CS1632, Lecture 19: Static Analysis, Part 3

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State Space Reduction Techniques

- State collapsing
- Heuristic state approximation
- Hash compaction
- Heap canonicalization
- Symbolic execution

Symbolic Execution

- Analysis of programs with unspecified inputs
  - Execute a program on symbolic inputs
- Symbolic states represent sets of concrete states
- For each path, build a path condition
  - Condition on inputs for the execution to follow that path
  - Check path condition satisfiability -- explore only feasible paths
- Symbolic state
  - Symbolic values/expressions for variables
  - Path condition
  - Program counter

Example: Standard Execution

Code that swaps 2 integers

```
int x, y;
if (x > y) {
  x = x + y;
  y = x – y;
  x = x – y;
  if (x > y)
    assert false;
}
```

Concrete Execution Path

x = 1, y = 0

1 > 0 ? true

x = 1 + 0 = 1

y = 1 – 0 = 1

x = 1 – 1 = 0

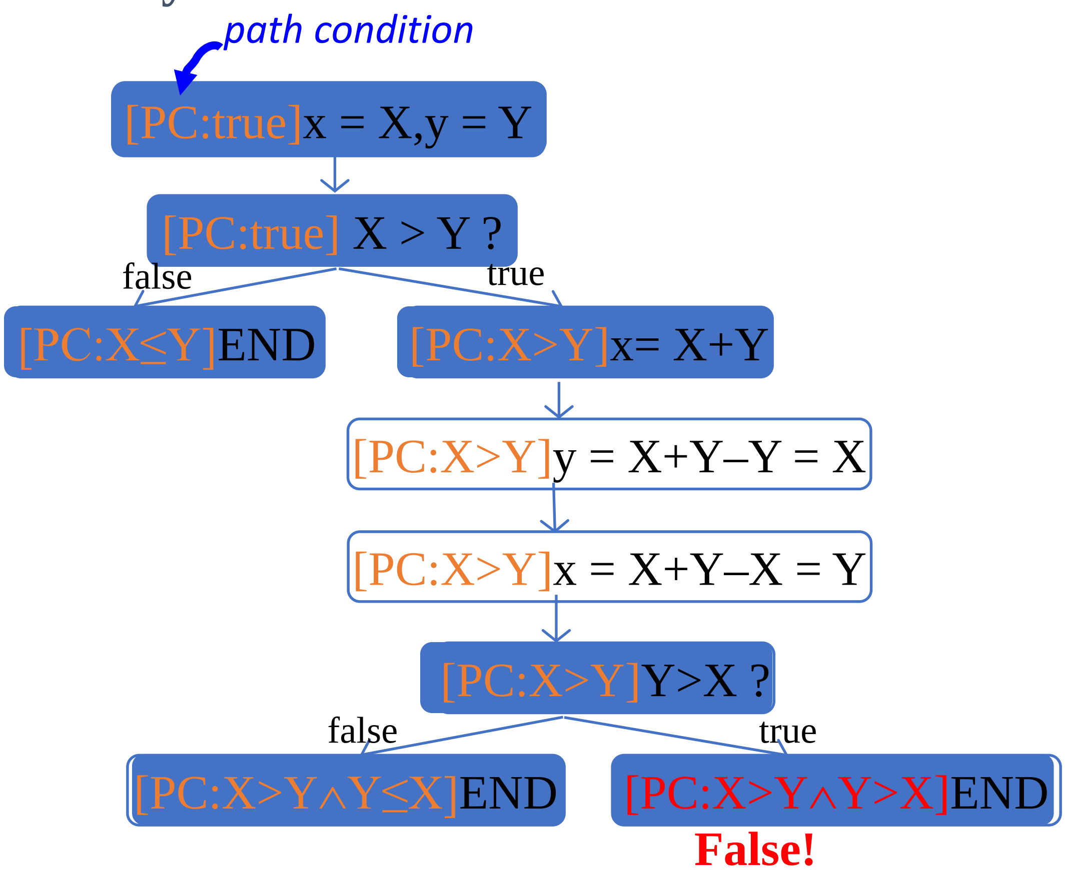
0 > 1 ? false

Example: Symbolic Execution

Code that swaps 2 integers:

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Symbolic Execution Tree:



Solve path conditions → test inputs

Questions

- What about loops?
- What about overflow?

