Ideas Proposal and Pitching for a Practical Application of Fractional/Nonlinear Circuits Analysis

This document is a copy of the original presentation delivered to Ain Shams University Professors during my enrollment.

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1. Motivations for Nonlinear and Fractional Circuit Analysis

a) Nonlinear :

- Generalization of the analysis techniques instead of focusing on the linear analysis techniques (Nodal analysis versus Modified Nodal Analysis).

- Diversity :Ideal Linear elements numbers (R,L,C) are very few compared to the ideal nonlinear elements (V-Sources, Memristors, diodes, op-amps) which have no linear counterparts or have limited linear operating regions or even can not be analyzed using Linear Techniques.

- Accuracy : Linear analysis fails to predict the effect of the devices limitations, like clipping due to saturation, jitter noise, frequency dependent parasitic elements.

- Nanoelectronics: Nanoscale elements exhibit nonlinear behavior dominantly due to the rise of the quantum effects like quantum tunneling, increased temperature per unit area.

- Diversity : Many devices operate based on the nonlinear models; Frequency synthesis, Mixers, Filtering, Op-Amps, Modulation, signal shaping.

- Modeling Theory : using classical linear R,L, and C elements solely will fail to model nonlinear behavior such as hysteresis or saturation, and the newly introduced nano-quantum effects such as Columb Blockade and Quantum Tunneling, thus using new elements will enrich the field of circuit modeling by introducing new basis elements (functions) to simulate the devices behaviors.

- Predicting New applications and elements : Chua’s circuit, Memristors were predicted by Leon Chua more than 40 years before its actual realization.

1. Motivations for Nonlinear and Fractional Circuit Analysis

b) Fractional:

- Generalization of the analysis techniques instead of focusing on the integer-order analysis techniques (Differential Algebraic Equations vs Fractional Differential Algebraic Equations).

- Growing Trend : More papers are published for the realization of fractional order devices (Varshney 2013 , Implementation of 1st and 3rd Order fractional order differentiators and integrators using switched capacitors , 2008 fractional order differentiator, Krishna 2011 Fabrication of a fractional order capacitor using polymer films)

- Possible benefits for Modeling : It could be easier and faster to use fractional-order models to express nonlinear effects rather than adding more integer-order devices to the model.

- Predicting New applications and elements: Fractional order controllers, the periodic table of electrical elements.

2. Main Topics and Combinations

- Selected Topics
  - Integer Order Elements $(\alpha, \beta) \neq \{(0,0),(0,-1),(-1,0)\}$, Integer Order Complex Elements
  - Fractional Elements, Fractional Complex-order elements.
  - Nonlinear (Fractional/Partial/Ordinary Differential) Algebraic System of Equations.
  - Nonlinear Frequency Response Methods : (Harmonic Balance, Generalized Frequency Response Function GRFs)
  - Nonlinear System Modeling : Volterra Series, Wiener Series/Model, Hammerstein Models.
  - Constrained Optimization.
  - Circuit Analysis Techniques : MNA, Tableau,Order Reduction, Circuit Partitioning, using Flux/Charge instead of voltage/current as design criteria.
  - Graph Theory.

2. Main Topics and Combinations

- Leon Chua is regarded as the father of nonlinear circuits.

- In his famous book "…" and literature, Professor Chua covered multiple topics in the field of nonlinear circuits analysis.

- The discussion is limited to lumped elements.

2. Main Topics and Combinations

- Analytical Methods
  - Laplace Transform
- Numerical Methods
  - Time Domain
  - Frequency Domain

2. Main Topics and Combinations

- Circuit Analysis Techniques
  - Tableau Analysis.
  - MNA.
  - Tellegen's Theorem.
  - Topological Analysis (Optional if time permits).
- Fractional Order Elements
  - Algebraic.
  - Dynamic (Optional if time permits).
  - LTI or Linear Time Variant.
  - Nonlinear Time invariant or Time Variant.
- DAE and Fractional DAE
- Special Function (Hypergeometric, Mettage-Leffler, Gamma, Beta, Incomplete Beta)
- Fractional Integral Transforms (Laplace, Mellin, Fourier)

3. Circuits Digraphs

- Digraphs of Elements and Circuits :
  - N-Terminal Element ⇨ Graph with N nodes and N-1 branches
  - Each terminal is represented by a node, and each branch current is represented by a directed branch from the +ve voltage node to the negative voltage node (Associated Current Reference)
  - For an n-terminal element, only n-1 terminal currents are linearly independent, the other can be calculated using KCL as the sum of the other currents ;

    $I_n = \sum_{k=1}^{n-1} I_k$

    Thus the no. of branch currents = n-1
  - By choosing a datum node for the element, all other terminal voltages will be calculated with reference to this datum node.
  - The resulting digraph will be formed of (n-nodes , b = n-1 branches)
  - The circuit digraph is formed by replacing each element with its digraph.
  - Disconnected Graphs

    Due to coupling (physical, mechanical,…), ex: Ideal Transfomers

    Each node can be hinged to represent multiple equivalent nodes (Representing a hinged element with I = 0).

    Inversely, all hinged nodes can be soldered to represent a single node, resembling selecting a common datum for each portion of the circuit (equivalent to selecting a common datum node for all portions).
  - The topology of the circuit is represented by an incidence matrix $\mathbf{A_a}$ (n-nodes x b-branches)
  - The linear independence of node and loop equations are guaranteed by choosing a reference node, resulting in a reduced incidence matrix **A** (n-1 nodes x b-branches)

3. Circuits Digraphs (Tellegen's Theorem)

- Multiport Theory
  - For an n-terminal element , there are n current and node-voltage variables, among which only n-1 currents and n-1 branch voltages (with one of them as a datum node) that are linearly independent.
  - A multi-port element can be constructed from a multi-terminal element by choosing a common datum node across all ports, and forming ports out of every node-datum pairs.

    Example :

    Resistor – 2 terminal – 1 Port.

    MOS – 4 Terminal – 3 or 2 Ports.
  - Thus for an n-terminal element, a multiport element can be formed with maximum no. of ports = n-1 (the case where a common terminal is set as datum and all terminals are treated as ports)
  - For each port there will be one port voltage and one port variable.
  - Thus for each n-port element, the number of available representations for n- port voltages and currents are (2n)C(n) , each representation utilizes n-dependent variables and n-independent variables

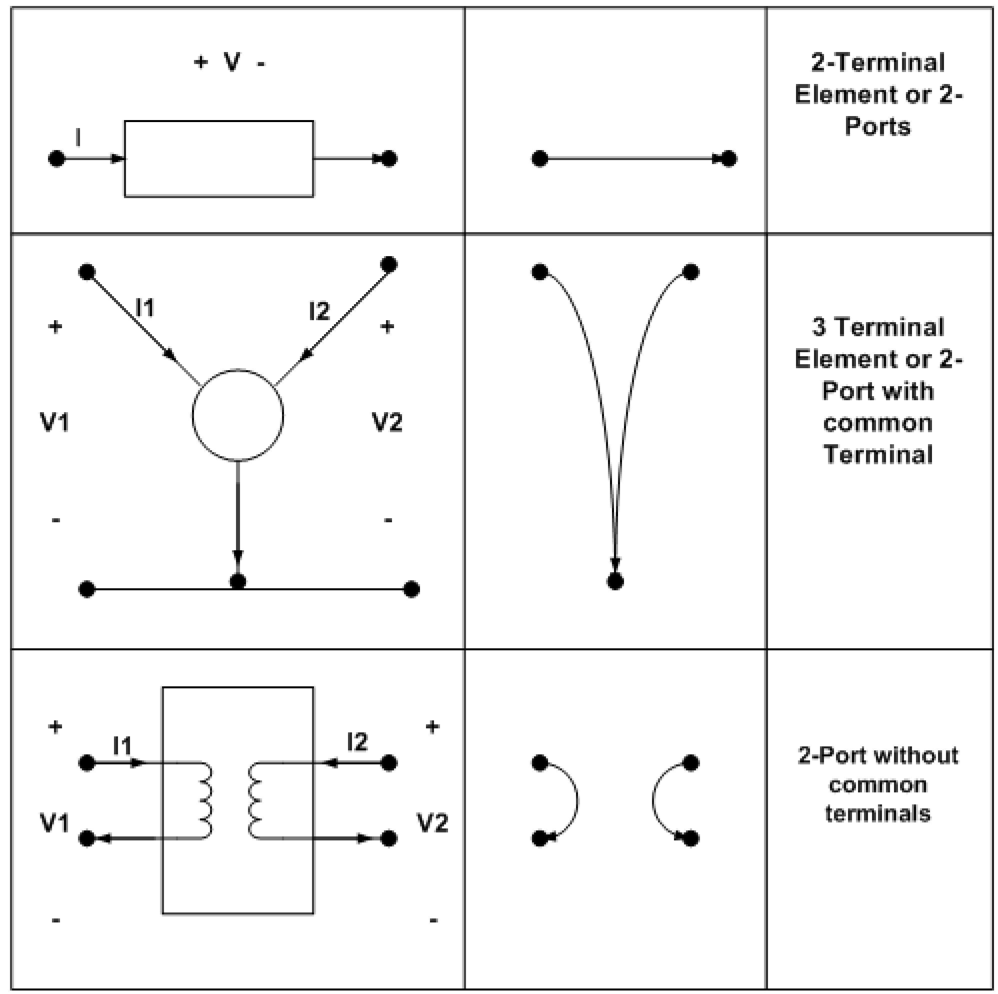
    Example:

    2-Port --- 4C2 = 6 representations

    4 Port variables = $\mathbf{V_1}$, $\mathbf{V_2}$, $\mathbf{I_1}$, $\mathbf{I_2}$

    Voltage controlled, Current controlled, Hybrid 1, Hybrid 2, Scattering, Transfer) , each representation uses a different set of 2 variables ($\mathbf{I_1}$ and $\mathbf{I_2}$ in terms of $\mathbf{V_{1\ and}}$ $\mathbf{V_2}$, $\mathbf{V_1}$ and $\mathbf{V_2}$ in terms of $\mathbf{I_{1\ and}}$ $\mathbf{I_{2,...}}$)

- Digraphs of some examples.



3. Circuits Digraphs

- When all the circuit's elements are replaced with their digraph, a circuit digraph is generated.
- Based on the circuit's digraph, the incidence Matrix A_a and the reduced incidence matrix A are generated, and the matrix forms of KVL and KCL are ;
  - KVL

$$\boldsymbol{V} = {A_a}^{\boldsymbol{T}}\, \boldsymbol{e}$$

$$\boldsymbol{V} = A^{\boldsymbol{T}}\, \boldsymbol{e}$$

Where $\boldsymbol{e}$ is the node voltages n-1 vector, $\boldsymbol{V}$ is branch voltages b vector.

  - KCL

$$A_a\boldsymbol{I} = \boldsymbol{0}$$

$$A\,\boldsymbol{I} = \boldsymbol{0}$$

Where $\boldsymbol{I}$ is the branch voltages b vector.

3. Circuits Digraphs

- Incidence Matrix A_a (n-Nodes x b-Branches)
  - $a_{i,j} = \begin{cases} 1 & If\ branc\quad j\ leaves\ node\ i \\ 1 & If\ branc\quad j\ enters\ node\ j \\ 0 & Else \end{cases}$
  - $i = 1, \ldots n\ ; j = 1, \ldots b$
- Reduced Incidence Matrix A(b-branches x n-1 nodes)
  - Selecting a reference (datum) node forms a new reduced matrix that is identical to the incidence matrix but with the row of the reference matrix eliminated.

