6.1

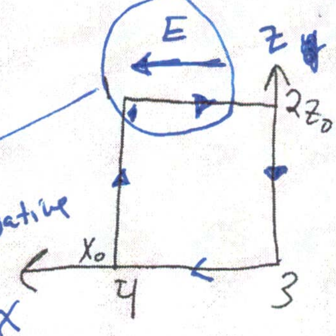
1. $\nabla \times E = \left(\cancel{\frac{dE_z}{dy}}^{0} - \cancel{\frac{dE_y}{dz}}^{0}\right)\hat{x} + \left(\frac{dE_x}{dz} - \cancel{\frac{dE_z}{dx}}^{0}\right)\hat{y} + \left(\cancel{\frac{dE_y}{dx}}^{0} - \cancel{\frac{dE_x}{dy}}^{0}\right)\hat{z}$

$= dE_x/dz\ \hat{y} = -K E_{0x} \sin(K_z z - \omega t)\hat{y}$

$-dB/dt = -K E_{0x} \sin(K_z z - \omega t)\hat{y} \rightarrow B = K E_{0x} \int \sin(K_z z - \omega t)\, dt$

$B = \frac{K}{\omega} E_{0x} \cos(K_z z - \omega t) = \frac{1}{c} E_{0x} \cos(K_z z - \omega t)\hat{y}$

2.



$\int_1^2 \vec{E} \cdot d\vec{l}$ is negative

$\oint E \cdot dl = \int_1^2 E \cdot dl + \int_2^3 E \cdot dl + \int_3^4 E \cdot dl + \int_4^1 E \cdot dl$

$\int_1^2 E \cdot dl = \int_1^2 E_{0x} \cos(K_z z - \omega t) dl$

dl = length of line segment $= x_2 - x_1 = x_0 - 0 = x_0$

$\int_1^2 E \cdot dl = E_{0x} x_0 \cos(K_z z_0 - \omega t)$

No change in $z \rightarrow z = z_0$

$\int_2^3 E \cdot dl = 0$ because dl is perpendicular to the E-field

Based on diagram & loop dir, this term is negative.

$\int_3^4 E \cdot dl = \int_3^4 E_{0x} \cos(K_z z - \omega t) dl$ $\quad dl = x_0 - 0 = x_0$ and no change in z, at $z = 0$

$\int_3^4 E \cdot dl = E_{0x} x_0 \cos(\omega t)$ $\quad (\cos(-\omega t) = \cos(\omega t))$

$\int_4^1 E \cdot dl = 0$ because dl is perpendicular to the E-field

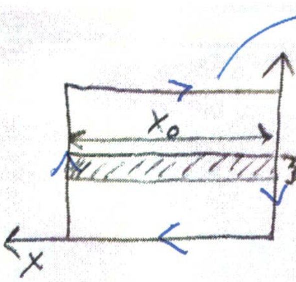
$\oint E \cdot dl = E_{0x} x_0 \cos(K_z z_0 - \omega t) + 0 + E_{0x} x_0 \cos(\omega t) + 0$

$= E_{0x} x_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right]$

X

\-

loop dir determines $\hat{n}$ dir for $\mathbb{B}\cdot\hat{n}\,da$



Area of E-field that contributes to B-flux $\quad dA = \text{length} \times \text{width} = x_0\,dz$

$$\Phi_B \int B\cdot dA = \int_0^{z_0} \frac{1}{c} E_{0x}\cos(k_z z - \omega t)\,x_0\,dz$$

$$= -\frac{1}{c}\frac{E_{0x}x_0}{k}\Big[\sin(-k_z z + \omega t)\Big]_0^{z_0} = -\frac{1}{c}\frac{E_{0x}x_0}{k}\Big[\sin(-k_z z_0 + \omega t) - \sin(\omega t)\Big]$$

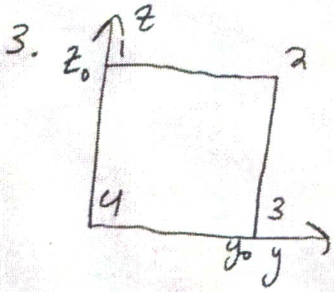
$\sin(-k_z z_0 + \omega t) = -\sin(k_z z_0 + \omega t)$

$$-\frac{d\Phi_B}{dt} = -\frac{1}{c}\frac{E_{0x}x_0}{k}\Big[\sin(k_z z - \omega t) + \sin(\omega t)\Big]\,d/dt$$

$$= \frac{1}{c}\frac{\omega}{k}E_{0x}x_0\Big[\cos(k_z z_0 - \omega t) + \cos(\omega t)\Big]$$

$$= E_{0x}x_0\Big[\cos(k_z z_0 - \omega t) + \cos(\omega t)\Big] = \oint E\cdot dl$$

3.



