

# Classification

# Classification

- Humans are good at classifying things into categories for example tasks such as filtering spam email messages.
- Manual classification suffices for small and simple data sets with only a few attributes, larger and more complex datasets require an automated solution.

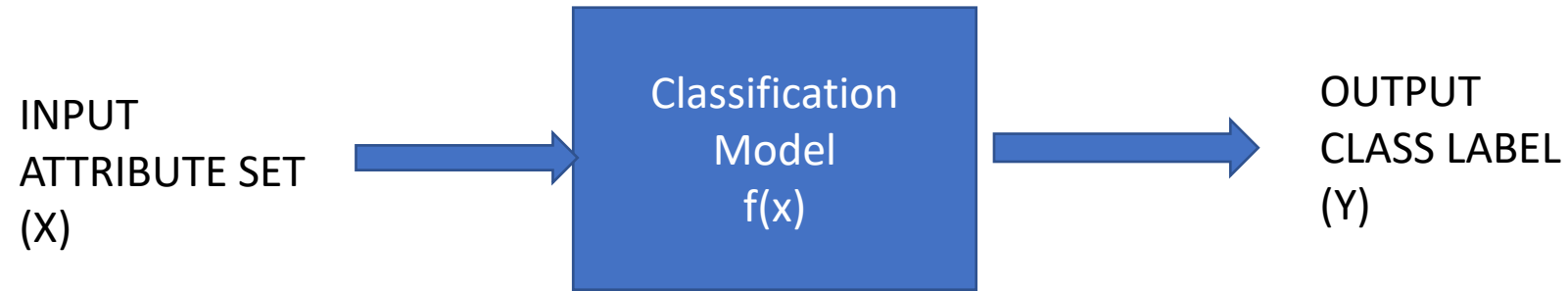
# Basic concepts

- The data for a classification task consists of a collection of instances (records).
- Each instance is characterized by a tuple  $(x,y)$  where  $x$  is the set of attribute values that describe the instance and  $y$  is the class label of the instance.
- The attribute set  $x$  can contain attributes of any type while the class label  $y$  must be categorical.

# Classification Model

- A classification model is an abstract representation of the relationship between the attribute set and the class label. The model can be represented in many ways eg as a tree, a probability table, or simply a vector of real valued parameters.
- Formally this can be expressed mathematically as a target function  $f$  that takes as input the attribute set  $x$  and produces output corresponding to the predicted class label. The model is said to classify an instance  $(x,y)$  correctly if  $f(x)=y$ .

# Illustration of a classification task



# Examples of classification tasks

- Spam filtering and tumor classification are **binary classification** where data can be classified in each of two instances.
- If the classes are more than two then this is a **multi-classification** problem.

Task	Attribute Set	Class label
Spam filtering	Features extracted from email message header and content	Spam or non-spam
Tumor identification	Features extracted from magnetic resonance imaging (MRI) scans	Malignant or benign
Galaxy classification	Features extracted from telescope images	Elliptical, spiral or irregular shape

# Example Vertebrate classification

- The table shows sample data for classifying vertebrates into mammals, reptiles, birds, fish, and amphibians. The attribute set includes characteristics of vertebrate such as its body temperature, skin cover and ability to fly. The data set can also be used for binary classification task such as mammal classification, by grouping the reptiles, birds, fish, and amphibians into a single category called non-mammals.

# Example Vertebrate Classification

Vertebrate Name	Body temperature	Skin cover	Gives birth	Aquatic creature	Aerial creature	Has legs	Hibernates	Class Label
human	Warm-blooded	hair	yes	no	no	yes	no	mammal
python	Cold-blooded	scales	no	no	no	no	yes	reptile
salmon	Cold-blooded	scales	no	yes	no	no	no	fish
whale	Warm-blooded	hair	yes	yes	no	no	no	mammal
frog	Cold-blooded	none	no	semi	no	yes	yes	amphibian
komodo	Cold-blooded	scales	no	no	no	yes	no	reptile
bat	Warm-blooded	hair	yes	no	yes	yes	yes	mammal
pigeon	Warm-blooded	feathers	no	no	yes	yes	no	bird
cat	Warm-blooded	hair	yes	no	no	yes	no	mammal
Leopard shark	Cold-blooded	scales	yes	yes	no	no	no	fish
turtle	Cold-blooded	scales	no	semi	no	yes	no	reptile
penguin	Warm-blooded	feathers	no	semi	no	yes	no	bird
porcupine	Warm-blooded	quills	yes	no	no	yes	yes	mammal
eel	Cold-blooded	scales	no	yes	no	no	no	fish
salamander	Cold-blooded	none	no	semi	no	yes	yes	amphibian



# Example Loan Borrower Classification

ID	Home owner	Marital status	Annual Income	Defaulted
1	yes	single	125K	no
2	no	married	100K	no
3	no	single	70K	no
4	yes	married	120K	no
5	no	divorced	95K	yes
6	no	married	60K	no
7	yes	divorced	220K	no
8	no	single	85K	yes
9	no	married	75K	no
10	no	single	90K	yes

Consider the problem of predicting whether a loan borrower will repay loan or default on the loan payment. The attribute set includes personal information of the borrower such as marital status and annual income, while the class label indicates whether the borrower had defaulted on the payments

# Classification model

- A classification model serves two important roles in data mining.
  - First it is used as a predictive model to classify previously unlabeled instances. A good classification model must provide accurate predictions with a fast response time.
  - Second it serves as a descriptive model to identify characteristics that distinguish instances from different classes. This is particularly useful for applications such as medical diagnosis where it is insufficient to have a model that makes a prediction without justifying how it reaches such a decision.

For example - The classification model induced from the vertebrate data set can be used to predict the class label for

Vertebrate Name	Body temperature	Skin cover	Gives birth	Aquatic creature	Aerial creature	Has legs	Hibernates	Class Label
Gila monster	cold-blooded	scales	no	no	no	yes	yes	?

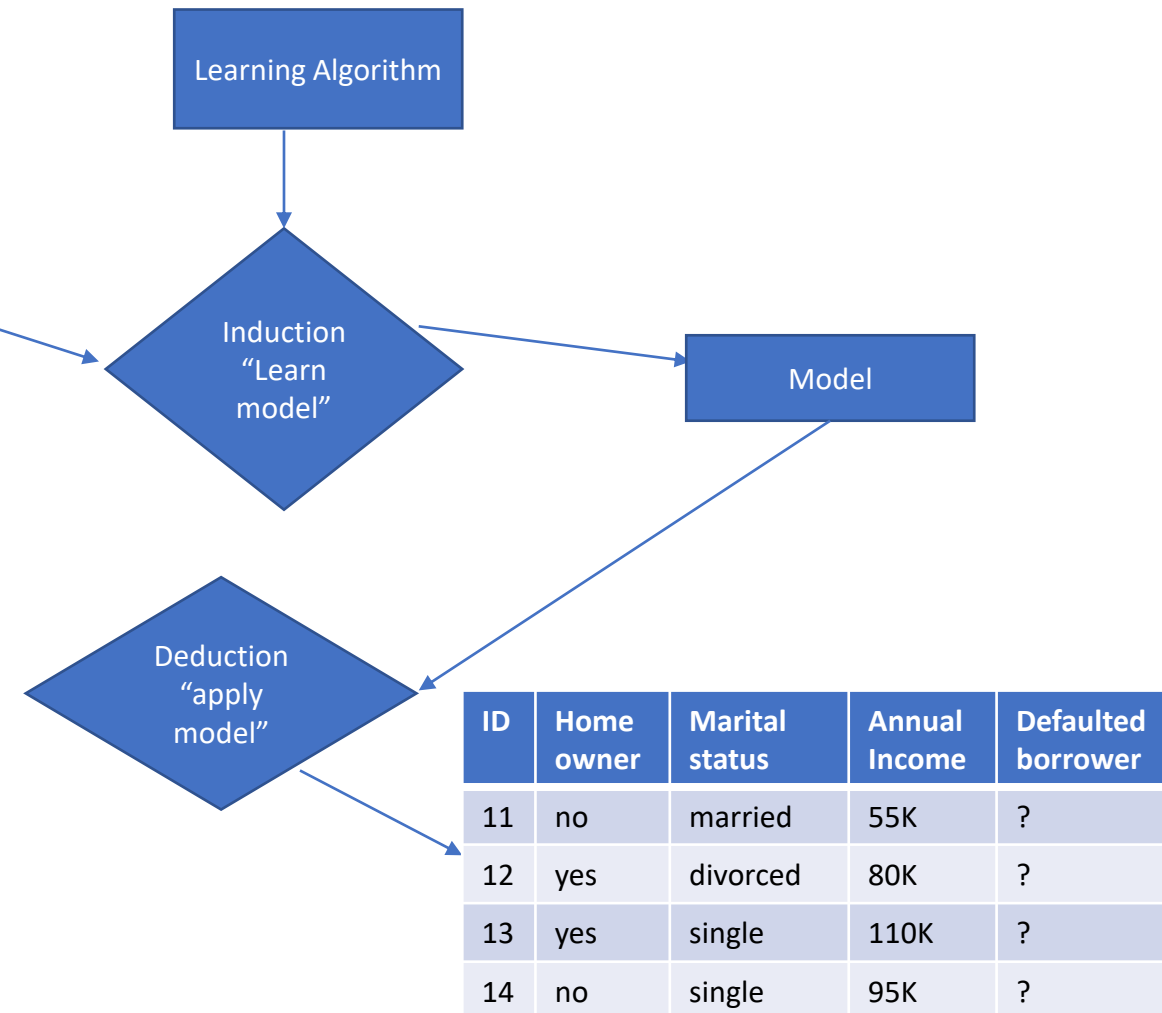
It can also be used as descriptive model to help determine the characteristics that define a vertebrate as a mammal, Reptile, a bird, a fish or an amphibian

# General Framework for Classification

- Classification is the task of assigning labels to unlabeled data instances, this is done by a classifier. The model is created using a set of instances known as the training set, which contains attribute values as well as class labels for each instance.
- The systematic approach for learning a classification model given a training set is known as a **learning algorithm**. The process of using a learning algorithm to build a classification model from the training data is known as **induction**. The process of applying a classification model on unseen test instances to predict their class labels is known as **deduction**.

# General framework for building a classification model

ID	Home owner	Marital status	Annual Income	Defaulted borrower
1	yes	single	125K	no
2	no	married	100K	no
3	no	single	70K	no
4	yes	married	120K	no
5	no	divorced	95K	yes
6	no	married	60K	no
7	yes	divorced	220K	no
8	no	single	85K	yes
9	no	married	75K	no
10	no	single	90K	yes



# Performance of classifier

- The performance of a classifier can be evaluated by comparing the predicted labels against the true labels of the instances.
- The information can be summarized in matrix called **confusion matrix**.
- Each entry  $f_{ij}$  denotes the number of instances from **class i predicted as class j**. For example,  $f_{01}$  is the number of instances from **class 0 incorrectly predicted as class 1**. The number of **correct** predictions made by the model is  $(f_{11} + f_{00})$  and the number of incorrect predictions is  $(f_{10} + f_{01})$

		Predicted Class	
		Class = 1	Class = 0
Actual Class	Class = 1	$f_{11}$	$f_{10}$
	Class = 0	$f_{01}$	$f_{00}$

Confusion Matrix

# Accuracy of classification algorithm

- Although the confusion matrix summarizes the data, combining this to a single number makes it more convenient to compare the relative performance of different models. This can be done using an evaluation metric such as accuracy, computed below:

$$\text{Accuracy} = \frac{\text{Number of correct predictions}}{\text{Total number of predictions}}$$

- For binary classification problems, the accuracy of a model is given by

$$\text{Accuracy} = \frac{f_{11} + f_{00}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

# Error Rate

- Error rate is another related metric, which is defined as follows for binary classification problems.

$$\text{Error rate} = \frac{\text{Number of incorrect predictions}}{\text{Total number of predictions}} = \frac{f_{10} + f_{01}}{f_{11} + f_{10} + f_{01} + f_{00}}$$

- The learning algorithms of most classification techniques are designed to **learn models that attain the highest accuracy**, or equivalently, the **lowest error** when applied to the test set.

# Classification Techniques

- Base Classifiers
  - Decision Tree based Methods
  - Nearest-neighbour
  - Naïve Bayes
  - Support Vector Machines (SVM)



# Decision Tree Classifier

- A simple **classification technique** is known as **Decision Tree classification**.
- To illustrate how a decision tree works, consider the classification problem of distinguishing mammals from non-mammals using the vertebrae data set
- How can we tell whether an animal is a mammal or not a mammal?
- One approach is to pose a series of questions about the characteristics of the species.
- The first question we may ask is whether the species is cold or warm blooded. If cold blooded then it is definitely not a mammal.

If it is cold blooded then it definitely not a mammal.

