1. $$\eta_{th} = \frac{(1+f)u_e^2 - u^2}{2fQ_R}$$

$\dot m_f \ll \dot m_a \Rightarrow f \ll 1.$

$$\eta_{th} = \frac{u_e^2 - u^2}{2fQ_R} = \frac{u_e^2 - u^2}{2c_p(T_{04} - T_{02})}$$

$u = M\sqrt{\gamma R T_a}$, $u_e = M_e\sqrt{\gamma R T_e} = M\sqrt{\gamma R T_6}$

for ideal ramjet, $M_e = M \Rightarrow u_e = M\sqrt{\gamma R T_6}$

$$\eta_{th} = \frac{M^2 \gamma R (T_6 - T_a)}{2c_p(T_{04} - T_{02})}$$

$$= \frac{M^2 \gamma R T_a \left(\frac{T_6}{T_a} - 1\right)}{2c_p T_{02}\left(\frac{T_{04}}{T_{02}} - 1\right)}$$

Since $\frac{P_{04}}{P_6} = \frac{P_{02}}{P_a}$

$$\left(\frac{T_{04}}{T_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_{02}}{T_a}\right)^{\frac{\gamma-1}{\gamma}}$$

$\frac{T_{04}}{T_6} = \frac{T_{02}}{T_a}$. on $\gamma = $ Const.

$\therefore \frac{T_{04}}{T_{02}} = \frac{T_6}{T_a}$.

$$\eta_{th} = \frac{\gamma R T_a M^2}{2c_p T_{02}} = \frac{\gamma R T_a M^2}{2\frac{\gamma R}{\gamma-1}T_{02}} = \frac{M^2(\gamma-1)}{2\left(\frac{T_{02}}{T_a}\right)}$$

$$\tau_r = \frac{T_{02}}{T_a} = 1 + \frac{\gamma-1}{2}M^2$$

$$\eta_{th} = \frac{\left(\frac{\gamma-1}{2}\right)M^2}{1 + \frac{\gamma-1}{2}M^2} = 1 - \frac{1}{1 + \frac{\gamma-1}{2}M^2}$$

$$\boxed{\eta_{th} = 1 - \frac{1}{\tau_r}}$$

2.

Diffuser : $M = 0$, for an ideal diffusion process,

$$T_a = T_{0a} = T_{02} = T_7$$

$$P_{02} = P_{0a} = P_a$$

Compressor : ideal compression, $\frac{T_{03}}{T_{02}} = \pi_c^{\frac{\gamma_c - 1}{\gamma_c}} \quad \left(\pi_c = \frac{P_{03}}{P_{02}}\right)$. (a)

Combustion chamber : no pres. loss in the combustion chamber and negligible 'f', $P_{04} = P_{03}$, $f \ll 1$

Turbine : $\frac{T_{04}}{T_{05}} = \pi_t^{\frac{\gamma_t - 1}{\gamma_t}}$, $\pi_t = P_{04}/P_{05}$

work developed by turbine = work consumed by compressor

$$W_t = W_c$$

for $f \ll 1$,

$$c_{p_t}(T_{04} - T_{05}) = c_{p_c}(T_{03} - T_{02}) \quad \text{(b)}$$

$$c_{p_t} T_{04}\left(1 - \frac{1}{\pi_t^{\frac{\gamma_t - 1}{\gamma_t}}}\right) = c_{p_c} T_{02}\left(\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right).$$

with $T_{02} = T_a$,

$$\frac{1}{\pi_t^{\frac{\gamma_t - 1}{\gamma_t}}} = 1 - \frac{c_{p_c}}{c_{p_n}} \frac{T_a}{T_{04}}\left(\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)$$

$$\pi_t = \frac{1}{\left[1 - \left(\frac{c_{p_c}}{c_{p_n}}\right) \frac{T_a}{T_{04}}\left(\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)\right]^{\frac{\gamma_t}{\gamma_t - 1}}}$$

Nozzle : For an unchoked nozzle & ideal expansion, jet speed is

$$V_j = \sqrt{2 c_{p_n} T_{06}\left[1 - \left(\frac{P_7}{P_{06}}\right)^{\frac{\gamma_n - 1}{\gamma_n}}\right]}$$

using (a) & (b) , $T_{05} = T_{04} - \frac{c_{p_c}}{c_{p_n}} T_a\left[\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right] \equiv T_{06}$

$\frac{P_7}{P_{06}} = \frac{P_7}{P_{02}} \times \frac{P_{02}}{P_{03}} \times \frac{P_{03}}{P_{04}} \times \frac{P_{04}}{P_{05}} \times \frac{P_{05}}{P_{06}}$, Thus, $P_7 = P_a = P_{02}$, $P_{03} = P_{04}$ & P_{05}

$$V_j = \sqrt{2 c_{p_n}\left[T_{04} - \frac{c_{p_c}}{c_{p_n}} T_a\left(\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1\right)\right]\left\{1 - \left(\frac{\pi_t}{\pi_c}\right)^{\frac{\gamma_n - 1}{\gamma_n}}\right\}}$$