3. Circuits Digraphs

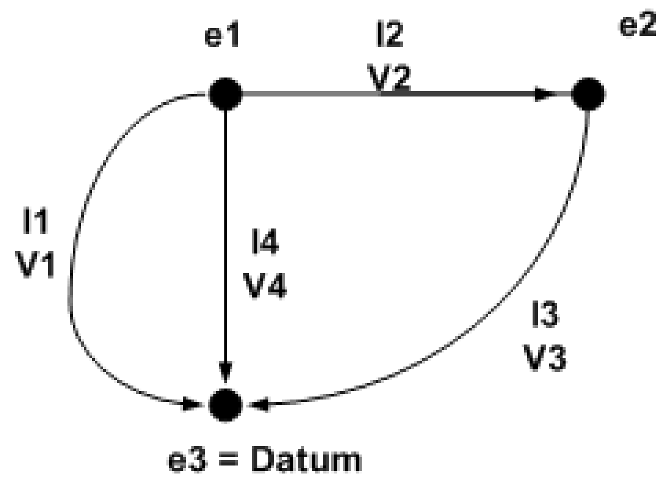
- Example : 3 nodes, 4 branches, then A is a 3x4 matrix

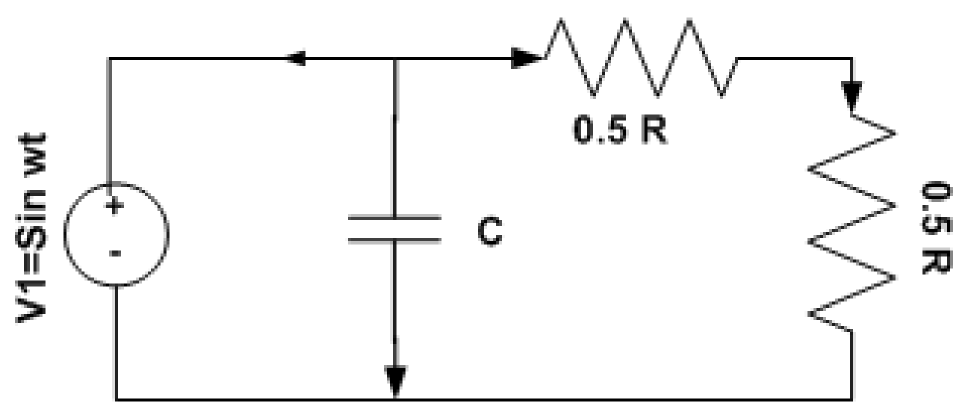
$$A_a = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

- Choosing node 3 as reference , the reduced matrix is ;

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

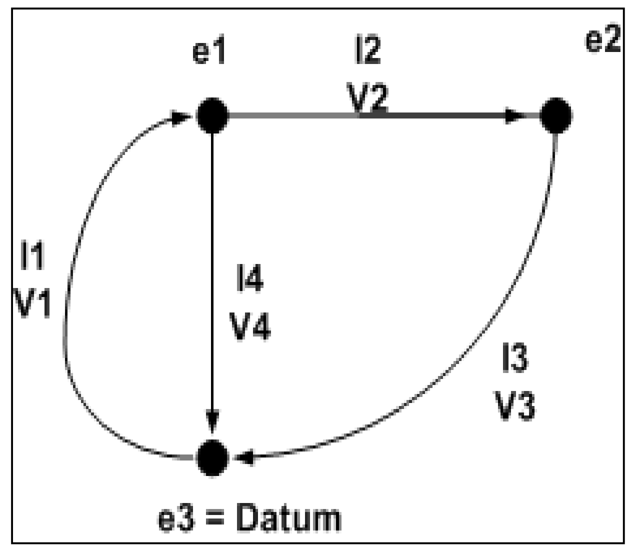




3. Circuits Digraphs

- $\boldsymbol{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} e_1 & e_3 \\ e_1 & e_2 \\ e_2 & e_3 \\ e_1 & e_3 \end{bmatrix} = \begin{bmatrix} & e_1 & \\ e_1 & & e_2 \\ & e_2 & \\ & e_1 & \end{bmatrix} =$
$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} [e_1 \quad e_2] = \boldsymbol{A}^T \boldsymbol{e}$

- $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \boldsymbol{AI} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$



3. Circuits Digraphs (Tellegen's Theorem)

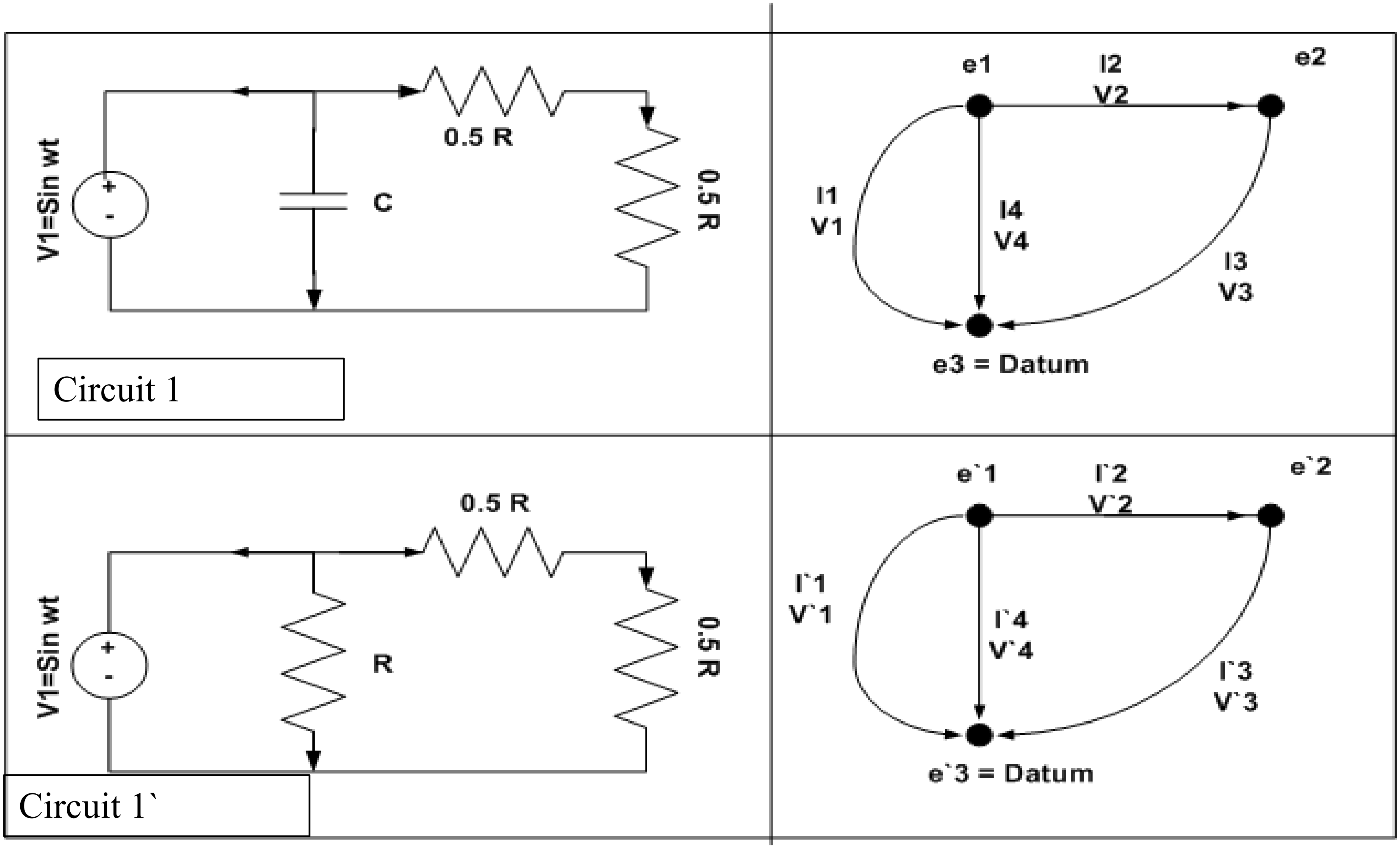
- Tellegen's Theorem
  - For arbitrary circuits with the same linear graph, using the associated current reference, if any **I(t)** satisfy KCL and any **V(t)** satisfy KVL, then;

    $\mathbf{V}^{T}\,\mathbf{I}= \mathbf{0}$
  - Conservation of Energy in Circuits is a special case of Tellegen's Theorem.
  - Topology Dependent.
  - Extended to **V(t)** and **I(t)** with any linear operator applied on them such as differentiation or integration.
  - Complementary form for Tellegen's Theorem ;

    For arbitrary circuits with the same linear graph, using the associated current reference, if any $\mathbf{I}^{<\beta>}(\mathbf{t})$ satisfy KCL and any $\mathbf{V}^{<\alpha>}(\mathbf{t})$ satisfy KVL, then;

    $\mathbf{V}^{<\alpha>\,T}\,\mathbf{I}^{<\beta>}= \mathbf{0}$

3. Circuits Digraphs



3. Circuits Digraphs

- For Circuit(1)
  - $V = \begin{bmatrix} Sin\ wt \\ \frac{Sin\ wt}{2} \\ \frac{Sin\ wt}{2} \\ Sin\ wt \end{bmatrix}$ and $I = \begin{bmatrix} (c\ Cos\ wt + \frac{Sin\ wt}{R}) \\ \frac{Sin\ wt}{R} \\ \frac{Sin\ wt}{R} \\ c\ Cos\ wt \end{bmatrix}$
  - $V^T\ I = 0$ (Conservation of Energy)
- For Circuit(1`)
  - V` $= \begin{bmatrix} Sin\ wt \\ \frac{Sin\ wt}{2} \\ \frac{Sin\ wt}{2} \\ Sin\ wt \end{bmatrix}$ and $I` = \begin{bmatrix} \frac{-2\ Sin\ wt}{R} \\ \frac{Sin\ wt}{R} \\ \frac{Sin\ wt}{R} \\ \frac{Sin\ wt}{R} \end{bmatrix}$
  - $V`^T\ I` = 0$ (Conservation of Energy)
- Using Tellegen's Theorem , although the 2 circuits share the same digraph but with different element (R instead of C) ;
  - $V`^T I = 0$ ; $V^T I` = 0$

3. Circuits Digraphs

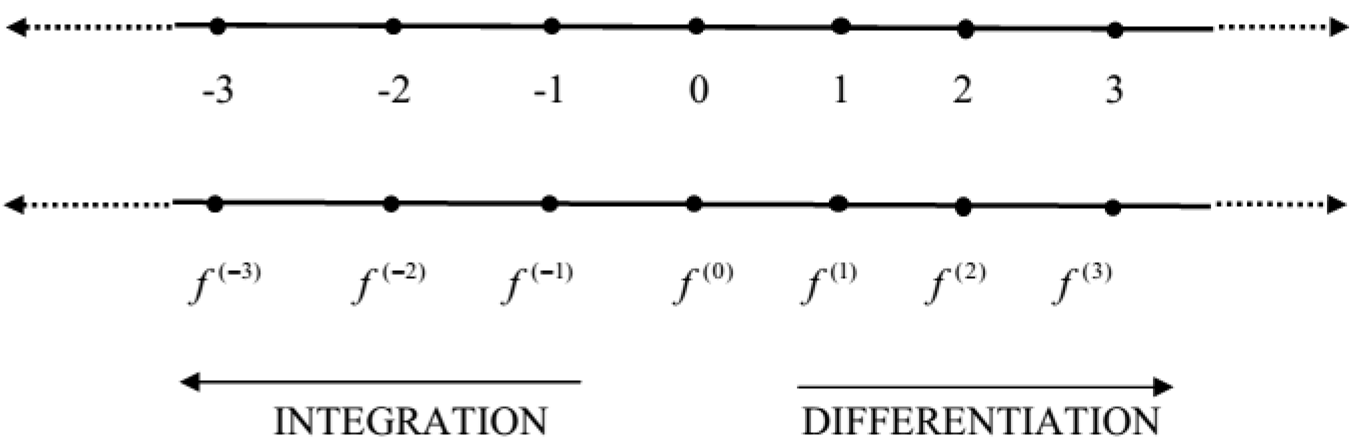
- For Circuit(1)
  - $V = \begin{bmatrix} Sin\ wt \\ \frac{Sin\ wt}{2} \\ \frac{Sin\ wt}{2} \\ Sin\ wt \end{bmatrix}$ and $I = \begin{bmatrix} (c\ Cos\ wt + \frac{Sin\ wt}{R}) \\ \frac{Sin\ wt}{R} \\ \frac{Sin\ wt}{R} \\ c\ Cos\ wt \end{bmatrix}$
  - $\frac{dV}{dt} = \begin{bmatrix} Cos\ wt \\ \frac{Cos\ wt}{2} \\ \frac{Cos\ wt}{2} \\ Cos\ wt \end{bmatrix}$ and $\frac{dI}{dt} = \begin{bmatrix} (\ \ c\ Sin\ wt + \frac{Cos\ wt}{R}) \\ \frac{Cos\ wt}{R} \\ \frac{Cos\ wt}{R} \\ c\ Sin\ wt \end{bmatrix}$
  - Again using Tellegen's Theorem ;
  - $\left(\frac{dV}{dt}\right)^T I^T = 0; V^T \frac{dI}{dt}^T = 0$

4. Integral and Differential Fractional Operators

- Rieman-Liouville Forward Integral Fractional Operator J for $\alpha > 0$
  - $${}_{t_0}\overset{\alpha}{J}_{t}\, \mathrm{f}(t) = f^{<-\alpha>} = \frac{\int_{t_0}^{t} (t-t')^{\alpha-1} f(t)\, dt'}{\Gamma(\alpha)}$$
  - $$f^{<-\alpha>} = \sum_{j=1}^{\alpha} \frac{f^{<-1>}(t_0)(t-t_0)^{j-1}}{(j-1)!} +$$
    $$\int_{t_0}^{t} \int_{t_0}^{t'_{j-1}} \ldots.. \int_{t_0}^{t'_3} \int_{t_0}^{t'_2} f(t'_1)\, dt'_1\, dt'_2 \ldots dt'_j$$
- De Caputo Differential Right Hand Operator D for $(n \quad 1) \leq \alpha < n$, n is a positive integer
  - $${}_{t_0}\overset{\alpha}{D}_{t}\, \mathrm{f(t)} = f^{<\alpha>} = \frac{\int_{t_0}^{t} (t-t')^{n-\alpha-1} f^{<n>}(\mathrm{t})\, dt'}{\Gamma(n-\alpha)}$$
  - Where $f^{<n>}(\mathrm{t}) = \frac{d^n\, \mathrm{f}(t)}{\mathrm{d}t^n} \Rightarrow f^{<0>}(\mathrm{t}) = f(\mathrm{t})$
- Both are Linear Operators.

