Overview

Linear vs branching time

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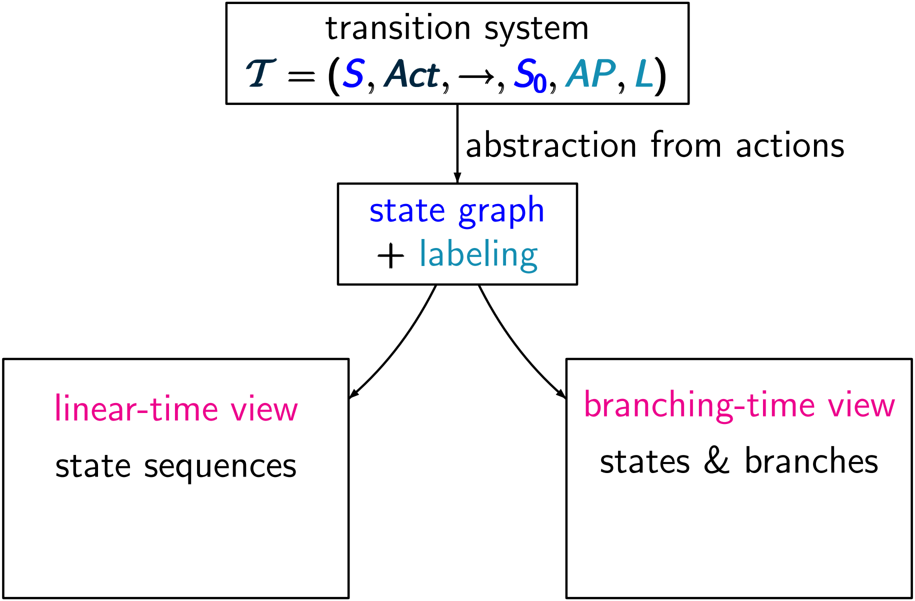


Linear vs branching time

transition system
$\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

abstraction from actions

state graph
+ labeling

projection on AP

linear-time view

state sequences
$\Downarrow$
traces

branching-time view

states & branches
$\Downarrow$
computation tree

Computation tree

Computation tree

The computation tree of a transition system $\mathcal{T} = (S, Act, \to, s_0, AP, L)$ arises by:

- unfolding into a tree
- abstraction from the actions
- projection of the states s to their labels $L(s) \subseteq AP$

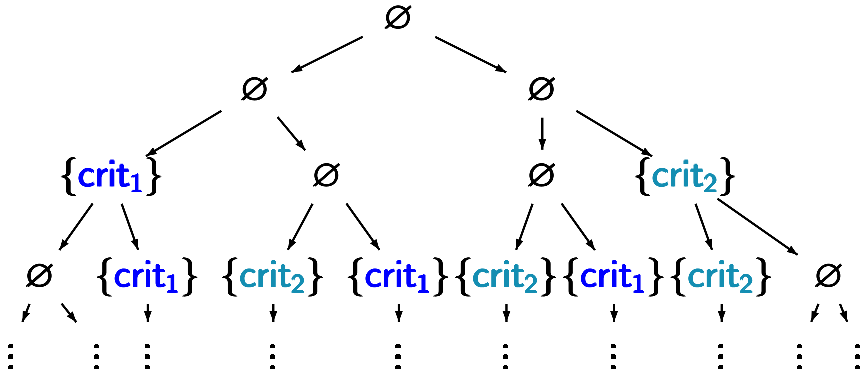
Computation tree

The computation tree of state s_0 in a transition system $\mathcal{T} = (S, Act, \to, S_0, AP, L)$ arises by:

- unfolding $\mathcal{T}_{s_0} = (S, Act, \to, s_0, AP, L)$ into a tree
- abstraction from the actions
- projection of the states s to their labels $L(s) \subseteq AP$

Example: computation tree

mutual exclusion with semaphore and $AP = \{crit_1, crit_2\}$:



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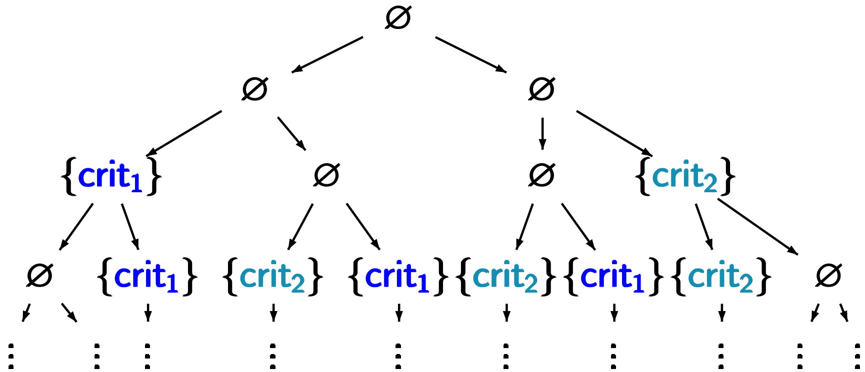

path $\langle nc_1, nc_2\rangle$ $\langle wait_1, nc_2\rangle$ $\langle crit_1, nc_2\rangle$ $\langle crit_1, wait_2\rangle$ ...

