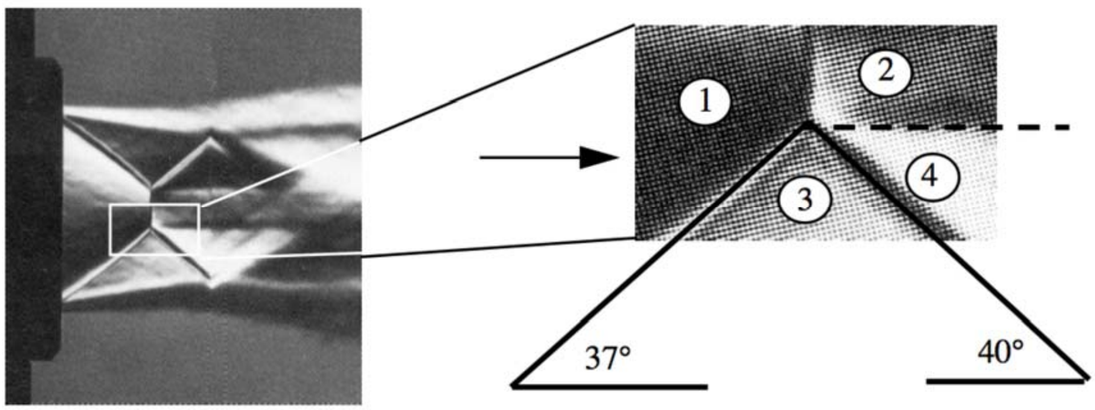
Compressible Aerodynamics (AE-311)

(2019-20 – I Semester)

Assignment 5 (<u>Due October 31, 2019</u>)

Problem 1: Figure below shows the flow of *helium* from a supersonic over-expanded round jet. If we restrict our attention to a small region near the intersection of the first two oblique shocks and the so-called Mach disc as shown in the blow-up, then we can use oblique shock theory to determine the flow properties near the shock intersection (despite the generally non-uniform 3-D nature of the rest of the flow). The shock angles with respect to the horizontal measured from the image are as shown.

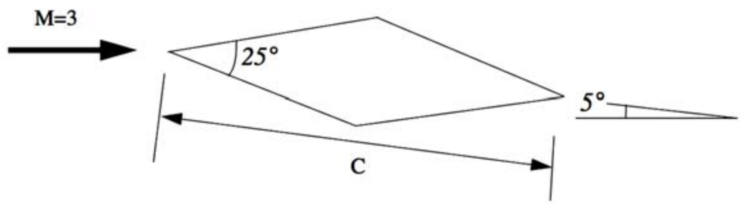


Determine the jet exit Mach number. *Hint, you will need to select a Mach number that balances the pressures in regions 2 and 4 with a dividing streamline that is very nearly horizontal as shown in the picture.*

2) Determine the Mach number in region 2.
3) Determine the flow angles and Mach numbers in regions 3 and 4.
4) Determine P_2/P_1 and P_4/P_1. How well do the static pressures match across the dividing streamline (dashed line) between regions 2 and 4?

Problem 2: A *<u>weak</u>* oblique shock makes an angle ε with Mach lines ahead of it (as done in class). Show that it makes the same angle with Mach lines downstream of it. So shock wave position is "average" of the Mach line positions on either side.

Problem 3: Figure below shows a symmetrical, diamond shaped airfoil at a 5° angle of attack in a supersonic flow of air.



a) Determine the lift and drag coefficients of the airfoil.

$$C_L = \frac{Lift/span}{\frac{1}{2}\rho U_\infty^2 C}\ ;\ C_D = \frac{Drag/span}{\frac{1}{2}\rho U_\infty^2 C}$$

b) What happens to the flow over the airfoil if the free-stream Mach number is decreased to 1.5? Compare your result with the lift and drag of a thin flat plate at 5° angle of attack and free-stream Mach number of 3.

Problem 4: Show that for a thin symmetrical airfoil, whose profile is a lens defined by two circular arcs, the drag coefficient is:

$$C_D = \frac{16}{3\sqrt{M_\infty^2 - 1}}\left(\frac{t}{c}\right)^2$$

Prove that for a given thickness ratio, t/c, the profile for minimum drag is a symmetrical diamond profile.

Problem 5: Show that the variation of total pressure across streamlines is given, for a perfect gas, by

$$-\frac{1}{\rho_o}\frac{dp_o}{dn} = \left(1 + \frac{\gamma - 1}{2}M^2\right)u\varsigma + \frac{\gamma - 1}{2}C_p M^2 \frac{dT_o}{dn}$$

And hence for incompressible flow ($M \to 0$), the total pressure gradient is related to the vorticity by

$$\frac{dp_o}{dn} = -\rho_o u\varsigma$$