ASSIGNMENT # 1 (additional)

Q1   Let the displacement field be given by :

$$u_1 = \bar{\alpha}\left(x_1^2 - 5x_2 x_3\right) ; \quad u_2 = \bar{\alpha}\left(2x_2^3 + 0.001\, x_1\right)$$

$$u_3 = \bar{\alpha}\left(10\, x_1 x_2 + 5 x_3\right).$$

For this displacement field, determine :

(a) The deformation gradient $[F]$, with $F_{ij} = \dfrac{\partial x_i}{\partial x_j} = \delta_{ij} + \dfrac{\partial u_i}{\partial x_j}$

(b) The infinitesimal strain $[E]$, with $E_{ij} = \frac{1}{2}\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\right)$

(c) At point $P = (1,0,1)$ obtain the principal strains in terms of $\bar{\alpha}$.

(d) If material is isotropic, with Young's modulus $E = 70\,GPa$, Poisson's ratio $\nu = 0.3$, determine the state of stress at P.

(e) Find the principal stresses at P (in terms of $\bar{\alpha}$).

(f) If the value of yield stress, σ_Y, is $300\,MPa$, what value of $\bar{\alpha}$ will lead to onset of yield at P?

(g) Check if point $Q = (-1,1,0)$ will yield or not for this value of $\bar{\alpha}$.

Q2   For an unswept, tapered wing of semi-span $7m$; with root chord of $1m$ and tip chord of $10\,cm$ length; determine the lift distribution to be used for design calculation.

Other data: All-up weight $W = 3$ tonne ; $n = 3$; Factor of Safety $FS = 1.5$.