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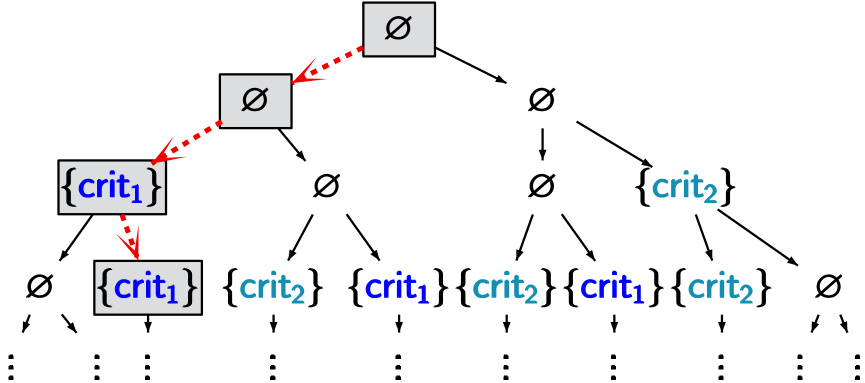

$$\begin{array}{lccccc}
\text{path} & \langle nc_1, nc_2\rangle & \langle wait_1, nc_2\rangle & \langle crit_1, nc_2\rangle & \langle crit_1, wait_2\rangle & \ldots \\
\big\downarrow & \big\downarrow & \big\downarrow & \big\downarrow & \big\downarrow & \\
\text{trace} & \varnothing & \varnothing & \{crit_1\} & \{crit_1\} & \ldots
\end{array}$$

Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation Tree Logic

- syntax and semantics of CTL $\longleftarrow$
- expressiveness of CTL and LTL
- CTL model checking
- fairness, counterexamples/witnesses
- CTL^+ and CTL^*

Equivalences and Abstraction

Computation Tree Logic (CTL)

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CTL (state) formulas:

$$\Phi \; ::= \; \mathit{true} \;\big|\; a \;\big|\; \Phi_1 \wedge \Phi_2 \;\big|\; \neg\Phi \;\big|\; \exists\varphi \;\big|\; \forall\varphi$$

CTL path formulas:

$$\varphi \; ::= \; \bigcirc\Phi \;\big|\; \Phi_1 \,\mathsf{U}\, \Phi_2$$

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eventually:

$$\exists\Diamond\Phi \quad \stackrel{\text{def}}{=} \quad \exists(\textit{true}\,\mathsf{U}\,\Phi)$$

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Syntax of CTL

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unconditional process fairness $\forall\Box\forall\Diamond crit_1 \wedge \forall\Box\forall\Diamond crit_2$

Example: 15-puzzle

6	8	2	12
4	1	13	5
	9	10	14
7	11	15	3

$\overset{?}{\rightsquigarrow}$

1	2	3	4
5	6	7	8
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$\uparrow$

states:	game configurations
transitions:	legal moves

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- transition system has $16! \approx 2 \cdot 10^{13}$ states
- representation as parallel system:

  $$\mathit{left} \,\|\, \mathit{up} \,\|\, \mathit{down} \,\|\, \mathit{right}$$

  with shared variables $\mathit{field}[i]$ for $i = 1, \ldots, 16$

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CTL specification:

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CTL specification: seeking for a **witness** for

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Semantics of CTL

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define a satisfaction relation $\models$ for CTL formulas over AP and a given TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

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Semantics of CTL

define a satisfaction relation $\models$ for CTL formulas over AP and a given TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$ without terminal states

- interpretation of state formulas over the states
- interpretation of path formulas over the paths (infinite path fragments)

Recall: semantics of LTL

for infinite path fragment $\pi = s_0\, s_1\, s_2\, \ldots$:

$$
\begin{array}{lll}
\pi \models \mathit{true} & & \\
\pi \models a & \text{iff} & s_0 \models a \text{ ,i.e., } a \in L(s_0) \\
\pi \models \varphi_1 \wedge \varphi_2 & \text{iff} & \pi \models \varphi_1 \text{ and } \pi \models \varphi_2 \\
\pi \models \neg\varphi & \text{iff} & \pi \not\models \varphi \\
\pi \models \bigcirc\varphi & \text{iff} & \mathit{suffix}(\pi, 1) = s_1\, s_2\, s_3\, \ldots \models \varphi \\
\pi \models \varphi_1 \,\mathsf{U}\, \varphi_2 & \text{iff} & \text{there exists } j \geq 0 \text{ such that}
\end{array}
$$

$$
\begin{array}{lll}
\mathit{suffix}(\pi, j) = s_j\; s_{j+1}\; s_{j+2}\; \ldots \models \varphi_2 & \text{and} \\
\mathit{suffix}(\pi, k) = s_k\, s_{k+1}\, s_{k+2}\, \ldots \models \varphi_1 & \text{for } 0 \leq k < j
\end{array}
$$

