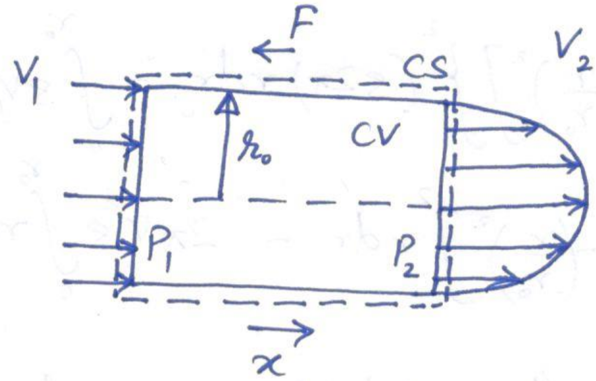
Soln 1



$$V_2 = V_{max}\left[1-\left(\frac{r}{r_o}\right)^2\right]$$

Control Volume Analysis (Steady Flow) :-

Mass Conservation :-

$$\frac{dm}{dt} = 0 = \int_{CS} (\vec{V}\cdot\vec{n})\rho\, dA \qquad dA = 2\pi r\, dr$$

$$\Rightarrow \cancel{\rho}\int_0^{r_o} V_{max}\left[1-\left(\frac{r}{r_o}\right)^2\right] 2\cancel{\pi} r\, dr - \cancel{\rho} V_1 \cancel{\pi} r_o^2 = 0$$

$$\Rightarrow 2V_{max}\int_0^{r_o}\left(r - \frac{r^3}{r_o^2}\right) dr = V_1 r_o^2$$

$$\Rightarrow 2V_{max}\left[\frac{r^2}{2} - \frac{r^4}{4r_o^2}\right]_0^{r_o} = V_1 r_o^2$$

$$\Rightarrow 2V_{max}\left[\frac{r_o^2}{2} - \frac{r_o^4}{4r_o^2}\right] = V_1 r_o^2$$

$$\Rightarrow 2V_{max}\left[\frac{r_o^2}{4}\right] = V_1 r_o^2$$

$$\Rightarrow V_{max} = 2V_1$$

Linear Momentum balance (x-direction)

$$\Sigma F_x = \int_{CS} V_x \rho (\vec{V}\cdot\vec{n})\, dA$$

$$\Rightarrow -F + P_1\pi r_o^2 - P_2\pi r_o^2 = \int_0^{r_o} 2V_1\left[1-\left(\frac{r}{r_o}\right)^2\right]\rho\left\{2V_1\left[1-\left(\frac{r}{r_o}\right)^2\right]\right\}2\pi r\, dr - \int^{r_o}\rho V_1^2\, 2\pi r\, dr$$

$$\text{R.H.S.} = \int_0^{r_0} \left\{ 2V_1 \left[1 - \left(\frac{r}{r_0}\right)^2 \right] \right\}^2 (\rho 2\pi) r\,dr - \int_0^{r_0} \rho V_1^2 2\pi r\,dr$$

$$= 8\pi V_1^2 \rho \int_0^{r_0} \left[1 - \left(\frac{r}{r_0}\right)^2 \right]^2 r\,dr - 2\pi V_1^2 \rho \int_0^{r_0} r\,dr$$

$$= 8\pi V_1^2 \rho \int_0^{r_0} \left(1 + \frac{r^4}{r_0^4} - \frac{2r^2}{r_0^2} \right) r\,dr - 2\pi V_1^2 \rho \left[\frac{r^2}{2} \right]_0^{r_0}$$

$$= 8\pi V_1 \rho \int_0^{r_0} \left(r + \frac{r^5}{r_0^4} - \frac{2r^3}{r_0^2} \right) dr - \pi r_0^2 V_1^2 \rho$$

$$= 8\pi V_1^2 \rho \left[\frac{r^2}{2} + \frac{r^6}{6r_0^4} - \frac{2r^4}{4r_0^2} \right]_0^{r_0} - \pi r_0^2 V_1^2 \rho$$

$$= 8\pi V_1^2 \rho \left[\cancel{\frac{r_0^2}{2}} + \frac{r_0^2}{6} - \cancel{\frac{r_0^2}{2}} \right] - \pi r_0^2 V_1^2 \rho$$

