Afterburning turbojet Engine operates at -

$M_a = 2.0$, $p_a = 10\,kPa$, $T_a = -45^\circ C$, $\gamma_c = 1.4$

$C_{p_c} = 1004\ J/kg\text{-}K$, π_d (Press. ratio across diffuser) $= 0.88$

π_c (Press. ratio across compressor) $= 12$, $e_c = 0.9$ (poly. eff.)

$\tau_\lambda = 8.0$, $Q_R = 42000\ kJ/kg$, $\eta_b = 0.98$, $\pi_b = 0.95$

$\gamma_t = 1.33$, $C_{p,t} = 1156\ J/kg\text{-}K$, $e_t = 0.82$ (poly. eff.), $\eta_m = 0.995$ (shaft eff.)

$\tau_{\lambda AB}$ = (Temp. ratio across AB) $= 11$, $Q_{R,AB} = 42000\ J/kg\text{-}K$

$\eta_{AB} = 0.98$, $\pi_{AB} = 0.93$, $\gamma_{AB} = 1.30$, $C_{p,AB} = 1243\ J/kg\text{-}K$

$\pi_n = 0.93$, $P_9 = P_0$, $\tau_\lambda \equiv \dfrac{C_{p_t} T_{04}}{C_{p_c} T_a}$

Ans!

$a, 1$ = same, $T_a = -45 + 273 = 228\ K$

$$a_1 = \sqrt{(\gamma_c - 1) C_{p_c} T_a} = \sqrt{0.4 \times 1004 \times 228} = 302.6\ m/s$$

$$u_1 = M_a \cdot a_1 = 605.2\ m/s.$$

Flight P_{0a}, T_{0a}

$$P_{0a} = P_{0_1} = P_a \left[1 + (\gamma_c - 1)\frac{M_a^2}{2}\right]^{\frac{\gamma_c}{\gamma_c - 1}} = 78.24\ kPa. \quad ①$$

$$T_{0a} = T_{0_1} = T_a \left[1 + (\gamma_c - 1)\frac{M_a^2}{2}\right] = 410.4\ K.$$

Since inlets are adiabatic, $T_{0_2} = T_{0_1} = 410.4\ K.$

$P_{0a} = P_{0_1} \to P_{0_2}$

$$P_{0_2} = P_{0_1} \cdot \pi_d = 78.24 \times 0.88 = 68.85\ kPa \quad ②$$

Compressor exit

$$P_{0_3} = P_{0_2} \cdot \pi_c = 68.85 \times 12 = 826.26\ kPa \quad ③$$

$$\tau_c = \pi_c^{\frac{\gamma_c}{e_c(\gamma_c - 1)}} = 2.201, \quad T_{0_3} = T_{0_2} \cdot \tau_c = 903.24\ K$$

Combustor exit.

$$\tau_\lambda = \frac{C_{p_t} T_{04}}{C_{p_c} T_a} \Rightarrow T_{0_4} = 1584.2\ K \quad ④$$

$$P_{0_4} = P_{0_3} \cdot \pi_b = 784.95\ kPa$$

energy eq: $$c_{p_c} T_{03} + f Q_R \eta_b = (1+f)\, c_{p_t} T_{04}$$

$$\Rightarrow f \cong 0.02357$$

Turbine exit: (using the balance between turbine & compressor)

$$c_{p_c}(T_{03} - T_{02}) = \eta_m (1+f)\, c_{p_t} (T_{04} - T_{05})$$

$$\Rightarrow T_{05} = T_{04} - \frac{c_{p_c}(T_{03} - T_{02})}{c_{p_t}\, \eta_m (1+f)} = 1163.85\ K$$

$$\pi_t = \tau_t^{\frac{\gamma_t}{(\gamma_t - 1) e_t}} = 0.2197$$

$$P_{05} = P_{04} \times \pi_t = 172.47\ kPa$$

5

AB exit.

$$P_{06} = P_{05} \times \pi_{AB} = 160.\ kPa$$

$$T_{06} = \frac{c_{p_c} T_a \tau_{\lambda,AB}}{c_{p,AB}} = 2025.8\ K$$

$$f_{AB} = \frac{(1+f)(h_{06} - h_{05})}{Q_{R_{AB}} \eta_{AB} - h_{06}} = 0.0311$$

6

Nozzle exit

$$P_{07} = P_{06} \times \pi_n = 149.17\ kPa$$

nozzle flow is adiabatic $\rightarrow$ $$T_{07} = T_{06} = 2025.8\ K$$

Since, $P_e = P_7 = P_a = 10\ kPa$, we can calculate M_e from.

$$M_7 = M_e = \sqrt{\frac{2}{\gamma_{AB} - 1}\left[\left(\frac{P_{07}}{P_7}\right)^{\frac{\gamma_{AB}-1}{\gamma_{AB}}} - 1\right]} = 2.402$$

$$T_7 = T_e = \frac{T_{07}}{1 + \frac{(\gamma_{AB} - 1)}{2} M_e^2} = 1085.8\ K$$

$$a_7 = a_e = \sqrt{(\gamma_{AB} - 1)\, c_{p_{AB}} T_7} = 636.3\ m/s$$

$$u_e = u_7 = M_e a_e = 1528.67\ m/s$$

7

specific thrust. When the nozzle is perfectly expanded.

$$\frac{T}{\dot{m}_a} = \left(1+f+f_{AB}\right)u_e - u_1$$

Non-dimensional specific thrust

$$\frac{T}{\dot{m}_a \cdot a_1} = \left(1+f+f_{AB}\right)\frac{u_e}{a_1} - M_a = 3.3275$$

$$TSFC = \frac{f+f_{AB}}{F/\dot{m}_a} = 54.2 \text{ mg/s/N}$$

The cycle thermal eff. is:

$$\eta_{th} = \frac{\left(1+f+f_{AB}\right)\frac{u_e^2}{2} - \frac{u_1^2}{2}}{fQ_R + f_{AB}Q_{R,AB}} = 0.4578$$

Performance parameters

cycle propulsive eff. - $\eta_p = \frac{(T/\dot{m}_a)u_1}{\left(1+f+f_{AB}\right)\frac{u_e^2}{2} - \frac{u^2}{2}}$

$= 0.5809$

approx. form of propulsive eff. -

$$\eta_p \approx \frac{2}{1+\frac{u_e}{u}} = 0.5672$$

Note $\rightarrow$

* given — (conditions)

e_c = polytropic efficiency of compresser
(it is differenet from isentropic eff.)

e_t = polytropic efficiency of turbine
(Not isentropic eff.)

$$\frac{P_{O_3}}{P_{O_2}} = \pi_c = \left(\frac{T_{O3}}{T_{O2}}\right)^{\frac{\gamma_c e_c}{\gamma-1}} = (\tau_c)^{\frac{\gamma_c e_c}{\gamma_c - 1}}$$: e_c = polytropic eff.

→ this is used in calculation.

isentropic efficiency (η_c) $= \dfrac{\pi_c^{\gamma_c/\gamma_c - 1} - 1}{\pi_c^{\frac{\gamma_c - 1}{\gamma_c e_c}} - 1}$

$$\frac{T_{O5}}{T_{O4}} = \tau_t \quad , \pi_t = \frac{P_{O5}}{P_{O4}} = \tau_t^{\frac{\gamma_t}{(\gamma_t - 1) e_t}}$$

$$\eta_t = \frac{1 - \tau_t}{1 - \tau_t^{1/e_t}}$$ | e_t = polytropic eff.