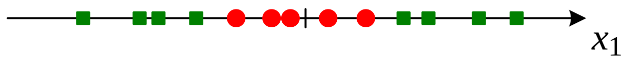
1. Consider the following one-dimensional data; that is, data with one attribute.



As we know these data are not linearly separable as there is no single point on the x_1-axis that separates the points.
In the following diagrams, we have introduced a new attribute $x_2 = x_1^2$ (diagram A) and a new attribute $x_2 = x_1^3$ (diagram B) to extend the data into the plane $\mathbb{R}^2$.

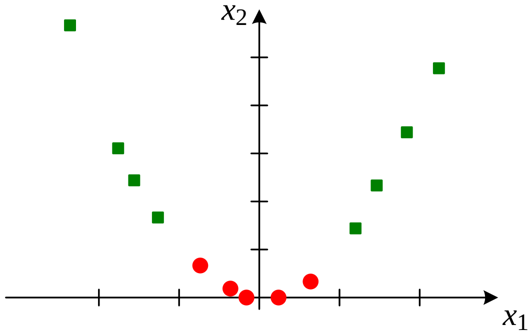


Diagram A

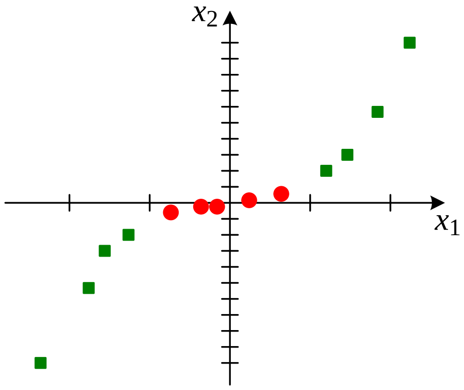
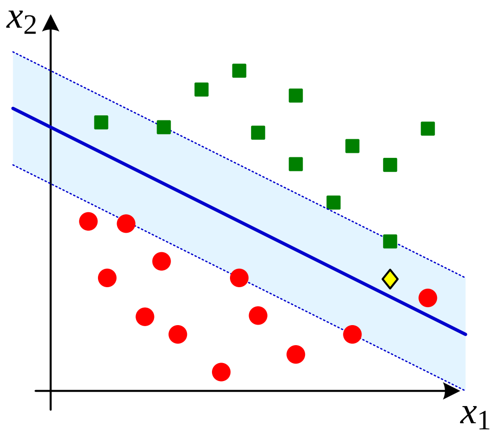


Diagram B

In each case, decide whether these data are linearly separable in the higher dimensional space $\mathbb{R}^2$.

2. Consider the following diagram showing a support vector classifier. The two classes are red circles (negative points) and green squares (positive points) – ignore the yellow diamond point initially.



(i) Is this a hard-margin or soft-margin classifier? Explain.
(ii) How many support vectors are there? Are they positive points or negative points?
(iii) How would the new yellow diamond point be classified, positive or negative?

3. A support vector classifier in $\mathbb{R}^2$ has vector equation $\boldsymbol{w} \cdot \boldsymbol{x} + b = 0$ where $\boldsymbol{w} = (-1, 2)$ and $b = 1$. Hence the non-vector equation is $-x_1 + 2x_2 + 1 = 0$. Assume that the equation has been scaled so that the 'margin barrier lines' containing support vectors satisfy $\boldsymbol{w} \cdot \boldsymbol{x} + b = \pm 1$.

(i) For each of the following points, determine whether the point is

(a) a positive point or negative point;

(b) a support vector (or not).

$$P = (2, 1);\;\; Q = (1, -1);\;\; R = (3, 2);\;\; S = (-2, -2);\;\; T = (-1, 3).$$

(ii) How would you classify the points $A = \left(\frac{1}{2}, -\frac{1}{2}\right) = (0.5, -0.5)$ and $B = (3, 1)$?

4. (i) An SVM classifier line in $\mathbb{R}^2$ is $(3, 4) \cdot (x_1, x_2) - 2 = 0$ or, equivalently, $3x_1 + 4x_2 - 2 = 0$.
What is the width of the margin?
What is the distance from the SVM classifier line to a support vector?

(ii) An SVM classifier plane in $\mathbb{R}^3$ is $(1, -2, 3) \cdot (x_1, x_2, x_3) + 5 = 0$ or, equivalently, $x_1 - 2x_2 + 3x_3 + 5 = 0$.
What is the width of the margin?