AE461A

Aircraft Design-I

Instructor: Prof. A K Ghosh

Term Report

by

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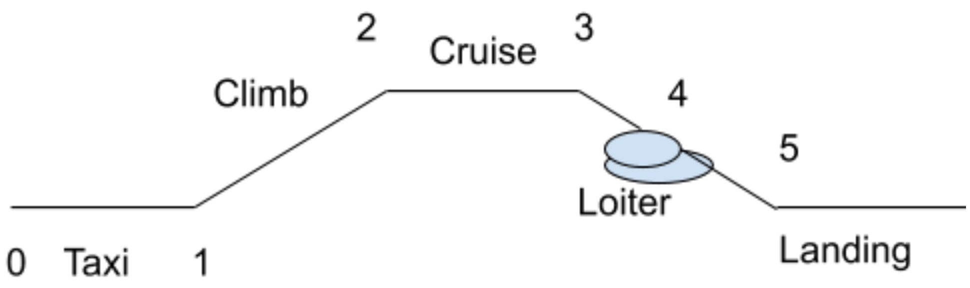
Contents:

1. Mission Profile:



1. Warmup + Taxi: 0 – 1
2. Climb: 1 - 2
3. Cruise: 2 - 3
4. Loiter: 3- 4
5. Landing: 4 - 5

Data Given:

- V_{stall} = 35 - 40 m/s
- V_{cruise} = 70 - 80 m/s
- Range (R) - 1500 km
- Endurance = 8 hrs
- Loiter Time = 30 minutes
- Rate of Climb = 5 m/s @ sea level
  3 m/s @ 8000ft
- Total Passengers = 4 + 2 crew (Passenger Weight: 80 kg + Baggage Weight: 15 kg / passenger)

2. Weight Estimation:

Formula to be used:

$$W_0 = \frac{W_{crew} + W_{payload}}{1 - \frac{W_E}{W_0} - \frac{W_F}{W_0}}$$

For the weight estimation we proceed as follows:

1. Take-off + Warm Up weight ratio: W1/W0 = 0.97
2. Climb weight ratio: W2/W1 = 0.985
3. Cruise Weight Ratio:

$$\frac{W_3}{W_2} = e^{-\frac{Rc}{\eta_p \left(\frac{L}{D}\right)_{max}}}$$

Where R =range, c = specific fuel consumption, η_p is the efficiency.
Assumed that the engine type is general aviation single-engine (piston with fixed pitch).
Assuming the cruise velocity to be 70 m/s and $\left(\frac{L}{D}\right)_{max}$ to be equal to 14
R = 1500 km , η_p = 0.8
From table

$$c = \frac{0.4 \times 9.81}{2.204 \times 0.745} = \frac{2.389}{3600} \text{ N/KW-s}$$

Putting the values

$$\frac{W_3}{W_2} = 0.925$$

4. Loiter Weight Ratio:

$$\frac{W_4}{W_3} = e^{\frac{-Ec}{L/D}}$$

Where E = Endurance, c is the specific fuel consumption
E = 30min = 0.5 hr
From table we have,

$$c = \frac{0.5 \times 9.81}{2.204 \times 0.745} = \frac{2.987}{1000} \frac{N}{Whr}$$

For Loiter, $\left(\frac{L}{D}\right)_{max} = 0.866 \times 14 = 13.856$
Substituting the above values, we get,

$$\frac{W_4}{W_3} = 0.990$$

5. Landing Weight Ratio:

$$\frac{W_5}{W_4} = 0.995$$

6. Total Weight Ratio:

$$\frac{W_5}{W_0} = \frac{W_1}{W_0} \times \frac{W_2}{W_1} \times \frac{W_3}{W_2} \times \frac{W_4}{W_3} \times \frac{W_5}{W_4} = 0.871$$

7. Fuel Ratio Estimation:
Considering 6% allowance for fuel,

$$\frac{W_F}{W_0} = 1.06(1 - 0.871) = 0.137$$

8. Empty Weight Ratio Estimation:

$$\frac{W_E}{W_0} = AW_0^c K_{VS}$$

Where A(metric) = 2.05(given in notes),

Kvs = 1, c = -0.18

$W_{crew} + W_{payload}$ = (6 * (80+15))kg = 570 kg

Substituting the above relation in the equation for total weight stated at the start we get the expression,

$$0.863W_0 - 2.05W_0^{0.82} - 570 = 0$$

Solving the above expression using online numerical equation solver we get,

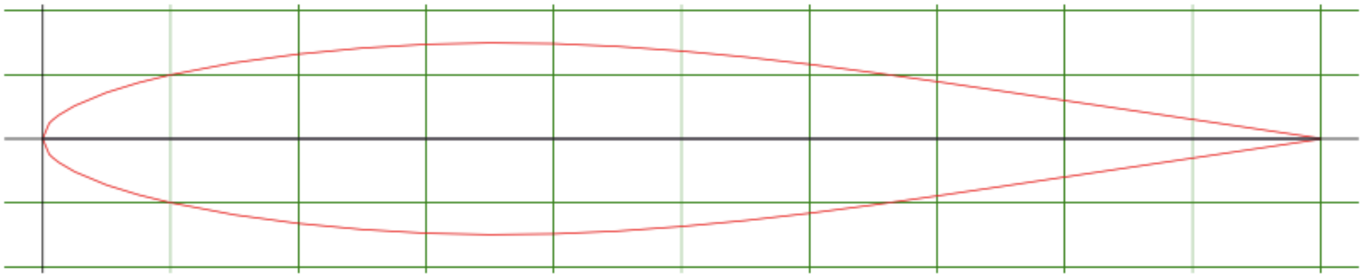
W_0=1738.814 kg

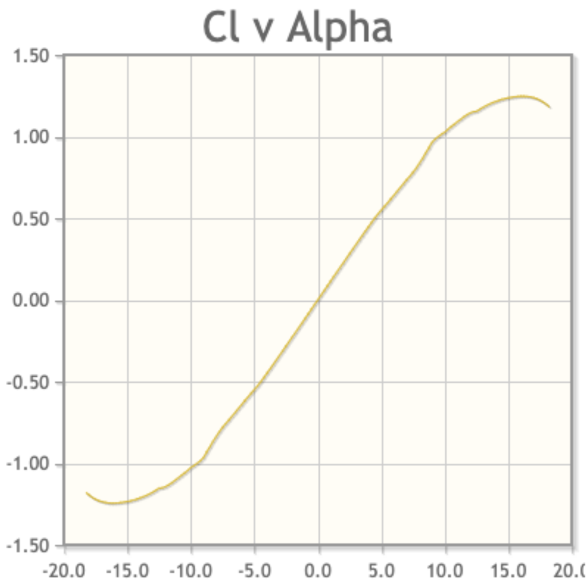
3. Airfoil Selection :

