11.1.1.

Voltage Law:

Loop 1: $Re[\tilde{V}_0 e^{j\omega t}] - L_1 \frac{d}{dt} Re[\tilde{I}_1 e^{j\omega t}] - \frac{1}{C}\int (Re[\tilde{I}_1 e^{j\omega t}] - Re[\tilde{I}_2 e^{j\omega t}])dt = 0$

$= Re[\tilde{V}_0 e^{j\omega t}] - L_1 (j\omega) Re[\tilde{I}_1 e^{j\omega t}] - \frac{1}{C}\int (Re[\tilde{I}_1 e^{j\omega t}] - Re[\tilde{I}_2 e^{j\omega t}])dt = 0$

$= Re[\tilde{V}_0 - j\omega L_1 \tilde{I}_1 - \frac{1}{C}\int (\tilde{I}_1 - \tilde{I}_2)dt] e^{j\omega t} = 0$

$= Re[\tilde{V}_0 - j\omega L_1 \tilde{I}_1 - \tilde{V}_1] e^{j\omega t} = 0$

$= \tilde{V}_0 - j\omega L_1 \tilde{I}_1 - \tilde{V}_1 = 0 \rightarrow \tilde{V}_1 = \tilde{V}_0 - j\omega L_1 \tilde{I}_1$

Loop 2: $-\frac{1}{C}\int (Re[\tilde{I}_2 e^{j\omega t}] - Re[\tilde{I}_1 e^{j\omega t}])dt - L_2 \frac{d}{dt} Re[\tilde{I}_2 e^{j\omega t}] - Z_L Re[\tilde{I}_2 e^{j\omega t}] = 0$

$= \frac{1}{C}\int (Re[\tilde{I}_2 e^{j\omega t}] - Re[\tilde{I}_1 e^{j\omega t}])dt - L_2 (j\omega) Re[\tilde{I}_2 e^{j\omega t}] - Z_L Re[\tilde{I}_2 e^{j\omega t}] = 0$

$= Re[\frac{1}{C}\int (\tilde{I}_2 - \tilde{I}_1)dt - L_2 (j\omega)\tilde{I}_2 - Z_L \tilde{I}_2] e^{j\omega t} = 0$

$= Re[-(-\tilde{V}_1) - j\omega L_2 \tilde{I}_2 - Z_L \tilde{I}_2] e^{j\omega t} = 0$

Should justify the -> step that led to this.

$= \tilde{V}_1 - j\omega L_2 \tilde{I}_2 - \tilde{V}_2 = 0 \rightarrow \tilde{V}_2 = \tilde{V}_1 - j\omega L_2 \tilde{I}_2$

Current Law:

Node 1: $\frac{1}{L_1}\int (Re[\tilde{V}_1 e^{j\omega t}] - Re[\tilde{V}_0 e^{j\omega t}])dt + \frac{1}{L_2}\int (Re[\tilde{V}_1 e^{j\omega t}] - Re[\tilde{V}_2 e^{j\omega t}])dt + C\frac{d}{dt} Re[\tilde{V}_1 e^{j\omega t}] = 0$

$= Re[\frac{1}{L_1}\int (\tilde{V}_1 - \tilde{V}_0)dt + \frac{1}{L_2}\int (\tilde{V}_1 - \tilde{V}_2)dt + C(j\omega)\tilde{V}_1] e^{j\omega t} = 0$

$= Re[-\tilde{I}_1 + \tilde{I}_2 + j\omega C \tilde{V}_1] e^{j\omega t} = 0$

$= -\tilde{I}_1 + \tilde{I}_2 + j\omega C V_1 = 0 \rightarrow \tilde{I}_2 = \tilde{I}_1 - j\omega C V_1$

Node 2: $\frac{1}{L_2}\int (Re[\tilde{V}_2 e^{j\omega t}] - Re[\tilde{V}_1 e^{j\omega t}])dt + \frac{Re[\tilde{V}_2 e^{j\omega t}]}{Z_L} = 0$

$= Re[\frac{1}{L_2}\int (\tilde{V}_2 - \tilde{V}_1)dt + \frac{\tilde{V}_2}{Z_L}] e^{j\omega t} = 0$

$= Re[-\tilde{I}_2 + \frac{\tilde{V}_2}{Z_L}] e^{j\omega t} = 0$

$= -\tilde{I}_2 + \frac{\tilde{V}_2}{Z_L} = 0 \rightarrow \tilde{I}_2 = \frac{\tilde{V}_2}{Z_L}$

11.1.2. $Z_1 = C \parallel (L+Z)$ where $L = j\omega L$, $C = \frac{1}{j\omega C}$ and $Z = Z_L$

$$Z_0 = L + Z_1 = L + C \parallel (L+Z) = L + \frac{C(L+Z)}{C+L+Z} = \frac{2LC + L^2 + LZ + CZ}{C+L+Z}$$

