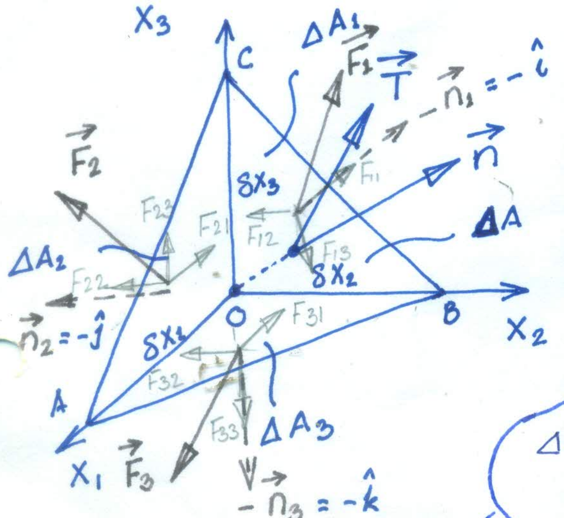
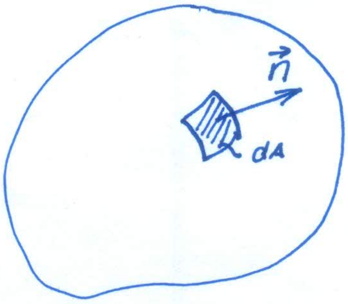
ALL ABOUT STRESS!





$\Delta A_1 = \frac{1}{2}\,\delta X_2 \cdot \delta X_3$; $\Delta A_2 = \frac{1}{2}\,\delta X_1 \cdot \delta X_3$;

$\Delta A_3 = \frac{1}{2}\,\delta X_1 \cdot \delta X_2$

$$\Delta A\,\vec{n} = \vec{\Delta A} = \frac{1}{2}(\vec{AB} \times \vec{AC})$$
$$= \frac{1}{2}\left(-\delta X_1\hat{i} + \delta X_2\hat{j}\right) \times \left(-\delta X_1\hat{i} + \delta X_3\hat{k}\right)$$
$$= \Delta A_1\,\hat{i} + \Delta A_2\,\hat{j} + \Delta A_3\,\hat{k}$$

Now $\Delta A_1 = \Delta A \cdot n_1$; $\Delta A_2 = \Delta A \cdot n_2$; $\Delta A_3 = \Delta A \cdot n_3$

where $\vec{n} = n_1\hat{i} + n_2\hat{j} + n_3\hat{k}$

$\therefore$ force balance:

$$\underset{\delta X_i \to 0}{Lt}\quad \vec{T}\,\cancel{\Delta A} + \vec{F_1}\,\underbrace{\Delta A_1}_{\cancel{\Delta A}\,n_1} + \vec{F_2}\,\underbrace{\Delta A_2}_{\cancel{\Delta A}\,n_2} + \vec{F_3}\,\underbrace{\Delta A_3}_{\cancel{\Delta A}\,n_3} + \frac{\Delta V}{\Delta A}\cdot\vec{f} = 0$$

$$\Rightarrow \vec{T} = -\left(\vec{F_1}\,n_1 + \vec{F_2}\,n_2 + \vec{F_3}\,n_3\right)$$

$\vec{F_1}$: $-F_{11}\hat{i} - F_{12}\hat{j} - F_{13}\hat{k}$; $\vec{F_2}$: $-F_{21}\hat{i} - F_{22}\hat{j} - F_{23}\hat{k}$; $\vec{F_3}$: $-F_{31}\hat{i} - F_{32}\hat{j} - F_{33}\hat{k}$

$$= \left(F_{11}n_1 + F_{21}n_2 + F_{31}n_3\right)\hat{i} + \left(F_{12}n_1 + F_{22}n_2 + F_{32}n_3\right)\hat{j}$$
$$+ \left(F_{13}n_1 + F_{23}n_2 + F_{33}n_3\right)\hat{k}$$
$$= T_1\hat{i} + T_2\hat{j} + T_3\hat{k}$$

$$\therefore \boxed{T_i = F_{ji}\, n_j}$$

stress (force per unit area)

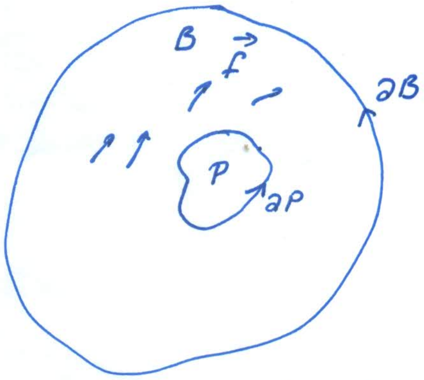
$$= \sigma_{ji}\, n_j = [\sigma]^T \{n\}$$

Cauchy Stress

Cauchy's hypothesis: The local, pointwise traction vector $\vec{T}$ is a linear function of the normal vector $\vec{n}$. $[\sigma]$ is the operator and is called the stress tensor.