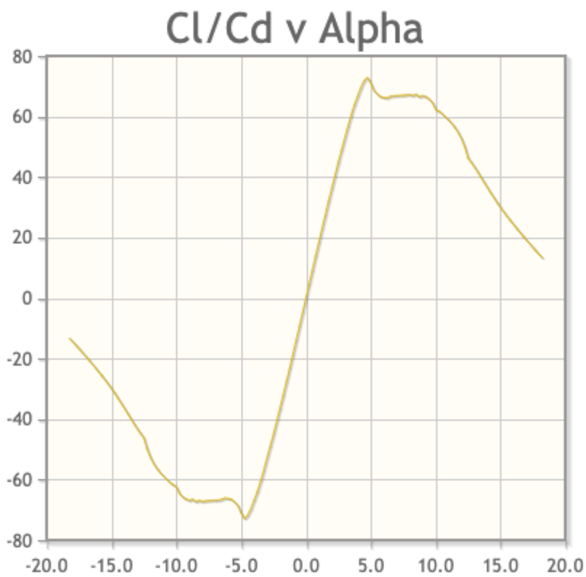
a). NACA 63015A

Max (t/c) = 15% at 35% Chord

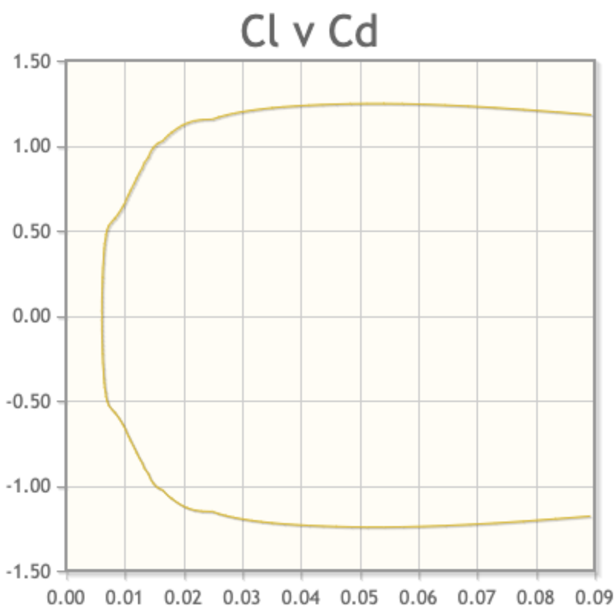
Max cambre = 0% at 0% Chord

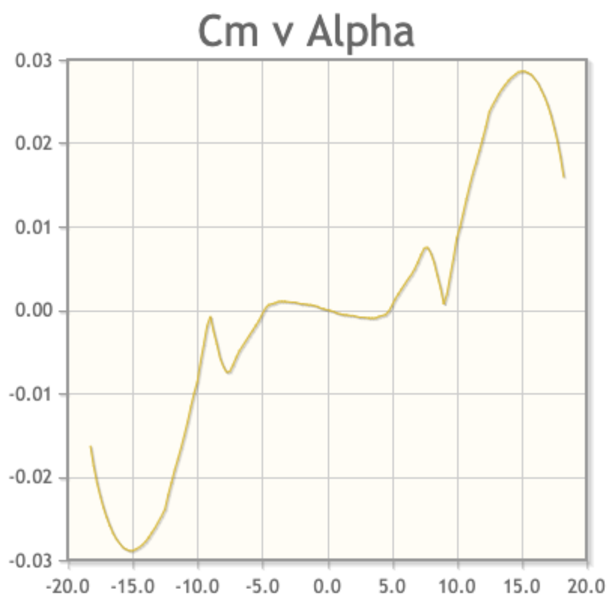




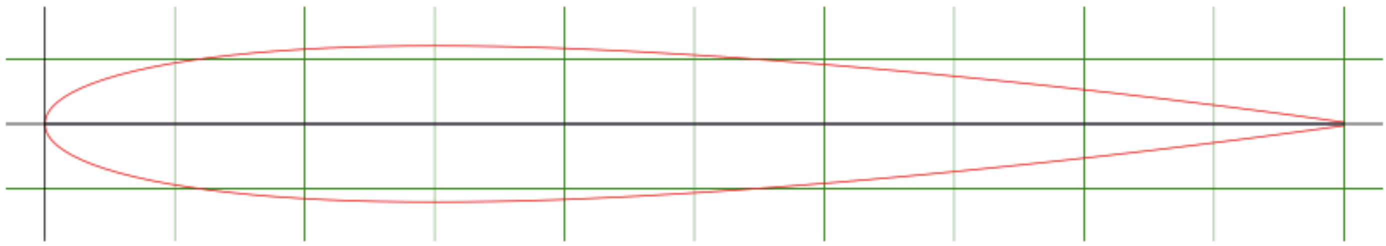


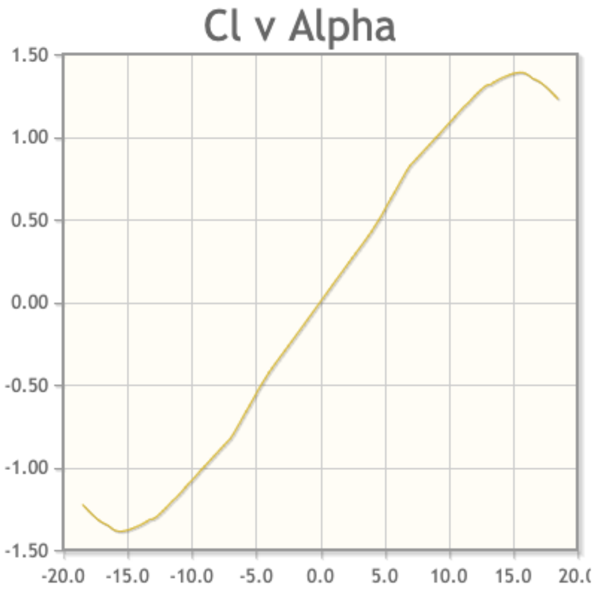
Cl max = 1.3178 at AoA = 16.5

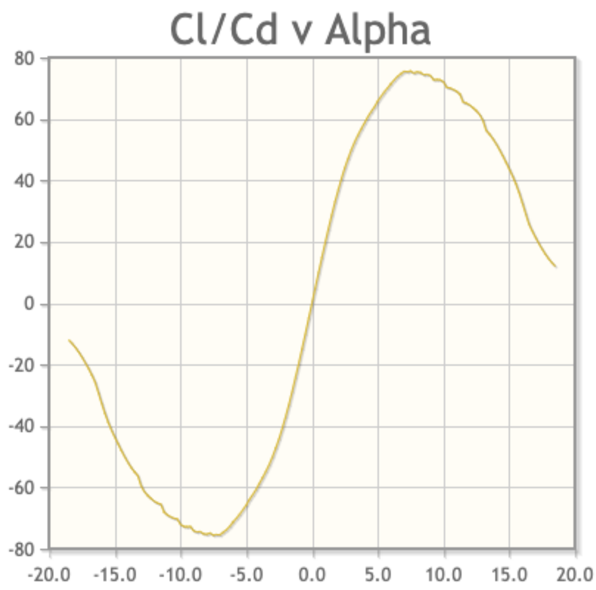




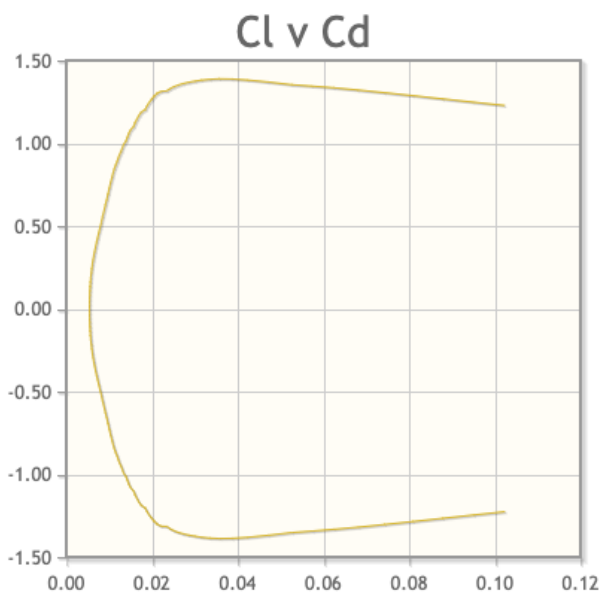
b). NACA 0012
Max (t/c) = 12% at 30% Chord
Max cambre = 0% at 0% Chord

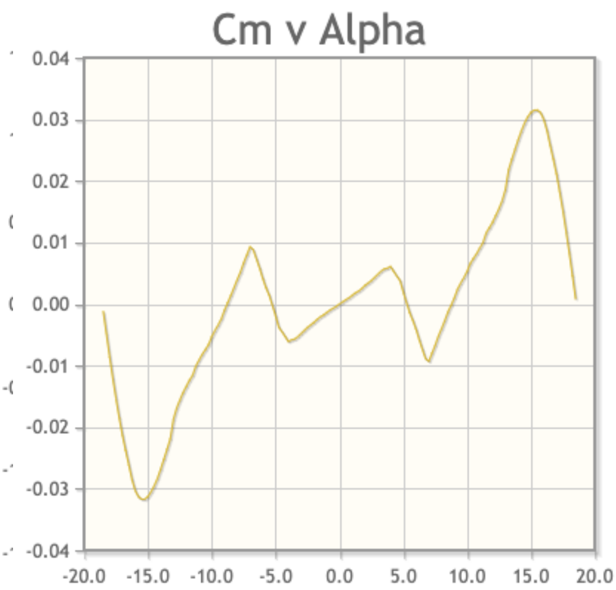




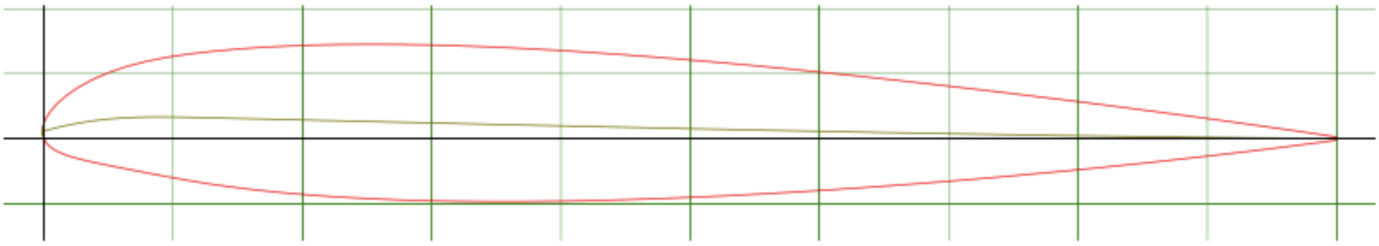


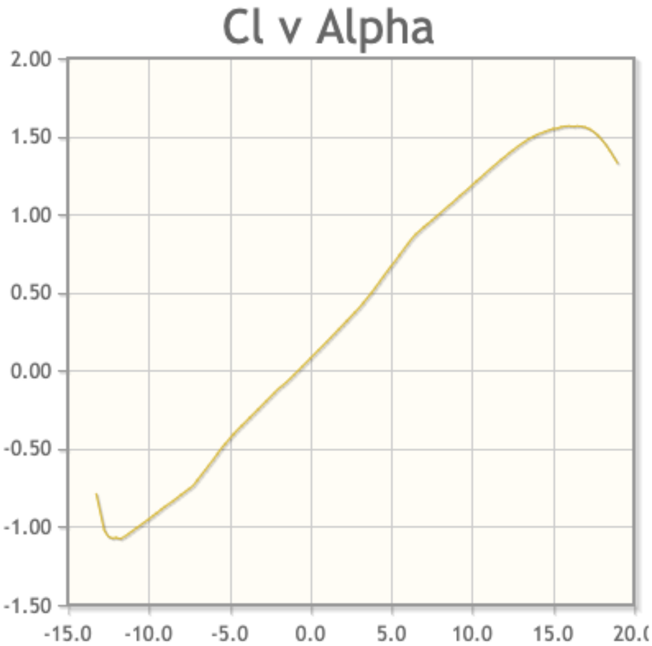
Cl max = 1.4511 at AoA = 16.5

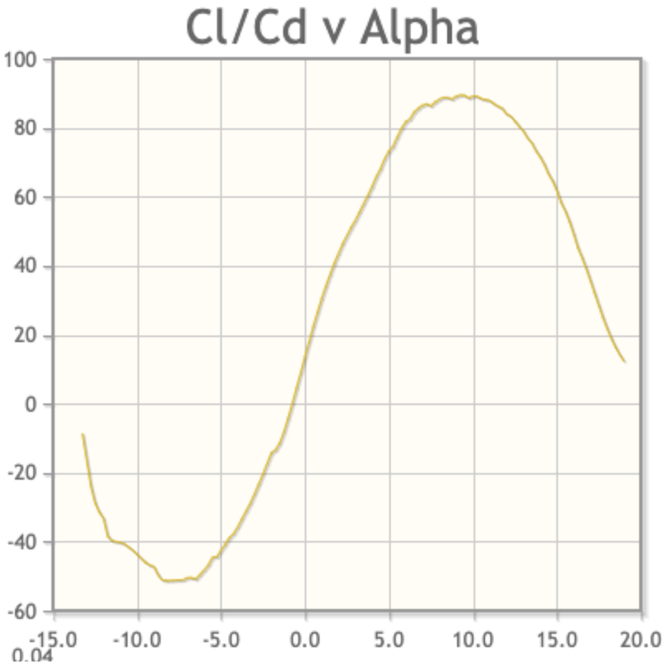




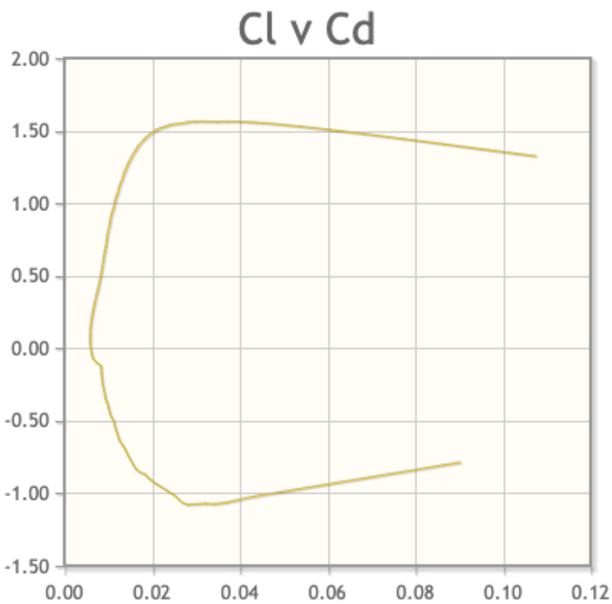
c). NACA 22112
Max (t/c) = 12% at 29.5% Chord
Max cambre = 0.8% at 9.6% Chord

