Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

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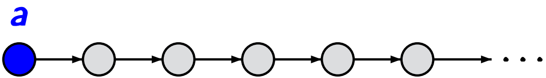

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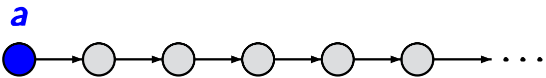

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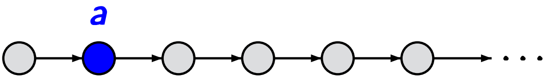

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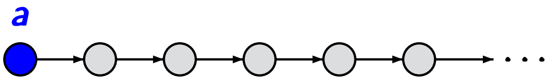

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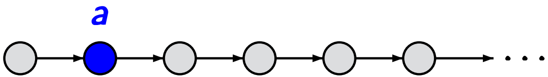

until operator
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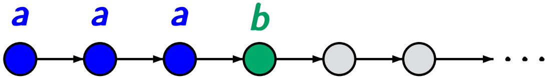

Derived operators in LTL

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derived operators:

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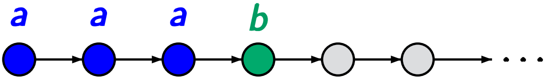
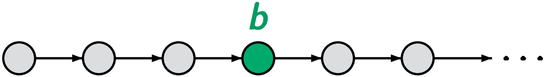


eventually
$\Diamond b$



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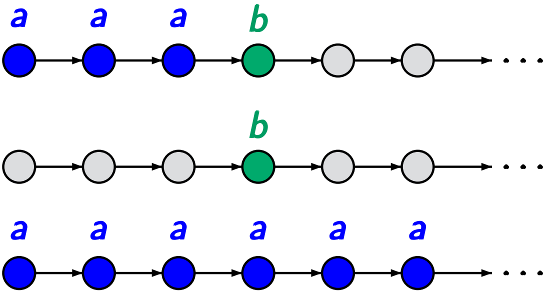
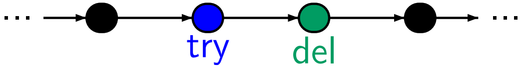


$$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\, \text{delivered})$$



Next $\bigcirc$, until U and eventually $\Diamond$

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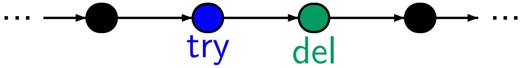

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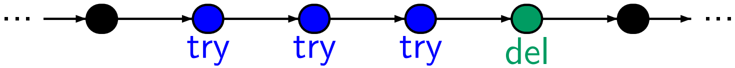

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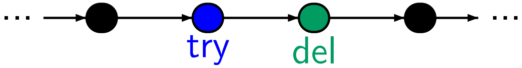

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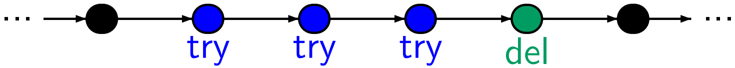

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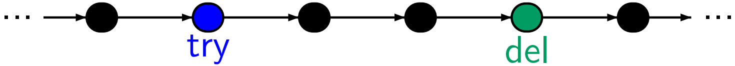

Examples for LTL formulas

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traffic light: $\Box(\mathit{yellow} \vee \bigcirc\neg \mathit{red})$

Infinitely often and eventually forever

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LTL-semantics

interpretation of **LTL** formulas over traces, i.e.,
infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$

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$\sigma \models \varphi_1 \mathsf{U} \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

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LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \left\{\sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi\right\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$:

$$\vdots$$

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$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that
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$\sigma \models \Box\varphi$	iff	for all $j \geq 0$ we have:
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LTL semantics over TS

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e.,
all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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LTL formula φ over AP

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interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

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$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

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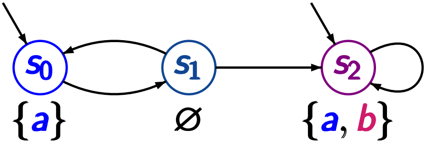
remind: LT property of an LTL formula:

$$Words(\varphi) = \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

Example: LTL-semantics over paths



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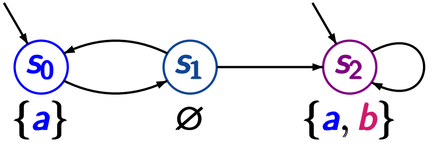

