Tail Design Proceedure

Step. 1

Fixed the design lift coefficient and required Static margin

$$(C_L)_{Design} = 0.4$$

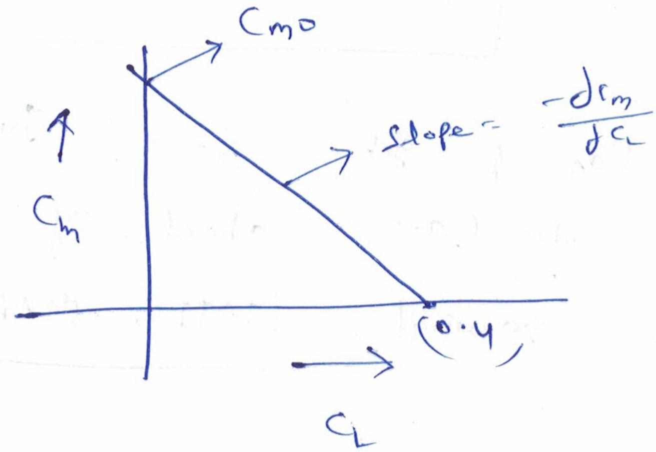
Static Margin (SM) = 15%

Using the relation

$$SM = -\frac{dC_m}{dC_L}$$

Step. 2

We get,



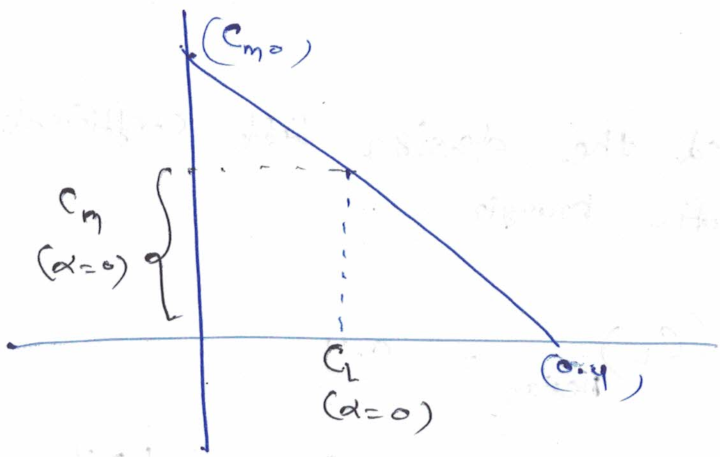
from above graph

C_m (at $C_L = 0$) is 0.06

or $\boxed{C_{mo} = 0.06}$

Step. 3

What should be C_m at $\alpha = 0$?



$$C_{L_{\alpha=0}} = C_{L_{\alpha w}} \left| \alpha_{L=0} \right|$$

where, $C_{L_{\alpha w}} = \dfrac{(C_{l\alpha})_{Airfoil}}{1 + \dfrac{(C_{l\alpha})_{Airfoil}}{\pi e AR}}$

$$C_{L_{\alpha=0}} = 0.2084$$

[See Next Page for calculation]

Now

using above graph

$$\boxed{C_m(\alpha=0) = 0.031}$$

Question: Can I get $(C_m = 0)$ Just Putting Aerodynamic center (a.c) ahead of CG to get the required STATIC MARGIN.

Step. 4 Take only wing ⇒

Pitching moment equation we can write.

$$C_m\Big|_{\alpha=0} = C_{mac} + C_{L_{0w}} \left[\frac{x_{cg}}{c} - \frac{x_{ac}}{\bar{c}} \right]$$

where,

$$C_{mac} = \left(C_{mac}\right)_{Airfoil} \left(\frac{AR}{AR+2}\right)$$

for the present case $\Rightarrow$

$$\alpha_{L=0} = -2.5^{\circ}$$

$$\left(C_{l\alpha}\right)_{Airfoil} = 5.9/\text{rad}$$

$$(AR) = 10$$

$$e = 0.8 \quad \text{(Assumed)}$$

so $\Rightarrow$

$$\left(C_{l\alpha}\right)_{w} = \frac{5.9}{1 + \frac{5.9}{\pi \times 0.8 \times 10}} = 4.778/\text{rad}$$