$$\oint B\cdot dl = \int_1^2 B\cdot dl + \int_2^3 B\cdot dl + \int_3^4 B\cdot dl + \int_4^1 B\cdot dl$$

$$\int_1^2 B\cdot dl = \int_1^2 \frac{1}{c}E_{0x}\cos(k_z z - \omega t)\,dl$$

$$dl = y_2 - y_1 = y_0 - 0 = y_0$$

$$\int_1^2 B\cdot dl = \frac{1}{c}E_{0x}\cos(k_z z_0 - \omega t)\,y_0$$

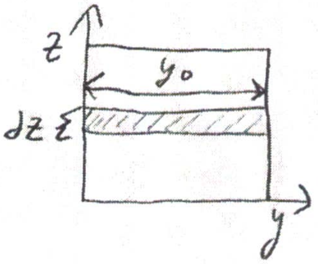
$\int_2^3 B\cdot dl = 0$ because dl is perpendicular to the B-field

$\int_3^4 B\cdot dl = \frac{1}{c}E_{0x}\cos(k_z z_1 - \omega t)\,y_0 \quad z_1 = 0$ so $\int_3^4 B\cdot dl = \frac{1}{c}E_{0x}\cos(\omega t)\,y_0$

$\int_4^1 B\cdot dl = 0$ because dl is perpendicular to the B-field

$$\oint B\cdot dl = \frac{1}{c}E_{0x}\cos(k_z z_0 - \omega t)\,y_0 + 0 + \frac{1}{c}E_{0x}\cos(\omega t)\,y_0 + 0$$

$$= \frac{1}{c}E_{0x}y_0\Big[\cos(k_z z_0 - \omega t) + \cos(\omega t)\Big]$$



Area of B-field that contributes to the E-flux

$$dA = y_0 dz$$

$$\phi E = \int E \cdot dA = \int_0^{z_0} E_{0x} \cos(K_z z - \omega t) y_0 \, dz$$

$$= -\frac{E_{0x} y_0}{K} \left[\sin(-K_z z + \omega t) \right]_0^{z_0} = -\frac{E_{0x} y_0}{K} \left[-\sin(K_z z_0 - \omega t) - \sin(\omega t) \right]$$

$$= \frac{E_{0x} y_0}{K} \left[\sin(K_z z_0 - \omega t) + \sin(\omega t) \right]$$

$$-\frac{d\phi_E}{dt} = -\frac{E_{0x} y_0}{K} \left[\sin(K_z z_0 - \omega t) + \sin(\omega t) \right] d/dt$$

$$= \frac{\omega}{K} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t) \right]$$

$$= c E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t) \right]$$

$$\frac{1}{c^2}(c E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t) \right] = \frac{1}{c} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t) \right]$$

$$\frac{1}{c} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t) \right] = \oint B \cdot dl$$

6.2. $E = E_{0x}(x,t)\hat{x} + E_{0y}(x,t)\hat{y} + E_{0z}(x,t)\hat{z}$

$B = B_{0x}(x,t)\hat{x} + B_{0y}(x,t)\hat{y} + B_{0z}(x,t)\hat{z}$

Show this using maxwell's eqns.

1. $\frac{d^2 f}{du^2} \rightarrow$ $\frac{\delta^2 E_y(x,t)}{dx^2} = k^2 E_y''(x,t)$, $\frac{d^2 E_y(x,t)}{dy^2} = \frac{d^2 E_y(x,t)}{dz^2} = 0$

$\frac{\delta^2 E_z(x,t)}{dx^2} = k^2 E_z''(x,t)$, $\frac{d^2 E_z(x,t)}{dy^2} = \frac{d^2 E_z(x,t)}{dz^2} = 0$

$\frac{d^2 B_y(x,t)}{dx^2} = k^2 B_y''(x,t)$, $\frac{d^2 B_y(x,t)}{dy^2} = \frac{d^2 B_y(x,t)}{dz^2} = 0$

$\frac{d^2 B_z(x,t)}{dx^2} = k^2 B_z''(x,t)$, $\frac{d^2 B_z(x,t)}{dy^2} = \frac{d^2 B_z(x,t)}{dz^2} = 0$

$\frac{d^2 f}{dt^2} \rightarrow$ $\frac{d^2 E_y(x,t)}{dt^2} = \omega^2 E_y''(x,t)$ $\frac{d^2 B_y(x,t)}{dt^2} = \omega^2 B_y''(x,t)$

$\frac{d^2 E_z(x,t)}{dt^2} = \omega^2 E_z''(x,t)$ $\frac{d^2 B_z(x,t)}{dt^2} = \omega^2 B_z''(x,t)$

$\frac{d^2 E_y(x,t)}{dx^2} = E_y''(x,t) = \frac{\omega^2}{k^2} E_y''(x,t) = \frac{1}{c^2} E_y''(x,t)$

$\frac{d^2 E_y(x,t)}{dy^2} = \frac{d^2 E_y(x,t)}{dz^2} = 0 \rightarrow \omega^2 E_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

Wave is propogating only in x direction

$\frac{d^2 B_z(x,t)}{dx^2} = E_z''(x,t) = \frac{\omega^2}{k^2} E_z''(x,t) = \frac{1}{c^2} E_z''(x,t)$

$\frac{d^2 E_z(x,t)}{dy^2} = \frac{d^2 E_z(x,t)}{dz^2} = 0 \rightarrow \omega^2 E_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

wave is propogating only in x direction

$\frac{d^2 B_y(x,t)}{dx^2} = B_y''(x,t) = \frac{\omega^2}{k^2} B_y''(x,t) = \frac{1}{c^2} B_y''(x,t)$