4. Integral and Differential Fractional Operators

$$\leftarrow \ldots \frac{d^{-3}f}{dt^{-3}}, \frac{d^{-2}f}{dt^{-2}}, \frac{d^{-1}f}{dt^{-1}}, f, \frac{df}{dt}, \frac{d^{2}f}{dt^{2}}, \frac{d^{3}f}{dt^{3}}, \ldots \rightarrow$$



Shantanu Das , Functional Fractional Calculus, Springer 2011

4. Integral and Differential Fractional Operators

Function $f(t)$	${}_{-\infty}D_t^{\alpha} f(t)$ Derivative
$H(t-a)$	$\begin{cases} \dfrac{(t-a)^{-\alpha}}{\Gamma(1-\alpha)}, (t>a) \\ 0, (t \le a) \end{cases}$
$H(t-a) f(t)$	$\begin{cases} {}_aD_t^{\alpha} f(t), (t>a) \\ 0, (t \le a) \end{cases}$
$e^{\lambda t}$	$\lambda^{\alpha} e^{\lambda t}$
$e^{\lambda t+\mu}$	$\lambda^{\alpha} e^{\lambda t+\mu}$
$\sin \lambda t$	$\lambda^{\alpha} \sin\left(\lambda t + \dfrac{\pi\alpha}{2}\right)$
$\cos \lambda t$	$\lambda^{\alpha} \cos\left(\lambda t + \dfrac{\pi\alpha}{2}\right)$
$e^{\lambda t} \sin \mu t$	$r^{\alpha} e^{\lambda t} \sin(\mu t + \alpha\phi)$ $r = \sqrt{\lambda^2 + \mu^2} \quad \tan\phi = \dfrac{\mu}{\lambda} \quad (\lambda, \mu > 0)$
$e^{\lambda t} \cos \mu t$	$r^{\alpha} e^{\lambda t} \cos(\mu t + \alpha\phi)$ $r = \sqrt{\lambda^2 + \mu^2} \quad \tan\phi = \dfrac{\mu}{\lambda} \quad (\lambda, \mu > 0)$

Shantanu Das , Functional Fractional Calculus, Springer 2011

5. Taxonomy of Electrical Elements

- From the definition of flux,charge,current and source and from the properties of the operators ;
  - $V^{<-\alpha>} = J^{\alpha}\, V(t) \qquad (\alpha > 0)$
  - $I^{<-\alpha>} = J^{\alpha}\, I(t) \qquad (\alpha > 0)$
  - $\varphi(\mathrm{t}) = \varphi(\mathrm{t}_0) + \underset{t_0 t}{\overset{1}{J}}\, V(t) = V^{<-1>}$
  - $\mathrm{q}(\mathrm{t}) = \mathrm{q}(\mathrm{t}_0) + \underset{t_0 t}{\overset{1}{J}}\, I(t) = I^{<-1>}$

5. Taxonomy of Electrical Elements

- 2-Terminal Lumped Elements are defined by the system of equations known as the constitutive realtion :

Assuming m-equations ,n-internal variables ;

$$f_1\left(V^{<-\gamma>},\ldots,V^{<-1>},V^{<0>},\ldots,V^{<\alpha>},I^{<-\delta>},\ldots,I^{<-1>},I^{<0>},\ldots,I^{<\beta>},X_1{}^{<K_1>},\ldots,X_l{}^{<K_l>},\ldots,X_n{}^{<K_n>},t\right)$$
$$f_2\left(V^{<-\gamma>},\ldots,V^{<-1>},V^{<0>},\ldots,V^{<\alpha>},I^{<-\delta>},\ldots,I^{<-1>},I^{<0>},\ldots,I^{<\beta>},X_1{}^{<K_1>},\ldots,X_l{}^{<K_l>},\ldots,X_n{}^{<K_n>},t\right)$$
$$\ldots\ldots$$
$$f_m\left(V^{<-\gamma>},\ldots,V^{<-1>},V^{<0>},\ldots,V^{<\alpha>},I^{<-\delta>},\ldots,I^{<-1>},I^{<0>},\ldots,I^{<\beta>},X_1{}^{<K_1>},\ldots,X_l{}^{<K_l>},\ldots,X_n{}^{<K_n>},t\right)$$

Where ;

V and I : terminal currents and voltages

X_j: Internal variables

α,β,γ,δ **:** Non Negative real numbers

K_j: Non negative real numbers for J <= l and non positive integer for j > l

5. Taxonomy of Electrical Elements

- PN-Junction Diode Higher Order Model (for including storage time, fall time,…) presented in Chua's Memristive Circuit model for PN-Junction Diodes :
  - $I = g_1(V, I, V^{<1>}, X_1, X_1^{<1>}, X_2^{<-1>})$
  - $X_1 = g_2(X_2^{<-1>})\left(I \quad g_3(X_2^{<-1>})\right)$
  - *M*=2, *N*=2 ,α= 1,β=γ =δ =0 ,*l*=*1*, $K_1 = \{0,1\}, K_2 = \{\ \ 1\}$

5. Taxonomy of Electrical Elements (Algebraic Elements)

- Resistors (0,0)
  - M=1, n=0, l=0, $\alpha = \beta = \gamma = \delta = 0$, f_1 is an algebraic equation.

    $f_1 = f_R(V, I, t) = 0$
- Capacitors (0,-1)
  - M=1, n=0, l=0, $\alpha = \gamma = 0, \delta = 1$, $\beta =$ omitted , f_1 is an algebraic equation.

    $f_1 = f_C(V, q = I^{<-1>}, t) = 0$

  By Definition, a voltage source E(t) can be classified as either a resistor or a capacitor, both with V=E(t) independent of the current/charge across its terminals.

  Similarily
- Inductors (-1,0)
  - M=1, n=0, l=0, $\beta = \delta = 0, \gamma = 1$, $\alpha =$ omitted , f_1 is an algebraic equation.

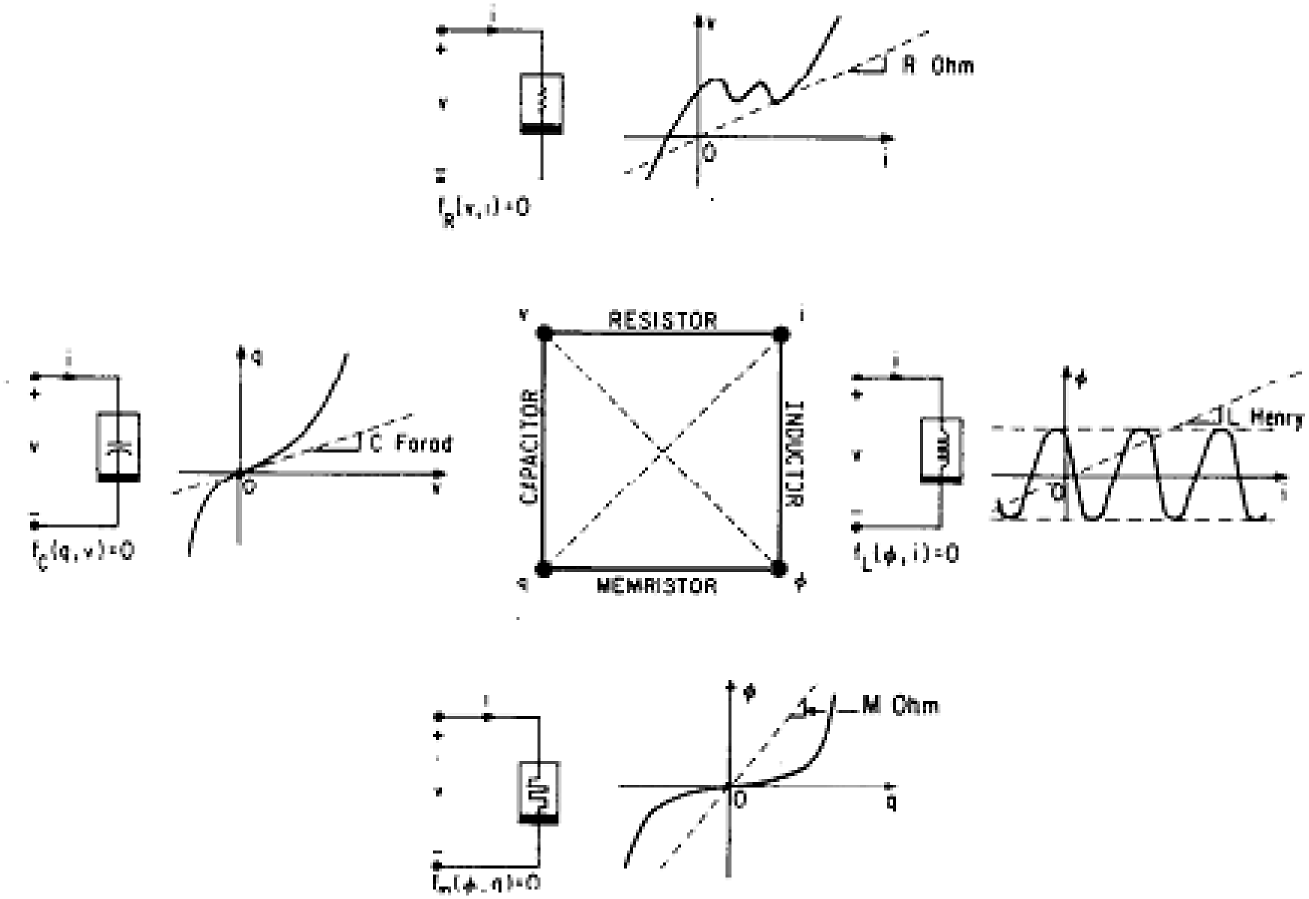
    $f_1 = f_L(I, \varphi = V^{<-1>}, t) = 0$

  By Definition, a current source $I_m(t)$ can be classified as either a resistor or a inductor, both with I= $I_m(t)$ independent of the voltage/flux across its terminals.
- Memristors (-1,-1)
  - M=1, n=0, l=0, $\gamma = \delta = 1$, $\alpha = \beta =$ omitted, f_1 is an algebraic equation.

    $f_1 = f_R(\varphi = V^{<-1>}, q = I^{<-1>}, t) = 0$

    This definition of memristor is different from HP Labs claimed memristor since their model utilizes an internal state variable x(t)

5. Taxonomy of Electrical Elements (Algebraic Elements)



6. Higher Integer and Fractional Order Elements

- Generalization into Fractional and Higher Order Algebraic Element (α ,β)
  - Characterized by the generalization of the previous CR for Algebraic function f ;

    $f(V^{<\alpha>}, I^{<\beta>}, t) = 0$
  - The locus of $(V(t)^{<\alpha>}, I(t)^{<\beta>})$ for an (α ,β) element is fixed and is independent of the input/test signal ccs.
  - The higher integer-order elements share similar properties in small-signal behavior the same order as it will be demonstrated in the next section.
  - For Multiport P-Port case, the scalar function f , the scalar variables V(t) and I(t) , the scalar parameters α and β are vectored to a Px1 vector.
- $Input^{<x>}$ Controlled Element
  - An element is considered $Input^{<x>}$ Controlled element if its CR is in the form of

    $Output^{<y>} = f_{Signle-Valued}(Input^{<x>})$
  - Example

    $q(t) = CV^2(t)$ is a voltage-controlled nonlinear Capacitor (0,-1) .

    $V(t) = \frac{1}{C} q^{0.5}(t)$ is not a charge-controlled nonlinear capacitor (0,-1).

    $q(t) = C(V(t)^{< \ 0.75>})^2$ is a fractional (-0.75) voltage controlled nonlinear Fractional Capacitor (Resisto Capacitor) (-0.75,-1) element.

    $I(t)^{<-0.5>} = C(V(t)^{< \ 0.75>})^2$ is a fractional (-0.75) voltage controlled nonlinear (-0.75,-0.5) element.
- Equivalence of Linear Time Invariant Forms of Fractional and Higher Order Algebraic Element (α ,β)
  - All LTI (α ,β) elements i.e. $f(V^{<\alpha>}, I^{<\beta>}, t) = a\,V^{<\alpha>} + b\,I^{<\beta>} = 0$ are equivalent and identical to each other due to the linearity and the inversibility of the fractional operators.

    LTI Memristor (-1,-1) : $V^{< \ 1>} = M\,I^{< \ 1>}$, by applying the differential operator D then $V^{<0>} = M\,I^{<0>}$ is equivalent to a Linear Resistor with Resistance M.
  - This is one of the points which mandates the study of nonlinear circuit theory in order to fully utilize the concepts of the higher order and fractional elements.

6. Higher Integer and Fractional Order Elements

- Complementary and Admissible Signal Pairs
  - Complementary : The set of possible values of $(V(t)^{<\alpha>}, I(t)^{<\beta>})$
  - Admissible : The set of possible (V(t),I(t)) admissible on the terminals of an element.