$$\left(C_{L0}\right)_{w} = 4.778 \times (|-2.5|)\ \pi/180$$

$$= 0.2084$$

and

$$C_{mac}\Big|_{wing} = -0.08 \times \frac{10}{10+2}$$

$$= -0.066$$

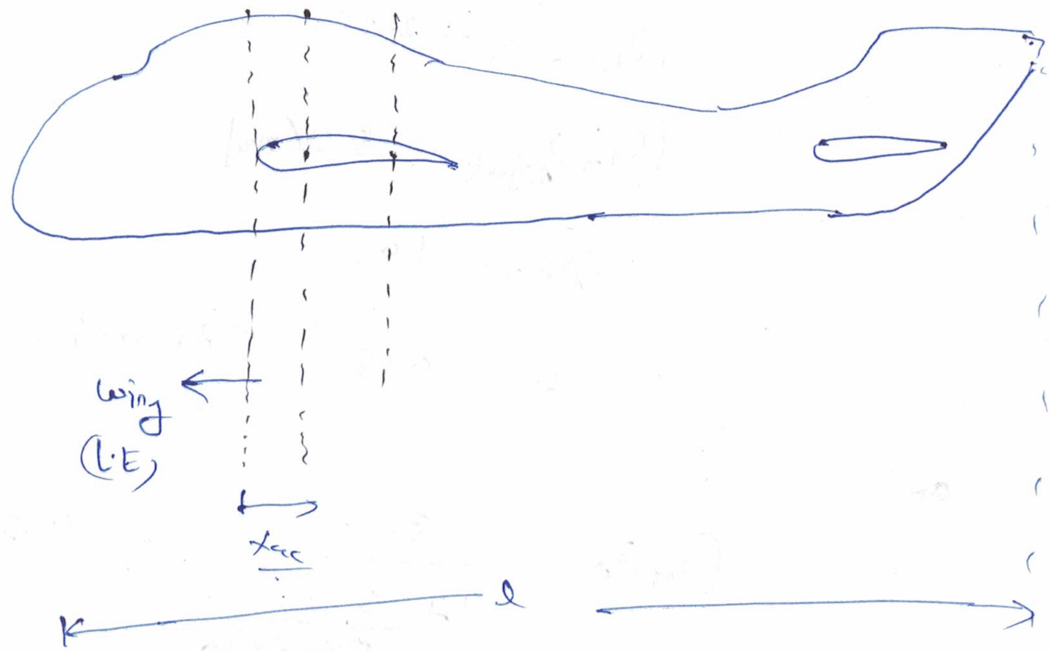
the $(X_{ac})_w$ will be at $\frac{\bar{c}}{4}$

$$\boxed{\frac{X_{ac}}{w} = \frac{\bar{c}}{4} = \frac{1.345}{4}}$$

Step. 5 How to locate the C.G ?

First,

locate the (C.G) at 40% of the total length of Aircraft



Let

x_{cg} = 35% from wing leading edge

[first guess]

$$x_{cg} = \frac{35}{100} \times \bar{c}$$

$$= \frac{35}{100} \times 1.345$$

$$\boxed{x_{cg} = 0.47075}$$

Now, using

$$C_m(\alpha = 0) = C_{m_{ac}} + C_{L_{0w}} \left[\frac{x_{cg}}{c} - \frac{x_{ac}}{\bar{c}} \right]$$

$$= -0.066 + 0.2084 \left[0.35 - 0.25 \right]$$

$$\boxed{C_m(\alpha = 0) = -0.04516}$$

It can be noticed that, we are not getting desired C_{m0}.

What should be done ?