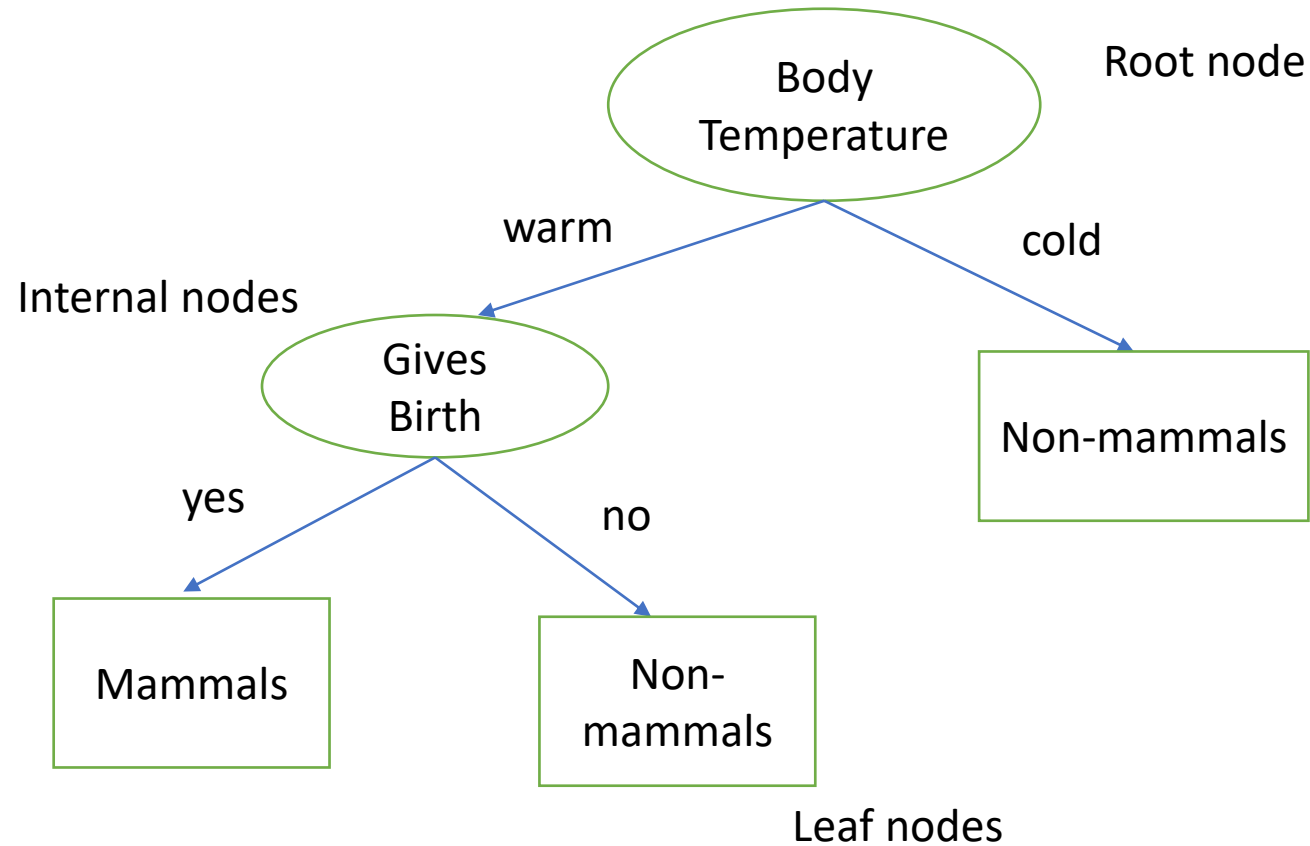
Otherwise it is a bird or a mammal. In the latter case, we need to ask a follow up question .

Do females of these species give birth to their young?

Those that give birth are definitely mammals, while those that do not are definitely not mammals.

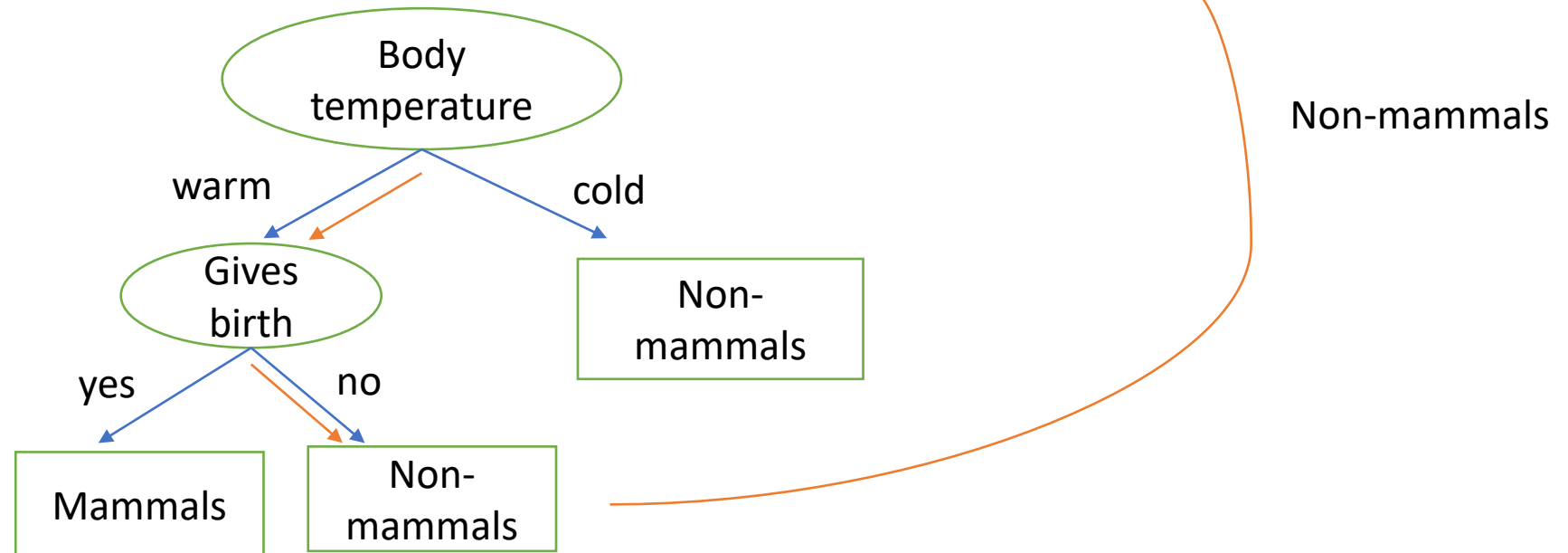
- This example illustrated solving a classification problem by asking a series of questions about the attributes of the test instances.
- Each time an answer is received then a follow up question is asked until the class label is decided. The series of questions and their possible answers can be organized into a hierarchical structure called a decision tree.
- The tree has three types of nodes
  - A root node
  - Internal nodes
  - Leaf nodes
- Every leaf node in the decision tree is associated with a class label.
- The non-leaf nodes, which include the root and internal nodes contain attribute test conditions that are typically defined using a single attribute.

- A decision tree for a mammal classification problem



- Classifying an unlabeled vertebrate.

Name	Body temperature	Gives birth	....	Class
Flamingo	Warm	No	....	?

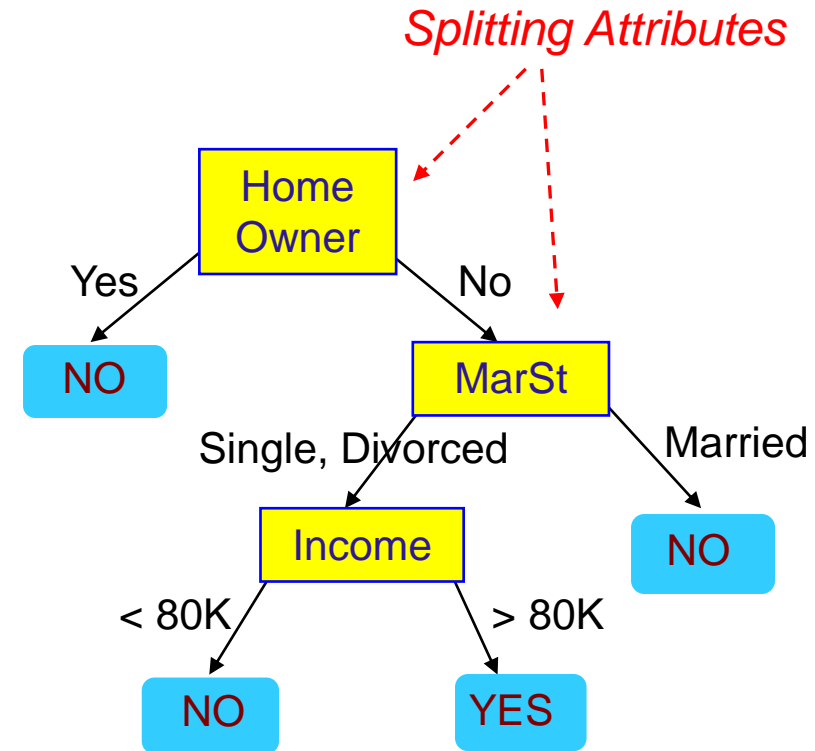


# Example of a Decision Tree

categorical  
categorical  
continuous  
class

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data

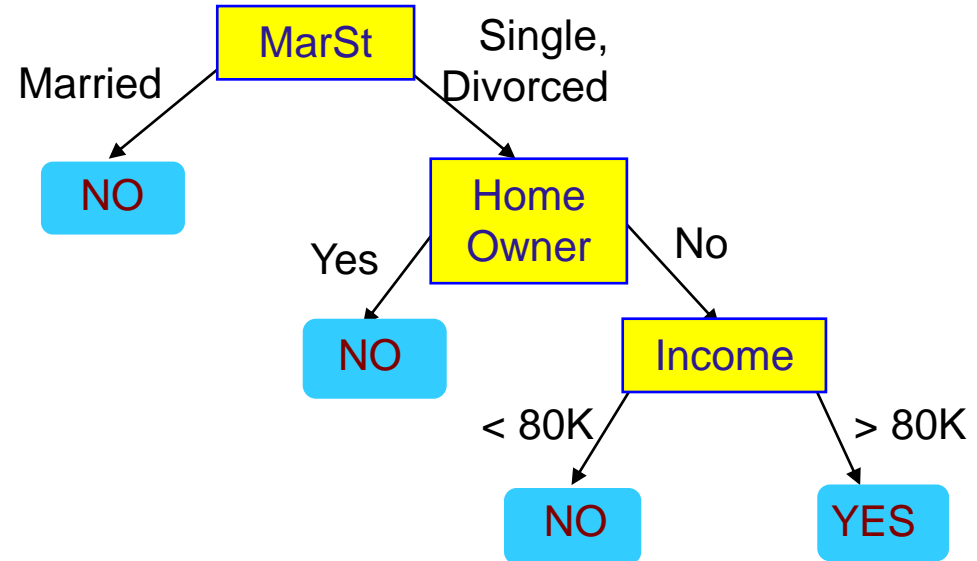


Model: Decision Tree

# Another Example of Decision Tree

categorical  
categorical  
continuous  
class

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

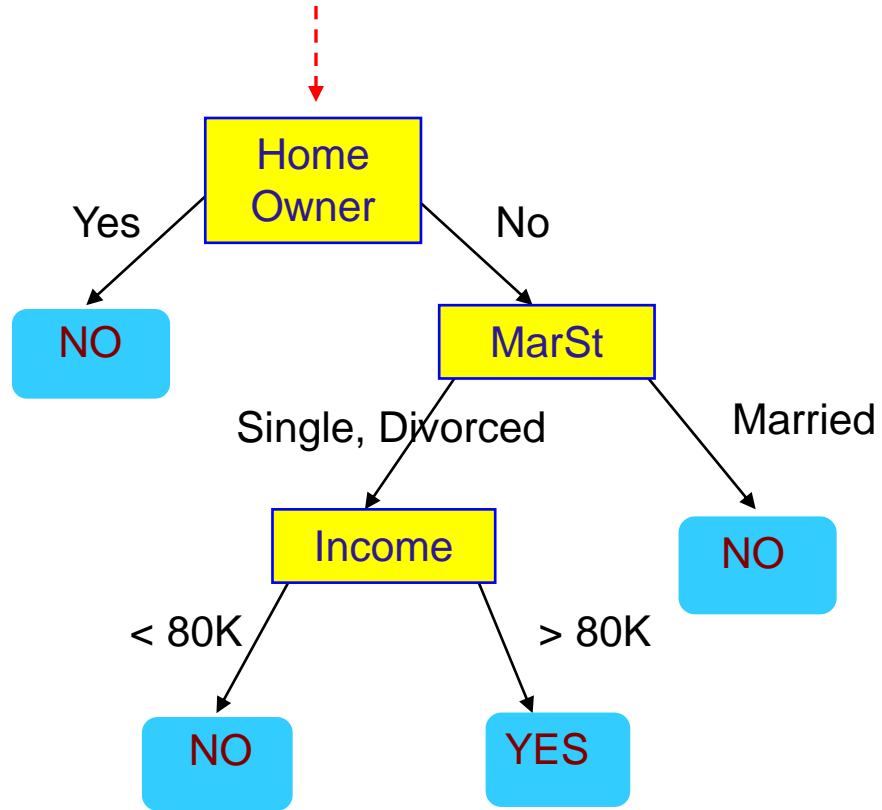


There could be more than one tree that fits the same data!

# Apply Model to Test Data

Test Data

Start from the root of tree.

