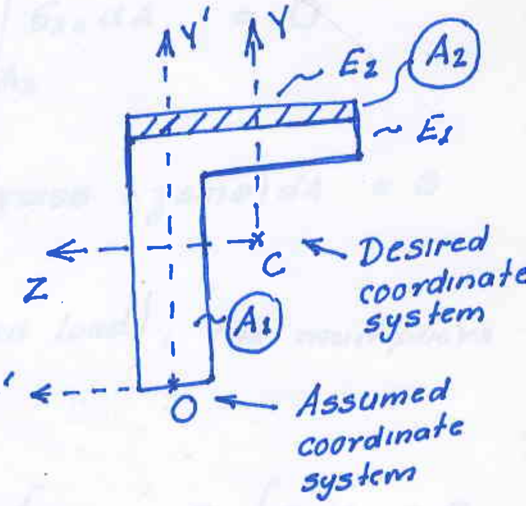
PRACTICAL CONSIDERATIONS

→ The cross-sections encountered in practice may be multi-material in construction

How to handle these ?



① Need to find the new NEUTRAL POINT, i.e. point of ZERO axial strain.

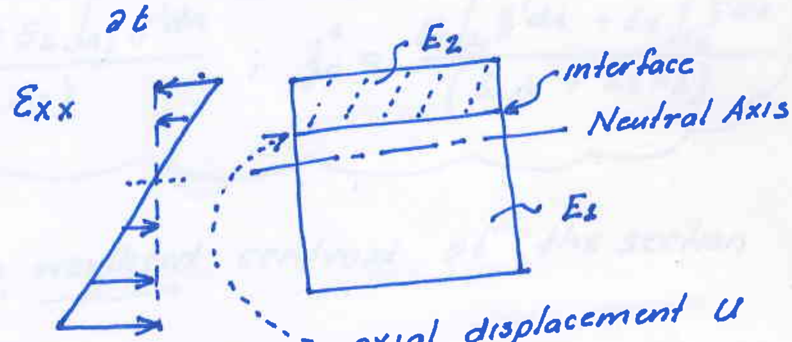
Will it be the area centroid ?

② Where do we start ?

Assumption : Strain varies linearly across the cross section, from compressive to tensile. (WHY ?)

~ This is fine as across the material interfaces the displacements are continuous, and hence the tangential derivative $\frac{\partial u}{\partial t}$ is also continuous

~ Also, the beam will still bend into an arc of a circle.



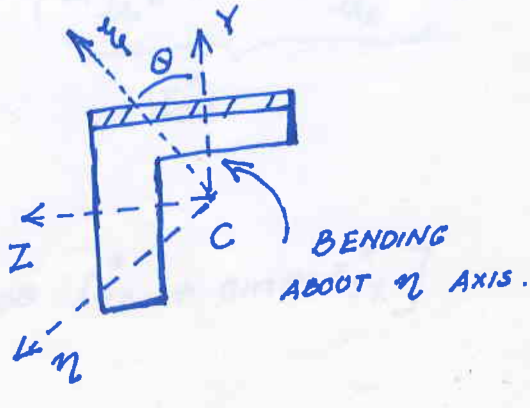
axial displacement u is continuous and so is $\frac{\partial u}{\partial x}$. Note that $\frac{\partial u}{\partial y}$ will not be continuous across the interface !

$\therefore \quad \varepsilon_{xx} = -\frac{\varepsilon}{R}$ where ε is the distance along the direction of bending.

or $\varepsilon_{xx} = -\frac{1}{R}\left(y\cos\theta + z\sin\theta\right)$

$$\Rightarrow \quad \sigma_{xx} = \begin{cases} E_1 \cdot \varepsilon_{xx} & \text{in material 1} \\ E_2 \cdot \varepsilon_{xx} & \text{in material 2} \end{cases}$$

~ (A)



Now, resultant axial force $F_x = 0$

$$\Rightarrow \int_A \sigma_{xx}\, dA = \int_{A_1} \sigma_{xx}\, dA + \int_{A_2} \sigma_{xx}\, dA = 0$$

$$\Rightarrow -\frac{1}{R}\int_{A_1} E_1 (y\cos\theta + z\sin\theta)\, dA - \frac{1}{R}\int_{A_2} E_2 (y\cos\theta + z\sin\theta)\, dA = 0$$

As θ can vary (depending on the applied load), the multipliers of each term should be zero, i.e.

$$E_1\int_{A_1} y\, dA + E_2\int_{A_2} y\, dA = 0 \quad ; \quad E_1\int_{A_1} z\, dA + E_2\int_{A_2} z\, dA = 0$$

where ξ, η, y, z are taken about the neutral-point.

