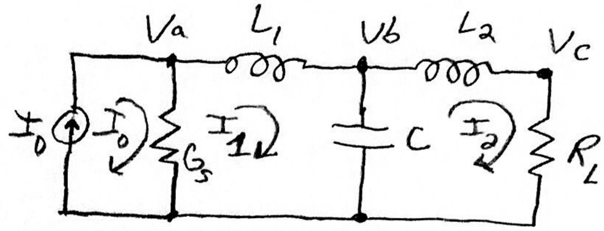
4.3b.



$$L_1 \frac{dI_1}{dt} + \frac{1}{C}\int (I_1 - I_2)\,dt + \frac{(I_1 - I_0)}{G_s} = 0$$

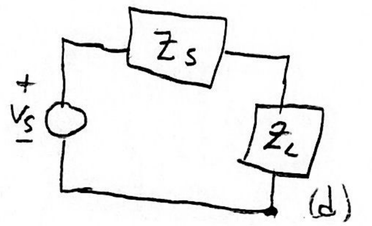
$$L_2 \frac{dI_2}{dt} + I_2 R_L + \frac{1}{C}\int (I_2 - I_1)\,dt = 0$$

Node V_a: $I_0 = V_a G_s + \frac{1}{L_1}\int (V_a - V_b)\,dt \rightarrow I_0 - V_a G_s - \frac{1}{L_1}\int (V_a - V_b)\,dt = 0$

Node V_b: $\frac{1}{L_1}\int (V_b - V_a)\,dt + C\frac{dV_b}{dt} + \frac{1}{L_2}\int (V_b - V_c)\,dt = 0$

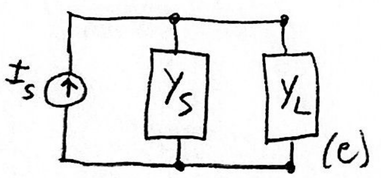
Node V_c: $\frac{V_c}{R_L} + \frac{1}{L_2}\int (V_c - V_b)\,dt = 0$

4.3c.



$I_L = \frac{V_s}{Z_s + Z_L}$ current through the load impedance

$V_L = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)$



$V_L = \frac{I_s}{Y_s + Y_L}$

$I_s = V_s Y_s$, $Y_s = Z_s^{-1}$ $\rightarrow$ $I_s = \frac{V_s}{Z_s}$

$$V_L = \frac{I_s}{Y_s + Y_L} \rightarrow \frac{(V_s/Z_s)}{(Z_s^{-1}) + (Z_L^{-1})} = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)$$

Current through load admittance:

$$I_L = V_L Y_L = V_s\left(\frac{Z_L}{Z_s + Z_L}\right)(Z_L^{-1}) = \frac{V_s}{Z_s + Z_L}$$

I_L is the same for the current flowing through the load impedance and the current through the load admittance.

V_L is also the same for the voltage across the load impedance and the voltage across the load admittance.

4.3 d. For the current in the circuit:

$$I = \frac{V_s}{Z_s + Z_L} \qquad \text{where } Z_s = R_s + jX_s \text{ and } Z_L = R_L + jX_L$$

$$I = \frac{V_s}{(R_s + jX_s) + (R_L + jX_L)} = \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

For power to the load, $W_L = \frac{R_L}{2} I I^*$

$$I^* = \left[\frac{V_s}{(R_s+R_L) + j(X_s+X_L)}\right]^* = \left[\frac{V_s}{(R_s+R_L) + j(X_s+X_L)}\right]\left[\frac{(R_s+R_L) - j(X_s+X_L)}{(R_s+R_L) - j(X_s+X_L)}\right]$$

$$I^* = \frac{V_s[(R_s+R_L) - j(X_s+X_L)]}{(R_s+R_L)^2 + (X_s+X_L)^2}$$

$$W_L = \frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L) + j(X_s+X_L)}\right]\left[\frac{V_s[(R_s+R_L) - j(X_s+X_L)]}{(R_s+R_L)^2 + (X_s+X_L)^2}\right]$$

$$= \frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2 + (X_s+X_L)^2}\right]$$

For max power to load: $\frac{dW_L}{dX_L} = 0$ and $\frac{dW_L}{dR_L} = 0$

$$\frac{dW_L}{dX_L} = \frac{d}{dX_L}\left(\frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2 + (X_s+X_L)^2}\right]\right) = 0$$

$$= \frac{-R_L V_s (X_s + X_L)}{[(R_s+R_L)^2 + (X_s+X_L)^2]^2} \rightarrow -R_L V_s (X_s + X_L) = 0, \quad X_L = -X_s$$

$$\frac{dW_L}{dR_L} = \frac{d}{dR_L}\left(\frac{R_L}{2}\left[\frac{V_s}{(R_s+R_L)^2 + (X_s+X_L)^2}\right]\right) = 0$$

$$= \frac{V_s}{2}\left[\frac{-2R_L(R_s+R_L) + [(R_s+R_L)^2 + (X_s+X_L)^2]}{[(R_s+R_L)^2 + (X_s+X_L)^2]^2}\right] = 0$$

$$-2R_L(R_s+R_L)+[(R_s+R_L)^2+(X_s+X_L)^2]=0$$

As $X_L=-X_s$,

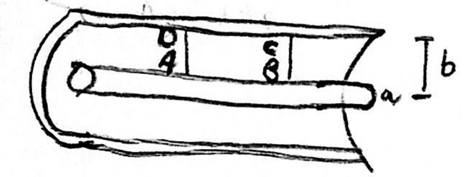
$$-2R_L(R_s+R_L)+[(R_s+R_L)^2+0]=0$$

$$-2R_LR_s-2R_L^2+R_s^2+R_L^2+2R_LR_s = R_s^2-R_L^2=0, \quad R_L=R_s$$

Max power to the load when $R_L=R_s$ and $X_L=-X_s$

$$Z_L=R_L+jX_L=R_s+j(-X_s)=R_s-jX_s=(R_s+jX_s)^*=Z_s^*$$

4.6e.



$$\oint_{ABCD} E\cdot dl=-j\omega\int_S B\cdot dS$$

$$\int_S B\cdot dS=IL_e\,dz$$

$$\oint_{ABCD} E\cdot dl=-j\omega IL_e\,dz$$

for $E_z=I(Z_{inner})_{outer}$ inner surface of the outer conductor

$$E_z=I\left(\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)$$

b = radius of outer conductor
R_{sb} = resistance
L_{ib} = inductance

for $E_z=-I(Z_{inner})_{inner}$ outer surface of inner conductor

$$E_z=-I\left(\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}\right)$$

a = radius of inner conductor
R_{sa} = resistance
L_{ia} = inductance

$$\oint_{ABCD} E\cdot dl=-j\omega IL_e\,dz$$

$$I\left(\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}\right)dz+V_{CB}+I\left(\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)dz-V_{DA}=-j\omega IL_e\,dz$$

$$V_{DA}-V_{CB}=I\left(j\omega L_e+\frac{R_{sa}}{2\pi a}+\frac{j\omega L_{ia}}{2\pi a}+\frac{R_{sb}}{2\pi b}+\frac{j\omega L_{ib}}{2\pi b}\right)dz$$

$$V_{DA}-V_{CB}=0$$

For current from C to D,

$$I_C - I_D = j\omega(C\,dz)V$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \ \frac{F}{m}$$

$$I_C - I_D = j\omega\left(\frac{2\pi\varepsilon\,dz}{\ln(b/a)}\right)V$$ matches capacitance shown in Fig. P4.6e