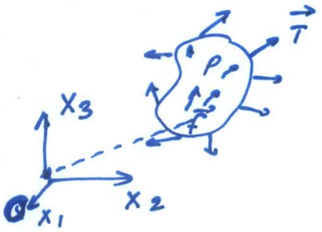
This relationship is true for any PORTION P of B, i.e.

$$\vec{T}\Big|_{\partial P} = [\sigma]^T \vec{n}\Big|_{\partial P}$$



Now the portion also has to be in equilibrium or

$$\int_P \vec{f}\, dV + \int_{\partial P} \vec{T}\, dA = \vec{0}$$



$$\Rightarrow \int_P \vec{f}\, dV + \int_{\partial P} [\sigma]^T \vec{n}\, dA = \vec{0}$$

$$\Downarrow$$

$$\int_P f_i\, dV + \int_{\partial P} \sigma_{ji}\, n_j\, dA = 0 \quad ; i = 1,2,3$$

$$\Rightarrow \int_P (f_i + \sigma_{ji,j})\, dV = 0$$

$\longleftarrow$ true for all arbitrary portions $\Rightarrow \boxed{f_i + \sigma_{ji,j} = 0}$

Force balance equation

Moment balance:

Take moment about origin

$$\Rightarrow \int_P \underbrace{\vec{r} \times \vec{f}}_{r_i f_j \vec{e}_k}\, dV + \int_{\partial P} \underbrace{\vec{r} \times \vec{T}}_{r_i T_j \vec{e}_k}\, dA = \vec{0}$$

(i, j, k in cyclic symmetry)

Let us look at a specific case:

$$\int_P r_1 f_2 \vec{e}_3 \, dV + \int_{\partial P} r_1 \underbrace{T_2}_{(\sigma_{12} n_1 + \sigma_{22} n_2 + \sigma_{32} n_3)} \vec{e}_3 \, dA$$

$$\downarrow$$

$$\int_P (\overset{X_1}{r_1} \sigma_{12})_{,1} + (\overset{X_1}{r_1} \sigma_{22})_{,2} + (\overset{X_1}{r_1} \sigma_{32})_{,3}$$

$$\int_P \left(\sigma_{12} + X_1 \underline{\sigma_{12,1}} + X_1 \underline{\sigma_{22,2}} + X_1 \underline{\sigma_{32,3}} \right) dV$$

$$\boxed{\int_P \left(\sigma_{12} + \cancel{X_1 (-f_2)} \right) dV} + \cancel{\int_P X_1 f_2 \, dV} = \int_P \sigma_{12} \, dV$$

Similarly, $\displaystyle\int_P - r_2 f_1 \vec{e}_3 \, dV + \int_{\partial P} - \overset{X_2}{r_2} \underbrace{T_1}_{(\sigma_{11} n_1 + \sigma_{21} n_2 + \sigma_{31} n_3)} \vec{e}_3 \, dA$

$$= \int_P \cancel{- X_2 f_1 \vec{e}_3} \qquad - \int_P \left\{ (\sigma_{11} X_2)_{,1} + (\sigma_{21} X_2)_{,2} + (\sigma_{31} X_2)_{,3} \right\} dV$$

$$- \int_P X_2 \left(\cancel{\overset{-f_1}{\sigma_{11,1} + \sigma_{21,2} + \sigma_{31,3}}} \right) dV - \int_P \sigma_{21} \, dV = \int_P - \sigma_{21} \, dV$$

$\Rightarrow$ Corresponding to $\vec{e}_3$, we have $\displaystyle\int_P (\sigma_{12} - \sigma_{21}) \, dV = 0$

$\Rightarrow \boxed{\sigma_{12} = \sigma_{21}}$ $\leftarrow$ Stress symmetry!

