Assignment-1

AE 321A

Question-1

Solution

first runway altitude, $h_1 = 1.5$ km

second " " , $h_2 = 2.5$ km

Static pressure @ $h_1 = 1.5$ km, $P_{s_1} = P_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}}$

where $T_0 = 288.16$ K

$T_1 = 288.16 - 6.5 \times 1.5 = 278.41$ K

$P_0 = 1.01325 \times 10^5$ N/m^2

So, $P_{s_1} = 1.01325 \times 10^5 \left(\frac{278.41}{288.16}\right)^{-\left(\frac{9.81}{-6.5\times10^{-3} \times 287}\right)}$

$= 1.01325 \times 10^5 (0.966)^{+5.2586}$

$$\boxed{P_{s_1} = 8.447 \times 10^4 \ N/m^2}$$

Using gas equation,

$P_{s_1} = \rho_1 R T_1$

$\rho_1 = \frac{P_{s_1}}{RT_1} = \frac{8.447 \times 10^4}{287 \times 278.41}$

$$\boxed{\rho_1 = 1.057 \ kg/m^3}$$

Velocity at first runway is given as 50 m/s, hence total pressure, $P_{total} = P_{s_1} + \frac{1}{2}\rho_1 V_1^2$

$$= 8.447 \times 10^4 + 0.5 \times 1.057 \times 50^2$$

$$\boxed{P_{total} = 8.579 \times 10^4 \ N/m^2}$$

Atmospharic properties @ $h_2 = 2.5$ km,

$$T_2 = 288.16 - 6.5 \times 2.5$$

$$= 271.91 \ K$$

$$P_{s_2} = P_0 \left(\frac{T_2}{T_0}\right)^{-\frac{g_0}{aR}}$$

$$= 1.01325 \times 10^5 \left(\frac{271.91}{288.16}\right)^{-\left(\frac{9.81}{-6.5 \times 10^{-3} \times 287}\right)}$$

$$= 1.01325 \times 10^5 (0.944)^{5.2586}$$

$$\boxed{P_{s_2} = 7.467 \times 10^4 \ N/m^2}$$

$$\rho_2 = \frac{P_{s_2}}{R T_2} = \frac{7.467 \times 10^4}{287 \times 271.91}$$

$$\boxed{\rho_2 = 0.957 \ kg/m^3}$$

~~If test case~~ to maintain the same total pressure at second runway, required velocity (V_2) would be given as -

$$P_{total} = P_{s_2} + \frac{1}{2}\rho_2 V_2^2$$

$$8.579\times10^4 = 7.467\times10^4 + 0.5\times0.957\times V_2^2$$

$$\boxed{V_2 = 152.44 \ m/s}$$

$$\% \text{ change} = \frac{V_2 - V_1}{V_1}\times100$$

$$= \frac{152.44 - 50}{50}\times100$$

$$= 204.88\ \%$$

Ans

Acceptable answer $\in [194.64,\ 215.12]\ \%$

Question - 2

Solution

$$t_{max} = \frac{3E_m}{2\sqrt[4]{3}\,V_R}(h_i - h_f)$$

Dynamic equations of gliding flight are given by -

$$L - W\cos\gamma = 0 \quad —(1)$$

$$D - W\sin\gamma = 0 \quad —(2)$$

Assuming $\gamma < 0$.

Kinematic equations are given as -

$$\dot{x} = V\cos\gamma \quad —(3)$$

$$\dot{h} = +V\sin\gamma \quad —(4)$$

From eq (1) & (2),

$$\tan \gamma = \frac{C_D}{C_L}$$

if γ is small.

$$\gamma = \frac{C_D}{C_L} \quad \text{—— (5)}$$

Sink rate is given as –

$$\dot{h}_s = -\dot{h}$$

$$\dot{h}_s = -V \sin \gamma$$

$$\dot{h}_s = -V\gamma \quad \text{—— (6)}$$

From eq (5) & (6), we get -

$$\dot{h}_s = -V \frac{C_D}{C_L}$$

$$= -\sqrt{\frac{2W}{\rho S C_L}} \cdot \frac{C_D}{C_L} \qquad [\because L \approx W]$$

$$= -\sqrt{\frac{2W}{\rho S}} \frac{C_D}{C_L^{3/2}}$$

$$(\dot{h}_s)_{min} = -\sqrt{\frac{2W}{\rho S}} \left(\frac{C_D}{C_L^{3/2}}\right)_{min}$$

We know, $\left(\frac{C_D}{C_L^{3/2}}\right)_{min}$ occurs, when

$$3C_{D0} = C_{Di} \implies C_L = \sqrt{\frac{3C_{D0}}{k_2}}$$

and $C_D = 4C_{D0}$

$$\left(\frac{C_D}{C_L^{3/2}}\right)_{min} = \frac{4C_{D0}}{\left(\frac{3C_{D0}}{k}\right)^{3/4}} = 4\sqrt[4]{\frac{C_{D0}k^3}{27}}$$

$$(\dot{h}_s)_{min} = -4\sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{C_{D0}k^3}{27}} \quad \text{--- (7)}$$