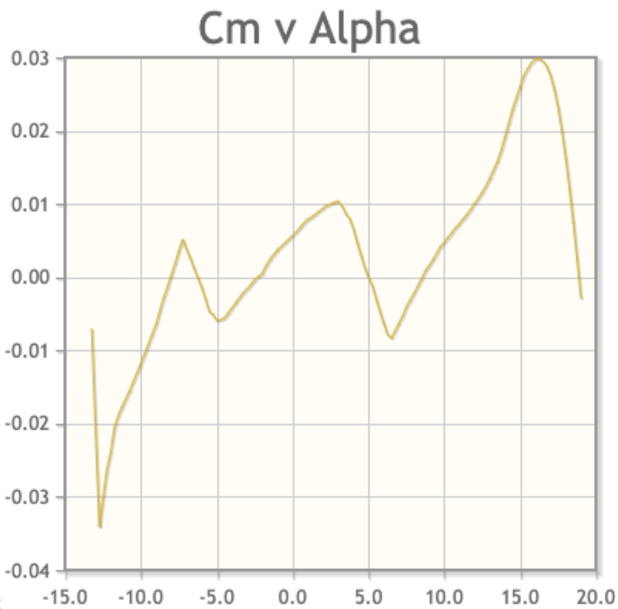




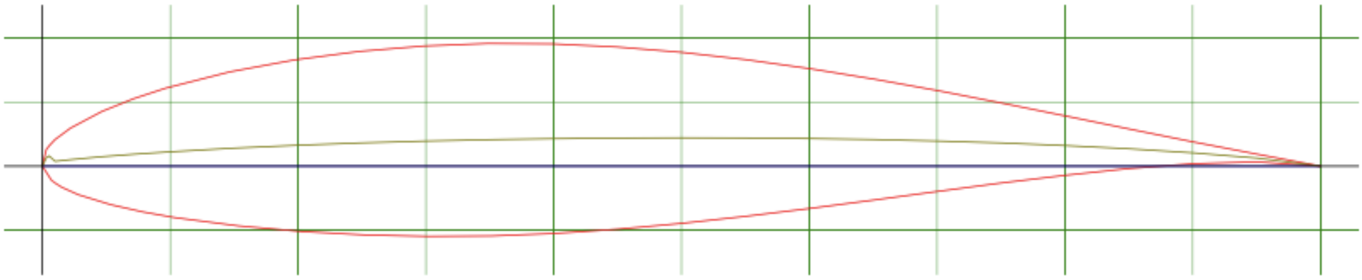


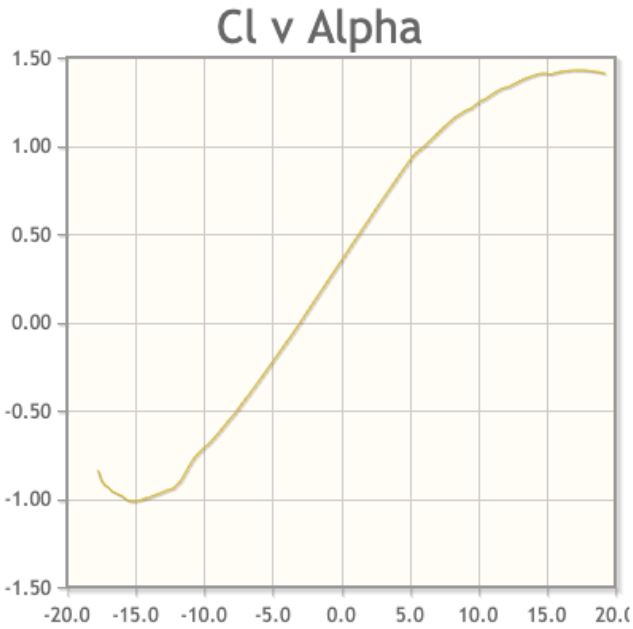
Cl max = 1.5576 at AoA = 16.25

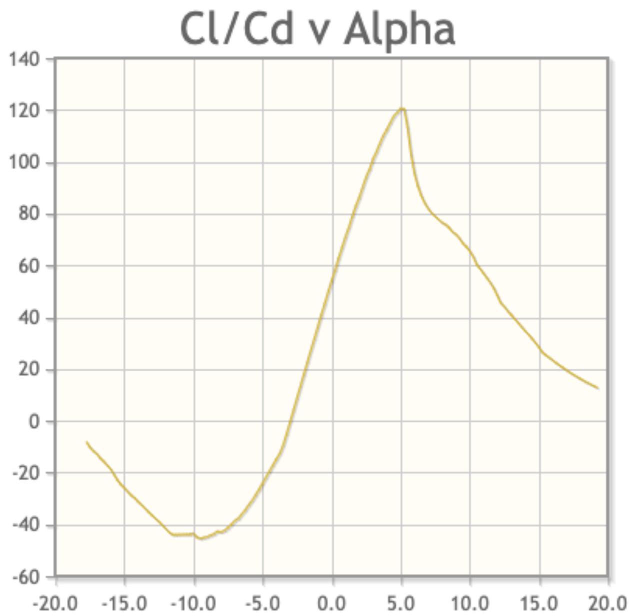




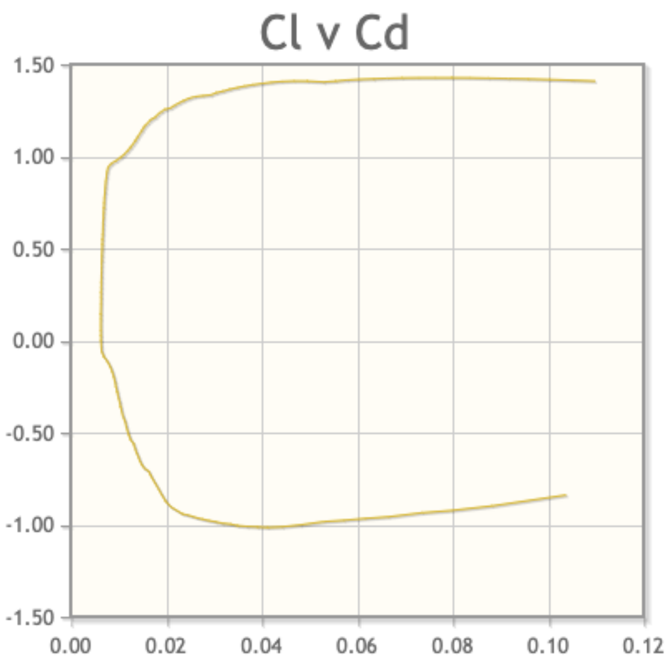
d). NACA 642415
Max (t/c) = 15% at 34.9% Chord
Max cambre = 0% at 0% Chord

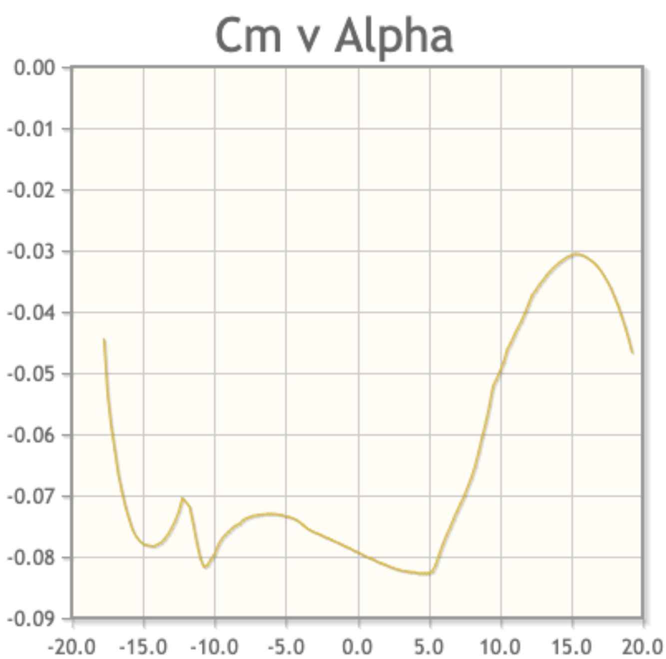






Cl max = 1.4935 at AoA = 17.5

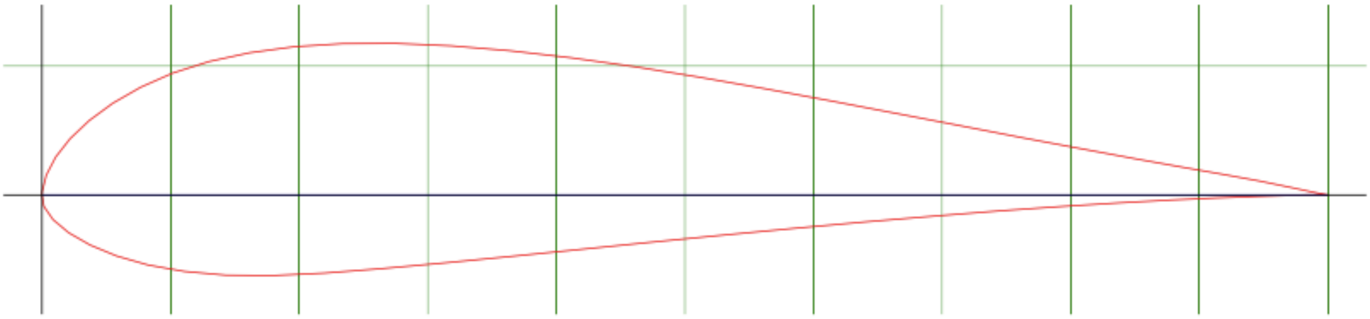


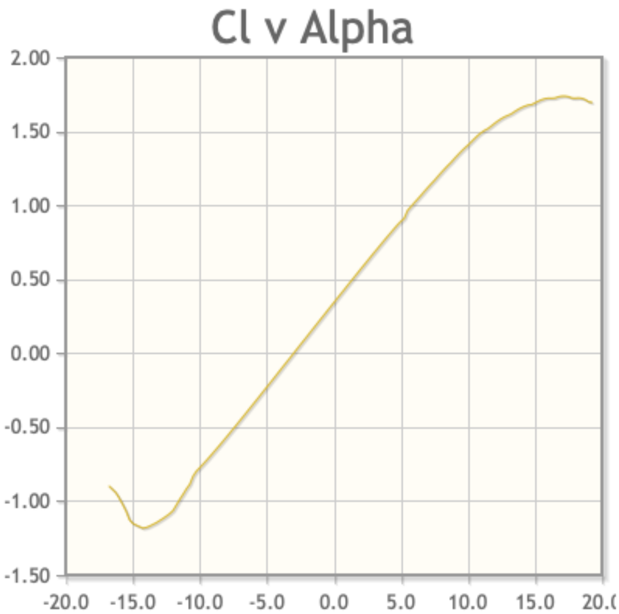


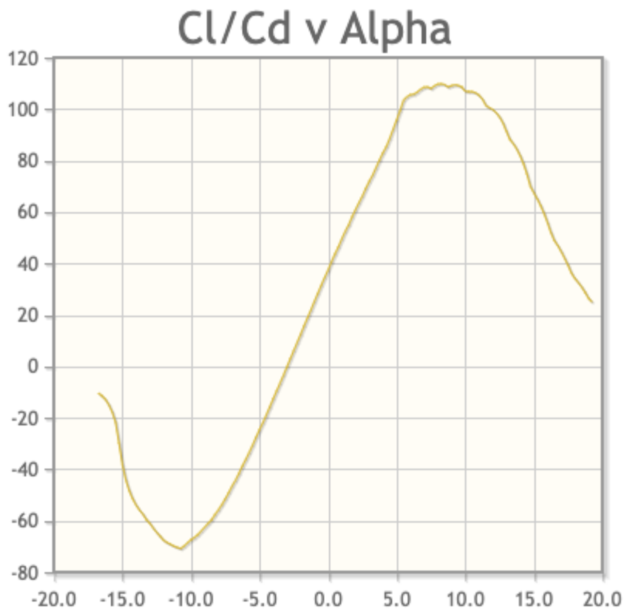
e). EPLER E1212:

Max thickness 17.7% at 23.4% chord.

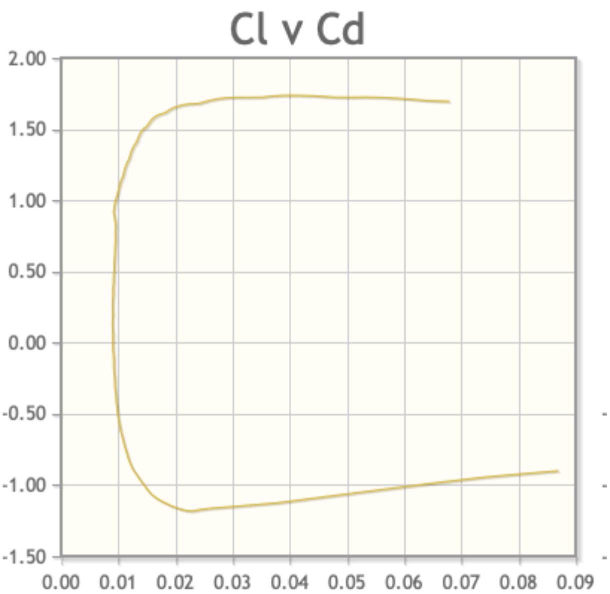
Max camber 3.2% at 36.4% chord

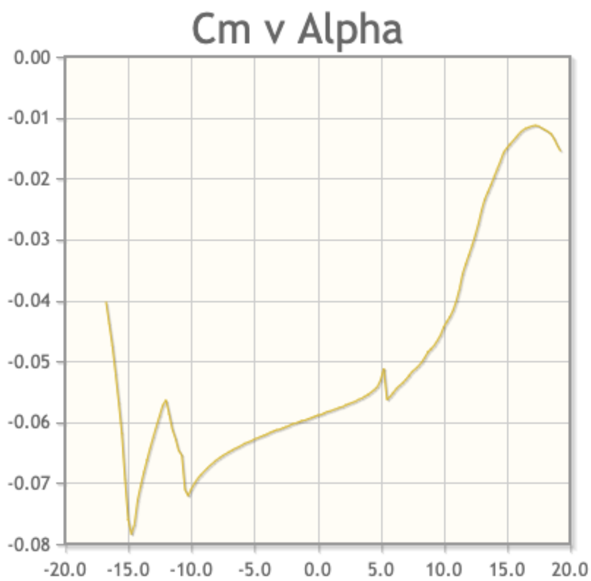






Cl max = 1.7368 at AoA = 17





WE CHOSE EPPLER E1212 (Because of its most preferable drag bucket).

4. **Wing Loading Calculation:**

$$C_{l\alpha}(3D) = \frac{C_{l\alpha}(2D)}{1 + \frac{C_{l\alpha}(2D)}{\pi e AR}} = \frac{5.89}{1 + \frac{5.89}{\pi(0.7)(8)}} = 4.412$$

$$C_{L0,3D} = 4.412 \times |zero\ lift\ angle|$$

The zero-lift angle is found to be -3 degrees (For EPPLER E1212). Hence

$$C_{L0,3D} = 0.231$$

We know, $C_l = C_{lo} + C_{l\alpha}\alpha$

For C_{lmax} , $\alpha = \alpha_{stall} = 17\ degrees$ = 0.297 radians

$$C_{lmax}(3D) = 1.54$$

A. $\boldsymbol{V_{stall}}$ **:**

C_{lmax}=1.54

$$\frac{W}{S} = 0.5\rho v^2 C_{lmax}$$

$\boldsymbol{V_{stall}}$ (ms^{-1})	**Altitude**	**Density** (kgm^{-3})	**Cl**	**Wing Loading** (kgm^{-2})
35	seal level	1.225	1.54	117.78606
35	8000ft	0.962	1.54	92.4981142
40	seal level	1.225	1.54	153.843017
40	8000ft	0.962	1.54	120.813863

B. $\boldsymbol{V_{cruise}}$ **@ 8000ft :**

$$\frac{W}{S} = 0.5\rho v^2 C_l$$

Here: C_l=0.31 (corresponding to the drag polar)

We know, $C_l = C_{lo} + C_{l\alpha}\alpha$

Substituting for C_l=0.31 and solving the above equation, we get

$$\alpha = 0.897\ degrees$$

V_{cruise} (ms^{-1})	Density (kgm^{-3})	A.R	Cl	Wing Loading (kgm^{-2})
70	0.961	8	0.31	74.40158002
75	0.961	8	0.31	85.40997706
80	0.961	8	0.31	97.1775739

C. V_{cruise} (corresponding to maximum endurance) :

$$\frac{W}{S} = 0.5\rho v^2 C_l$$

$$C_l = \sqrt{3\pi eARC_{Do}}$$

V_{cruise} (ms^{-1})	Density (kgm^{-3})	Altitude	A.R	Cl	Wing Loading (kgm^{-2})
70	1.225	sea level	8	1.027	314.1981397
70	0.961	8000ft	8	1.027	246.4852345
75	1.225	sea level	8	1.027	360.6866399
75	0.961	8000ft	8	1.027	282.9549885
80	1.225	sea level	8	1.027	410.3812436
80	0.961	8000ft	8	1.027	321.9398981