OR $\boxed{\sigma_{ij} = \sigma_{ji}}$ $\Rightarrow [\sigma]^T = [\sigma]$

This means that stress tensor is symmetric $\Rightarrow$

when we look for those specific directions $\vec{n}$ such that

$\boxed{\vec{T} = [\sigma] \vec{n} = \lambda \vec{n}}$ ~ i.e. $\vec{T}$ directed along $\vec{n}$

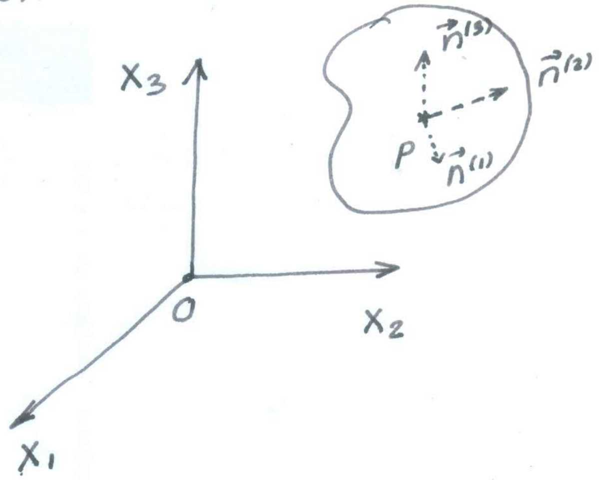
$\Rightarrow$ $\lambda_i \rightarrow$ EIGENVALUE, $\vec{n}^{(i)} \rightarrow$ EIGENVECTOR

$\underbrace{\text{3 perpendicular directions}}$ ~ PRINCIPAL DIRECTIONS

$\lambda_i \rightarrow$ PRINCIPAL STRESSES

$\vec{n}^{(i)} \rightarrow$ PRINCIPAL DIRECTION

x_1^P, x_2^P, x_3^P ~ forms a rotated coordinate system.



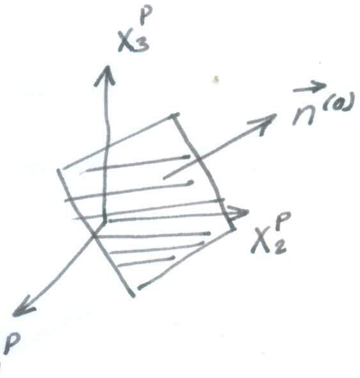
OCTAHEDRAL PLANE:

Equally inclined to the 3 principal directions,

$$\Rightarrow \quad \vec{n}^{(0)} = \frac{1}{\sqrt{3}}(i' + j' + k')$$

as $n_1^{(0)} = n_2^{(0)} = n_3^{(0)}$ and

$$n_1^{(0)2} + n_2^{(0)2} + n_3^{(0)2} = 1$$



On this plane, the normal stress is:

$$\vec{T}\cdot\vec{n}^{(0)} = T_1 n_1^{(0)} + T_2 n_2^{(0)} + T_3 n_3^{(0)} = T_n$$

$$= (\sigma_{11} n_1^{(0)} + \cancel{\sigma_{21} n_2^{(0)}} + \cancel{\sigma_{31} n_3^{(0)}})\, n_1^{(0)} + (\cancel{\sigma_{12} n_1^{(0)}} + \sigma_{22} n_2^{(0)} + \cancel{\sigma_{32} n_3^{(0)}})\, n_2^{(0)} + (\cancel{\sigma_{13} n_1^{(0)}} + \cancel{\sigma_{23} n_2^{(0)}} + \sigma_{33} n_3^{(0)})\, n_3^{(0)}$$

$$= \lambda_1 n_1^{(0)2} + \lambda_2 n_2^{(0)2} + \lambda_3 n_3^{(0)2} = \frac{(\lambda_1 + \lambda_2 + \lambda_3)}{3}$$

$$= \sigma_H \leftarrow \text{HYDROSTATIC STRESS}.$$

The shear part is $\vec{T} - \vec{T}\cdot\vec{n}^{(0)}\,\vec{n}$ or T_s

$$|T_s|^2 + T_n^2 = |\vec{T}|^2 \Rightarrow |T_s|^2 = (\sigma_{11}^2 n_1^{(0)2} + \sigma_{22}^2 n_2^{(0)2} + \sigma_{33}^2 n_3^{(0)2}) - \left(\frac{(\lambda_1 + \lambda_2 + \lambda_3)}{3}\right)^2 = \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{3} - \frac{(\lambda_1 + \lambda_2 + \lambda_3)^2}{9}$$

$$= \frac{2}{9}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) - \frac{2}{9}(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$$

$$= \frac{1}{9}\left[(\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2\right]$$

$$\Rightarrow |T_s^{(o)}| = \sqrt{\frac{1}{9}\left((\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2\right)}$$