$\frac{d^2 B_y(x,t)}{dy^2} = \frac{d^2 B_y(x,t)}{dz^2} = 0 \rightarrow \omega^2 B_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

" "

$\frac{d^2 B_z(x,t)}{dx^2} = B_z''(x,t) = \omega^2/k^2 B_z''(x,t) = \frac{1}{c^2} B_z''(x,t)$

$\frac{d^2 B_z}{dy^2} = d^2 B_z/dz^2 = 0 \rightarrow \omega^2 B_z''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

" "

2. $\frac{d^2E_x(x,t)}{dx^2} = \frac{d^2B_x(x,t)}{dx^2} = 0$ The wave propogates in the x-direction but does not vary in that direction

$\frac{d^2E_x(x,t)}{dt^2} = \omega^2 E_x''(x,t)$ $\frac{d^2B_x(x,t)}{dt^2} = \omega^2 B_x''(x,t)$

$\omega^2 E_x''(x,t) = \omega^2 B_x''(x,t) = 0$

The partial derivatives of E_x and B_x wrt y and z are 0 since the waves do not propogate in that direction. That leaves:

$E_x''(x,t) = B_x''(x,t) = 0$

3. $$\nabla^2 E = \nabla^2 \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \frac{d^2E_x}{dx^2} + \frac{d^2E_x}{dy^2} + \frac{d^2E_x}{dz^2} \\ \frac{d^2E_y}{dx^2} + \frac{d^2E_y}{dy^2} + \frac{d^2E_y}{dz^2} \\ \frac{d^2E_z}{dx^2} + \frac{d^2E_z}{dy^2} + \frac{d^2E_z}{dz^2} \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ K^2E_y''(x,t)+0+0 \\ K^2E_z''(x,t)+0+0 \end{bmatrix}$$

$$\frac{1}{c^2}\frac{d^2E}{dt^2} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2E_y''(x,t) \\ \omega^2E_z''(x,t) \end{bmatrix} \rightarrow \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2/k^2 E_y''(x,t) \\ \omega^2/k^2 E_z''(x,t) \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ c^2E_y''(x,t) \\ c^2E_z''(x,t) \end{bmatrix}$$

$$\nabla^2 B = \nabla^2 \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \frac{d^2B_x}{dx^2} + \frac{d^2B_x}{dy^2} + \frac{d^2B_x}{dz^2} \\ \frac{d^2B_y}{dx^2} + \frac{d^2B_y}{dy^2} + \frac{d^2B_y}{dz^2} \\ \frac{d^2B_z}{dx^2} + \frac{d^2B_z}{dy^2} + \frac{d^2B_z}{dz^2} \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ K^2B_y''(x,t)+0+0 \\ K^2B_z''(x,t)+0+0 \end{bmatrix}$$

$$\frac{1}{c^2}\frac{d^2B}{dt} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2B_y''(x,t) \\ \omega^2B_z''(x,t) \end{bmatrix} \rightarrow \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2/k^2 B_y''(x,t) \\ \omega^2/k^2 B_z''(x,t) \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ c^2B_y''(x,t) \\ c^2B_z''(x,t) \end{bmatrix}$$

6.3.

1. $E = E_{0x} \cos(k_z z - \omega t + \delta_x)\hat{x}$

$$\nabla^2 E = \nabla^2 \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 + 0 - k_z^2 E_{0x} \cos(k_z z - \omega t + \delta_x) \\ 0 + 0 + 0 \\ 0 + 0 + 0 \end{bmatrix}$$

$$= \frac{1}{c^2} \frac{d^2 E(x,t)}{dt^2} = \frac{1}{c^2} \begin{bmatrix} -\omega^2 E_{0x} \cos(k_z z - \omega t + \delta_x) \\ 0 \\ 0 \end{bmatrix}$$

$$-k_z^2 E_{0x} \cos(k_z z - \omega t + \delta_x) = \frac{1}{c^2}\left[-\omega^2 E_{0x} \cos(k_z z - \omega t + \delta_x)\right]\hat{x}$$

$$E_{0x} \cos(k_z z - \omega t + \delta_x) = \frac{1}{c^2}\left[c^2 E_{0x} \cos(k_z z - \omega t + \delta_x)\right]\hat{x}$$

$$B = B_{0x} \cos(k_z z - \omega t\ \delta_x')\hat{x} + B_{0y} \cos(k_z z - \omega t + \delta_y')\hat{y}$$

$$\nabla^2 B = \nabla^2 \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 0 + 0 - k_z^2 B_{0x} \cos(k_z z - \omega t + \delta_x') \\ 0 + 0 - k_z^2 B_{0x} \cos(k_z z - \omega t + \delta_y') \\ 0 + 0 + 0 \end{bmatrix}$$

$$\frac{1}{c^2} \frac{d^2 B(x,t)}{dt^2} = \frac{1}{c^2} \begin{bmatrix} -\omega^2 B_{0x} \cos(k_z z - \omega t + \delta_x') \\ -\omega^2 B_{0x} \cos(k_z z - \omega t + \delta_y') \\ 0 \end{bmatrix}$$

