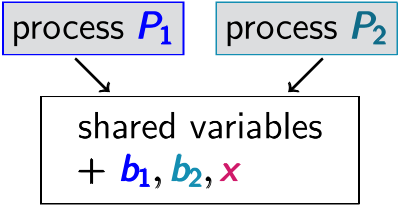
Peterson algorithm for mutual exclusion



Peterson algorithm for mutual exclusion

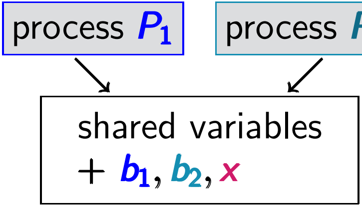


Peterson algorithm for mutual exclusion



b_1, b_2 Boolean variables, $x \in \{1, 2\}$

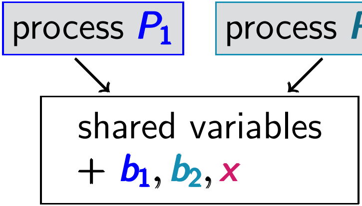
Peterson algorithm for mutual exclusion

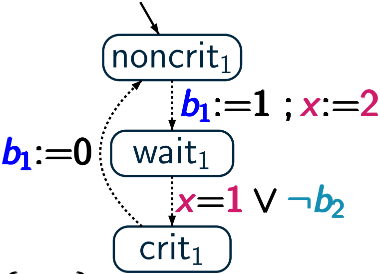


b_1, b_2 Boolean variables, $x \in \{1, 2\}$

```
LOOP FOREVER                          (* protocol for P1 *)
    noncritical actions;
    b1:=1 ; x:=2;
    AWAIT  x=1 ∨ ¬b2  DO  critical section OD
    b1:=0
END LOOP
```

Peterson algorithm for mutual exclusion





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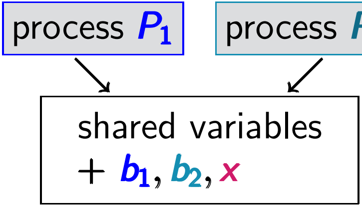
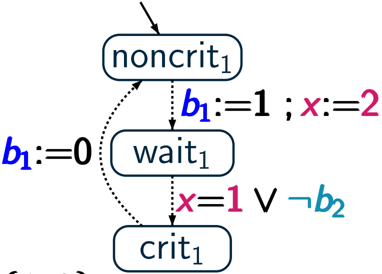
Peterson algorithm for mutual exclusion





b_1, b_2 Boolean variables, $x \in \{1, 2\}$

LOOP FOREVER (* protocol for P_1 *)

  noncritical actions;

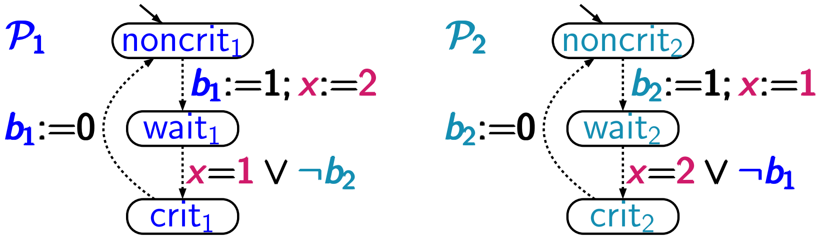
  atomic$\{b_1 := 1 \; ; \; x := 2\}$;

  AWAIT $x = 1 \vee \neg b_2$ DO critical section OD

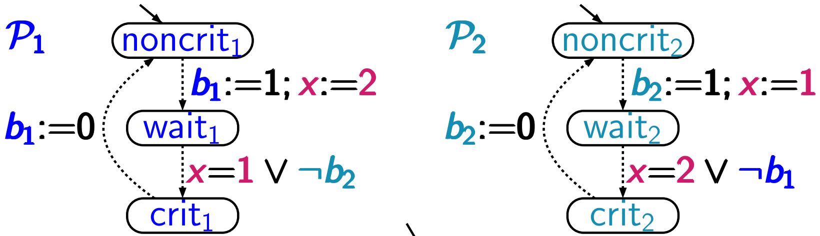
  $b_1 := 0$

END LOOP

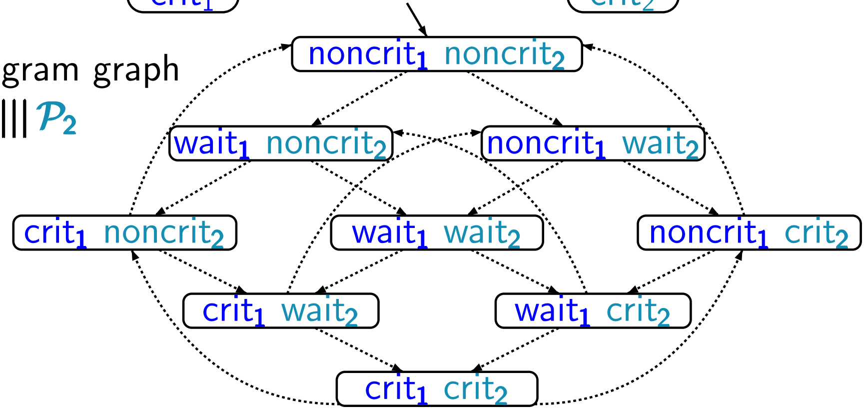
Program graphs for Peterson algorithm



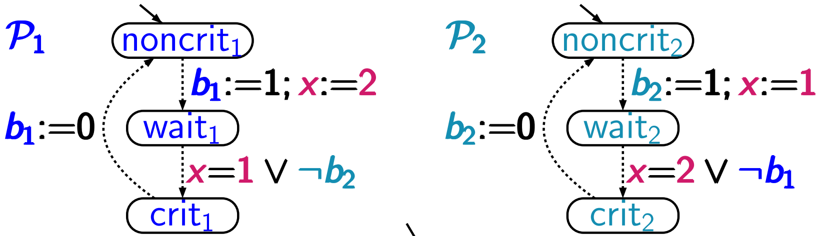
Program graphs for Peterson algorithm



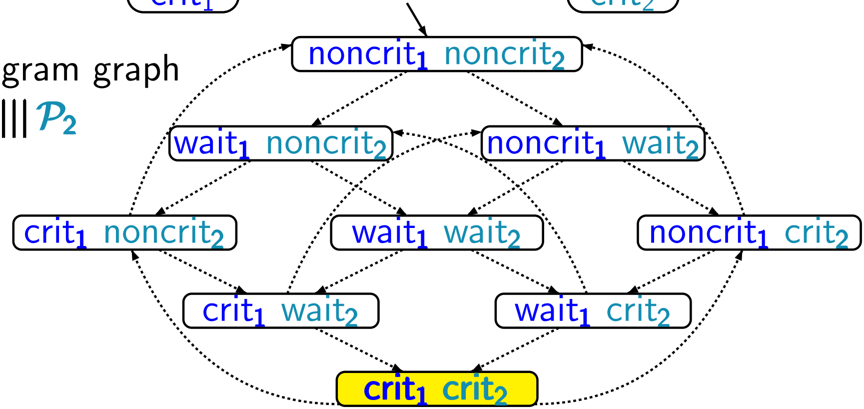
program graph
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



