Algebra Everything is Handwavy!

A, a, b, c

are constants

Think:

$x^T A x \sim A x^2$

$x^T x \sim x^2$

$\underbrace{Ax}_{\text{vector}} \cdot \underbrace{(b - Ax)}_{\text{vector}} = x^T A^T (b - Ax)$

Derivatives Notation in the Textbooks you'll find

this type of notation a lot ∂X.

this means with respect to the matrix or

vector. i.e. $\frac{\partial c}{\partial x}$ is a vetor too

and for example $\left[\frac{\partial c}{\partial x}\right]_k = \frac{\partial c}{\partial x_k}$ the

partial derivative of c

w.r.t. k.

$\frac{\partial}{\partial X}$ where x is a vector / Matrix

means we take the derivative

w.r.t. to all values in X

$$\partial A = 0$$

$$\partial(aX) = a\,\partial X$$

$$\partial(X+Y) = \partial X + \partial Y$$

$$\sim \frac{dax}{dx}$$

$$\frac{\partial x^T a}{\partial x} = \frac{\partial a^T x}{\partial x} = a \quad \sim \frac{\partial(ax)}{\partial x}$$

$$\frac{\partial a^T X b}{\partial X} = ab^T \quad \sim \frac{\partial((ab)x)}{dx}$$

$$\frac{\partial a^T X^T b}{\partial X} = ba^T$$

Think:

$$\frac{\partial \|x\|_2^2}{\partial x} = \frac{\partial \| x^T x\|_2}{\partial x}$$

$$= \begin{bmatrix} \frac{\partial \|x^T x\|_2}{\partial x_1} \\ \frac{\partial \|x^T x\|_2}{\partial x_2} \\ \vdots \\ \frac{\partial \| x^T x}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x_1} \sum_{j=1}^{n} x_j^2 \\ \vdots \\ \frac{\partial}{\partial x_i} \sum_{j=1}^{n} x_j^2 \\ \vdots \\ \frac{\partial}{\partial x_n} \sum_{j=1}^{n} x_j^2 \end{bmatrix} = \begin{bmatrix} 2x_1 \\ 2x_2 \\ \vdots \\ 2x_i \\ \vdots \\ 2x_n \end{bmatrix} = \nabla \|x\|_2^2$$

with Symmetric Matrix S

$$\frac{\partial}{\partial x}(b-Ax)^T S(b-Ax) = -2A^T S(b-Ax)$$

Think ~

$$\frac{\partial}{\partial x} S\,y(x)^2 = 2S\,y(x)\cdot y'(x)$$

Try: $\frac{\partial}{\partial x}(b-x)^T W(b-x)$:

if not symmetric more weird but don't think you'll need