$$-k_z^2 B_{0x} \cos(k_z z - \omega t + \delta_x') = \frac{1}{c^2}\left[-\omega^2 B_{0x} \cos(k_z z - \omega t + \delta_x')\right]\hat{x}$$

$$B_{0x} \cos(k_z z - \omega t + \delta_x') = \frac{1}{c^2}\left[c^2 B_{0x} \cos(k_z z - \omega t + \delta_x')\right]\hat{x}$$

$$-k_z^2 B_{0y} \cos(k_z z + \omega t + \delta_y') = \frac{1}{c^2}\left[-\omega^2 B_{0y} \cos(k_z z - \omega t + \delta_y')\right]\hat{y}$$

$$B_{0y} \cos(k_z z - \omega t + \delta_y') = \frac{1}{c^2}\left[c^2 B_{0y} \cos(k_z z - \omega t + \delta_y')\right]\hat{y}$$

2. $\nabla \times E = -\partial B/\partial t$

$$\nabla \times E = \left(\cancel{\partial E_z/\partial y} - \cancel{\partial E_y/\partial z}\right)\hat{x} + \left(\partial E_x/\partial z - \cancel{\partial E_z/\partial x}\right)\hat{y} + \left(\cancel{\partial E_y/\partial x} - \cancel{\partial E_x/\partial y}\right)\hat{z}$$

$$\nabla \times E = \partial E_x/\partial z \ \hat{y} = K_z E_{0x} \sin(K_z z - \omega t + \delta_x)\hat{y}$$

$$-\partial B/\partial t = -B_{0x}\cos(K_z z - \omega t + \delta'_x)\hat{x}\ \partial/\partial t - B_{0y}\cos(K_z z - \omega t + \delta'_y)\hat{y}\ \partial/\partial t$$

$$= \omega B_{0x}\sin(K_z z - \omega t + \delta'_x)\hat{x} + \omega B_{0y}\sin(K_z z - \omega t + \delta'_y)\hat{y}$$

$$B_{0x} = 0, \quad \delta x = \delta' y, \quad c = |\omega/K|, \quad E_{0x} \propto B_{0y}$$

$$E_{0x}\sin(K_z z - \omega t + \delta_x) = \frac{1}{c}\left[\frac{\omega}{K} B_{0y}\sin(K_z z - \omega t + \delta'_y)\right]\hat{y}$$

State prop. const.

3. $E = E_{0y}\cos(K_z z - \omega t + \delta_y)\hat{y}$

$$\nabla \times E = -\partial E_y/\partial z\ \hat{x} = -K_z E_{0y}\sin(K_z z - \omega t + \delta_y)\hat{x}$$

$$-\partial B/\partial t = \omega B_{0x}\sin(K_z z - \omega t + \delta'_x)\hat{x} + \omega B_{0y}\sin(K_z z - \omega t + \delta'_y)\hat{y}$$

$$B_{0y} = 0, \quad \delta_y = \delta'_y, \quad c = |\omega/K|, \quad E_{0x} \propto B_{0y}$$

$$E_{0y}\sin(K_z z - \omega t + \delta_y) = \frac{1}{c}\left[\frac{\omega}{K} B_{0y}\sin(K_z z - \omega t + \delta'_y)\right]\hat{x}$$

4. $B = \frac{1}{c}\hat{K} \times E = \begin{matrix} K_x & K_y & K_z \\ E_x & E_y & E_z \end{matrix}$

$$\frac{1}{c}\left[(K_y E_z - K_z E_y)\hat{x} - (K_x E_z - K_z E_x)\hat{y} + (K_x E_y - K_y E_x)\hat{z}\right]$$

For $E = E_{0x}\cos(K_z z - \omega t + \delta_x)\hat{x}$

$$B = \frac{1}{c}\left[K_z E_x \hat{y} - K_y E_x \hat{z}\right] \quad K_y = 0 \text{ so } B = \frac{1}{c}\left[K_z E_x \hat{y}\right] = \frac{1}{c} K_z E_{0x}\cos(K_z z - \omega t + \delta_x)\hat{y}$$

For $E = E_{0y}\cos(K_z z - \omega t + \delta_y)\hat{y}$

$$B = \frac{1}{c}\left[-K_z E_y \hat{x} + K_x E_y \hat{z}\right] \quad K_x = 0 \text{ so } B = \frac{1}{c}\left[-K_z E_y \hat{x}\right] = -\frac{1}{c} K_z E_{0y}\cos(K_z z - \omega t + \delta_y)\hat{x}$$

6.4 $\nabla \times E = -\partial B/\partial t$

$$\nabla \times E = Re[e^{-i\omega t} \nabla \times \tilde{E} e^{ik\cdot r}] = Re[e^{-i\omega t}(ik_y \tilde{E}_z e^{ik\cdot r} - \tilde{E}_y (ik_z) e^{ik\cdot r})\hat{x}]$$

$$-\partial B/\partial t = -Re[\tilde{B}_x(-i\omega) e^{-i\omega t} e^{ik\cdot r}]\hat{x}$$

$$\nabla \times E = -\partial B/\partial t = Re[\cancel{e^{-i\omega t}}(\cancel{i}k_y \tilde{E}_z \cancel{e^{ik\cdot r}} - \tilde{E}_y(\cancel{i}k_z)\cancel{e^{ik\cdot r}})\hat{x}] = \cancel{-}Re[\tilde{B}_x(\cancel{-i}\omega)\cancel{e^{-i\omega t}}\cancel{e^{ik\cdot r}}\hat{x}]$$

$$= Re[(\tilde{E}_z k_y - \tilde{E}_y k_z)\hat{x} = \tilde{B}_x(\omega)\hat{x}]$$