D. V_{cruise} (corresponding to maximum range) :

$$\frac{W}{S} = 0.5\rho v^2 C_l$$

$$C_l = \sqrt{\pi eARC_{Do}}$$

V_{cruise} (ms^{-1})	Density (kgm^{-3})	Altitude	A.R	Cl	Wing Loading (kgm^{-2})
70	1.225	sea level	8	0.593	181.4211264
70	0.961	8000ft	8	0.593	142.3230224
75	1.225	sea level	8	0.593	208.2640482
75	0.961	8000ft	8	0.593	163.3810206
80	1.225	sea level	8	0.593	236.9582059
80	0.961	8000ft	8	0.593	185.8912946

5. Tail Setting Angle and Neutral Point:

Formulae:

$$\frac{N_p}{\bar{c}} = \frac{X_{ac,w}}{\bar{c}} + \eta V_H \frac{C_{L,\alpha t}}{C_{L,\alpha w}}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right)$$

$$C_{m0} = C_{m0,w} + \eta V_H C_{L,\alpha t}\,(\epsilon_0 + i_w - i_t)$$

$$C_{m0,w} = C_{m,ac} + C_{L0}(x_{CG} - x_{ac})$$

Where $\bar{c}$ is the mean chord.
We set static margin as 13% of chord of wing.
Setting V_H = 0.7, $\eta = 1$, AR of Tail = 4 and AR of wing = 8

For EPPLER E1212 airfoil we have,

$$C_{L\alpha,3D} = 4.412$$

Now for cruise we have,

$$C_{L,trim} = \frac{2W}{\rho V^2 S} = \frac{2 \times 1661.35 \times 9.8}{0.961 \times 22.33 \times 70 \times 70} = 0.31$$

Now the lift coefficient at zero angle of attack is given as

$$C_{L0,3D} = 4.412 \times |zero\ lift\ angle|$$

The zero-lift angle is found to be -3 degrees (For EPPLER E1212). Hence

$$C_{L0,3D} = 0.231$$

Now $C_{m0} = 0.13 \times (0.31 - 0.231) = 0.0103$

For EPPLER E1212 we have, $C_{m,ac} = 0.058$

For tail NACA 0012 airfoil is chosen. For this airfoil $C_{l\alpha}(2D) = 6.28$

$$C_{L\alpha,3D} = \frac{C_{l\alpha}(2D)}{1 + \frac{C_{l\alpha}(2D)}{\pi e AR}} = 3.66$$

Also,

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{C_{L\alpha,3D(wing)}}{\pi (AR)_w} = \frac{4.412}{3.14 \times 8} = 0.351$$

Putting these values, we have

$$\frac{N_p}{\bar{c}} = 0.25 + 1 \times 0.7 \times \frac{3.66}{4.412}(1 - 0.351) = 0.63$$

To have static margin as 13% we must have

$$\frac{X_{CG}}{\bar{c}} = 0.5$$

Now , $C_{m0,w} = C_{m,ac} + C_{L0}(x_{CG} - x_{ac}) = -0.058 + 0.231 \times 0.25 = -2.5 \times 10^{-4}$

Setting

$$i_w = 0$$

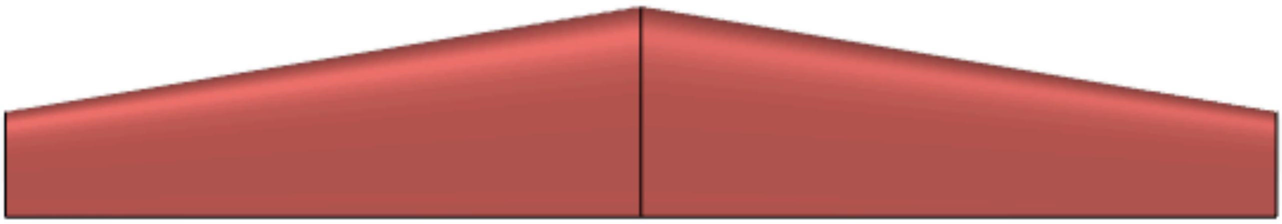
$$\epsilon_0 = \frac{2C_{L0}}{\pi(AR)_w} = \frac{0.231}{8\times3.14} = 0.0184$$

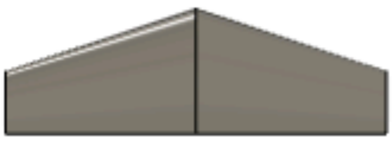
Putting these values, we have,

$$i_t = 0.0184 - \frac{0.0103 + 2.5 \times 10^{-4}}{1 \times 0.7 \times 3.66} = 0.81\ degrees$$

6. <u>Wing Planform:</u>

Taper Ratio = 0.5 for both wing and tail.





7. Wing Sizing:

(a) For cruise, we have

$$\frac{W}{S} = 74.402\ kgm^{-2}$$

$$W_{cruise} = 1661.35\ kg$$

Which gives, $S = 22.33\ m^2$

The aspect ratio of the wing is set to be 8.
So, the span can be found as,

$$b = \sqrt{22.33 \times 8} = 13.37\ m$$

We set the taper ratio $\tau = 0.5$

The root chord and tip chord are then given as,

$$c_{root} = \frac{2S}{(1+\tau)b} = 2.22\ m$$

$$c_{tip} = 1.11\ m$$

The mean aerodynamic chord is

$$\bar{c} = \frac{2}{3} c_{root} \frac{1+\tau+\tau^2}{1+\tau} = 1.73m$$

(b) Flap chord = 0.25× local chord width
Flap Span = 0.6×(b/2) = 4.011 m
Aileron chord = 0.25×local chord
Aileron span = 0.4×(b/2) = 2.674 m

8. Horizontal Tail Sizing:

(a) We have V_H = 0.7 and AR of Tail = 4
Now,

$$V_H = \frac{S_t l_t}{S_w c_w} = 0.7$$

The distance between the aerodynamic center of the tail and the CG,

l_t= 0.49×13.37 = 6.5513 m

Putting these values,

$$S_t = \frac{0.7 \times 22.33 \times 1.67}{6.5513} = 3.984\ m^2$$

Tail span,

$$b_t = \sqrt{3.984 \times 4} = 4m$$

Similarly, root chord and tip chord are given as

$$c_{root} = \frac{2S}{(1+\tau)b} = 1.328\ m$$

(Assuming taper ratio $\tau = 0.5$) $$c_{tip} = 0.664\ m$$

Mean aerodynamic chord is,

$$\bar{c} = \frac{2}{3} c_{root} \frac{1 + \tau + \tau^2}{1 + \tau} = 1.03\ m$$

(b) Assume that the elevator extends the whole tail span and is rectangular in shape. Elevator surface area,

$$S_e = 0.35 \times 3.984 = 1.3944\ m$$

Now, elevator chord,

$$c_e = \frac{1.3944}{4} = 0.3486$$

9. Vertical Tail Sizing:

(a) Vertical tail volume ratio is set to be

$$V_V = \frac{S_V l_t}{S_w b_w} = 0.03$$

From the above formula, we have

$$S_V = \frac{0.03 \times 22.33}{0.49} = 1.367\ m^2$$

Assuming aspect ratio of vertical tail as 1.7

$$\frac{{b_{tv}}^2}{S_v} = 1.7$$

$$b_{tv} = 1.524\ m$$

From area of trapezium and assuming taper ratio, $\tau = 0.5$

$$1.367 = \frac{1}{2}\left(c_{root} + c_{tip}\right) b_{tv} = \frac{1}{2}\left(c_{root} + \frac{c_{root}}{2}\right) 1.524$$

$$c_{root} = 1.196 \quad and \quad c_{tip} = 0.598$$

(b) Assuming rudder to be 50% of the vertical tail, the surface area for the rudder is

$$S_r = 0.6835\, m^2$$

Assume that the rudder extends the whole vertical tail span and is rectangular in shape.
Now, rudder chord,

$$c_r = \frac{0.6835}{1.524} = 0.448\, m$$

10. Power Requirement

Given that the Rate of Climb = 5m/s at sea level
We know that,

$$ROC = \frac{excess\ power}{W}$$

$$\Rightarrow TV - DV = ROC \times W$$

$$\Rightarrow P_A - P_R = 5 \times 1661.35 \times 9.8$$

$$= 81.406\, KW$$

The power available is to be calculated as if the flight climbs from the cruise condition to ensure the maximum value of the power available and choose the engine as per the requirement. So, the weight is taken to be the cruise weight here.
Now,

$$P_R = D.V = \frac{1}{2}\rho v^2 S(C_{D0} + kC_L^2)V$$

For cruise we have, $C_L = 0.31$, and

$$k = \frac{1}{\pi e(AR)} = 0.0568$$

We know,

$$C_D = C_{D0} + kC_L^2$$

$$C_D = 0.020+0.0568\times0.31\times0.31 = 0.0255$$

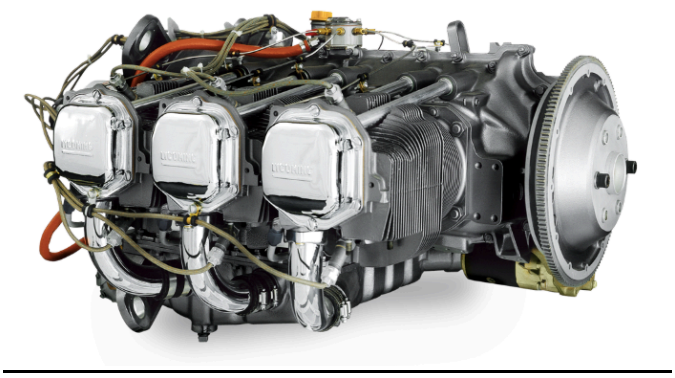
So, we have the available power as,

$$P_A = 81.406 + 0.5 \times 0.961 \times 70 \times 70 \times 22.33 \times 0.0255 \times 70$$
$$P_A = 175.25\, KW$$

Reserving 30% power as emergency,

Engine power = 1.3×175.25 = 227.825 KW = 305.52 hp

11. Engine Selection:



TIO-540-AJ1A

MODEL TIO-540-AJ1A	**FUEL PUMP** AN Drive Vane	**SPARK PLUG** RHB37E	**TURBO CHARGER** 46P22250
VOLTAGE 24	**AIR MFG** CESSNA	**AIR MODEL** T-206	**AIR TYPE** STATIONAIR, TURBO
HP 310	**VARB CONTROLLER** 48B22314	**MOUNT PREP** BED	**MAGNETO** 6361 (2) Pressurized
STARTER RING GEAR 31M22246	**THERMO BYPASS** 53E22144	**VACUUM PUMP** AN	**WASTE GATE** LW-18861
PROP GOV L FRONT	**REMARKS** SAME AS 10269 EXCEPT INCLUDES TWO FUEL DRAIN MANIFOLD ASSY'S P/ N/ 63M26585 AS OPPOSED TO ALL PREVIOUS CONFIGUATIONS THAT INCLUDED ONE 63M22386 AND ONE 63M22564 FUEL DRAIN MANIFOLD ASSY'S	**OIL FILTER** Y	

