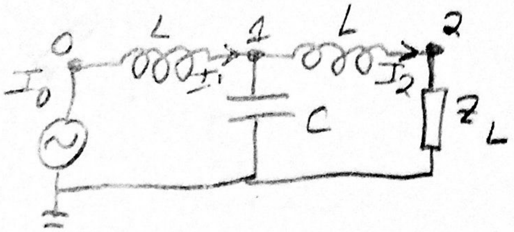


11.1.1. Kirchhoff's voltage law:

Loop 1: $\tilde{V}_0 - L_1 \frac{d\tilde{I}_1}{dt} - \frac{1}{C}\int(I_1 - I_2)\,dt = 0$

$\tilde{V}_0 - L_1 \frac{d}{dt}\tilde{I}_1 e^{j(\omega t+\theta)} - \tilde{V}_1 = 0$

$\tilde{V}_0 - L_1 \tilde{I}_1 (j\omega) - \tilde{V}_1 = 0$

$\rightarrow \tilde{V}_1 = \tilde{V}_0 - j\omega L_1 \tilde{I}_1$

Loop 2: $-\frac{1}{C}\int(I_2 - I_1)\,dt - L_2 \frac{d\tilde{I}_2}{dt} - Z_L \tilde{I}_2 = 0$

$-(-\tilde{V}_1) - L_2 \frac{d}{dt}\tilde{I}_2 e^{j(\omega t+\theta)} - \tilde{V}_2 = 0$

$\tilde{V}_1 - L_2 \tilde{I}_2 (j\omega) - \tilde{V}_2 = 0 \rightarrow \tilde{V}_2 = \tilde{V}_1 - j\omega L_2 \tilde{I}_2$

Kirchoff's Current Law:

Node 1: $\frac{1}{L_1}\int(V_1 - V_0)\,dt + \frac{1}{L_2}\int(V_1 - V_2)\,dt + C\frac{dV_1}{dt} = 0$

$-\tilde{I}_1 + \tilde{I}_2 + C \frac{d}{dt}\tilde{V}_1 e^{j(\omega t+\theta)} = 0$

$-\tilde{I}_1 + \tilde{I}_2 + C\tilde{V}_1(j\omega) = 0 \rightarrow \tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1$

Node 2: $\frac{1}{L_2}\int(V_2 - V_1)\,dt + \frac{\tilde{V}_2}{Z_L} = 0$

$-\tilde{I}_2 + \frac{\tilde{V}_2}{Z_L} = 0 \rightarrow \tilde{I}_2 = \frac{\tilde{V}_2}{Z_L}$

11.1.2.

$Z_1 = Z_C \,||\, (Z_L + Z_L) \rightarrow Z_C \,||\, 2Z_L$

$= \frac{Z_C \cdot 2Z_L}{Z_C + 2Z_L}$

$Z_0 = Z_L + Z_1 = Z_L + \frac{Z_C \cdot 2Z_L}{Z_C + 2Z_L}$

11.1.3.

$$\tilde{V}_0 = V_0,\ Z_0 = Z_L + \frac{2Z_C Z_L}{2Z_L + Z_C}$$

$$\tilde{I}_1 = \frac{\tilde{V}_0}{Z_0} = \frac{V_0}{Z_L + \frac{2Z_L Z_C}{2Z_L + Z_C}} = \frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L}$$

$$\tilde{V}_1 = \tilde{V}_0 - j\omega L \tilde{I}_1 = V_0 - j\omega L\left[\frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L}\right] \quad Z_L = j\omega L \rightarrow \tilde{V}_1 = \frac{2V_0 Z_C}{2Z_L + 3Z_C}$$

$$\tilde{I}_2 = \tilde{I}_1 - j\omega C \tilde{V}_1 = \frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L} - j\omega C\left[\frac{2V_0 Z_C}{2Z_L + 3Z_C}\right]$$

$$Z_C = \frac{1}{j\omega C} \rightarrow \tilde{I}_2 = \frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L} - \frac{2V_0}{2Z_L + 3Z_C} = \frac{V_0 Z_C}{Z_L(3Z + 2Z_L)}$$

$$\tilde{V}_2 = \tilde{V}_1 - j\omega L \tilde{I}_2 = \frac{2V_0 Z_C}{2Z_L + 3Z_C} - j\omega L\left[\frac{V_0 Z_C}{Z_L(3Z_C + 2Z_L)}\right] = \frac{V_0 Z_C}{2Z_L + 3Z_C}$$

$$\text{* check: } \tilde{I}_2 = \frac{\tilde{V}_2}{Z_L} = \frac{V_0 Z_C}{Z_L(2Z_L + 3Z_C)} = \tilde{I}_2$$

$$V_1(t) = L_1 \frac{dI_1}{dt} e^{j(\omega t + \theta)} = j\omega L_1 I_1 = Z_L\left[\frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L}\right] = \frac{V_0(Z_C + 2Z_L)}{2Z_L + 3Z_C}$$

$$\text{*check: } V_1(t) = V_0 - V_1 = V_0 - \frac{2V_0 Z_C}{2Z_L + 3Z_C} = \frac{V_0(Z_C + 2Z_L)}{2Z_L + 3Z_C}$$

$$I_1(t) = C_1 \frac{dV_1}{dt} e^{j(\omega t + \theta)} = j\omega C_1 V_1 = \frac{1}{Z_C}\left[\frac{2V_0 Z_C}{2Z_L + 3Z_C}\right] = \frac{2V_0}{2Z_L + 3Z_C}$$