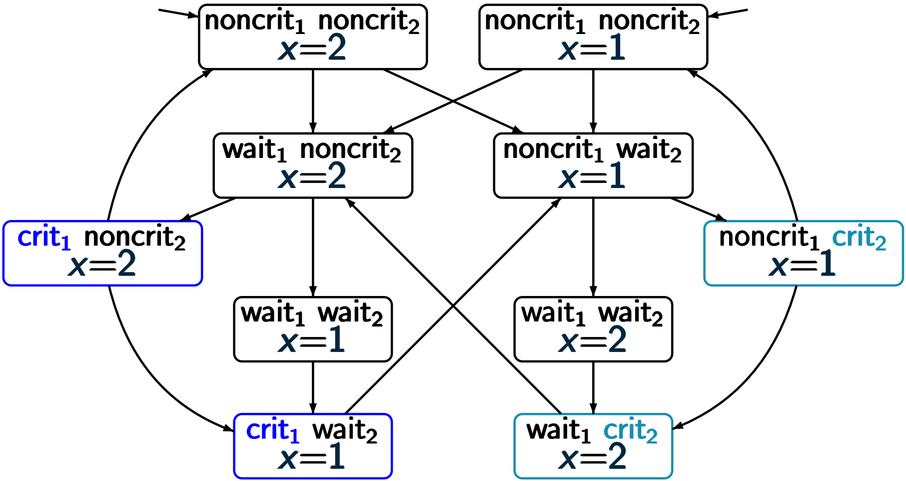
Program graphs for Peterson algorithm



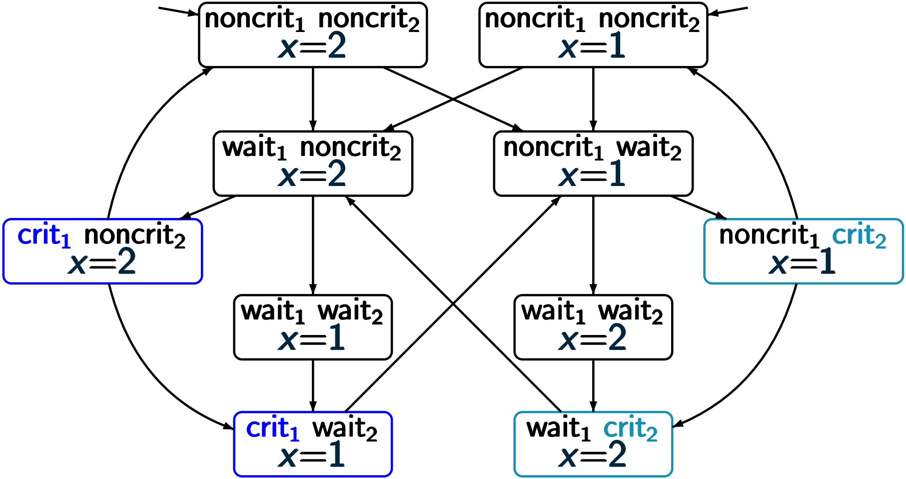
program graph
$\mathcal{P}_1 \,|||\, \mathcal{P}_2$