$$= \frac{4}{3} \pi r_0^2 V_1^2 \rho - \pi r_0^2 V_1^2 \rho$$

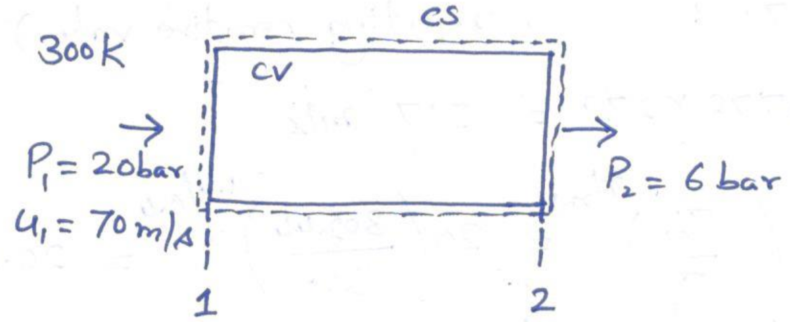
$$= \frac{1}{3} \pi r_0^2 V_1^2 \rho$$

$$\Rightarrow \quad -F + P_1 \pi r_0^2 - P_2 \pi r_0^2 = \frac{1}{3} \pi r_0^2 V_1^2 \rho$$

$$\Rightarrow \quad \boxed{F = \pi r_0^2 \left[P_1 - P_2 - \frac{1}{3} \rho V_1^2 \right]}$$

Ans.

Soln 2 a)



Mass Conservation :

$$\rho_1 \cancel{A_1} u_1 = \rho_2 \cancel{A_2} u_2$$

$$\rho_1 u_1 = \rho_2 u_2 \qquad (\because A_1 = A_2)$$

$$\frac{P_1 u_1}{\cancel{R} T_1} = \frac{P_2 u_2}{\cancel{R} T_2}$$

$$\Rightarrow \quad \frac{20 \times 10^5 \times 70}{300} = \frac{6 \times 10^5 \times u_2}{T_2}$$

$$\Rightarrow \quad \boxed{\frac{u_2}{T_2} = 0.778} \qquad - (1)$$

Energy Conservation

Adiabatic Flow :- $T_{01} = T_{02}$

$$T_1 + \frac{u_1^2}{2C_p} = T_2 + \frac{u_2^2}{2C_p}$$

$$300 + \frac{70^2}{2 \times 1005} = T_2 + \frac{u_2^2}{2 \times 1005}$$

$$302.44 = T_2 + \frac{u_2^2}{2010} \qquad (\text{use } u_2 = 0.778\, T_2)$$

$$\Rightarrow \quad 302.44 = T_2 + \frac{0.605}{2010} T_2^2$$

$$\Rightarrow \quad T_2^2 + 3322.3\, T_2 - 1004801 = 0$$

$$T_2 = \frac{-3322.31 \pm \sqrt{3322.31^2 + 4 \times 1004801}}{2}$$

$\Rightarrow \quad T_2 = 279 \text{ K}$ (rejecting negative value)

$\Rightarrow \quad u_2 = 0.778 \times 279 = 217 \text{ m/s}$

$$P_{01} = P_1 \left(\frac{T_{01}}{T_1}\right)^{\gamma/\gamma-1} = 20\left(\frac{302.44}{300}\right)^{1.4/0.4} = 20.58 \text{ bar}$$

$$P_{02} = P_2 \left(\frac{T_{02}=T_{01}}{T_2}\right)^{\gamma/\gamma-1} = 6\left(\frac{302.44}{279}\right)^{1.4/0.4} = 7.96 \text{ bar}$$

$$\boxed{\frac{P_{02}}{P_{01}} = 0.39}$$

Ans.

b) Isothermal compressibility, $\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$, $v = \frac{RT}{p}$, $\therefore \left(\frac{\partial v}{\partial p}\right)_T = -\frac{RT}{p^2}$

$$\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{p}{RT}\right)\left(-\frac{RT}{p^2}\right) = \frac{1}{p}$$

The pressure is same for all the gases, so

$$p = 4 \text{ bar} = 4\times10^5 \text{ Pa} = 4\times10^5 \text{ N/m}^2$$

$$\boxed{\tau_T = 1/4\times10^5 = 2.5\times10^{-6} \text{ m}^2/\text{N}}$$

Ans.

Isentropic process pv^γ = constant. Differentiate to obtain,

$$dp\, v^\gamma + \gamma p v^{\gamma-1} dv = 0$$

$$-\left(\frac{1}{v}\right)\left(\frac{\partial v}{\partial p}\right) = \frac{1}{\gamma p}$$

$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s = \frac{1}{\gamma p}$$

Calculate τ_s in (m^2/N) for each gas using p & γ.