12. Fuel Tank Sizing :

$$W_o = 1738.814\ kgs$$

For Eppler E1212 $\left(\frac{t}{c}\right)_{max} = 17.7\ \%\ of\ MAC = 0.306\ m$

For our aircraft,

$$\frac{W_f}{W_o} = 0.137$$

$$\Rightarrow W_f = 0.137 \times 1738.814 = 238.2175\ kgs$$

$$\rho_f = 804\ kgm^{-3} \qquad \text{(Fuel Type -> Jet A-1)}$$

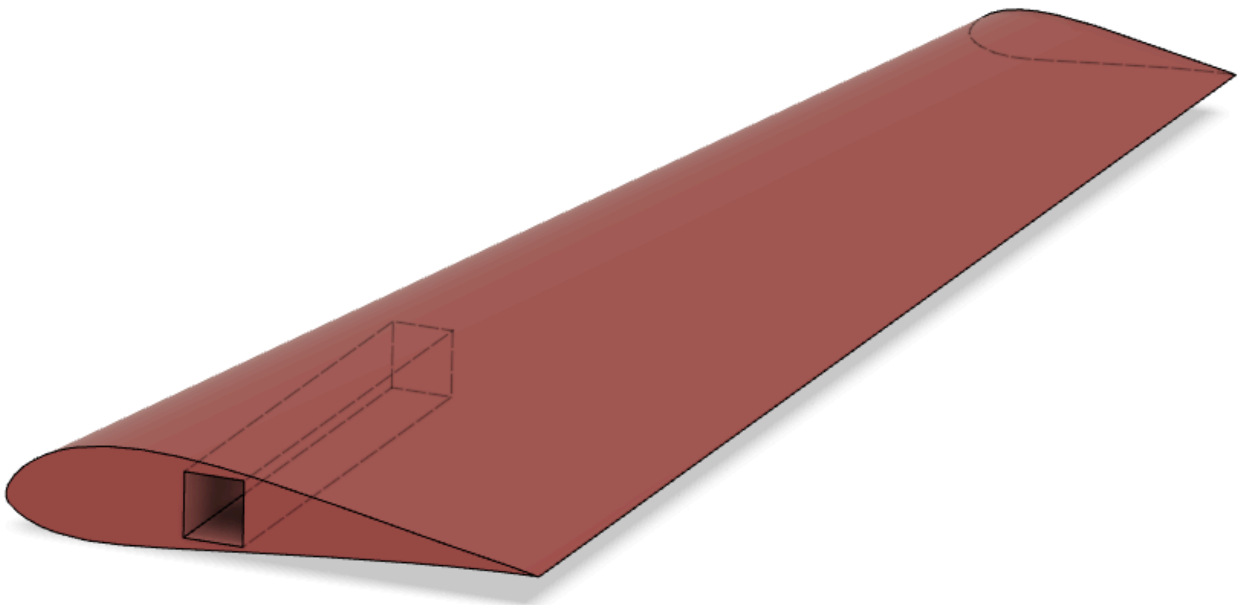
$$\Rightarrow Volume\ of\ fuel = V_f = \frac{238.2175}{804} = 0.2963 m^3$$

To maintain the CG conserved we'll distribute the fuel equally, half in the right side wing and half in the left side wing. We'll construct two tanks of equal size and place them in the wings.

$$Volume\ of\ one\ fuel\ tank = V_{tank} = \frac{0.2963}{2} = 0.14815\ m^3$$

After iterating over several dimensions of tanks so that it would fit inside the wing, we finally took the dimensions of tank as

$$l \times b \times h = 2m \times 0.277m \times 0.267m\ .$$



13. <u>Lift off distance calculation:</u>

$$S_{\boldsymbol{LO}} = \frac{{V_{LO}}^2\left(\frac{W}{g}\right)}{2\{T - [D + \mu(W - L)]\}_{avg}}$$

$$V_{LO} = 1.2(V_{stall})$$

$$V_{LO} = 1.2 \times \sqrt{\frac{2 \times 1738.814 \times 9.8}{1.225 \times 22.33 \times 1.57}} = 34.48\ \ ms^{-1}$$

Average is calculated at 0.7 (V_{LO})

Where, $\phi = \frac{\left(16\frac{h}{b}\right)^2}{1+\left(16\frac{h}{b}\right)^2}$ h=height of wing from ground

Taking, h=2m

$$\phi = \frac{\left(16\frac{2}{13.37}\right)^2}{1+\left(16\frac{2}{13.37}\right)^2} = 0.852$$

Now, thrust is assumed to be given by 60% of total power required for cruise,

$$T = 0.6 \times \frac{175250}{34.48 \times 0.7} = 4356\ N$$

$$D = 0.5\rho v^2 S(C_{D_o} + \phi K C_L^2)$$

Assume $C_L = 0$ for ground roll.

$$D = 0.5(1.225)(0.7 \times 34.48)^2(22.33)(0.02)$$

$$D = 159.35\ N$$

$$L = 0$$

Now,

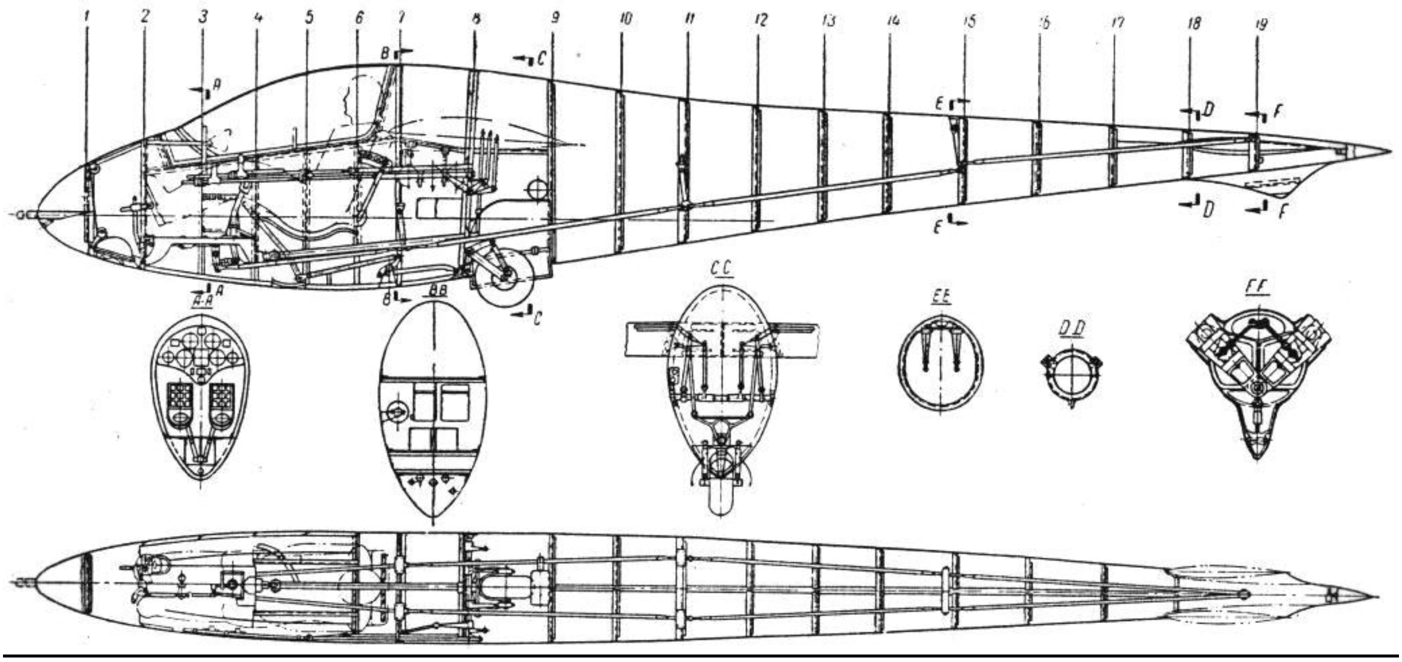
$$S_{\boldsymbol{LO}} = \frac{{V_{LO}}^2\left(\frac{W}{g}\right)}{2\{T - [D + \mu(W - L)]\}_{avg}}$$

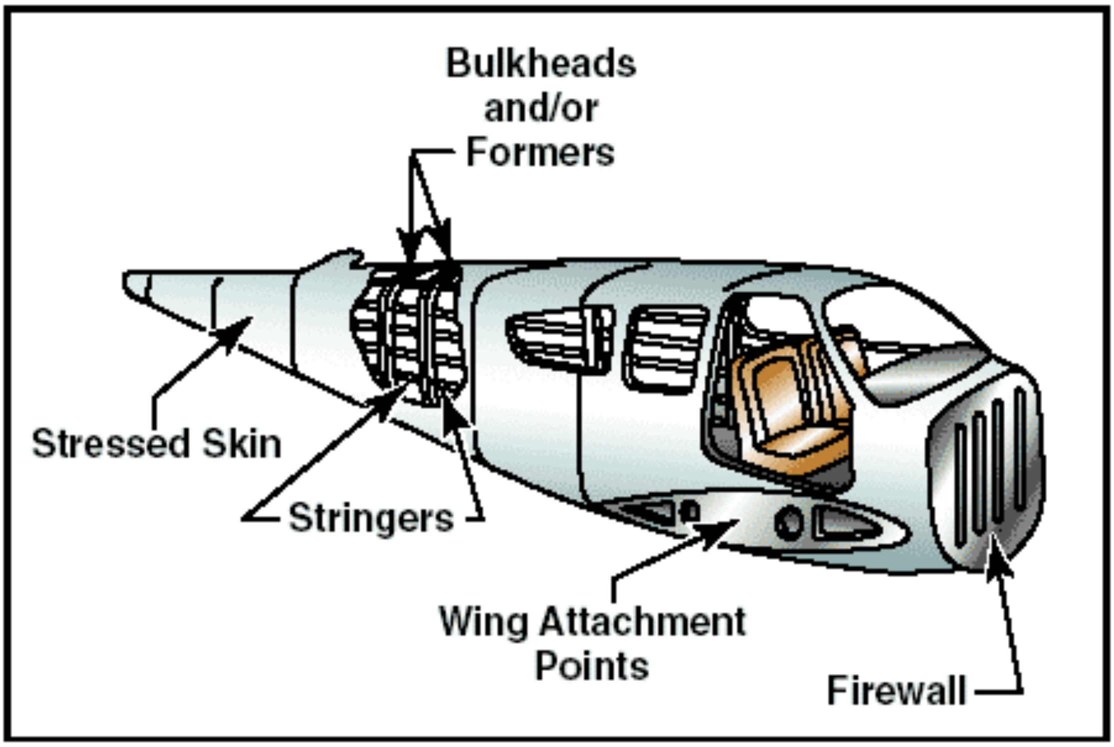
$$S_{\boldsymbol{LO}} = \frac{(34.48)^2 \times 1738.814}{2\{4356 - [159.35 + 0.04(1738.814 \times 9.8)]\}_{avg}}$$

$$S_{\boldsymbol{LO}} = 294.05\ m$$

14. Fuselage Shape :

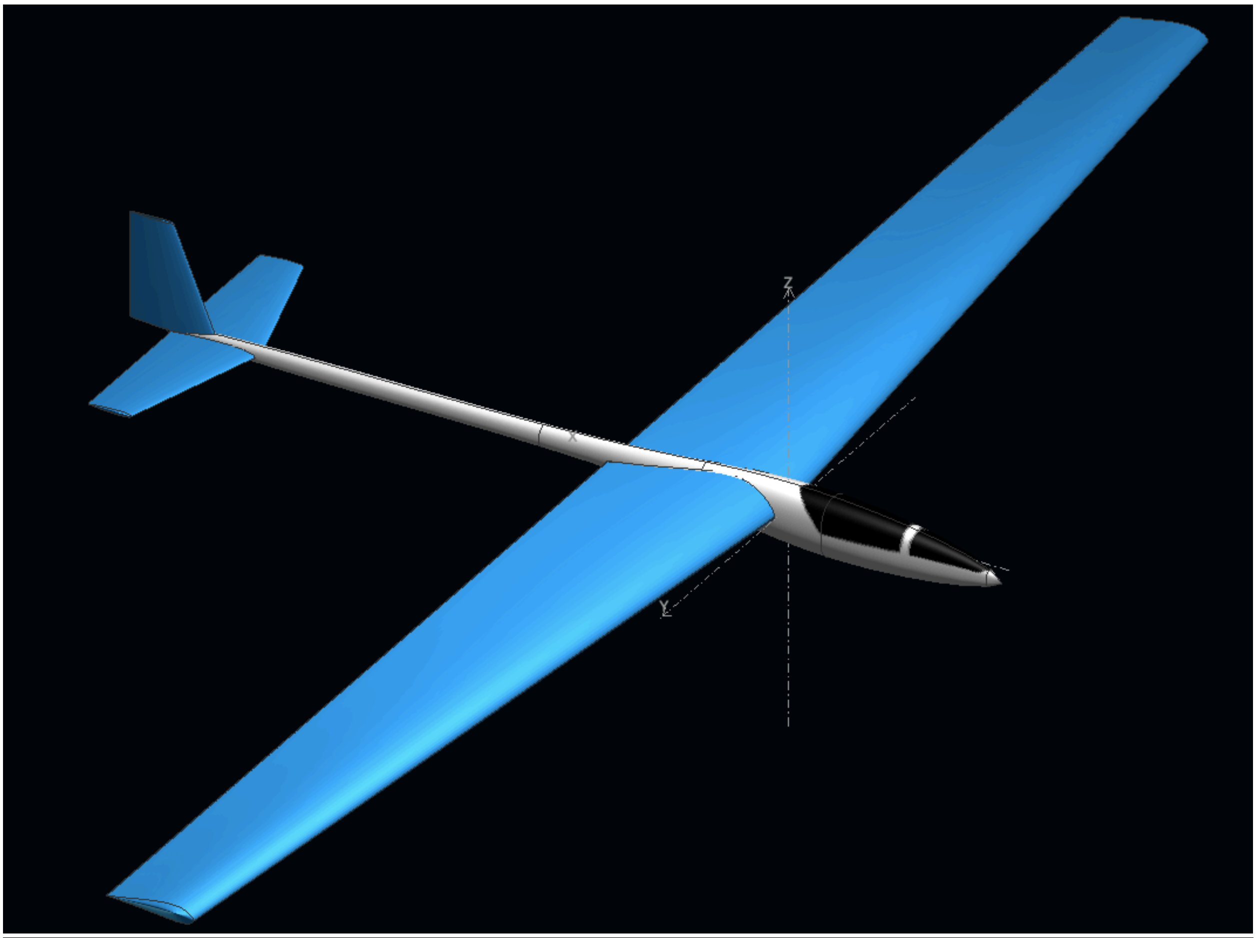
Semi-Monocoque Fuselage.