TS for the Peterson algorithm



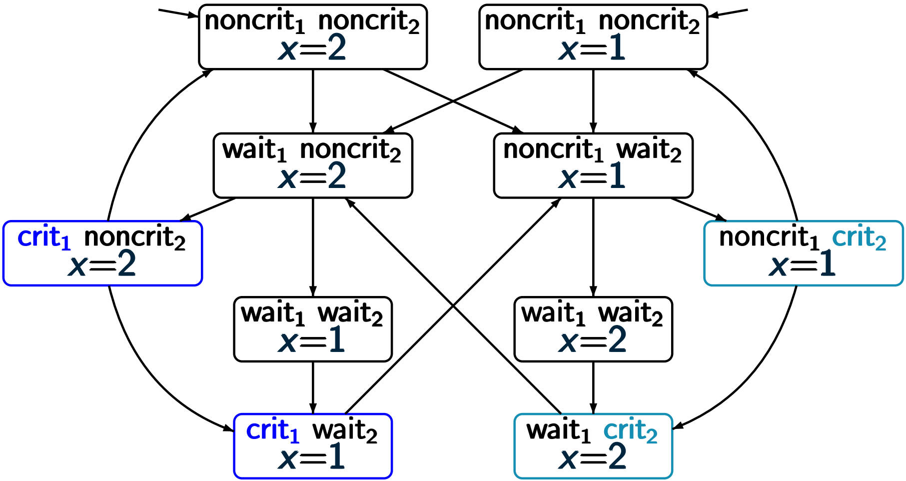
TS for the Peterson algorithm



value of b_1 is given by $\mathbf{wait_1} \vee \mathbf{crit_1}$

value of b_2 is given by $\mathbf{wait_2} \vee \mathbf{crit_2}$

TS for the Peterson algorithm

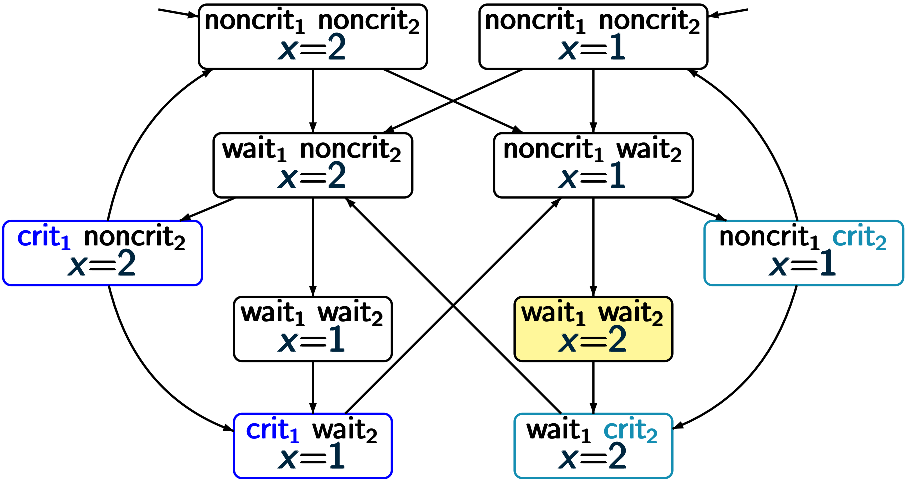


value of b_1 is given by **wait$_1$** $\vee$ **crit$_1$**

value of b_2 is given by **wait$_2$** $\vee$ **crit$_2$**

+ unreachable states

TS for the Peterson algorithm

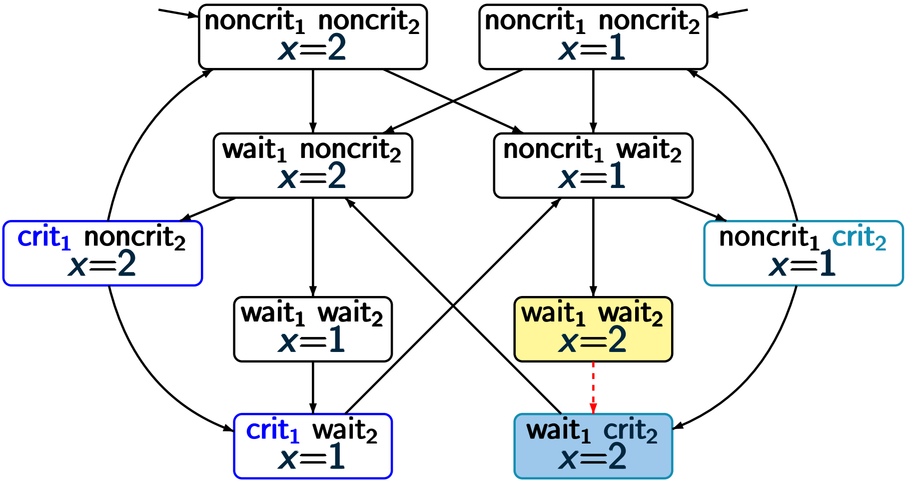


value of b_1 is given by **wait$_1$** $\vee$ **crit$_1$**

value of b_2 is given by **wait$_2$** $\vee$ **crit$_2$**

+ unreachable states

TS for the Peterson algorithm



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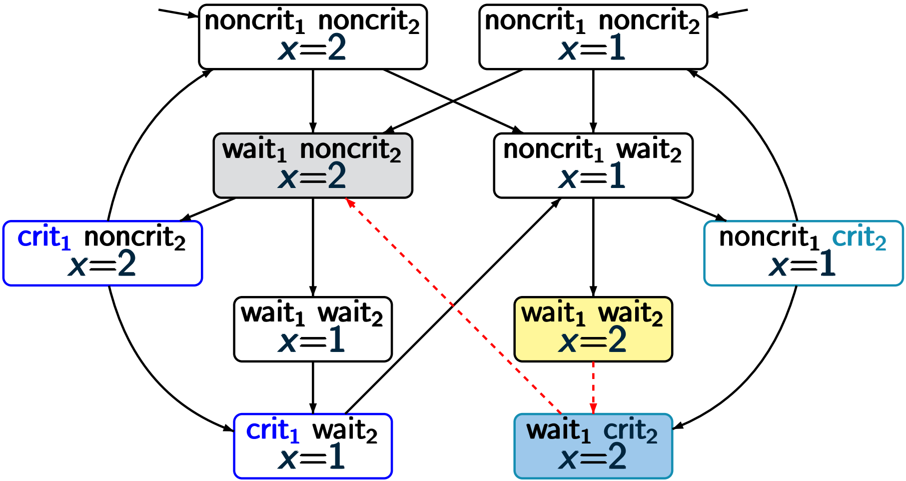
+ unreachable states

TS for the Peterson algorithm



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+ unreachable states

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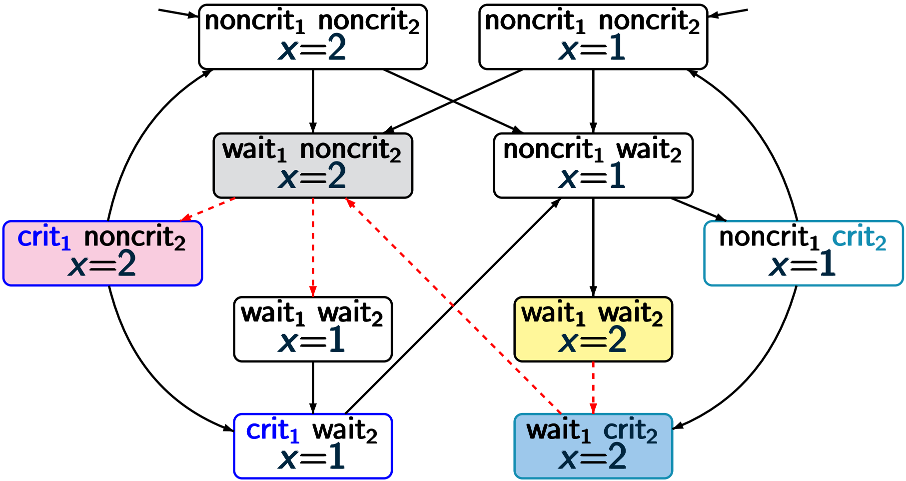

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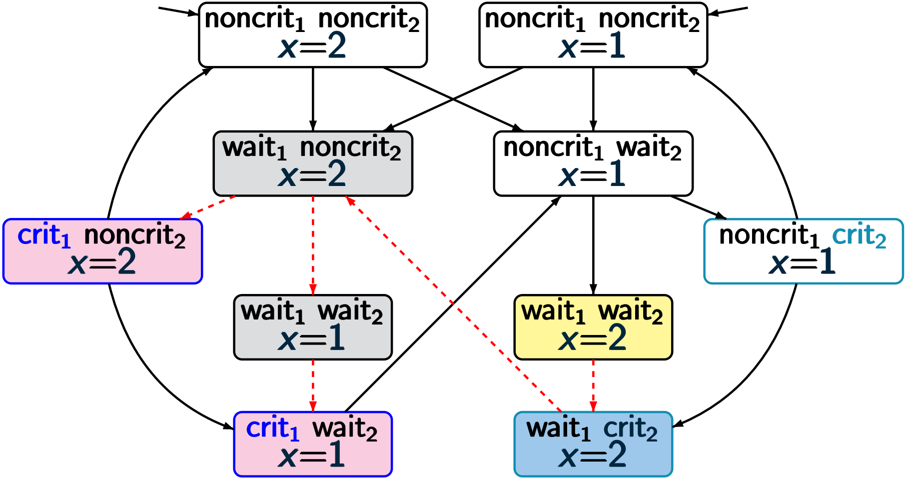
TS for the Peterson algorithm



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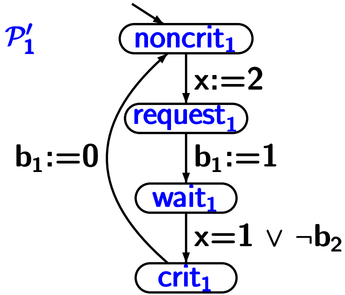
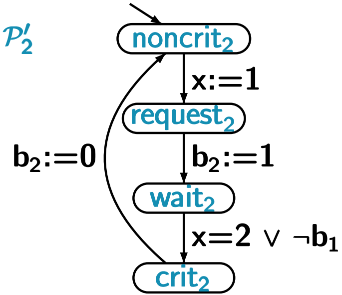
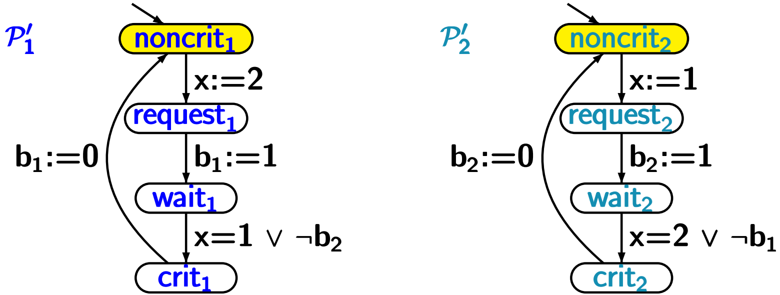
Variant of Peterson algorithm





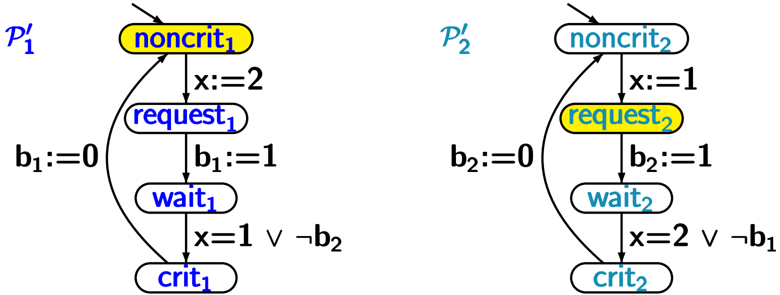
Variant of Peterson algorithm



possible executions

noncrit$_1$ noncrit$_2$ $x=1$ $\neg b_1$ $\neg b_2$

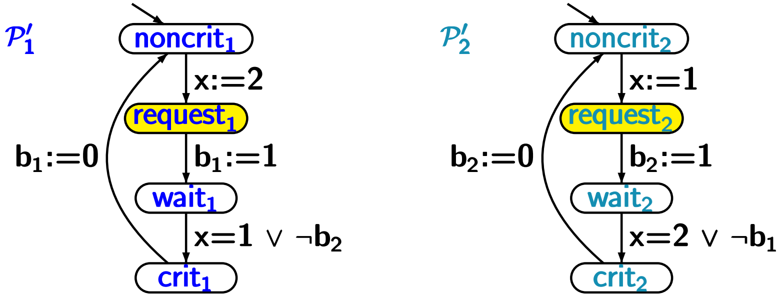
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	$x=1$	$\neg b_1$	$\neg b_2$
noncrit$_1$	request$_2$	$x=1$	$\neg b_1$	$\neg b_2$