Endurance can be given as,

$$\int_0^t dt = \int_{h_i}^{h_f}\frac{dh}{\dot{h}_s}$$

$$t_{max} = -\int_{h_i}^{h_f}\frac{dh}{\sqrt{\frac{2W}{\rho S}}\left(\frac{C_D}{C_L^{3/2}}\right)}$$

$$t = \sqrt{\frac{\rho S}{2W}}\left(\frac{C_L^{3/2}}{C_D}\right)(h_i - h_f)$$

$$t_{max} = \sqrt{\frac{\rho S}{2W}}\left(\frac{C_L^{3/2}}{C_D}\right)_{max}(h_i - h_f)$$

$$t_{max} = \frac{1}{4}\sqrt{\frac{\rho S}{2W}}\sqrt[4]{\frac{27}{C_{D0}k^3}}(h_i - h_f) \quad \text{--- (8)}$$

we know,

$$E_m = \frac{1}{2\sqrt{kC_{D0}}} \quad \text{--- (9)}$$

and

$$V_R = \sqrt{\frac{2W}{\rho S}}\sqrt[4]{\frac{k}{C_{D0}}} \quad \text{--- (10)}$$

Rearranging the equation - (8), we get -

$$t_{max} = \frac{1}{4}\sqrt{\frac{\rho S}{2W}} \cdot \sqrt[4]{\frac{27\, k^{3}}{C_{D0}\, k^{4} \times 3}}\,(h_i - h_f)$$

$$= \frac{1}{4}\sqrt{\frac{\rho S}{2W}} \cdot \frac{3}{k}\sqrt[4]{\frac{k}{3 C_{D0}}}\,(h_i - h_f)$$

$$= \frac{\frac{1}{4} \times \frac{3}{k} \cdot \frac{1}{\sqrt[4]{3}} \cdot \sqrt[4]{\frac{k}{C_{D0}}} \times \sqrt[4]{\frac{k}{C_{D0}}}}{\sqrt{\frac{2W}{\rho S}} \times \sqrt[4]{\frac{k}{C_{D0}}}}\,(h_i - h_f)$$

$$= \frac{\frac{1}{2} \times \frac{1}{\sqrt[4]{3}} \times \frac{3}{2\sqrt{k C_{D0}}}}{V_R}\,(h_i - h_f)$$

$$\boxed{t_{max} = \frac{3\, E_m}{2\sqrt[4]{3}\, V_R}(h_i - h_f)}$$

<u>Question - 3</u>

$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$

<u>Solution</u> We have to ~~show~~ show that dynamic pressure at ~~climb~~ max^m ~~rate~~ climb rate is give by -

$$q = \frac{T}{6 S C_{D0}} + \sqrt{\left(\frac{T}{6 S C_{D0}}\right)^2 + \frac{W^2}{3\pi e AR S^2 C_{D0}}}$$

Climb rate can be given as -

$$R/C = V\sin\gamma = \frac{V(T-D)}{W}$$

$$R/C = \frac{TV}{W} - \frac{VD}{W}$$

$$= \frac{TV}{W} - \frac{V\,\frac{1}{2}\rho V^2 S\left(C_{D0} + k C_L^2\right)}{W}$$

$$= \frac{TV}{W} - \frac{\frac{1}{2}\rho S V^3\left(C_{D0} + k\left(\frac{2W}{\rho S V^2}\right)^2\right)}{W}$$ [Assuming $L \approx W$]

$$R/C = \frac{TV}{W} - \frac{\rho V^3 C_{D0} S}{2W} - \frac{2kW}{\rho S V}$$

$$\frac{d(R/C)}{dV} = \frac{T}{W} - \frac{3\rho V^2 S C_{D0}}{2W} + \frac{2kW}{\rho S V^2}$$

$$\therefore \quad \frac{T}{W} = \left(\frac{3\rho S C_{D0}}{2W} V^4 + \frac{2kW}{\rho S}\right)\frac{1}{V^2}$$

$$\frac{3\rho S C_{D0}}{2W} V^4 - \frac{T}{W} V^2 + \frac{2kW}{\rho S} = 0$$

$$V^4 - \frac{2T}{3\rho S C_{D0}} V^2 + \frac{4kW^2}{3\rho^2 S^2 C_{D0}} = 0$$

$$V^2 = \frac{1}{2}\left[\frac{2T}{3\rho S C_{D0}} \pm \sqrt{\left(\frac{2T}{3\rho S C_{D0}}\right)^2 + \frac{16kW^2}{3\rho^2 S^2 C_{D0}}}\right]$$

$$V^2 = 2\left[\frac{T}{6\rho S C_{D0}} \pm \frac{1}{\rho}\sqrt{\left(\frac{T}{6 S C_{D0}}\right)^2 + \frac{kW^2}{3S^2 C_{D0}}}\right]$$