Air $\longrightarrow$ $\tau_s = 1.786\times10^{-6} \text{ m}^2/\text{N}$

Argon $\longrightarrow$ $\tau_s = 1.506\times10^{-6} \text{ m}^2/\text{N}$

Carbon Dioxide $\rightarrow$ $\tau_s = 1.953\times10^{-6} \text{ m}^2/\text{N}$

Ans.

Soln 3 $\quad T_a = 239.5\,K$, $P_a = 0.383$ bar , $M_a = 1.5$

$$u = M_a\sqrt{\gamma R T_a} = 1.5\sqrt{1.4\times 287\times 239.5} = 465.32\ m/s$$

Intake / Diffuser :

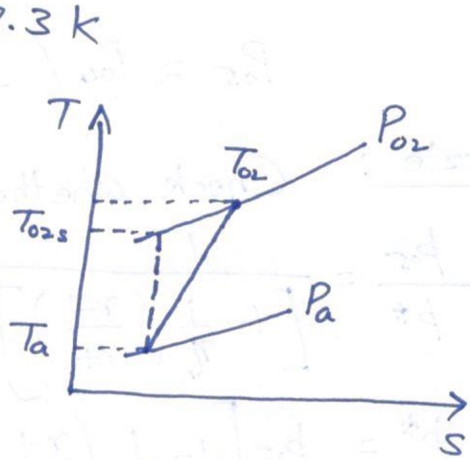
$$T_{01} = T_1\left(1+\frac{\gamma-1}{2}M_a^2\right)$$

$T_{01} = T_{02}$ (adiabatic process) & $T_1 = T_a$

$$\Rightarrow\ T_{02} = T_a\left(1+\frac{\gamma-1}{2}M_a^2\right)$$

$$\Rightarrow\ T_{02} = 239.5\left(1+\frac{1.4-1}{2}\times 1.5^2\right) = 347.3\ K$$

$$\eta_d = \frac{h_{02s}-h_a}{h_{02}-h_a} = \frac{T_{02s}-T_a}{T_{02}-T_a} = 0.85$$

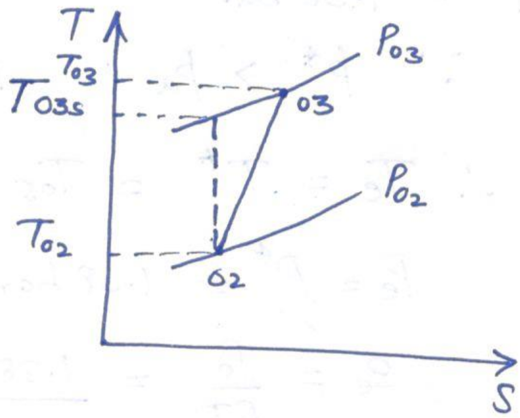


$$0.85 = \frac{T_{02s}-239.5}{347.3-239.5}$$

$$\Rightarrow\ T_{02s} = 331.1\ K$$

$$\frac{P_{02}}{P_a} = \left(\frac{T_{02s}}{T_a}\right)^{\gamma/\gamma-1} \Rightarrow P_{02} = 0.383\left(\frac{331.1}{239.5}\right)^{1.4/1.4-1} = 1.190\ bar$$

Compressor :



$$P_{03} = r_p P_{02} = 10\times 1.190 = 11.90\ bar$$

$$T_{03s} = T_{02}\left(\frac{P_{03}}{P_{02}}\right)^{\gamma-1/\gamma} = 347.3\,(10)^{\frac{1.4-1}{1.4}}$$

$$\Rightarrow\ T_{03s} = 670.53\ K$$

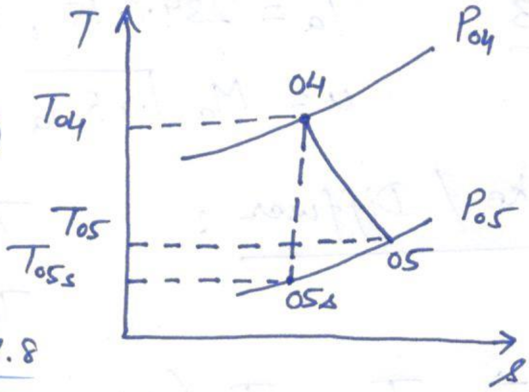
$$\eta_c = \frac{T_{03s}-T_{02}}{T_{03}-T_{02}} \Rightarrow 0.82 = \frac{670.53-347.3}{\cancel{347.3}\ T_{03}-347.3}$$

$$\Rightarrow\ T_{03} = 741.48\ K$$

Combustor : $\quad T_{04} = 1400\ K$

$$P_{04} = \left(1-\frac{\Delta P_b}{P_{03}}\right)P_{03} = (1-0.06)\ 11.90 = 11.186\ bar$$

