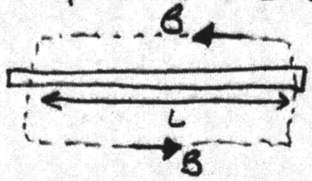
9.1.

1. Treating the top of the duct as a sheet:

top sheet $\vec{K}_1 \odot$

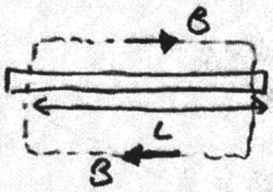


$$\oint B \cdot dl = \mu_0 I_{encl}$$

$$B(2 \cdot L) = \mu_0 (K_1 L) \rightarrow B = \frac{\mu_0 K_1}{2} \text{ under sheet}$$

$$B = -\frac{\mu_0 K_1}{2} \text{ above sheet}$$

Bottom sheet $\vec{K}_1 \otimes$



$$\oint B \cdot dl = \mu_0 I_{encl}$$

$$B(2 \cdot L) = \mu_0 (K_1 L)$$

$$B = -\frac{\mu_0 K_1}{2} \text{ above sheet}$$

$$B = \frac{\mu_0 K_1}{2} \text{ below sheet}$$

$$B_{total} = -\frac{\mu_0 K_1}{2} + \frac{\mu_0 K_1}{2} = 0 \text{ outside}$$ ✓

$$B_{total} = \frac{\mu_0 K_1}{2} + \frac{\mu_0 K_1}{2} = \mu_0 K_1 \text{ inside}$$ ✓

2. $\Phi_m = \int B \cdot dA = \int \mu_0 K_1 \, dA = \mu_0 K_1 (h_1 w)$

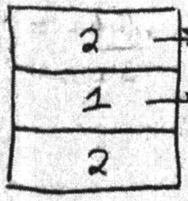
$\Phi_m = \mu_0 K_1 h_1 w$

$$-L_1 \frac{dI_1}{dt} = -\frac{d\Phi_m}{dt}$$

$$L_1 = \frac{d\Phi_m}{dt} \cdot \frac{dt}{dI_1} = \frac{\Phi_m}{I_1}$$

$$L_1 = \frac{\mu_0 K_1 h_1 w}{(K_1 L)} = \frac{\mu_0 h_1 w}{L} = \frac{\mu_0 A_1}{L}$$ ✓

3.



$\rightarrow B_{tot_2} = B_2 = B$

$\rightarrow B_{tot_1} = B_1 + B_2 = 2B$

$$\mathcal{E}_1 = -\frac{d\Phi_m}{dt} = -\frac{d(B_{tot_1} \cdot A_1)}{dt} = -\frac{d\, 2BA_1}{dt}$$

$$\mathcal{E}_2 = -\frac{d\Phi_m}{dt} = -\frac{d}{dt}(B_{tot_2} \cdot A_2 + B_{tot_1} \cdot A_1) = -\frac{d}{dt}(B(A_2 - A_1) + 2B \cdot A_1)$$

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 = -\frac{d\, 2BA_1}{dt} - \frac{d\, BA_2}{dt} + \frac{d}{dt} BA_1 - \frac{d}{dt} 2BA_1$$

$$\mathcal{E} = -\frac{d\, 3BA_1}{dt} - \frac{dBA_2}{dt} = \frac{-d}{dt}(3BA_1 + BA_2) \checkmark$$

$$\mathcal{E} = -3\mu_0 K_1 (h_1 w) - \mu_0 K_2 (h_2 w)$$

$$\mathcal{E} = -\frac{3\mu_0 K_1 L \cdot A_1}{L} - \frac{\mu_0 K_2 \cdot L \cdot A_2}{L}$$

$$\mathcal{E} = -L_n \frac{dI_n}{dt}$$

$$L = \frac{\mathcal{E}}{I} = \frac{\mathcal{E}}{K_n L} = \frac{3\mu_0 A_1}{L} + \frac{\mu_0 A_2}{L}$$

4. $$L_{ext} = -\mathcal{E}_1 \cdot \left(\frac{dt}{dI_1}\right) = -\frac{d(2BA_1)}{dt} \cdot \frac{dt}{dI}$$

$$= \frac{2BA_1}{2I} = \frac{BA_1}{I}$$

$$L_{int} = -\mathcal{E}_2 \cdot \left(\frac{dt}{dI}\right) = -\frac{d(B_{21}(A_2 - A_1))}{dt} \cdot \frac{dt}{dI} = \frac{B(A_2 - A_1)}{I}$$

$$L_{total} = L_{ext} + L_{int} = \frac{BA_1}{I} + \frac{BA_2}{I} - \frac{BA_1}{I} = \frac{BA_2}{I} \times$$

5. Based on these results, the inductance computed in #4 is smaller because it was missing the effect of the flux from the first duct on the second duct.