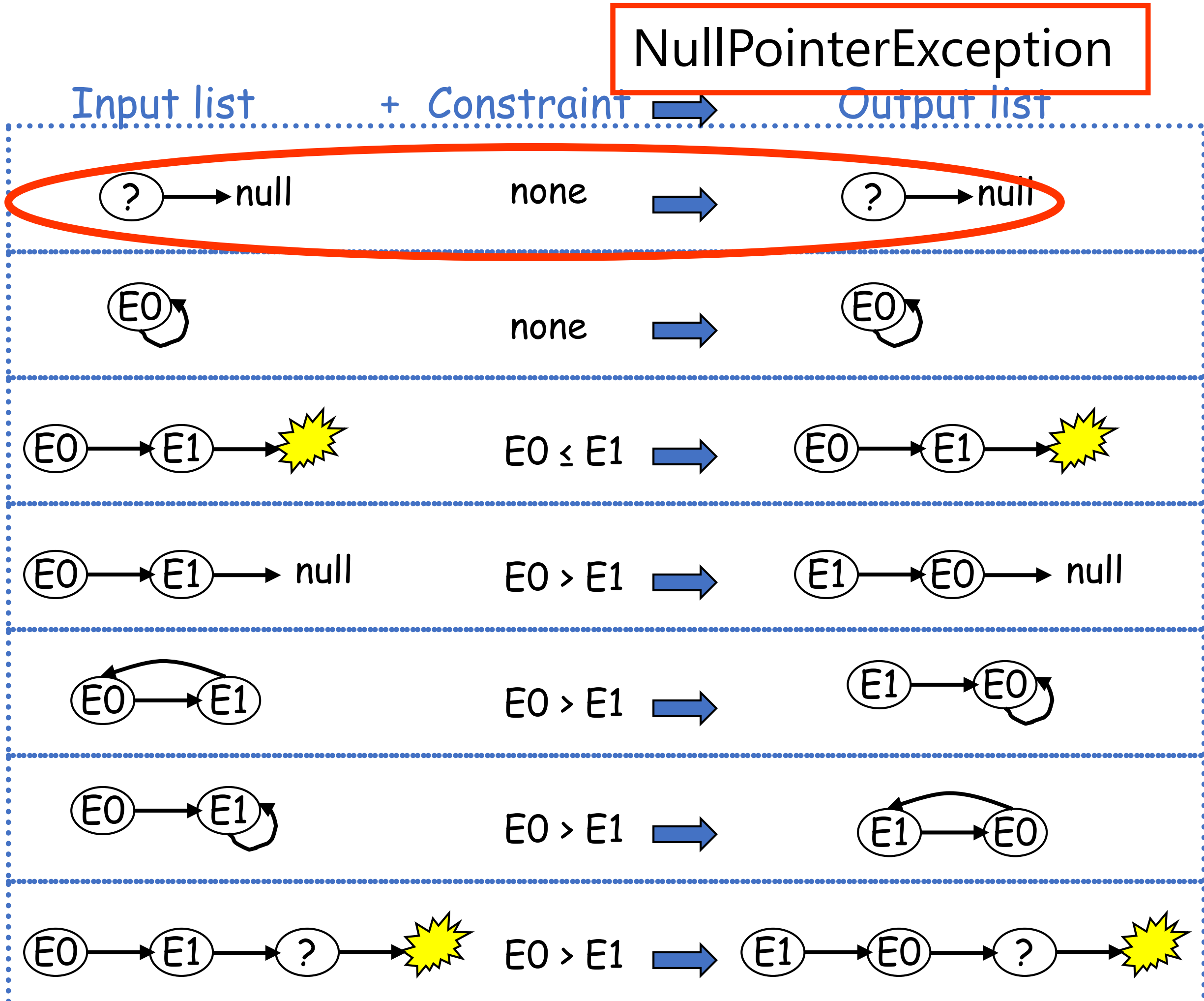
- What about multi-threading?
- What about data structures?

Generalized Symbolic Execution [TACAS'03]

- Handles dynamically allocated data structures and multi-threading
- Key elements:
  - Lazy initialization for input data structures
  - Standard model checker (Java PathFinder) for multi-threading
- Model Checker
  - Analyzes thread inter-leavings
  - Leverages optimizations
    - Symmetry and partial order reductions, abstraction etc.
  - Generates and explores the symbolic execution tree
  - Explores different heap configurations explicitly -- non-determinism handles aliasing

Example Analysis

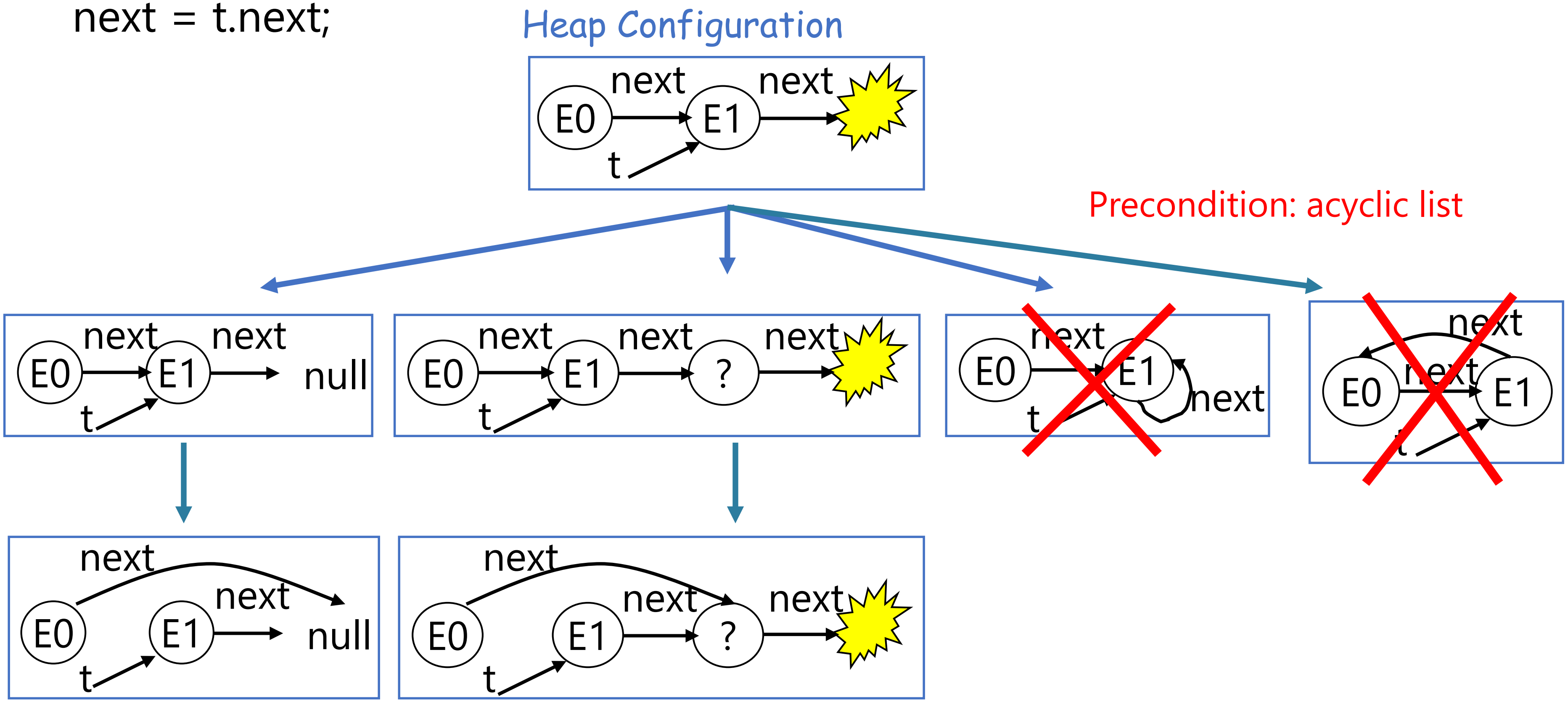
```
class Node {
    int elem;
    Node next;

    Node swapNode() {
      if (next != null)
        if (elem > next.elem) {
            Node t = next;
            next = t.next;
            t.next = this;
            return t;
        }
      return this;
    }
}
```