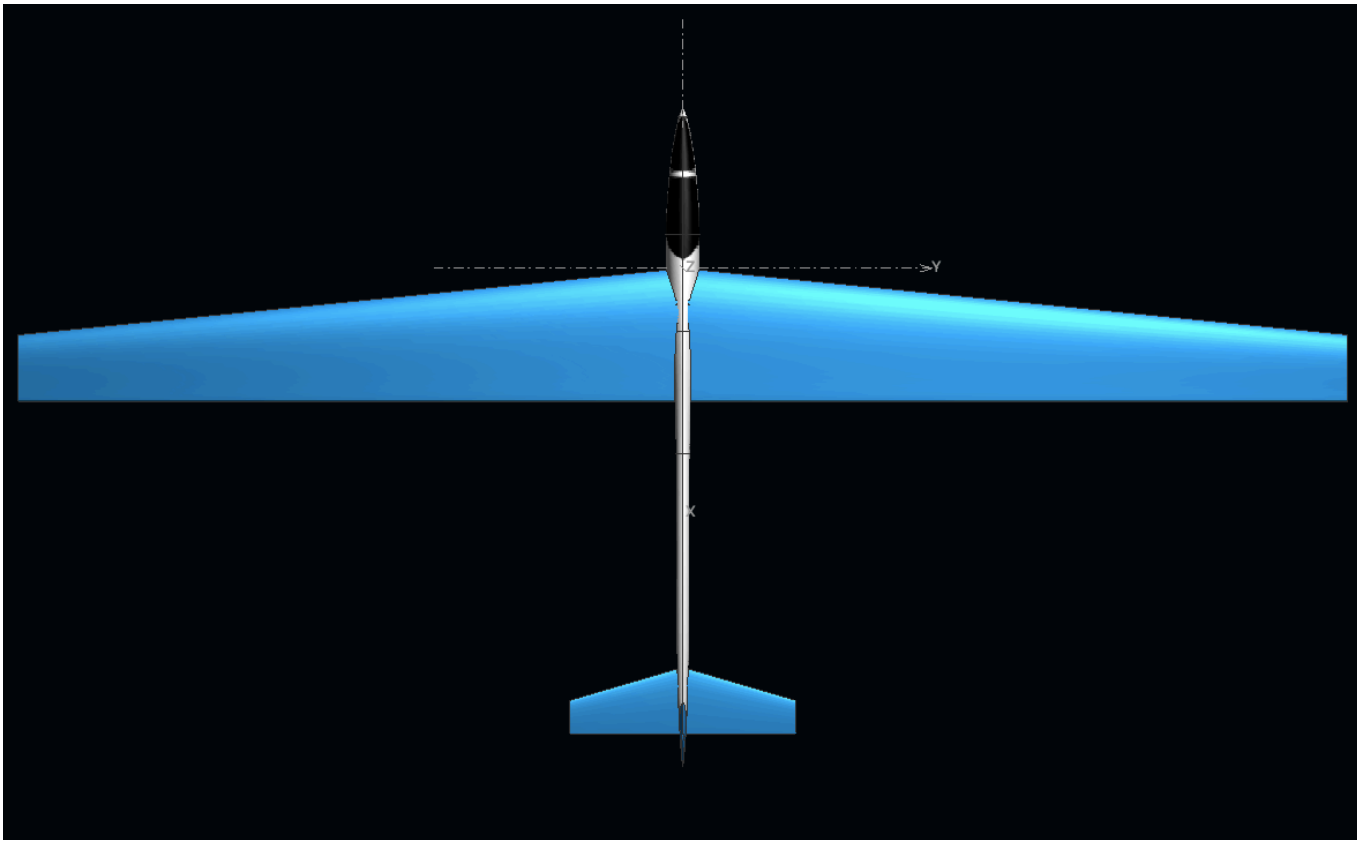


15.**Layout Diagram :**

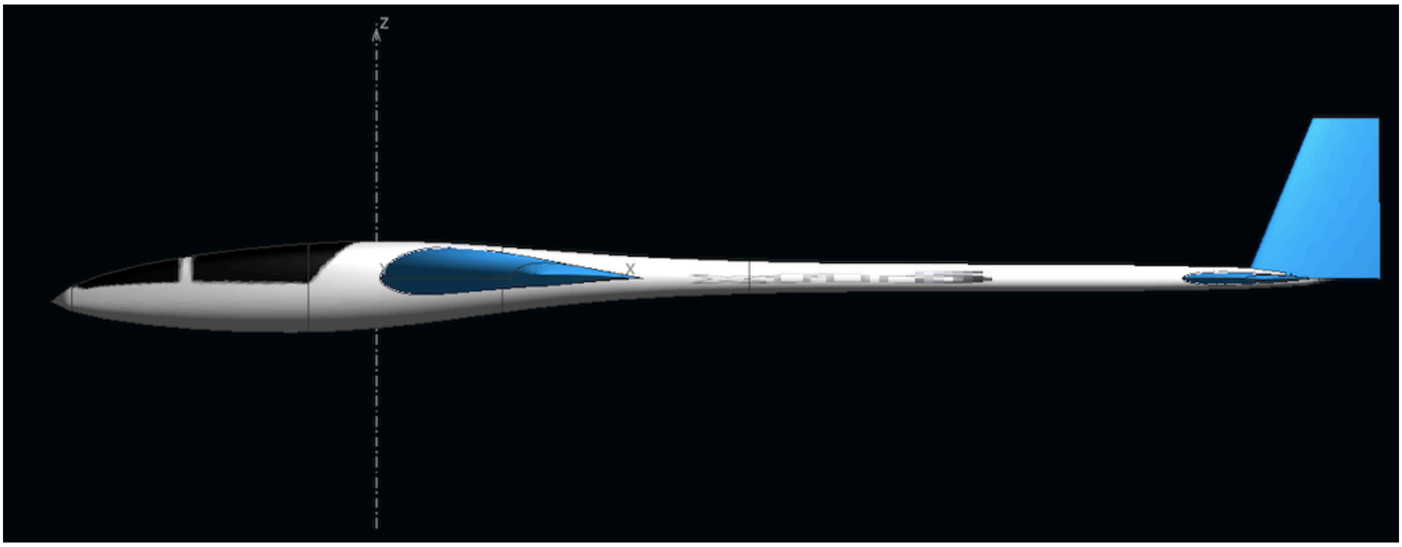
3D VIEW :



TOP VIEW :



SIDE VIEW :



16. Estimation of Aircraft Derivatives

• C_{L_u}, C_{D_u} *and* C_{m_u}

The stability derivatives C_{Lu}, C_{Du} and C_{Mu} are changes in lift, drag and pitching moment coefficients, respectively, with airspeed. These changes at low speeds are really Reynolds number effects and an usually be **considered to be zero.**

• $C_{L_\alpha}, C_{D_\alpha}$ *and* C_{m_α}

The stability derivative $C_{L\alpha}$ is the change in lift coefficient with angle of attack and is commonly known as the lift curve slope. The lift curve slope per degree is written in terms of 2-D airfoil lift curve slope, $(C_{L\alpha})_\infty$ as

$$\frac{dC_L}{d\alpha} = \frac{\left(\frac{dC_L}{d\alpha}\right)_\infty}{1 + \left(\frac{dC_L}{d\alpha}\right)_\infty \frac{57.3}{\pi e_1 AR}}$$

Typical values for $(C_{L\alpha})_\infty$ per degree range from about 0.110 for thin airfoil to about 0.115 for thick airfoil. The tail contribution is written as

$$\left(\frac{dC_L}{d\alpha}\right)_{tail} = C_{L_{\alpha_t}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{S_t}{S_w}\eta_t$$

Total $(C_{L\alpha})$ is written as

$$\left(\frac{dC_L}{d\alpha}\right)_{total} = C_{L_{\alpha_w}} + C_{L_{\alpha_{fuse}}} + C_{L_{\alpha_{tail}}}\left(1 - \frac{d\varepsilon}{d\alpha}\right)\frac{S_t}{S_w}\eta_t$$

$$= 4.412 + 0 + 3.66(1 - 0.351)\left(\frac{3.984}{22.33}\right) \times 1$$

$$= 4.835795547 \textit{ per radian}$$

Typical values of ($C_{L\alpha}$) for light airplanes fall in the range of 4.0 to 7.0 per radian, depending on the type of wing used.

The stability derivative $C_{D\alpha}$ is the change in drag coefficient with varying angle of attack. We have,

$$C_{D\alpha_{wing}} = \frac{dC_{d_0}}{d\alpha} + \frac{2C_L}{\pi eAR} C_{L\alpha}$$

For cruise, we have

$$= 0 + \left(\frac{2 \times 0.31}{\pi \times 0.7 \times 8}\right) 4.84$$
$$C_{D\alpha_{wing}} = 0.170420455 \; per\ radian$$

For small angles of attack, the first term vanishes; however, for flight modes other than cruise, it could have significant value. The wing $C_{D\alpha}$, in most cases, serves as a good approximation of the total $C_{D\alpha}$.
The value of $C_{D\alpha}$ calculated for Cessna 182 is 0.126 per radian.

$C_{M\alpha}$ is perhaps the most important derivative related to longitudinal stability and control, since it establishes the natural frequency of the short period mode and is a major factor in determining the response of airframe to elevator motions and gusts. Usually, a large negative value is desired (-0.5 to -1.0 for light airplanes), but if it is too large, the required elevator effectiveness may become unreasonably high. We have,

$$C_{m_\alpha} = \frac{dC_m}{dC_L} \times C_{L_\alpha} = -0.13 \times 4.836 = -0.629 \; per\ radian$$

A typical value of $C_{M\alpha}$ is the one associated with the Cessna 182, that is -0.885. (for c.g. at 26% of mean aerodynamic chord). Calculated earlier.

- ## $C_{L_{\dot\alpha}}, C_{D_{\dot\alpha}}$ and $C_{m_{\dot\alpha}}$

This derivative is the change in lift coefficient with the rate of change of angle of attack. It arises from a type of 'plunging' motion along the z-axis, during which the angle of pitch, θ, remains zero. We have,

$$C_{L_{\dot\alpha}} = \frac{\partial C_L}{\partial\left(\frac{c\dot\alpha}{2U}\right)} = 2C_{L_{\alpha_t}} \varepsilon_\alpha \frac{\bar{\ell}_t}{c} \frac{S_t}{S_w} \eta_t$$

where l_t` is the distance between the wing quarter chord and the horizontal tail quarter chord.

$$C_{L_{\dot\alpha}} = 2 \times 3.66 \times 0.351 \times \left(\frac{6.1188}{1.73}\right)\left(\frac{3.984}{22.33}\right) \times 1$$

$$C_{L_{\dot\alpha}} = 1.621 \; per\ radian$$

A typical range of values for **$C_{L\dot\alpha}$ for light airplanes is 1.5 to 3.0.** For Cessna 182, it is found to be 1.74.
The stability derivative $C_{D\dot\alpha}$ is the change in drag coefficient with rate of change of angle of attack. For light airplanes, the drag variation due to both the effects are negligible. Consequently, **$C_{D\dot\alpha}$ is taken to be zero.**

The derivative $C_{M\dot{\alpha}}$ is the change in pitching moment w.r.t time rate change if angle of attack. We have,

$$C_{m_{\dot{\alpha}}} = \frac{\partial C_m}{\partial\left(\frac{2U}{c}\dot{\alpha}\right)} = -2C_{L_{\alpha_t}}\ \varepsilon_\alpha \left(\frac{\ell_t}{c}\right)\left(\frac{\ell_t'}{c}\right)\frac{S_t}{S_w}\eta_t$$

$$C_{M_{\dot{\alpha}}} = -2 \times 3.66 \times 0.351 \times \left(\frac{6.1188}{1.73}\right)\left(\frac{6.5513}{1.73}\right)\left(\frac{3.984}{22.33}\right) \times 1$$

$$C_{M_{\dot{\alpha}}} = -5.51002768\ per\ radian$$

A typical range of $C_{M\dot{\alpha}}$ for a light airplane is -3.0 to -7.0. For the Cessna 182, it is calculated to be -5.24.