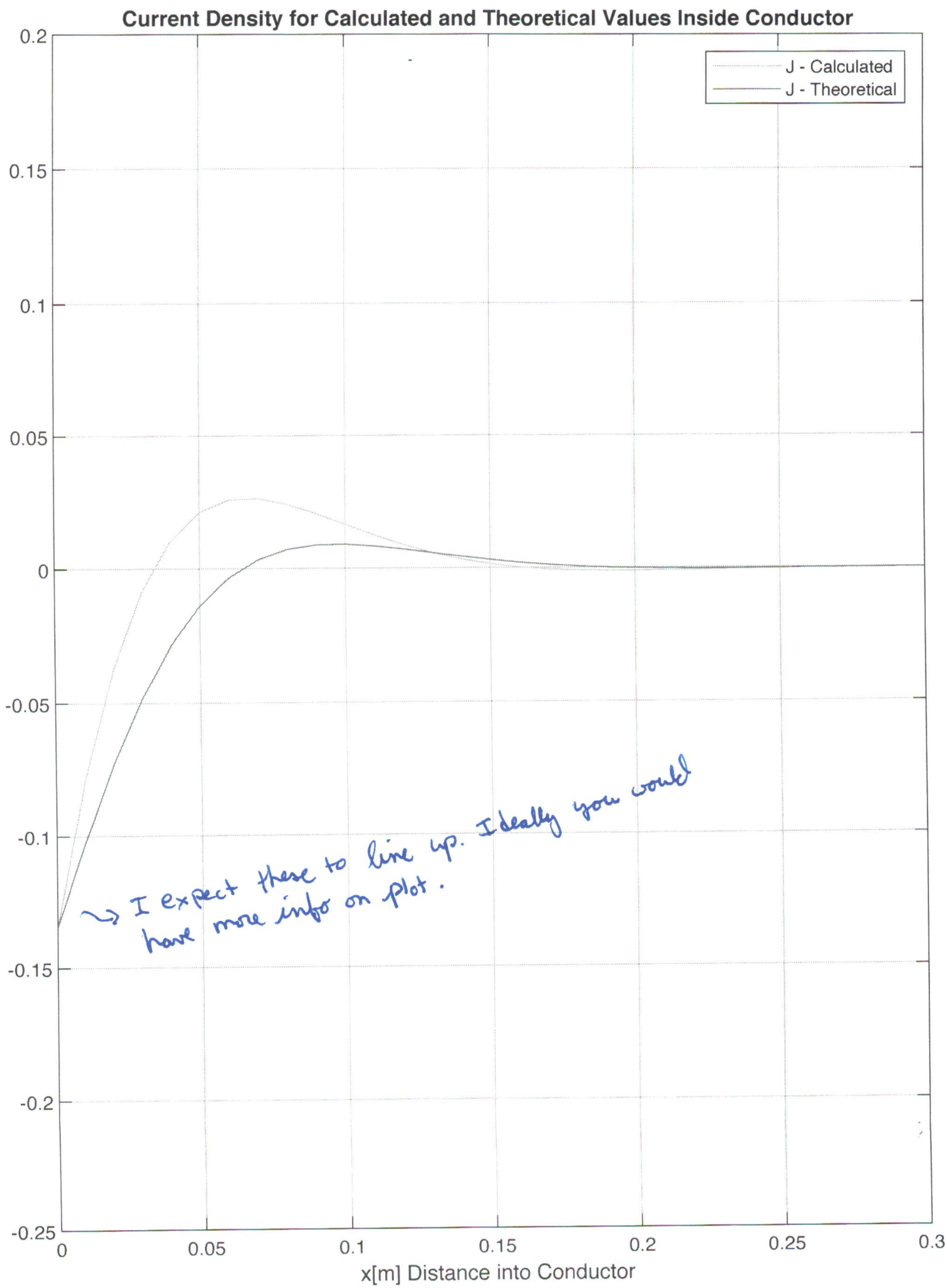
$$= Re\left[\frac{\hat{k} \times \tilde{E}}{c} = \tilde{B}\right]$$

$Re\left[\frac{\hat{k}\times\tilde{E}}{c} = \tilde{B}\right]$ and $Im\left[\frac{\hat{k}\times\tilde{E}}{c} = \tilde{B}\right]$ are in phase