Lazy Initialization (illustration)

consider executing

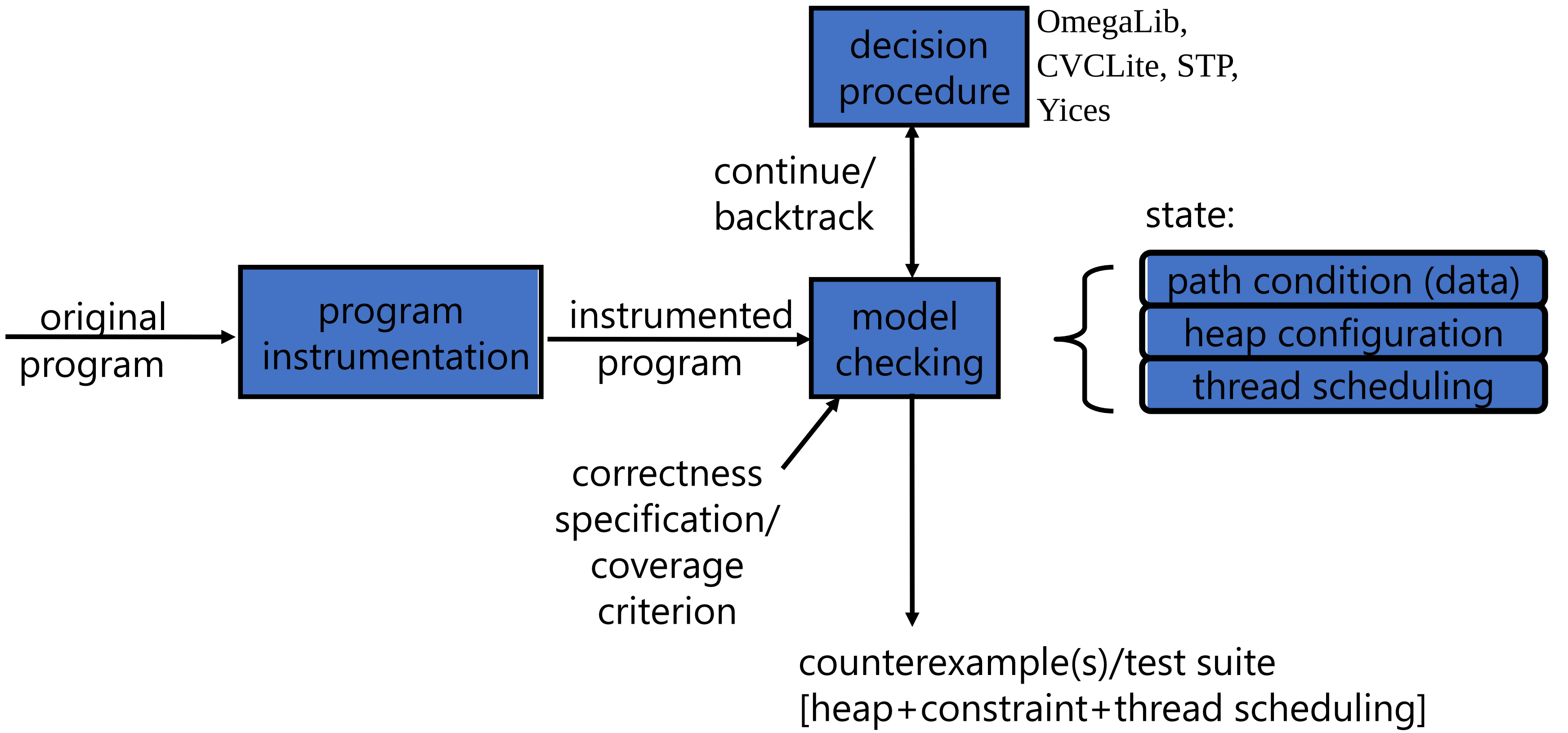
    next = t.next;