- ## C_{L_q}, C_{D_q} *and* C_{m_q}

The stability derivative C_{Lq} represents the change in airplane lift with the varying pitching velocity while the angle of attack of the airplane as a whole remains constant. Contributions are made by both wing and horizontal tail, but the tail has an important contribution. We have,

$$C_{L_q} = \left.\frac{\partial C_L}{\partial\left(\frac{cq}{2U}\right)}\right|_{wing} + \left.\frac{\partial C_L}{\partial\left(\frac{cq}{2U}\right)}\right|_{tail} = 2\frac{x'}{c}C_{L_\alpha} + 2\frac{\ell_t}{c}C_{L_{\alpha_t}}\frac{S_t}{S_w}\eta_t$$

$$= 0 + \left\{2 \times \left(\frac{6.1188}{1.73}\right) \times 3.66 \times \left(\frac{3.984}{22.33}\right) \times 1\right\}$$

$$C_{L_q} = 4.619147629 \quad per\ radian$$

For Cessna 182, C_{Lq} is calculated to be 3.9 after neglecting the wing contribution because it has the c.g. near the quarter chord.

The stability derivative C_{Dq} is the change in drag of the airplane with varying pitching velocity while the angle of attack as a whole remains constant. It has contributions from both the wing and the fuselage but both contributions are very small. It is taken to be zero because of its small magnitude and unimportant nature in analysing flight dynamics.

$$C_{D_q} = 0$$

The stability derivative C_{Mq} is the change in pitching moment due to change in pitching velocity. The wing contribution to C_{Mq} either opposes or increases the pitching motion, depending on the c.g. location; however, this is relatively insignificant compared to the tail contribution. We have,

$$C_{m_q} = -\frac{2x'}{c^2}|x'|\,C_{L_\alpha} - \frac{2\ell_t^2}{c^2}C_{L_{\alpha_t}}\frac{S_t}{S_w}\eta_t$$

$$C_{m_q} = -\frac{2(1.73 \times 0.25)^2}{1.73^2}(4.418) - \frac{2(6.5513)^2}{1.73^2}(3.66)\left(\frac{3.984}{22.33}\right) \times 1$$

$$C_{m_q} = -19.2808\ per\ radian$$

where |x`| is the magnitude of distance from c.g. to wing quarter chord.

The value of C_{Mq} calculated for Cessna 182 is -12.43.

- $C_{L_{\delta_E}}, C_{D_{\delta_E}}\ and\ C_{m_{\delta_E}}$

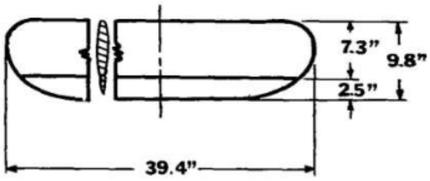
The change in lift coefficient due to elevator deflection is the stability derivative $C_{L\delta e}$. For light airplanes with relatively large tails, **$C_{L\delta e}$ may be taken on values between 0.3 to 0.5 per radian.** For most light airplanes we have,

$$C_{l_{\delta e}} = \frac{S_t}{S_w}|\eta|C_{l_{\alpha,t}}\tau$$

$$C_{l_{\delta e}} = 0.326\ per\ radian$$

The value calculated for Cessna 182 is 0.427.

The change in drag coefficient due to a change in elevator angle is the control surface stability derivative $C_{D\delta e}$.



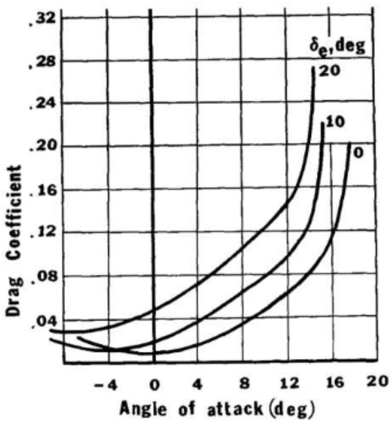


Figure 20. Drag coefficient against angle of attack at various elevator deflections for tail surface 3.

The numerical value of $C_{D_{\delta e}}$ can be taken from the plots of C_D Vs α for different elevator deflections for each tail surface; however, the values of $C_{D_{\delta e}}$ must be multiplied by $\frac{S_t}{S}$ so that $C_{D_{\delta e}}$ will be based on wing area.

The stability derivative $C_{M\delta e}$ is the change in pitching moment coefficient with changes in elevator deflection. $C_{M\delta e}$ is evaluated without allowing the airplane to rotate or any other parameters to change; thus, $C_{M\delta e}$ is really the moment produced by $C_{L\delta e}$. We have,

$$C_{m_{\delta e}} = -V_h\eta C_{l_{\alpha,t}}\tau$$

$$C_{m_{\delta e}} = -1.23\ per\ radian$$

For Cessna 182, it is calculated to be -1.28.

• C_{y_β}, C_{l_β} *and* C_{n_β}

The stability derivative $C_{y\beta}$ is the change in side force caused by a variation in sideslip angle. The major contribution to $C_{y\beta}$ comes from the vertical tail, with a smaller contribution from the fuselage and a nearly negligible contribution from the wing. **Typical value is around -0.31 per radian.**

$$(C_{y\beta})_{wing} = C_L^2 \frac{6 \tan \Lambda \sin \Lambda}{\pi\, AR(AR + 4 \cos \Lambda)}$$

For vertical tail $(C_{L_\alpha})_{2D} = 6.28$

AR = 1.7

$(C_{L_\alpha})_{3D} = 2.343$

$(C_{y_\beta})_{wing} = 0$ for zero sweep.

$$(C_{y\beta})_{tail} = -k\,(C_{L\alpha})_v \left(1 + \frac{\partial\sigma}{\partial\beta}\right) \frac{q_v}{q} \frac{S_v}{S_w} .$$

$$\left(1 + \frac{\partial\sigma}{\partial\beta}\right) \frac{q_v}{q} = .724 + 1.53 \left(\frac{S_v}{S_w}\right) + .4 \frac{z_w}{d} + .009\,(AR),$$

z_w = distance, parallel to z-axis, from wing root quarter-chord point to fuselage centerline,
d = maximum diameter of fuselage.

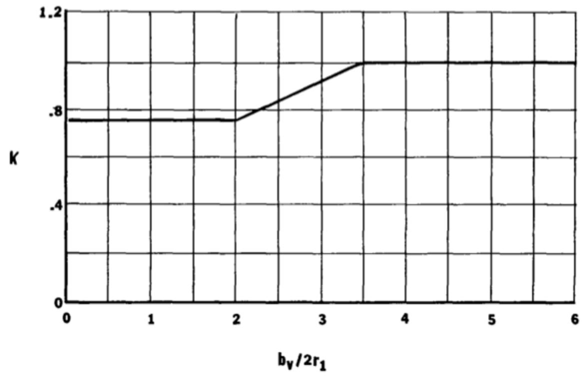


Figure 24. Values for k as a function of the ratio of vertical tail span to fuselage diameter in the tail region.

Value of k corresponding to $\frac{1.524}{0.7} = 2.177$ is 0.8 and $z_w = 0$.

Substituting values we get,

$$(C_{y_\beta}) = -0.1\ per\ radian$$

The stability derivative $C_{l\beta}$, normally referred to as "effective dihedral derivative" is the change in rolling moment coefficient caused by variation in sideslip angle. **Typical values of $C_{l\beta}$ range from -0.03 to -0.12 per radian.**

$$(C_{\ell_\beta})_{total} = (C_{\ell\beta})_w + (C_{\ell\beta})_v + (\Delta C_{\ell_\beta})_1 + (\Delta C_{\ell_\beta})_2 + (C_{\ell\beta})_{w,\Gamma=0}$$

$$(C_{\ell_\beta})_{w,\,\Gamma=0} = C_L\left[- \frac{k(.71\,\lambda + .29)}{AR\,\lambda} + .05\right] \text{ per radian}$$

k = 1.0 for straight wing tips

$$(C_{l_\beta})_{wing} = -0.0344\ per\ radian$$

$$(C_{\ell_\beta})_v = -a_v \frac{S_v}{S_w} \frac{z_v}{b_w} \eta_v = -(C_{n\beta})_v \frac{z_v}{\ell_v}$$

z_v = distance from the center of pressure of the vertical tail to the airplane's x-axis (positive for vertical tail above the x-axis).

$$z_v = 0.4(b_v) = 0.635 \text{ m}$$

$$(C_{l_\beta})_v = -0.006 \; per \; radian$$

Substituting the values we get,

$$(C_{l_\beta})_{total} = -0.04 \; per \; radian$$

The stability derivative $C_{n\beta}$, often called the "weathercock" or static directional derivative, is the change in yawing moment coefficient resulting from the sideslip angle. The vertical tail, fuselage, and wing contribute to $C_{n\beta}$, with the vertical tail the dominant factor. **For light aircraft, typical value of $C_{n\beta}$ seem to range from 0.03 to 0.12 per radian.**

$$(C_{n\beta})_{total} = (C_{n\beta})_w + (C_{n\beta})_{fus} + (C_{n\beta})_v + \Delta_1 C_{n\beta} + \Delta_2 C_{n\beta}$$

$$(C_{n_\beta})_{wing} \approx 0 \qquad (C_{n\beta})_w = \frac{C_L^{\,2}}{4\pi AR} \text{ per radian}$$

$$(C_{n\beta})_{fus} = \frac{-.96 \, K_\beta}{57.3} \left(\frac{S_s}{S_w}\right) \left(\frac{\ell_b}{b_w}\right) \left(\frac{h_1}{h_2}\right)^{1/2} \left(\frac{w_2}{w_1}\right)^{1/3}$$

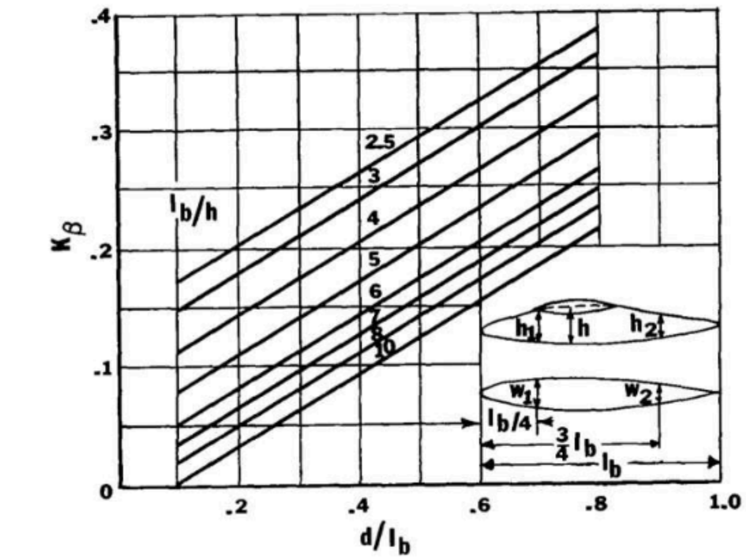


Figure 27. Empirical constant K_β as a function of fineness ratio and c.g. location.

Assuming $h_1 = h_2$ and $w_1 \approx 4w_2$ from wing airfoil.

$$(C_{n\beta})_v = a_v \frac{S_v}{S_w} \frac{\ell_v}{b_w} \eta_v$$

$$(C_{n_\beta})_v = 0.07$$

From graph, corresponding to

$$\frac{d}{l_b} = \frac{2.5}{0.7 \times 13.37} = 0.267$$

And $\frac{b}{h} = \frac{13.37}{1.5} = 8.91$

We get $K_\beta = 0.1$

Assuming the fuselage is cylindrical, lateral surface projection area,
$S_s = 14.038 \text{ m}^2$

$$(C_{n_\beta})_{fus} = -4.4644 \times 10^{-4} \; per \; radian$$

$$(C_{n_\beta})_{total} = 0.069 \; per \; radian$$

• C_{y_p}, C_{l_p} *and* C_{n_p}

The stability derivative C_{yp} is the change in side force resulting from rolling velocity, with the vertical tail the main contributor. It is relatively insignificant and commonly neglected.

$$C_{y_p} = 0$$

The stability derivative C_{lp}, the roll damping derivative, is the change in rolling moment coefficient due to variation on rolling velocity. This derivative is the principal determinant of the damping-in-roll characteristics of the aircraft. **Typical values range from -0.25 to -0.60 per radian**.

$$(C_{\ell_p})_{total} = [(C_{\ell_p})_{a_o} = 2\pi]\left[\frac{AR + 4}{(\frac{2\pi}{(a_o)_w})AR + 4}\right] - \frac{1}{8} C_D \longrightarrow \text{Wing}$$

$$+ 0.5 \frac{S_h}{S_w}(\frac{b_h}{b_w})^2 \ |(C_{\ell_p})_{a_o=2\pi}| \left[\frac{AR + 4}{(\frac{2\pi}{(a_o)_h})AR + 4}\right] \longrightarrow \text{Horizontal Tail}$$

$$+ 2.0 \ (\frac{z_V}{b_w}) \left[\frac{z_V}{b_w} - (\frac{z_V}{b_w})_{\alpha} = 0^o\right] (C_{y_\beta})_{tail} \ . \longrightarrow \text{Vertical Tail}$$

Where, $\left[(C_{l_p})a_o = 2\pi\right] = -0.48$ (From Graph)

$$z_v \approx 0.635 \text{ m}$$

Substituting values in the above equation,

$$(C_{l_p})_{total} = -0.3796 \ per \ radian$$

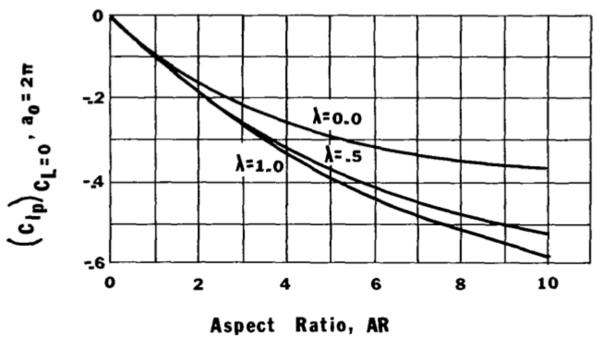


Figure 34. Wing contribution to C_{ℓ_p} for wing with zero or small sweep.

