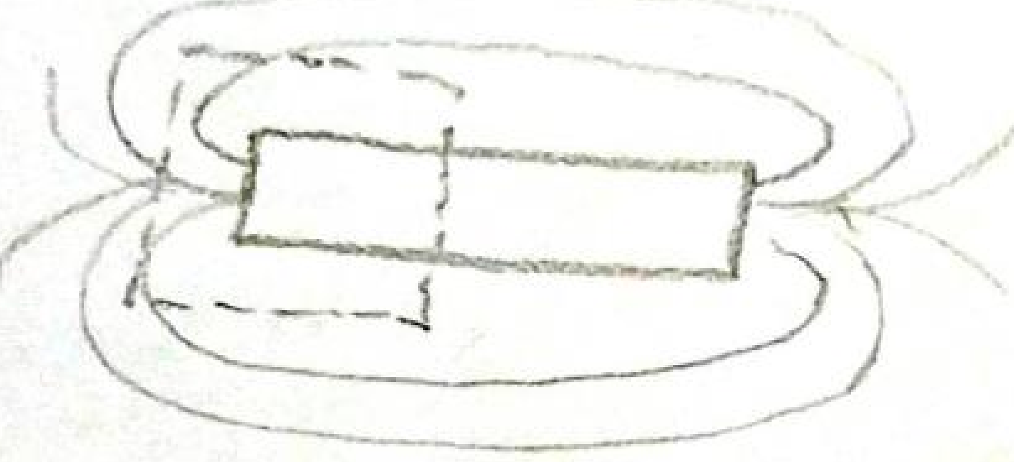
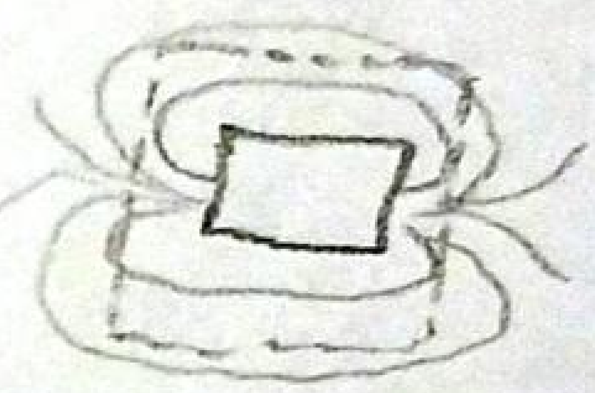
5.1

1. $\vec{\nabla} \cdot \vec{B} = 0$ This states that the divergence of the magnetic field at any location is 0, which means the same amount of magnetic field is flowing into a defined region as flowing out of it. If we look at a space around the pole of a dipole magnet, it may look like a positive ~~divergence~~ (or negative) divergence, (1), but if this magnet were to be cut into a size smaller than our area, we would see that the divergence is, in fact, still 0 (2).

(1) 

(2) 

2. $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ This states that the curl of the magnetic field is proportional to the current density. The magnetic field rotates about the direction of current, In the case of a line current, the magnetic field rotates perpendicular to the wire (3). If ~~a~~ the curl were taken of another point along the $\vec{B}$ field where there is no enclosed current, the curl would be 0.

(3) 

$\vec{\nabla} \times \vec{B} = 0$ since $\vec{J} = 0$ within the region

3. $\vec{B} = -\vec{\nabla}\varphi_m$ This describes the magnetic field in the case of no currents, such as a permanent magnet, In this case, $\vec{\nabla}\times\vec{B} = 0$ with no current but still holds true for a scalar product.