[redacted]

3

Case I : The mass flow rate is constant = 40 kg/s

$T_{momentum} = \dot{m}_a(1+f)U_e = 40\times 1.02\times 600 = 24480\ N$

$T_{pressure} = A_e(P_e-P_a) = 0.25(200-30.8)\times 10^3 = 42300\ N$

$T_{gross} = T_{momentum} + T_{pressure} = 66780\ N$

The momentum drag for flight speed varying from 500–6000 km/h is written as : $D_{momentum} = \dot{m}_a U = 40\times \frac{U\,(km/h)}{3.6}$

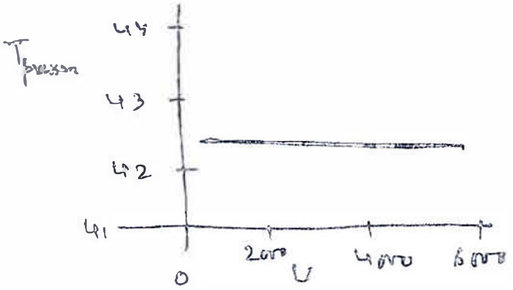
$= 11.11\times U\ (N)$

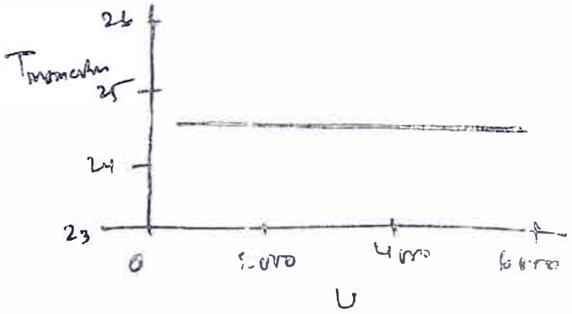
Which is a linear relation in the flight speed U.

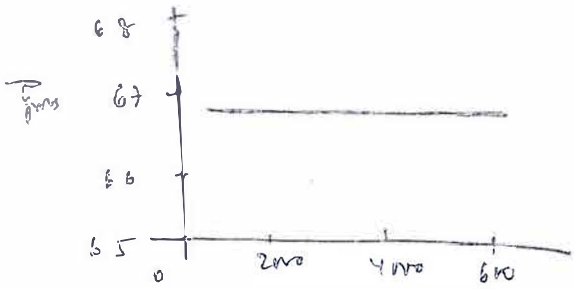
The net thrust is : $T_{net} = T_{gross} - D_{momentum}$

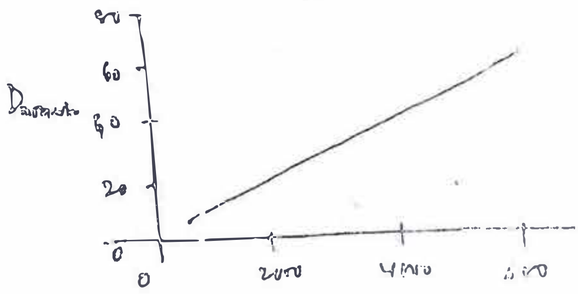
$= 66780 - 11.11\times U\ (N)$

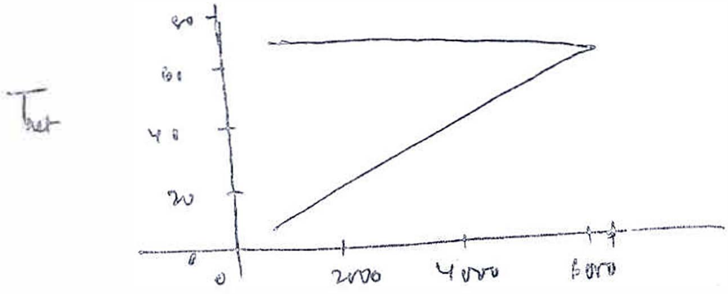
The net thrust varies linearly with the flight speed too.











Case 2: The mass flow rate varies linearly with the fligh sp according to the relation:

$$\dot{m}_a = \rho_a U A_i = \frac{P_a}{RT_a} U A_i = \frac{30.8\times10^3}{287\times229.74}\times\frac{U}{3.6}\times 0.235 \ \text{kg/s}$$

$$= 0.03049\times U \ \text{kg/s}$$

$$T_{momentum} = \dot{m}_a(1+f)u_e = 18.66\times U \ \text{N}$$

$$T_{pressure} = A_e(P_e - P_a) = 42300 \ \text{N}$$

$$T_{gross} = 18.66\times U + 42300 \ \text{N}$$

$$D_{momentum} = \dot{m}_a U = 8.47\times10^{-3}\, U^2 \ \text{N} \rightarrow \text{quadratic relation}$$

$$T_{net} = (42300 + 18.66\times U - 8.47\times10^{-3}\times U^2) \ \text{N}$$