Implementation

- Symbolic execution of Java programs
- Code instrumentation
  - Programs instrumented to enable JPF to perform symbolic execution
  - Replace concrete operations with calls to methods that implement symbolic operations
  - General: could use/leverage any model checker
- Decision procedures used to check satisfiability of path conditions
  - Omega library for integer linear constraints
  - CVCLite, STP (Stanford), Yices (SRI)

Implementation via Instrumentation



Symbolic PathFinder

- No longer uses code instrumentation
- Implements a non-standard interpreter of byte-codes
  - Enables JPF to perform symbolic analysis
  - Replaces standard byte-code execution with non-standard symbolic execution
- During execution checks for assert violations, run-time errors, etc.
- Symbolic information:
  - Stored in attributes associated with the program data
  - Propagated dynamically during symbolic execution
- Choice generators and listeners:
  - Non-deterministic choices handle branching conditions
  - Listeners print results: path conditions, test vectors/sequences
- Native peers model native libraries:
  - Capture `Math` calls and send them to the constraint solver
- Generic interface for multiple decision procedures
  - Choco, IASolver, CVC3, Yices, HAMPI, CORAL [NFM11], etc.

Example: IADD

Concrete execution of IADD byte-code:

```
public class IADD extends
  Instruction { …
 public Instruction execute(…
  ThreadInfo th){
  int v1 = th.pop();
  int v2 = th.pop();
  th.push(v1+v2,…);
  return getNext(th);
 }
}
```

Symbolic execution of IADD byte-code:

```
public class IADD extends
   ….bytecode.IADD { …
 public Instruction execute(…
   ThreadInfo th){
   Expression sym_v1 = ….getOperandAttr(0);
   Expression sym_v2 = ….getOperandAttr(1);
   if (sym_v1 == null && sym_v2 == null)
     // both values are concrete
     return super.execute(… th);
   else {
     int v1 = th.pop();
     int v2 = th.pop();
     th.push(0,…); // don't care
     …
     ….setOperandAttr(Expression._plus(
    sym_v1,sym_v2));
     return getNext(th);
   }
 }
}
```

Complex mathematical constraints

- Model-level interpretation of calls to math functions

`$x + 1` $\longrightarrow$ *Math.sin* $\longrightarrow$ `sin($x + 1)`

Symbolic expression
(un-interpreted function)
denoting the result value of the call

CORAL solver [NFM'11]

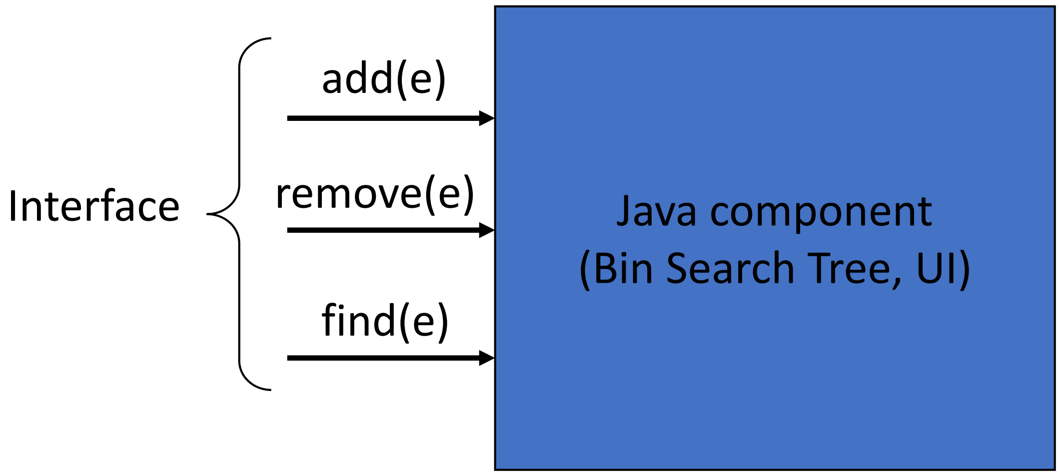
- Target applications:
  - Symbolic execution of programs that manipulate floating-p
  - Use floating-point arithmetic
  - Call specific math functions (from java.lang.Math)

Common in software from NASA

$\mathrm{sqrt}(\exp(x+z))) < \mathrm{pow}(z, x) \wedge x>0 \wedge y>1 \wedge z>1 \wedge y<x+2 \wedge w=x+2$ ⇨ Coral Solver ⇨ {x=4.31, y=6.08, z=9.51, w=6.31}

- Meta-heuristic solver
  - Distance-based fitness function
- Particle swarm optimization (PSO)
  - Search simulates movements in a group of animals
  - Used opt4j library (see opt4j.sourceforge.net)

Applications: Test Input and Sequence Generation



Generated test sequence:

```
BinTree t = new BinTree();
t.add(1);
t.add(2);
t.remove(1);
```

- SymbolicSequenceListener generates JUnit tests
- JUnit tests can be run directly by the developers
- Measure coverage (e.g. MC/DC)

Dynamic Techniques

- Classic symbolic execution is a static technique
- Dynamic techniques
  - Collect symbolic constraints during concrete executions
  - DART = Directed Automated Random Testing
  - Concolic (Concrete Symbolic) testing

DART = Directed Automated Random Testing

- Dynamic test generation
  - Run the program starting with some random inputs
  - Gather symbolic constraints on inputs at conditional statements
  - Use a constraint solver to generate new test inputs
  - Repeat the process until a specific program path or statement is reached (classic dynamic test generation [Korel90])
  - Or repeat the process to attempt to cover ALL feasible program paths (DART [Godefroid et al PLDI'05])
- Detect crashes, assert violations, runtime errors etc.