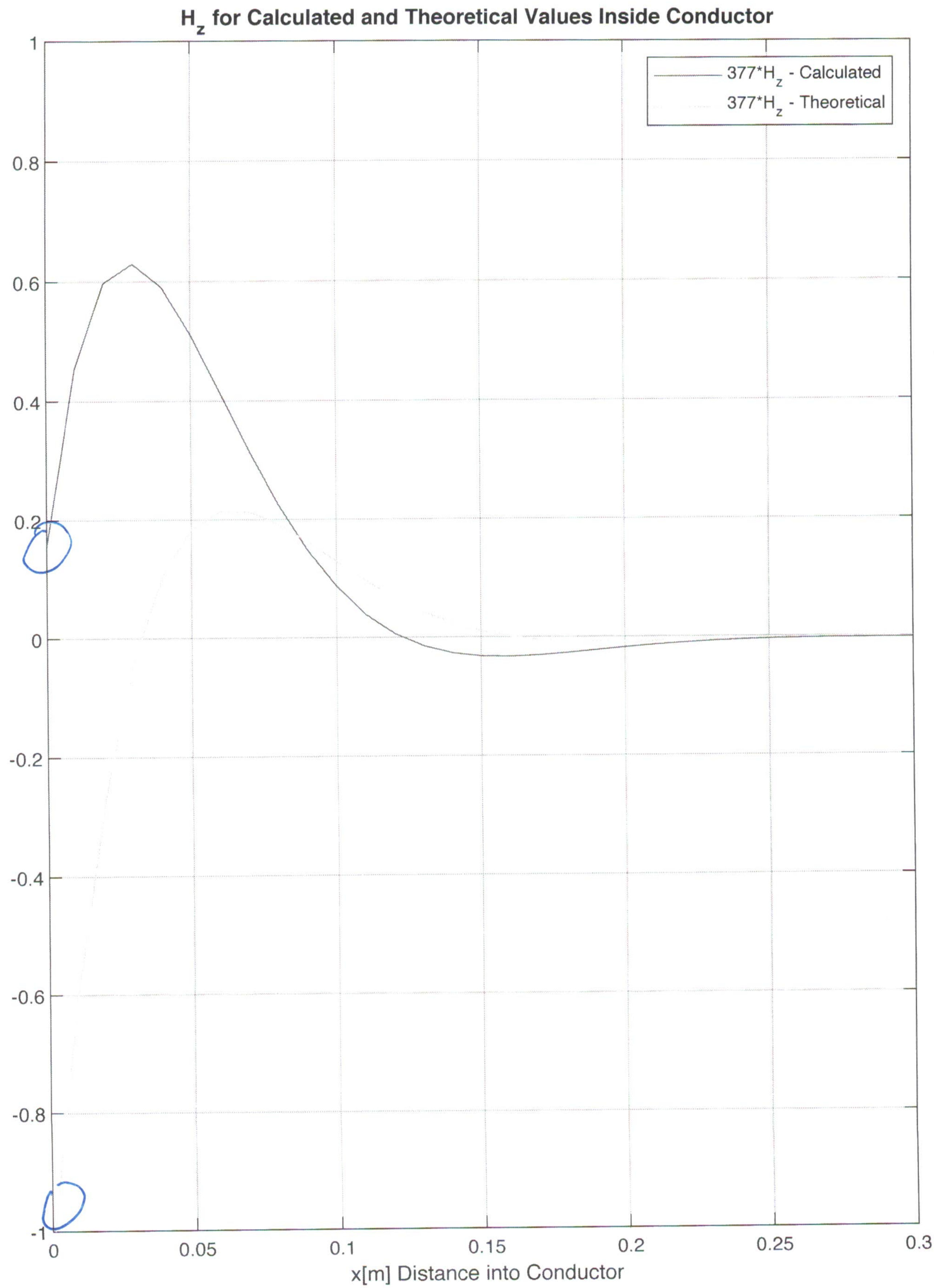
so Re and Im are dropped

$B = \frac{1}{c}\hat{k} \times E$ and $\tilde{B} = \frac{1}{c}\hat{k} \times \tilde{E}$