    Example : Short Circuit (V=0)

    Any I is admissible, but only V=0 is admissible.
  - Generally for an algebraic element (α ,β), only the locus of the complementary signal pairs will fully describe its properties (i.e. applying a known V will yield a fixed known value(s) of I that are independent of the waveform of V.

    Example : C (0,-1)

    The locus (V(t),I(t)) depends on the shape of the testing signal V(t), while the locus of $(V(t), I(t)^{< \ 1>})$ will remain fixed and independent of V(t), so it fully characterizes the capacitor , so the CR of a capacitor(0,-1) is $f(V, I^{< \ 1>}$, t) = 0

7. Fractional Laplace Transform

- As a generalization of its integer counterparts, Laplace transform of the fractional Caputo Differential and RL Integral operators are defined by ;

- $$\left({}_{t_0}\overset{\alpha}{D}{}_{t} f(t)\right) = s^{\alpha} F(s) \quad \sum_{k=1}^{N} s^{\alpha-k} \frac{d^{k-1} f(0^{+})}{dt^{k-1}}$$

  $(n \quad 1) < \alpha < n \;\&\; n \in Z^{+}$

- $$\left({}_{t_0}\overset{\alpha}{J}{}_{t} f(t)\right) = s^{-\alpha} F(s)$$

7. Fractional Laplace Transform

- In this table, the fractional elements act as hybrids between the integer order elements
- For example
  - (0,-1) is an integer capacitor C (0,-1) is a capacitor with γ=1 and CFD small signal CCs.
  - (0,0) is an integer resistor R (0,0) is a resistor with γ=0 and PFDR small signal CCs.
  - (-1,-1) is a memristor with γ= 0 and PFDR small signal CCs
  - (-1,0) is a inductor with γ=1 and CFD small signal CCs.

  - (0,0< β <-1) is a Fractional capacitor with γ= β and hybrid C-R CCs that exists between the R and C (between the γ=**0** and $\gamma = 1$ axis, so it exhibits both properties depending on the value of the value of the operator power a .
  - The small signal impedance in that case will have both terms of Positive Resistive and Capacitive frequency dependent impedance.
  - $Z_Q(jw) = m_Q(jw)^{\beta}$

    $R(w) = m_Q w^{\beta}(\,Cos(\beta\pi/2))$

    $X(w) = m_Q w^{\beta}(\,Sin(\beta\pi/2))$

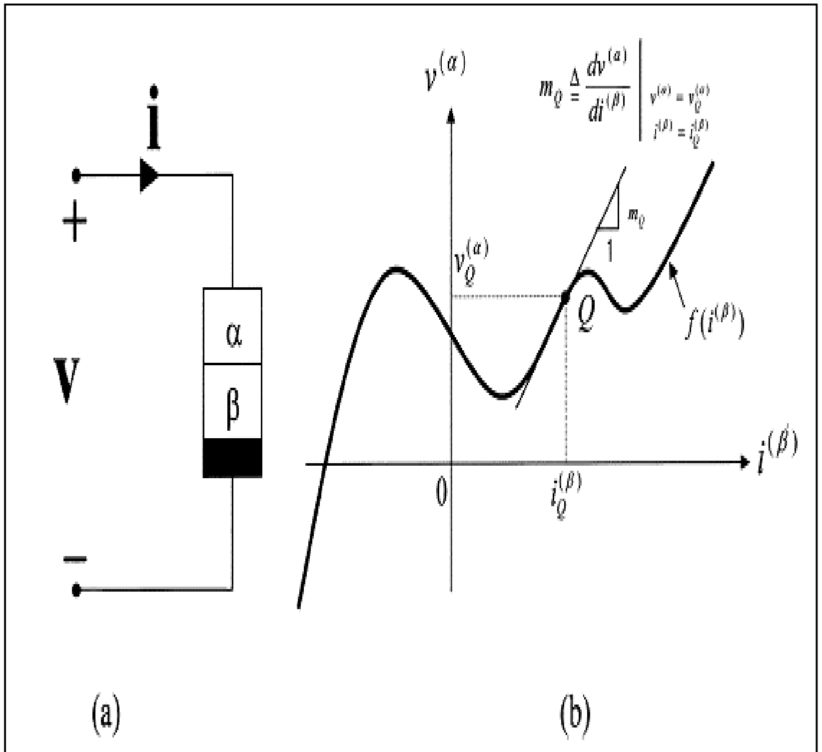
  - (0<α<-1,0) similarily is a Fractional Inductor with γ=- α and hybrid L-R CCs.

$$Z(s) = \frac{V(s)}{I(s)} = Ks^{\beta-\alpha} = Ks^{\gamma}$$

α	β	Element Type
0	0	Resistor
-1	0	Inductor
0	-1	Capacitor
-1	-1	Memristor

7. Fractional Laplace Transform (Small Signal Model)

- Algebraic Element {**α , β** } has Q-point at($V_Q^{<\alpha>}$, $I_Q^{<\beta>}$), assuming 2-components one for large signal Q and the other for the small signal q.
- For a current-controlled element $V^{<\alpha>} = f(\,I_Q^{<\beta>})$, if f is differentiable at Q, then from Taylor's expansion around Q ;
  - $V^{<\alpha>} = f\left(I^{<\beta>}\right) = V_Q^{<\alpha>} + V_q^{<\alpha>}$
    $I_Q^{<\beta>} + m_Q\left(I^{<\beta>} \quad I_Q^{<\beta>}\right) = I_Q^{<\beta>} + m_Q\, I_q^{<\beta>}$
    $m_Q = \frac{d\,V^{<\alpha>}}{d\,I^{<\beta>}}\Big|_{(V_Q^{<\alpha>},\ I_Q^{<\beta>})}$ *Slope at Q Point*
- Then the small signal components are re related by ;
  - $V_q^{<\alpha>} = m_Q I_q^{<\beta>}$
- Taking Laplace Transform
  - $s^{\alpha} V_q\ (s) = m_Q s^{\beta} I_q\ (s)$



CR of an algebraic element ,Leon Chua, Nonlinear Circuit Foundations for Nanodevices : Part 1 the four-element torus

7. Fractional Laplace Transform (Small Signal Model)

- Small Signal Impedance at Q
  - $Z_Q(s) = \frac{V_q(s)}{I_q(s)} = m_Q s^{\beta-\alpha}$

    β α= γ the order of the element.
  - $Z_Q(jw) = m_Q(jw)^{\beta-\alpha} = m_Q w^{\beta-\alpha} j^{\beta-\alpha} = m_Q w^{\beta-\alpha} e^{\frac{j(\beta-\alpha)\pi}{2}} = m_Q w^{\gamma}(\,Cos(\gamma\pi/2) + j\,Sin(\gamma\pi/2) = R(w) + j\,X(w)$
  - $R(w) = m_Q w^{\gamma}(\,Cos(\gamma\pi/2))$
  - $X(w) = m_Q w^{\gamma}(\,Sin(\gamma\pi/2))$
- For voltage-controlled elements, the derivation is similar to get the small signal conductance.
- Frequency Response of Current-Controlled Elements
  - According to the value of γ =β α , all the 2-terminal algebraic electrical elements share the same behavior.

7. Fractional Laplace Transform (Small Signal Model)

- 1) Positive Frequency Dependent Resistors (m_Q>0)
  - $Z_Q(jw) = m_Q(jw)^\gamma = R(w) > 0$
  - $R(w) = m_Q w^\gamma(\ Cos(\gamma\pi/2)) > 0$
  - $X(w) = 0 = m_Q w^\gamma(\ Sin(\gamma\pi/2))$

    γ =± 2n , n = even integer

    γ = ..,-8,-4,0,4,8,..
- 2) Negative Frequency Dependent Resistors(m_Q>0)
  - $Z_Q(jw) = m_Q(jw)^\gamma = R(w) < 0$
  - $R(w) = m_Q w^\gamma(\ Cos(\gamma\pi/2)) < 0$
  - $X(w) = 0 = m_Q w^\gamma(\ Sin(\gamma\pi/2))$

    γ =± 2n , n = odd integer

    γ = ..,-6,-2,2,6,10,..

7. Fractional Laplace Transform (Small Signal Model)

- 3) Frequency Dependent Inductor (m_Q>0)
  - $Z_Q(jw) = m_Q(jw)^{\gamma} = j\,X(w) = jw\,L(w)$
  - $R(w) = m_Q w^{\gamma}(\,Cos(\gamma\pi/2)) = 0$
  - $X(w) = w\,L(w) = m_Q w^{\gamma}(\,Sin(\gamma\pi/2)) > 0$

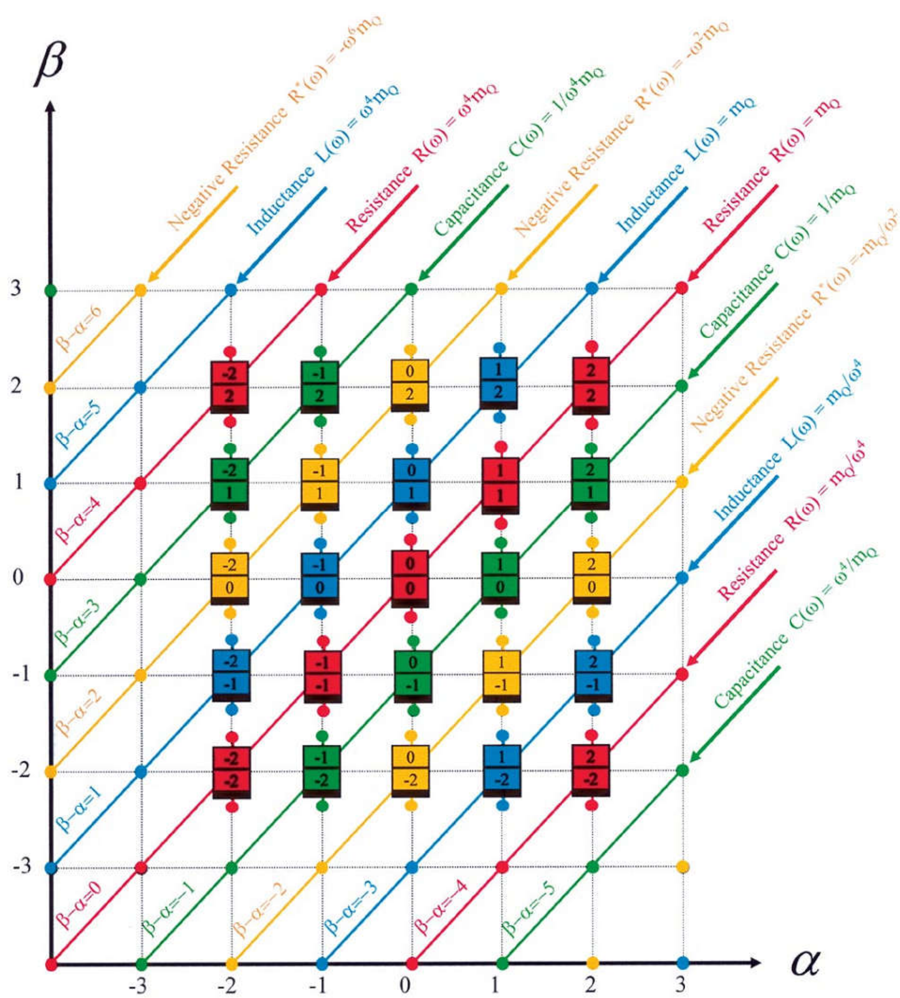
    $\gamma = (\quad 1)^{n}(2n+1)$, n = any integer.

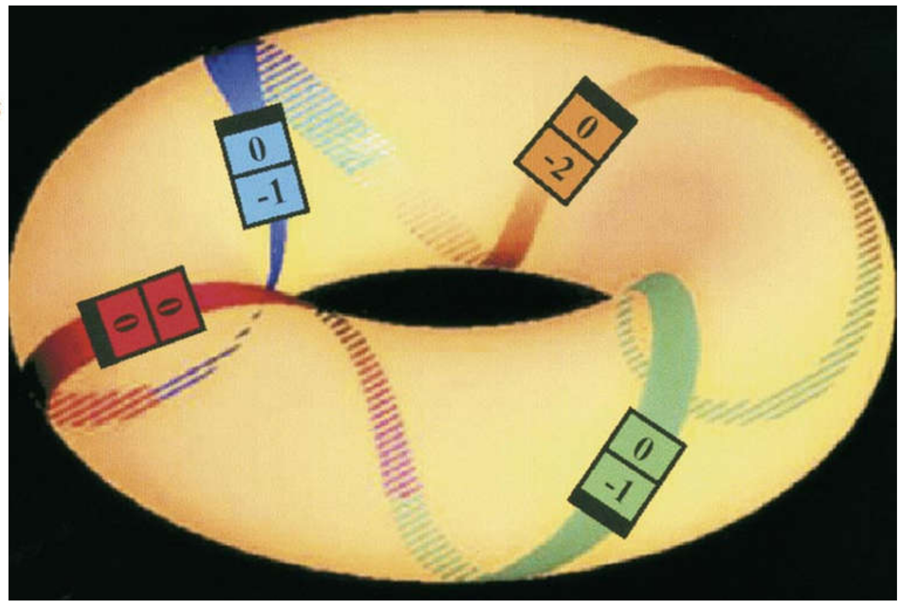
    γ =.., 3,1,5,6, ...
- 4) Frequency Dependent Capacitor (m_Q>0)
  - $Z_Q(jw) = m_Q(jw)^{\gamma} = j\,X(w) = \quad j\frac{1}{w\,C(w)}$
  - $R(w) = m_Q w^{\gamma}(\,Cos(\gamma\pi/2)) = 0$
  - $X(w) = \frac{1}{w\,C(w)} = m_Q w^{\beta-\alpha}(\,Sin(\gamma\pi/2)) > 0$

    $\gamma = (\quad 1)^{n+1}(2n+1)$, n = any integer.