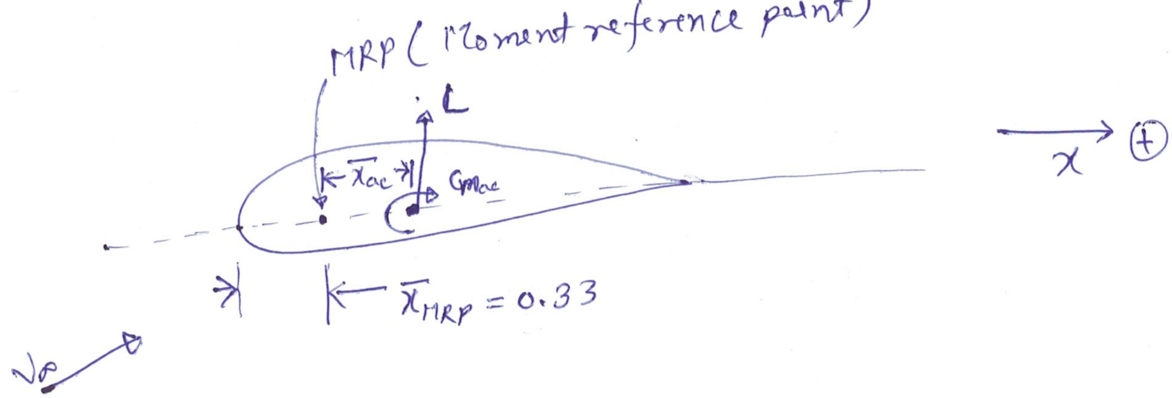
$$V^2 = \frac{2}{\rho}\left[\frac{T}{6 S C_{D0}} \pm \sqrt{\left(\frac{T}{6 S C_{D0}}\right)^2 + \frac{W^2}{3\pi e AR S^2 C_{D0}}}\right]$$

$$\boxed{q = \frac{1}{2}\rho V^2 = \frac{T}{6 S C_{D0}} + \sqrt{\left(\frac{T}{6 S C_{D0}}\right)^2 + \frac{W^2}{3\pi e AR S^2 C_{D0}}}}$$

Since, +ve sign belongs to $(R/C)_{max}$.

Question - 4

Solution



$$C_{MRP} = C_{m_{ac}} - \bar{x}_{ac}\, C_l$$

from the given table -

$$C_{MRP} = -0.02 \quad @ \quad C_l = 0.2$$

$$-0.02 = C_{mac} - \bar{x}_{ac} \times 0.2 \qquad \text{— (1)}$$

from the given table,

$$C_{MRP} = 0 \quad @ \quad C_l = 0.4$$

$$0 = C_{mac} - \bar{x}_{ac} \times 0.4$$

$$C_{mac} = 0.4\bar{x}_{ac} \qquad \text{— (2)}$$

From eq (1) & (2), we can write,

$$-0.02 = 0.4\,\bar{x}_{ac} - 0.2\,\bar{x}_{ac}$$

$$\boxed{\bar{x}_{ac} = -0.1}$$ from moment reference point (MRP).

$$C_{mac} = 0.4 \times (-0.1)$$

$$\boxed{C_{mac} = -0.04}$$

Location of center of pressure @ $C_l = 0.5$

$$\bar{x}_{cp} = \bar{x}_{ac} - \frac{C_{mac}}{C_l}$$

$$= -0.1 - \frac{-0.04}{0.5}$$

$\boxed{\bar{x}_{cp} = -0.02}$ from moment Reference point (MRP)

Aerodynamic center location from the leading-edge of aerofail $= \bar{x}_{MRP} + \bar{x}_{ac}$

$$= 0.33 - 0.1$$

$$= 0.23$$

Ans

Acceptable Answer $\in [0.22, 0.24]$

Location of center of pressure from the leading-edge of the aerofail $= \bar{x}_{MRP} + \bar{x}_{cp}$

$$= 0.33 - 0.02$$

$$= 0.31$$

Ans

Acceptable answer $\in [0.29, 0.33]$

Question - 5

Solution

Specifications of given glider as follows,

$$W = 5500\ N$$

$$W/S = 500\ N/m^2$$

$$C_D = 0.01 + 0.022\, C_L^2$$

$$C_{D0} = 0.01\ , \quad k = 0.022$$

$$\Delta h = 500\ m$$

(a)

$$R_{max} = \frac{\Delta h}{\sqrt{4kC_{D0}}} = \frac{500}{\sqrt{4\times 0.022\times 0.01}}$$

$$= \underline{16.855\ Km}$$ Acceptable Answer $\in [16.01, 17.7]$ km

(b)

$$t_{max} = \sqrt{\frac{\rho}{2(W/S)}} \times \frac{1}{4} \cdot \sqrt[4]{\frac{27}{k^3 C_{D0}}}\ \Delta h$$

$$= \sqrt{\frac{1.225}{2\times 500}} \times \frac{1}{4} \times \sqrt[4]{\frac{27}{0.022^3 \times 0.01}} \times 500$$

$$= 552.08\ sec$$

or $= \underline{9.2\ minutes}$ Acceptable Answer $\in [8.74, 9.66]$ min

(c)

$$\gamma_{min} = \sqrt{4kC_{D0}} = \sqrt{4\times 0.022\times 0.01} = 0.0297\ rad$$

or $\underline{\gamma_{min} = 1.70^\circ}$ Acceptable Answer $\in [1.62, 1.79]$ deg