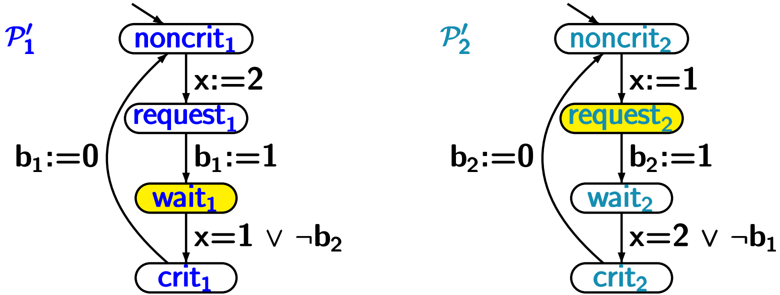
Variant of Peterson algorithm



possible executions

$noncrit_1$	$noncrit_2$	$x=1$	$\neg b_1$	$\neg b_2$
$noncrit_1$	$request_2$	$x=1$	$\neg\ b_1$	$\neg b_2$
$request_1$	$request_2$	$x=2$	$\neg b_1$	$\neg b_2$

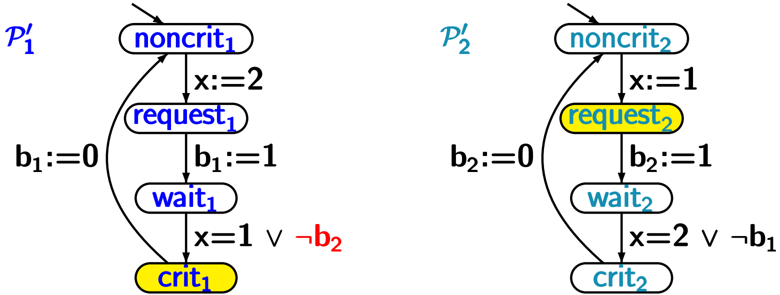
Variant of Peterson algorithm



possible executions

$noncrit_1$	$noncrit_2$	$x=1$	$\neg b_1$	$\neg b_2$
$noncrit_1$	$request_2$	$x=1$	$\neg b_1$	$\neg b_2$
$request_1$	$request_2$	$x=2$	$\neg b_1$	$\neg b_2$
$wait_1$	$request_2$	$x=2$	b_1	$\neg b_2$

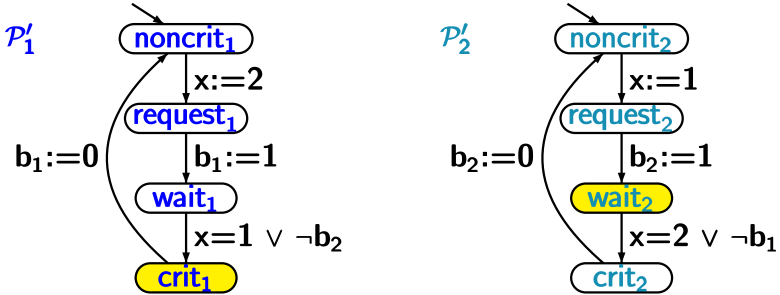
Variant of Peterson algorithm



possible executions

$noncrit_1$	$noncrit_2$	$x=1$	$\neg b_1$	$\neg b_2$
$noncrit_1$	$request_2$	$x=1$	$\neg b_1$	$\neg b_2$
$request_1$	$request_2$	$x=2$	$\neg b_1$	$\neg b_2$
$wait_1$	$request_2$	$x=2$	b_1	$\neg b_2$
$crit_1$	$request_2$	$x=2$	b_1	$\neg b_2$

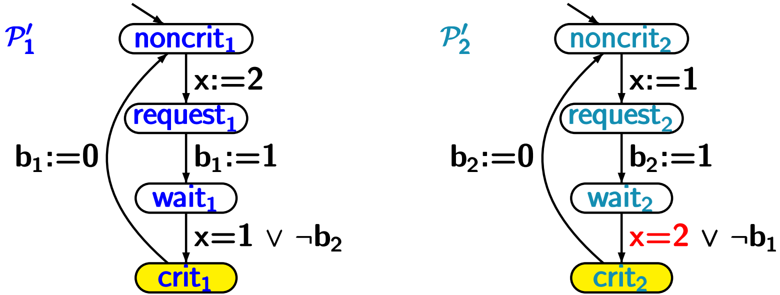
Variant of Peterson algorithm



possible executions

noncrit$_1$	noncrit$_2$	x=1	$\neg b_1$	$\neg b_2$
noncrit$_1$	request$_2$	x=1	$\neg b_1$	$\neg b_2$
request$_1$	request$_2$	x=2	$\neg b_1$	$\neg b_2$
wait$_1$	request$_2$	x=2	b_1	$\neg b_2$
crit$_1$	request$_2$	x=2	b_1	$\neg b_2$
crit$_1$	wait$_2$	x=2	b_1	b_2

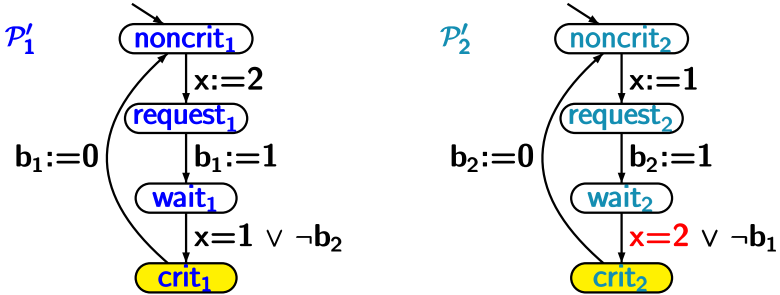
Variant of Peterson algorithm



possible executions

$noncrit_1$	$noncrit_2$	$x=1$	$\neg b_1$	$\neg b_2$
$noncrit_1$	$request_2$	$x=1$	$\neg b_1$	$\neg b_2$
$request_1$	$request_2$	$x=2$	$\neg b_1$	$\neg b_2$
$wait_1$	$request_2$	$x=2$	b_1	$\neg b_2$
$crit_1$	$request_2$	$x=2$	b_1	$\neg b_2$
$crit_1$	$wait_2$	$x=2$	b_1	b_2
$crit_1$	$crit_2$	$x=2$	b_1	b_2

Variant of Peterson algorithm incorrect!



possible executions

$noncrit_1$	$noncrit_2$	$x=1$	$\neg b_1$	$\neg b_2$
$noncrit_1$	$request_2$	$x=1$	$\neg b_1$	$\neg b_2$
$request_1$	$request_2$	$x=2$	$\neg b_1$	$\neg b_2$
$wait_1$	$request_2$	$x=2$	b_1	$\neg b_2$
$crit_1$	$request_2$	$x=2$	b_1	$\neg b_2$
$crit_1$	$wait_2$	$x=2$	b_1	b_2
$crit_1$	$crit_2$	$x=2$	b_1	b_2

