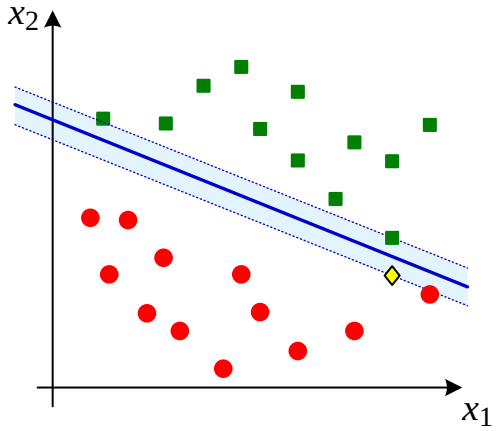
Solutions

1. Diagram A: the data is linearly separable (as in class).
   Diagram B: the data is not linearly separable – there is no straight line that separates the points.

2. (i) This is a soft margin classifier (because there are data points in the margin region).
   (ii) There are five support vectors – two positive and three negative.
   (iii) The yellow diamond would be classified as a positive point as it is on the positive side of the classifier line.
   However, if we were to make a hard margin classifier, the yellow diamond would probably be classified as a negative point – see the diagram below.



3. (i) P : substituting gives $-x_1 + 2x_2 + 1 = -2 + 2 + 1 = 1$ so this is a positive support vector.
   Q : substituting gives $-x_1 + 2x_2 + 1 = -1 - 2 + 1 = -2$ so this is a negative vector, but not a support vector.
   R : substituting gives $-x_1 + 2x_2 + 1 = -3 + 4 + 1 = 2$ so this is a positive vector, but not a support vector.
   S : substituting gives $-x_1 + 2x_2 + 1 = 2 - 4 + 1 = -1$ so this is a negative support vector.
   T : substituting gives $-x_1 + 2x_2 + 1 = 1 + 6 + 1 = 8$ so this is a positive vector, but not a support vector.
   (ii) A : substituting gives $-x_1 + 2x_2 + 1 = -0.5 - 1 + 1 = -0.5$ so we would classify this as a negative point (even though it is within the margin).
   B : substituting gives $-x_1 + 2x_2 + 1 = -3 + 2 + 1 = 0$ so this lies on the support classifier line and so cannot be classified.

4. (i) The margin is

$$\frac{2}{\|\boldsymbol{w}\|} = \frac{2}{\sqrt{3^2 + 4^2}} = \frac{2}{5}.$$

   The distance from the SVM classifier to a support vector is half the margin:

$$\frac{1}{\|\boldsymbol{w}\|} = \frac{1}{\sqrt{3^2 + 4^2}} = \frac{1}{5}.$$

   (ii) The margin is

$$\frac{2}{\|\boldsymbol{w}\|} = \frac{2}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{2}{\sqrt{14}}.$$