9.2.1.

$$\tilde{V}_n(z) = \tilde{V}_n^{+} e^{-j\beta_n z} + \tilde{V}_n^{-} e^{j\beta_n z}$$

$$= \tilde{V}_n^{+} e^{-j\beta_n z}\left(1 + \frac{\tilde{V}_n^{-}}{\tilde{V}_n^{+}} e^{2j\beta_n z}\right) = \tilde{V}_n^{+} e^{-j\beta_n z}(1 + \tilde{\Gamma}(z))$$

$$\tilde{I}_n(z) = \frac{1}{Z_n}\left(\tilde{V}_n^{+} e^{-j\beta_n z} - \tilde{V}_n^{-} e^{j\beta_n z}\right)$$

$$= \frac{\tilde{V}_n^{+}}{Z_n} e^{-j\beta_n z}\left(1 - \frac{\tilde{V}_n^{-}}{\tilde{V}_n^{+}} e^{2j\beta_n z}\right) = \frac{\tilde{V}_n^{+}}{Z_n} e^{-j\beta_n z}(1 - \tilde{\Gamma}(z))$$

where $\tilde{\Gamma}(z) = \frac{\tilde{V}_n^{-}}{\tilde{V}_n^{+}} e^{2j\beta_n z}$

Impedance $Z_n(z) = \frac{\tilde{V}_n(z)}{\tilde{I}_n(z)} = Z_n\left[\frac{1 + \tilde{\Gamma}_n(z)}{1 - \tilde{\Gamma}_n(z)}\right]$

At the boundary:

$$\tilde{V}_0(z_{01}) = \tilde{V}_1(z_{01}) \text{ and } \tilde{I}_0(z_{01}) = \tilde{I}_1(z_{01})$$

$$\tilde{V}_0(z_{01}) = \tilde{V}_0^{+} e^{-j\beta_0 z_{01}}(1 + \tilde{\Gamma}_0(z_{01}))$$

$$\tilde{V}_1(z_{01}) = \tilde{V}_1^{+} e^{-j\beta_1 z_{01}}(1 + \tilde{\Gamma}_1(z_{01}))$$

$$\tilde{I}_0(z_{01}) = \frac{\tilde{V}_0^{+}}{Z_0} e^{-j\beta_0 z_{01}}(1 - \tilde{\Gamma}_0(z_{01}))$$

$$\tilde{I}_1(z_{01}) = \frac{\tilde{V}_1^{+}}{Z_1} e^{-j\beta_1 z_{01}}(1 - \tilde{\Gamma}_1(z_{01}))$$

$$\tilde{Z}_0(z_{01}) = \frac{\tilde{V}_0(z_{01})}{\tilde{I}_0(z_{01})} = \frac{\tilde{V}_0^{+} e^{-j\beta_0 z_{01}}(1 + \tilde{\Gamma}_0(z_{01}))}{\left(\frac{\tilde{V}_0^{+}}{Z_0}\right) e^{-j\beta_0 z_{01}}(1 - \tilde{\Gamma}_0(z_{01}))} = Z_0\left[\frac{1 + \tilde{\Gamma}_0(z_{01})}{1 - \tilde{\Gamma}(z_{01})}\right]$$

Since $\tilde{V}_0(z_{01}) = \tilde{V}_1(z_{01})$ and $\tilde{I}_0(z_{01}) = \tilde{V}_1(z_{01})$

$$\tilde{Z}_0(z_{01}) = \tilde{Z}_1(z_{01})$$

$$Z_0\left[\frac{1+\tilde{\Gamma}_0(z_{01})}{1-\tilde{\Gamma}_0(z_{01})}\right]=Z_1$$

$$\tilde{\Gamma}_0(z_{01})=\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}$$

$$\tilde{\Gamma}_0(z)=\frac{\tilde{V}_0^-}{\tilde{V}_0^+}e^{2j\beta_0 z}\rightarrow\tilde{V}_0^-(z)=\tilde{V}_0^+e^{2j\beta_0 z_{01}}\left[1+\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}\right]$$ since $\tilde{V}_0^+$ is known

$$\tilde{\Gamma}_1(z)=\frac{\tilde{V}_1^-}{\tilde{V}_1^+}e^{2j\beta_1 z}\rightarrow\tilde{V}_1^+(z)=\tilde{V}_1^-e^{2j\beta_1 z_{12}}\left[1+\frac{Z_2(z_{12})+Z_1}{Z_2(z_{12})-Z_1}\right]$$

$$\tilde{V}_2^+(z)=\tilde{V}_2^-e^{2j\beta_2 z_{23}}\left[1+\frac{Z_3(z_{23})+Z_2}{Z_3(z_{23})-Z_2}\right]$$

$$\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}e^{-j\beta_0 z_{01}}=\tilde{\Gamma}_0(z_{01})\rightarrow\frac{\tilde{V}_0^-(z)}{\tilde{V}_0^+(z)}=e^{j\beta_0 z_{01}}\left[\frac{Z_1(z_{01})-Z_0}{Z_1(z_{01})+Z_0}\right]$$

$$\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}e^{-j\beta_1 z_{12}}=\tilde{\Gamma}_1(z_{12})\rightarrow\frac{\tilde{V}_1^-(z)}{\tilde{V}_1^+(z)}=e^{j\beta_1 z_{12}}\left[\frac{Z_2(z_{12})-Z_1}{Z_2(z_{12})+Z_1}\right]$$

9.2.2.

1. $v(z,t)=V^+[\cos(\omega t-\beta z)+\rho\cos(\omega t+\beta z)]$

   $\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$

   $v(z,t)=V^+[\cos(\omega t)\cos(\beta z)+\sin(\omega t)\sin(\beta z)+\rho\cos(\omega t)\cos(\beta z)-\rho\sin(\omega t)\sin(\beta z)]$

   $v(z,t)=V^+[(1+\rho)\cos(\omega t)\cos(\beta z)+(1-\rho)\sin(\omega t)\sin(\beta z)]$

   $v(z,t)=A\cos(\omega t)\cos(\beta z)+B\sin(\omega t)\sin(\beta z)$ where $A=V^+(1+\rho)$ and $B=V^+(1-\rho)$

3. This plot shows 2 oppositely travelling waves which will produce a standing wave. The VSWR can be calculated from the wave amplitudes to determine the impedance mismatch and describe how efficiently radio-frequency power is transmitted.