Operators for parallelism and communication

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  - interleaving for independent actions
  - synchronization over actions in *Syn*

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- synchronous message passing $\longleftarrow$ data abstract
  - ∗ operator $\|_{Syn}$ for TS
  - ∗ interleaving for independent actions
  - ∗ synchronization over actions in Syn
- channel systems
  communication via shared variables + via channels

Synchronous message passing

Synchronous message passing

$\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, \ldots)$, $\mathcal{T}_2 = (S_2, Act_2, \rightarrow_2, \ldots)$ TS

$Syn \subseteq Act_1 \cap Act_2$ set of synchronization actions

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composite transition system:

$$\mathcal{T}_1 \,\|_{Syn}\, \mathcal{T}_2 = (S_1 \times S_2, Act_1 \cup Act_2, \rightarrow, \ldots)$$

for modeling the concurrent execution of $\mathcal{T}_1$ and $\mathcal{T}_2$ with synchronization over all actions in Syn

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interleaving for all actions $\alpha \in Act_i \setminus Syn$:

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2 \rangle} \qquad \frac{s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1, s_2' \rangle}$$

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handshaking (rendezvous) for all $\alpha \in Syn$:

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handshaking (rendezvous) for all $\alpha \in Syn$:

$$\frac{s_1 \xrightarrow{\alpha}_1 s_1' \quad \wedge \quad s_2 \xrightarrow{\alpha}_2 s_2'}{\langle s_1, s_2 \rangle \xrightarrow{\alpha} \langle s_1', s_2' \rangle}$$

Mutual exclusion

by synchronous message passing

Mutual exclusion

by synchronous message passing using an arbiter

Mutual exclusion with an arbiter

protocol for process P_i

```
LOOP FOREVER DO
      noncritical actions
      request
      critical section
      release
      noncritical actions
OD
```

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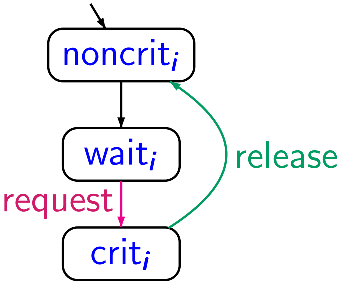
transition system $\mathcal{T}_i$



Mutual exclusion with an arbiter

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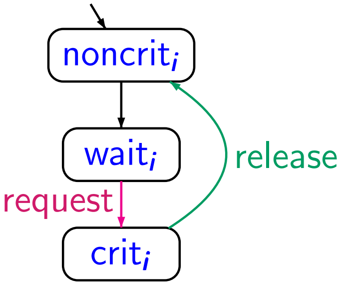
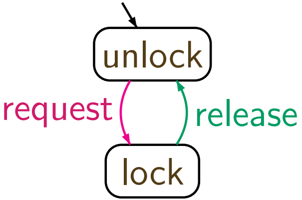


Arbiter:

selects nondeterministically a synchronization partner $\mathcal{T}_1$ or $\mathcal{T}_2$



Mutual exclusion with an arbiter

$$(\mathcal{T}_1 \; ||| \; \mathcal{T}_2) \; \|_{\mathit{Syn}} \; \mathit{Arbiter} \text{ where } \mathit{Syn} = \{\mathsf{request}, \mathsf{release}\}$$

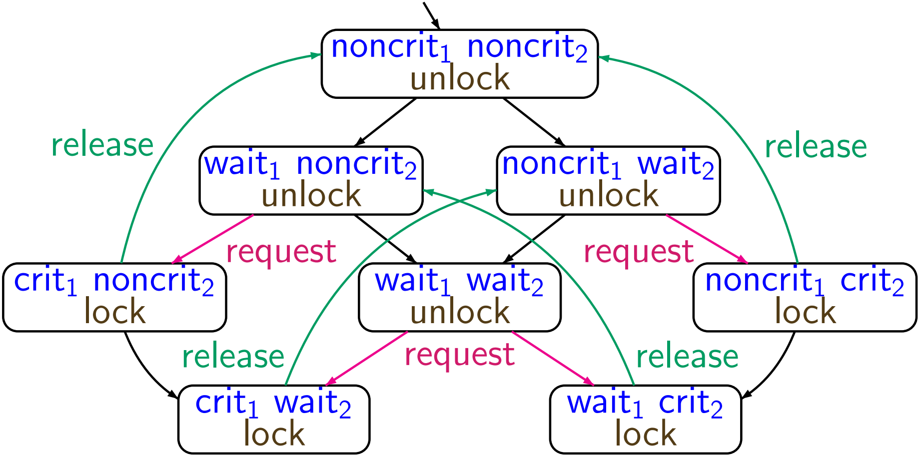
$$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \;\|_{Syn}\; \textit{Arbiter} \text{ where } Syn = \{\text{request}, \text{release}\}$$

"pure" interleaving for TS

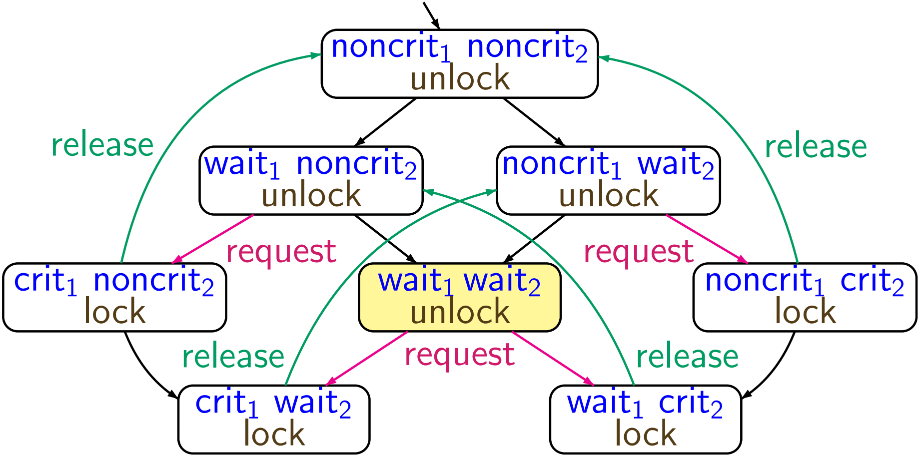
handshaking for actions request and release

Mutual exclusion with an arbiter

$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \parallel_{Syn} \textit{Arbiter}$ where $Syn = \{\text{request}, \text{release}\}$



$(\mathcal{T}_1 \,|||\, \mathcal{T}_2) \parallel_{Syn} \textit{Arbiter}$ where $Syn = \{\text{request}, \text{release}\}$



nondeterministic choice: who enters the critical section**?**

Synchronous message passing

synchronization operator $\|_{Syn}$ for
three or more processes

Synchronous message passing

$$\left.\begin{array}{rcl} \mathcal{T}_1 & = & (S_1, Act_1, \rightarrow_1, \ldots) \\ \mathcal{T}_2 & = & (S_2, Act_2, \rightarrow_2, \ldots) \\ \mathcal{T}_3 & = & (S_3, Act_3, \rightarrow_3, \ldots) \\ \mathcal{T}_4 & = & (S_4, Act_4, \rightarrow_4, \ldots) \end{array}\right\} \text{ transition systems}$$

$$\begin{array}{rcl} \vdots & & \vdots \end{array}$$

Synchronous message passing

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$$\vdots \qquad\qquad \vdots$$

for $Syn \subseteq Act_1 \cup Act_2 \cup Act_3 \cup Act_4 \cup \ldots$

$$\boxed{\begin{array}{l} \mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \parallel_{Syn} \mathcal{T}_3 \parallel_{Syn} \mathcal{T}_4 \parallel_{Syn} \ldots \overset{\text{def}}{=} \\ \qquad \Big(((\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2) \parallel_{Syn} \mathcal{T}_3) \parallel_{Syn} \mathcal{T}_4\Big) \parallel_{Syn} \ldots \end{array}}$$