Satisfaction relation for path formulas

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semantics of derived operators:

$$\pi \models \Diamond \Phi \quad \text{iff} \quad \text{there exists } j \geq 0 \text{ with } \; s_j \models \Phi$$

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\pi \models \Diamond \Phi & \text{iff} & \text{there exists } j \geq 0 \text{ with } \quad s_j \models \Phi \\
\pi \models \Box \Phi & \text{iff} & \text{for all } j \geq 0 \text{ we have: } \quad s_j \models \Phi
\end{array}
$$

Satisfaction relation for state formulas

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$s \models true$

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$s \models a \qquad \text{iff} \quad a \in L(s)$

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$s \models \Phi_1 \wedge \Phi_2$ iff $s \models \Phi_1$ and $s \models \Phi_2$

Satisfaction relation for state formulas

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$$s \models \neg\Phi \quad \text{iff} \quad s \not\models \Phi$$

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s \models \exists \varphi & \text{iff} & \text{there is a path } \pi \in \mathit{Paths}(s) \\
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Satisfaction relation for state formulas

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$s \models \exists\varphi$	iff	there is a path $\pi \in \mathit{Paths}(s)$ s.t. $\pi \models \varphi$
$s \models \forall\varphi$	iff	for each path $\pi \in \mathit{Paths}(s)$: $\pi \models \varphi$

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satisfaction set for state formula Φ:

$$\mathit{Sat}(\Phi) \stackrel{\text{def}}{=} \{s \in S : s \models \Phi\}$$

Interpretation of CTL formulas over a TS

Interpretation of CTL formulas over a TS

satisfaction of state formulas over a TS $\mathcal{T}$:

$$\mathcal{T} \models \Phi \quad \text{iff} \quad S_0 \subseteq Sat(\Phi)$$

where S_0 is the set of initial states

recall: $Sat(\Phi) \ = \ \{ s \in S : s \models \Phi \}$

Interpretation of CTL formulas over a TS

satisfaction of state formulas over a TS $\mathcal{T}$:

$$\mathcal{T} \models \Phi \quad \text{iff} \quad S_0 \subseteq Sat(\Phi)$$

$$\text{iff} \quad s_0 \models \Phi \text{ for all initial states } s_0 \text{ of } \mathcal{T}$$

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recall: $Sat(\Phi) \;=\; \{s \in S : s \models \Phi\}$

Semantics of the next operator

$s \models \exists \bigcirc \Phi$ iff there exists $\pi = s\, s_1\, s_2 \ldots \in Paths(s)$ s.t. $\pi \models \bigcirc \Phi$

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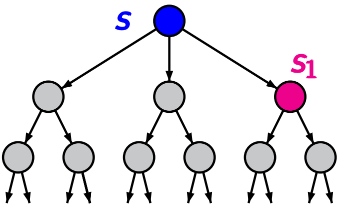
$\exists \bigcirc \Phi$



$Post(s) \cap Sat(\Phi) \neq \varnothing$

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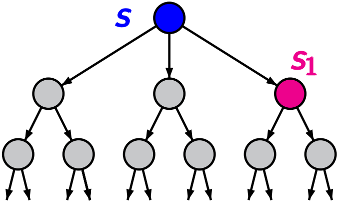

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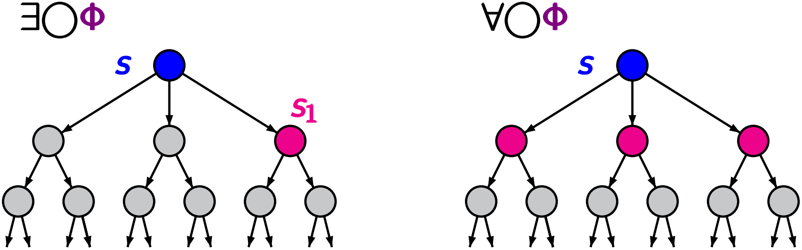
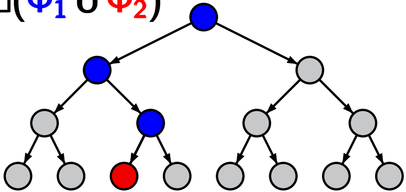