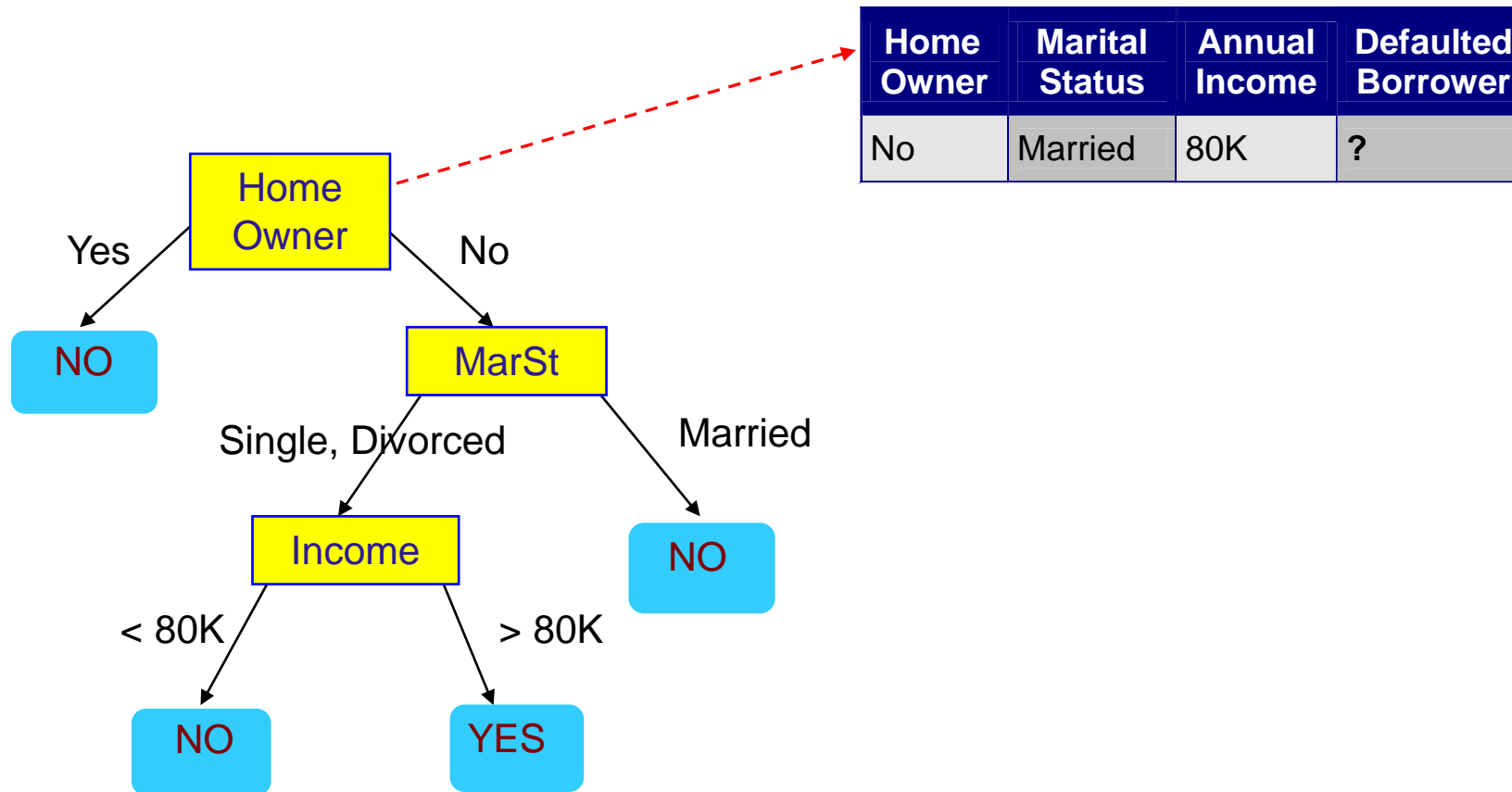


Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



# Apply Model to Test Data

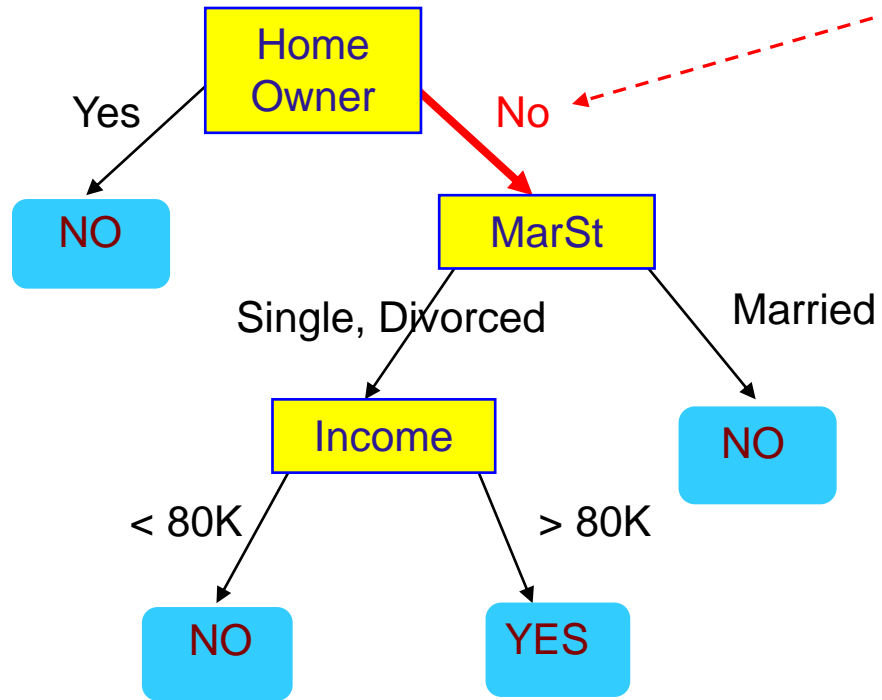
Test Data



# Apply Model to Test Data

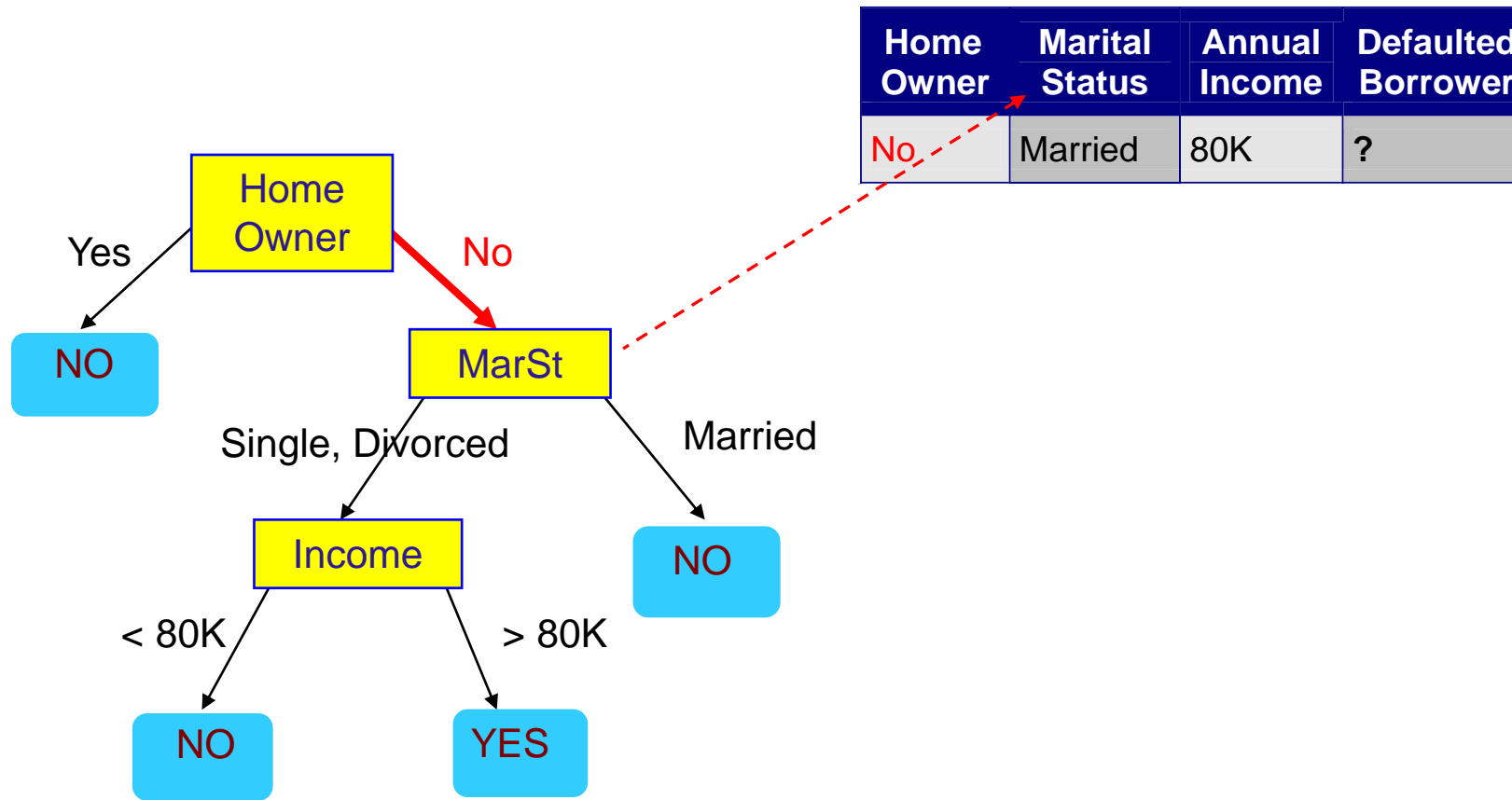
Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



# Apply Model to Test Data

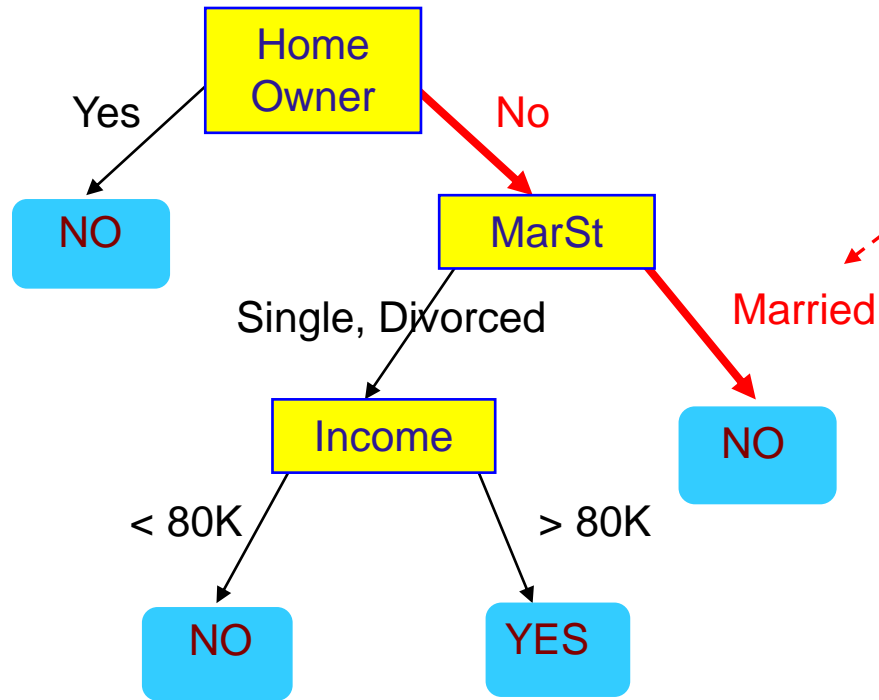
Test Data



# Apply Model to Test Data

Test Data

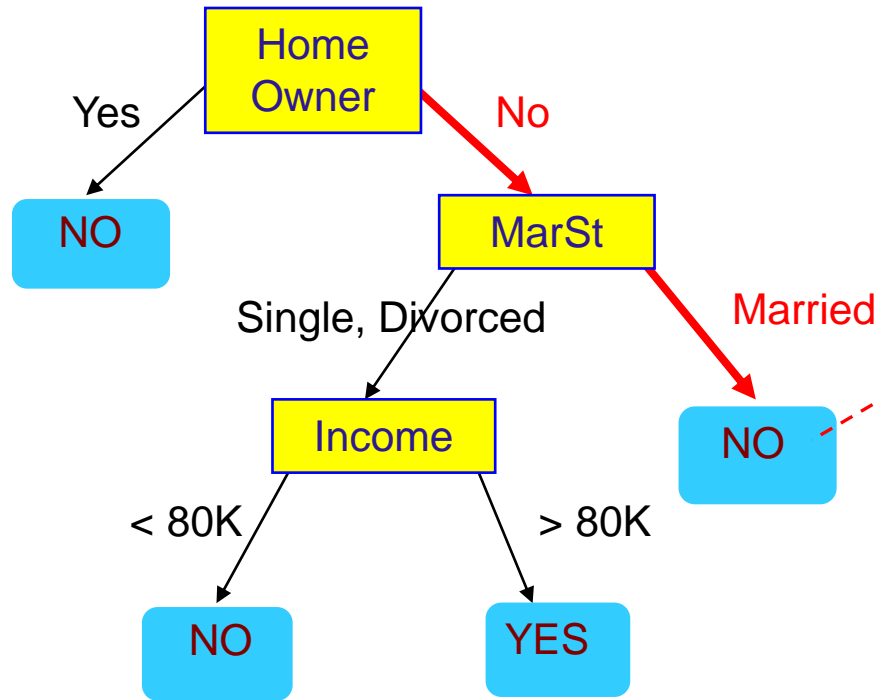
Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



# Apply Model to Test Data

Test Data

Home Owner	Marital Status	Annual Income	Defaulted Borrower
No	Married	80K	?



Assign Defaulted to  
"No"

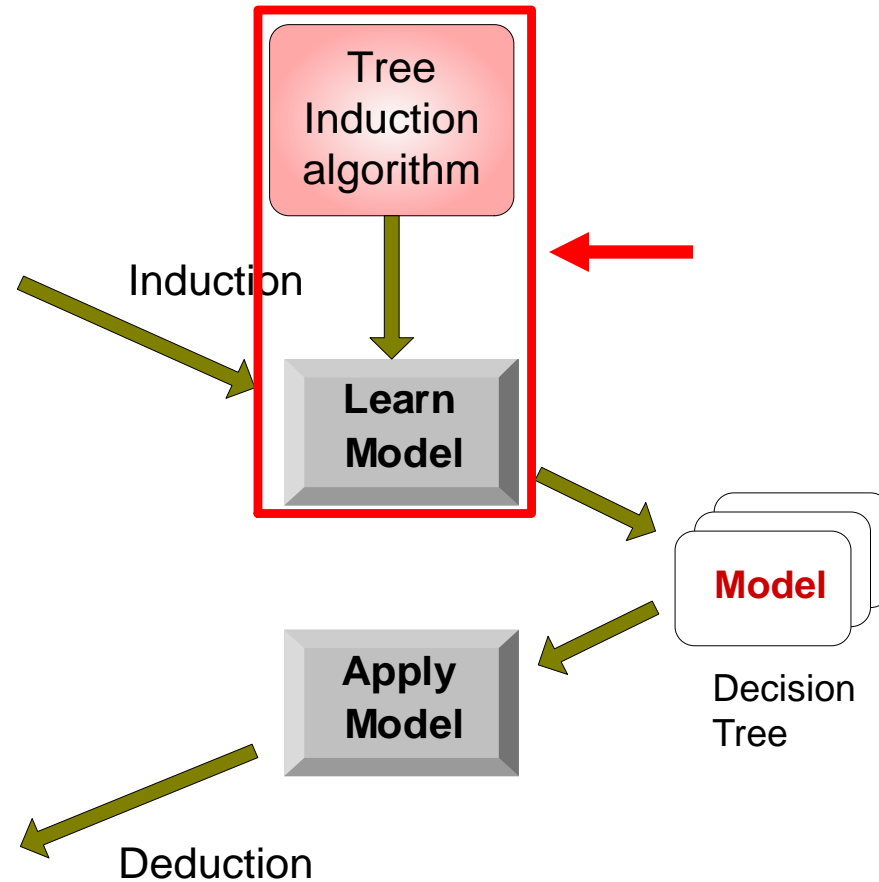
# Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

Training Set

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



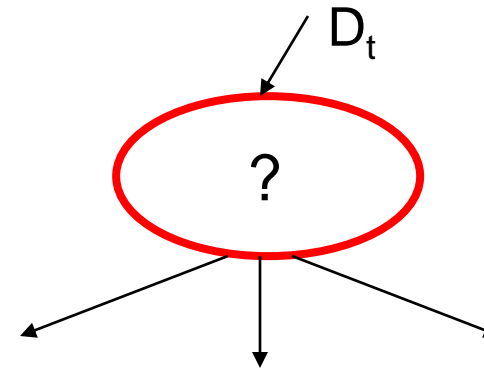
# A basic algorithm to build a decision tree

- One of the earliest algorithms is the Hunt's Algorithm, which is the basis for many current implementation of decision tree algorithms ID3, C4.5 and CART.
- In Hunt's algorithm a decision tree is grown in a recursive fashion, the tree initially contains a single root node that is associated with all the training instances.

