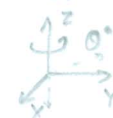
Question Set #1

Q1 We have shown in class that $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. Also, traction (or stress) vector $\vec{T} = [\sigma]\vec{n}$. Use these expression to derive corresponding strain and stress tensor components in a ROTATED coordinate system $x_1' - x_2' - x_3'$ given by $\vec{x}' = [Q]\vec{x}$, where $[Q]$ is the rotation matrix with components Q_{ij}.

Special case of rotation about z-axis



$$\begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Q2 If the state of stress is given by $\sigma_{11} = 10$ MPa; $\sigma_{12} = 5$ MPa; $\sigma_{22} = 20$ MPa; $\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$. Obtain the principal stresses σ_I, σ_{II}, $\sigma_{III} = 0$. Give the corresponding directions.

$$\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix} \begin{Bmatrix} n_1 \\ n_2 \end{Bmatrix} = \lambda \begin{Bmatrix} n_1 \\ n_2 \end{Bmatrix}$$

$(\lambda^{(1)}, \vec{n}^{(1)})$, $(\lambda^{(2)}, \vec{n}^{(2)})$

$(0, \vec{n}^{(3)} = (0,0,1))$

$\tau_{max} = ?$ $\sigma_{RMS} = ?$

Q3 For the problem statement given in problem 2, determine the state of strain (in both the original and principal coordinates) when the material is isotropic with $E = 70$ GPa, $\nu = 0.3$.

4 If the material properties are given by $E = 210$ GPa, $\nu = 0.3$ determine the following:

(a) The Lame's constants λ and μ.

(b) Elements of the compliance matrix and stiffness matrix

(c) Elements of the compliance and stiffness matrix under the plane strain AND plane stress assumptions.

5 If ~~Poisson's ratio~~ Young's modulus $E > 0$, what are the admissible values of ν? (Hint: Look at volumetric expansion).

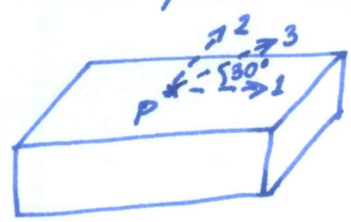
Q6: Go back to your MOS book (Crandell & Dahl), and from chapter 5, find out the shape the Mises curve for a plane stress problem ($\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$). If $\sigma_Y = 500$ MPa, write a MATLAB program to generate the YIELD surface for various combinations of principal stresses σ_1 and σ_2. Also, draw the Tresca Yield surface for the same.

Q7: Extend the code to draw yield surfaces for the 3D-state of stress given by the principal state $\sigma_1, \sigma_2, \sigma_3$. Again use $\sigma_Y = 500$ MPa.

Q8 If the state of strain at a surface point is given by

$\varepsilon_1 = 10^{-4}$; $\varepsilon_2 = 2 \times 10^{-5}$;

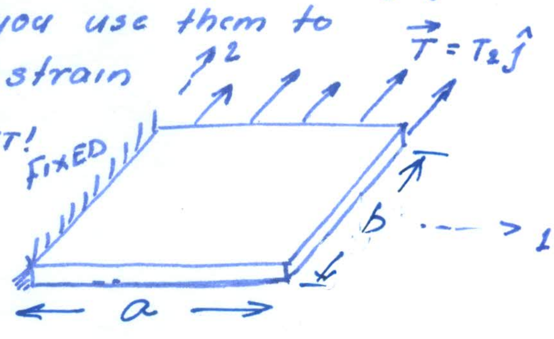
$\varepsilon_3 = 3 \times 10^{-5}$ (direction 3 is at 30° to direction 1), determine the surface state of strain $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$.



Assuming plane stress case, determine the corresponding surface stress $\sigma_{11}, \sigma_{22}, \sigma_{12}$.

Q9 If you are given a thin plate and as many strain gages as you want (or rossettes). How will you use them to obtain a good picture of the state of strain almost everywhere? PLAN AN EXPERIMENT!

Suggest a way to work out the displacement field from your measurements.



* ALSO DO PROBLEMS OF CHAPTER 1 OF MEGSON.