Directed Search

```
int x, y;
if (x > y) {
  x = x + y;
  y = x – y;
  x = x – y;
  if (x > y)
    assert false;
}
```

Concrete Execution	Symbolic Execution	Path Constraint
x = 0, y = 0	create symbolic variables x, y	

Directed Search

```
int x, y;
if (x > y) {
  x = x + y;
  y = x – y;
  x = x – y;
  if (x > y)
    assert false;
}
```

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
		$x \leq y$
$x = 0, y = 0$		

Solve: $!(x \leq y)$

Solution: $x=1, y=0$

Directed Search

Concrete Execution	Symbolic Execution	Path Constraint
x = 1, y = 0	create symbolic variables x, y	

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Directed Search

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
x = 1, y = 0		x > y

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Directed Search

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
		x > y
x = 1, y = 0	x = x+y	

Directed Search

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
		x > y
x = 1, y = 1	x = x+y y = x	

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Directed Search

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
		x > y
	y = x	
x = 0, y = 1	x = y	

Directed Search

```
int x, y;

if (x > y) {

  x = x + y;

  y = x – y;

  x = x – y;

  if (x > y)

    assert false;

}
```

Concrete Execution	Symbolic Execution	Path Constraint
	create symbolic variables x, y	
		x > y
	y = x	
	x = y	
x = 0, y = 1		y ≤ x

Solve: x> y AND !(y≤x)

Impossible: DONE!

Dynamic Test Generation

- Very popular
- Implemented and extended in many interesting ways
- Many tools
  - PEX, SAGE, CUTE, jCUTE, CREST, SPLAT, etc
- Many applications
  - Bug finding, security, web and database applications, etc.
- EXE (Stanford Univ. [Cadar et al TISSEC 2008])
  - Related dynamic approach to symbolic execution

Mixed Concrete-Symbolic Solving [ISSTA'11]

- Use un-interpreted functions for external library calls
- Split path condition PC into:
  - simplePC – solvable constraints
  - complexPC – non-linear constraints with un-interpreted functions
- Solve simplePC
  - Use obtained solutions to simplify complexPC
  - Check the result again for satisfiability

Example (assume hash(x) = 10 *x):

PC: $\underbrace{X>3 \wedge Y>10}_{\text{simplePC}} \wedge \underbrace{Y=hash(X)}_{\text{complexPC}}$

Solve simplePC; use solution X=4 to compute h(4)=40

Simplify complexPC: Y=40

Solve again: $X>3 \wedge Y>10 \wedge Y=40$ Satisfiable!

Challenge

Path explosion

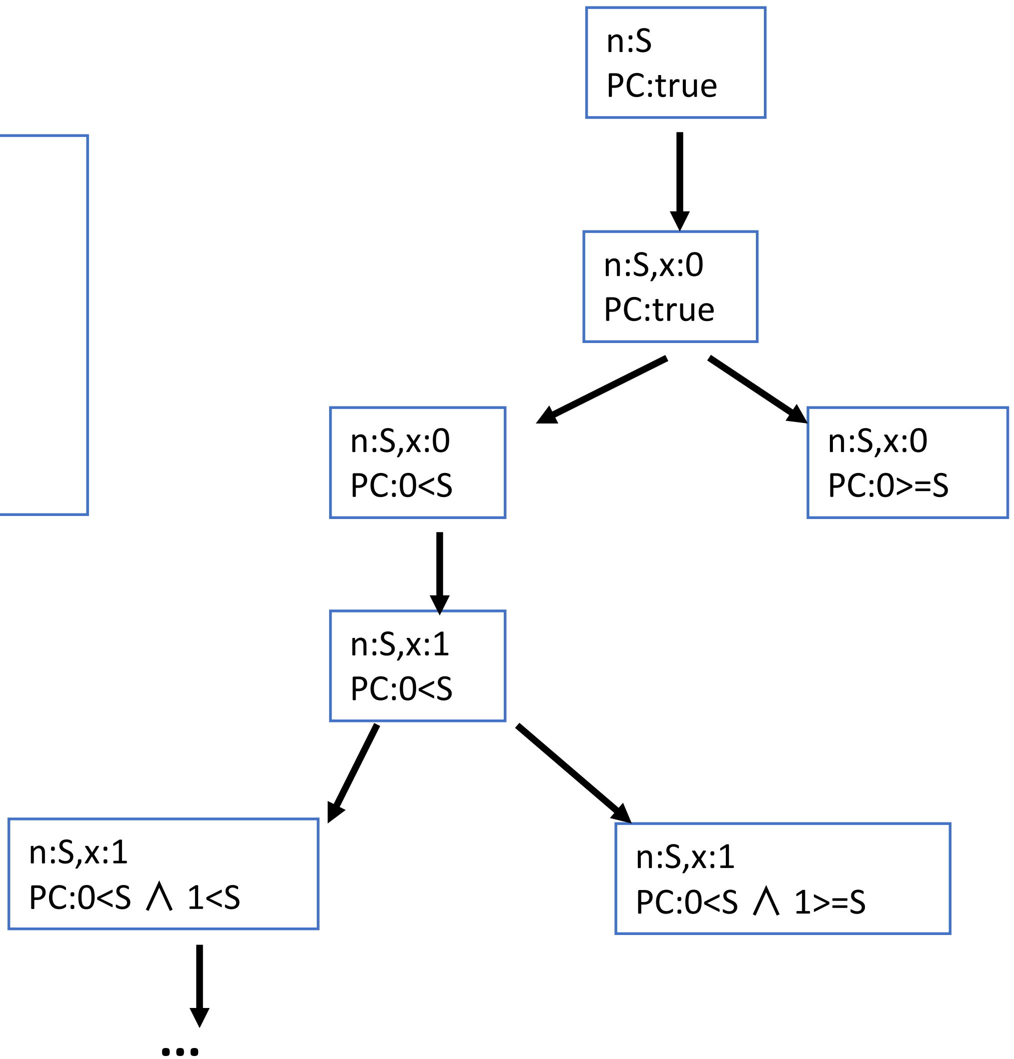
Symbolic execution of a program may result in a very large, possibly infinite number of paths