# General Structure of Hunt's Algorithm

- Let  $D_t$  be the set of training records that reach a node  $t$
- General Procedure:
  - If  $D_t$  contains records that belong the same class  $y_t$ , then  $t$  is a leaf node labeled as  $y_t$
  - If  $D_t$  contains records that belong to more than one class, use an attribute test to split the data into smaller subsets. Recursively apply the procedure to each subset.

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes





# Hunt's Algorithm

Defaulted = No

(7,3)

(a)

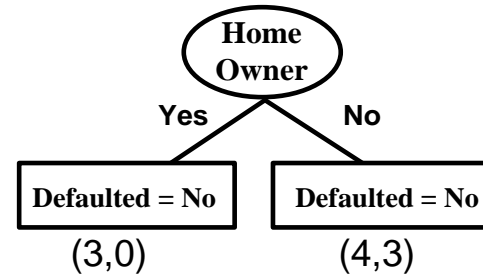
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5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Hunt's Algorithm

Defaulted = No

(7,3)

(a)



(b)

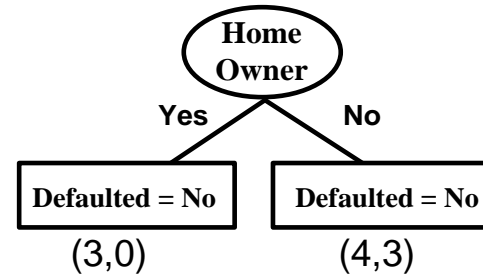
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7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Hunt's Algorithm

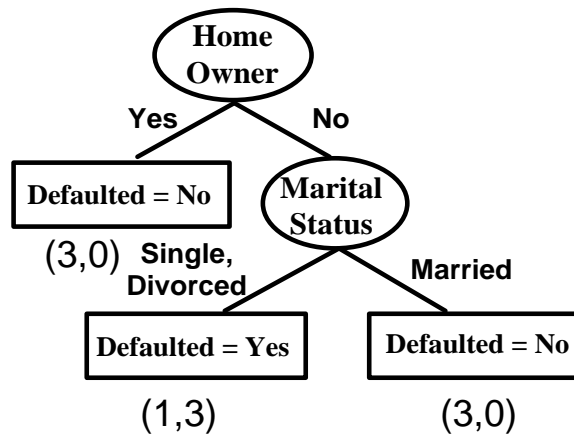


(7,3)

(a)



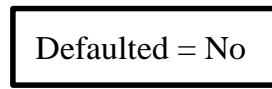
(b)



(c)

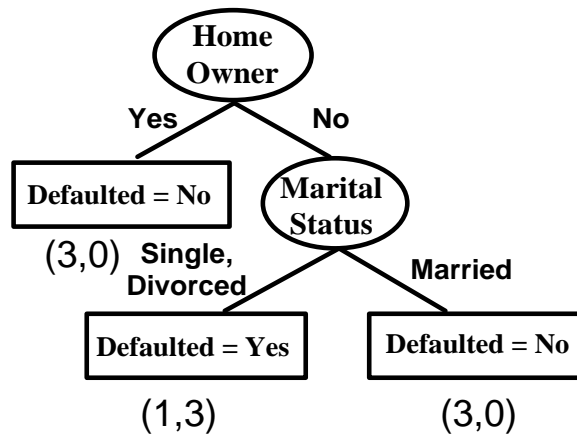
ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
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3	No	Single	70K	No
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7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Hunt's Algorithm

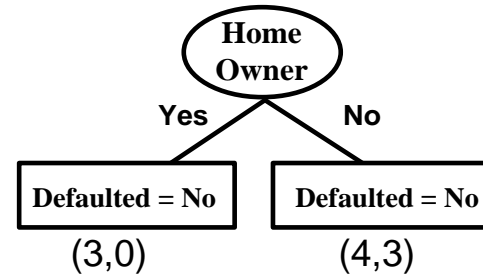


(7,3)

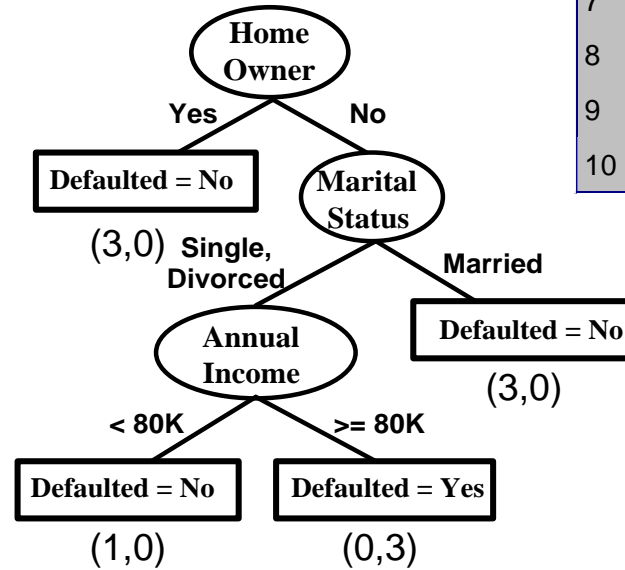
(a)



(c)



(b)



(d)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Design Issues of Decision Tree Induction

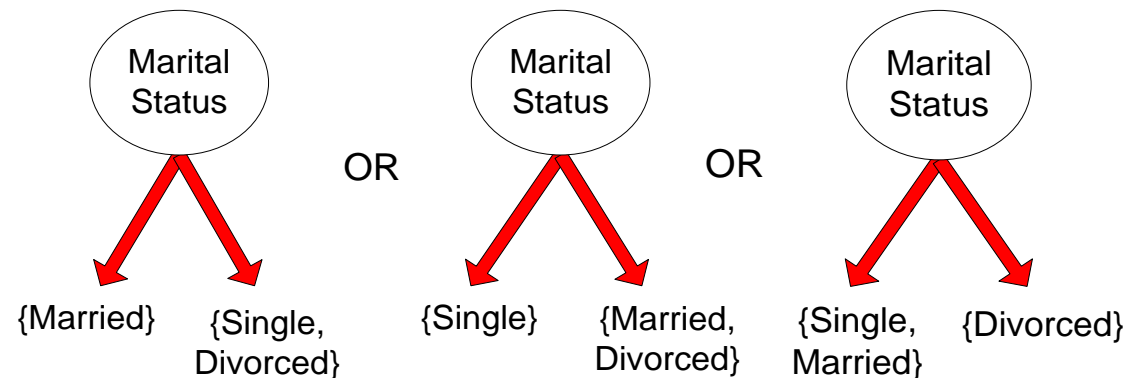
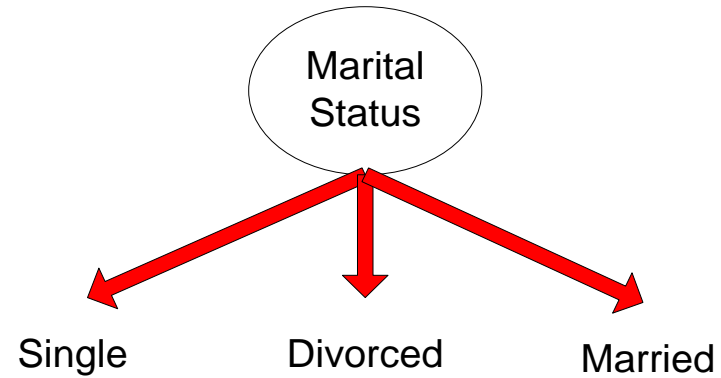
- How should training records be split?
  - Method for specifying test condition
    - depending on attribute types
  - Measure for evaluating the goodness of a test condition
- How should the splitting procedure stop?
  - Stop splitting if all the records belong to the same class or have identical attribute values

# Methods for Expressing Attribute Test Conditions

- Depends on attribute types
  - Binary - The test condition for a binary attribute generates two potential outcomes.
  - Nominal - A nominal attributes can have many values its attribute test condition can be expressed in two ways either as a multiway split or a binary split
  - Ordinal – this can be expressed as a multiway split or a binary split
  - Continuous – this can be expressed as a multiway split or a binary split
- Depends on number of ways to split
  - 2-way split
  - Multi-way split

# Test Condition for Nominal Attributes

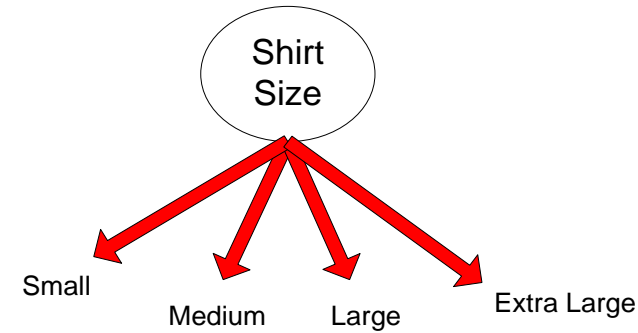
- **Multi-way split:**
  - Use as many partitions as distinct values.
- **Binary split:**
  - Divides values into two subsets



# Test Condition for Ordinal Attributes

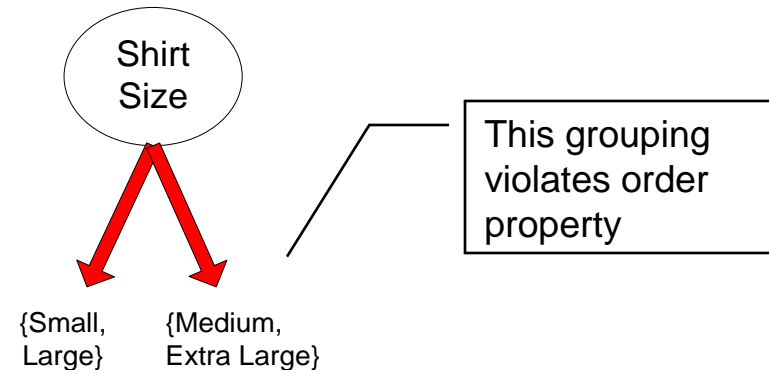
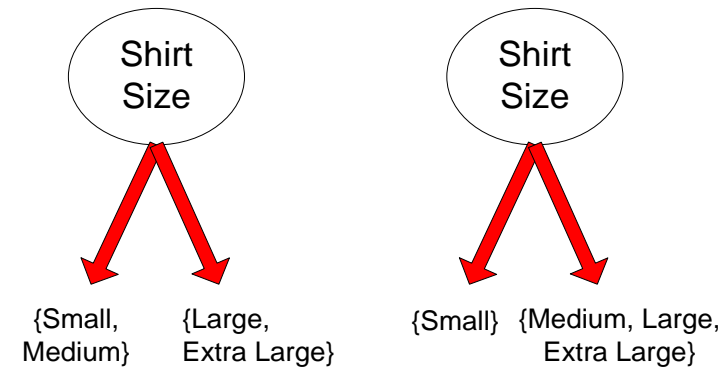
- **Multi-way split:**

- Use as many partitions as distinct values



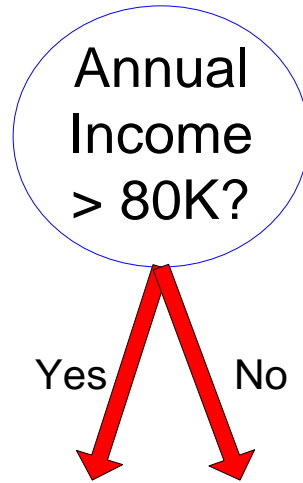
- **Binary split:**

- Divides values into two subsets
- Preserve order property among attribute values

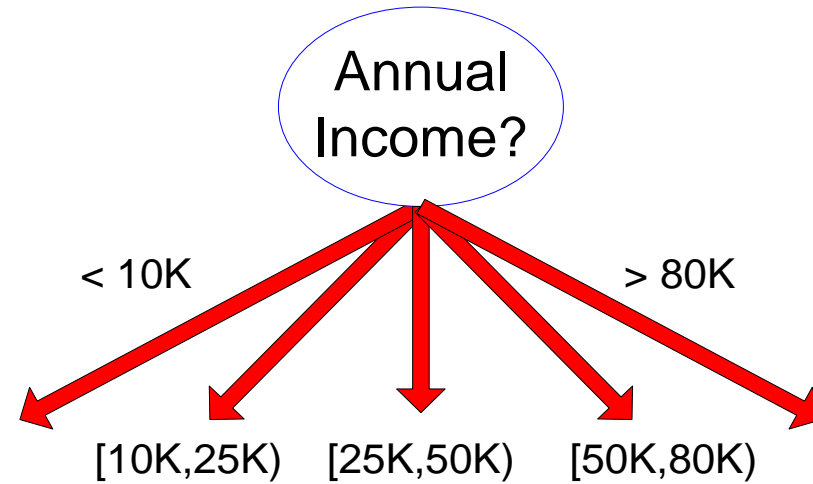




# Test Condition for Continuous Attributes



(i) Binary split



(ii) Multi-way split

# Splitting Based on Continuous Attributes

- Different ways of handling continuous attributes
  - **Discretization** to form an ordinal categorical attribute

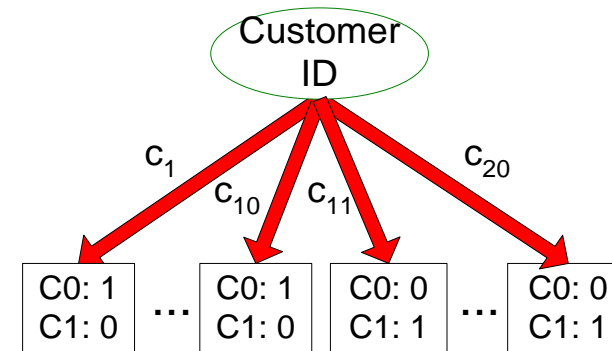
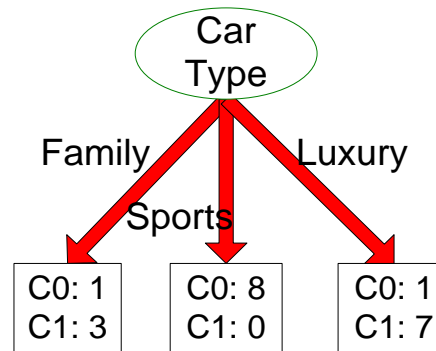
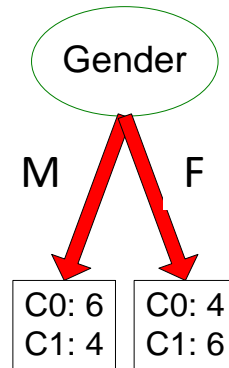
Ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.