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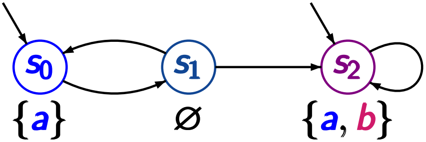

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$\pi \models a$

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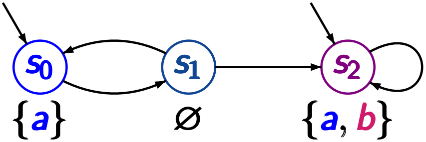

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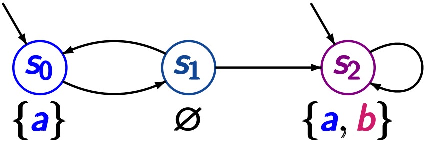

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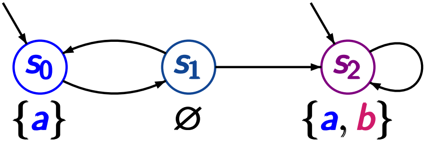
$\pi \models \bigcirc(\neg a \wedge \neg b)$

Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ as $L(s_0) = \{a\}$

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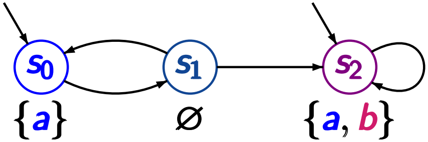

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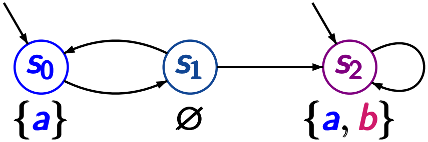

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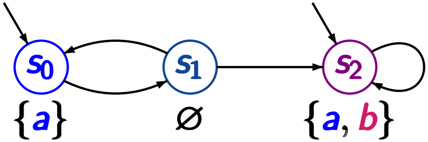

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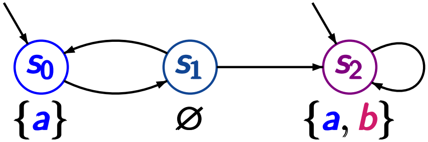

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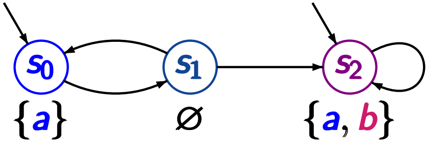

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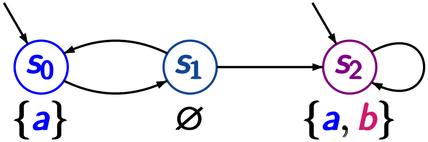
Correct or wrong ?



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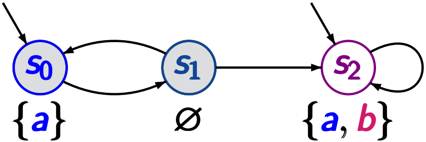

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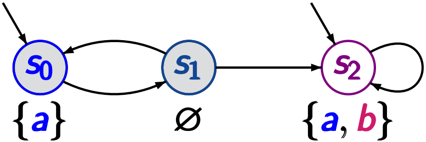
Correct or wrong ?

LTLSF3.1-7



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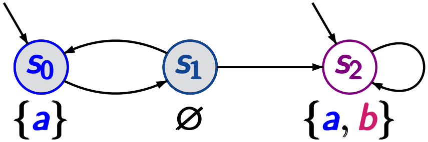

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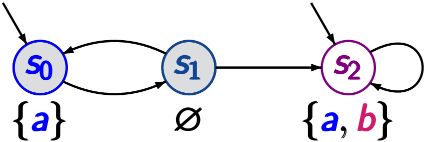

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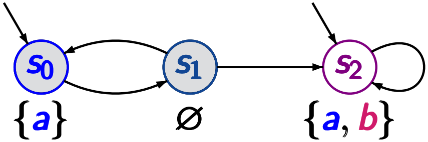

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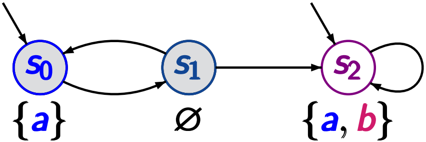
Correct or wrong ?