Turbine : $(T_{04} - T_{05})\eta_m = (T_{03} - T_{02})$



$$\Rightarrow (1400 - T_{05})\, 0.98 = (741.48 - 347.3)$$

$$\Rightarrow \quad T_{05} = 997.8 \text{ K}$$

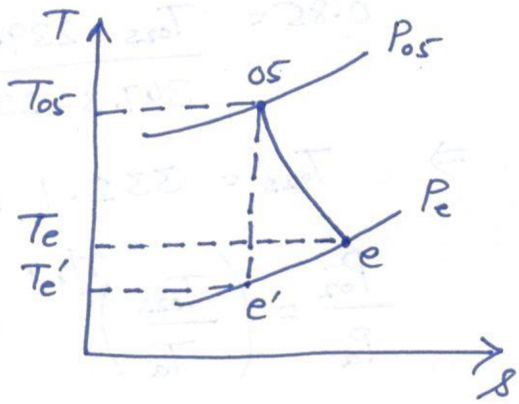
$$\eta_t = \frac{T_{04} - T_{05}}{T_{04} - T_{05s}} \Rightarrow 0.87 = \frac{1400 - 997.8}{1400 - T_{05s}}$$

$$\Rightarrow \quad T_{05s} = 937.7 \text{ K}$$

$$P_{05} = P_{04}\left(\frac{T_{05s}}{T_{04}}\right)^{\gamma/\gamma-1} = 11.86\left(\frac{937.7}{1400}\right)^{1.4/1.4-1} = 2.75 \text{ bar}$$

Nozzle : Check whether Nozzle is Choked or not!

$$\frac{p_{05}}{p^*} = \frac{1}{\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/\gamma-1}}$$



$$\Rightarrow p^* = p_{05}\left[1 - \frac{1}{\eta_n}\left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\gamma/\gamma-1}$$

$$p^* = 2.75\left[1 - \frac{1}{0.93}\left(\frac{1.4-1}{1.4+1}\right)\right]^{1.4/1.4-1}$$

$$p^* = 1.38 \text{ bar}$$

$\because \; p^* > p_a$ Nozzle is Choked!

$$\therefore \; T_e = T^* = T_{05}\frac{2}{\gamma+1} = \frac{997.8 \times 2}{2.4} = 831.5 \text{ K}$$

$$P_e = P^* = 1.38 \text{ bar}$$

$$\rho_e = \frac{P_e}{RT_e} = \frac{1.38\times10^5}{287 \times 831.5} = 0.578 \text{ kg/m}^3$$

$$u_e = \sqrt{\gamma R T_e} = \sqrt{1.4 \times 287 \times 831.5} = 578 \text{ m/s}$$

$$\text{Specific Thrust} = \frac{\mathcal{T}}{\dot{m}_a} = (u_e - u) + \frac{P_e - P_a}{\rho_e u_e}$$

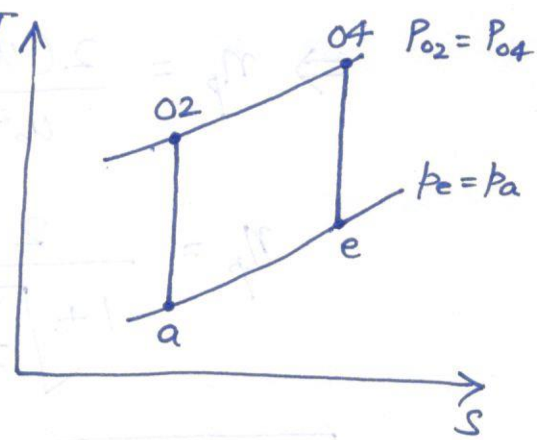
$$= (578 - 465.32) + \frac{(1.38 - 0.383)\times10^5}{0.578 \times 578}$$

$$\boxed{\text{Specific Thrust} = 411.11 \text{ Ns/kg}}$$

Ans.