    - Static – discretize once at the beginning
    - Dynamic – repeat at each node
  - **Binary Decision**:  $(A < v)$  or  $(A \geq v)$ 
    - consider all possible splits and finds the best cut
    - can be more compute intensive

# How to determine the Best Split

Before Splitting: 10 records of class 0,  
10 records of class 1

Customer Id	Gender	Car Type	Shirt Size	Class
1	M	Family	Small	C0
2	M	Sports	Medium	C0
3	M	Sports	Medium	C0
4	M	Sports	Large	C0
5	M	Sports	Extra Large	C0
6	M	Sports	Extra Large	C0
7	F	Sports	Small	C0
8	F	Sports	Small	C0
9	F	Sports	Medium	C0
10	F	Luxury	Large	C0
11	M	Family	Large	C1
12	M	Family	Extra Large	C1
13	M	Family	Medium	C1
14	M	Luxury	Extra Large	C1
15	F	Luxury	Small	C1
16	F	Luxury	Small	C1
17	F	Luxury	Medium	C1
18	F	Luxury	Medium	C1
19	F	Luxury	Medium	C1
20	F	Luxury	Large	C1



Which test condition is the best?

# How to determine the Best Split

- Greedy approach:
  - Nodes with **purser** class distribution are preferred
- Need a measure of node impurity:

C0: 5
C1: 5

High degree of impurity

C0: 9
C1: 1

Low degree of impurity

# Measures for selecting an attribute test condition

- There are many measures that can be used to determine the goodness of an attribute test condition.
- These measures try to give preference to attribute test conditions that partition the training instances into purer subsets in the child nodes, which mostly have the same class labels.
- Having purer nodes is useful since a node that has all of its training instances from the same class labels is a leaf node.

# Impurity Measures for a single node

- The impurity of a node measures how dissimilar the class labels are for the instances belonging to a common node.
- Following are examples of measures that can be used to evaluate the impurity of node  $t$ :
  - *Entropy* =  $-\sum_{i=1}^c p_i(t) \log_2 p_i(t)$
  - *Gini index* =  $1 - \sum_{i=1}^c p_i(t)^2$
  - *Classification error* =  $1 - \max[p_i(t)]$
- Where  $p_i(t)$  is the relative frequency of training instances that belong to class  $i$  at node  $t$ ,  $c$  is the total number of classes.

# Example

- Below are the examples of calculating these

$$Entropy = - \sum_{i=1}^c p_i(t) \log_2 p_i(t)$$

$$Gini\ index = 1 - \sum_{i=1}^c p_i(t)^2$$

$$Classification\ error = 1 - \max[p_i(t)]$$

Node N <sub>1</sub>	Count
Class = 0	0
Class =1	6

$$\begin{aligned} Gini &= 1 - (0/6)^2 - (6/6)^2 = 0 \\ Entropy &= -(0/6)\log_2(0/6) - (6/6)\log_2(6/6) = 0 \\ Error &= 1 - \max[0/6, 6/6] = 0 \end{aligned}$$

Node N <sub>2</sub>	Count
Class = 0	1
Class =1	5

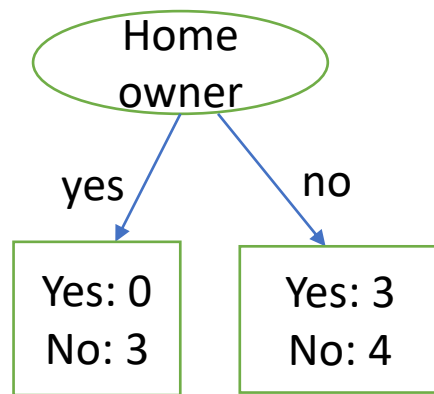
$$\begin{aligned} Gini &= 1 - (1/6)^2 - (5/6)^2 = 0.278 \\ Entropy &= -(1/6)\log_2(1/6) - (5/6)\log_2(5/6) = 0.650 \\ Error &= 1 - \max[1/6, 5/6] = 0.167 \end{aligned}$$

Node N <sub>3</sub>	Count
Class = 0	3
Class =1	3

$$\begin{aligned} Gini &= 1 - (3/6)^2 - (3/6)^2 = 0.5 \\ Entropy &= -(3/6)\log_2(3/6) - (3/6)\log_2(3/6) = 1 \\ Error &= 1 - \max[3/6, 3/6] = 0.5 \end{aligned}$$

# Example [Entropy]

- Consider the candidate attribute test condition shown in the figure for the loan borrower classification problem. Splitting on the Home Owner attribute will generate two child nodes whose weighted entropy can be calculated as:



Entropy( home owner, yes)

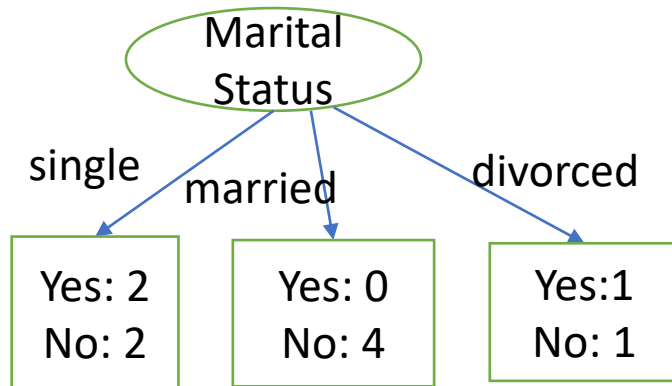
$$= - 0/3 \log_2 0/3 - 3/3 \log_2 3/3 = 0$$

Entropy( home owner, no)

$$= - 3/7 \log_2 3/7 - 4/7 \log_2 4/7 = 0.985$$



- Consider the candidate attribute test condition shown in the figure for the Marital Status classification problem. Splitting on the Marital Status attribute will generate two child nodes whose weighted entropy can be calculated as:



Entropy( MartSt, single)

$$= - 2/4 \log_2 2/4 - 2/4 \log_2 2/4 = 1$$

Entropy( MartSt, married)

$$= - 0/4 \log_2 0/7 - 4/4 \log_2 4/4 = 0$$

Entropy( MartSt, divorced)

$$= - 1/2 \log_2 1/2 - 1/2 \log_2 1/2 = 1$$

# Identifying the best attribute test condition

- To determine the **goodness of an attribute test condition** we need to compare the **degree of impurity of the parent node (before splitting)** with the **weighted degree of the impurity child nodes (after splitting)**. The **larger** their difference the **better** the test condition. This difference is also termed as the gain in purity of an attribute test condition

$$\Delta = I(\text{parent}) - I(\text{children})$$

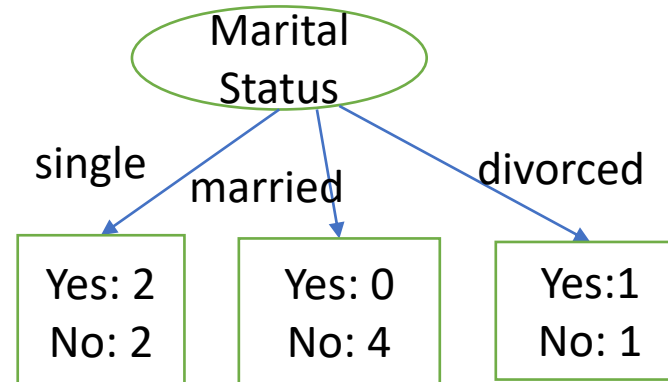
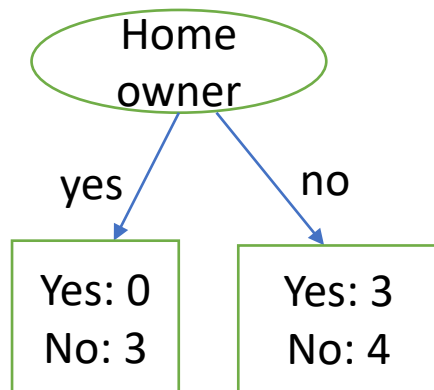
Where  $I(\text{parent})$  is the impurity of the node before splitting and  $I(\text{children})$  is the weighted impurity of the node after splitting

This is commonly known as **information gain**

# Splitting of Qualitative Attributes

Consider the first two candidate splits shown in the figure involving qualitative attributes Home Owner and Marital Status. The initial class distribution at the parent node is that there are 3 instances of class Yes and 7 instances of class No in the training data. Thus:

$$I(\text{parent}) = -\frac{3}{10}\log_2\frac{3}{10} - \frac{7}{10}\log_2\frac{7}{10} = 0.881$$

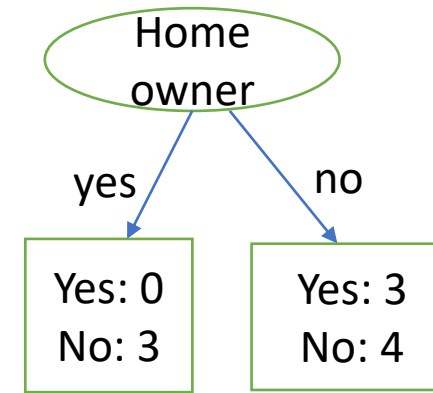


- Splitting on the home owner

$$I(\text{Home owner} = \text{yes}) = -\frac{0}{3}\log_2 \frac{0}{3} - \frac{3}{3}\log_2 \frac{3}{3} = 0$$

$$I(\text{Home owner} = \text{no}) = -\frac{3}{7}\log_2 \frac{3}{7} - \frac{4}{7}\log_2 \frac{4}{7} = 0.985$$

$$I(\text{Home owner}) = \frac{3}{10} \times 0 + \frac{7}{10} \times 0.985 = 0.690$$



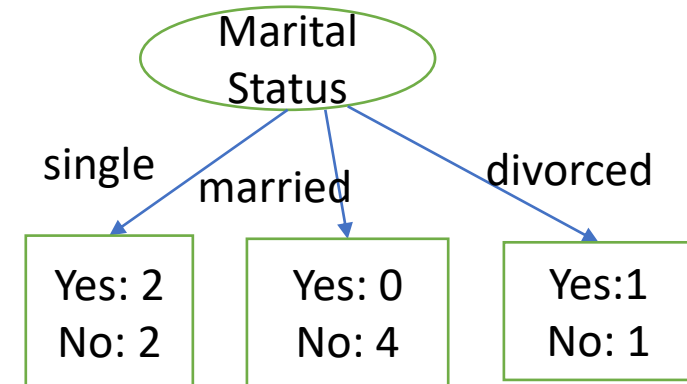
- Splitting on the marital status

$$I(\text{Marital Status} = \text{Single}) = -\frac{2}{4}\log_2 \frac{2}{4} - \frac{2}{4}\log_2 \frac{2}{4} = 1$$

$$I(\text{Marital Status} = \text{Married}) = -\frac{0}{4}\log_2 \frac{0}{4} - \frac{4}{4}\log_2 \frac{4}{4} = 0$$

$$I(\text{Marital Status} = \text{Divorced}) = -\frac{1}{2}\log_2 \frac{1}{2} - \frac{1}{2}\log_2 \frac{1}{2} = 1$$

$$I(\text{Marital status}) = \frac{4}{10} \times 1 + \frac{4}{10} \times 0 + \frac{2}{10} \times 1 = 0.6$$



$$\Delta = I(\textit{parent}) - I(\textit{children})$$

$$\Delta(\textit{Home Owner}) = 0.881 - 0.690 = 0.191$$

$$\Delta(\textit{Marital Status}) = 0.881 - 0.6 = 0.281$$

Marital status has the larger difference

# Binary Splitting of Qualitative Attributes

- Consider building a decision tree using only binary splits and the Gini index as the impurity measure. Consider the diagram below which shows four candidate splitting for the Home Owner and Marital Status Attributes.

	Parent
No	7
Yes	3
Gini = 0.420	

$$\text{Gini index} = 1 - \sum_{i=1}^c p_i(t)^2$$

Home Owner

Yes No

	N1	N2
No	3	4
Yes	0	3
Gini	0	0.490
Weighted Gini = 0.343		

Marital Status

Single Married, Divorced

	N1	N2
No	2	5
Yes	2	1
Gini	0.5	0.278
Weighted Gini = 0.367		

Marital Status

Single, Married Divorced

	N1	N2
No	6	1
Yes	2	1
Gini	0.375	0.5
Weighted Gini = 0.400		

Marital Status

Single, Divorced Married

	N1	N2
No	3	4
Yes	3	0
Gini	0.5	0
Weighted Gini = 0.3		

- The Gini index of the parent node before splitting is

$$1 - \left(\frac{3}{10}\right)^2 - \left(\frac{7}{10}\right)^2 = 0.420$$

If Home Owner is chosen as the splitting attribute, the Gini index for the child nodes  $N_1$  and  $N_2$  are 0 and 0.490, respectively. The weighted index for the children is

$$\frac{3}{10} \times 0 + \frac{7}{10} \times 0.490 = 0.343$$

The gain in Home Owner as the splitting attribute is  $0.420 - 0.343 = 0.077$ .

# Binary Splitting of Quantitative Attributes

- Consider the problem of identifying the best binary split for Annual Income. The training instances are first sorted based on their Annual Income. The candidate split is given by a midpoint split between every adjacent sorted values £55,000, £65,000, £72,500 and so on.

Class	No		No		No		Yes		Yes		Yes		No		No		No		No			
	Annual Income																					
	60		70		75		85		90		95		100		120		125		220			
	55		65		72		80		87		92		97		110		122		172		230	
	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>		
Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0		
No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
Gini	0.420		0.400		0.375		0.343		0.417		0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

ID	Home Owner	Marital Status	Annual Income	Defaulted
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



# Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Sorted Values →

Class	No	No	No	Yes	Yes	Yes	No	No	No	No
Annual Income										
	60	70	75	85	90	95	100	120	125	220

# Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing gini index
  - Choose the split position that has the least gini index

Sorted Values Split Positions	→	Class	No	No	No	Yes	Yes	Yes	No	No	No	No
		Annual Income										
	→	60	70	75	85	90	95	100	120	125	220	
	→	55	65	72	80	87	92	97	110	122	172	230
		<=	>	<=	>	<=	>	<=	>	<=	>	<=

# Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
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  - Choose the split position that has the least gini index

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# Continuous Attributes: Computing Gini Index...