(d)

$$(h_s)_{min} = 4\sqrt{\frac{2(W/S)}{\rho}}\ \sqrt[4]{\frac{k^3 C_{D0}}{27}} = 4\sqrt{\frac{2\times 500}{1.225}} \times \sqrt[4]{\frac{0.022^3\times 0.01}{27}}$$

$$= \underline{0.906\ m/s}$$ Acceptable Answer $\in [0.86, 0.95]$ m/s

Question - 6

Solution A piston-prop aircraft has following specifications —

$$W/S = 1400 \ N/m^2, \quad S = 24 \ m^2$$

$$C_D = 0.025 + 0.05 \, C_L^2$$

$$C_{D0} = 0.025, \quad k = 0.05$$

$$C_{L\,max} = 1.5$$

$$P_s = 700 \ kW$$

$$\eta_p = 0.85$$

$$W = \frac{W}{S} \times S = 1400 \times 24 = 33600 \ N$$

(b) maximum rate of climb for propeller aircraft —

$$(R/C)_{max} = \frac{P_a - P_{R\,min}}{W}$$

$$P_{R\,min} = W \sqrt{\frac{2W}{\rho S}} \left(\frac{C_D}{C_L^{3/2}}\right)_{min}$$

$$= W \sqrt{\frac{2W}{\rho S}} \cdot 4 \cdot \sqrt[4]{\frac{k^3 C_{D0}}{27}}$$

$$= 33600 \sqrt{\frac{33600 \times 2}{1.225 \times 24}} \times 4 \times \sqrt[4]{\frac{0.05^3 \times 0.025}{27}}$$

$$= 118.573 \ kW$$

$$(R/C)_{max} = \frac{\eta_p P_s - P_{R\,min}}{W} = \frac{0.85 \times 700 \times 10^3 - 118.573 \times 10^3}{33600}$$

$$= 14.18 \ m/s$$ Ans Acceptable Answer $\in [13.5, 14.9]$ m/s

$$(C_L)_{R/C,\,max} = \sqrt{\frac{3 C_{D0}}{k}} = \sqrt{\frac{3 \times 0.025}{0.05}} = 1.22$$ Ans

Acceptable Answer $\in [1.16, 1.28]$

$$V_{R/C,max} = \sqrt{\frac{2(W/S)}{\rho\,(C_L)_{R/C,max}}}$$

$$= \sqrt{\frac{2\times 1400}{1.225\times 1.22}}$$

$$= \underline{43.28 \text{ m/s}}$$

Ans

Acceptable Answer $\in [41.12, 45.44]$ m/s

(a) Maximum climb angle, γ_{max} can be obtained using numerial method or graphically.

$$\gamma = \frac{P_a}{WV} - \frac{\rho S V^2 C_{D_0}}{2W} - \frac{2Wk}{\rho S V^2}$$

$$\gamma = \frac{\eta_p P_s}{WV} - \frac{\rho S V^2 C_{D_0}}{2W} - \frac{2Wk}{\rho S V^2}$$

$$\gamma = \frac{0.85\times 700\times 10^3}{33600\ V} - \frac{1.225\times 24\times 0.025\ V^2}{2\times 33600}$$

$$- \frac{2\times 33600\times 0.05}{1.225\times 24\ V^2}$$

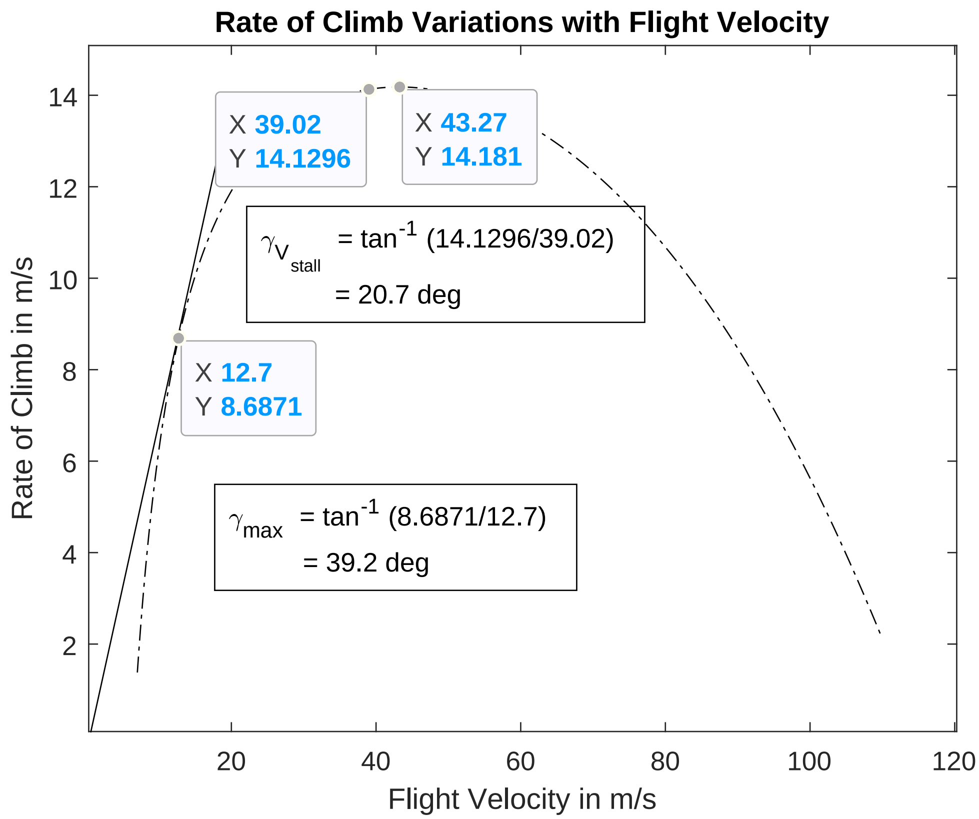
$$\gamma = \frac{17.708}{V} - \underline{1.094\times 10^{-5}}\ V^2 - \frac{114.286}{V^2}$$