Soln 4 As can be seen,

$$T_{oa} = T_{o2}$$
$$T_{o4} = T_{oe}$$



$$\frac{T_{o2}}{T_a} = \frac{T_{oa}}{T_a} = \tau_r$$

$$\frac{T_{o4}}{T_{oa}} = \frac{T_{o4}}{T_{o2}} = \tau_\lambda$$

Also,
$$\frac{T_{o2}}{T_a} = \frac{T_{o4}}{T_e} = \left(\frac{P_{o2} = P_{o4}}{P_a = P_e}\right)^{\frac{\gamma-1}{\gamma}}$$

a) Thermal Efficiency, η_{th}

$$\eta_{th} = \frac{\dot{m}_a\left(\frac{u_e^2}{2} - \frac{u^2}{2}\right)}{\dot{m}_f Q_R} = \frac{\dot{m}_a\left(\frac{u_e^2}{2} - \frac{u^2}{2}\right)}{\dot{m}_a c_p (T_{o4} - T_{o2})} = \frac{\frac{u_e^2 - u^2}{2}}{c_p (T_{o4} - T_{o2})}$$

$$h_{oe} = h_e + \frac{u_e^2}{2} \Rightarrow u_e = \sqrt{2c_p(T_{oe} - T_e)} = \sqrt{2c_p(T_{o4} - T_e)}$$

$$h_{oa} = h_a + \frac{u^2}{2} \Rightarrow u = \sqrt{2c_p(T_{oa} - T_a)} = \sqrt{2c_p(T_{o2} - T_a)}$$

$$\Rightarrow \eta_{th} = \frac{\cancel{2c_p}(T_{o4} - T_e) - \cancel{2c_p}(T_{o2} - T_a)}{\cancel{2c_p}(T_{o4} - T_{o2})} = \frac{(T_{o4} - T_e) - (T_{o2} - T_a)}{(T_{o4} - T_{o2})}$$

$$\boxed{\eta_{th} = \frac{T_{o4}\left(1 - \frac{1}{\tau_r}\right) - T_{o2}\left(1 - \frac{1}{\tau_r}\right)}{T_{o4} - T_{o2}} = 1 - \frac{1}{\tau_r}}$$

b) Propulsion Efficiency, η_p

$$\eta_p = \frac{\mathcal{T} u}{\dot{m}_a\left(\frac{u_e^2}{2} - \frac{u^2}{2}\right)} = \frac{\dot{m}_a (u_e - u) u}{\dot{m}_a\left(\frac{u_e^2}{2} - \frac{u^2}{2}\right)}$$

$$\Rightarrow \eta_p = \frac{2(u_e-u)u}{u_e^2-u^2} = \frac{2}{1+\frac{u_e}{u}}$$

$$\eta_p = \frac{2}{1+\sqrt{\frac{T_{04}-T_e}{T_{02}-T_a}}} = \frac{2}{1+\sqrt{\frac{T_{04}\left(1-\frac{1}{\tau_r}\right)}{T_{02}\left(1-\frac{1}{\tau_r}\right)}}}$$

$$\boxed{\eta_p = \frac{2}{1+\sqrt{\frac{T_{04}}{T_{02}}}} = \frac{2}{1+\sqrt{\tau_\lambda}}}$$

c) Overall Efficiency, η_o

$$\eta_o = \frac{\mathcal{T}u}{\dot{m}_f Q_R} = \eta_{th}\,\eta_p$$

$$\Rightarrow \eta_o = \left(1-\frac{1}{\tau_r}\right)\left(\frac{2}{1+\sqrt{\tau_\lambda}}\right)$$

$$\boxed{\eta_o = \frac{2(\tau_r-1)}{\tau_r(1+\sqrt{\tau_\lambda})}}$$

Soln 5 $\quad P_a = 0.383$ bar, $T_a = 239.5$ K, $M_a = 0.7$

$$u = M_a\sqrt{\gamma R T_a} = 0.7\sqrt{1.4\times 287\times 239.5} = 217.15 \text{ m/s}$$

(1)-(2) Diffuser: $\quad T_1 = T_a = 239.5$ K

$P_1 = P_a = 0.383$ bar, Also $T_{oa} = T_{o1}$

$$T_{oa} = T_{o1} = T_1\left(1+\frac{\gamma-1}{2}M_a^2\right) = 239.5\left(1+\frac{1.4-1}{2}\times 0.7^2\right) = 262.97\text{ K}$$