    γ =.., 5, 1,3,7, ...

7. Fractional Laplace Transform (Small Signal Model)





Extensions from the classic RLC elements to a periodic table of elements. Leon Chua 2003, Nonlinear Circuit Foundations for Nanodevices, Part 1 The four element Torus

7. Fractional Laplace Transform (Small Signal Model) : Modeling Example

- To model the nonlinear frequency response of an element with Integer elements ;
  - PFDR : $m\,w^{\gamma}\ (m > 0, \gamma = \quad , \quad 4,0,4, \ldots)$
  - NFDR: $m\,w^{\gamma}(m > 0, \gamma = \quad , \quad 2,2,6, \ldots)$
  - L: γ=-7,-3,1,5,9

    j $m\,w^{2n+1}(m > 0, n\ even)$

    j $m\,w^{-(2n+1)}(m > 0\ , n\ odd)$
  - C: γ=,-9,-5,-1,3,7

    -j $m\,w^{-(2n+1)}(m > 0, n\ even)$

    -j $m\,w^{2n+1}(m > 0, n\ odd)$
- Some of the elements that can not be modeled directly are the elements with even powers of w.

7. Fractional Laplace Transform (Small Signal Model) : Modeling Example

- Using Series connections, model $Z(w) = j(\, aw \quad bw^3 + cw^5), (a, b, c > 0)$
  - $jaw \;\rightarrow L \;\; wit \;\; n = 0 \; or \; n = \quad 1 \;\rightarrow \gamma = 1$
  - $\quad jbw^3 \rightarrow C \;\; wit \;\; n = \quad 2 \; or \; n = 1 \;\rightarrow \gamma = 3$
  - $jcw^5 \rightarrow L \;\; wit \;\; n = 2 \; or \, .. \rightarrow \gamma = 5$
- So any series combination of elements chosen from the diagonal axes $\gamma = 1{,}3{,}5$ will adequately model Z(w) frequency response near the Q-Point.
- However no series combination of integer order elements will model the element with $Z(w) = j(a + bw^2 \quad cw^4)$ using Integer-order elements
  - $ja \;\rightarrow L \;\; wit \;\; n = \quad 0.5 \;\rightarrow \gamma = 0 \; but \; in \; t \;\; at \; case \; ja \rightarrow R$

8. Fractional Capacitor

- Fractional Capacitor (Ivo Petras, Fractional Order Nonlinear Systems, Modeling and Analysis 2011)
  - Described by the lumped element representation
    M=1, n=0, l=0, $\alpha = \beta = \gamma = 0,$ 0< δ <1
  - CR of Lumped : $f_1 = f_C\left(V, I^{<-\delta>}, t\right) = 0$
  - As an algebraic element ;

    C(0, $1 > \beta > 0) \Rightarrow I(t) = 1/K\, {}_{0}D_{t}^{-\beta} V(t)$ or $V(t) = K\, {}_{0}J_{t}^{-\beta} I(t)$ $at\ zeroed\ initial\ conditions$

    β=0⇒ K= R[ohm] (Purely Resistive)
    β=-1⇒ K=1/ C [Farad^{-1}](Purely Capacitive)

    Example : C(0,-0.5) is described by ;
    $f_C(V, I^{<-0.5>}, t)$ =0
    Linear Time-invariant Integral
    - $I(t)^{<-0.5>} = C\, V(t) = \frac{\int_0^t (t-t`)^{-0.5} I(t)\, dt`}{\Gamma(0.5)}$

    The differential form ;
    - $I(t) = CV(t)^{<0.5>} = C\,\frac{d^{0.5}V(t)}{dt^{0.5}} = \frac{C\int_0^t (t-t`)^{-0.5}\, \frac{dV(t`)}{dt`}\, dt`}{\Gamma(0.5)}$

  - The fractional capacitor model is the accurate model for ultracapacitors impedance in low and medium frequencies and trnsients, which are high energy storage devices used in photovoltaicsystems, biomedical sensors and energy harvesting (Measurement of Supercapacitor Fractional Oder Model Parameters, Todd J. Freeborn).

8. Fractional Capacitor

- Step Response
  - $V(t)=U(t)$,from the table ;

$$ {}_{0}D_{t}^{\beta} U(t) = \frac{t^{-\beta}}{\Gamma(1-\beta)} $$

Then

$$ I(t) = K^{-1}\, {}_{0}D_{t}^{-\beta} V(t) = K^{-1} \frac{t^{\beta}}{\Gamma(1+\beta)} \left(\ 1 < \beta < 0 \right) $$

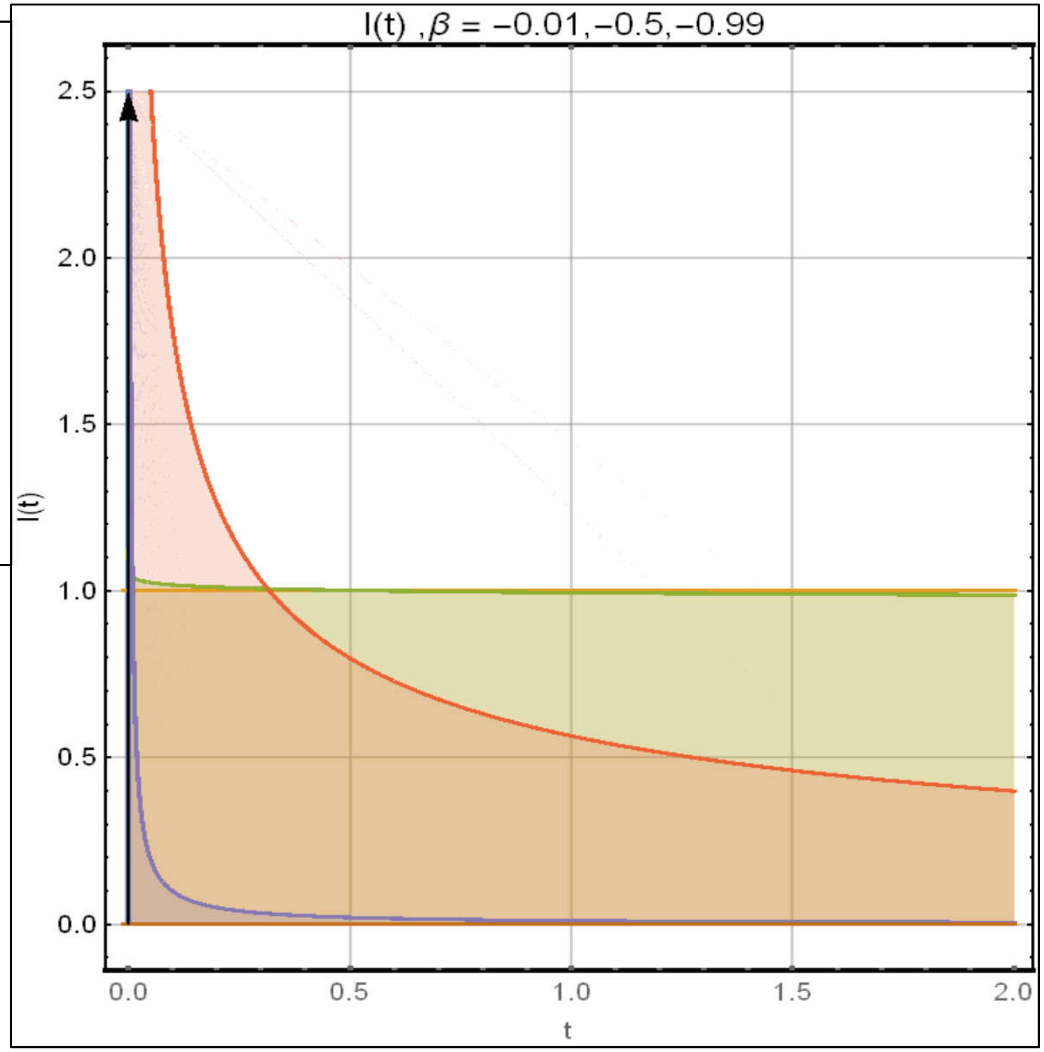
$$ I(t) = K^{-1}\, {}_{0}D_{t}^{1} V(t) = K^{-1} \delta(t) \qquad (\beta = \ 1 \rightarrow Capacitive) $$

$$ I(t) = K^{-1}\, {}_{0}D_{t}^{0} V(t) = K^{-1} U(t) \qquad (\beta = 0 \ \rightarrow Resistive) $$

8. Fractional Capacitor

I(t) Step Response with β = **0,-0.01,-0.5,-0.99,-1 and K=1**

-As $\beta = 0$, V(t) = I(t) so the step response is a unit function , as α increases , the element tends to exhibit hybrid behavior between R and C, till β **= -1 ,** at this point the step response becomes the impulse function.



8. Fractional Capacitor (Frequency Response)

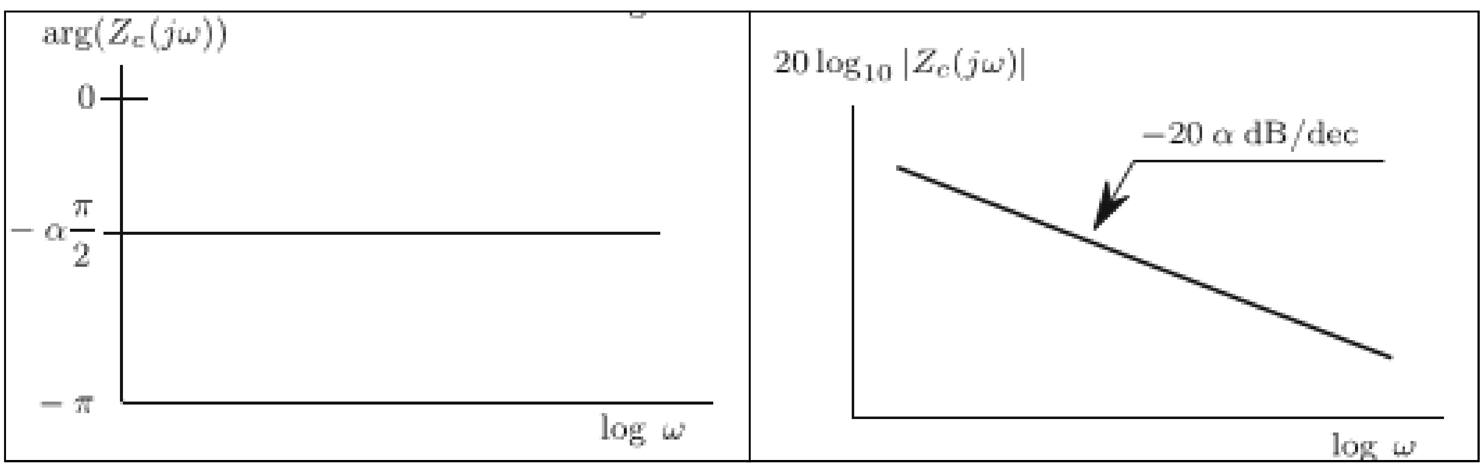
- Applying Laplace transform to $I(t) = K^{-1} {}_0D_t^{-\beta} V(t)$

$$I(s) = K^{-1}s^{-\beta}V(s) \rightarrow Z(s) = \frac{V(s)}{I(s)} = Ks^{\beta}$$

$$Z(jw) = K(jw)^{\beta} = Kw^{\beta}e^{j\frac{\beta\pi}{2}}$$, let K=1

$$20\ Log\left(\left|w^{\beta}e^{j\frac{\beta\pi}{2}}\right|\right) = \beta\ (20\ Log\ w\ )\ \text{and}\ Arg\left(w^{\beta}e^{j\frac{\beta\pi}{2}}\right) = \frac{\beta\pi}{2}$$

  - Since $\beta \epsilon\ (0,\quad 1)$, if we let β=-α, then the bode plot is characterized by a gain (-ve gain -> Loss) of -20 α per decade , and $\frac{-\alpha\,\pi}{2}$ fixed phase.



