2.1.1 $V=0$ when $y=0$

$V=0$ when $y=a$

$V=0$ when $x=b$

$V=V_0$ when $x=0$

$$V=(A\cosh(m(b-x))+B\sinh(m(b-x))(C\cos(my)+D\sin(my))$$

For $V=0$ when $y=0$,

$D\sin(my)=0$ and $C\cos(my)=0$ if $C=0$

For $V=0$ when $y=a$,

$D\sin(\frac{n\pi y}{a})=D\sin(n\pi)=0$, so $m=\frac{n\pi}{a}$

$$V=(A\cosh(\tfrac{n\pi}{a}(b-x))+B\sinh(\tfrac{n\pi}{a}(b-x))D\sin(\tfrac{n\pi y}{a})$$

For $V=0$ when $x=b$,

$(b-x)$ is used instead of x

$B\sinh(\frac{n\pi}{a}(0))=0$ and $A\cosh(\frac{n\pi}{a}(0))=0$ if $A=0$

$$V=BD\sinh(\tfrac{n\pi}{a}(b-x))\sin(\tfrac{n\pi}{a}y)$$

$$V_0=\sum_{n=1}C_n\sinh\left(\frac{n\pi b}{a}\right)\sin\left(\frac{n\pi}{a}y\right) \text{ for } x=b$$

$$C_n\sinh\left(\frac{n\pi b}{a}\right)=\frac{2}{a}\int_0^a V_0\sin\left(\frac{n\pi y}{a}\right)dy$$

$$C_n=\frac{2}{a\sinh\left(\frac{n\pi b}{a}\right)}\int_0^a V_0\sin\left(\frac{n\pi y}{a}\right)dy$$

$$C_n=\frac{2V_0}{a\sinh\left(\frac{n\pi b}{a}\right)}\frac{a}{n\pi}\left[-\cos\left(\frac{n\pi y}{a}\right)\right]_0^a$$

will be 0 with even n and 2 with odd n

$$C_n=\frac{4V_0}{n\pi\sinh\left(\frac{n\pi b}{a}\right)}$$

$$\Phi_r=\frac{4V_r}{\pi}\sum_{n=1,3,5\ldots}\frac{\sinh\left(\frac{n\pi(b-x)}{a}\right)\sin\left(\frac{n\pi y}{a}\right)}{n\sinh\left(\frac{n\pi b}{a}\right)}$$ where $a=y_0$ and $b=x_0$

2.1.2 $V=0$ when $x=0$
$V=0$ when $x=b$
$V=0$ when $y=a$
$V=V_0$ when $y=0$

$$(A\cos(mx)+B\sin(mx))(C\cosh(m(a-y))+D\sinh(m(a-y)))$$

At $x=0$, $V=0$ when $A=0$
At $x=b$, $V=0$ when $m=n\pi/b$
At $y=a$, $V=0$ when $C=0$

$$V=BD\sin\left(\frac{n\pi x}{b}\right)\sinh\left(\frac{n\pi}{b}(a-y)\right)$$

At $y=0$, $V=V_0$

$$V_0=\sum_{n=1} C_n \sin\left(\frac{n\pi x}{b}\right)\sinh\left(\frac{n\pi a}{b}\right)$$

$$C_n=\frac{4V_0}{n\pi\sinh\left(\frac{n\pi a}{b}\right)}$$

$$\Phi_b=\frac{4V_b}{\pi}\sum_{n=1,3,5,\ldots}\frac{\sinh\left(\frac{n\pi(a-y)}{b}\right)\sin\left(\frac{n\pi x}{b}\right)}{n\sinh\left(\frac{n\pi a}{b}\right)}$$

2.1.3 $V=0$ when $x=0$
$V=0$ when $x=b$
$V=0$ when $y=0$
$V=V_0$ when $y=a$

$$V=(A\cos(mx)+B\sin(mx))(C\cosh(my)+D\sinh(my))$$

At $x=0$, $V=0$ when $A=0$
At $x=b$, $V=0$ when $m=\frac{n\pi}{b}$
At $y=0$, $V=0$ when $C=0$
At $y=a$,

$$V_0=\sum_{n=1} C_n \sinh\left(\frac{n\pi a}{b}\right)\sin\left(\frac{n\pi x}{b}\right)$$

$$C_n=\frac{4V_0}{n\pi\sinh\left(\frac{n\pi a}{b}\right)}$$

$$\Phi_t=\frac{4V_t}{\pi}\sum_{n=1,3,5,\ldots}\frac{\sinh\left(\frac{n\pi y}{b}\right)\sin\left(\frac{n\pi x}{b}\right)}{n\sinh\left(\frac{n\pi a}{b}\right)}$$

2.1.4 $V=0$ when $y=0$

$V=0$ when $y=a$

$V=0$ when $x=0$

$V=V_0$ when $x=b$

$$V=(A\cosh(mx)+B\sinh(mx))(C\cos(my)+D\sin(my))$$

when $y=0$, $V=0$ when $C=0$

when $y=a$, $V=0$ when $m=\frac{n\pi}{a}$

when $x=0$, $V=0$ when $A=0$

when $x=b$,

$$V_0=\sum_{n=1} C_n \sinh\left(\frac{n\pi b}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

$$C_n=\frac{4V_0}{n\pi\sinh\left(\frac{n\pi b}{a}\right)}$$

$$\Phi_r=\frac{4V_r}{\pi}\sum_{n=1,3,5\ldots}\frac{\sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)}{n\sinh\left(\frac{n\pi b}{a}\right)}$$

2.1.5. $\Phi(x,y)=\Phi_L(x,y)+\Phi_r(x,y)+\Phi_t(x,y)+\Phi_b(x,y)$

$\Phi_L, \Phi_r, \Phi_t, \Phi_b$ are all solutions to Laplace's equation and can be added together without messing up the boundary conditions,

2.1.6. $$\Phi=\frac{4V_0}{\pi}\sum_{n=1,3,5\ldots}\left[\frac{\sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)+\sinh\left(\frac{n\pi(b-x)}{a}\right)\sin\left(\frac{n\pi y}{a}\right)}{n\sinh(n\pi b/a)}+\frac{\sinh\left(\frac{n\pi y}{b}\right)\sin\left(\frac{n\pi x}{b}\right)+\sinh\left(\frac{n\pi(a-y)}{b}\right)\sin\left(\frac{n\pi x}{b}\right)}{n\sinh(n\pi a/b)}\right]$$

At $x=0$, $$\Phi=\frac{4V_0}{\pi}\sum_{n=1,3,5\ldots}\frac{\sinh(n\pi b/a)\sin(n\pi y/a)}{n\sinh(n\pi b/a)}=\frac{4V_0}{\pi}\sum_{n=1,3,5\ldots}\frac{\sin(n\pi y/a)}{n}$$

This is the same for $x=b$, $y=0$, and $y=a$

We can apply $E=-\nabla\Phi$ to see that the E-field is denser along the edges.