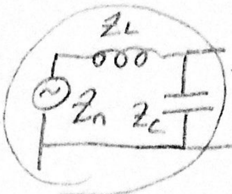
$$\text{*check: } I_1(t) = I_1 - I_2 = \frac{V_0(Z_C + 2Z_L)}{2Z_L^2 + 3Z_C Z_L} - \frac{V_0 Z_C}{Z_L(3Z_C + 2Z_L)} = \frac{2V_0}{2Z_L + 3Z_C}$$

$$V_2(t) = L_2 \frac{dI_2}{dt} e^{j(\omega t + \theta)} = j\omega L_2 I_2 = Z_L\left[\frac{V_0 Z_C}{Z_L(3Z_C + 2Z_L)}\right] = \frac{V_0 Z_C}{3Z_C + 2Z_L}$$

$$\text{*check: } V_2(t) = V_1 - V_2 = \frac{2V_0 Z_C}{2Z_L + 3Z_C} - \frac{V_0 Z_C}{2Z_L + 3Z_C} = \frac{V_0 Z_C}{3Z_C + 2Z_L}$$

$$I_2(t) = C_2 \frac{dV_2}{dt} e^{j(\omega t + \theta)} = j\omega C_2 V_2 = \frac{1}{Z_C}\left[\frac{V_0 Z_C}{2Z_L + 3Z_C}\right] = \frac{V_0}{2Z_L + 3Z_C}$$

11.1.4.

1. 

Z_{n-1}

$$Z_n = Z_L + Z_c \| Z_{n-1} = Z_L + \frac{Z_{n-1} \cdot Z_c}{Z_{n-1} + Z_c}$$

$$Z_n = \omega L + \frac{\left(\frac{1}{\omega C} \cdot Z_{n-1}\right)}{Z_{n-1} + \left(\frac{1}{\omega C}\right)} = \omega L + \frac{Z_{n-1}}{Z_{n-1}(\omega C)+1}$$

2. $$\left(I_1 - \frac{V_1}{2Z_L}\right) \cdot \frac{Z_c}{2} = \left[\frac{V_0(Z_c + 2Z_L)}{Z_L(2Z_L+3Z_c)} - \frac{2V_0 Z_c}{2Z_L+3Z_c} \cdot \frac{1}{2Z_L}\right] \cdot \frac{Z_c}{2}$$

$$= \left[\frac{V_0 Z_c}{Z_L(2Z_L+3Z_c)} + \frac{2V_0 \cancel{Z_L}}{\cancel{Z_L}(2Z_L+3Z_c)} - \frac{2V_0 Z_c}{2Z_L+3Z_c}\left(\frac{1}{2Z_L}\right)\right] \cdot \frac{Z_c}{2}$$

$$= \left[\frac{V_0 Z_c}{Z_L(2Z_L+3Z_c)} + \frac{2V_0}{2Z_L+3Z_c} - \frac{V_0 Z_c}{Z_L(Z_L+3Z_c)}\right] \cdot \frac{Z_c}{2}$$

$$= \frac{2V_0}{2Z_L+3Z_c} \cdot \frac{Z_c}{2} = \frac{V_0 Z_c}{2Z_L+3Z_c} = V_2 \longrightarrow V_{n+1} = \left(I_n - \frac{V_n}{2 \cdot Z_L}\right) \cdot \frac{Z_c}{2}$$

3. $$I_1 - \frac{2}{Z_c} \cdot V_2 = \frac{V_0(Z_c + 2Z_L)}{Z_L(2Z_L+3Z_c)} - \frac{2}{Z_c} \cdot \frac{V_0 Z_c}{2Z_L+3Z_c}$$

$$= \frac{V_0 Z_c}{Z_L(2Z_L+3Z_c)} + \frac{2V_0 \cancel{Z_L}}{\cancel{Z_L}(2Z_L+3Z_c)} - \frac{2V_0}{2Z_L+3Z_c} = \frac{V_0 Z_c}{Z_L(2Z_L+3Z_c)} = I_2$$

$$I_{n+1} = I_n - \frac{2}{Z_c} \cdot V_{n+1}$$