$Post(s) \cap Sat(\Phi) \neq \varnothing$ $\qquad$ $Post(s) \subseteq Sat(\Phi)$

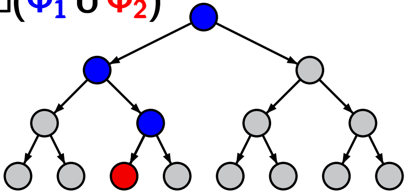
Semantics of until

$\exists(\Phi_1 \mathsf{U} \Phi_2)$

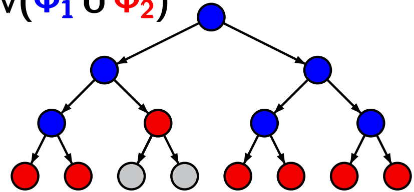


Semantics of until

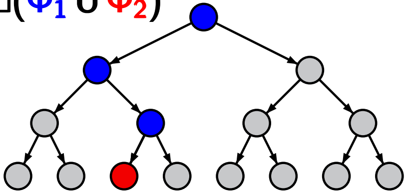
$\exists(\Phi_1 \, \mathsf{U} \, \Phi_2)$



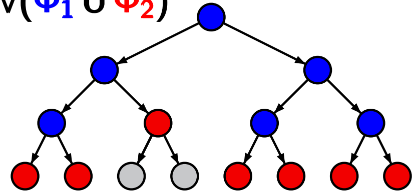
$\forall(\Phi_1 \, \mathsf{U} \, \Phi_2)$



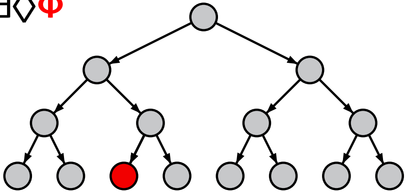
$\exists(\Phi_1 \, \mathsf{U} \, \Phi_2)$