Synchronous message passing

$$\left.\begin{array}{lcl}\mathcal{T}_1 & = & (S_1, Act_1, \rightarrow_1, \ldots) \\ \mathcal{T}_2 & = & (S_2, Act_2, \rightarrow_2, \ldots) \\ \mathcal{T}_3 & = & (S_3, Act_3, \rightarrow_3, \ldots) \\ \mathcal{T}_4 & = & (S_4, Act_4, \rightarrow_4, \ldots) \\ \vdots & & \qquad\quad \vdots \end{array}\right\} \text{transition systems}$$

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or any other order of paranthesis

Synchronous message passing

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where, e.g., $\mathcal{T}_1 \parallel_{Syn} \mathcal{T}_2 \overset{\text{def}}{=} \mathcal{T}_1 \parallel_H \mathcal{T}_2$

with $H = Syn \cap Act_1 \cap Act_2$

Parallel operator $\|$

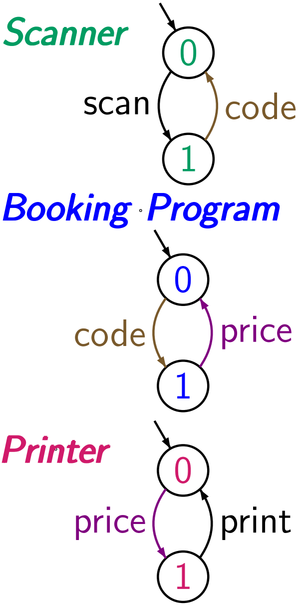
$$\begin{array}{lcl} \mathcal{T}_1 & = & (S_1, Act_1, \rightarrow_1, \ldots) \\ \mathcal{T}_2 & = & (S_2, Act_2, \rightarrow_2, \ldots) \\ \mathcal{T}_3 & = & (S_3, Act_3, \rightarrow_3, \ldots) \\ \mathcal{T}_4 & = & (S_4, Act_4, \rightarrow_4, \ldots) \\ \vdots & & \vdots \end{array}$$

transition systems s.t. $Act_i \cap Act_j \cap Act_k = \emptyset$ if i, j, k are pairwise distinct

$$\boxed{\begin{array}{l} \mathcal{T}_1 \,\|\, \mathcal{T}_2 \,\|\, \mathcal{T}_3 \,\|\, \mathcal{T}_4 \,\|\, \ldots \stackrel{\text{def}}{=} \\ \quad (((\mathcal{T}_1 \,\|_{Syn_{1,2}}\, \mathcal{T}_2) \,\|_{Syn_{1,2,3}}\, \mathcal{T}_3) \,\|_{Syn_{1,2,3,4}}\, \mathcal{T}_4) \ldots \end{array}}$$

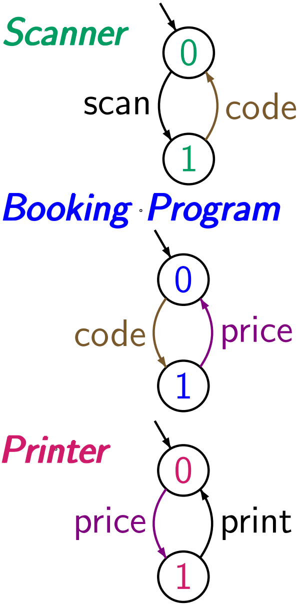
where

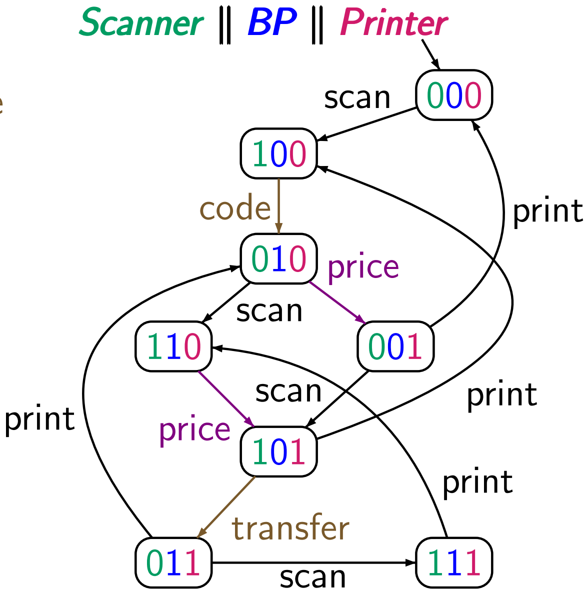
$$\begin{array}{lcl} Syn_{1,2} & = & Act_1 \cap Act_2 \\ Syn_{1,2,3} & = & (Act_1 \cup Act_2) \cap Act_3 \\ Syn_{1,2,3,4} & = & (Act_1 \cup Act_2 \cup Act_3) \cap Act_4 \\ \vdots & & \vdots \end{array}$$