From the figure, $y = y' - y_c^* \; ; \; z = z' - z_c^*$

$$\Rightarrow y_c^* = \frac{E_1\int_{A_1} y'\, dA + E_2\int_{A_2} y'\, dA}{(E_1 A_1 + E_2 A_2)} \quad ; \quad z_c^* = \frac{E_1\int_{A_1} z'\, dA + E_2\int_{A_2} z'\, dA}{(E_1 A_1 + E_2 A_2)}$$

This gives us the modulus weighted centroid of the section

WHAT ABOUT THE EXPRESSION FOR σ_{xx} ?

$$M_z = -\int_A \sigma_{xx} \cdot y\, dA = \frac{1}{R}\left[\int_{A_1} E_1 (y\cos\theta + z\sin\theta)\, y\, dA + \int_{A_2} E_2 (y\cos\theta + z\sin\theta)\cdot y\, dA\right] = \frac{1}{R}\left[\cos\theta \underbrace{\left(E_1\int_{A_1} y^2\, dA + E_2\int_{A_2} y^2\, dA\right)}_{I_{zz}^*} + \sin\theta \underbrace{\left(E_1\int_{A_1} yz\, dA + E_2\int_{A_2} yz\, dA\right)}_{I_{yz}^*}\right]$$

$$M_y = \int_A \sigma_{xx} \cdot z\, dA = -\frac{1}{R}\left[\cos\theta\, I_{yz}^* + \sin\theta\, I_{yy}^*\right]$$

Thus,

$$\begin{Bmatrix} M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} -I^*_{yz} & -I^*_{yy} \\ I^*_{zz} & I^*_{yz} \end{bmatrix} \begin{Bmatrix} \cos\theta/R \\ \sin\theta/R \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \cos\theta/R \\ \sin\theta/R \end{Bmatrix} = \frac{1}{\underbrace{(I^*_{yy} I^*_{zz} - I^{*2}_{yz})}_{\Delta^*}} \begin{bmatrix} I^*_{yz} & I^*_{yy} \\ -I^*_{zz} & -I^*_{yz} \end{bmatrix} \begin{Bmatrix} M_y \\ M_z \end{Bmatrix}$$

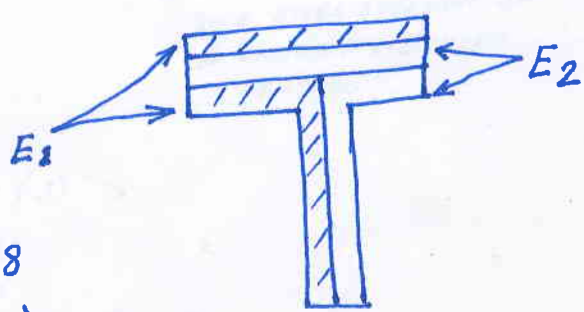
$$\Rightarrow \varepsilon_{xx} = -\left[\frac{I^*_{yz} M_y + I^*_{yy} M_z}{\Delta^*} \cdot y + \frac{-I^*_{zz} M_y - I^*_{yz} M_z}{\Delta^*} z \right]$$

σ_{xx} will come from (A), depending on which material (E_1, E_2) the desired point (y, z) lies in.

THOUGHT EXERCISES

1) Determine the equations of equilibrium in terms of v, w.

2) Take the case of a bi-material beam with cross-section as given. Suggest how should one proceed with the problem formulation.



Do. (MEGSON)

Problems 16.1 – 16.4, 16.6, 16.8

< See solved examples >