8. Fractional Capacitor (Frequency and Transient Response)

- Steady State Sinusoidal Response
  - $V(t) = Sin\ (w_i t)$, from the properties of fractional derivation of Sinusoidal wave ;
  - $I(t) = {}_0D_t^{-\beta} Sin\ (w_i t) = \dfrac{t^{1+\beta}\, w\ {}_1F_2\left[1, \frac{2+\beta}{2}, \frac{3+\beta}{2}, -\frac{t^2 w^2}{4}\right]}{\Gamma[2+\beta]}$

    ${}_1F_2[a, b, c; t]$ is the 1-2 HyperGeometric generalized function.
- Frequency Response

  $V(jw) = \quad \pi j(\sigma(w \quad w_i) \quad \sigma(w + w_i))$Then

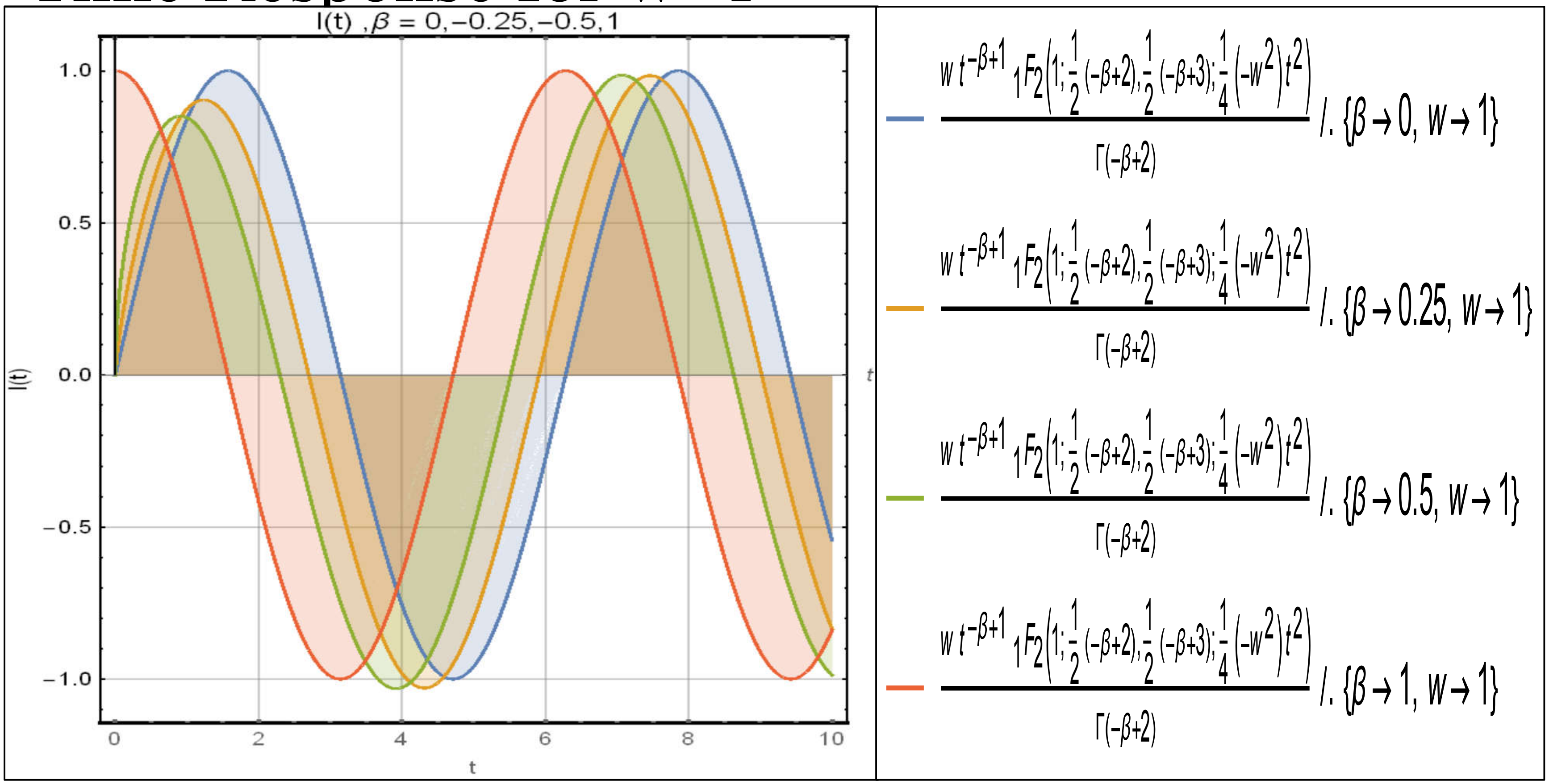
  $I(jw) = \frac{V(jw)}{Z(jw)} = \pi w^{\beta}(\sigma(w \quad w_i) \quad \sigma(w + w_i))e^{j\frac{(\beta-1)\pi}{2}}$

  Magnitude at w = log w_i : 20 Log$\left(\pi w^{\beta}\right)$ = 20 Log(π) + 20 β (Log w) = 9.95 + 20 β (Log w)

  Phase at w = log w_i : $\frac{(\beta-1)\pi}{2}$

8. Fractional Capacitor (Transient Response)

- Time Response for w=1



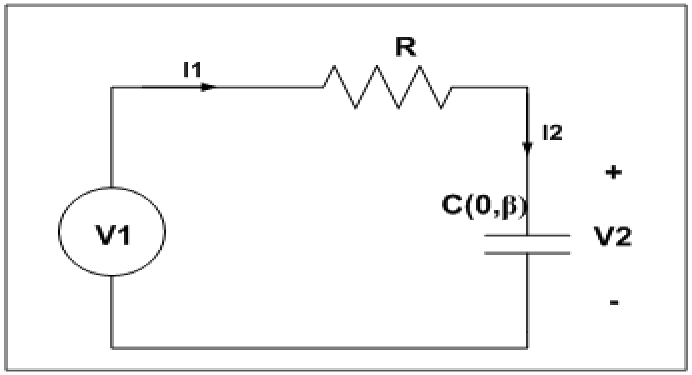
9. LTI Fractional Systems

- Using the classical RLC elements in circuit analysis, the output of the circuit analysis techniques such as ; Loop analysis, mesh analysis, Modified Nodal Analaysis, Tableau Analysis , all lead to a system of Differential Algebraic Equations.
- The generalization of electrical elements leads to the introduction of Fractional Differential Equation and Fractional Differential Algebraic Equations in the field of circuit analysis.
- The General form of a Fractional DAE is ;
  - $D^{\boldsymbol{q}}\boldsymbol{X} = \boldsymbol{f}(\boldsymbol{X}) \Rightarrow \boldsymbol{g}(\boldsymbol{X}, D^{\boldsymbol{q}}\boldsymbol{X}, \boldsymbol{h}(t), t)$

    $\boldsymbol{q} = [q_1, q_2, q_3, .. q_n]^T, \boldsymbol{f} = [f_1, f_2, ... f_m], \boldsymbol{X} = [X_1(t), ... X_n(t)] \epsilon R^n$
- The LTI Fractional Systems are expressed by ;
  - $\sum_{l=0}^{N} a_l D^{\alpha_l} y(t) = \sum_{k=0}^{M} b_k D^{\beta_k} x(t)$
- The state-space representation of LTI Fractional systems is in the form of ;

9. LTI Fractional Systems : Example – Generalized Series RC Circuit

- Time-Domain
  - $V_1(t) = I_1(t)R + V_2(t)$
  - $I_2(t) = I_1(t)\ (t) = C\ {}_0D_t^{-\beta} V_2(t)$
  - $V_1(t) = RC\ {}_0D_t^{-\beta} V_2(t) + V_2(t)$



The solution of this fractional linear ODE is given by Kaczorek 2008 ;

$$V_2(t) = \int_0^t \sum_{k=0}^{\infty} \frac{(-1)^{-k}(\mathrm{RC})^{-k-1}(t-\tau)^{-\ \beta(k+1)-1}}{\Gamma(-\ \beta(k+1))} V_1(t)\, d\tau$$

- Laplace Transform

$$V_1(s) = \left(RC\ s^{-\beta} + 1\right)V_2(s) \rightarrow V_2(s) = \frac{\frac{V_1(s)}{RC}}{\left(s^{-\beta}+\frac{1}{RC}\right)} \rightarrow V_2(t) = \quad {}^{-1}\left[\frac{\frac{V_1(s)}{RC}}{\left(s^{-\beta}+\frac{1}{RC}\right)}\right] = \frac{1}{RC}(V_1(t)$$

$$t^{\boldsymbol{\beta}} E_{-\boldsymbol{\beta},-\boldsymbol{\beta}}(\ \ (RC)^{-1}t^{-\boldsymbol{\beta}})$$

$E_{a,b}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\mathrm{ak}+b)}$ is known as Mittage Leffler Function (a,b).

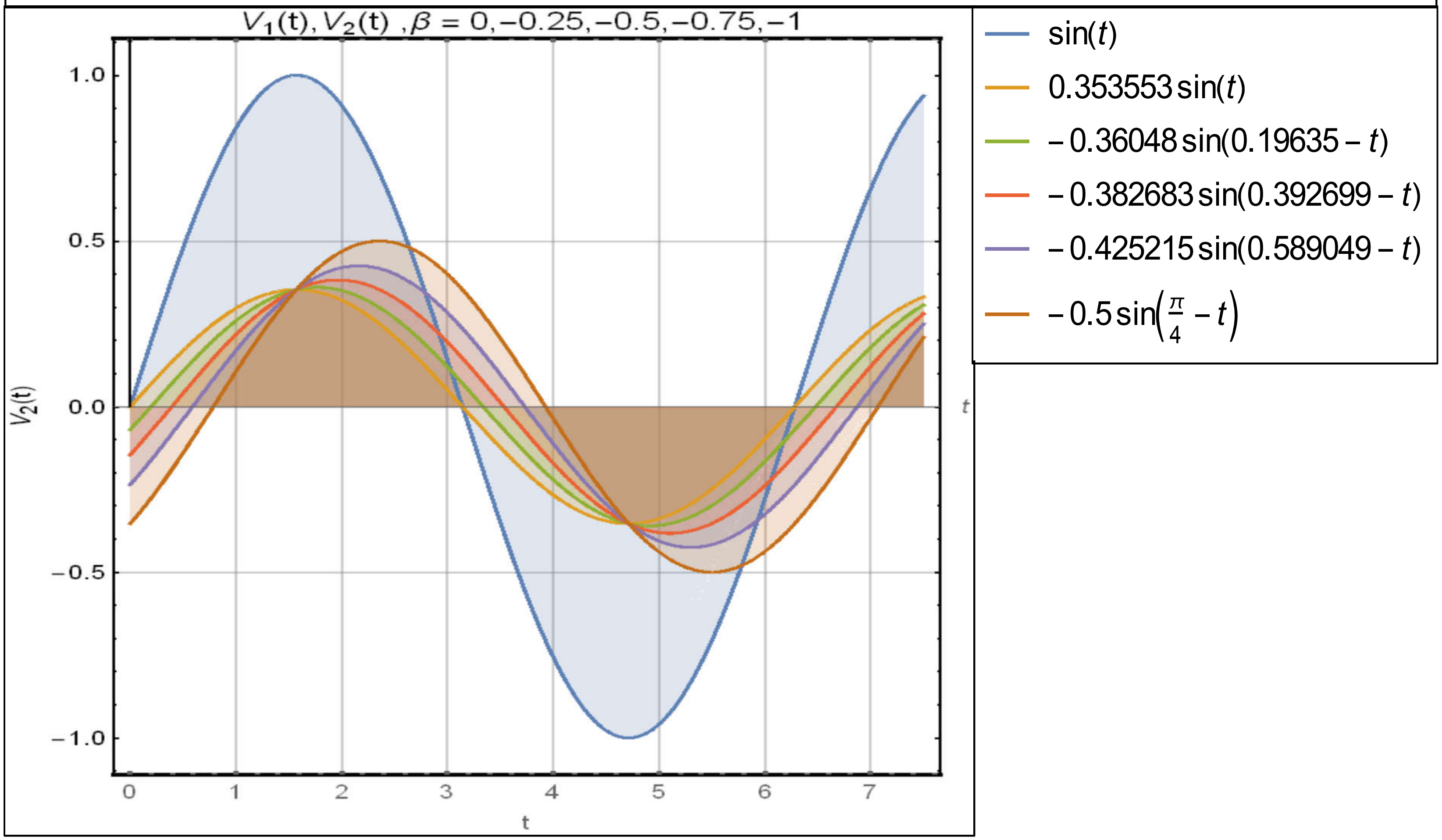
0

9. LTI Fractional Systems : Example – Generalized Series RC Circuit

- Steady State Sinusoidal Response
  - $Z(jw) = R + (jw)^{\beta}C = R + Cw^{\beta}e^{\frac{j\beta\pi}{2}} = R + Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right) + jCw^{\beta}Sin\left(\frac{\beta\pi}{2}\right) = \sqrt{(R + Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right))^2 + \left(Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)\right)^2}\; e^{j\,tan^{-1}\left(\frac{Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)}{R+Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right)}\right)}$
  - $I_1(jw) = \frac{V_1(jw)}{Z(jw)}$
  - $V_2(jw) = \frac{V_1(jw)}{Z(jw)}Z_2(jw) = \frac{V_1(jw)}{\sqrt{(R+Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right))^2+\left(Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)\right)^2}}Cw^{\beta}e^{j\left(\frac{\beta\pi}{2}-tan^{-1}\left(\frac{Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)}{R+Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right)}\right)\right)}$
  - For $V_1(t) = Sin\,(wt)$, $V_2(t) = \frac{Cw^{\beta}}{\sqrt{(R+Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right))^2+\left(Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)\right)^2}}Sin\left(wt + \left(\frac{(\beta)\pi}{2} - tan^{-1}\left(\frac{Cw^{\beta}Sin\left(\frac{\beta\pi}{2}\right)}{R+Cw^{\beta}Cos\left(\frac{\beta\pi}{2}\right)}\right)\right)\right)$