$\forall(\Phi_1 \, \mathsf{U} \, \Phi_2)$

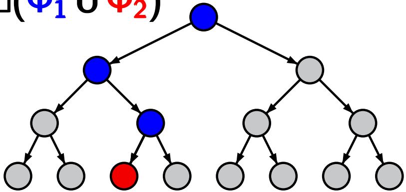


$\exists\Diamond\Phi$

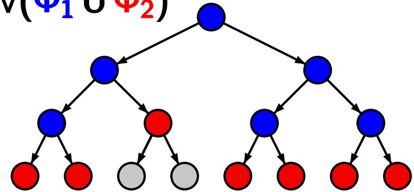


Semantics of until and eventually

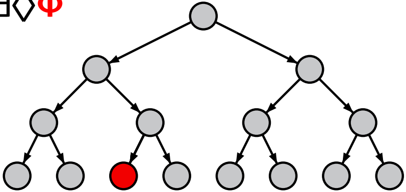
$\exists(\Phi_1 \mathsf{U} \Phi_2)$



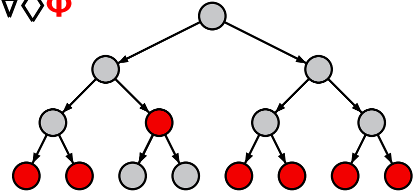
$\forall(\Phi_1 \mathsf{U} \Phi_2)$



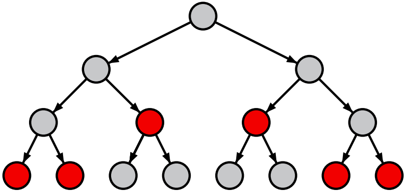
$\exists\Diamond\Phi$



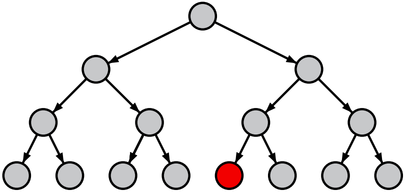
$\forall\Diamond\Phi$



$\forall\Diamond\Phi$

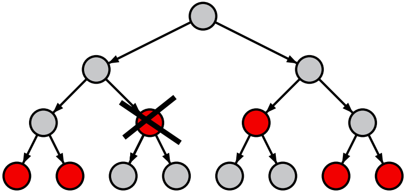


$\exists\Diamond\Phi$

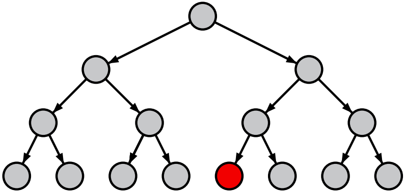


Semantics of eventually and always

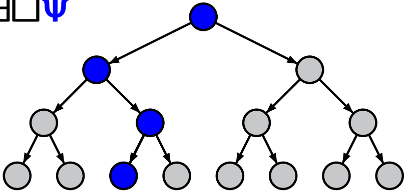
$\neg\forall\Diamond\Phi$



$\exists\Diamond\Phi$

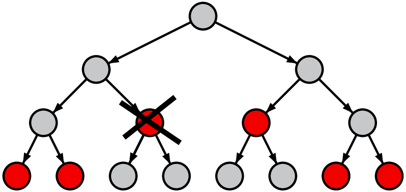


$\exists\Box\Psi$

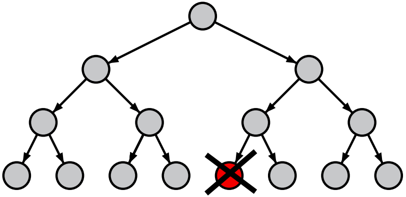


Semantics of eventually and always

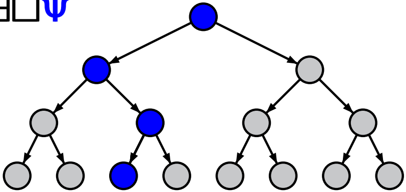
$\neg\forall\Diamond\Phi$



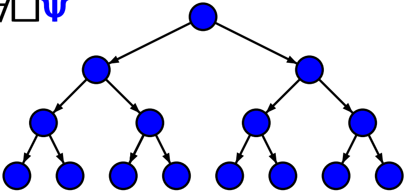
$\neg\exists\Diamond\Phi$



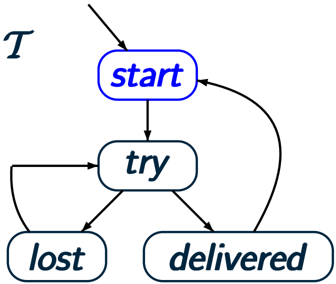
$\exists\Box\Psi$

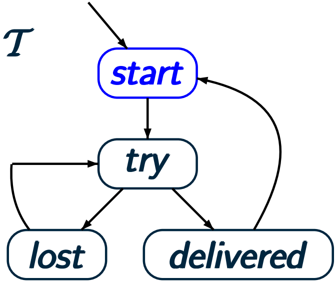


$\forall\Box\Psi$



Example for CTL semantics

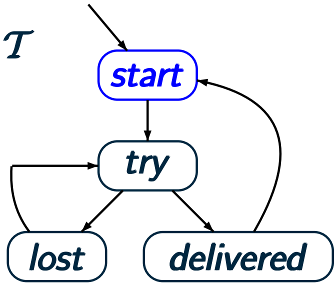




CTL formula

$\Phi = \forall\Box\forall\Diamond start$

Example for CTL semantics

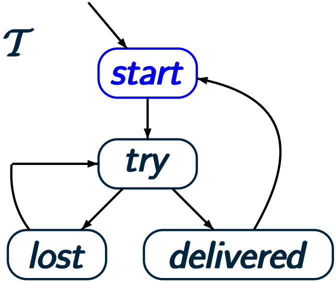


CTL formula

$\Phi = \forall\Box \boxed{\forall\Diamond start}$

$Sat(\forall\Diamond start) = \textbf{?}$

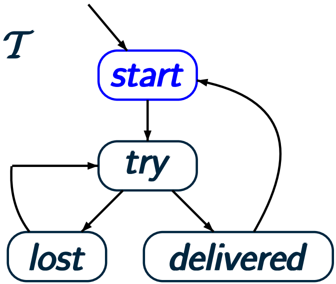
Example for CTL semantics



CTL formula

$\Phi = \forall\Box \boxed{\forall\Diamond start}$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

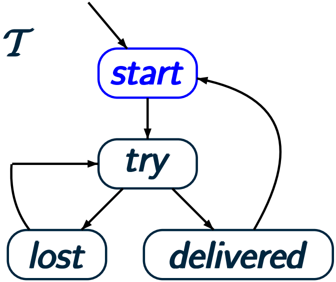


CTL formula

$$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

Example for CTL semantics



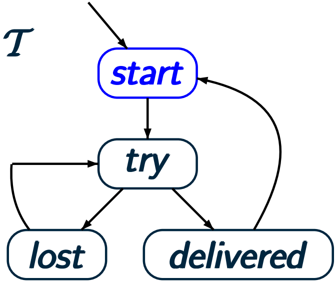
CTL formula

$$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

$$Sat(\Phi) = \varnothing$$

Example for CTL semantics



$\mathcal{T} \not\models \forall\Box\forall\Diamond start$

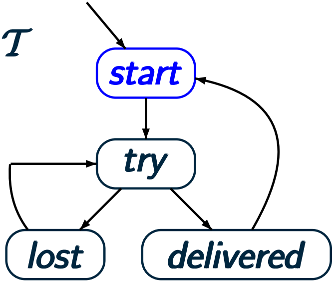
CTL formula

$\Phi = \forall\Box\forall\Diamond start \quad \widehat{=} \quad \forall\Box(start \vee delivered)$

$$Sat(\forall\Diamond start) = \{start, delivered\}$$

$$Sat(\Phi) = \varnothing$$

Example for CTL semantics



$\mathcal{T} \not\models \forall\Box\forall\Diamond \textit{start}$

"infinitely often *start*"

