

Need more discussion of figures and why they provide a justification.

1. For an infinite ladder network, $Z = Z_L + Z_C \parallel Z = Z_L + \frac{Z_C Z}{Z + Z_C}$
 $\rightarrow Z = \frac{1}{2} (Z_L + \sqrt{Z_L^2 + 4Z_L Z_C})$

$$Z = \frac{1}{2} (i\omega L + \sqrt{-\omega^2 L^2 + 4(L/C)})$$

If each section of the transmission line has a length of δx , $L = L' \delta x$ and $C = C' \delta x$

$$\text{so } Z = \frac{1}{2} (i\omega L' \delta x + \sqrt{-(\omega L' \delta x)^2 + 4(L'/C')})$$

$$\text{As } \delta x \rightarrow 0, Z = \frac{1}{2} \pm \sqrt{4(L'/C')} = \sqrt{L'/C'}$$

For this to be true, load impedance must equal characteristic impedance: $Z = Z_{\text{load}} = \sqrt{L'/C'}$

2. When $Z_{\text{load}} \neq \sqrt{L'/C'}$, the load is mismatched. A $1/4$ transmission line can be used to match the load with a line of characteristic impedance Z_0 . The characteristic impedance of the transmission line to match Z_0 would be

$$Z_{\text{in}} = Z_0' \left[\frac{Z_L + jZ_0' \tan \beta l}{Z_0' + jZ_L \tan \beta l} \right]$$

$$\beta l = (2\pi/\lambda)(1/4) = \pi/2 \text{ so } \tan \beta l \rightarrow \infty \text{ and } Z_{\text{in}} = Z_0' \left(\frac{Z_0'}{Z_L} \right)$$

$$\text{For a matched system, } Z_{\text{in}} = Z_0 \text{ so } Z_{\text{in}} = Z_0 = \frac{(Z_0')^2}{Z_L}$$
$$Z_0' = \sqrt{Z_0 Z_L}$$

Therefore, a ladder network solution with $Z_L \neq \sqrt{L'/C'}$ would require a transmission line of characteristic impedance

$Z_0' = \sqrt{Z_0 Z_L}$ in order to act as a continuous line with characteristic impedance of Z_0 .

3. For $Z = \frac{1}{2} (i\omega L \pm \sqrt{-\omega^2 L^2 + 4(L/c)})$

When ω is small, $(L/c) \gg (\omega L)$ and the discrete approximation becomes $Z = \frac{1}{2} \sqrt{4(L/c)} = \sqrt{L/c}$

Which is the characteristic impedance of an infinitely long continuous transmission line.