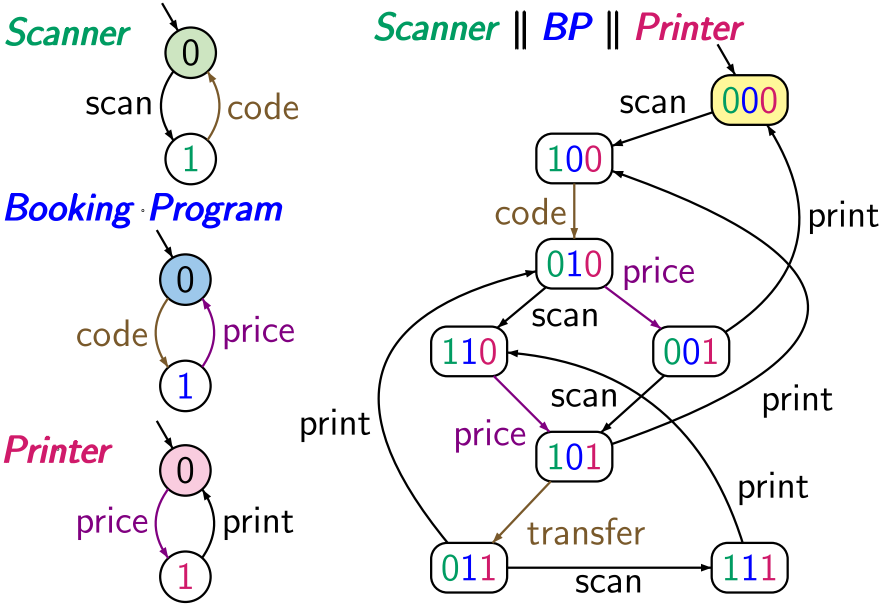
Scanner $\|$ *BP* $\|$ *Printer*

Booking system in supermarket

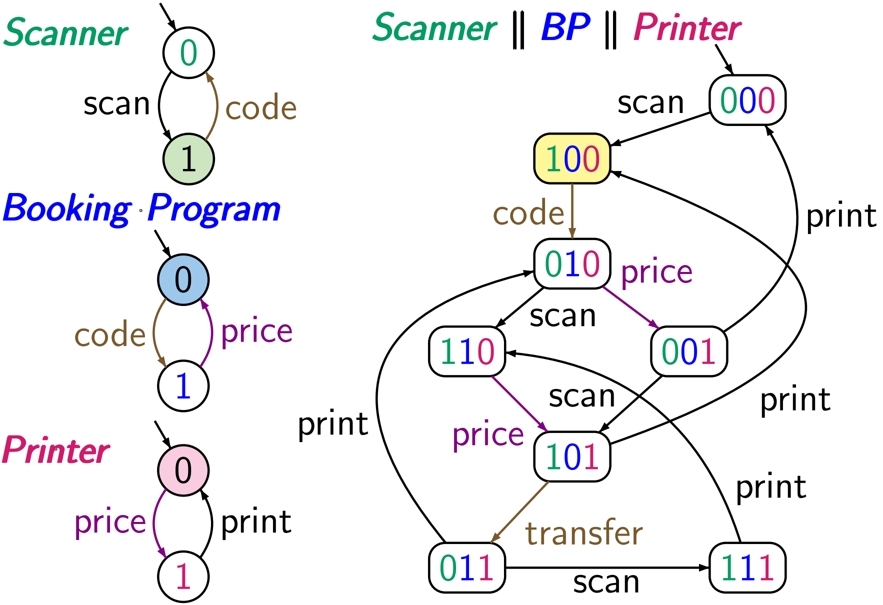




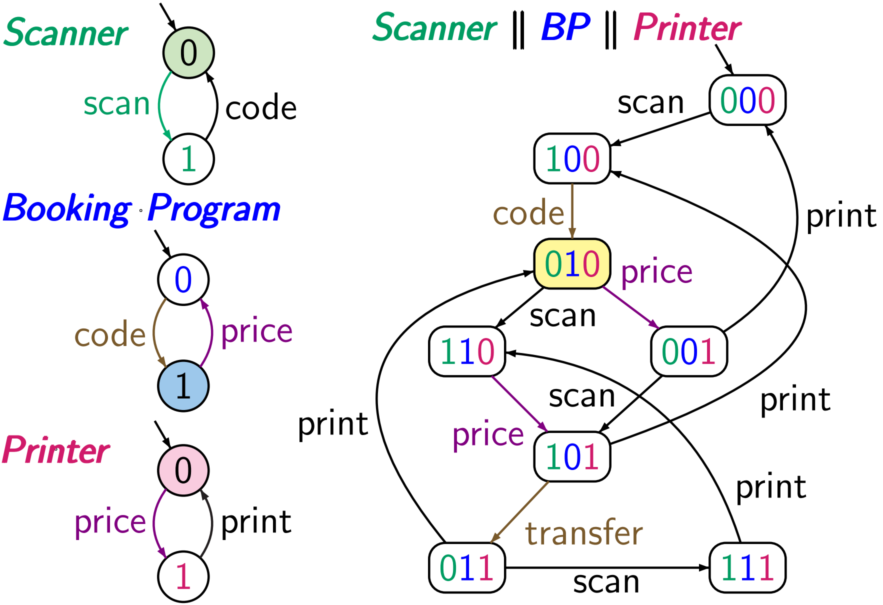
Booking system in supermarket

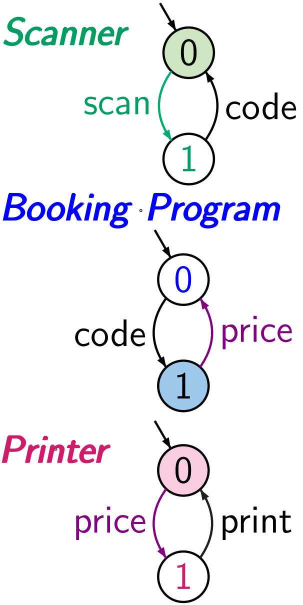


Booking system in supermarket

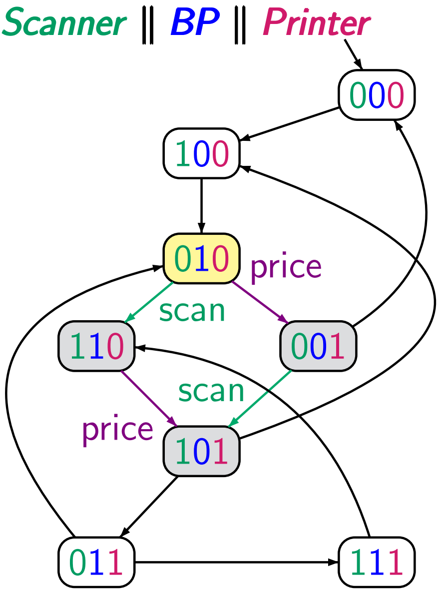


Booking system in supermarket





Scanner
0
scan
code
1
Booking Program
0
code
price
1
Printer
0
price
print
1



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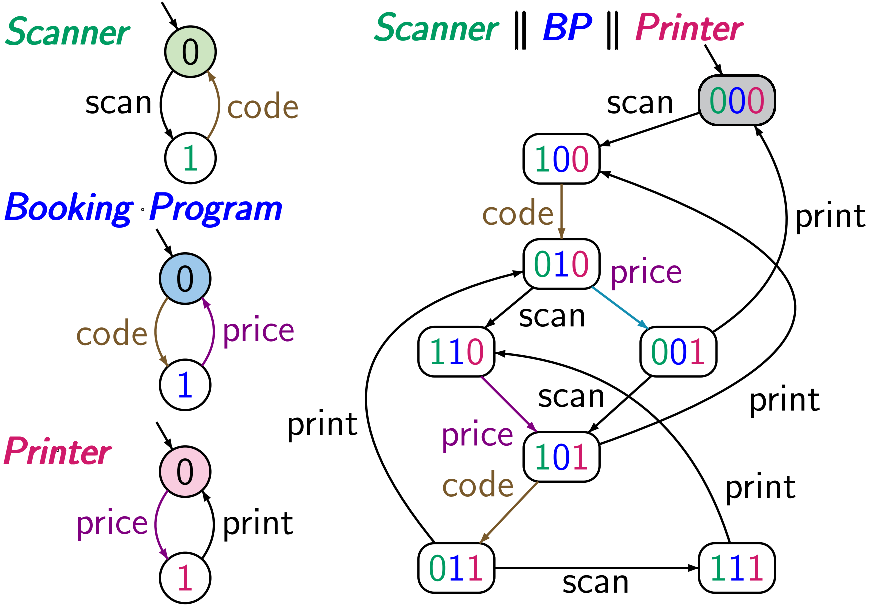
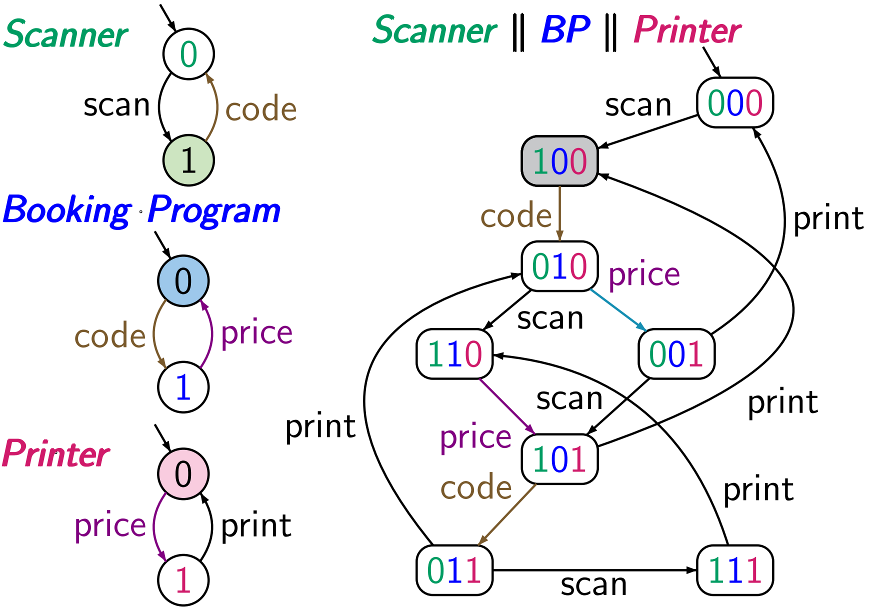
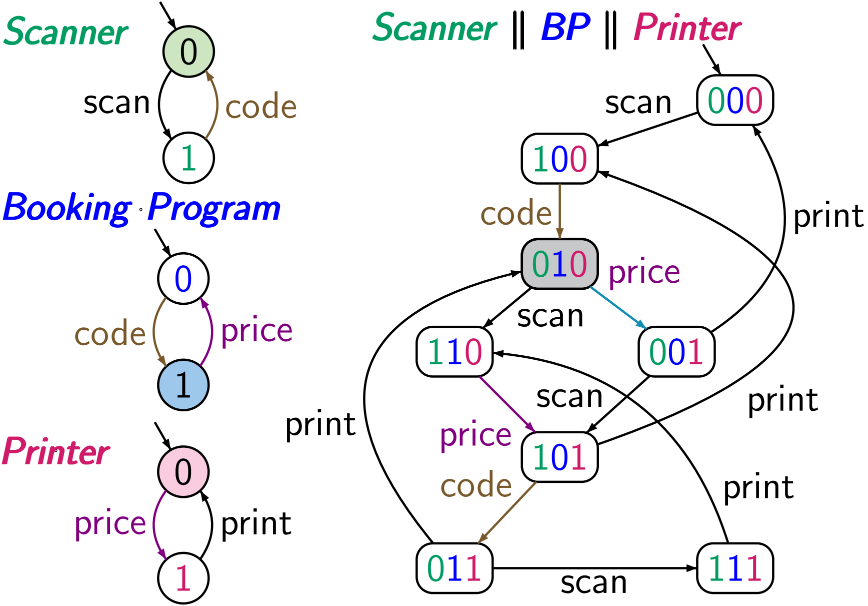
Booking system in supermarket