UNI-AXIAL LOAD $\Rightarrow \lambda_1 \neq 0, \ \lambda_2 = \lambda_3 = 0$

$$\Rightarrow |T_s^{(o)}| = \sqrt{\frac{1}{9}(2\lambda_1^2)} = \sqrt{\frac{2}{9}}\,\lambda_1$$

$\Rightarrow$ At critical failure in multi-dimensional stress state the critical octahedral shear is constant, i.e.

$$|T_s^{(o)}|_c = T_{s,c} = \frac{\sqrt{2}}{3}\lambda_{1,c} = \frac{\sqrt{2}}{3}\sigma_Y$$

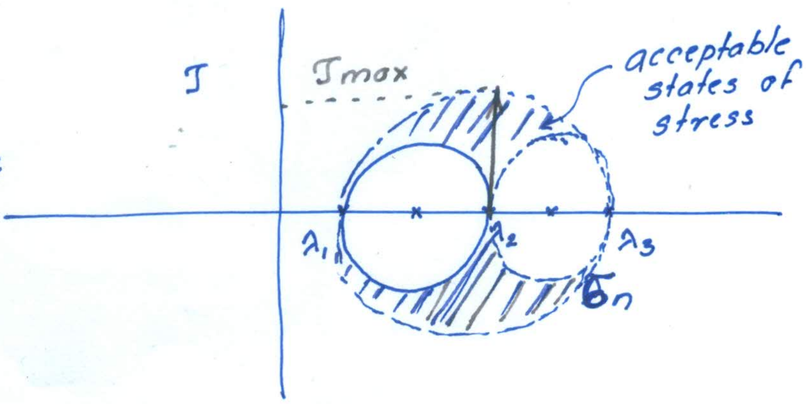
$$\Rightarrow \boxed{\sqrt{\frac{1}{2}\left\{(\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2\right\}} = \sigma_Y}$$

MISES CRITERION

* NOTE: Since 1970 it is established that for DUCTILE metals (steel, Al-alloys) σ_H also plays a role, especially at higher temperatures, in initiation and growth of damage.

3D MOHR's CIRCLE:

$\lambda_1, \lambda_2, \lambda_3$ define the radii of the 3 circles.



ISOTROPIC MATERIAL : Constitutive relationship for linear elasticity

RELATE STRESS TO STRAIN : (HOOKE'S LAW!)

$$\boxed{\sigma_{ij} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\mu\, \varepsilon_{ij}}$$

Takes care of rotation independence of relationship, as ε_{kk} is an invariant

$$\therefore\ \sigma_{i'j'} = \lambda\, \varepsilon_{k'k'}\, \delta_{ij} + 2\mu\, \varepsilon_{i'j'} = \lambda\, \varepsilon_{kk}\, \delta_{ij} + 2\mu\, \varepsilon_{i'j'}$$

$\lambda, \mu \rightarrow$ LAME'S CONSTANT

OR

$$\sigma_{kk} = 3\lambda\, \varepsilon_{kk} + 2\mu\, \varepsilon_{kk} \Rightarrow \varepsilon_{kk} = \frac{\sigma_{kk}}{(3\lambda + 2\mu)}.$$

$$\Rightarrow 2\mu\, \varepsilon_{ij} = \sigma_{ij} - \frac{\lambda\, \sigma_{kk}}{(3\lambda+2\mu)} \cdot \delta_{ij}$$

$$\Rightarrow \varepsilon_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{(3\lambda+2\mu)(2\mu)}\sigma_{kk} \cdot \delta_{ij} \quad \underbrace{+ \alpha\, \delta_{ij}\, \Delta T}_{\text{for thermal}}$$

$$\varepsilon_{11} = \frac{\sigma_{11}}{2\mu} - \frac{\lambda}{2\mu(3\lambda+2\mu)} \cdot (\sigma_{11} + \sigma_{22} + \sigma_{33})$$

$$\varepsilon_{12} = \frac{1}{2\mu}\sigma_{12}$$

shear modulus

$$\varepsilon_{11} = \overbrace{\frac{1 \cdot \sigma_{11}}{2\mu(3\lambda+2\mu)}(2\lambda + 2\mu)}^{\frac{1}{E}} - \overbrace{\frac{\lambda}{2\mu(3\lambda+2\mu)}}^{\frac{\nu}{E}}(\sigma_{22} + \sigma_{33})$$

$\Rightarrow$ Young's modulus $\boxed{E = \frac{\mu(3\lambda + 2\mu)}{(\lambda+\mu)}}$

Poisson's ratio $\boxed{\nu = \frac{\lambda}{2(\lambda+\mu)}}$

CHECK ALGEBRA!