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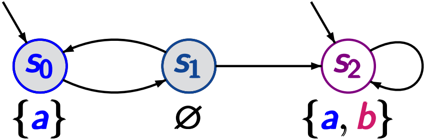

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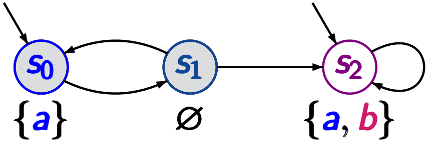

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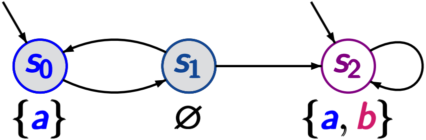
$\pi \models \Box a$?

Correct or wrong ?



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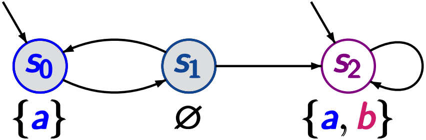

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$\pi \models \Box \Diamond a$?

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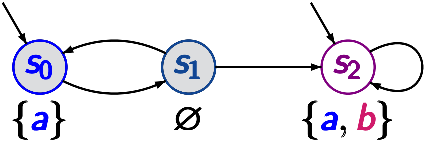

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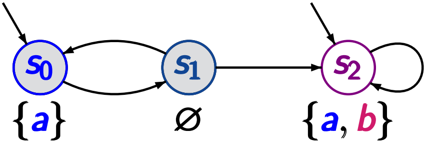

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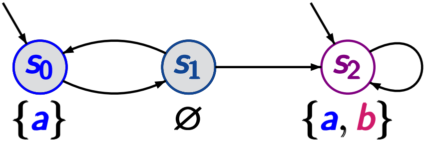

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$\pi \not\models \Diamond\Box a$ $\qquad$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \Box\Diamond\varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j A_{j+1} A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box\varphi$	iff	for almost all $j \geq 0$ we have: $A_j A_{j+1} A_{j+2} \ldots \models \varphi$

LTL semantics over the states of a TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)$$

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↑

satisfaction relation for LT properties

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Interpretation of LTL formulas over TS

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$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

Interpretation of LTL formulas over TS

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LTL formula φ over AP

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given: TS $\mathcal{T} = (S, Act, \to, S_0, AP, L)$
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LTL formula φ over AP

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\end{aligned}$$

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$$

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LTL formula φ over AP

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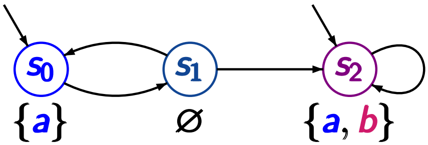
satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$$AP = \{a, b\}$$

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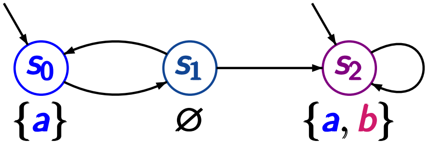

$AP = \{a, b\}$

$\mathcal{T} \models a$

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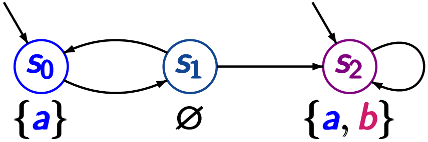
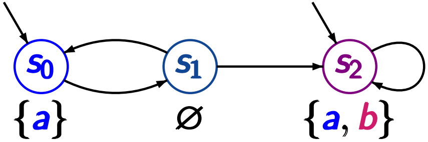


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

Which formulas hold for $\mathcal{T}$?



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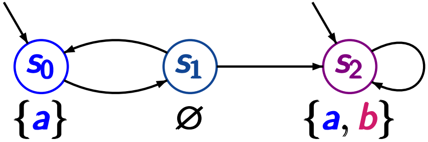
$\mathcal{T} \models \Diamond\Box a$

Which formulas hold for $\mathcal{T}$?



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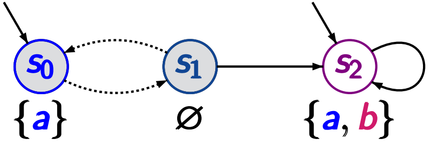

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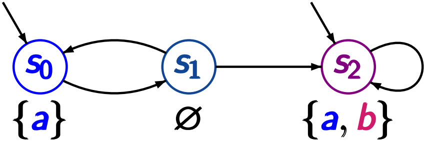

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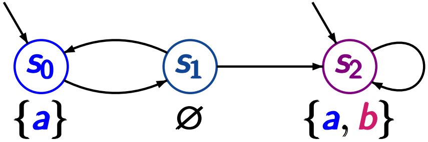

