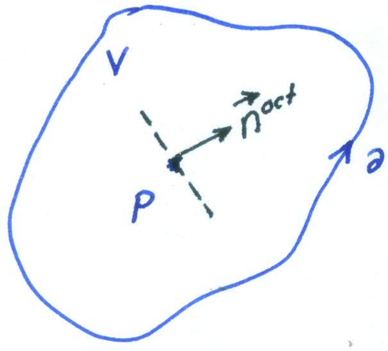
A NOTE ON YIELDING



← Generic point P

← Cutting plane with normal $\vec{n}^{oct} = (\pm 1/\sqrt{3}, \pm 1/\sqrt{3}, \pm 1/\sqrt{3})$ with respect to $\underbrace{X_1^P - X_2^P - X_3^P}_{\text{principal stress axes}}$

– In principal coordinate system at P, state of stress is:

$$[\sigma] = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

– On the cutting plane with normal $\vec{n}^{oct}$, we have

$$t_n = \vec{t} \cdot \vec{n}^{oct} = t_1 n_1 + t_2 n_2 + t_3 n_3$$

$$t_i = \sigma_{ji} n_j \Rightarrow t_i = \lambda_j \delta_{ij} n_i \Rightarrow t_1 = \lambda_1 n_1 \; ; t_2 = \lambda_2 n_2$$

$$t_3 = \lambda_3 n_3$$

$$\Rightarrow t_n = \lambda_1 n_1^2 + \lambda_2 n_2^2 + \lambda_3 n_3^2 = \frac{\lambda_1 + \lambda_2 + \lambda_3}{3} = \sigma_H$$

σ_H → hydrostatic stress ~ does not cause failure for the crystalline materials (actually failure is at HIGH value of σ_H).

$\therefore$ t_s^{oct} = <u>shear component of</u> $\vec{t}$, such that

$$(t_s^{oct})^2 = |\vec{t}|^2 - t_n^2 = (\lambda_1^2 n_1^2 + \lambda_2^2 n_2^2 + \lambda_3^2 n_3^2) - t_n^2$$

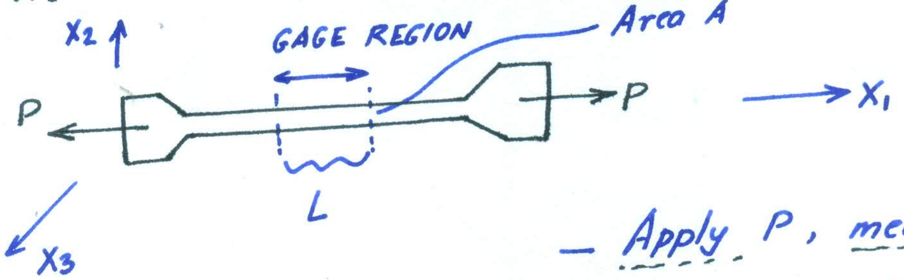
$$= \frac{(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}{3} - \left(\frac{\lambda_1 + \lambda_2 + \lambda_3}{3}\right)^2$$

$$= \frac{1}{9}\left[(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2\right]$$

* This plane is the OCTAHEDRAL plane, on which "failure" happens due to ONLY t_s, as $t_n = \sigma_H$ does not cause failure

* $\boxed{t_s^{oct} \leq t_s^{c}}$ can thus be a yield criterion which will work for the 3D-state of stress – its now about comparing two numbers!

– If this is true for $[\sigma]$ due to any loading, then it is true for a 1D-experiment also!



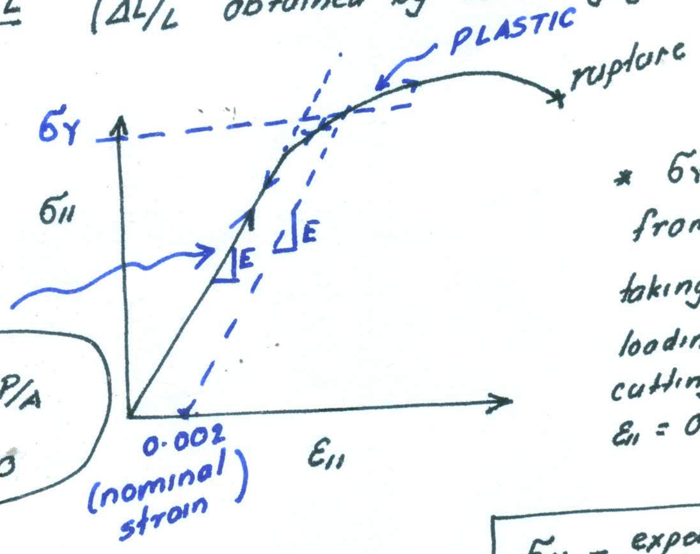
– Apply P, measure ΔL in gage region (at centre).

