Compressible Aerodynamics (AE-311) (2019-20 – I Semester)

Assignment 4

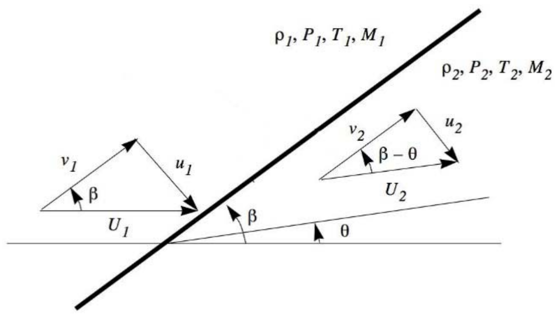
(Oblique shocks and Prandtl Meyer Expansion)

Due: 22 October 2019

Problem 1:

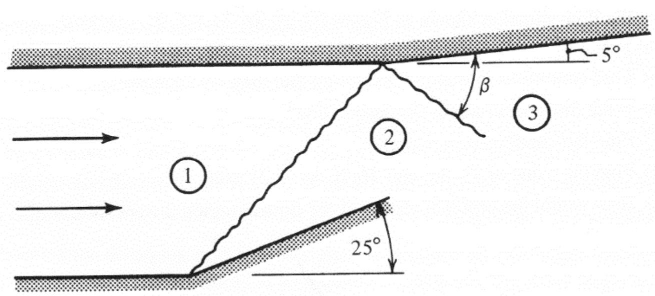
For an oblique shock wave shown below derive the modified Prandtl's relation:

$$u_1 u_2 = (a^*)^2 - (\frac{\gamma - 1}{\gamma + 1}) v_1^2$$



Problem 2:

A uniform flow of air has a Mach number of 3.3. The bottom of the duct is bent upwards at a 25° angle. At the point where the shock intersects the upper wall, the boundary is bent 5° upward as shown in the figure. Assume that the flow is supersonic throughout the system. Compute M_3, P_3/P_1, T_3/T_1, and β.



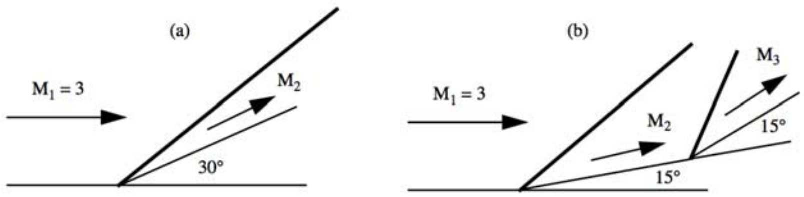
Problem 3:

For a given Prandtl-Meyer expansion, the upstream Mach number is 3.0 and the pressure ratio across the wave is $P_2/P_1 = 0.4$. Calculate the angles of the forward and rearward Mach lines of the expansion fan relative to the free stream direction.

Problem 4:

Figure below shows supersonic flow of air turned through an angle of 30°. The freestream Mach number is 3.0. In case (a) the turning is accomplished by a single 30° wedge, whereas in case (b) the turning is accomplished by two 15° wedges in tandem. Determine the stagnation pressure change in each case:

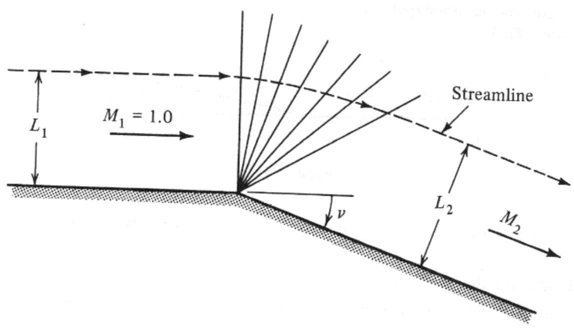
$(P_{t2}/P_{t1})_{\text{case (a)}}$ and $(P_{t3}/P_{t1})_{\text{case (b)}}$ and comment on the relative merit of one design over the other.



Problem 5:

Flow initially at a Mach number of unity, expands around a corner through angle ν and reaches Mach number M_2 (see figure below). Lengths L_1 and L_2 are measured perpendicular to the wall and measure the distance out to the same streamline as shown.

(a) Derive an equation for the ratio $L_2/L_1 = f(M_2,\gamma)$.
(b) If $M_1 = 1.0$, $M_2 = 1.79$, and $\gamma = 1.67$, compute the ratio L_2/L_1.



Problem 6:

Consider the intersection of two shocks of opposite families, as sketched in figure below.. For $M_1 = 3.0$, $P_1 = 1$ atm, $\theta_2 = 20°$, and $\theta_3 = 15°$. Calculate the pressures in regions 4 and 4', and the flow direction Φ, behind the refracted shocks.