CTL formula

$$\Phi = \forall\Box\forall\Diamond \textit{start} \quad \widehat{=} \quad \forall\Box(\textit{start} \vee \textit{delivered})$$

$$\textit{Sat}(\forall\Diamond \textit{start}) = \{\textit{start}, \textit{delivered}\}$$

$$\textit{Sat}(\Phi) = \varnothing$$

Weak until W

Weak until W

in **LTL**: $\varphi \,\mathbf{W}\, \psi \overset{\text{def}}{=} (\varphi \,\mathbf{U}\, \psi) \vee \Box\varphi$

in **CTL**: **?**

Weak until W

in **LTL**: $\varphi \, \mathsf{W} \, \psi \stackrel{\text{def}}{=} (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi$

duality of **U** and **W**:

$$\neg(\varphi \, \mathsf{U} \, \psi) \equiv (\varphi \wedge \neg\psi) \, \mathsf{W} \, (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \, \mathsf{W} \, \psi) \equiv (\varphi \wedge \neg\psi) \, \mathsf{U} \, (\neg\varphi \wedge \neg\psi)$$

in **CTL**: **?**

in **LTL**: $\varphi \, \mathbf{W} \, \psi \stackrel{\text{def}}{=} (\varphi \, \mathbf{U} \, \psi) \vee \Box\varphi$

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$$\neg(\varphi \, \mathbf{W} \, \psi) \equiv (\varphi \wedge \neg\psi) \, \mathbf{U} \, (\neg\varphi \wedge \neg\psi)$$

definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\Phi \, \mathbf{W} \, \Psi) \stackrel{\text{def}}{=} \neg\forall((\Phi \wedge \neg\Psi) \, \mathbf{U} \, (\neg\Phi \wedge \neg\Psi))$$

Weak until W

in **LTL**: $\varphi \, \mathsf{W} \, \psi \stackrel{\text{def}}{=} (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi$

duality of **U** and **W**:

$$\neg(\varphi \, \mathsf{U} \, \psi) \equiv (\varphi \wedge \neg\psi) \, \mathsf{W} \, (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \, \mathsf{W} \, \psi) \equiv (\varphi \wedge \neg\psi) \, \mathsf{U} \, (\neg\varphi \wedge \neg\psi)$$

definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\Phi \, \mathsf{W} \, \Psi) \stackrel{\text{def}}{=} \neg\forall((\Phi \wedge \neg\Psi) \, \mathsf{U} \, (\neg\Phi \wedge \neg\Psi))$$

$$\forall(\Phi \, \mathsf{W} \, \Psi) \stackrel{\text{def}}{=} \neg\exists((\Phi \wedge \neg\Psi) \, \mathsf{U} \, (\neg\Phi \wedge \neg\Psi))$$

Weak until W in CTL

definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\Phi \mathsf{W} \Psi) \stackrel{\text{def}}{=} \neg\forall(\,(\Phi \wedge \neg\Psi)\,\mathsf{U}\,(\neg\Phi \wedge \neg\Psi)\,)$$

$$\forall(\Phi \mathsf{W} \Psi) \stackrel{\text{def}}{=} \neg\exists(\,(\Phi \wedge \neg\Psi)\,\mathsf{U}\,(\neg\Phi \wedge \neg\Psi)\,)$$

Weak until W in CTL

definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\Phi \, \mathsf{W} \, \Psi) \overset{\text{def}}{=} \neg\forall\big((\Phi \land \neg\Psi) \, \mathsf{U} \, (\neg\Phi \land \neg\Psi)\big)$$

$$\forall(\Phi \, \mathsf{W} \, \Psi) \overset{\text{def}}{=} \neg\exists\big((\Phi \land \neg\Psi) \, \mathsf{U} \, (\neg\Phi \land \neg\Psi)\big)$$

note that:

$$\exists(\Phi \, \mathsf{W} \, \Psi) \equiv \exists(\Phi \, \mathsf{U} \, \Psi) \lor \exists\Box\Phi$$

Weak until W in CTL

definition of **W** in **CTL** on the basis of duality rules:

$$\exists(\Phi \, \mathsf{W} \, \Psi) \stackrel{\text{def}}{=} \neg\forall\big((\Phi \wedge \neg\Psi) \, \mathsf{U} \, (\neg\Phi \wedge \neg\Psi)\big)$$

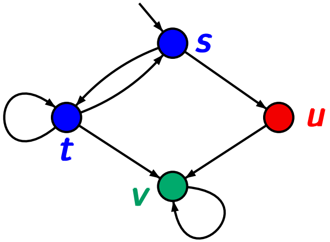
$$\forall(\Phi \, \mathsf{W} \, \Psi) \stackrel{\text{def}}{=} \neg\exists\big((\Phi \wedge \neg\Psi) \, \mathsf{U} \, (\neg\Phi \wedge \neg\Psi)\big)$$

note that:

$$\exists(\Phi \, \mathsf{W} \, \Psi) \equiv \exists(\Phi \, \mathsf{U} \, \Psi) \vee \exists\Box\Phi$$

$$\forall(\Phi \, \mathsf{W} \, \Psi) \not\equiv \forall(\Phi \, \mathsf{U} \, \Psi) \vee \forall\Box\Phi$$