$T_{o2} = T_{o1}$ (adiabatic process with no work transfer)

$\Rightarrow T_{o2} = 262.97$ K

$$\because \eta_d = 1 \Rightarrow p_{o2} = p_a\left(\frac{T_{o2}}{T_a}\right)^{\gamma/\gamma-1} = 0.383\left(\frac{262.97}{239.5}\right)^{1.4/1.4-1} = 0.531\text{ bar}$$

(2)-(3) Fan: $\quad \eta_f = 1 \qquad p_{o3} = p_{o2}(FPR) = 0.531\times 2 = 1.062$ bar

$$T_{o3} = T_{o2}(FPR)^{\gamma-1/\gamma} = 262.97\times(2)^{1.4-1/1.4} = 320.56\text{ K}$$

(3)-(4) Compressor: $\quad \eta_c = 1$

$$p_{o4} = r_p\, p_{o3} = 10\times 1.062 = 10.62\text{ bar}$$

$$T_{o4} = T_{o3}(r_p)^{\gamma-1/\gamma} = 320.56\times(10)^{1.4-1/1.4} = 618.90\text{ K}$$

(4)-(5) Combustor:

$$\Delta P_b = 0 \Rightarrow p_{o5} = p_{o4} = 10.62\text{ bar}$$

$$T_{o5} = 1500\text{ K}$$

(5)-(6) H.P. Turbine: $\quad \eta_t = \eta_m = 1 \Rightarrow T_{o6} = T_{o5} - (T_{o4} - T_{o3})$

$$T_{o6} = 1500 - (618.9 - 320.56) = 1201.66\text{ K}$$

$$p_{o6} = p_{o5}\left(\frac{T_{o6}}{T_{o5}}\right)^{\gamma/\gamma-1} = 10.62\left(\frac{1201.66}{1500}\right)^{1.4/1.4-1} = 4.89\text{ bar}$$

(6)-(7) L.P. Turbine: $\quad \eta_t = \eta_m = 1$

$$T_{o7} = T_{o6} - (B+1)(T_{o3} - T_{o2}) = 1201.66 - (4+1)(320.56 - 262.97)$$

$\Rightarrow T_{o7} = 913.7$ K

$$p_{o7} = p_{o6}\left(\frac{T_{o7}}{T_{o6}}\right)^{\gamma/\gamma-1} = 4.89\left(\frac{913.7}{1201.66}\right)^{1.4/1.4-1} = 1.875\text{ bar}$$

(7)-(8) Hot Nozzle: $\quad p^* = p_{o7}\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} = 1.875\left(\frac{2}{2.4}\right)^{1.4/0.4} = 0.99$ bar

$p^* > p_a \Rightarrow$ Nozzle is Choked

$$p_{ec} = p^* = 0.99 \text{ bar}$$

$$T_{ec} = T^* = T_{07}\left(\frac{2}{\gamma+1}\right) = 913.7 \times \left(\frac{2}{2.4}\right) = 761.42 \text{ K}$$

$$u_{ec} = \sqrt{\gamma R T_{ec}} = \sqrt{1.4 \times 287 \times 761.42} = 553.12 \text{ m/s}$$

(3)-(9) Bypass Nozzle (Cold Nozzle) : $p^* = p_{03}\left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} = 1.062\left(\frac{2}{2.4}\right)^{1.4/0.4}$

$$p^* = 0.561 \text{ bar} > p_a \Rightarrow \text{Nozzle is Choked}$$

$$p_{eb} = p^* = 0.561 \text{ bar}$$

$$T_{eb} = T^* = T_{03}\left(\frac{2}{\gamma+1}\right) = 320.56\left(\frac{2}{2.4}\right) = 267.13 \text{ K}$$

$$u_{eb} = \sqrt{\gamma R T_{eb}} = \sqrt{1.4 \times 287 \times 267.13} = 327.62 \text{ m/s}$$

Specific Thrust : $\frac{\mathcal{T}}{\dot{m}_a} = \frac{u_{ec}}{(1+B)} + \frac{B}{(1+B)} u_{eb} - u + \frac{(p_{ec} - p_a) A_8}{\dot{m}_a} + \frac{(p_{eb} - p_a) A_9}{\dot{m}_a}$ —(1)