Problem: loops and recursion

Example Code

```
void test(int n) {
      int x = 0;
      while(x < n)
        x = x + 1;
}
```

Infinite symbolic execution tree



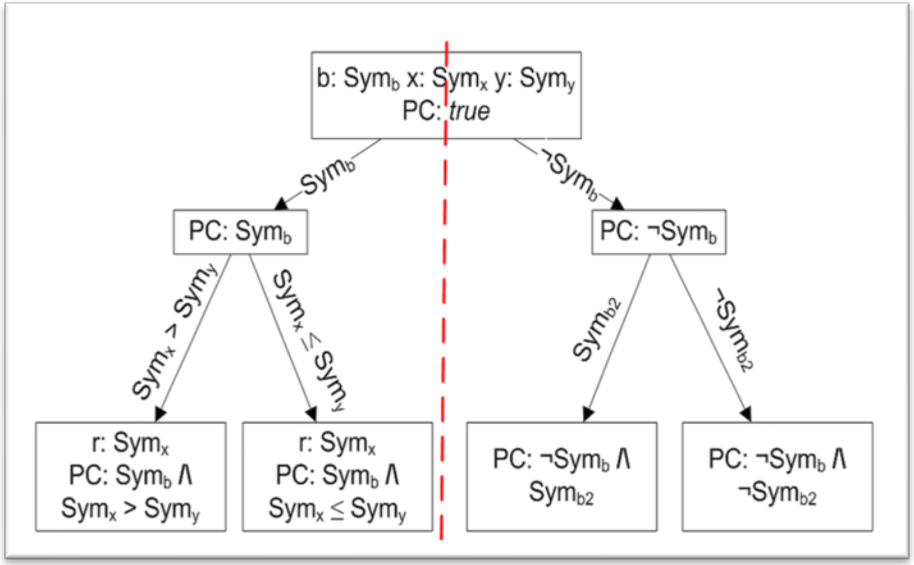
Solutions

- Dealing with loops and recursion
  - Put bound on search depth or on number of PCs
  - Stop search when desired coverage achieved
  - Loop abstraction [Saxena et al ISSTA'09] [Godefroid ISSTA'11]
- Solutions addressing path explosion
  - Parallel Symbolic Execution
  - Abstract State Matching
  - Compositional DART = SMART

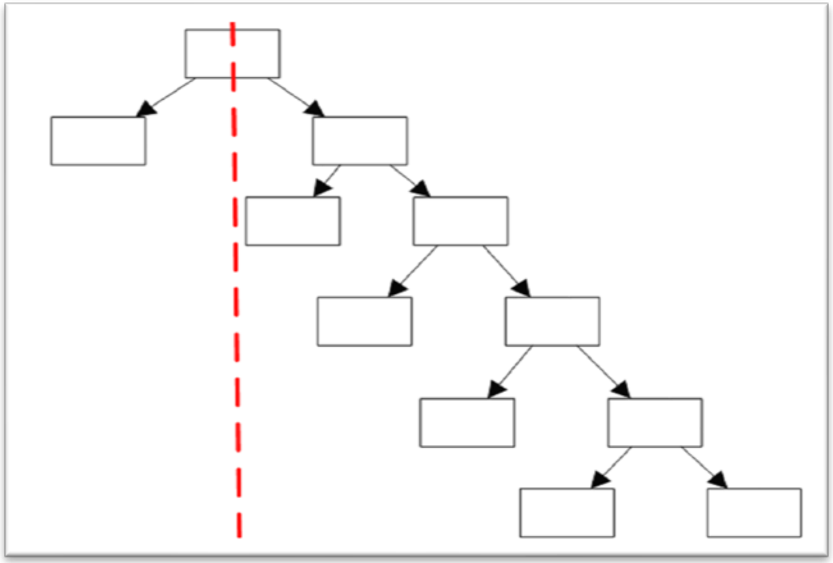
Parallel Symbolic Execution

- Path explosion
  - Increases exponentially with the number of inputs specified as symbolic
  - Very expensive in terms of time (weeks, months)
- Solution
  - Speed-up symbolic execution using parallel or distributed techniques
- Symbolic execution is amenable to parallelization
  - No sharing between sub-trees