11.1.3. $\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0} = \frac{V_0(C+L+Z)}{L(2C+L+Z)+CZ}$

$$\tilde{V}_1 = \tilde{V}_0 - L\tilde{I}_1 = V_0 - \frac{V_0 \cdot L[C+L+Z]}{L[2C+L+Z]+CZ} = \frac{V_0[CL+CZ]}{L[2C+L+Z]+CZ}$$

$$\tilde{I}_2 = \tilde{I}_1 - \frac{1}{C}\tilde{V}_1 = \frac{V_0[C+L+Z]}{L[2C+L+Z]+CZ} - \frac{V_0 C[L+Z]}{C[L(2C+L+Z)+CZ]}$$

$$= \frac{V_0 C}{L(2C+L+Z)+CZ}$$

$$\tilde{V}_2 = \tilde{V}_1 - L\tilde{I}_2 = \frac{V_0[CL+CZ]}{L[2C+L+Z]+CZ} - \frac{V_0 CL}{L(2C+L+Z)+CZ} = \frac{V_0 CZ}{L(2C+L+Z)+CZ}$$

Check: $\tilde{I}_2 = \frac{\tilde{V}_2}{Z} = \frac{V_0 CZ}{Z[L(2C+L+Z)+CZ]} = \frac{V_0 C}{L(2C+L+Z)+CZ}$

$$\tilde{V}_1(t) = \tilde{V}_1 e^{j\omega t} = \frac{V_0[CL+CZ]e^{j\omega t}}{L[2C+L+Z]+CZ}$$

1/Z_c

Check: $\tilde{I}_2 = \tilde{I}_1 - C\frac{d}{dt}\tilde{V}_1(t) = \frac{V_0(C+L+Z)}{L(2C+L+Z)+CZ} - \overset{=1/C}{j\omega C}\left[\frac{V_0[CL+CZ]}{L[2C+L+Z]+CZ}\right]$

$$\tilde{I}_2 = \frac{V_0(C+L+Z)}{L(2C+L+Z)+CZ} - \frac{V_0 C[L+Z]}{C[L(2C+L+Z)+CZ]} = \frac{V_0 C}{L(2C+L+Z)+CZ}$$

$$\tilde{I}_1(t) = \tilde{I}_1 e^{j\omega t} = \frac{V_0(C+L+Z)e^{j\omega t}}{L(2C+L+Z)+CZ}$$

Z_{inductor }

Check: $\tilde{V}_1 = \tilde{V}_0 - L\frac{d}{dt}\tilde{I}_1(t) = V_0 - \overset{=L}{j\omega L}\left[\frac{V_0(C+L+Z)}{L(2C+L+Z)+CZ}\right] = \frac{V_0[CL+CZ]}{L(2C+L+Z)+CZ}$

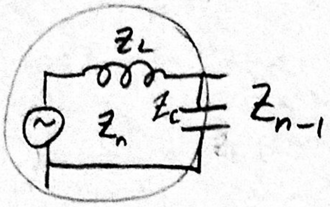
$$\tilde{I}_2(t) = \tilde{I}_2 e^{j\omega t} = \frac{V_0 C e^{j\omega t}}{L(2C+L+Z)+CZ}$$

$$\tilde{V}_2(t) = \tilde{V}_2 e^{j\omega t} = \frac{V_0 C Z e^{j\omega t}}{L(2C+L+Z)+CZ}$$

Check: $\tilde{V}_2 = \tilde{V}_1 - L\frac{d}{dt}\tilde{I}_2(t) = \frac{V_0[CL+CZ]}{L[2C+L+Z]+CZ} - \overset{=L}{j\omega L}\left[\frac{V_0 C}{L(2C+L+Z)+CZ}\right]$

$$= \frac{V_0 C Z}{L(2C+L+Z)+CZ}$$

11.1.4. 1.



$$Z_n = Z_L + Z_c \| Z_{n-1} = Z_L + \frac{Z_{n-1}\cdot Z_c}{Z_{n-1}+Z_c}$$

$$Z_n = j\omega L + \frac{\left(\frac{1}{j\omega C}\cdot Z_{n-1}\right)}{Z_{n-1}+\left(\frac{1}{j\omega C}\right)} = j\omega L + \frac{Z_{n-1}}{Z_{n-1}(j\omega C)+1}$$

2. $\tilde{V}_2 = \tilde{V}_1 - j\omega L\tilde{I}_2$

$\tilde{V}_2 = \tilde{V}_1 - j\omega L[\tilde{I}_1 - j\omega C\tilde{V}_1]$

$\tilde{V}_2 = \tilde{V}_1[1-\omega^2 CL] - j\omega L\tilde{I}_1$

$\tilde{V}_{n+1} = \tilde{V}_n[1-\omega^2 CL] - j\omega C\tilde{I}_n$

3. $\tilde{I}_2 = \tilde{I}_1 - j\omega C\tilde{V}_1$

$\tilde{I}_2 = \tilde{I}_1 - j\omega C[\tilde{V}_2 + j\omega L\tilde{I}_2]$

$\tilde{I}_2 = \tilde{I}_1 - j\omega C\tilde{V}_2 + \omega^2 CL\tilde{I}_2$

$\tilde{I}_2(1-\omega^2 CL) = \tilde{I}_1 - j\omega C\tilde{V}_2$

$$\tilde{I}_2 = \frac{\tilde{I}_1 - j\omega C\tilde{V}_2}{(1-\omega^2 CL)} \rightarrow \tilde{I}_{n+1} = \frac{\tilde{I}_n - j\omega C\tilde{V}_{n+1}}{(1-\omega^2 CL)}$$