1. For an infinite ladder network, $Z = Z_L + Z_C \| Z = Z_L + \frac{Z_C Z}{Z + Z_C}$

$$\rightarrow Z = \frac{1}{2}\left(Z_L \pm \sqrt{Z_L^2 + 4 Z_L Z_C}\right)$$

$$Z = \frac{1}{2}\left(i\omega L \pm \sqrt{-\omega^2 L^2 + 4(L/C)}\right)$$

If each section of the transmission line has a length of δx, $L = L'\delta x$ and $C = C'\delta x$

$$\text{so } Z = \frac{1}{2}\left(i\omega L'\delta x \pm \sqrt{-(\omega L'\delta x)^2 + 4(L/C)}\right)$$

As $\delta x \rightarrow 0$, $Z = \frac{1}{2} \pm \sqrt{4(L/C)} = \sqrt{L/C}$

For this to be true, load impedance must equal characteristic impedance: $Z = Z_{Load} = \sqrt{L/C}$

2. When $Z_{Load} \neq \sqrt{L/C}$, the load is mismatched. A $\lambda/4$ transmission line can be used to match the load with a line of characteristic impedance Z_0. The characteristic impedance of the transmission line to match Z_0 would be

$$Z_{in} = Z_0' \left[\frac{Z_L + j Z_0' \tan \beta l}{Z_0' + j Z_L \tan \beta l}\right]$$

$\beta l = \left(\frac{2\pi}{\lambda}\right)(\lambda/4) = \pi/2$ so $\tan\beta l \rightarrow \infty$ and $Z_{in} = Z_0'\left(\frac{Z_0'}{Z_L}\right)$

For a matched system, $Z_{in} = Z_0$ so $Z_{in} = Z_0 = \frac{(Z_0')^2}{Z_L}$

$$Z_0' = \sqrt{Z_0 Z_L}$$

Therefore, a ladder network solution with $Z_L \neq \sqrt{L/C}$ would require a transmission line of characteristic impedance $Z_0' = \sqrt{Z_0 Z_L}$ in order to act as a continuous line with characteristic impedance of Z_0.

3. For $Z = \frac{1}{2}\left(i\omega L \pm \sqrt{-\omega^2 L^2 + 4(L/C)}\right)$

When ω is small, $(L/C) \gg (\omega L)$ and the discrete approximation becomes $Z = \frac{1}{2}\sqrt{4(L/C)} = \sqrt{L/C}$

Which is the characteristic impedance of an infinitely long continuous transmission line.