- For efficient computation: for each attribute,
  - Sort the attribute on values
  - Linearly scan these values, each time updating the count matrix and computing Gini index
  - Choose the split position that has the least Gini index

Sorted Values Split Positions	→	Annual Income																					
	→	60		70		75		85		90		95		100		120		125		220			
		55		65		72		80		87		92		97		110		122		172		230	
		<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>	<=	>
	Yes	0	3	0	3	0	3	0	3	1	2	2	1	3	0	3	0	3	0	3	0	3	0
	No	0	7	1	6	2	5	3	4	3	4	3	4	3	4	4	3	5	2	6	1	7	0
	Gini	0.420		0.400		0.375		0.343		0.417		0.400		<u>0.300</u>		0.343		0.375		0.400		0.420	

# Building a decision tree

Using the loan borrower data, let's build a decision tree using **entropy** to measure impurity.

From the root node, there are three possible attributes to split on:

- **Home owner**: yes, no
- **Marital status**: 3-way split (single, married, divorced) or a binary split
- **Annual income**:  $\leq n$ ,  $> n$  (for  $n$  to be found)

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

Root node: No – 7; Yes – 3.

$$\text{Entropy} = -\frac{7}{10}\log_2\frac{7}{10} - \frac{3}{10}\log_2\frac{3}{10} = 0.881$$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

## Splitting on **home owner**:

Yes: no – 3; yes – 0

$$\text{entropy} = -\frac{3}{3}\log_2\frac{3}{3} - \frac{0}{3}\log_2\frac{0}{3} = 0$$

No: no – 4; yes – 3

$$\text{entropy} = -\frac{4}{7}\log_2\frac{4}{7} - \frac{3}{7}\log_2\frac{3}{7} = 0.985$$

$$\text{Weighted entropy} = \frac{3}{10} \times 0 + \frac{7}{10} \times 0.985 = 0.690$$

$$\text{Information gain} = 0.881 - 0.690 = 0.192$$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes



# Building a decision tree

Splitting on **marital status**: 4 options

**3-way split: single, married, divorced**

Single: no – 2; yes – 2: entropy = 1

Married: no – 4; yes – 0: entropy = 0

Divorced: no – 1; yes = 1: entropy = 1

$$\text{Weighted entropy} = \frac{4}{10} \times 1 + \frac{4}{10} \times 0 + \frac{2}{10} \times 1 = 0.6$$

$$\text{Information gain} = 0.881 - 0.6 = 0.281$$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

Splitting on marital status: 4 options

binary split: **single, married/divorced**

Single: no – 2; yes – 2: entropy = 1

Married/divorced: no – 5; yes – 1:

$$\text{entropy} = -\frac{5}{6}\log_2\frac{5}{6} - \frac{1}{6}\log_2\frac{1}{6} = 0.650$$

$$\text{Weighted entropy} = \frac{4}{10} \times 1 + \frac{6}{10} \times 0.650 = 0.790$$

$$\text{Information gain} = 0.881 - 0.790 = 0.091$$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

Splitting on marital status: 4 options

binary split: **single/married, divorced**

Single/married: no – 6; yes – 2:

$$\text{entropy} = -\frac{6}{8}\log_2\frac{6}{8} - \frac{2}{8}\log_2\frac{2}{8} = 0.811$$

Divorced: no – 1; yes – 1: entropy = 1

$$\text{Weighted entropy} = \frac{8}{10} \times 0.811 + \frac{2}{10} \times 1 = 0.849$$

$$\text{Information gain} = 0.881 - 0.849 = 0.032$$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

Splitting on marital status: 4 options

binary split: **single/divorced, married**

Single/divorced: no – 3; yes – 3: entropy = 1

Married: no – 4; yes – 0: entropy = 0

$$\text{Weighted entropy} = \frac{6}{10} \times 1 + \frac{4}{10} \times 0 = 0.6$$

$$\text{Information gain} = 0.881 - 0.6 = 0.281$$

**Note:** 3-way and **single/divorced, married** both have gain = 0.281

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

# Building a decision tree

## Splitting on annual income

Split	<55	>55	<65	>65	<72	>72	<80	>80	<87	>87	<92	>92
No	0	7	1	6	2	5	3	4	3	4	3	4
Yes	0	3	0	3	0	3	0	3	1	2	2	1
Entropy	-	0.881	0	0.918	0	0.954	0	0.985	0.811	0.981	0.971	0.722
Weighted	0.881		0.826		0.764		0.690		0.875		0.846	

Split	<97	>97	<110	>110	<122	>122	<172	>172	<230	>230		
No	3	4	4	3	5	2	6	1	7	0		
Yes	3	0	3	0	3	0	3	0	3	0		
Entropy	1	0	0.985	0	0.954	0	0.918	0	0.881	-		
Weighted	0.6		0.690		0.764		0.826		0.881			

Information gain **0.281**

# Building a decision tree

## Summary

Split marital status: single, married, divorced

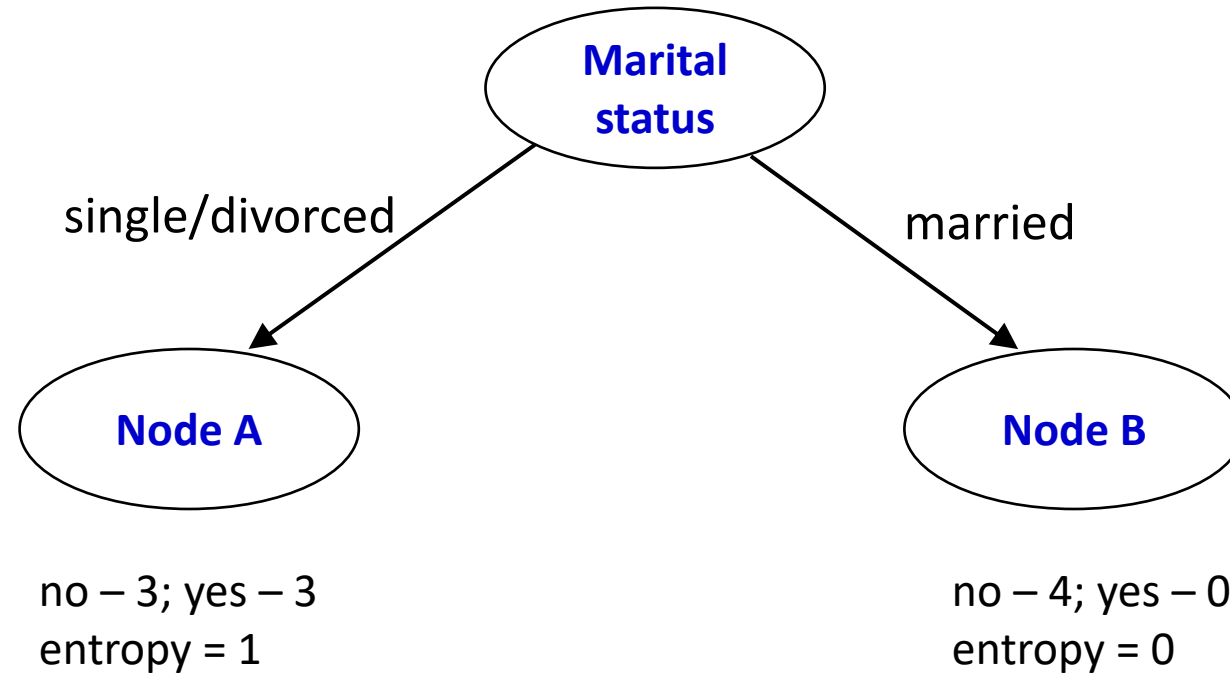
Split marital status: single/divorced, married

Split annual income:  $\leq 97$ ,  $> 97$

Each has information gain = 0.281.

Choose: marital status: single/divorced, married

# Building a decision tree: level 1



**Node A:** split on home owner or income    **Node B:** leaf node: No

# Node A: single/divorced

## Splitting on **home owner**

Yes: no – 2; yes – 0: entropy = 0

No: no – 1; yes – 3: entropy = 0.811

Weighted entropy =  $\frac{2}{6} \times 0 + \frac{4}{6} \times 0.811 = 0.541$

Information gain =  $1.0 - 0.541 = 0.459$

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
3	No	Single	70K	No
5	No	Divorced	95K	Yes
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
10	No	Single	90K	Yes



# Node A: single/divorced

## Splitting on annual income

Best split is: income  $\leq 110$ , income  $> 110$

$\leq 110$ : no – 1; yes – 3: entropy = 0.811

$> 110$ : no – 2; yes – 0: entropy = 0

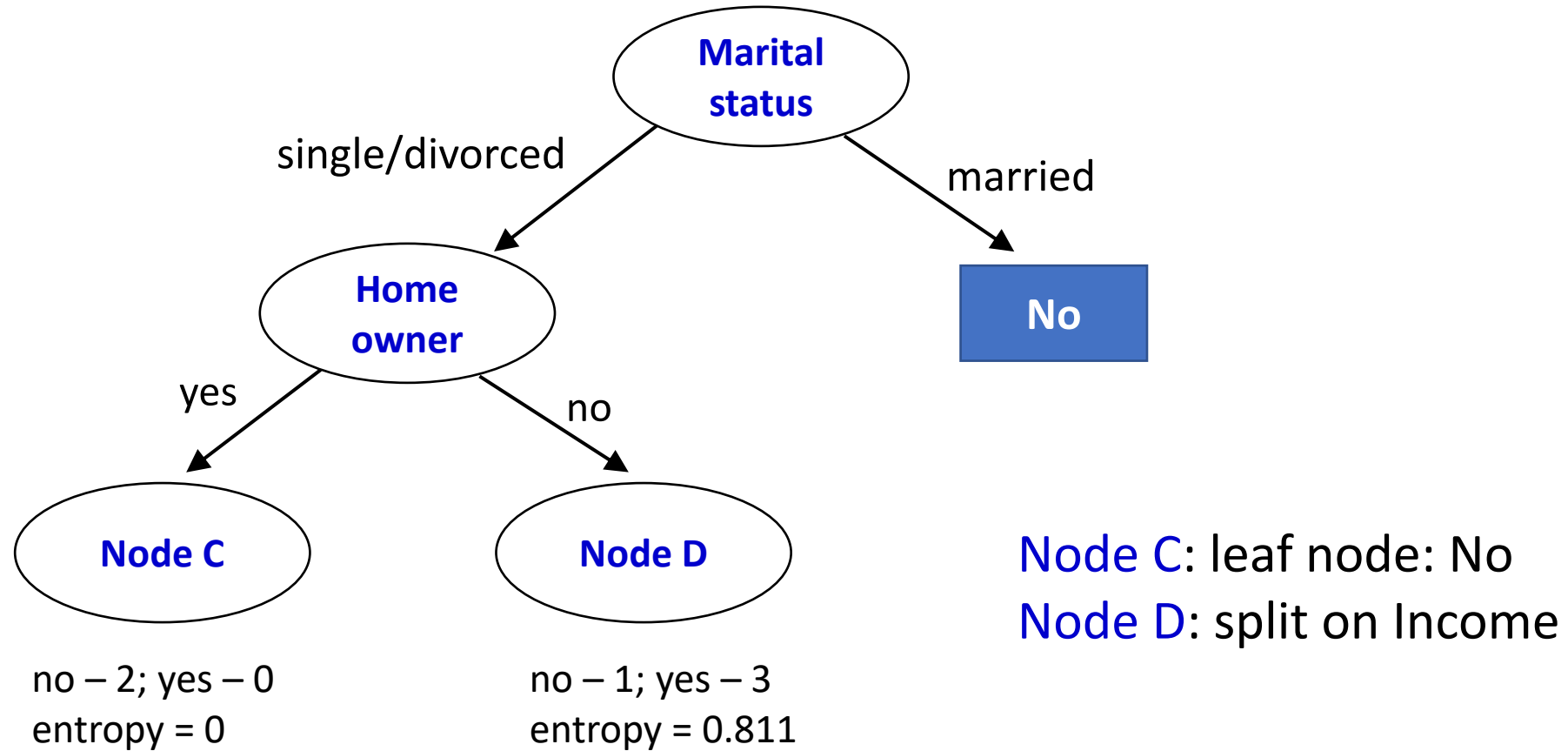
$$\text{Weighted entropy} = \frac{4}{6} \times 0.811 + \frac{2}{6} \times 0 = 0.541$$

$$\text{Information gain} = 1.0 - 0.541 = 0.459$$

Information gain is same splitting on home owner or income; choose **home owner**

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
1	Yes	Single	125K	No
3	No	Single	70K	No
5	No	Divorced	95K	Yes
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
10	No	Single	90K	Yes