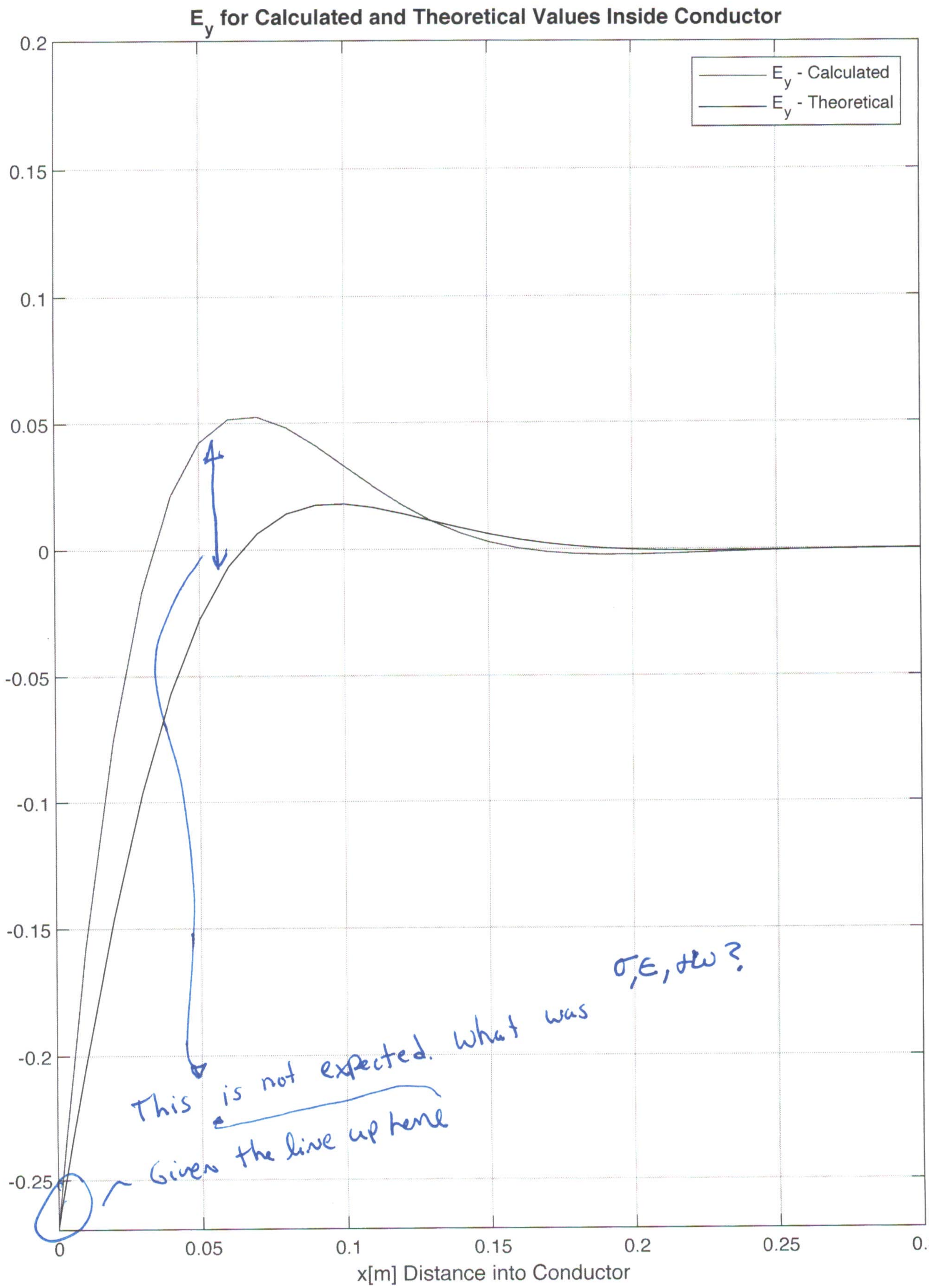
What does "in phase" mean in this context?



Current Density for Calculated and Theoretical Values Inside Conductor
J - Calculated
J - Theoretical
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.05
0.1
0.15
0.2
0.25
0.3
x[m] Distance into Conductor
I expect these to line up. Ideally you would have more info on plot.

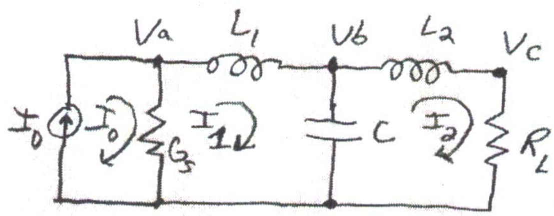


H_z for Calculated and Theoretical Values Inside Conductor
377*H_z - Calculated
377*H_z - Theoretical
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.05
0.1
0.15
0.2
0.25
0.3
x[m] Distance into Conductor



E_y for Calculated and Theoretical Values Inside Conductor
E_y - Calculated
E_y - Theoretical
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
0
0.05
0.1
0.15
0.2
0.25
0.3
x[m] Distance into Conductor
This is not expected. What was σ, ϵ, ω?
Given the line up here

4.3b.



$$L_1 \frac{dI_1}{dt} + \frac{1}{C}\int (I_1 - I_2)\,dt + \frac{(I_1 - I_0)}{G_s} = 0$$

$$L_2 \frac{dI_2}{dt} + I_2 R_L + \frac{1}{C}\int (I_2 - I_1)\,dt = 0$$

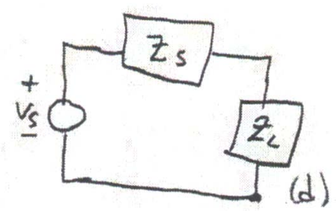
Ideally you would show or comment about result being same as V source system. (point of problem)

Node V_a: $I_0 = V_a G_s + \frac{1}{L_1}\int (V_a - V_b)\,dt \rightarrow I_0 - V_a G_s - \frac{1}{L_1}\int (V_a - V_b)\,dt = 0$

Node V_b: $\frac{1}{L_1}\int (V_b - V_a)\,dt + C\frac{dV_b}{dt} + \frac{1}{L_2}\int (V_b - V_c)\,dt = 0$

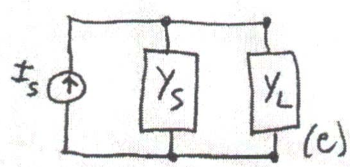
Node V_c: $\frac{V_c}{R_L} + \frac{1}{L_2}\int (V_c - V_b)\,dt = 0$

4.3c.