Booking system in supermarket





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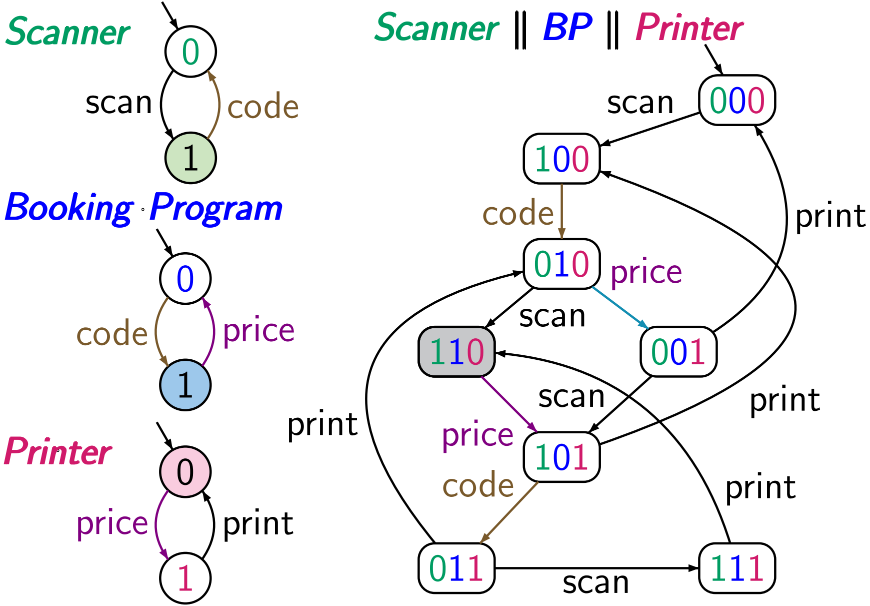

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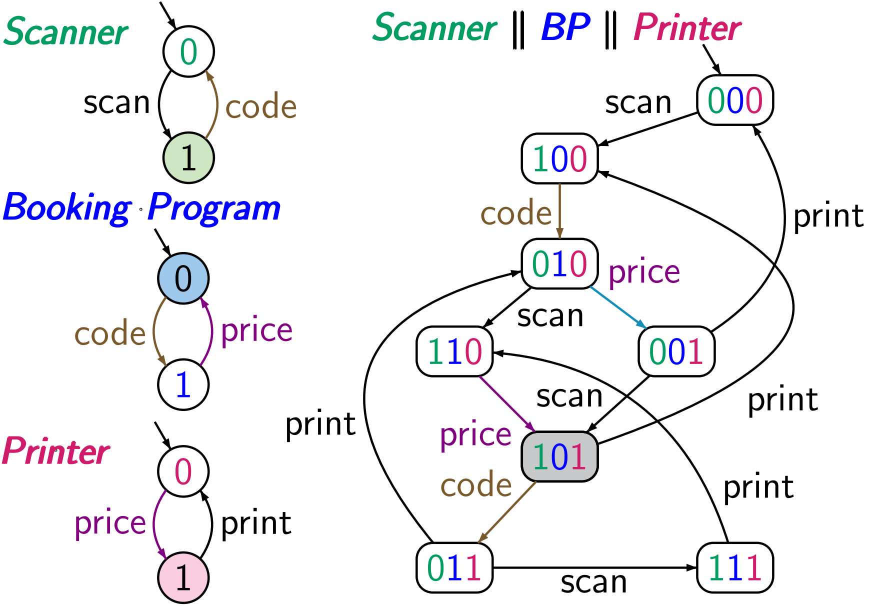
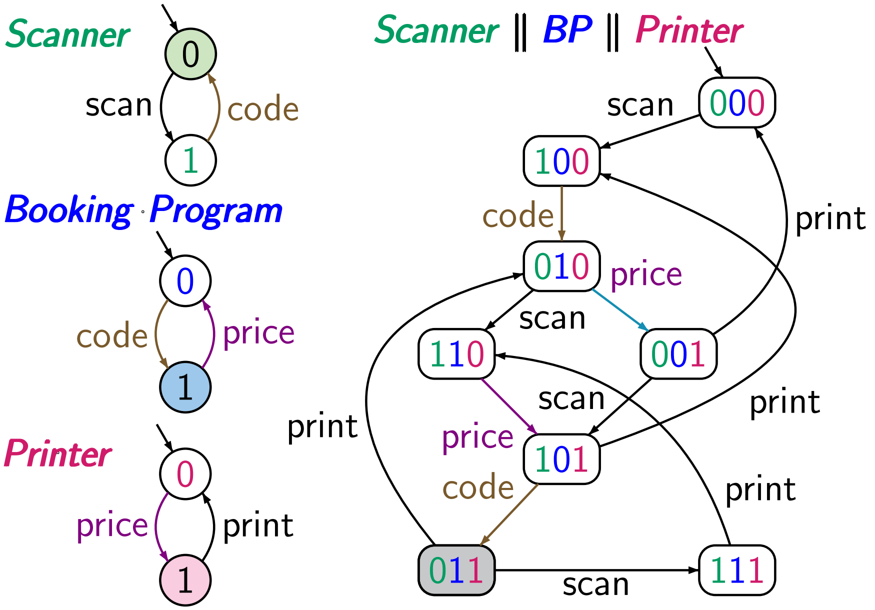
Booking system in supermarket



Booking system in supermarket



Interleaving