# Building a decision tree: level 2



# Node D: split on income

## Splitting on annual income

Best split is:  $\text{income} \leq 77$ ,  $\text{income} > 77$

$\leq 77$ : no – 1; yes – 0: entropy = 0

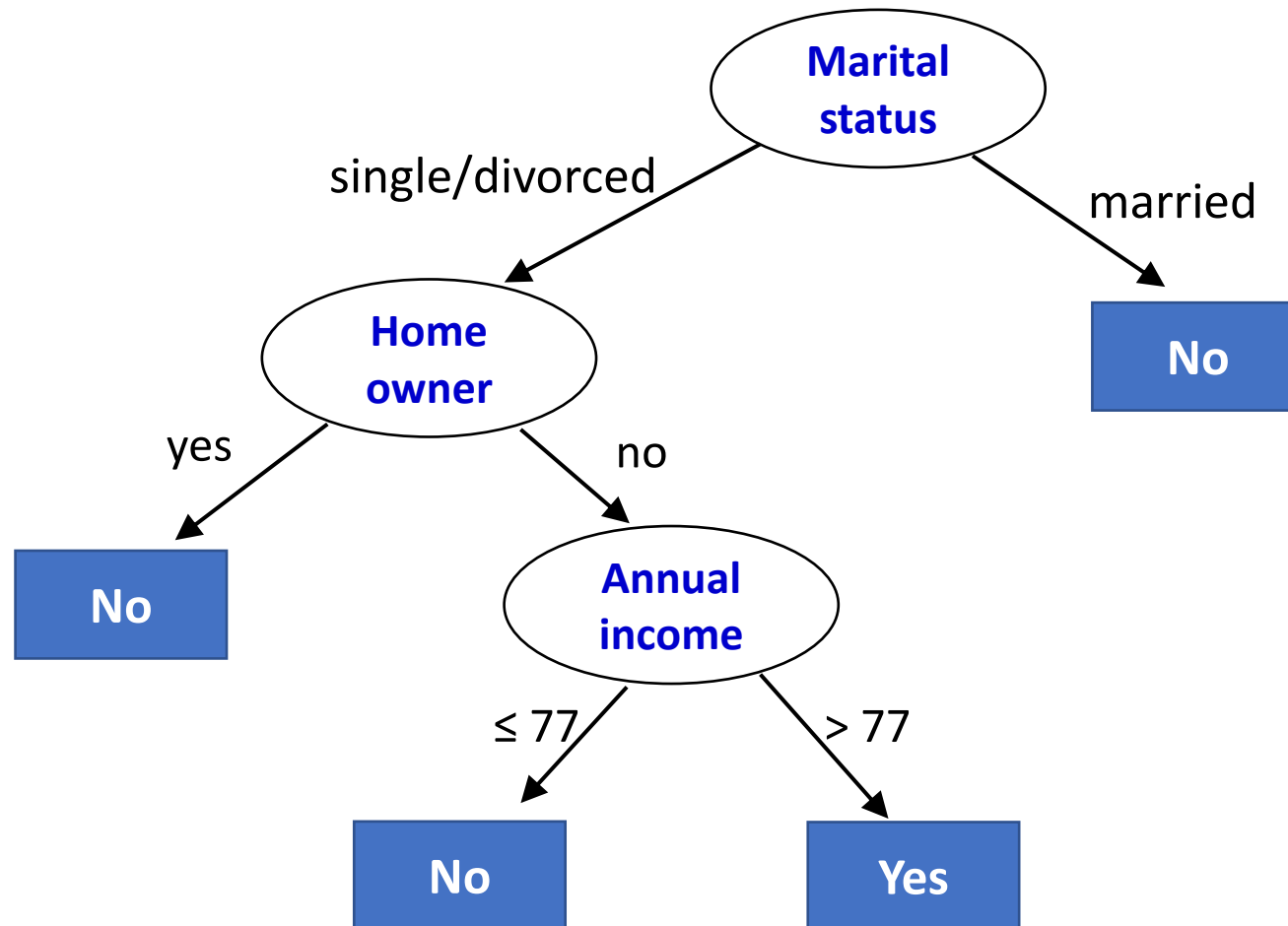
$> 77$ : no – 0; yes – 3: entropy = 0

Weighted entropy = 0; information gain =  $0.811 - 0 = 0.811$

This gives the final tree.

ID	Home Owner	Marital Status	Annual Income	Defaulted Borrower
3	No	Single	70K	No
5	No	Divorced	95K	Yes
8	No	Single	85K	Yes
10	No	Single	90K	Yes

# Building a decision tree: the final tree



# Evaluation of Classification Algorithms

## -Training Set and Test Set

- Machine learning algorithms are evaluated to determine their performance accuracy. Machine learning algorithms performance accuracy depends on how well they are able to predict new unseen data. If the algorithm doesn't generalize then it will not perform well on new unseen data.
- Two problems are associated with algorithms that do not generalize: Overfitting and Underfitting.

# Overfitting and Underfitting

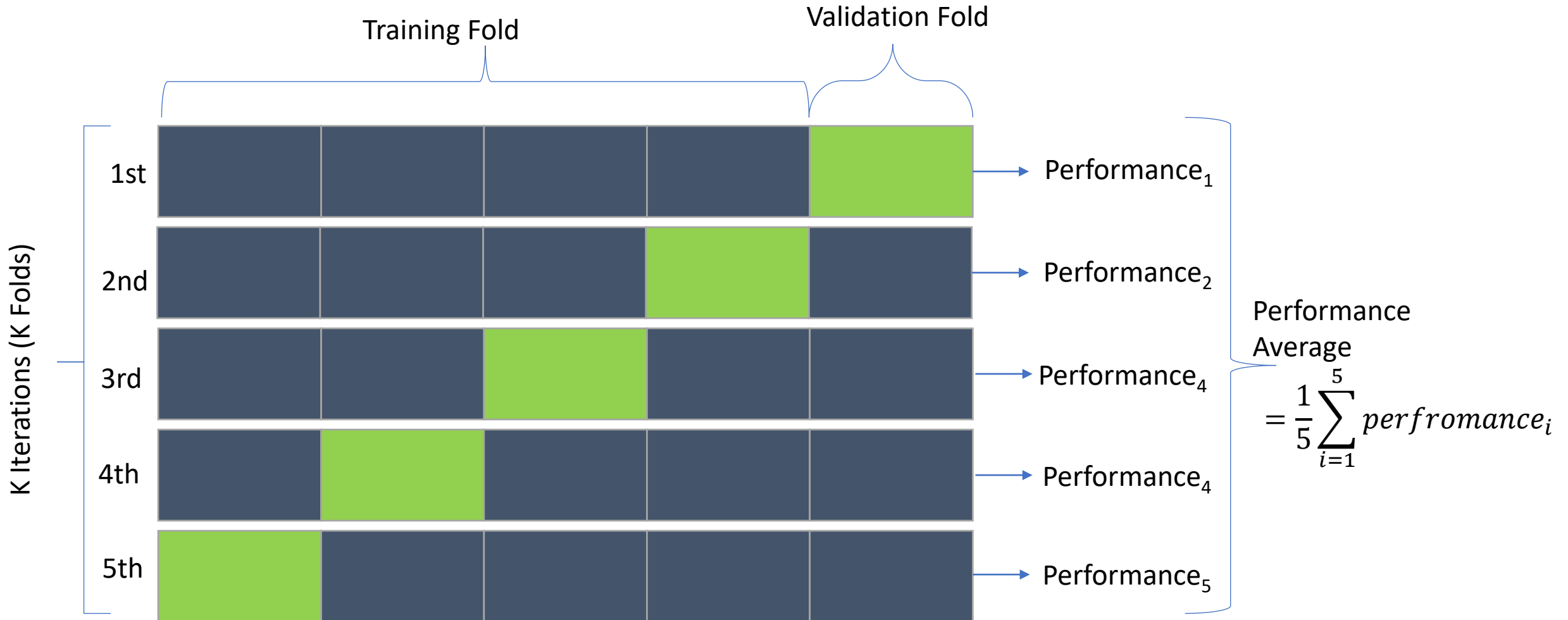
- Overfitting – the concept in machine learning when the machine learning model fits exactly to its training data. When this happens the model does not perform well on new unseen data.
- Underfitting – the model has not been trained for enough time or the input variables are not significant enough to draw any conclusions between the input and the output values.

# Training set and test set

- To overcome this problem some of the data is used to train the data and some of the data is used to test the data. This is usually a split of 80% training data and 20% test data.
- Overfitting, when the data trained does badly on the test set. Accuracy is lower when tested on the test data.
- Underfitting, when the algorithm has poor performance on both the training data and test data.

# K-Fold Cross Validation

K-Fold cross validation is a resampling technique. The data is split into k sets of training and test folds. The performance is averaged over all sets.



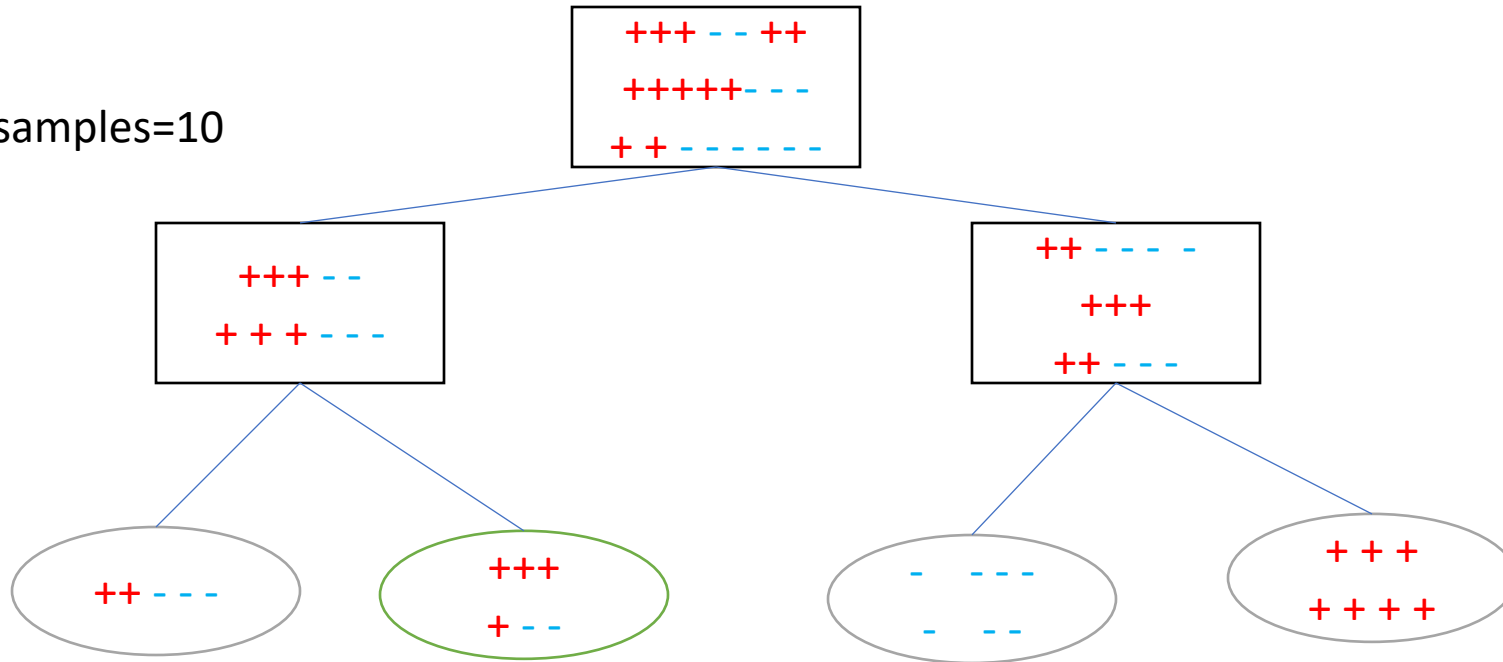


# Optimizing (Pruning) the Decision Tree

- Decision tree optimization or pruning techniques are used to avoid overfitting and underfitting.
1. Minimum number of samples for a node split
  2. Minimum samples for a leaf node
  3. Maximum depth of the tree

# Minimum Samples for a Node split

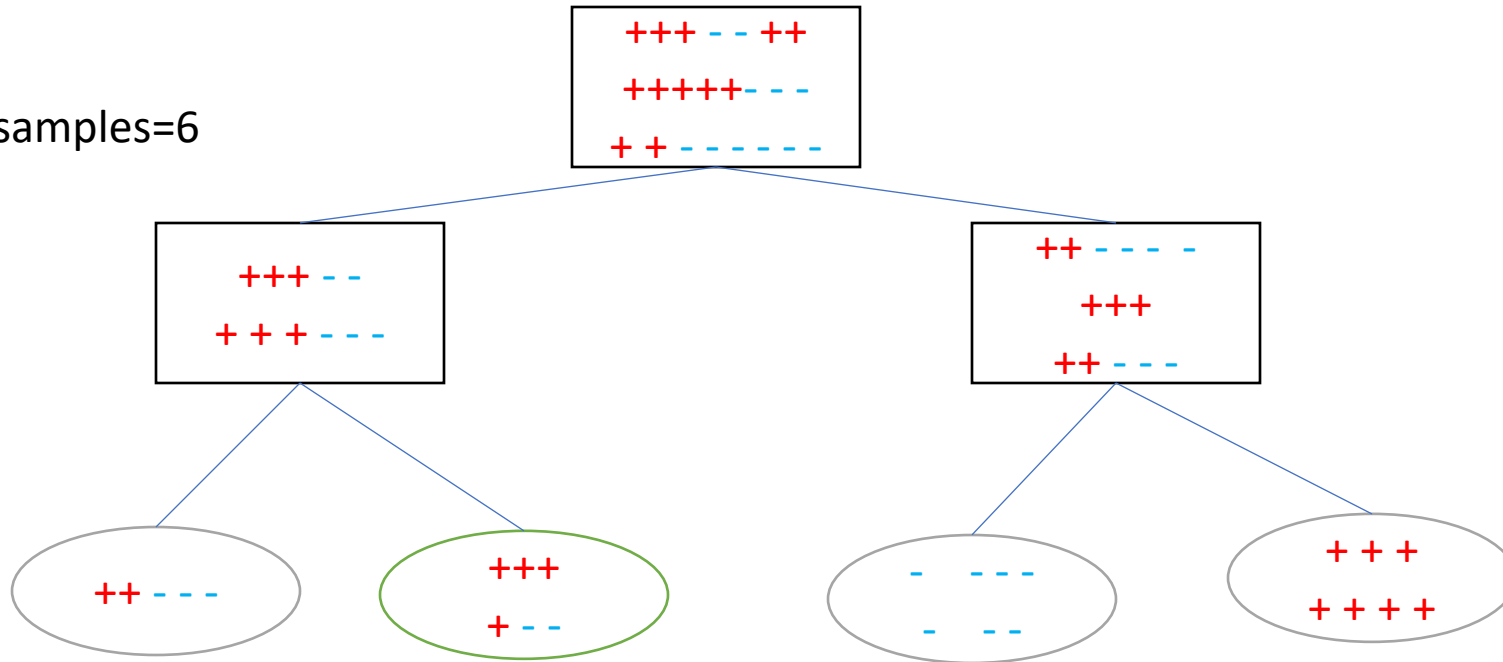
Minimum samples=10



In python the attribute is used with a decision tree classifier  
**min\_samples\_split**