* SHOW THAT $-1 < \nu < 1/2$ IS A NECESSARY MATERIAL CONSTRAINT, WITH $E > 0$

So, we get

$$\varepsilon_{11}^{E} = \frac{1}{E}\left(\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})\right)$$

$$\varepsilon_{22}^{E} = \frac{1}{E}\left(\sigma_{22} - \nu(\sigma_{11} + \sigma_{33})\right)$$

$$\varepsilon_{33}^{E} = \frac{1}{E}\left(\sigma_{33} - \nu(\sigma_{11} + \sigma_{22})\right)$$

elastic

$$\varepsilon_{12}^{E} = \frac{1}{2\mu}\sigma_{12}\,; \quad \varepsilon_{13}^{E} = \frac{1}{2\mu}\sigma_{13}\,; \quad \varepsilon_{23}^{E} = \frac{1}{2\mu}\sigma_{23}\,;$$

What is $2\mu = ?$ $\boxed{2\mu = \frac{E}{(1+\nu)}}$ ← SHOW THIS!

* How are E, ν or λ, μ MEASURED in the laboratory?
<FIND IT OUT> ~ ASTM standards ← testing standards.

TYPICAL E: Steel ~ 210 GPa, Al ~ 70 GPa } ν : ~0.3

THERMAL STRAIN:

Who gives temperature at every material point?

$$+\, C\,\frac{\partial T}{\partial t} - k\,\Delta T = r$$ ← Heat conduction

with T or q_n (heat flux) given on boundary!

$\boxed{\text{ASSUMPTION}}$: ΔT is UNIFORM everywhere

Thermal strain $\varepsilon_{ij}^{T} = \underbrace{\alpha}_{\text{coefficient of thermal expansion}} \Delta T\, \delta_{ij}$

Then TOTAL STRAIN $\boxed{\varepsilon_{ij} = \varepsilon_{ij}^{E} + \varepsilon_{ij}^{T}}$

OR

$$\boxed{\varepsilon_{11} = \frac{1}{E}\left(\sigma_{11} - \nu\left(\sigma_{22} + \sigma_{33}\right)\right) + \alpha\,\Delta T}$$

ε_{22}, ε_{33} follow similar forms

$\boxed{\varepsilon_{12} = \frac{1}{2\mu}\sigma_{12}}$; $\varepsilon_{13} = \frac{1}{2\mu}\sigma_{13}$; $\varepsilon_{23} = \frac{1}{2\mu}\sigma_{12}$

No thermal influence in SHEAR!

* Note that if thermal expansion is constrained, i.e. if $\varepsilon_{11} = 0$, while the material is being heated, then

$$\varepsilon_{11} = \varepsilon_{11}^{E} + \varepsilon_{11}^{P} = 0 \Rightarrow \boxed{\varepsilon_{11}^{E} = -\varepsilon_{11}^{P}}$$

$$\varepsilon_{ij} = \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{2\mu(3\lambda+2\mu)}(\sigma_{kk})\,\delta_{ij} + \alpha\,\Delta T\,\delta_{ij}$$

$$\Rightarrow (\varepsilon_{ij} - \alpha\,\Delta T\,\delta_{ij}) = \varepsilon_{ij}^{E} = \frac{1}{2\mu}\sigma_{ij} - \frac{\lambda}{2\mu(3\lambda+2\mu)}\sigma_{kk}\,\delta_{ij}$$

$$\Rightarrow \sigma_{ij} = \lambda\left(\varepsilon_{kk}^{E}\right)\delta_{ij} + 2\mu\,\varepsilon_{ij}^{E} = \lambda\left(\varepsilon_{kk} - 3\alpha\,\Delta T\right)\delta_{ij} + 2\mu\left(\varepsilon_{ij} - \alpha\,\Delta T\,\delta_{ij}\right)$$

