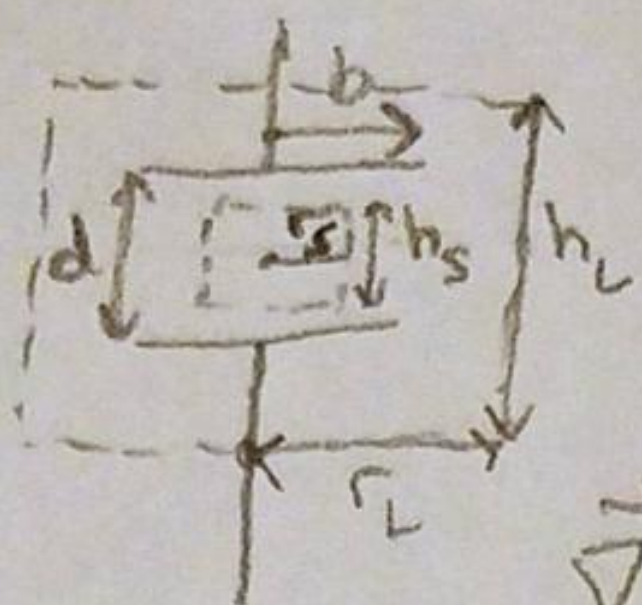


For the "small cylinder":

$$5.2 \quad \int_V \left[ \frac{d}{dt} \left( \frac{\vec{B} \cdot \vec{H}}{2} \right) + \frac{d}{dt} \left( \frac{\vec{D} \cdot \vec{E}}{2} \right) + \vec{E} \cdot \vec{J} \right] dV = - \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S}$$



$$E = \frac{Q}{\epsilon_0 S}$$

$$V = \frac{Qd}{\epsilon_0 S} \rightarrow \frac{Q}{\epsilon_0 S} = \frac{V}{d}$$

$$\hookrightarrow E = \frac{V}{d} \quad d = h_s, \text{ so } E = \frac{V}{h_s}$$

$$\vec{\nabla} \times \vec{H} = \epsilon_0 \frac{d\vec{E}}{dt} = \frac{\epsilon_0}{h_s} \frac{dV}{dt}$$

$$\oint_S \vec{H} \cdot d\vec{\ell} = \vec{H} \cdot 2\pi r_s = \frac{\epsilon_0}{h_s} \pi r_s^2 \frac{dV}{dt} \rightarrow H = \frac{\epsilon_0 r_s}{2h_s} \frac{dV}{dt}$$

$$\vec{E} \times \vec{H} = -\frac{\epsilon_0 r_s}{2h_s^2} V \frac{dV}{dt}$$

$$- \oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = 2\pi r_s h_s \left( \frac{\epsilon_0 r_s}{2h_s^2} V \frac{dV}{dt} \right) = \frac{d}{dt} \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{h_s} \right]$$

$$\int_V \left[ \frac{d}{dt} \left( \frac{\vec{B} \cdot \vec{H}}{2} \right) \right] dV = \int_V \left[ \frac{d}{dt} \left( \frac{\mu_0 \vec{H} \cdot \vec{H}}{2} \right) \right] dV = \frac{d}{dt} \left[ \frac{\mu_0}{2} \left( \frac{\epsilon_0}{2h_s} \frac{dV}{dt} \right)^2 \int_0^{r_s} r^2 2\pi r dr \right]$$

$$= \frac{d}{dt} \left[ \frac{\mu_0 \epsilon_0 r_s^2}{16} \left( \frac{\pi r_s^2 \epsilon_0}{h_s} \right) \left( \frac{dV}{dt} \right)^2 \right]$$

$$\int_V \left[ \frac{d}{dt} \left( \frac{\vec{D} \cdot \vec{E}}{2} \right) \right] dV = \int_V \left[ \frac{d}{dt} \left( \frac{\epsilon_0 \vec{E} \cdot \vec{E}}{2} \right) \right] dV$$

$$= \frac{d}{dt} \left[ \frac{1}{2} \epsilon_0 (h_s \pi r_s^2) \left( \frac{V}{h_s} \right)^2 \right] = \frac{d}{dt} \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{2h_s} \right]$$

$$\int_V \vec{E} \cdot \vec{J} dV = 0 \text{ because no current is flowing into the "small cylinder"}$$

So we have:

$$\frac{d}{dt} \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{h_s} \right] = \frac{d}{dt} \left[ \frac{\mu_0 \epsilon_0 r_s^2}{8} \left( \frac{\pi r_s^2 \epsilon_0}{2h_s} \right) \left( \frac{dV}{dt} \right)^2 \right] + \frac{d}{dt} \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{2h_s} \right] + 0$$

When  $\frac{\mu_0 \epsilon_0 r_s^2}{8} \left( \frac{dV}{dt} \right)^2 \ll V^2$ , the equation becomes:

$$\frac{d}{dt} \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{h_s} \right] = \frac{d}{dt} \left( \frac{1}{2} \right) \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{h_s} \right] + \frac{d}{dt} \left( \frac{1}{2} \right) \left[ \frac{\pi r_s^2 \epsilon_0 V^2}{h_s} \right]$$

Assuming no charges on the plate at  $t=0$  and that  $E = \frac{Q}{\epsilon_0 S}$



For the "large cylinder," the  $\vec{E}$  field is 0 outside  
of the capacitor,  $-\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S} = 0$