# Minimum Samples for a leaf node

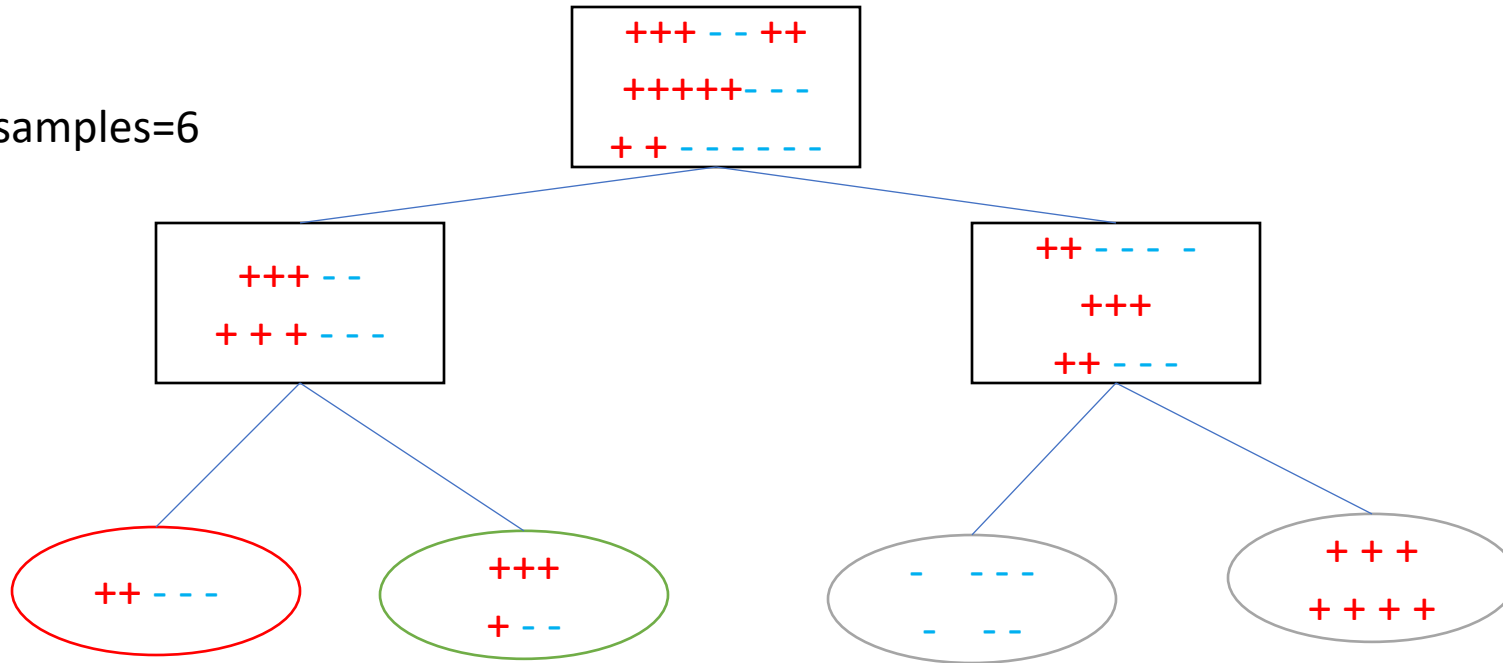
Minimum samples=6



In python the attribute is used with a decision tree classifier  
**min\_samples\_leaf**

# Minimum Samples for a leaf node

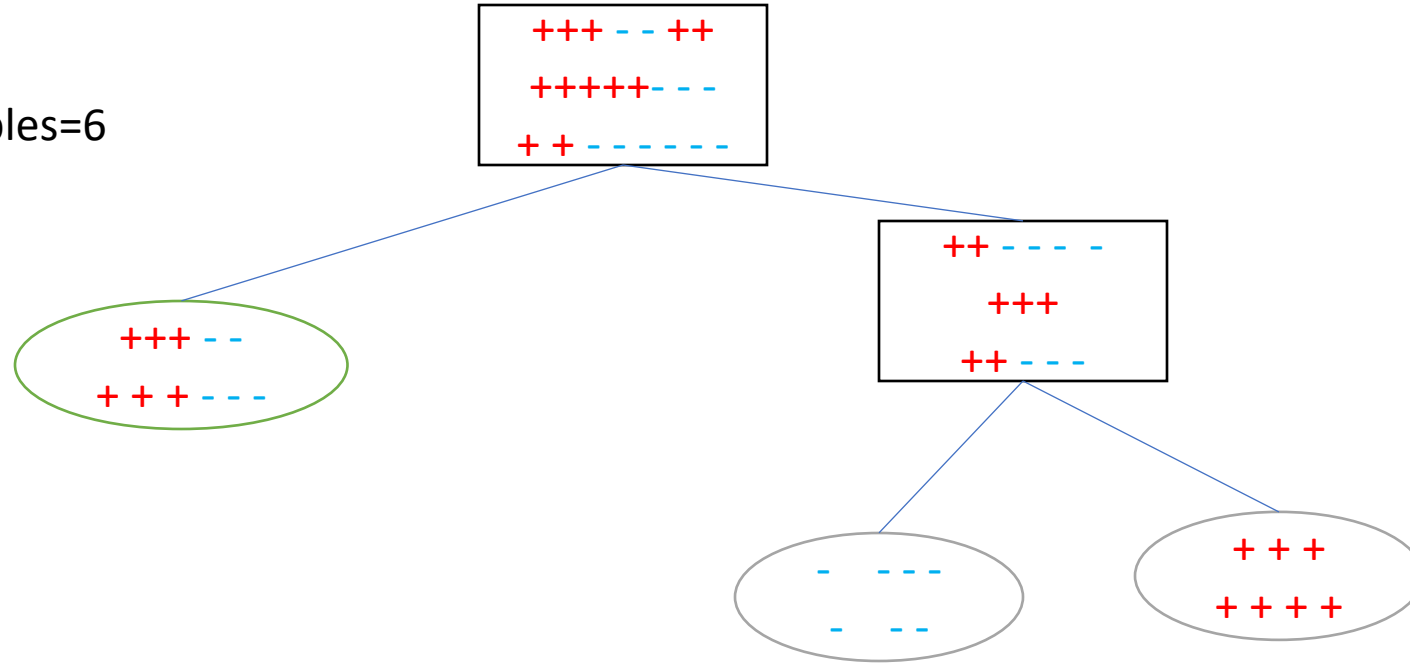
Minimum samples=6



In python the attribute is used with a decision tree classifier  
**min\_samples\_leaf**

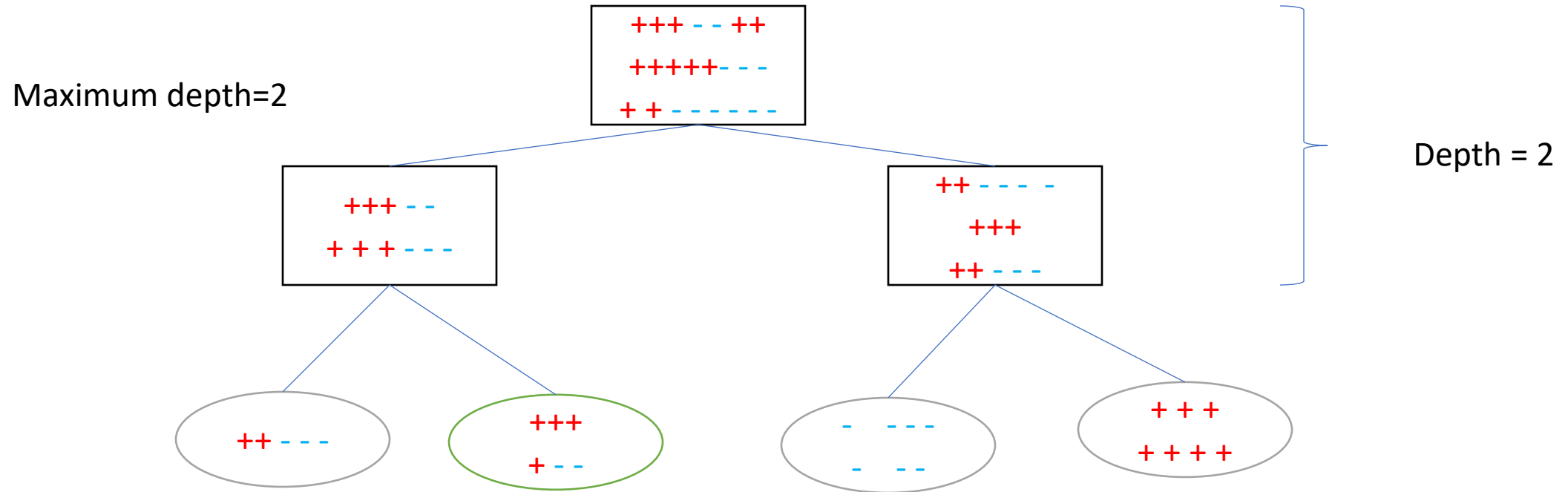
# Minimum Samples for a leaf node

Minimum samples=6



In python the attribute is used with a decision tree classifier  
**min\_samples\_leaf**

# Maximum Depth of the Tree



In python the attribute is used with a decision tree classifier  
**max\_depth**

# Implementing Decision Trees

Import relevant Python packages used to create a decision tree

- `import pandas as pd`
- `import numpy as np`
- `import matplotlib.pyplot as plt`
- `from sklearn import tree`
- `from sklearn.model_selection import train_test_split`
- `from sklearn.tree import DecisionTreeClassifier`
- `from sklearn import metrics`
- `from sklearn.model_selection import cross_val_score, KFold`

# Pandas

Reading the data file into pandas using the `read_csv()` function

```
data= pd.read_csv('data_file.csv')
```

Printing first four records

```
data.head()
```



Assigning the attributes and values of the dataset to variables x and y

```
y = data['class_label']
```

```
X = data.drop(['class_label'], axis=1)
```

```
#convert the string values to number
```

```
d={'Iris-setosa':0,'Iris-versicolor':1,'Iris-virginica':2}
```

```
data['species']=data['species'].map(d)
```

Create a training set with 80% and test set with 20% of the samples

- `x_train,x_test,y_train,y_test =train_test_split(X,y,test_size=0.20)`

Creating an instance of the decision tree classifier

```
dt=DecisionTreeClassifier()
```

## Decision tree classifier parameters

- **criterion**{*"gini", "entropy", "log\_loss"*}, *default="gini"*
- **max\_depth** {*int, default=None*} The maximum depth of the tree. If None, then nodes are expanded until all leaves are pure
- **min\_samples\_split** {*int or float, default=2*} The minimum number of samples required to split an internal node.
- **min\_samples\_leaf** {*int or float, default=1*} The minimum number of samples required to be at a leaf node.

# Decision tree classifier with example parameters

- `dt=DecisionTreeClassifier(criterion='entropy',max_depth=3,min_samples_split=6, min_samples_leaf=6)`

## Fitting the training data

```
dt.fit(x_train,y_train)
```

# Prediction accuracy

- `y_pred = dt.predict(x_test)`
- `print("Accuracy:", metrics.accuracy_score(y_test, y_pred))`

# K-Fold Accuracy prediction

- `kf=KFold(n_splits=5)`
- `score=cross_val_score(dt,X,y,cv=kf)`
- `print(score.mean())`

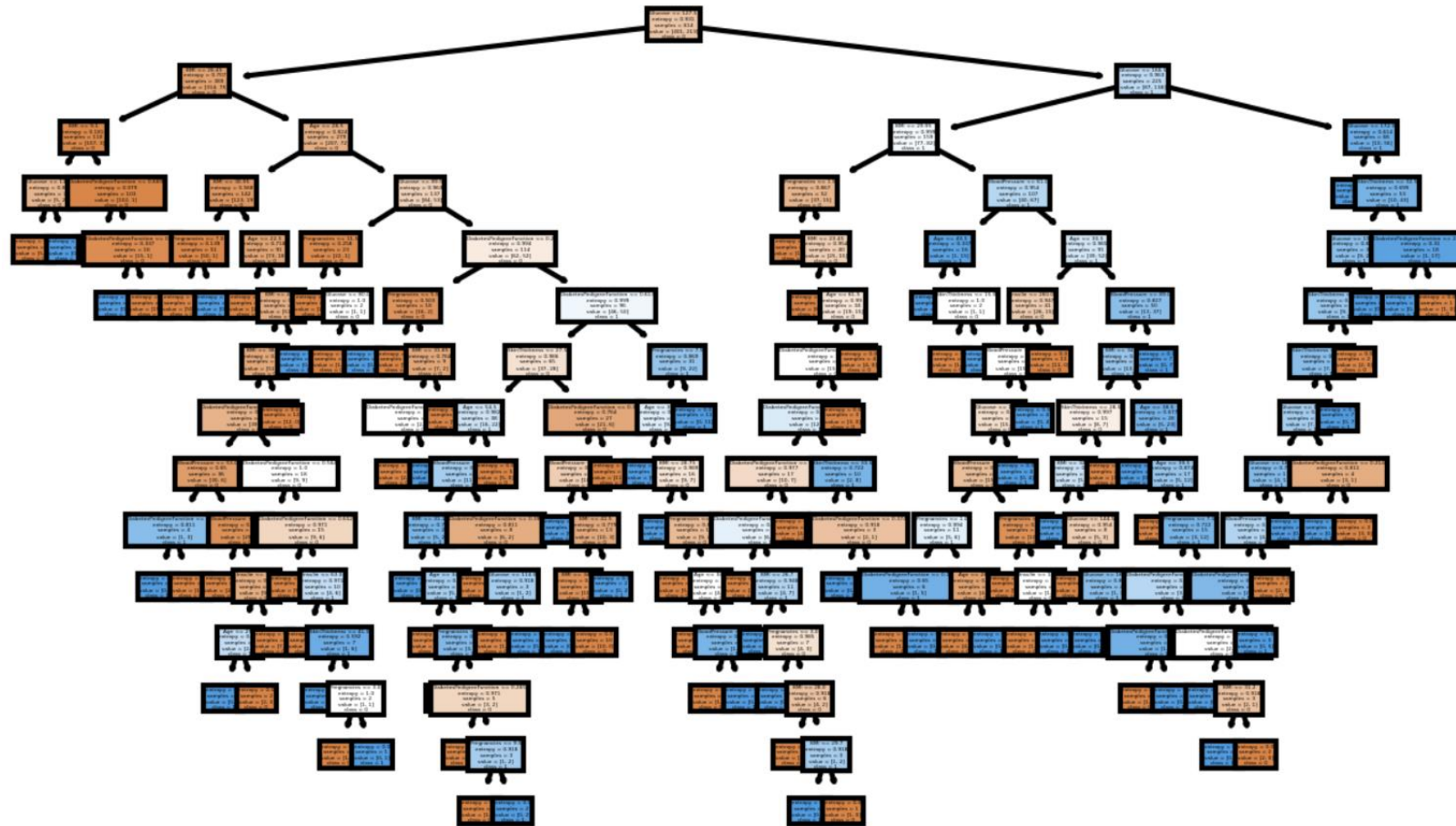
# Displaying the decision tree

```
fig, axes = plt.subplots(nrows = 1,ncols = 1,figsize =  
(5,3), dpi=300)
```

```
tree.plot_tree(dt,feature_names = ['A','B','C'],  
               class_names=['0','1','2'],  
               filled = True);
```

```
fig.savefig('imagename.png')
```

# Decision Tree before Optimization (pruning)





# Decision Tree after pruning