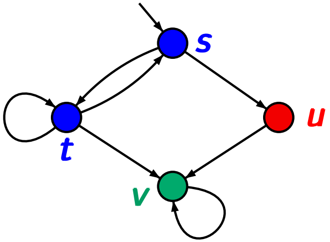
Weak until W in CTL



● $\widehat{=}$ $\{a\}$
● $\widehat{=}$ $\{b\}$
● $\widehat{=}$ $\{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \mathsf{W} c)$ **?**

Weak until W in CTL



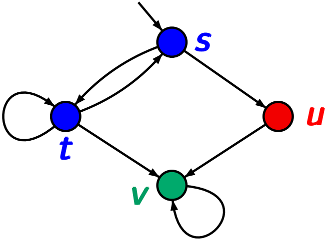
$\bullet \mathrel{\widehat{=}} \{a\}$

$\bullet \mathrel{\widehat{=}} \{b\}$

$\bullet \mathrel{\widehat{=}} \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \mathsf{W}\, c)$

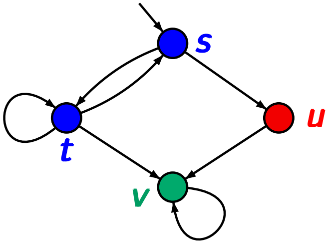
Weak until W in CTL



$\bullet \mathrel{\widehat{=}} \{a\}$
$\bullet \mathrel{\widehat{=}} \{b\}$
$\bullet \mathrel{\widehat{=}} \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \mathsf{W} c)$ $\surd$ as $s\, s_1\, s_2 \ldots \models \Diamond\exists(a \mathsf{W} c)$

Weak until W in CTL

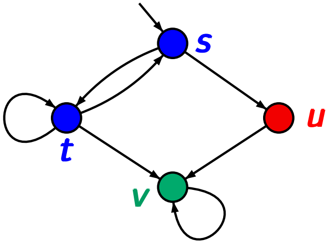


● $\widehat{=}$ $\{a\}$
● $\widehat{=}$ $\{b\}$
● $\widehat{=}$ $\{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \,\mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \,\mathsf{W}\, c)$

$\mathcal{T} \models \exists(a \,\mathsf{W}\, \exists\Box b)$ **?**

Weak until W in CTL



$\bullet \ \widehat{=}\ \{a\}$

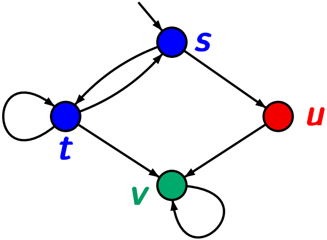
$\bullet \ \widehat{=}\ \{b\}$

$\bullet \ \widehat{=}\ \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \ \mathsf{W}\ c)$ $\surd$ as $s \models \exists(a \ \mathsf{W}\ c)$

$\mathcal{T} \models \exists(a \ \mathsf{W}\ \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

Weak until W in CTL



$\bullet \mathrel{\widehat{=}} \{a\}$

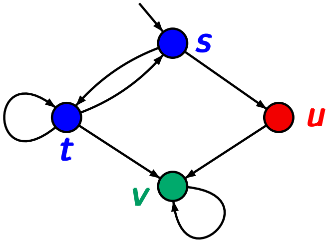
$\bullet \mathrel{\widehat{=}} \{b\}$

$\bullet \mathrel{\widehat{=}} \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \,\mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \,\mathsf{W}\, c)$

$\mathcal{T} \models \exists(a \,\mathsf{W}\, \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \,\mathsf{W}\, (a \wedge b))$ **?**



$\bullet \;\widehat{=}\; \{a\}$

$\bullet \;\widehat{=}\; \{b\}$

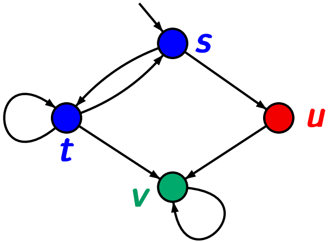
$\bullet \;\widehat{=}\; \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \mathsf{W} c)$ $\surd$ as $s \models \exists(a \mathsf{W} c)$

$\mathcal{T} \models \exists(a \mathsf{W} \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \mathsf{W} (a \wedge b))$ $\surd$

Weak until W in CTL



$\bullet \mathrel{\widehat{=}} \{a\}$ (blue)
$\bullet \mathrel{\widehat{=}} \{b\}$ (red)
$\bullet \mathrel{\widehat{=}} \{c\}$ (green)

$\mathcal{T} \models \forall\Diamond\exists(a \,\mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \,\mathsf{W}\, c)$