$I_L = \frac{V_s}{Z_s + Z_L}$ current through the load impedance

$V_L = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)$



$V_L = \frac{I_s}{Y_s + Y_L}$

$I_s = V_s Y_s,\quad Y_s = Z_s^{-1} \rightarrow I_s = \frac{V_s}{Z_s}$

$$V_L = \frac{I_s}{Y_s + Y_L} \rightarrow \frac{(V_s/Z_s)}{(Z_s^{-1}) + (Z_L^{-1})} = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)$$

Current through load admittance:

$$I_L = V_L Y_L = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)(Z_L^{-1}) = \frac{V_s}{Z_s + Z_L}$$

I_L is the same for the current flowing through the load impedance and the current through the load admittance.

V_L is also the same for the voltage across the load impedance and the voltage across the load admittance.

4.3 d. For the current in the circuit:

$$I = \frac{V_s}{Z_s + Z_L} \qquad \text{where } Z_s = R_s + jX_s \text{ and } Z_L = R_L + jX_L$$

$$I = \frac{V_s}{(R_s + jX_s) + (R_L + jX_L)} = \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

For power to the load, $W_L = \frac{R_L}{2} I I^*$ → can you derive?

$$I^* = \left[\frac{V_s}{(R_s+R_L)+j(X_s+X_L)}\right]^* = \left[\frac{V_s}{(R_s+R_L)+j(X_s+X_L)}\right]\left[\frac{(R_s+R_L)-j(X_s+X_L)}{(R_s+R_L)-j(X_s+X_L)}\right]$$

$$I^* = \frac{V_s\left[(R_s+R_L)-j(X_s+X_L)\right]}{(R_s+R_L)^2+(X_s+X_L)^2}$$

$$W_L = \frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)+j(X_s+X_L)}\right]\left[\frac{V_s\left[(R_s+R_L)-j(X_s+X_L)\right]}{(R_s+R_L)^2+(X_s+X_L)^2}\right]$$

$$= \frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2+(X_s+X_L)^2}\right]$$

For max power to load: $\frac{dW_L}{dX_L} = 0$ and $\frac{dW_L}{dR_L} = 0$ & surface $W(R_L^*, X_L^*)$ is concave down.

$$\frac{dW_L}{dX_L} = \frac{d}{dX_L}\left(\frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2+(X_s+X_L)^2}\right]\right) = 0$$

$$= \frac{-R_L V_s (X_s+X_L)}{\left[(R_s+R_L)^2+(X_s+X_L)^2\right]^2} \rightarrow -R_L V_s (X_s+X_L) = 0, \quad X_L = -X_s$$

$$\frac{dW_L}{dR_L} = \frac{d}{dR_L}\left(\frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2+(X_s+X_L)^2}\right]\right) = 0$$

$$= \frac{V_s}{2}\left[\frac{-2R_L(R_s+R_L)+\left[(R_s+R_L)^2+(X_s+X_L)^2\right]}{\left[(R_s+R_L)^2+(X_s+X_L)^2\right]^2}\right] = 0$$

$$-2R_L(R_S+R_L)+[(R_S+R_L)^2+(X_S+X_L)^2]=0$$

As $X_L=-X_S$,

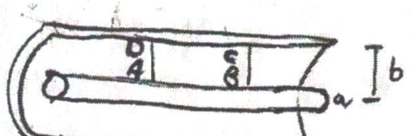
$$-2R_L(R_S+R_L)+[(R_S+R_L)^2+0]=0$$

$$-2R_LR_S-2R_L^2+R_S^2+R_L^2+2R_LR_S = R_S^2-R_L^2=0, \quad R_L=R_S$$ ✓

Max power to the load when $R_L=R_S$ and $X_L=-X_S$ ✓ ✓

$$Z_L = R_L+jX_L = R_S+j(-X_S) = R_S-jX_S = (R_S+jX_S)^* = Z_S^*$$

4.6e.



$$\oint_{ABCD} E\cdot dl = -j\omega\int_S B\cdot dS$$

$$\int_S B\cdot dS = IL_e\,dz$$

$$\oint_{ABCD} E\cdot dl = -j\omega IL_e\,dz$$

for $E_z = I(Z_{inner})_{outer}$ inner surface of the outer conductor

$$E_z = I\left(\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)$$

b = radius of outer conductor

R_{sb} = resistance

L_{ib} = inductance

why this eqn? usually $R=\frac{1}{\sigma}\frac{L}{A}$

for $E_z = -I(Z_{inner})_{inner}$ outer surface of inner conductor

$$E_z = -I\left(\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}\right)$$

a = radius of inner conductor

R_{sa} = resistance

L_{ia} = inductance

$$\oint_{ABCD} E\cdot dl = -j\omega IL_e\,dz$$ ✓

$$I\left(\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}\right)dz + V_{CB} + I\left(\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)dz - V_{DA} = -j\omega IL_e\,dz$$

$$V_{DA}-V_{CB} = I\underbrace{\left(j\omega L_e+\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}+\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)}dz$$

$$V_{DA}-V_{CB} = \cancel{0} \times$$

$$= dV \qquad \frac{dV}{dz} = (\qquad)$$

For current from C to D,

$$I_C - I_D = j\omega(C\,dz)V$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \frac{F}{m}$$

$$I_C - I_D = j\omega\left(\frac{2\pi\varepsilon\,dz}{\ln(b/a)}\right)V$$ matches capacitance shown in Fig. P4.6e