6.2. $E = E_{0x}(x,t)\hat{x} + E_{0y}(x,t)\hat{y} + E_{0z}(x,t)\hat{z}$

$B = B_{0x}(x,t)\hat{x} + B_{0y}(x,t)\hat{y} + B_{0z}(x,t)\hat{z}$

1. $\frac{d^2 f}{du^2} \rightarrow$ $\frac{d^2 E_y(x,t)}{dx^2} = k^2 E_y''(x,t)$, $\frac{d^2 E_y(x,t)}{dy^2} = \frac{d^2 E_y(x,t)}{dz^2} = 0$

$\frac{d^2 E_z(x,t)}{dx^2} = k^2 E_z''(x,t)$, $\frac{d^2 E_z(x,t)}{dy^2} = \frac{d^2 E_z(x,t)}{dz^2} = 0$

$\frac{d^2 B_y(x,t)}{dx^2} = k^2 B_y''(x,t)$, $\frac{d^2 B_y(x,t)}{dy^2} = \frac{d^2 B_y(x,t)}{dz^2} = 0$

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$\frac{d^2 f}{dt^2} \rightarrow$ $\frac{d^2 E_y(x,t)}{dt^2} = \omega^2 E_y''(x,t)$ $\frac{d^2 B_y(x,t)}{dt^2} = \omega^2 B_y''(x,t)$

$\frac{d^2 E_z(x,t)}{dt^2} = \omega^2 E_z''(x,t)$ $\frac{d^2 B_z(x,t)}{dt^2} = \omega^2 B_z''(x,t)$

$\frac{d^2 E_y(x,t)}{dx^2} = E_y''(x,t) = \frac{\omega^2}{k^2} E_y''(x,t) = \frac{1}{c^2} E_y''(x,t)$

$\frac{d^2 E_y(x,t)}{dy^2} = \frac{d^2 E_y(x,t)}{dz^2} = 0 \rightarrow \omega^2 E_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

Wave is propogating only in x direction

$\frac{d^2 B_z(x,t)}{dx^2} = E_z''(x,t) = \frac{\omega^2}{k^2} E_z''(x,t) = \frac{1}{c^2} E_z''(x,t)$

$\frac{d^2 E_z(x,t)}{dy^2} = \frac{d^2 E_z(x,t)}{dz^2} = 0 \rightarrow \omega^2 E_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

wave is propogating only in x direction

$\frac{d^2 B_y(x,t)}{dx^2} = B_y''(x,t) = \frac{\omega^2}{k^2} B_y''(x,t) = \frac{1}{c^2} B_y''(x,t)$

$\frac{d^2 B_y(x,t)}{dy^2} = \frac{d^2 B_y(x,t)}{dz^2} = 0 \rightarrow \omega^2 B_y''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

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$\frac{d^2 B_z(x,t)}{dx^2} = B_z''(x,t) = \omega^2/k^2 B_z''(x,t) = \frac{1}{c^2} B_z''(x,t)$

$d^2 B_z/dy^2 = d^2 B_z/dz^2 = 0 \rightarrow \omega^2 B_z''(x,t) = 0$ and $\omega^2 E_z''(x,t) = 0$

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2. $\frac{d^2 E_x(x,t)}{dx^2} = \frac{d^2 B_x(x,t)}{dx^2} = 0$ The wave propogates in the x-direction but does not vary in that direction

$$\frac{d^2 E_x(x,t)}{dt^2} = \omega^2 E_x''(x,t) \qquad \frac{d^2 B_x(x,t)}{dt^2} = \omega^2 B_x''(x,t)$$

$$\omega^2 E_x''(x,t) = \omega^2 B_x''(x,t) = 0$$

The partial derivatives of E_x and B_x wrt y and z are 0 since the waves do not propogate in that direction. That leaves:

$$E_x''(x,t) = B_x''(x,t) = 0$$

3. $$\nabla^2 E = \nabla^2 \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \frac{d^2 E_x}{dx^2} + \frac{d^2 E_x}{dy^2} + \frac{d^2 E_x}{dz^2} \\ \frac{d^2 E_y}{dx^2} + \frac{d^2 E_y}{dy^2} + \frac{d^2 E_y}{dz^2} \\ \frac{d^2 E_z}{dx^2} + \frac{d^2 E_z}{dy^2} + \frac{d^2 E_z}{dz^2} \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ K^2 E_y''(x,t)+0+0 \\ K^2 E_z''(x,t)+0+0 \end{bmatrix}$$

$$\frac{1}{c^2}\frac{d^2 E}{dt^2} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2 E_y''(x,t) \\ \omega^2 E_z''(x,t) \end{bmatrix} \rightarrow \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2/k^2 E_y''(x,t) \\ \omega^2/k^2 E_z''(x,t) \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ c^2 E_y''(x,t) \\ c^2 E_z''(x,t) \end{bmatrix}$$

$$\nabla^2 B = \nabla^2 \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \frac{d^2 B_x}{dx^2} + \frac{d^2 B_x}{dy^2} + \frac{d^2 B_x}{dz^2} \\ \frac{d^2 B_y}{dx^2} + \frac{d^2 B_y}{dy^2} + \frac{d^2 B_y}{dz^2} \\ \frac{d^2 B_z}{dx^2} + \frac{d^2 B_z}{dy^2} + \frac{d^2 B_z}{dz^2} \end{bmatrix} = \begin{bmatrix} 0+0+0 \\ K^2 B_y''(x,t)+0+0 \\ K^2 B_z''(x,t)+0+0 \end{bmatrix}$$

$$\frac{1}{c^2}\frac{d^2 B}{dt} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2 B_y''(x,t) \\ \omega^2 B_z''(x,t) \end{bmatrix} \rightarrow \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ \omega^2/k^2 B_y''(x,t) \\ \omega^2/k^2 B_z''(x,t) \end{bmatrix} = \frac{1}{c^2}\begin{bmatrix} 0 \\ c^2 B_y''(x,t) \\ c^2 B_z''(x,t) \end{bmatrix}$$