– Here we get (recorded using a LOAD CELL)

$\sigma_{11} = P/A$; $\sigma_{ij} = 0$ for all other stress components

$\varepsilon_{11} = \dfrac{\Delta L}{L}$ ($\Delta L/L$ obtained by strain gage or extensometer)

– Plot:



ELASTIC

$\Rightarrow$ $\lambda_1 = \sigma_{11} = P/A$
$\lambda_2 = \lambda_3 = 0$

* σ_Y obtained from graph by taking line // to loading line and cutting ε_{11} axis at $\varepsilon_{11} = 0.002$.

$\boxed{\sigma_Y \text{ – experimental; material dependent}}$

$$\therefore \left(t_s^{oct}\right)^2 = \frac{1}{9}\left[(\sigma_Y - 0)^2 + 0 + (0 - \sigma_Y)^2\right] = \frac{2}{9}\sigma_Y^2$$

$$\Rightarrow \frac{1}{9}\left[(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2\right] = \frac{2}{9}\sigma_Y^2$$

$$\Rightarrow \underbrace{\frac{1}{2}\left[(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_3 - \lambda_1)^2\right]}_{\sigma_{RMS}^2 \text{ or } \sigma_{von-Mises}^2} = \sigma_Y^2$$

∴ Mises yield criterion:

$$\boxed{\begin{array}{ll} \sigma_{RMS} < \sigma_Y & \Rightarrow \text{NO YIELDING} \\ \text{at } \sigma_{RMS} = \sigma_Y & \Rightarrow \text{YIELDING} \end{array}}$$

Similarly, we have another criterion: TRESCA YIELD CRITERION:

Here $\tau_{max} = \text{max. shear} = \dfrac{\lambda_{max} - \lambda_{min}}{2}$

with $\tau_{max} < \tau^c \Rightarrow$ no yielding

For 1D-example: $\tau_{max} = \dfrac{\sigma_Y - 0}{2}$

$\Rightarrow \tau_{max} < \dfrac{\sigma_Y}{2}$ gives no yielding:

$$\boxed{\tau_{max} = \frac{\sigma_Y}{2} \Rightarrow \text{yielding.}}$$

At yield

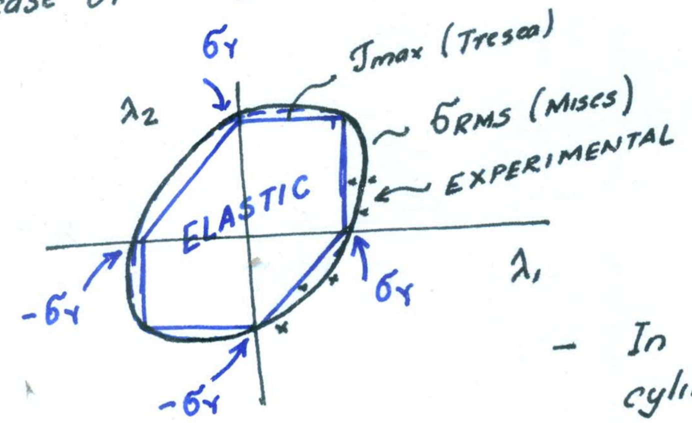
$\tau_{oct} = \dfrac{\sqrt{2}}{3}\sigma_Y$

$\tau_{max} = \dfrac{1}{2}\sigma_Y$

For ductile materials τ_{oct} does the job better!

How different are these?

2D-case or $\lambda_3 = 0$; $\lambda_1, \lambda_2 \neq 0$



YIELD SURFACE

– In 3D this becomes a cylinder

Experimental data ⟶ closer to Mises criterion

σ_Y	Material
~ 400 MPa	Al-alloy
~ 400–1600 MPa	Steel
~ 800 MPa	Ti-alloy
~ 1–3 GPa	Carbon fiber
1.15 GPa	Spider silk. ⟵ think about it!