Now, we will placed the tail to meet the dinsired requirement :

$$\left(\begin{array}{c} C_{m0} = 0.031 \\ SM = 15\% \end{array} \right)$$

Step. 6

Now the pitching moment equation becomes

$$C_{m0} = C_{mac} + C_{L0w}\left[\frac{X_{cg}}{c} - \frac{X_{ac}}{c}\right] + \eta V_H C_{L\alpha t}\,(\varepsilon_0 + i_w - i_t)$$

where,

η = tail efficiency factor

V_H = tail volume ratio

$C_{L\alpha t}$ = lift curve slope of tail.

i_w = wing setting angle

i_t = tail setting angle

$\varepsilon_0 = \frac{2C_{L0}}{\pi AR}$ = downwash angle at $\alpha_L = 0$

Take NACA 0009 convert 2D to 3D

(Assumed)

for the present case,

$$\boxed{\begin{array}{l} \eta = 0.9,\quad V_H = 0.6,\quad C_{L\alpha t} = 3.8\,/\text{rad} \\ \varepsilon_0 = 0.76^\circ,\quad i_w = 0,\quad i_t = ? \end{array}}$$

Substituting these value in C_m equation we get -

$$0.031 = -0.066 + 0.2084 \left[0.35 - 0.25\right]$$
$$+ 0.9 \times 0.6 \times 3.8 \left[0.76 + 0 - i_t\right) \pi/180$$

$$\boxed{i_t = -1.36^\circ}$$

Question ⇒ The above i_t is a final value ?

Answer ⇒ No

This will be iterative till we achieve, $C_{m0} = 0.031$

and $S.M = 15\%$

Step. 7 Neutral point calculation ⇒

$$C_{m_\alpha} = C_{m_{\alpha f}} + C_{L\alpha}\left[\frac{x_{cg}}{c} - \frac{x_{ac}}{c}\right] - \eta V_H\, C_{L\alpha t}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

at Neutral point $x_{cg} = x_{NP}$ and $C_{m_\alpha} = 0$

Hence,
$$\frac{x_{NP}}{c} = \frac{x_{ac}}{c} - \frac{C_{m_{\alpha f}}}{C_{L\alpha w}} + \eta V_H \frac{C_{L\alpha t}}{C_{L\alpha w}}\left(1 - \frac{d\epsilon}{d\alpha}\right)$$

for the present case,

$$(C_{m_\alpha})_f = 0 \quad [\text{fuselage contribution}]$$

$$\frac{x_{NP}}{c} = 0.33625 + 0.9 \times 0.6 \times \frac{3.8}{4.778} \times (1 - 0.3041)$$

$$\boxed{\frac{x_{NP}}{c} = 0.6350}$$ where, $\frac{d\epsilon}{d\alpha} = \frac{2 C_{L_{\alpha w}}}{\pi AR} = 0.3041$

Step - 8

Check the Static margin $\Rightarrow$

$$SM = \bar{x}_{NP} - \bar{x}_{cg}$$

$$= 0.6350 - 0.35$$

$$= 0.285$$

$$SM = 28.5\% > 15\% \text{ (required SM)}$$

What should we do to get, $SM = 15\%$

→ we will keep changing the location of x_{cg}.
(This will be iterative process).

Step - 9 $\Rightarrow$ Iteration $\Rightarrow$

Let $\frac{x_{cg}}{c} = 0.45$

Repeating the calculation we get,

$$\boxed{i_t = -0.78^\circ}$$

Now,

$SM = \bar{x}_{NP} - \bar{x}_{cg}$

$= 0.63 - 0.45$

$= 0.18$

$$\boxed{SM = 18\%}$$

for SM 15%

SM $\bar{x}_{cg}$ = $\bar{x}_{NP} - \bar{x}_{cg}$

$= 0.63 - 0.15$

$$\boxed{\bar{x}_{cg} = 0.48}$$

at this $(\bar{x}_{cg})$

$$\boxed{i_t = -0.61^\circ}$$

So, tail setting angle should be -0.61° to get ($C_{m0} = 0.031$) and $x_{cg} = 0.48$ to get 15% static margin.