$\checkmark$

9.2.1.

$$\tilde{V}_n(z) = \tilde{V}_n^+ e^{-j\beta_n z} + \tilde{V}_n^- e^{j\beta_n z}$$

$$= \tilde{V}_n^+ e^{-j\beta_n z}\left(1 + \frac{\tilde{V}_n^-}{\tilde{V}_n^+} e^{2j\beta_n z}\right) = \tilde{V}_n^+ e^{-j\beta_n z}(1 + \tilde{\Gamma}(z))$$

$$\tilde{I}_n(z) = \frac{1}{Z_n}(\tilde{V}_n^+ e^{-j\beta_n z} - \tilde{V}_n^- e^{j\beta_n z})$$

$$= \frac{\tilde{V}_n^+}{Z_n} e^{-j\beta_n z}\left(1 - \frac{\tilde{V}_n^-}{\tilde{V}_n^+} e^{2j\beta_n z}\right) = \frac{\tilde{V}_n^+}{Z_n} e^{-j\beta_n z}(1 - \tilde{\Gamma}(z))$$

where $\tilde{\Gamma}(z) = \frac{\tilde{V}_n^-}{\tilde{V}_n^+} e^{2j\beta_n z}$

Impedance $Z_n(z) = \frac{\tilde{V}_n(z)}{\tilde{I}_n(z)} = Z_n\left[\frac{1 + \tilde{\Gamma}_n(z)}{1 - \tilde{\Gamma}_n(z)}\right]$ ✓

At the boundary:

$\tilde{V}_0(z_{01}) = \tilde{V}_1(z_{01})$ and $\tilde{I}_0(z_{01}) = \tilde{I}_1(z_{01})$

$$\tilde{V}_0(z_{01}) = \tilde{V}_0^+ e^{-j\beta_0 z_{01}}(1 + \tilde{\Gamma}_0(z_{01}))$$

$$\tilde{V}_1(z_{01}) = \tilde{V}_1^+ e^{-j\beta_1 z_{01}}(1 + \tilde{\Gamma}_1(z_{01}))$$

$$\tilde{I}_0(z_{01}) = \frac{\tilde{V}_0^+}{Z_0} e^{-j\beta_0 z_{01}}(1 - \tilde{\Gamma}_0(z_{01}))$$

$$\tilde{I}_1(z_{01}) = \frac{\tilde{V}_1^+}{Z_1} e^{-j\beta_1 z_{01}}(1 - \tilde{\Gamma}_1(z_{01}))$$

✓

$$\tilde{Z}_0(z_{01}) = \frac{\tilde{V}_0(z_{01})}{\tilde{I}_0(z_{01})} = \frac{\tilde{V}_0^+ e^{-j\beta_0 z_{01}}(1 + \tilde{\Gamma}_0(z_{01}))}{\left(\frac{\tilde{V}_0^+}{Z_0}\right) e^{-j\beta_0 z_{01}}(1 - \tilde{\Gamma}_0(z_{01}))} = Z_0\left[\frac{1 + \tilde{\Gamma}_0(z_{01})}{1 - \tilde{\Gamma}(z_{01})}\right]$$

Since $\tilde{V}_0(z_{01}) = \tilde{V}_1(z_{01})$ and $\tilde{I}_0(z_{01}) = \tilde{V}_1(z_{01})$

$\tilde{Z}_0(z_{01}) = \tilde{Z}_1(z_{01})$ ✓

$$Z_0\left[\frac{1+\tilde{\Gamma}_0(z_{01})}{1-\tilde{\Gamma}_0(z_{01})}\right]=Z_1$$

$$\tilde{\Gamma}_0(z_{01})=\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}$$ ✓

$$\tilde{\Gamma}_0(z)=\frac{\tilde{V}_0^-}{\tilde{V}_0^+}e^{2j\beta_0 z}\rightarrow\tilde{V}_0^-(z)=\tilde{V}_0^+e^{2j\beta_0 z_{01}}\left[1+\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}\right]$$ since $\tilde{V}_0^+$ is Known

$$\tilde{\Gamma}_1(z)=\frac{\tilde{V}_1^-}{\tilde{V}_1^+}e^{2j\beta_1 z}\rightarrow\tilde{V}_1^+(z)=\tilde{V}_1^-e^{2j\beta_1 z_{12}}\left[1+\frac{Z_2(z_{12})+Z_1}{Z_2(z_{12})-Z_1}\right]$$

$$\tilde{V}_2^+(z)=\tilde{V}_2^-e^{2j\beta_2 z_{23}}\left[1+\frac{Z_3(z_{23})+Z_2}{Z_3(z_{23})-Z_2}\right]$$

$$\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}e^{-j\beta_0 z_{01}}=\tilde{\Gamma}_0(z_{01})\rightarrow\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}=e^{j\beta_0 z_{01}}\left[\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}\right]$$ ✓

$$\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}e^{-j\beta_1 z_{12}}=\tilde{\Gamma}_1(z_{12})\rightarrow\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}=e^{j\beta_1 z_{12}}\left[\frac{Z_2(z_{12})-Z_1}{Z_2(z_{12})+Z_1}\right]$$

9.2.2.

1. $V(z,t)=V^+[\cos(\omega t-\beta z)+\rho\cos(\omega t+\beta z)]$

   $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

   $V(z,t)=V^+[\cos(\omega t)\cos(\beta z)+\sin(\omega t)\sin(\beta z)+\rho\cos(\omega t)\cos(\beta z)-\rho\sin(\omega t)\sin(\beta z)]$

   $V(z,t)=V^+[(1+\rho)\cos(\omega t)\cos(\beta z)+(1-\rho)\sin(\omega t)\sin(\beta z)]$

   $V(z,t)=A\cos(\omega t)\cos(\beta z)+B\sin(\omega t)\sin(\beta z)$ where $A=V^+(1+\rho)$ and $B=V^+(1-\rho)$ ✓

3. This plot shows 2 oppositely travelling waves which will produce a standing wave. The VSWR can be calculated from the wave amplitudes to determine the impedance mismatch and describe how efficiently radio-frequency power is transmitted. ✓