9. LTI Fractional Systems : Example – Generalized Series RC Circuit

Numerical Example $C = 1, R = 1, w = 1, \beta$=-0.5



10.Stability for Rational Linear Fractional DE

- The CCs equation of a Linear Fractional DE is in the form of ;
  - $\sum_{j=0}^{N} a_j s^{\alpha_j} = 0$

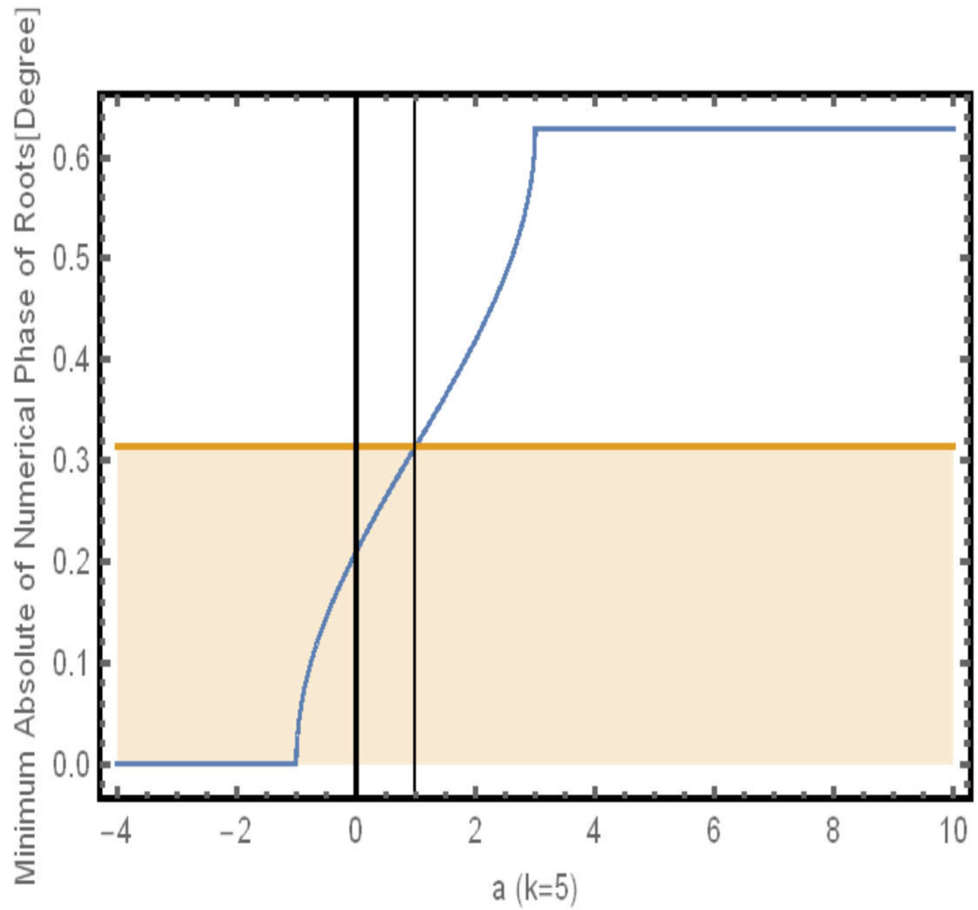
    For a rational system ; $\alpha_j = \frac{j}{m}$ and $m \in Z$
  - Making $\sum_{j=0}^{N} a_j s^{\frac{j}{m}} = \sum_{j=0}^{N} a_j W^j$ where $W = s^{\frac{1}{m}}$
  - Thus converting the equation into a polynomial in W , an algorithm is listed in (On the Stability of linear systems with fractional-order elements ; A.G. Radwan)
    1) Calculate the complex roots of W in W-domain ,$W_i = |W_i| e^{\theta_i}$
    2) Find the minimum absolute function of the phases Min $[|\theta_i|]$
    3) Stability Criterion : Min $[|\theta_i|] > \frac{\pi}{2m}$
    4) Oscillation Criterion : Min $[|\theta_i|] = \frac{\pi}{2m}$
    5) Unstability Criterion : Min $[|\theta_i|] < \frac{\pi}{2m}$
    6) Roots can be translated back to s-domain by applying the inverse transformation $s = W^m$ on all W roots in the region $|\theta_i| < \frac{\pi}{m}$

10.Stability for Rational Linear Fractional DE

- Example :
  - $D^2 f + a\, Df \quad D^{\frac{k}{5}} + D^{\frac{1}{5}} f + f = g(t)$
  - CCs : $s^2 + as \quad s^{\frac{k}{5}} + s^{\frac{1}{5}} + 1 = 0$
  - Transform $W = s^{1/5} \Rightarrow s = W^5$
  - Then : $W^{10} + aW^5 \quad W^k + W + 1 = 0$
  - Oscillation Condition : $\text{Min}\left[\text{Abs}[\theta_i]\right] = \frac{\pi}{2\ \ 5}$
  - Solved Numerically to extract 10 complex roots, by getting the absolute phase of each root and plotting the minimum ;

10.Stability for Rational Linear Fractional DE

- The plot is for the minimum absolute phase of roots, as a function of a with K=5
- The critical phase = $\pi/10$.
- The oscillation condition is the intersection of both plots (i.e. a   1)
- The shaded region is the unstable region (i.e. any value of a<1 will model an unstable system)
- The stable region is the upper region (i.e. any value of a>1 will model a stable system)



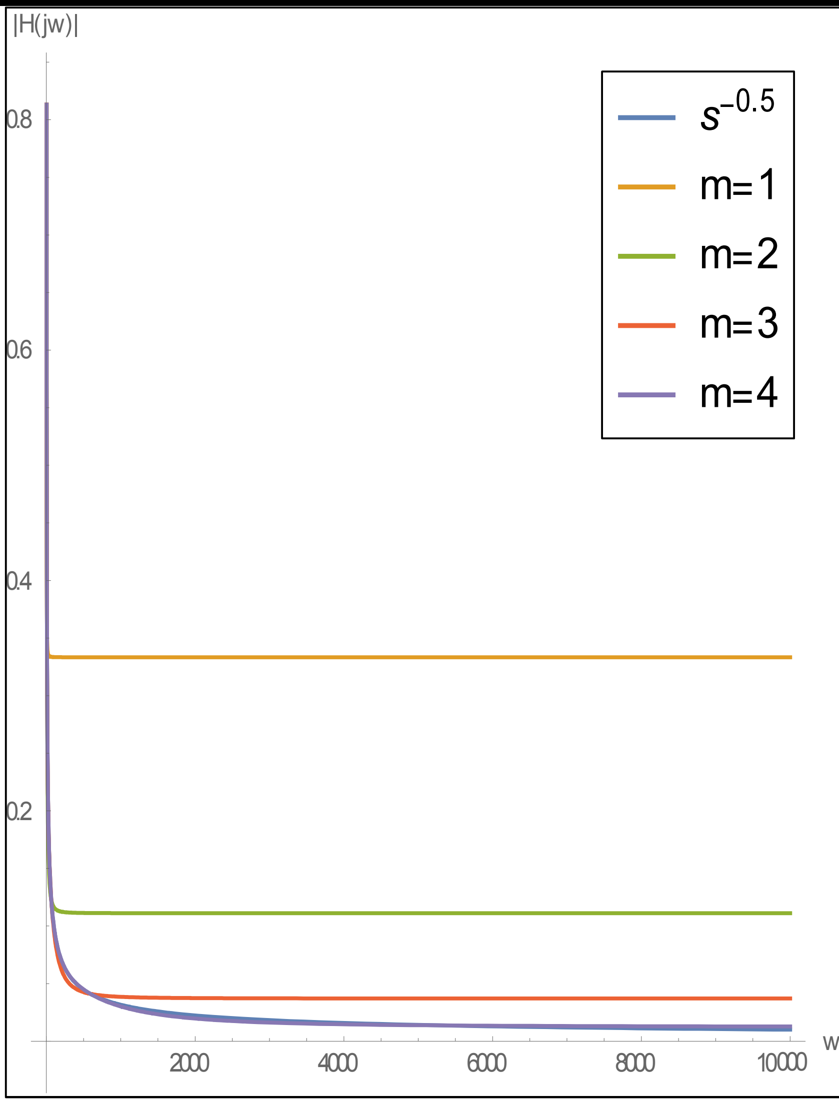
11. Synthesis/Realization of Fractional Order Elements

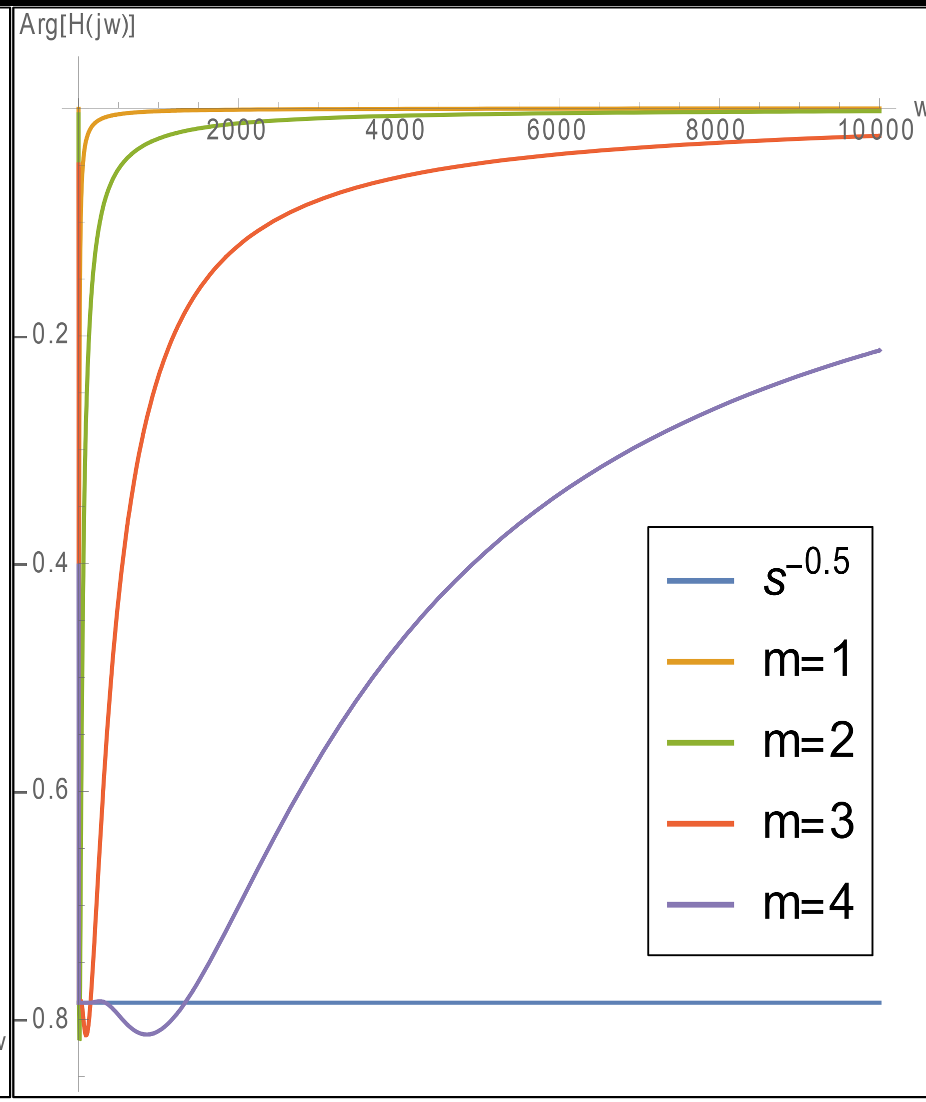
- Comparison in (Comparative Study for Various Fractional Order System Realization Methods ; Sandip A Mehta)
- As the general form of
- Using approximation methods for continuous time will be considered
  - Partial Fraction Expansion of Passive Elements
  - Self Similar Trees
  - Oustaloup Recursive Approximation
  - Modified oustaloup Recursive Approximation
  - Matsuda's Method
  - Carlson's Method
  - High Frequency Continued Fraction Expansion
  - Low-Frequency Continued Fraction Expansion
- Based on the results of this paper, both Oustaloup and Modified oustaloup methods gave the best performances and closer resemblance to the theoretical characteristics, but both provide good performance for a range of frequencies

11. Synthesis/Realization of Fractional Order Elements (Carlson's Method)

- Presented in (Approximation of Fractional Capacitors $(1/s)^n$ by a Regular Newton Process ; Carlson and Halijak)
- Used For realization a fractional element in the s-domain with Ccs ; $s^{1/n}$ $n \in Z^+$ i.e. (Fractional Capcitors) for low-frequency.
- The algorithm is summarized as ;
  - $X_{<m+1>} = F(X_{<m>}) = X_{<m>} \frac{(n-1)X_{<m>}+(n+1)s^{-1}}{(n+1)X_{<m>}+(n-1)s^{-1}}$ $and\ X_0 > 0$
  - For m > 2 , the model is bulky and unrealizable, so the common realizations are fo m=1,2 while m>2 can be used for modeling .
  - As the frequency increases, the error margin in phase increases dramatically even for higher order iterations.