EXAMPLE WITH PRINCIPAL STRESSES:

Let displacement field $\vec{u}(x_1, x_2, x_3)$ be given as:

$$u_1 = (3x_1^2 - 2x_1x_3) \times 10^{-3}\,m$$

$$u_2 = (6x_1x_3) \times 10^{-3}\,m$$

$$u_3 = 3x_3^2 \times 10^{-3}\,m.$$

$\therefore$ The strain components are given as:

$$\varepsilon_{11} = u_{1,1} = (6x_1 - 2x_3) \times 10^{-3}$$

$$\varepsilon_{22} = u_{2,2} = 0$$

$$\varepsilon_{33} = u_{3,3} = 6x_3 \times 10^{-3}$$

$$2\varepsilon_{12} = \gamma_{12} = u_{1,2} + u_{2,1} = (0 + 6x_3) \times 10^{-3} = 6x_3 \times 10^{-3}$$

$$2\varepsilon_{13} = \gamma_{13} = u_{1,3} + u_{3,1} = (-2x_1 + 0) \times 10^{-3}$$

$$2\varepsilon_{23} = \gamma_{23} = u_{2,3} + u_{3,2} = (6x_1 + 0) \times 10^{-3}$$

Assume that material is isotropic, with Young's modulus $E = 70\,GPa$; $\nu = 0.3$. Then the state of stress is given by

$$\sigma_{11} = \frac{E}{(1+\nu)(1-2\nu)}\left(\varepsilon_{11}(1-\nu) + \nu(\varepsilon_{22} + \varepsilon_{33})\right)$$

$$\sigma_{22} = \frac{E}{(1+\nu)(1-2\nu)}\left(\varepsilon_{22}(1-\nu) + \nu(\varepsilon_{11} + \varepsilon_{33})\right)$$

$$\sigma_{33} = \frac{E}{(1+\nu)(1-2\nu)}\left(\varepsilon_{33}(1-\nu) + \nu(\varepsilon_{11} + \varepsilon_{22})\right)$$

$$\sigma_{13} = \frac{E}{2(1+\nu)}(\gamma_{13}); \quad \sigma_{23} = \frac{E}{2(1+\nu)}\gamma_{23}; \quad \sigma_{12} = \frac{E}{2(1+\nu)}\gamma_{12}$$

* Note that $\sigma_{ij}(x_1, x_2, x_3)$ is a function of position.

Let us take 2 points to elaborate:

$P_1 = (0,0,1)$; $P_2 = (1,1,0)$

At P_1:

$\varepsilon_{11} = -2\times10^{-3}$; $\varepsilon_{22} = 0$; $\varepsilon_{33} = 6\times10^{-3}$; $\gamma_{12} = 6\times10^{-3}$; $\gamma_{13} = \gamma_{23} = 0$

At P_2:

$\varepsilon_{11} = 6\times10^{-3}$; $\varepsilon_{22} = 0$; $\varepsilon_{33} = 0$; $\gamma_{12} = 0$; $\gamma_{13} = -2\times10^{-3}$;

$\gamma_{23} = 6\times10^{-3}$

At P_1:

$$\sigma_{11} = \frac{E}{1.3\times0.4}\left(\varepsilon_{11}\times0.7 + 0.3(\varepsilon_{22}+\varepsilon_{33})\right) = \frac{25E}{13}\left(-1.4\times10^{-3} + 1.8\times10^{-3}\right)$$

$$= \frac{25E\times0.4\times10^{-3}}{13} = \frac{10E}{13}\times10^{-3} = \frac{10\times70\times10^{9}\times10^{-3}}{13} \approx 5.384\times10^{7} \approx 53.8\text{ MPa}$$

$$\sigma_{22} = \frac{25E}{13}\left(0 + 0.3\times(-2\times10^{-3}+6\times10^{-3})\right) = \frac{25E}{13}\times1.2\times10^{-3} = 16.154\times10^{7} \approx 161.5\text{ MPa}$$

$$\sigma_{33} = \frac{25E}{13}\left(0.7\times6\times10^{-3} + 0.3(-2\times10^{-3}+0)\right) = \frac{25E}{13}\times3.6\times10^{-3} = 484.6\text{ MPa}$$

$$\sigma_{12} = \frac{E}{2.6}\times6\times10^{-3} = \frac{30E}{13}\times10^{-3} \doteq 161.5\text{ MPa}$$