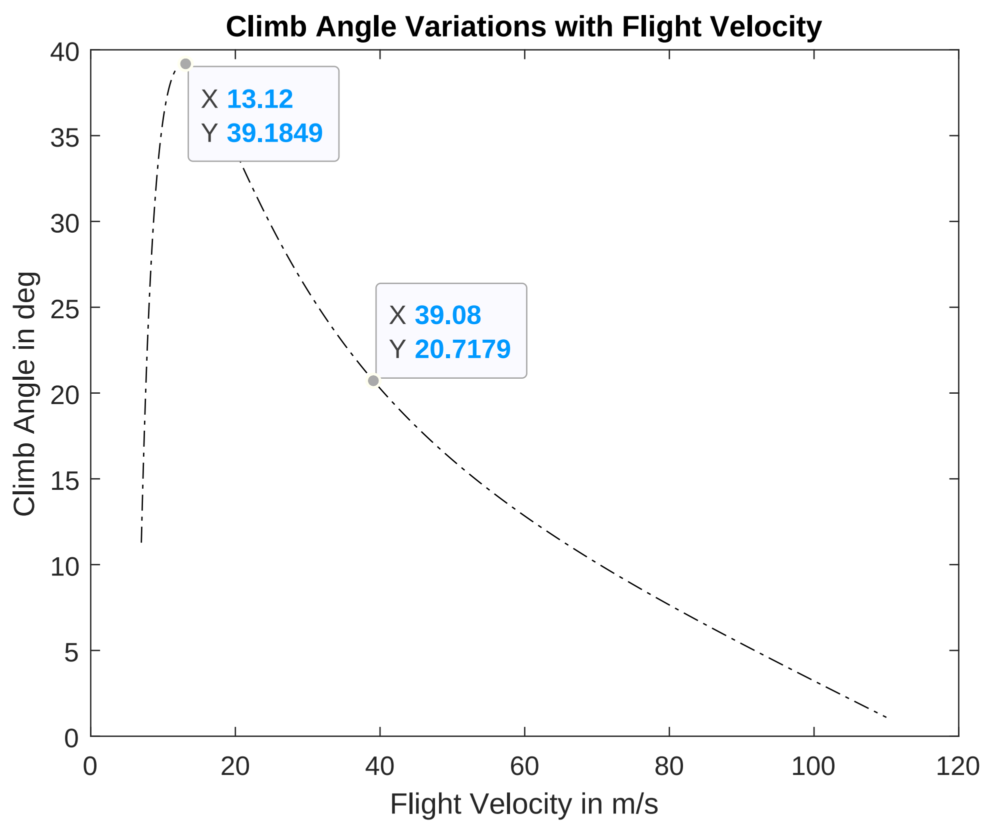
$$\frac{d\gamma}{dV} = -\frac{17.708}{V^2} - 2.188\times 10^{-5}\ V + \frac{228.572}{V^3}$$

$$0 = -17.708\ V^{\cancel{3}} - 2.188\times 10^{-5}\ V^4 + 228.572$$

$$2.188\times 10^{-5}\ V^4 + 17.708\ V - 228.57\cancel{7} = 0$$

Can not be solve analytically.


Rate of Climb Variations with Flight Velocity
Rate of Climb in m/s
Flight Velocity in m/s
X 39.02
Y 14.1296
X 43.27
Y 14.181
X 12.7
Y 8.6871
$\gamma_{V_{stall}}$ = tan^{-1} (14.1296/39.02)
= 20.7 deg
γ_{max} = tan^{-1} (8.6871/12.7)
= 39.2 deg
2
4
6
8
10
12
14
20
40
60
80
100
120


Climb Angle Variations with Flight Velocity
Climb Angle in deg
Flight Velocity in m/s
X 13.12
Y 39.1849
X 39.08
Y 20.7179
0
5
10
15
20
25
30
35
40
0
20
40
60
80
100
120

Solving ~~geo~~ graphically →

$\gamma_{max} = 39.2°$ @ $V_{\gamma_{max}} = 13.2$ m/s

$$V_{stall} = \sqrt{\frac{2W}{\rho S C_{L_{max}}}} = \sqrt{\frac{2 \times 33600}{1.225 \times 24 \times 1.5}} = 39.04 \text{ m/s}$$

$V_{\gamma_{max}} < V_{stall}$, Hence, γ_{max} will occur at V_{stall}.

