8.1.

1. For $0 < s < a$

$$j = \frac{I}{A} = \frac{I}{\pi a^2}$$

$$I_{enc} = j(\pi s^2) = \frac{I}{\pi a^2}(\pi s^2) = \frac{I s^2}{a^2}$$

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$B \cdot 2\pi s = \mu_0 \left(\frac{I s^2}{a^2}\right) \rightarrow B = \frac{\mu_0 I s}{2\pi a^2}$$

Flux linkage $\Phi_a = \int_0^a \frac{\mu_0 I s}{2\pi a^2} ds = \frac{\mu_0 I}{2\pi a^2}\left[\frac{s^2}{2}\right]_0^a = \frac{\mu_0 I}{4\pi}$

$$\frac{L_{ia}}{l} = \frac{\Phi}{Il} = \frac{\mu_0}{4\pi l}$$

For $b < s < c$

$$j = \frac{I}{A} = \frac{I}{\pi(c^2-b^2)}$$

$$I_{outer} = I - I\left[\frac{\pi(s^2-b^2)}{\pi(c^2-b^2)}\right] = I\left[\frac{c^2-s^2}{c^2-b^2}\right]$$

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$B \cdot 2\pi s = \mu_0\left(I\left[\frac{c^2-s^2}{c^2-b^2}\right]\right) \rightarrow B = \frac{\mu_0 I}{2\pi s}\left[\frac{c^2-s^2}{c^2-b^2}\right]$$

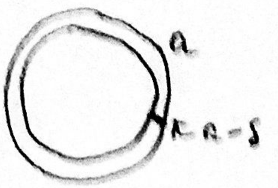
$$\Phi_b \int_b^c B \cdot ds = \int_b^c \frac{\mu_0 I}{2\pi s}\left[\frac{c^2-s^2}{c^2-b^2}\right] ds = \frac{\mu_0 I}{2\pi(c^2-b^2)}\left[c^2 \ln\left(\frac{c}{b}\right) - \frac{(c^2-b^2)}{2}\right]$$

$$= \frac{\mu_0 I}{2\pi(c^2-b^2)}\left(c^2 \ln\left(\frac{c}{b}\right)\right) + \underbrace{\frac{\mu_0 I}{4\pi}}_{\text{internal}}$$

$$\frac{L_{ib}}{l} = \frac{\Phi}{Il} = \frac{\mu_0}{4\pi L}$$

$$\frac{L_{i\,total}}{l} = \frac{L_{ia}}{l} + \frac{L_{ib}}{l} = \frac{\mu_0}{2\pi l}$$

For $0<s<a$



$$j=\frac{I}{A}=\frac{I}{\pi(a^2-(a-\delta)^2)}=\frac{I}{\pi(2a\delta-\delta^2)}=\frac{I}{2\pi a\delta(1-\delta/2a)}$$

$$I^{enc}=\frac{I\pi s^2}{2\pi a\delta(1-\delta/2a)}=\frac{Is^2}{2a\delta(1-\delta/2a)}$$

$$B=\frac{\mu_0 I s^2}{(2\pi s)2a\delta(1-\delta/2a)}=\frac{\mu_0 I s}{4\pi a\delta(1-\delta/2a)}$$

$$\Phi_a\int_{a-\delta}^{a}\frac{\mu_0 I s}{4\pi a\delta(1-\delta/2a)}\,ds=\frac{\mu_0 I}{4\pi a\delta(1-\delta/2a)}\left[\frac{s^2}{2}\right]_{a-\delta}^{a}$$

$$=\frac{\mu_0 I}{4\pi a\delta(1-\delta/2a)}\left[\frac{a^2}{2}-\frac{(a-\delta)^2}{2}\right]=\frac{\mu_0 I}{4\pi a\delta(1-\delta/2a)}\left[a\delta-\delta^2/2\right]$$

$$=\frac{\mu_0 I}{4\pi a\delta(1-\delta/2a)}(a\delta)\left[1-\delta/2a\right]=\frac{\mu_0 I}{4\pi}$$

$$\frac{L_{ia}}{l}=\frac{\phi}{Il}=\frac{\mu_0}{4\pi l}$$

For $b<s<c$

$$j=\frac{I}{A}=\frac{I}{\pi((b+\delta)^2-b^2)}=\frac{I}{\pi(2b\delta+\delta^2)}=\frac{I}{2\pi b\delta(1+\delta/2b)}$$

$$I^{enc}=I-I\left[\frac{\pi(s^2-b^2)}{2\pi b\delta(1+\delta/2b)}\right]=I\left[\frac{(b+\delta)^2-s^2}{2b\delta(1+\delta/2b)}\right]$$

$$B=\frac{\mu_0 I}{2\pi s}\left[\frac{(b+\delta)^2-s^2}{2b\delta(1+\delta/2b)}\right]$$

$$\Phi_b=\int_{b}^{b+\delta}\frac{\mu_0 I}{2\pi s}\left[\frac{(b+\delta)^2-s^2}{2b\delta(1+\delta/2b)}\right]ds=\frac{\mu_0 I}{4\pi b\delta(1+\delta/2b)}\left[(b+\delta)^2\ln\left(\frac{b+\delta}{b}\right)-\frac{(b+\delta)^2-b^2}{2}\right]$$

$$=\frac{\mu_0 I}{2\pi(2b\delta+\delta^2)}(b+\delta)^2\ln\left(\frac{b+\delta}{a}\right)+\frac{\mu_0 I}{4\pi}$$

$$\frac{L_{ib}}{l}=\frac{\phi}{I}=\frac{\mu_0}{4\pi l}$$

2. for $a<s<b$

$$\oint B \cdot dl = \mu_0 I$$

$$B \cdot 2\pi s = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi s}$$

$$\Phi = \int_a^b \frac{\mu_0 I}{2\pi s} ds = \frac{\mu_0 I}{2\pi} [\ln(s)]_a^b = \frac{\mu_0 I}{2\pi} \ln(b/a)$$

$$\frac{L_0}{L} = \frac{\Phi}{I} = \frac{\mu_0 \ln(b/a)}{2\pi L}$$

$$\frac{L_{ob}}{L} = \frac{\mu_0 c^2}{2\pi(c^2-b^2)L} \ln\left(\frac{c}{b}\right) \text{ from part 1}$$

3. For $a<s<b$

$$L = \frac{\mu_0}{2\pi} \ln(b/a)$$

If $b = a+\delta$, $L = \frac{\mu_0}{2\pi} \ln(a+\delta/a) = \frac{\mu_0}{2\pi} \ln(1+\delta/a)$

for $\delta/a \ll 1$, $\ln(1+z) \approx z$

$$L = \frac{\mu_0}{2\pi}(\delta/a) = \frac{\mu_0 \delta}{2\pi a}$$

$$\frac{L}{l} = \frac{\mu_0 \delta}{2\pi a l} = \frac{\mu_0 \delta}{A}$$ where A = area of planes