Balancing partitions



Nicely Balanced – linear speedup



Poorly Balanced – no speedup

- Solutions
  - Simple static partitioning [ISSTA'10]
  - Dynamic partitioning [Andrew King's Masters Thesis at KSU, Cloud9 at EPFL, Fujitsu]

Simple Static Partitioning

- Static partitioning of tree with light dynamic load balancing
  - Flexible, little communication overhead
- Constraint-based partitioning
  - Constraints used as initial pre-conditions
  - Constraints are disjoint and complete
- Approach
  - Shallow symbolic execution => produces large number of constraints
  - Constraints selection – according to frequency of variables
  - Combinatorial partition creation
- Intuition
  - Commonly used variables likely to partition state space in useful ways
- Results
  - maximum analysis time speedup of 90x observed using 128 workers and a maximum test generation time speedup of 70x observed using 64 workers.

Abstract State Matching

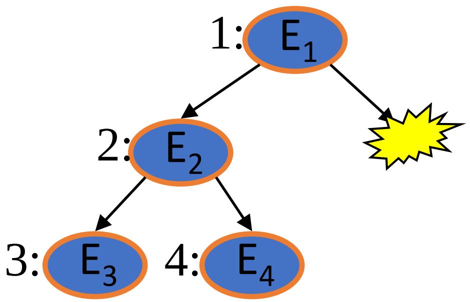
- State matching – subsumption checking [SPIN'06, J. STTT 2008]
  - Obtained through DFS traversal of "rooted" heap configurations
    - Roots are program variables pointing to the heap
    - Unique labeling for "matched" nodes
    - Check logical implication between numeric constraints
  - Not enough to ensure termination
- Abstraction
  - Store abstract versions of explored symbolic states
  - Use subsumption checking to determine if an abstract state is re-visited
  - Decide if the search should continue or backtrack

Abstract State Matching

- Enables analysis of under-approximation of program behavior
- Preserves errors to safety properties -- useful for testing
- Automated support for two abstractions (inspired by shape analysis [TVLA]
  - Singly linked lists
  - Arrays

State Matching: Subsumption Checking

Stored state:

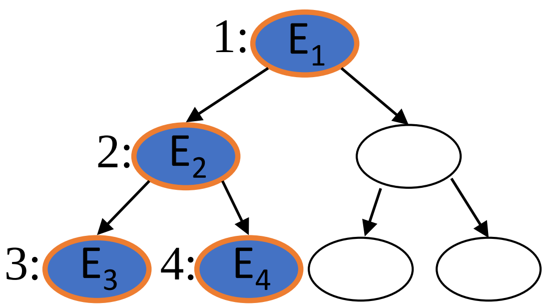


$E_1 > E_2 \wedge$
$E_2 > E_3 \wedge$
$E_2 \leq E_4 \wedge$
$E_1 > E_4$

Set of concrete states represented by stored state

$\Uparrow$

New state:



$E_1 > E_2 \wedge$
$E_2 > E_3 \wedge$
$E_2 < E_4 \wedge$
$E_1 > E_4$

Set of concrete states represented by new state

Normalized using existential quantifier elimination

Abstractions for Lists and Arrays

- Shape abstraction for singly linked lists
  - Summarize contiguous list elements not pointed to by program variables into summary nodes
  - Valuation of a summary node
    - Union of valuations of summarized nodes
  - Subsumption checking between abstracted states
    - Same algorithm as subsumption checking for symbolic states
    - Treat summary node as an “ordinary” node
- Abstraction for arrays
  - Represent array as a singly linked list
  - Abstraction similar to shape abstraction for linked lists

Compositional DART [POPL'07]

- Idea: compositional dynamic test generation
  - use summaries of individual functions like in inter-procedural static analysis
  - if f calls g, analyze g separately, summarize the results, and use g's summary when analyzing f
  - A summary $\phi(g)$ is a disjunction of path constraints expressed in terms of input pre-conditions and output post-conditions:

    $\phi(g) = \vee\phi(w)$, with $\phi(w) = pre(w) \wedge post(w)$
- g's outputs are treated as symbolic inputs to calling function f
- SMART: Top-down strategy to compute summaries on a demand-driven basis from concrete calling contexts
- Same path coverage as DART but can be exponentially faster!
- Follow-up work
  - Anand et al. [TACAS'08], Godefroid et al. [POPL'10]

Example

```
int is_positive(int x) {
  if (x>0) return 1;
  return 0;
}
#define N 100
void top (int s[N]) {// N inputs
 int i, cnt=0;
 for (i=0;i<N; i++)
   cnt=cnt+is_positive(s[i]);
 if (cnt == 3) error(); // (*)
 return;
}
```