$$\sigma_{13} = \frac{E}{2.6}\times0 = 0 \;;\; \sigma_{23} = 0$$

OR

$$\begin{bmatrix} \frac{10E}{13}\times10^{-3} & \frac{30E}{13}\times10^{-3} & 0 \\ \frac{30E}{13}\times10^{-3} & \frac{30E}{13}\times10^{-3} & 0 \\ 0 & 0 & \frac{90E}{13}\times10^{-3} \end{bmatrix} = [\sigma] = \begin{bmatrix} 53.8 & 161.5 & 0 \\ 161.5 & 161.5 & 0 \\ 0 & 0 & 484.6 \end{bmatrix}\times10^{6}$$

$[\sigma]\vec{n} = \lambda\vec{n}$ is the eigenvalue problem corresponding to this state of stress.

$$\therefore \det\left|[\sigma]-\lambda[I]\right| = 0 \Rightarrow (484.6-\lambda)\left[(53.8-\lambda)(161.5-\lambda) - (161.5)^2\right] = 0$$

$$\lambda^2 - \lambda\underbrace{(161.5+53.8)}_{215.3} + \underbrace{53.8\times161.5 - (161.5)^2}_{-161.5\times107.7} = 0$$

$\therefore$ $\lambda_1 = 484.6$ MPa ; $\lambda_2 = 277.9$ MPa ; $\lambda_3 = -62.6$ MPa

Now that we have the principal stresses, we can compute the octahedral shear stress $\tau^{oct} = \sqrt{\frac{1}{9}\left((\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2\right)}$

$$\Rightarrow \quad \tau_{oct} = \sqrt{\frac{1}{9}\left[\underbrace{(484.6-277.9)^2}_{42724.9}+\underbrace{(277.9+62.6)^2}_{115940.3}+\underbrace{(484.6+62.6)^2}_{299427.8}\right]}$$

$$= 225.6 \text{ MPa}.$$

Hydrostatic stress $\sigma_H = \frac{1}{3}(\lambda_1+\lambda_2+\lambda_3) = 233.3$ MPa

Mises criterion $\Rightarrow$ we check for yielding, i.e. if

$$\sigma_{RMS} = \sqrt{\frac{1}{2}\left[(\lambda_1-\lambda_2)^2+(\lambda_2-\lambda_3)^2+(\lambda_3-\lambda_1)^2\right]} \leq \sigma_Y$$

$$\sqrt{9/2}\;\tau_{oct} = 2.12\,\tau_{oct} = \underline{478.6 \text{ MPa}},$$

For Al-alloys we have $\sigma_Y \approx \underline{324 \text{ MPa}}$ (2024-Al alloy)

$\therefore \quad \sigma_{RMS} > \sigma_Y$ — hence material has yielded.

* Note that we can also write

$$\sigma_{RMS} = \sqrt{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6\left(\sigma_{12}^2+\sigma_{13}^2+\sigma_{23}^2\right)\right]}$$

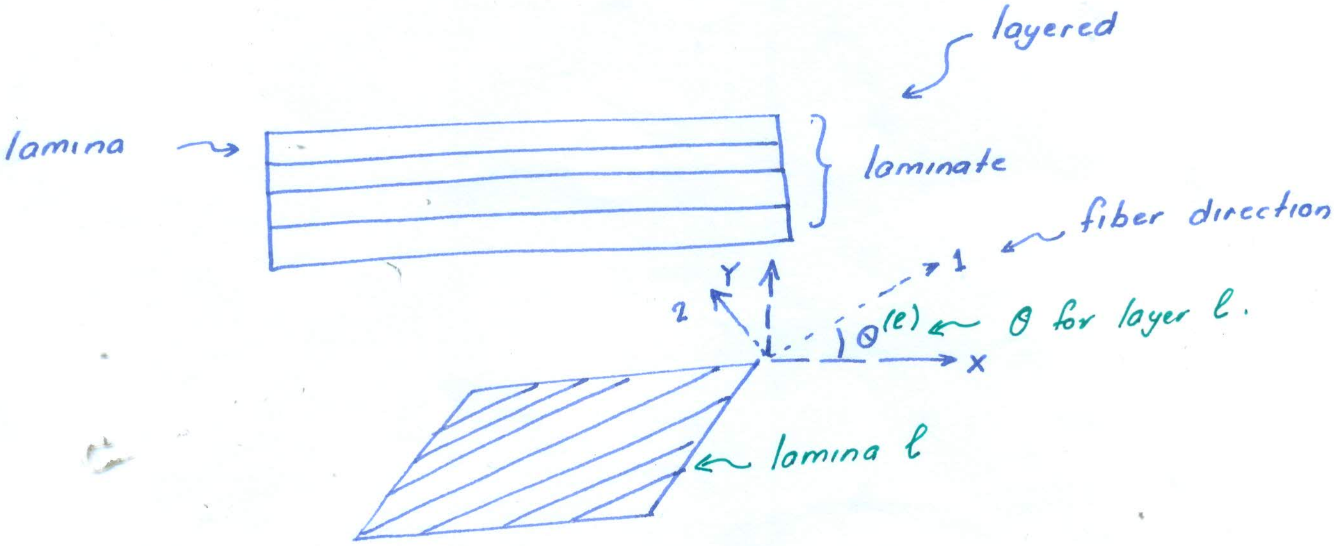
$$= \frac{1}{2}\left[\underbrace{(53.8-161.5)^2}_{11599.29}+\underbrace{(161.5-684.6)^2}_{104393.6}+\underbrace{(684.6-53.8)^2}_{185588.6}+\underbrace{6(161.5^2+0)}_{156493.5}\right] = \underline{\underline{478.6 \text{ MPa}}}$$

EASIER! As eigenvalues need not be computed!

* DO THE SAME EXERCISE FOR POINT P2.

Composites

- Tailorable, high specific strength and stiffness
- Good fatigue life and corrosion resistant



→ Thin structures, hence plane stress assumption works

→ Effective properties needed ~ tests, formulae.

3D :

$$\varepsilon_{11} = \frac{\sigma_1}{E_1} - \frac{\nu_{21}}{E_2}\sigma_2 - \frac{\nu_{31}}{E_3}\sigma_3$$

$$\varepsilon_2 = -\frac{\nu_{12}}{E_1}\sigma_1 + \frac{\sigma_2}{E_2} - \frac{\nu_{32}}{E_3}\sigma_3$$

$$\varepsilon_3 = -\frac{\nu_{13}}{E_1}\sigma_1 - \frac{\nu_{23}}{E_2}\sigma_2 + \frac{\sigma_3}{E_3}$$

$$\gamma_{23} = \frac{\sigma_{23}}{G_{23}} ; \quad \gamma_{13} = \frac{\sigma_{13}}{G_{13}} ; \quad \gamma_{12} = \frac{\sigma_{12}}{G_{12}}$$

$$\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i} \quad \Big| \quad \nu_{ij} \neq \nu_{ji}$$

$$\boxed{\nu_{ij}\frac{E_j}{E_i} = \nu_{ji}}$$

The properties are given in the 1-2-3 system. To convert back to X-Y-Z system we use coordinate rotation.

How do we do this?

Note that we can write the above in the form

$[\varepsilon^{(1)}] = [S^{(1)}][\sigma^{(1)}]$ or $\varepsilon^1_{ij} = S^1_{ijkl}\,\sigma^1_{kl}$

$[C^{(1)}] = [S^{(1)}]^{-1}$

But writing the stress tensor as a vector

$\{\sigma^{(1)}\} = [C^{(1)}]\{\varepsilon^{(1)}\}$ with $\{\sigma^{(1)}\} = [T_1]\{\sigma^{(x)}\}$

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & (m^2-n^2) \end{bmatrix}$$

$m = \cos\theta$
$n = \sin\theta$

Also, $\{\varepsilon^{(1)}\} = [T_2]\{\varepsilon^{(x)}\}$

Noting that normally we use γ_{ij} in the shear case $(\gamma_{ij} = 2\,\varepsilon_{ij})$

$[T_2]$ has all entries same as $[T_1]$, except:

$\boxed{T_{2\,6,1} = 2\,T_{1\,6,1}\ ;\ T_{2\,6,2} = 2\,T_{1\,6,2}}$; $T_{2\,1,6} = \frac{1}{2}\,T_{1\,1,6}$;

$T_{2\,2,6} = \frac{1}{2}\,T_{1\,2,6}$.

Thus we get:

$$\{\sigma^{(x)}\} = \underbrace{[T_1^{-1}][C^{(1)}][T_2]}_{[C^{(x)}]}\{\varepsilon^{(x)}\}$$

* Write a small matlab code to do this part for various values of θ.