$\gamma_{max} = 20.7°$ @ $V_{stall} = 39.04$ m/s

Acceptable Answer $\in [19.7°, 21.7]$ deg @ $V_{stall} \in [37.09, 40.99]$ m/s

Question - 7 turbojet

Solution

$W = 20000$ N

$W/S = 1000 \ N/m^2$

$T_0 = 3500$ N

$C_D = 0.015 + 0.024 \ C_L^2$

$C_{L_{max}} = 1.4$

$C_{D0} = 0.015$, $k = 0.024$

(a)

$$V = \sqrt{\frac{W}{\rho S C_{D0}}} \sqrt{T/W \pm \sqrt{(T/W)^2 - 4k C_{D0}}}$$

$$= \sqrt{\frac{1000}{1.225 \times 0.015}} \sqrt{\frac{3500}{20000} \pm \sqrt{\left(\frac{3500}{20000}\right)^2 - 4 \times 0.024 \times 0.015}}$$

$$V = 233.28 \sqrt{0.175 \pm 0.171}$$

$$V_{min} = 233.28 \sqrt{0.175 - 0.171} = 14.754 \text{ m/s}$$

$$V_{max} = 137.219 \text{ m/s} = 233.28 \sqrt{0.175 + 0.171}$$

Acceptable Answer $V_{min} \in [14.02, 15.49]$ m/s, $V_{max} \in [130.36, 144.08]$ m/s

(b)

$$T_{Rmin} = \frac{W}{\left(C_L/C_D\right)_{max}}$$

$$= \frac{W}{E_m}$$

$$E_m = \frac{1}{\sqrt{4 k C_{D0}}} = \frac{1}{\sqrt{4 \times 0.024 \times 0.015}}$$

$$E_m = 26.35$$

$$T_{Rmin} = \frac{20000}{26.35} = 759.01 \text{ N}$$ Ans

Acceptable Answer $\in [721.06, 796.96]$ N

Corresponding velocity, $V_{T_{Rmin}} = \sqrt{\frac{2(W/S)}{\rho}} \cdot \sqrt[4]{\frac{k}{C_{D0}}}$

$$= \sqrt{\frac{2 \times 1000}{1.225}} \times \sqrt[4]{\frac{0.024}{0.015}}$$

$$= 45.44 \text{ m/s}$$

Acceptable Answer $\in [43.17, 47.71]$ m/s

(c) $V_{stall} = \sqrt{\frac{2(W/S)}{\rho \, C_{Lmax}}} = \sqrt{\frac{2 \times 1000}{1.225 \times 1.4}}$

$$= 34.\cancel{1}4 \text{ m/s}$$

$V_{stall} < V_{T_{Rmin}}$

Hence, $V_{T_{Rmin}}$ is the minimum stable velocity for level flight.

Acceptable Answer $\in [43.17, 47.71]$ m/s

Question-8

Solution

$W/S = 1500\ N/m^2$

$S = 25\ m^2$

$C_D = 0.025 + 0.05\, C_L^2$

$C_{L,max} = 1.5$

$P_{s_0} = 750\ kW$

$\eta_p = 0.85$

$W = W/S \times S$
$= 1500 \times 25$
$= 37500\ N$

(a) Solving graphically →

~~Plot~~ ~~and~~

$V_{min} = 7.26\ m/s$, $V_{max} = 115.7\ m/s$

Acceptable Answer → $V_{min} \in [6.9, 7.62]\ m/s$, $V_{max} \in [109.9, 121.5]\ m/s$

(b) $P_{R_{min}} = ?$, $V_{PR,min} = ?$ $(C_L)_{PR_{min}} = ?$

$$P_{R_{min}} = \sqrt{\frac{2W^3}{\rho S}} \times 4 \times \sqrt[4]{\frac{k^3 C_{D_0}}{27}}$$

$$= \sqrt{\frac{2 \times 37500^3}{1.225 \times 25}} \times 4 \times \sqrt[4]{\frac{0.05^3 \times 0.025}{27}}$$

$$= \underline{136.916\ kW}$$

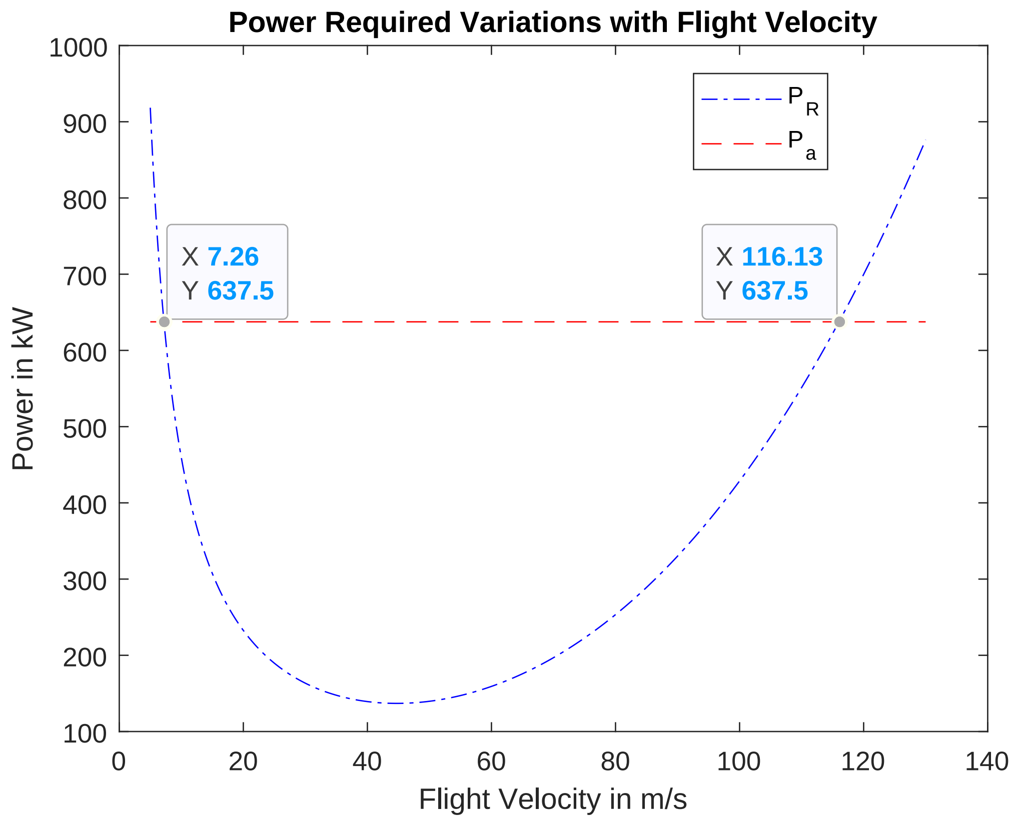
Acceptable Answer $\in [130.07, 143.762]$ kW

$$(C_L)_{PR_{min}} = \sqrt{\frac{3C_{D_0}}{k}} = \sqrt{\frac{3 \times 0.025}{0.05}} = \underline{1.22}$$

Acceptable Answer $\in [1.16, 1.28]$

$$V_{PR_{min}} = \sqrt{\frac{2\, W/S}{\rho\, (C_L)_{PR_{min}}}} = \sqrt{\frac{2 \times 1500}{1.225 \times 1.22}} = \underline{44.80\ m/s}$$

Acceptable Answer $\in [42.56, 47.04]$ m/s


Power Required Variations with Flight Velocity
1000
900
800
700
600
500
400
300
200
100
0
20
40
60
80
100
120
140
Power in kW
Flight Velocity in m/s
P_R
P_a
X 7.26
Y 637.5
X 116.13
Y 637.5

Question - 9

Solution

$W_i = 10000N$

$S = 28\ m^2$

$C_D = 0.015 + 0.025\, C_L^2$

$T_0 = 2300\ N$

$C_0 = 1.1\ N/Nh = 3.056\times10^{-4}\ N/Ns$

(a) Fuel load & cruise velocity for Breguet range of 200 km

Breguet range would be maximum when $E = E_m$ $\left(\frac{C_L}{C_D}\right)_{max}$

$$C_L = \sqrt{\frac{C_{D0}}{k}} = \sqrt{\frac{0.015}{0.025}} = 0.775$$

$$E_m = \frac{1}{\sqrt{4kC_{D0}}} = \frac{1}{\sqrt{4\times0.025\times0.015}} = 25.82$$

$$V_R = \sqrt{\frac{2W}{\rho S C_L}} = \sqrt{\frac{2\times10000}{1.225\times28\times0.775}}$$

$$= \underline{27.43\ m/s}$$

$$R_{max} = \frac{V_R E_m}{C_t} \ln\left(\frac{W_i}{W_f}\right)$$

$$200\times10^3 = \frac{27.43\times25.82}{3.056\times10^{-4}} \ln\left(\frac{W_i}{W_f}\right)$$

$$\frac{W_i}{W_f} = 1.09$$

$$W_f = \frac{W_i}{1.09} = 0.917\ W_i$$

Acceptable Answer $\in [788.5, 871.5]$ N

Fuel Load required $= W_i - W_f = W_i - 0.917 W_i = 0.083 W_i$

$= 0.083\times10000 = \underline{830\ N}$

(b) gain in altitude for a constant velocity range of 200 km.

from the previous section -

$$\frac{W_i}{W_f} = 1.09$$

$$\sigma_f = \left(\frac{W_f}{W_i}\right) \sigma_i$$

$$= \frac{1}{1.09} \times 1 = 0.917$$

$\sigma_i = 1$ at sea level

$$\Delta h = 44.3\left(1 - \sigma_f^{0.235}\right)$$

$$= 44.3\left(1 - 0.917^{0.235}\right)$$

$$= 0.8929 \text{ km}$$

or $\underline{892.9 \text{ m}}$ Acceptable Answer $\in [848.3, 937.5]$ m

(c) Fuel Load required to cover a range of 200 km with constant-altitude ~~profi~~ program.

$$R_{max} = \frac{2}{C_t}\sqrt{\frac{2}{\rho S}}\sqrt[4]{\frac{27}{256\, C_{D_0}^3 k}}\left[\sqrt{W_i} - \sqrt{W_f}\right]$$

$$200 \times 10^3 = \frac{2}{3.056 \times 10^{-4}} \times \sqrt{\frac{2}{1.225 \times 28}} \cdot \sqrt[4]{\frac{27}{256 \times 0.015^3 \times 0.025}}\left[\sqrt{W_i} - \sqrt{W_f}\right]$$

$$= 52841.128\left[\sqrt{10000} - \sqrt{W_f}\right]$$

$$W_f = 9257.34 \text{ N}$$

$$\text{Fuel load required} = W_i - W_f = 10000 - 9257.34$$

$$= \underline{742.66 \text{ N}}$$

Acceptable Answer $\in [705.53, 779.79]$ N

(d) Fuel weight = 803 N , Endurance = ?

$W_f = W_i - W_{fuel}$

$= 10000 - 803$

$= 9197$ N

$$t_{max} = \frac{1}{C_t} E_m \ln\left(\frac{W_i}{W_f}\right)$$

$$= \frac{1}{3.056\times10^{-4}} \times 25.82 \times \ln\left(\frac{10000}{9197}\right)$$

$= 7072.428$ s

or $= 1.965$ hr

Acceptable Answer $\in [1.867, 2.063]$ hr

Question - 10

Solution Propeller airplane-

$W_i = 55000$N

$W/S = 2000$ N/m²

$AR = 6.5$

$C_{D_0} = 0.021$

$e = 0.920$

$P_s = 700$ kW

$\eta_p = 0.82$

$C_p = 3.3$ N/kWhr

$$S = \frac{W_i}{W/S} = \frac{55000}{2000}$$

$= \cancel{2.75}$

$= 27.5$ m²

Assume aircraft operates at 2.4 km ($\sigma = 0.7892$)

(a) Fuel load for 1500 km range.

$$R_{max} = \frac{\eta_p}{C_p} E_m \ln\left(\frac{W_i}{W_f}\right)$$

$$E_m = \frac{1}{\sqrt{4kC_{D_0}}} = \frac{1}{\sqrt{4 \times 0.053 \times 0.021}}$$

$\left[k = \frac{1}{\pi e AR} = \frac{1}{\pi \times 0.92 \times 6.5} = 0.053\right.$

$$E_m = 14.99$$

$$C_p = 3.3 \text{ N/kWh} = \frac{3.3}{10^3 \times 3600} = 9.167 \times 10^{-7} \text{ N/Ws}$$

$$1500 \times 10^3 = R_{max} = \frac{0.82}{9.167 \times 10^{-7}} \times 14.99 \times \ln\left(\frac{W_i}{W_f}\right)$$

$$\frac{W_i}{W_f} = 1.118$$

$$W_f = 0.894\, W_i$$

Fuel load required $= W_i - W_f = W_i - 0.894\, W_i$

$= 0.106\, W_i = 0.106 \times 55000$

$= 5830\text{N}$

Acceptable Answer $\in [5538.5, 6121.5]$ N

(b) to remain in air for 7 h @ h = 2.4 km

$$\sigma = 0.7892$$

$$\rho = \rho_0 \times \sigma = 1.225 \times 0.7892 = 0.967 \text{ kg/m}^3$$

$$t_{max} = \frac{2\eta_p}{C_p}\left(\frac{C_L^{3/2}}{C_D}\right)_{max}\sqrt{\frac{\rho S}{2W}}\left[\frac{1}{\sqrt{W_f}} - \frac{1}{\sqrt{W_i}}\right]$$

$$t_{max} = \frac{2\eta_p}{C_p} \times \frac{1}{4}\sqrt[4]{\frac{27}{k^3 C_{D_0}}}\sqrt{\frac{\rho S}{2}}\left[\frac{1}{\sqrt{W_f}} - \frac{1}{\sqrt{W_i}}\right]$$

$$7\times 3600 = \frac{2\times 0.82}{9.167\times 10^{-7}} \times \frac{1}{4} \times \sqrt[4]{\frac{27}{0.053^3\times 0.021}}\sqrt{\frac{0.967\times 27.5}{2}}\left[\frac{1}{\sqrt{W_f}} - \frac{1}{\sqrt{W_i}}\right]$$

$$2.85\times 10^{-4} = \frac{1}{\sqrt{W_f}} - \frac{1}{\sqrt{55000}}$$

$$W_f = 48324.28 \text{ N}$$

$$\text{fuel load required} = W_i - W_f$$

$$= 55000 - 48324.28$$

$$= \underline{6675.72 \text{ N}}$$

Acceptable Answer $\in [6341.93, 7276.53]$ N