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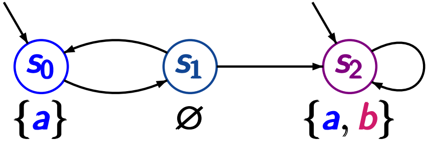

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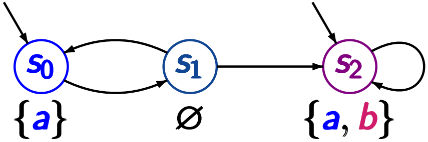

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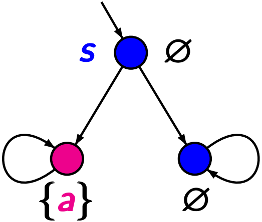

$s \not\models \Diamond a$ and $s \not\models \neg\Diamond a$

LTL-formulas for MUTEX protocols

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LTL formulas over $AP = \{\mathsf{wait}_1, \mathsf{crit}_1, \mathsf{wait}_2, \mathsf{crit}_2\}$

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$$\varphi_{mutex} = ?$$

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Expansion laws

Expansion law for U

until: $\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$

Expansion laws for U and $\Diamond$

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Expansion laws for U, $\Diamond$ and $\Box$

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always: $\Box \psi \equiv$ **?**

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$$\begin{aligned} \Box\psi &= \neg\Diamond\neg\psi \\ &\equiv \neg(\neg\psi \;\vee\; \bigcirc\Diamond\neg\psi) \longleftarrow \boxed{\text{expansion law for } \Diamond} \end{aligned}$$

Expansion laws for U, $\Diamond$ and $\Box$

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&\equiv \psi \;\wedge\; \neg\bigcirc\Diamond\neg\psi \quad \longleftarrow \boxed{\text{double negation}}
\end{aligned}
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&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \quad \longleftarrow \boxed{\text{self duality of } \bigcirc}
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&\equiv \psi \wedge \bigcirc\neg\Diamond\neg\psi \\
&\equiv \psi \wedge \bigcirc\Box\psi \quad \longleftarrow \boxed{\text{definition of } \Box}
\end{aligned}
$$

The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv ?$$

The weak until operator W

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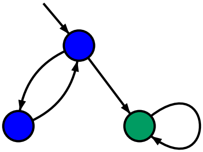
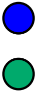
Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?





$\widehat{=}$ $\{a\}$

$\widehat{=}$ $\{b\}$

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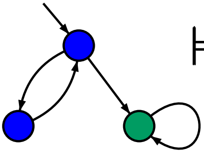

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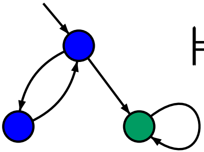

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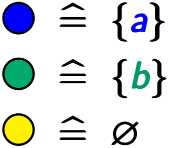

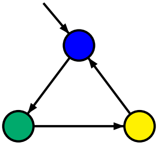

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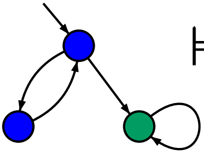
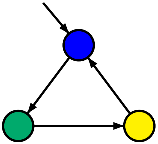


$\not\models a \,\mathsf{W}\, b$

● $\mathrel{\widehat{=}} \{a\}$

● $\mathrel{\widehat{=}} \{b\}$

● $\mathrel{\widehat{=}} \varnothing$



$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)

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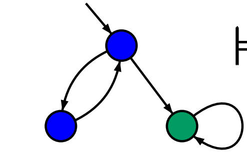

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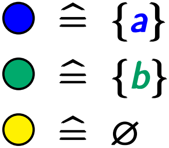

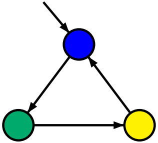

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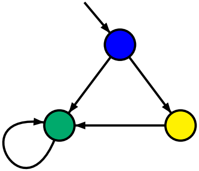

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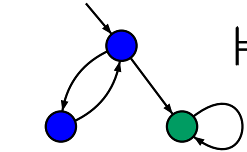

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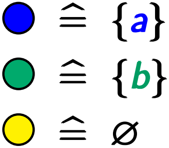

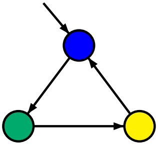

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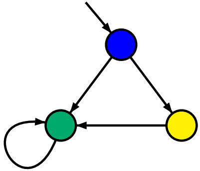

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