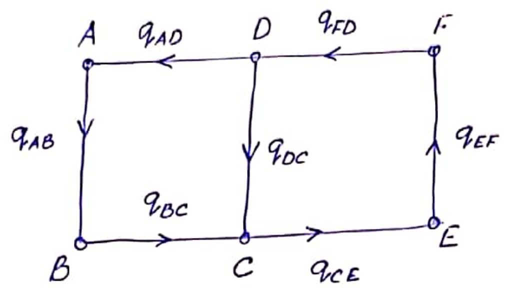
MULTI-CELLED SECTION + TORSION (Proofs) (Added) ①



- Shear flow in each arm is a constant
- Equilibrium at nodes A, B, E, F gives:

$q_{AD} = q_{AB}$ (at A)

$q_{AB} = q_{BC}$ (at B) $\Rightarrow q_{AB} = q_{AD} = q_{BC} = q_1$

Similarly,

$q_{CE} = q_{EF}$ (at E)

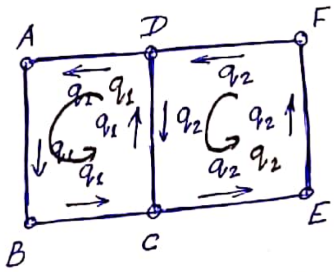
$q_{EF} = q_{FD}$ (at F)

$\Big\} \; q_{CE} = q_{EF} = q_{FD} = q_2$

At node C: $\quad q_{DC} + q_{BC} = q_{CE} \Rightarrow q_{DC} = q_2 - q_1$

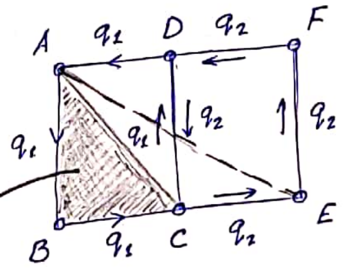
(shear flowing in = shear flowing out).

PICTURE:



← Each cell can be thought to have its own constant shear flow q_i

TORQUE BALANCE:
(Lets take about A).



Torque at A due to:

q_1 in BC ; q_2 in CE ;
q_2 in EF ; q_1 in CD , $-q_2$ in CD

$+\Delta T \rightarrow \underbrace{q_1 (\text{area } \Delta ABC)\times 2}_{\text{in arm } BC}$; $\underbrace{q_1 (\text{area of } \Delta ACD)\times 2}_{\text{in arm } CD}$; $\underbrace{q_2 (\text{area of } \Delta CEA)\times 2}_{\text{arm } CE}$

$\underbrace{q_2 (\text{area } \Delta FEA)\times 2}_{\text{arm } FE}$;

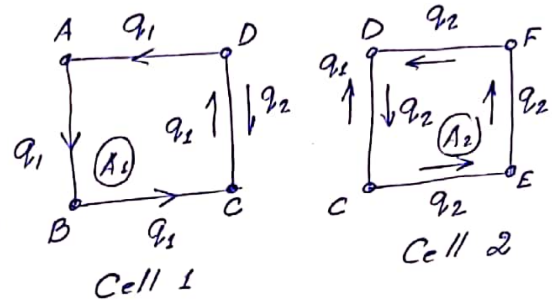
$-\Delta T \rightarrow -\underbrace{q_2 (\text{area } \Delta DCA)}_{\text{arm } CD}$

$$\therefore T = 2\Big[q_1 \underbrace{(\Delta ABC + \Delta ACD)}_{\text{area } A_1 = \text{area of } ABCD} \Big] + 2\Big[q_2 \underbrace{(\overbrace{\Delta CEA + \Delta FEA}^{\text{area of } COFE + \text{area } \Delta ACD} - \Delta ACD)}_{\text{area } A_2 = \text{area of } CDFE} \Big]$$

$$\therefore \boxed{T = 2A_1 q_1 + 2A_2 q_2} \quad \text{——— (1)}$$

Cell Rotation :

$$\gamma_{xs}\Big|_{①} = \frac{\partial u}{\partial s} + \frac{\partial v_t}{\partial x}$$ ← tangential displacement



$v_t = r_1\, \theta_1(x)$ ← about its own centre of rotation

$$\Rightarrow \frac{q_{xs}|_{①}}{Gt} = \frac{\partial u}{\partial s} + r_1 \alpha_1$$

($\alpha_1 = \frac{d\theta_1}{dx}$ ← rate of twist of cell 1)

$$\Rightarrow \oint_{C_1} \frac{q_{xs}}{Gt} ds = 0 + \alpha_1 (2A_1) \Rightarrow \alpha_1 = \frac{1}{2A_1} \oint \frac{q_{xs}}{Gt} ds$$

$$\Rightarrow \alpha_1 = \frac{1}{2A_1}\left[q_{AD} \int_{AD} \frac{ds}{Gt} + q_{AB} \int \frac{ds}{Gt} + q_{BC} \int \frac{ds}{Gt} + q_{CD} \int \frac{ds}{Gt} \right]$$

$$= \frac{1}{2A_1}\left[q_1 \left(\frac{l_{AD}}{Gt_{AD}}\right) + q_1 \left(\frac{l_{AB}}{Gt_{AB}}\right) + q_1 \left(\frac{l_{BC}}{Gt_{BC}}\right) + (q_1 - q_2)\left(\frac{l_{CD}}{Gt_{CD}}\right) \right] \quad \text{——— (2)}$$

Similarly, $$2A_2 \alpha_2 = \oint_{C_2} \frac{q_{xs}}{Gt} ds \Rightarrow \alpha_2 = \frac{1}{2A_2}\left[q_{CE} \frac{l_{CE}}{Gt_{CE}} + q_{EF} \frac{l_{EF}}{Gt_{EF}} + q_{FD} \frac{l_{FD}}{Gt_{FD}} + q_{DC} \frac{l_{DC}}{Gt_{DC}} \right]$$

$$= \frac{1}{2A_2}\left[q_2 \left(\frac{l_{CE}}{Gt_{CE}}\right) + q_2 \left(\frac{l_{EF}}{Gt_{EF}}\right) + q_2 \left(\frac{l_{FD}}{Gt_{FD}}\right) + q_2 \left(\frac{l_{CD}}{Gt_{CD}}\right) - q_1 \left(\underbrace{\frac{l_{CD}}{Gt_{CD}}}\right) \right] \quad \text{——— (3)}$$

material, thickness can be different in each limb

Now, the section rotates in unison, or, $\alpha_1 = \alpha_2 = \alpha$

CRUX OF THE MODEL

($\theta_1 = \theta_2 = \theta$)

$$\Rightarrow \frac{1}{2A_1}\left[q_1 \left(\frac{l_{AD}}{Gt_{AD}} + \frac{l_{AB}}{Gt_{AB}} + \frac{l_{BC}}{Gt_{BC}} + \frac{l_{CD}}{Gt_{CD}} \right) - q_2 \frac{l_{CD}}{Gt_{CD}} \right]$$
$$= \frac{1}{2A_2}\left[q_2 \left(\frac{l_{CE}}{Gt_{CE}} + \frac{l_{EF}}{Gt_{EF}} + \frac{l_{FD}}{Gt_{FD}} + \frac{l_{CD}}{Gt_{CD}} \right) - q_1 \left(\frac{l_{CD}}{Gt_{CD}}\right) \right] \quad \text{——— (4)}$$

→ From 1 and 4 we can get q_1, q_2

→ Given q_1, q_2 we can find α in terms of T, G; dimensions of limbs

⇓

J comes out of this