This a good Boundary?

How about this one?

Question: if we move some points a tiny bit, for which does the Decision Boundary change ? & which not?

Want:
fit a street between the classes that is as wide as possible

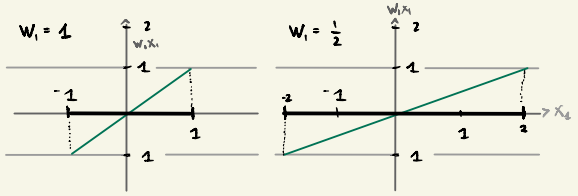
sv = support vector they "support" the hyperplane

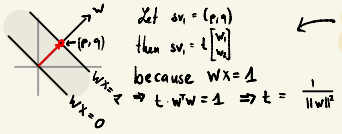
How?

Choose w such that $w^T x - b = 1$ goes through sv_1, and $w^T x - b = -1$ goes through sv_2.

Maximum Margin:
maximize the distance from sv_1, sv_2 to the Hyperplane $\Leftrightarrow$ minimize $\|w\|$ under conditions $y_i(w^T x_i - b) \geq 1$

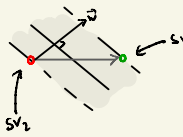
Intuitive picture
smaller w $\Rightarrow$ wider margin





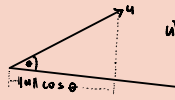
Let $sv_1 = (p,q)$
then $sv_1 = t\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$
because $wx = 1$
$\Rightarrow t \cdot w^T w = 1 \Rightarrow t = \frac{1}{\|w\|^2}$

← Simple case hyperplane through origin. Below more general.

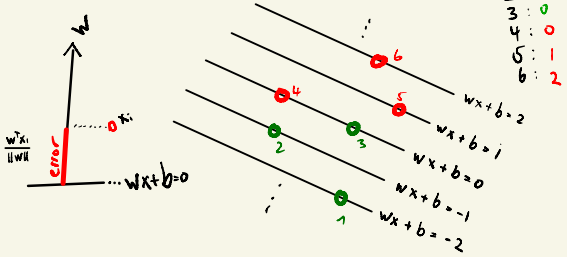


$$\text{Margin} = (sv_1 - sv_2)^T \frac{w}{\|w\|}$$
$$= sv_1^T \frac{w}{\|w\|} - sv_2^T \frac{w}{\|w\|}$$
$$= \frac{b+1}{\|w\|} - \frac{b-1}{\|w\|}$$
$$= \frac{2}{\|w\|}$$

Reminder: Dot Products & Projections

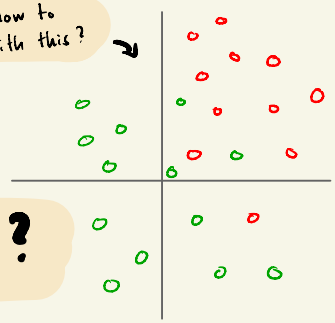


$u^T v = \|u\| \|v\| \cos\theta$



Problem!
← impossible if classes are not linearly separable

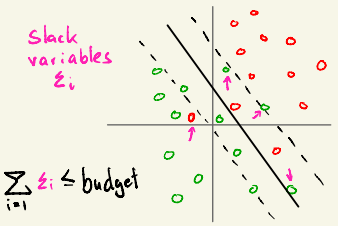
So..., how to deal with this?



IDEA!

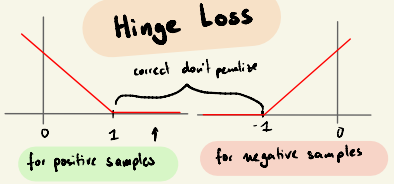
How?

Cut classifier some slack and give it a budget for margin violations and misclassifications



$\sum_{i=1} \xi_i \leq$ budget

Hinge Loss



Look familiar? →

Before (i.e. Ridge)

minimize $A + \lambda B$ bigger λ more weight on B

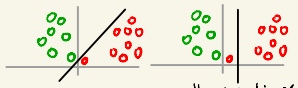
now

minimize $CA + B$ smaller C more weight on B

$\Rightarrow C \cong \frac{1}{\lambda}$

Soft Margin:
minimize:
$$\frac{1}{n} C \sum \left(\max(0, 1 - y_i(w^T x_i - b)) + \|w\|^2 \right.$$

Bonus!
also less sensitive to outliers!



$\Rightarrow$ big C → small budget small C → bigger budget → wider margin

TLDR: