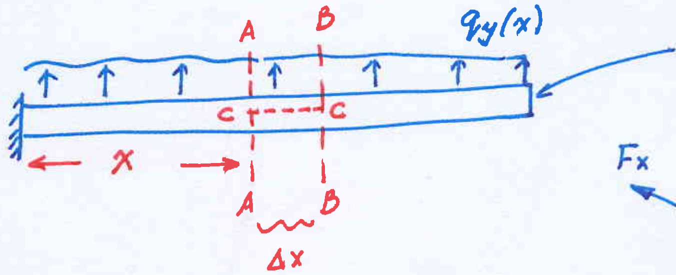
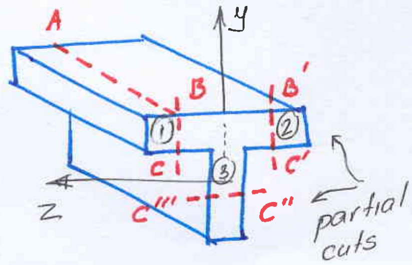
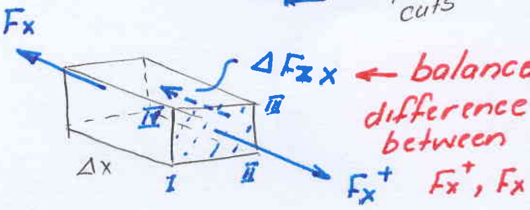
SHEAR DUE TO BENDING

→ Uniform bending, M_y (or M_z) a constant, then no shea[r]
Shear force $V_y, V_z = 0$ ⇒ NO ISSUE WITH SHEAR

→ Non-uniform bending, i.e. M_y, M_z not constant, or
$V_y, V_z \neq 0$.
~ Shear stress σ_{xy}, σ_{xz} obtained as a post-processing of σ_{xx}. How?







* Note that since the section is thin, we will make cuts THROUGH the cross-section, i.e. cuts like BC, B'C', C''C'''

Axial force imbalance

$F_x^+ - F_x \neq 0$ ⇒ the exposed cut face (AB, AB', AC'') should develop a resultant force ΔF_{xs} (s is z or y dependi[ng] on the normal to the cutting plane), such that

$$\Delta F_{xs} = F_x^+ - F_x = \frac{d F_x}{dx}\Delta x = \frac{d}{dx}\int_{A_{I,II,III,IV}} \sigma_{xx}\, dA$$

The front face with vertices I, II, III, IV will be the frontal area A

$$\Rightarrow \quad \Delta F_{xs} = \int_A \frac{d}{dx}(\sigma_{xx})\, dA \qquad \text{—— } (A \text{ is assumed to be unchanged})$$

But we have shown that

$$\sigma_{xx} = M_y\left(\frac{-y I_{yz} + z I_{zz}}{\Delta}\right) + M_z\left(\frac{-y I_{yy} + z I_{yz}}{\Delta}\right)$$

$$\Rightarrow \quad \sigma_{xx,x} = M_{y,x}\left(\frac{-y I_{yz} + z I_{zz}}{\Delta}\right) + M_{z,x}\left(\frac{-y I_{yy} + z I_{yz}}{\Delta}\right) \qquad \text{—— } (*$$

But from equilibrium

$$M_{z,x} = -V_y \; ; \quad M_{y,x} = V_z$$

$\Rightarrow$ (*) becomes :

$$\sigma_{xx,x} = V_z \left(\frac{-y I_{yz} + z I_{zz}}{\Delta} \right) - V_y \left(\frac{-y I_{yy} + z I_{yz}}{\Delta} \right) \quad \cdots (*_1)$$

$$\therefore \frac{\Delta F_{xs}}{\Delta x} = -V_z \frac{I_{yz}}{\Delta} \underbrace{\int_A y\, dA}_{Q_z} + V_z \frac{I_{zz}}{\Delta} \underbrace{\int_A z\, dA}_{Q_y} + V_y \frac{I_{yy}}{\Delta} \int_A y\, dA - V_z \frac{I_{yz}}{\Delta} \int_A z\, dA$$

$Q_z = \int_A y\, dA$ is 1^{st} moment of the area A about the z-axis (in the centroidal coordinate system)

$= y_c|_A \cdot A$ $\quad \langle y_c|_A \rightarrow$ y-location of centroid of $A \rangle$

$Q_y = \int_A z\, dA$ is 1^{st} moment of area A about the y-axis

$= z_c|_A \cdot A$ $\quad \langle z_c|_A \rightarrow$ z-location of centroid of $A \rangle$

$$\therefore \underbrace{\Delta F_{xs}}_{\downarrow} = V_z \left(-\frac{I_{yz} Q_z}{\Delta} + \frac{I_{zz} Q_y}{\Delta} \right) + V_y \left(\frac{I_{yy} Q_z - I_{yz} Q_y}{\Delta} \right) \Delta x \quad \text{---} (**)$$

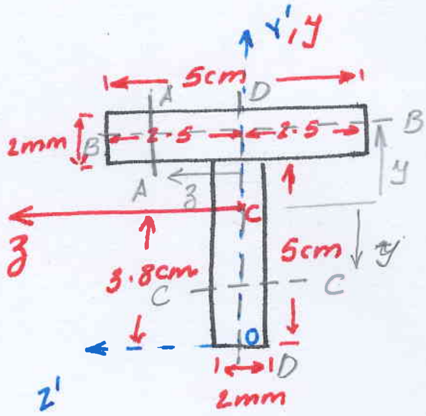
$\Delta F_{xs} \approx \underbrace{\sigma_{xs}} t_{loc} \cdot \Delta x$ $\longrightarrow$ t_{loc} is thickness at the given location

σ_{xs} is assumed to be constant through the thickness t_{loc}.

$$\Rightarrow \boxed{\sigma_{xs} \cdot t_{loc} = V_z \left(\frac{-I_{yz} Q_z + I_{zz} Q_y}{\Delta} \right) + V_y \left(\frac{I_{yy} Q_z - I_{yz} Q_y}{\Delta} \right)}$$

* NOTE THAT THE DEFINITION USES A CUT WITH A NEGATIVE NORMAL.

* FOR A CUT WITH POSITIVE NORMAL, DIRECTION OF ΔF_{xs} HAS TO BE IN +VE X-DIRECTION



EXAMPLE: T- section made of steel

Centroid: Due to symmetry $z_c' = 0$

$$y_c' \cdot A = y_{c_1}' A_1 + y_{c_2}' \cdot A_2$$

$$\Rightarrow y_c' = \frac{25 \times \cancel{(50\times 2)} + 51 \times \cancel{(50\times 2)}}{2 \times \cancel{(50\times 2)}}$$

$$= \frac{76}{2} = 38 \text{ mm}$$

Due to symmetry, $I_{yz} = 0 \Rightarrow \sigma_{xx} = \frac{M_y \cdot z}{I_{yy}} - \frac{M_z \cdot y}{I_{zz}}$

$$\Rightarrow \sigma_{xs} \cdot t_{loc} = \frac{V_z Q_y}{I_{yy}} + \frac{V_y Q_z}{I_{zz}}$$

Let us make cuts at AA, BB, CC, DD (for example)

It now boils down to finding the following:

(a) $s \rightarrow$ for AA it is z; for BB it is y; for CC it is y; for DD it is z.

(b) $t_{loc} \rightarrow$ for AA $t_{loc} = 2$mm; for BB $t_{loc} = 50$mm; for CC $t_{loc} = 2$ mm; for DD $t_{loc} = 52$ mm;

(c) Q_y: $Q_y^{AA} = (25+z)/2 \times (25-z) \cdot \times 2 \quad mm^3$

$$= 625 - z^2 \quad mm^3$$

$$Q_y^{BB} = (0) \times 50 \times (14-y) = 0$$

$$Q_y^{CC} = (0) + (0) = 0$$

$$Q_y^{DD} = \underbrace{12.5 \times 25 \times 2}_{\text{top member}} + \underbrace{0.5 \times (12-y) \times 1}_{\text{bottom segment}} = 625 + 6 - 0.5y = 631 - 0.5y$$

(d) Q_z: $Q_z^{AA} = 13 \times (25-z) \times 2$; $Q_z^{BB} = (14+y)/2 \times (14-y) \times 50$

$$Q_z^{CC} = \underbrace{\underbrace{13 \times 50 \times 2}_{\text{top part}} + \underbrace{\frac{(12+y)}{2} \times (12-y) \times 2}_{\text{bottom leg}}}_{1300 + 144 - y^2} ; \quad Q_z^{DD} = \underbrace{13 \times 25 \times 2}_{\text{top part}} + \underbrace{(13) \times 50 \times 1}_{\text{bottom leg}}$$

$Q_z^{CC} = 1444 - y^2 \rightsquigarrow$ ZERO at $y = -38$ mm (i.e. at bottom

$\sim$ varies QUADRATICALLY

NOTE: $Q_y^{BB} = 0$; $Q_z^{BB} = (196 - y^2) \times 25 = \underbrace{4900 - 25y^2}_{(a)}$ $(3600 \le (a) \le 4900)$

$0 \le Q_z^{BB} \le 1300$

$Q_y^{AA} = 625 - z^2 \Rightarrow 0 \le Q_y^{AA} \le 625$

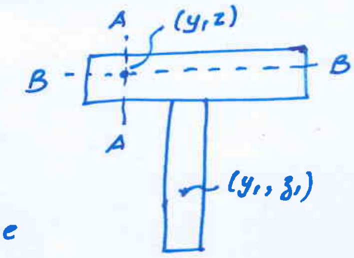
$Q_z^{AA} = 650 - 26z \Rightarrow 0 \le Q_z^{AA} \le 650$

$$\Rightarrow \sigma_{xz}\big|_{AA} = \sigma_{xs}\big|_{AA} \le \frac{V_z \cdot \overbrace{625}^{312.5}}{I_{YY} \times 2} + \frac{V_y \cdot \overbrace{650}^{325}}{I_{zz} \times 2}$$

$$\sigma_{xs}\big|_{BB} = \sigma_{xY}\big|_{BB} \le V_z \cdot 0 + \frac{V_y \cdot \overbrace{4900}^{98}}{I_{zz} \times 50}$$

∴ At the intersection of AA, BB, i.e. the point (y, z)

$\sigma_{xz} > \sigma_{xY}$ ~~but not significantly~~



If V_z is only force, then you will get $\sigma_{xY} = 0$ and σ_{xz} becomes the shear stress.

* A similar analysis with CC, DD gives for the point at the intersection of CC, DD :

$\sigma_{xY} > \sigma_{xz}$

The fact that $\sigma_{xz} \sim 6 \times \sigma_{xY}$ at (y, z)

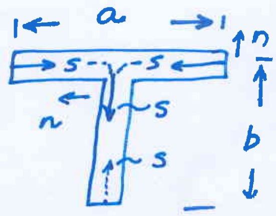
$\sigma_{xY} \sim 6 \times \sigma_{xz}$ at (y_1, z_1)

allows us to make a further assumption:

That along the coordinate s, $\sigma_{xs} \gg \sigma_{xn}$ and hence

σ_{xs} is considered only and

$\sigma_{xn} \approx 0$ is taken



, $b \sim a$

* Because $Q_y, Q_z \sim a^3$; $t_{loc} \sim a$

$$\Rightarrow \frac{Q_y, Q_z}{t_{loc}} \sim a^2$$

$$\sigma_{xx} \sim \frac{a}{I_{yy}, I_{zz}} \sim a^{-3}$$

$$\sigma_{xs} \sim \frac{a^2}{I_{yy}, I_{zz}} \sim a^{-2}$$

$$\frac{\sigma_{xx}}{\sigma_{xs}} \sim \frac{1}{a}$$

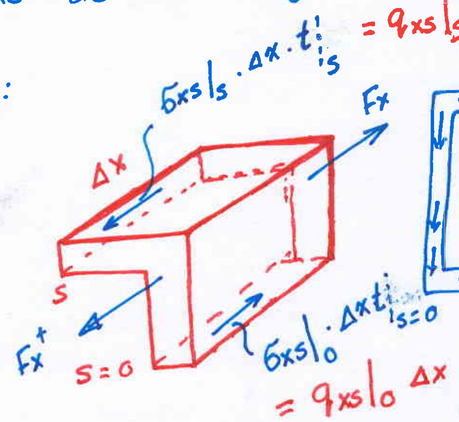
LARGE!

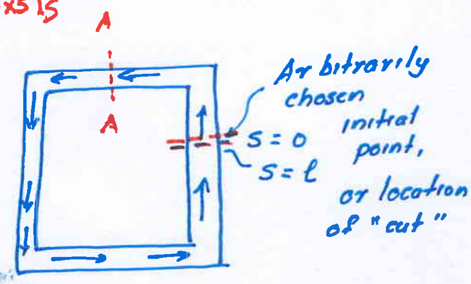
∴ Now we have a clear picture of shear due to bending and σ_{xs} is the dominant shear for THIN sections

$$\sigma_{xs} \cdot t_{loc} = q_{xs} \leftarrow \text{SHEAR FLOW}$$

* Very smartly we looked at open sections, for which the shear stress was known on the boundary, hence we had to deal with ΔF_{xs} as the only unknown.

CLOSED SECTIONS :





∴

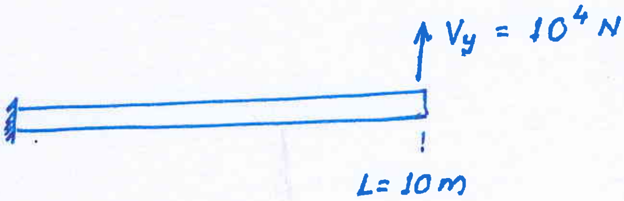
$$(F_x^+ - F_x) + (q_{xs}|_s - \underline{\underline{q_{xs}|_0}})\Delta x = 0$$

UNKNOWN

* q_{xs} in positive direction for face with positive normal

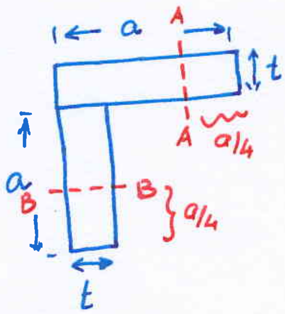
* NEED ANOTHER EQUATION!
~ FROM ROTATION? ⇒ Torsion will give us this equation...

MINI - ASSIGNMENT 4

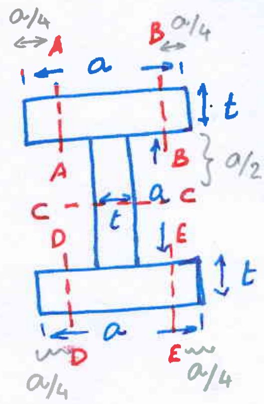


with cross-section :

(i)



(ii)



(a) Find σ_{xs} , q_{xs} at the sections AA, BB, CC, DD, EE (as marked) for the 2 sections

(b) Where is σ_{xs} MAXIMOM ?