The stability derivative C_{np} is the change in yawing moment caused by rolling, with the wing and vertical tail the main contributors. Dutch rolling damping is influenced by C_{np} in that the larger its negative value, the less Dutch Roll damping. **Typical values range from -0.01 to -0.10 per radian.**

$$(C_{n_p})_w = -\frac{C_L}{8} \qquad (C_{n_p})_w = -0.039 \ per \ radian$$

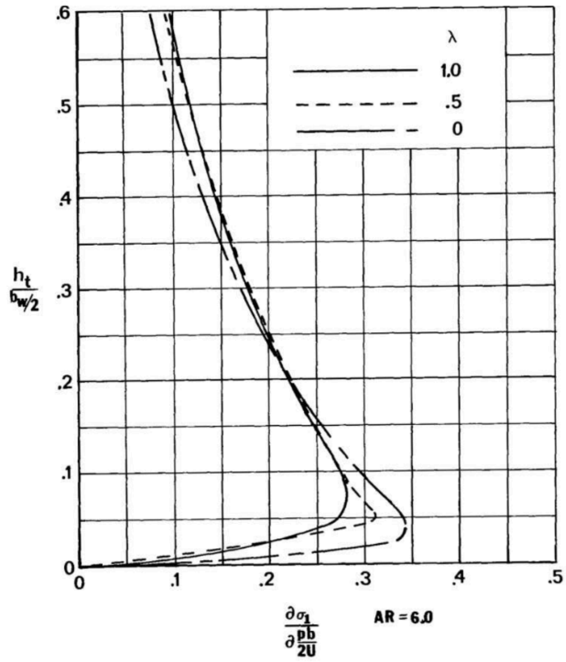
$$(C_{n_p})_v = 57.3 \ a_v \frac{S_v}{S_w}\frac{1}{b_w}(z_v \sin\alpha + \ell_v \cos\alpha)\left[\frac{2}{b_w}(z_v \cos\alpha - \ell_v \sin\alpha) - (\frac{\partial\sigma_1}{\partial\frac{pb}{2U}} + \frac{\partial\sigma_2}{\partial\frac{pb}{2U}})\right]$$

Alpha for cruise is 0.897 degree, so alpha is approximates as zero.
For low alpha, the second side-wash factor reduces to zero.
The first side-wash factor is obtained from graph is 0.275 corresponding to $\frac{2h_t}{b_w} = 0.1$
Where h_t is the distance from the wing centreline to the center of pressure of vertical tail = 0.635 m.

Substituting values, we get,

$$(C_{n_p})_v = -0.72 \; per \; radian$$

$$(C_{np})_{total} = -0.76 \; per \; radian$$



C_{y_r}, C_{l_r} and C_{n_r}

The stability derivative C_{yr} is the change in side force resulting from a change in yawing velocity. **C_{yr} usually has a small positive value**.

$$(C_{y_r})_w = .143\ C_L - .05.$$

$$(C_{y_r})_{tail} = -2\ \frac{\ell_v}{b}(C_{y_\beta})_{tail},$$

$$(C_{y_r})_{wing} = 0.143 \times 0.31 - 0.05 = -0.006 \; per \; radian$$

$$(C_{y_r})_{wing} = 0.098 \; per \; radian$$

$$C_{y_r} = 0.092 \; per \; radian$$

The change in rolling moment due to variation in yawing velocity constitutes the stability derivative C_{lr}. The tail contribution may be either positive or negative, depending on tail geometry and angle of attack of the airplane. Although it has little effect in Dutch Roll damping, it is quite important to the spiral mode. For spiral stability, it is desirable that C_{lr} be as small a positive number as possible. **Typical values range from 0.04 per radian to 0.12 per radian.**

$$(C_{\ell_r})_{wing} = C_L/4$$

$$(C_{\ell_r})_{tail} = -2\ \frac{\ell_v}{b_w}\ \frac{z_v}{b_w}\ (C_{y_\beta})_{tail},$$

$$(C_{y_r})_{wing} = 0.0775 \; per \; radian$$

$$(C_{y_r})_{tail} = 0.005 \; per \; radian$$

$$C_{l_r} = 0.082 \; per \; radian$$

C_{nr} is the change in yawing moment due to variation on yawing velocity. The derivative C_{nr} is the main contributor to the damping of the Dutch roll mode and also plays a significant role in determining spiral stability.

$$(C_{n_r})_{wing} = -0.02\, C_L^2 - 0.3\, C_{D_0} \qquad (C_{n_r})_{tail} = 2\left(\frac{\ell_v}{b_w}\right)^2 (C_{y\beta})_{tail}$$

$$(C_{n_r})_{wing} = -0.008 \; per\ radian \qquad (C_{n_r})_{tail} = -0.048\, per\ radian$$

$$C_{n_r} = -0.056\, per\ radian$$

- ## $C_{y_{\delta_A}}, C_{l_{\delta_A}}$ and $C_{n_{\delta_A}}$

The stability derivative $C_{y\delta A}$ is the change in side force coefficient with variation in aileron deflection. For most conventional light aircraft, **this derivative is zero**; however, for an airframe with low AR and highly swept wings, it may have a value other than zero.

$$C_{y_{\delta A}} = 0$$

The stability derivative $C_{l\delta A}$, known as the aileron effectiveness or "aileron power", is the variation in rolling moment coefficient with change in aileron deflection. The value of $C_{l\delta A}$ is normally found from the spanwise distribution data. **Typical values of $C_{l\delta A}$ range from 0.1 to 0.25 per radian.**

$$C_{\ell_{\delta A}} = \frac{2(C_{L_\alpha})_w\, \tau}{S_w b_w} \int_a^b c\, y\, dy$$

$$c = c_R \left[1 - \frac{y}{b_w/2}(1 - \lambda)\right],$$

$$c_R = \text{root chord.}$$

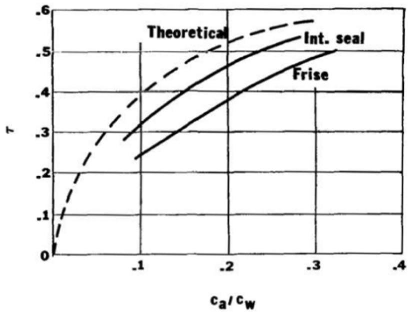


Figure 41. Values for τ as a function of aileron chord to wing chord ratio.

The value of τ is obtained from the graph for $\frac{c_a}{c_w} = 0.16,\ \tau = 0.45$

$$\int_{4.1467}^{6.0165} 2.22\left[1 - \frac{y(0.5)}{6.685}\right] y dy = -0.235$$

Substituting the above values we get,

$$C_{l_{\delta A}} = 0.172\, per\ radian$$

The stability derivative $C_{n\delta A}$, the change in yawing moment coefficient with variation in aileron deflection, results from the difference between drag on the up and down aileron. The desired value of $C_{n\delta A}$ is either zero or very positive value. **Typical values of $C_{n\delta A}$ range from -0.004 to -0.09 per radian.**

$$C_{n_{\delta_A}} = 2K\, C_L\, C_{\ell_{\delta_A}}$$

The value of K is obtained from the graph, $K = -0.16$
Substituting the values we get,

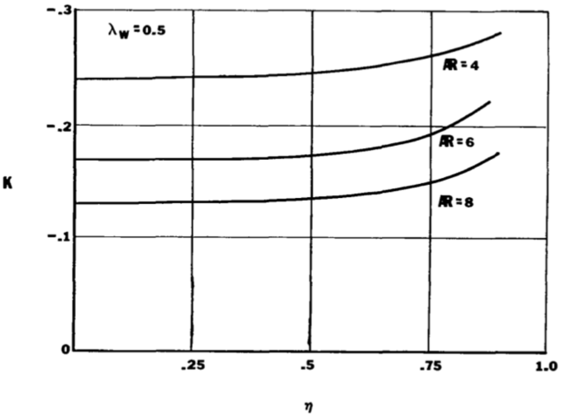


Figure 43. Empirical factor K as a function of η for taper ratio = 0.5.

$$\eta = \frac{Y_i}{b_w/2} = \frac{\text{spanwise distance from centerline to the inboard edge of the control surface}}{\text{semispan}}$$

$$\eta = \frac{3.4785}{6.685} = 0.52$$

$$C_{n_{\delta A}} = -0.017 \; per \; radian$$

• $C_{y_{\delta_R}}, C_{l_{\delta_R}}$ and $C_{n_{\delta_R}}$

The stability derivative $C_{y\delta R}$ is the change in side force resulting from rudder deflection. For an airplane without autopilot, the effect of $C_{y\delta R}$ is relatively unimportant to lateral stability and often is assumed equal to zero. **Typical values of $C_{y\delta R}$ range from 0.12 per radian to 0.24 per radian.**

$$C_{y_{\delta_R}} = a_V \, \tau \, \frac{S_V}{S_W}$$

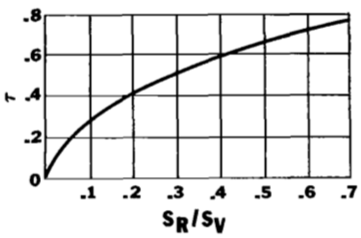


Figure 47. Values for τ as a function of rudder area to vertical tail area ratio.

From graph $\tau = 0.65$
Substituting values,

$$C_{y_{\delta R}} = 0.093 \; per \; radian$$

The stability derivative $C_{l\delta R}$ is th variation in rolling moment coefficient with change in rudder deflection. **For conventional light aircraft, this derivative is only of minor importance and is usually neglected.**

$$C_{\ell_{\delta_R}} = a_V \, \tau \, \frac{S_V}{S} \frac{z_V}{b_W}$$

Substituting values we get,

$$C_{l_{\delta R}} = 0.004 \; per \; radian$$

The stability derivative $C_{n\delta R}$ is th variation in yawing moment coefficient with change in rudder deflection..
The value of $C_{n\delta R}$ is normally on the order of -0.06 per radian but may vary greatly, depending on the airframe configuration.

$$C_{n_{\delta_R}} = - a_V \, \tau \, \frac{S_V}{S_W} \frac{\ell_V}{b_W} \eta_V$$

$$C_{n_{\delta R}} = -0.046 \; per \; radian$$

Longitudinal Derivatives:

Derivative	Calculated Value	Estimated Range
C_{L_α}	4.836	4 to 7
C_{D_α}	0.17	0.126
C_{m_α}	-0.629	-0.885
$C_{L_{\dot{\alpha}}}$	1.621	1.74
$C_{D_{\dot{\alpha}}}$	0	0
$C_{M_{\dot{\alpha}}}$	-5.51	-5.24
C_{L_q}	4.619	3.9
C_{D_q}	0	0
C_{m_q}	-19.28	-12.43
$C_{L_{\delta e}}$	0.326	0.427
$C_{D_{\delta e}}$	0.01	-
$C_{m_{\delta e}}$	-1.23	-1.28

Lateral Derivatives:

Derivative	Calculated Value	Estimated Range
C_{y_β}	-0.1	-0.31
C_{l_β}	-0.04	-0.03 to -0.12
C_{n_β}	0.074	0.03 to 0.12
C_{y_p}	0	0
C_{l_p}	-0.3796	-0.25 to -0.6
C_{n_p}	-0.76	-0.01 to -0.1
C_{y_r}	0.098	-0.3 to 0.3
C_{l_r}	0.082	0.04 to 0.12
C_{n_r}	-0.056	-0.05 to -0.14
$C_{y_{\delta A}}$	0	0
$C_{l_{\delta A}}$	0.172	0.1 to 0.25
$C_{n_{\delta A}}$	-0.017	-0.004 to -0.09
$C_{y_{\delta R}}$	0.093	0.12 to 0.24
$C_{l_{\delta R}}$	0.004	0
$C_{n_{\delta R}}$	-0.046	-0.06