

## Homework Problems

1. Evaluate the Integrals.

(a)  $\int \left( t \ln t \mathbf{i} + t \sin(t^2) \mathbf{j} + \frac{5t^2 - t - 2}{t^3 + t^2} \mathbf{k} \right) dt$

Solution:  $\left( \frac{1}{2}t^2 \ln t - \frac{1}{4}t^2 + c_1 \right) \mathbf{i} - \left( \frac{1}{2} \cos(t^2) + c_2 \right) \mathbf{j} + \left( \ln t + \frac{2}{t} + 4 \ln(t+1) + c_3 \right) \mathbf{k}$  or  
 $\left( \frac{1}{2}t^2 \ln t - \frac{1}{4}t^2 \right) \mathbf{i} - \left( \frac{1}{2} \cos(t^2) \right) \mathbf{j} + \left( \ln t + \frac{2}{t} + 4 \ln(t+1) \right) \mathbf{k} + \vec{c}$

(b)  $\int_{-\pi/4}^{\pi/4} (\sin t \mathbf{i} + (1 + \cos t) \mathbf{j} + \sec^2 t \mathbf{k}) dt$

Solution:  $\frac{\pi + 2\sqrt{2}}{2} \mathbf{j} + 2 \mathbf{k}$

(c)  $\int_0^{\pi/2} (\cos t \mathbf{i} - \sin(2t) \mathbf{j} + \sin^2 t \mathbf{k}) dt$

Solution:  $\mathbf{i} - \mathbf{j} + \frac{\pi}{4} \mathbf{k}$

(d)  $\int_1^{\ln 3} (te^t \mathbf{i} + e^t \mathbf{j} + \ln t \mathbf{k}) dt$

Solution:  $(3 \ln 3 - 3) \mathbf{i} + (3 - e) \mathbf{j} + (\ln 3 \cdot \ln(\ln 3) - \ln 3 + 1) \mathbf{k}$

2. Solve the initial value problems for the vector function  $\mathbf{r}(t)$ .

(a)  $\frac{d\mathbf{r}}{dt} = -t \mathbf{i} - t \mathbf{j} - t \mathbf{k} \quad ; \quad \mathbf{r}(0) = \mathbf{i} + 2 \mathbf{j} + 3 \mathbf{k}$

Solution:  $\left( 1 - \frac{1}{2}t^2 \right) \mathbf{i} + \left( 2 - \frac{1}{2}t^2 \right) \mathbf{j} + \left( 3 - \frac{1}{2}t^2 \right) \mathbf{k}$

(b)  $\frac{d\mathbf{r}}{dt} = \frac{3}{2} \sqrt{t+1} \mathbf{i} + e^{-t} \mathbf{j} + \frac{1}{t+1} \mathbf{k} \quad ; \quad \mathbf{r}(0) = \mathbf{k}$

Solution:  $\left( (t+1)\sqrt{t+1} - 1 \right) \mathbf{i} + \left( -e^{-t} + 1 \right) \mathbf{j} + \left( \ln(t+1) + 1 \right) \mathbf{k}$

(c)  $\frac{d^2\mathbf{r}}{dt^2} = -32 \mathbf{k} \quad ; \quad \mathbf{r}(0) = 100 \mathbf{k} \quad ; \quad \mathbf{r}'(0) = 8 \mathbf{i} + 8 \mathbf{j}$

Solution:  $8t \mathbf{i} + 8t \mathbf{j} + (100 - 16t^2) \mathbf{k}$

(d)  $\frac{d^2\mathbf{r}}{dt^2} = -\cos t \mathbf{i} - 12 \sin(2t) \mathbf{j} - 9.8 \mathbf{k} \quad ; \quad \mathbf{r}(0) = 10 \mathbf{j} + 150 \mathbf{k} \quad ; \quad \mathbf{r}'(0) = \mathbf{i} + 6 \mathbf{j}$

Solution:  $(\cos t + t - 1) \mathbf{i} + (3 \sin(2t) + 10) \mathbf{j} + (150 - 4.9t^2) \mathbf{k}$

3. A projectile is fired at a speed of 800 m/sec at an angle of  $\frac{\pi}{3}$  radians. How long will it take to get 12 km downrange?

Solution: 30 seconds.

4. A projectile is fired with an initial speed of 500 m/sec at an angle of elevation of  $\frac{\pi}{4}$  radians.

(a) When will the projectile strike the ground?

Solution: approximately 72.2 seconds

(b) How far downrange will the projectile be when it strikes the ground?

Solution: approximately 25,510 km

(c) How high overhead is the projectile when it is 5 km downrange?

Solution: approximately 4020 m

(d) What is the greatest height reached by the projectile?

Solution: approximately 6378 m

5. A spring gun at ground level fires a marble at an angle of  $\frac{\pi}{4}$  radians. The marble lands 10 m away. What was the marble's initial speed?

Solution: approximately 9.9 m/sec