Realizing the fact that, $\dot{m}_c = \rho_{ec} u_{ec} A_8 = \frac{\dot{m}_a}{(1+B)} \Rightarrow \dot{m}_a = (1+B)\rho_{ec} u_{ec} A_8$

$$\dot{m}_b = \rho_{eb} u_{eb} A_9 = \frac{\dot{m}_a B}{(1+B)} \Rightarrow \dot{m}_a = \rho_{eb} u_{eb} A_9 \frac{(1+B)}{B}$$

Plugging $\dot{m}_a$ from above in Eq. (1) above to eliminate A_8 and A_9

$$\text{Specific Thrust} = \frac{\mathcal{T}}{\dot{m}_a} = \frac{u_{ec}}{(1+B)} + \frac{B}{(1+B)} u_{eb} - u + \frac{(p_{ec} - p_a)}{(1+B)\rho_{ec} u_{ec}} + \frac{(p_{eb} - p_a) B}{(1+B)\rho_{eb} u_{eb}}$$

$$\rho_{ec} = \frac{p_{ec}}{R T_{ec}} = \frac{0.99 \times 10^5}{287 \times 761.42} = 0.453 \text{ kg/m}^3$$

$$\rho_{eb} = \frac{p_{eb}}{R T_{eb}} = \frac{0.561 \times 10^5}{287 \times 267.13} = 0.732 \text{ kg/m}^3$$

$$\text{Specific Thrust} = \frac{553.12}{5} + \frac{4}{5} \times 327.62 - 217.15 + \frac{(0.99 - 0.383) \times 10^5}{5 \times 0.453 \times 553.12} + \frac{(0.561 - 0.383) \times 10^5 \times 4}{5 \times 0.732 \times 327.62}$$

$$\boxed{\text{Specific Thrust} = 263.39 \text{ Ns/kg}}$$

Ans.

Soln 6

Continuity Equation :

$$\rho_1 u_1 = \rho_2 u_2 \qquad -①$$

$$\Rightarrow u_2 = u_1 \left(\frac{\rho_1}{\rho_2}\right) \qquad -②$$

Momentum Equation :

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \qquad -③$$

Substitute Eq. ② into Eq. ③ ,

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 \left(\frac{\rho_1 u_1}{\rho_2}\right)^2 \qquad -④$$

$$\Rightarrow u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_2}{\rho_1}\right) \qquad -⑤$$

Alternatively , writing Eq. ① as ,

$$u_1 = u_2 \left(\frac{\rho_2}{\rho_1}\right)$$

And again substituting into Eq. ③ , this time solving for u_2 , we obtain ,

$$u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1}\left(\frac{\rho_1}{\rho_2}\right) \qquad -⑥$$

Energy Equation :

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \qquad -⑦$$

Recalling , $h = e + \frac{p}{\rho}$, we have

$$e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2} \qquad -⑧$$

Substituting Eqns. 5 and 6 into Eq. 8

$$e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}\left[\frac{p_2-p_1}{\rho_2-\rho_1}\left(\frac{\rho_2}{\rho_1}\right)\right] = e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}\left[\frac{p_2-p_1}{\rho_2-\rho_1}\left(\frac{\rho_1}{\rho_2}\right)\right] \quad \text{—} 9$$

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{1}{2}\left(\frac{p_2-p_1}{\rho_2-\rho_1}\right)\left\{\frac{\rho_2}{\rho_1} - \frac{\rho_1}{\rho_2}\right\}$$

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{1}{2}\left(\frac{p_2-p_1}{\rho_2-\rho_1}\right)\frac{(\rho_2^2-\rho_1^2)}{\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} + \frac{1}{2}\left(\frac{p_2-p_1}{\cancel{\rho_2-\rho_1}}\right)\frac{\cancel{(\rho_2-\rho_1)}(\rho_2+\rho_1)}{\rho_1\rho_2}$$

$$e_2 - e_1 = \frac{p_1(\rho_2-\rho_1) + p_2(\rho_2-\rho_1)}{2\rho_1\rho_2}$$

$$e_2 - e_1 = \left(\frac{p_1+p_2}{2}\right)\left(\frac{\rho_2-\rho_1}{\rho_1\rho_2}\right)$$

$$e_2 - e_1 = \left(\frac{p_1+p_2}{2}\right)\left[\frac{1}{\rho_1} - \frac{1}{\rho_2}\right]$$

$$\boxed{e_2 - e_1 = \frac{p_1+p_2}{2}(v_1 - v_2)}$$