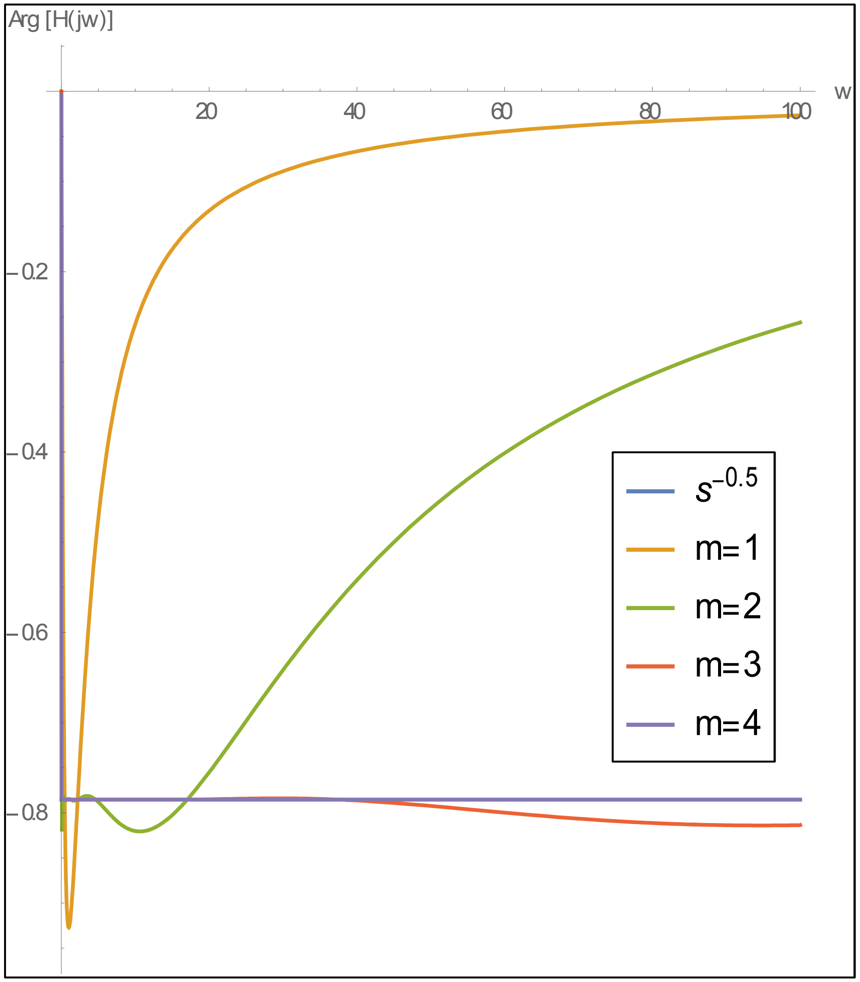
11. Synthesis/Realization of Fractional Order Elements (Carlson's Method)





11. Synthesis/Realization of Fractional Order Elements (Carlson's Method)

- $H_1(s) = \frac{s+3}{3s+1} = \frac{1}{3} + \frac{1}{\frac{3}{8}+\frac{9}{8}s}$
  - Achievable with a resistance 0.3 ohm in parallel with a series resistance 0.375 ohm and Capacitor with 1.125 Farad.
- $H_2(s) = \frac{9+84s+126s^2+36s^3+s^4}{1+36s+126s^2+84s^3+9s^4} = \frac{1}{9} + \frac{8}{9(1+3s)} + \frac{8(3+10s+3s^2)}{3(1+33s+27s^2+3s^3)}$
  - Achievable with a ladder network of R, L and C components.



12. Student Progress (Covered Topics and References)

- 1. Nonlinear Circuits Analysis :
  - Leon Chua, Charles A. Desoer : Linear and Nonlinear Circuits 1987.

    Leon Chua, Multiple papers from literature including;

      Device Modeling via Basic Nonlinear Circuit Elements

      Nonlinear Circuit Foundations for Nanodevices

      A theory of Algebraic N-Ports

11. Student Progress (Covered Topics and References)

- 2. Memristive Systems
  - Leon Chua, Multiple papers from literature including;
    - Memristor the missing circuit element.
    - Circuit Elements with Memory : Memristors, Memcapacitors and Meminductors.
    - Memristive Devices and Systems
    - What are Memristor, Memcapacitor and Meminductor
  - Dimitri Strukov, Gregory Snider, Duncan Stewart
    - The missing memristor found
  - Williams ; The missing memristor found
  - Ram Kaji, Maheshwar Pd Sah ; Composite Memristance of Parallel and Serial Memristor Circuits.
  - Yogesh N Joglekar, Stephen J. Wolf ; The elusive Memristor
  - Drakakis, Yaliraki ; Memristor and Bernoulli's Dynamics

11. Student Progress (Covered Topics and References)

- 3. Fractional Calculus (Multiple topics)
  - Ivo Petras ; Fractiona-Order Circuit Elements with Memory.
  - Ivo Petras ; Fractiona-Order Nonlinear Systems : Modeling, Analysis and Simulation.
  - Uchailkin V. ; Fractional Derivatives for Physicists and Engineers, Vol 1 Background and Theory.
  - Kenneth Miller, Bertram Ross ; Fractional Calculus and Fractional Differential Equations.
  - Ignor Podlubny, Kenneth Thimann ; Fractional Differential Equations.

11. Student Progress (Covered Topics and References)

- 4. Fractional Elements

  Mohamed E. Fouda, Ahmed G. Radwan ; Fractional Order Memristor Response Under DC and Periodic signals.

  Mohamed E. Fouda, Ahmed G. Radwan ; On the fractional-order memristor model

  Tenreiro Machado ; Fractional Generalization of Memristors and Higher Order Elements (Complex-Order)

  Ivo Petras, YangQuan Chen ; Fractional Order Circuit Elements with Memory

  Ivo Petras ; Fractional Order Memristive Systems

11. Student Progress (Covered Topics and References)

- 5. Memistors

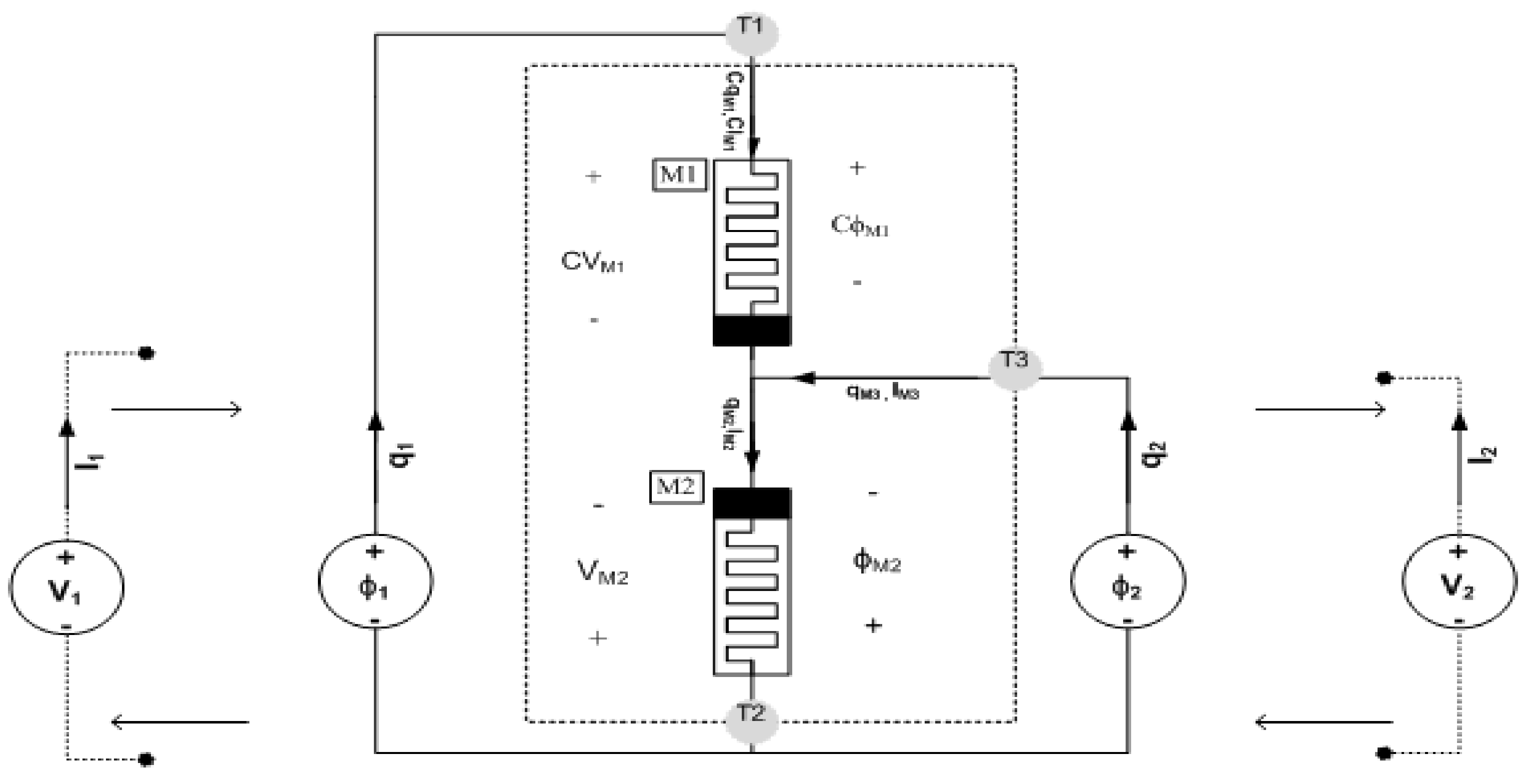
  Shyam Prasad Adhikri, Hyongsulk Kim ; Why are memristor and Memistor different devices.

  Qiang Xia, Mathhew D. Pickett et al. ; Two and Three Terminal Resistive Switches , Nano-scale memristors and memistors.

  S. Thakoar, A. Moopenn et al. ; Solid State Thin Film Memistor for Electronic Neural Networks.

11. Student Progress (Covered Topics and References)

- Simulation : Using Wolfram Mathematica 10
  - Flux-Charge Model for 2-Port Series Memristors



13. Possible Applications

- 1) Optimization of a chosen oscillator Model using Practical/Measured values by generalizing the RLC components and choosing the best set of (α, β) that minimize the errors in simulations using numerical interpolation and optimization techniques.
  - As in the case of fractances, capacitors exhibit fractional behavior in reality, so some sources of errors in the oscillators as those generated from the sensitivity analysis of a component might be accurately modeled by a fractional element instead of an integer-order element.
  - A comparison can be made by the integer-order model, the fractional-order theoretical mode, and the realization of the fractional-order theoretical model using one of the synthesis methods.
- 2) Analysis and Simulation of oscillators using fractional-order elements.
  - The focus will be on generalizing a specific type of oscillators to a fractional order by introducing fractional elements and possibly their simulations using one of the synthesis methods(ex; Wein Oscillators generalized in "Design Equations for Fractional-order sinusoidal oscillators" ; A.G. Radwan 2007).

12. Possible Applications

- 3) Focusing on the noise sources and their effects in fractional oscillators.
  - The focus will be on the numerical simulation of the effect of noise sources but in the case of using fractional elements in the oscillators.
- 4) Non-linear Frequency response techniques for nonlinear oscillators.
  - The focus will be on the analytical analysis of a selected nonlinear oscillator, using integer or fractional circuits, the orientation will be towards nonlinear frequency methods such as the describing function and harmonic balance.
- 5) Sensitivity Analysis for a chosen oscillator using Tellegen's Theorem.
  - Tellegen's theorem is used for the sensitivity analysis of circuits, so a possible orientation is to focus the research on utilizing different forms of Tellegen's theorem (Fractional form for example).
- 6) Analysis of Time-Variable or Complex Order Fractional elements.
  - This method should be the hardest and the most difficult, it generalizes the already complicated real fractional order elements into Complex order and/or time-dependent order elements.
  - Should this part be selected, the focus will be only for the linear algebraic elements.

13. Q&A