6.1

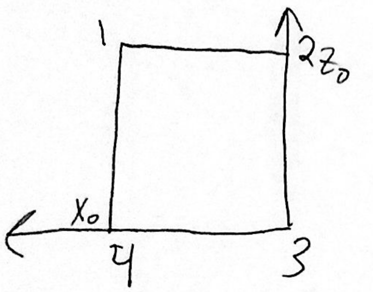
1. $$\nabla \times E = \left(\cancelto{0}{\frac{\partial E_z}{\partial y}} - \cancelto{0}{\frac{\partial E_y}{\partial z}}\right)\hat{x} + \left(\frac{\partial E_x}{\partial z} - \cancelto{0}{\frac{\partial E_z}{\partial x}}\right)\hat{y} + \left(\cancelto{0}{\frac{\partial E_y}{\partial x}} - \cancelto{0}{\frac{\partial E_x}{\partial y}}\right)\hat{z}$$

$$= \partial E_x/\partial z \;\hat{y} = -K E_{0x} \sin(K_z z - \omega t)\hat{y}$$

$$-\partial B/\partial t = -K E_{0x} \sin(K_z z - \omega t)\hat{y} \rightarrow B = K E_{0x} \int \sin(K_z z - \omega t)\, dt$$

$$B = \frac{K}{\omega} E_{0x} \cos(K_z z - \omega t) = \frac{1}{c} E_{0x} \cos(K_z z - \omega t)\hat{y}$$

2.



$$\oint E\cdot dl = \int_1^2 E\cdot dl + \int_2^3 E\cdot dl + \int_3^4 E\cdot dl + \int_4^1 E\cdot dl$$

$$\int_1^2 E\cdot dl = \int_1^2 E_{0x} \cos(K_z z - \omega t)\, dl$$

dl = length of line segment $= x_2 - x_1 = x_0 - 0 = x_0$

$$\int_1^2 E\cdot dl = E_{0x} x_0 \cos(K_z z_0 - \omega t)$$

No change in $z \rightarrow z = z_0$

$\int_2^3 E\cdot dl = 0$ because dl is perpendicular to the E-field

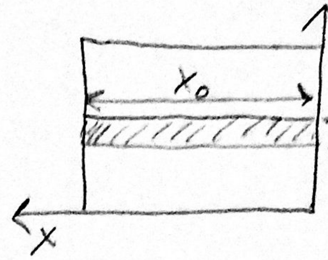
$\int_3^4 E\cdot dl = \int_3^4 E_{0x} \cos(K_z z - \omega t)\, dl$ $\quad dl = x_0 - 0 = x_0$ and no change in z, at $z = 0$

$$\int_3^4 E\cdot dl = E_{0x} x_0 \cos(\omega t) \qquad (\cos(-\omega t) = \cos(\omega t))$$

$\int_4^1 E\cdot dl = 0$ because dl is perpendicular to the E-field

$$\oint E\cdot dl = E_{0x} x_0 \cos(K_z z_0 - \omega t) + 0 + E_{0x} x_0 \cos(\omega t) + 0$$

$$= E_{0x} x_0 [\cos(K_z z_0 - \omega t) + \cos(\omega t)]$$



dz Area of E-field that contributes to B-flux $\quad dA = \text{length} \times \text{width} = x_0\, dz$

$$\Phi_B \int B \cdot dA = \int_0^{z_0} \frac{1}{c} E_{0x} \cos(K_z z - \omega t) x_0\, dz$$

$$= -\frac{1}{c}\frac{E_{0x} x_0}{K}\Big[\sin(-K_z z + \omega t)\Big]_0^{z_0} = -\frac{1}{c}\frac{E_{0x} x_0}{K}\Big[\sin(-K_z z_0 + \omega t) - \sin(\omega t)\Big]$$

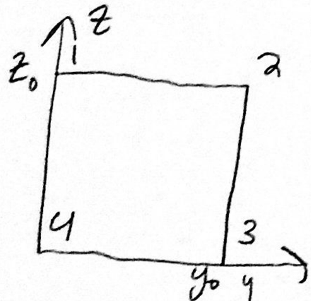
$$\sin(-K_z z_0 + \omega t) = -\sin(K_z z_0 + \omega t)$$

$$-\frac{d\Phi_B}{dt} = -\frac{1}{c}\frac{E_{0x} x_0}{K}\Big[\sin(K_z z - \omega t) + \sin(\omega t)\Big]\, d/dt$$

$$= \frac{1}{c}\frac{\omega}{K} E_{0x} x_0 \Big[\cos(K_z z_0 - \omega t) + \cos(\omega t)\Big]$$

$$= E_{0x} x_0 \Big[\cos(K_z z_0 - \omega t) + \cos(\omega t)\Big] = \oint E \cdot dl$$

3.



$$\oint B \cdot dl = \int_1^2 B \cdot dl + \int_2^3 B \cdot dl + \int_3^4 B \cdot dl + \int_4^1 B \cdot dl$$

$$\int_1^2 B \cdot dl = \int_1^2 \frac{1}{c} E_{0x} \cos(K_z z - \omega t)\, dl$$

$$dl = y_2 - y_1 = y_0 - 0 = y_0$$

$$\int_1^2 B \cdot dl = \frac{1}{c} E_{0x} \cos(K_z z_0 - \omega t) y_0$$

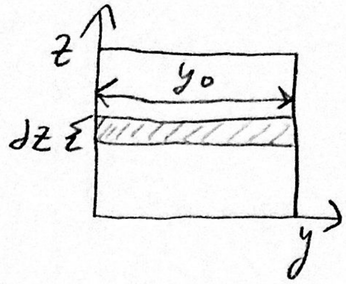
$\int_2^3 B \cdot dl = 0$ because dl is perpendicular to the B-field

$\int_3^4 B \cdot dl = \frac{1}{c} E_{0x} \cos(K_z z_1 - \omega t) y_0 \quad z_1 = 0$ so $\int_3^4 B \cdot dl = \frac{1}{c} E_{0x} \cos(\omega t) y_0$

$\int_4^1 B \cdot dl = 0$ because dl is perpendicular to the B-field

$$\oint B \cdot dl = \frac{1}{c} E_{0x} \cos(K_z z_0 - \omega t) y_0 + 0 + \frac{1}{c} E_{0x} \cos(\omega t) y_0 + 0$$

$$= \frac{1}{c} E_{0x} y_0 \Big[\cos(K_z z_0 - \omega t) + \cos(\omega t)\Big]$$



Area of B-field that contributes to the E-flux

$$dA = y_0 dz$$

$$\phi E = \int E \cdot dA = \int_0^{z_0} E_{0x} \cos(K_z z - \omega t) y_0 \, dz$$

$$= -\frac{E_{0x} y_0}{K} \left[\sin(-K_z z + \omega t)\right]_0^{z_0} = -\frac{E_{0x} y_0}{K}\left[-\sin(K_z z_0 - \omega t) - \sin(\omega t)\right]$$

$$= \frac{E_{0x} y_0}{K}\left[\sin(K_z z_0 - \omega t) + \sin(\omega t)\right]$$

$$-\frac{d\phi_E}{dt} = -\frac{E_{0x} y_0}{K}\left[\sin(K_z z_0 - \omega t) + \sin(\omega t)\right] d/dt$$

$$= \frac{\omega}{K} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right]$$

$$= c E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right]$$

$$\frac{1}{c^2}(c E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right] = \frac{1}{c} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right]$$

$$\frac{1}{c} E_{0x} y_0 \left[\cos(K_z z_0 - \omega t) + \cos(\omega t)\right] = \oint B \cdot dl$$