Program P = {top, is_positive} has 2^N feasible paths

DART will perform 2^N runs

SMART will perform only 4 runs

- 2 to compute summary

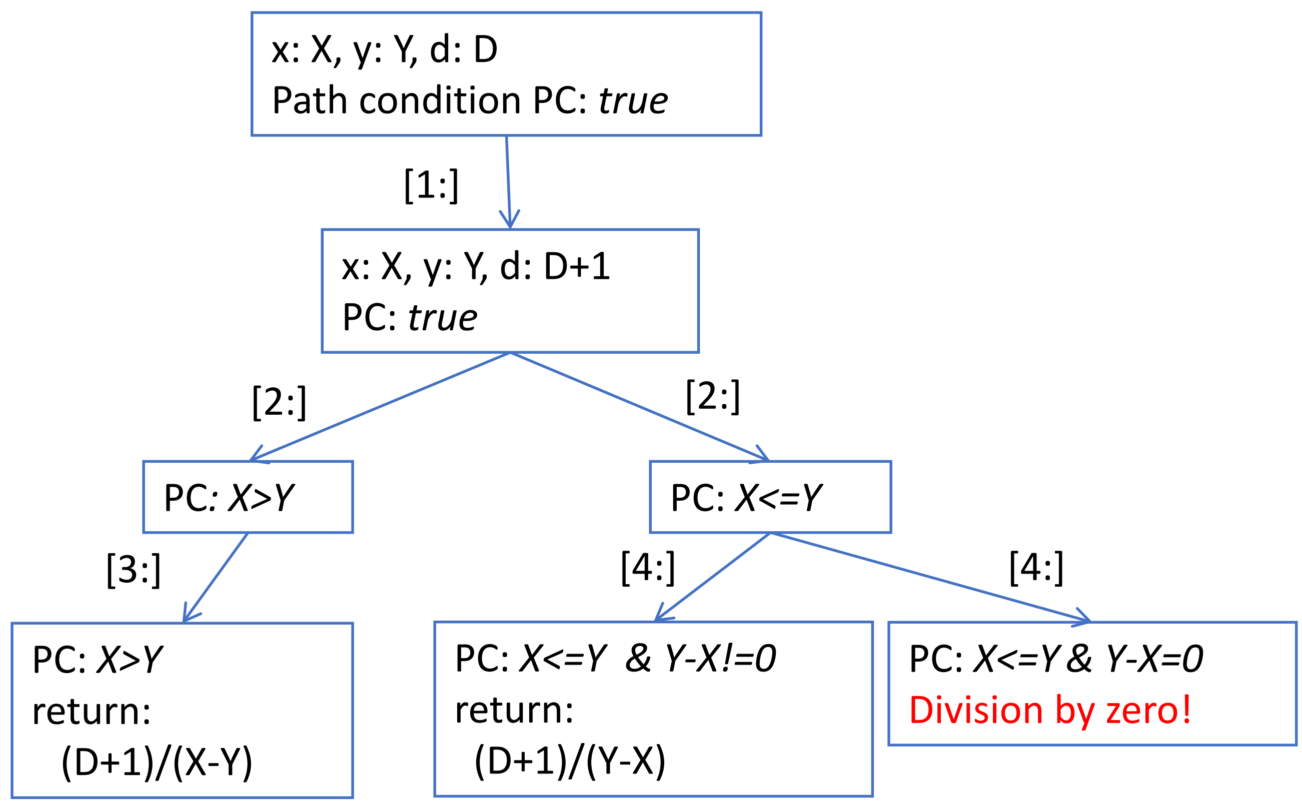
  $\phi(\text{is\_positive}) = (x>0 \wedge ret=1) \vee (x\leq 0 \wedge ret=0)$

- 2 to execute both branches of (*) by solving:

  $[(s[0]>0 \wedge ret_0=1) \vee (s[0]\leq 0 \wedge ret_0=0)] \wedge$
  $[(s[1]>0 \wedge ret_1=1) \vee (s[1]\leq 0 \wedge ret_1=0)] \wedge \cdots \wedge$
  $[(s[N\text{-}1]>0 \wedge ret_{N-1}=1) \vee (s[N-1]\leq 0 \wedge ret_{N-1}=0)] \wedge$
  $(ret_0+ret_1+ \cdots + ret_{N-1}=3)$

Applications – An Example

Symbolic execution tree:



Method m:

```
1: d=d+1;
2:  if (x > y)
3:    return d / (x-y);
     else
4:    return d / (y-x);
```

Solve path conditions → test inputs

Auto-generated JUnit Tests

```
@Test public void t1() {
   m(1, 0, 1);
}
@Test public void t2() {
   m(0, 1, 1);
}
@Test public void t3() {
   m(1, 1, 1);
}
```

Pass ✔

Pass ✔

Fail ✗ PC: $X<=Y$ & $Y-X=0 \Leftrightarrow X=Y$

Achieves full path coverage

Invariant Generation

- Pre-condition:
  - “x!=y”
- Post-condition:
  - “\result==((x>y) ? (d+1)/(x-y) : (d+1)/(y-x))”

Symbolic Execution and Software Testing

- Tools, many open-source
  - NASA's Symbolic (Java) Pathfinder
    http://babelfish.arc.nasa.gov/trac/jpf/wiki/projects/jpf-symbc
  - UIUC's CUTE and jCUTE
    http://osl.cs.uiuc.edu/~ksen/cute
  - Stanford's KLEE
    http://klee.llvm.org/
  - UC Berkeley's CREST and BitBlaze
    http://code.google.com/p/crest
  - Microsoft's Pex, SAGE, YOGI, PREfix
    http://research.microsoft.com/en-us/projects/pex/
    http://research.microsoft.com/en-us/projects/yogi
  - IBM's Apollo, Parasoft's testing tools etc.