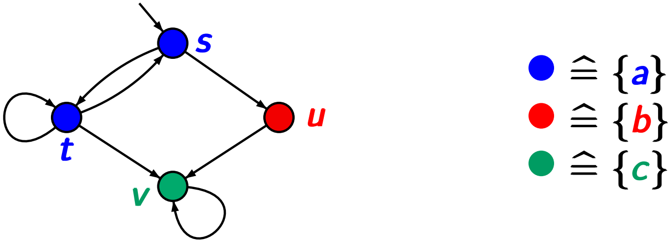
$\mathcal{T} \models \exists(a \,\mathsf{W}\, \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \,\mathsf{W}\, (a \wedge b))$ $\surd$

$\uparrow$

three types of paths: $(s\,t)^\omega$ or $(s\,t)^+\,v^\omega$ or $(s\,t)^*\,s\,u\,v^\omega$

Weak until W in CTL



$\mathcal{T} \models \forall\Diamond\exists(a \, \mathsf{W} \, c)$ $\surd$ as $s \models \exists(a \, \mathsf{W} \, c)$

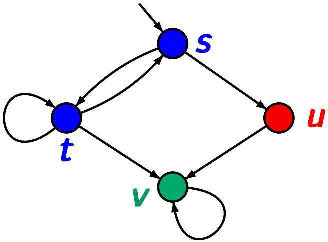
$\mathcal{T} \models \exists(a \, \mathsf{W} \, \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

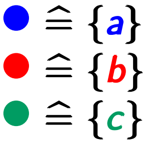
$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \, \mathsf{W} \, (a \wedge b))$ $\surd$

three types of paths: $(s\,t)^\omega$ or $(s\,t)^+ \, v^\omega$ or $(s\,t)^* \, s\,u\,v^\omega$

in all three cases: $\pi \models \Box\exists\bigcirc(b \vee c)$

Weak until W in CTL

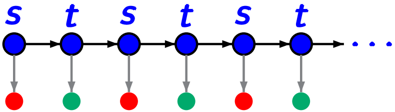




$\mathcal{T} \models \forall\Diamond\exists(a \,\mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \,\mathsf{W}\, c)$

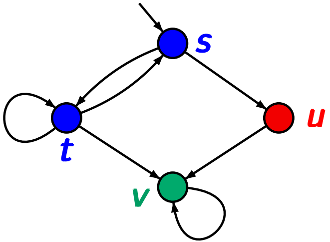
$\mathcal{T} \models \exists(a \,\mathsf{W}\, \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \,\mathsf{W}\, (a \wedge b))$ $\surd$



$\models \Box\exists\bigcirc(b \vee c)$

Weak until W in CTL



$\bullet \;\widehat{=}\; \{a\}$

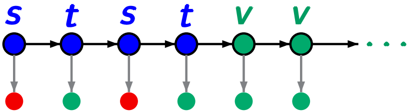
$\bullet \;\widehat{=}\; \{b\}$

$\bullet \;\widehat{=}\; \{c\}$

$\mathcal{T} \models \forall\Diamond\exists(a \,\mathsf{W}\, c)$ $\surd$ as $s \models \exists(a \,\mathsf{W}\, c)$

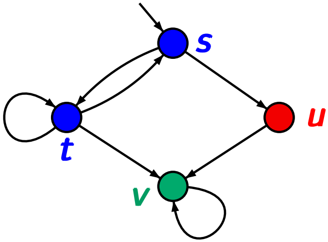
$\mathcal{T} \models \exists(a \,\mathsf{W}\, \exists\Box b)$ $\surd$ as $s \models \exists\Box a$

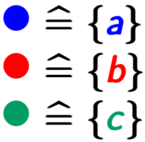
$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \,\mathsf{W}\, (a \wedge b))$ $\surd$



$\models \Box\exists\bigcirc(b \vee c)$

Weak until W in CTL

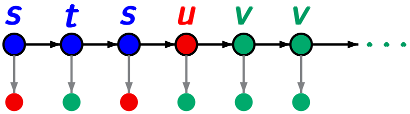




$\mathcal{T} \models \forall\Diamond\exists(a \, \mathsf{W} \, c)$ √ as $s \models \exists(a \, \mathsf{W} \, c)$

$\mathcal{T} \models \exists(a \, \mathsf{W} \, \exists\Box b)$ √ as $s \models \exists\Box a$

$\mathcal{T} \models \forall((\exists\bigcirc(b \vee c)) \, \mathsf{W} \, (a \wedge b))$ √



$\models \Box\exists\bigcirc(b \vee c)$

Expansion laws

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$$\exists(\Phi \, \mathsf{U} \, \Psi) \equiv \Psi \vee (\Phi \wedge \exists\bigcirc\exists(\Phi \, \mathsf{U} \, \Psi))$$

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$$\forall(\Phi \,\mathsf{U}\, \Psi) \equiv \ ?$$

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$$\exists\Diamond\Psi \;\equiv\; \Psi \vee \exists\bigcirc\exists\Diamond\Psi$$

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$$\exists\Box\Phi \equiv ?$$

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