The Stability Derivatives.

(Area) $S = 10.86\ m^2$

$b = 10.4\ m$ (span)

$C_{D0} = 0.0081$ (Aspect ratio) $AR = 10$

$K = 0.044$

$\bar{c} = 1.05\ m$ (chord)

$\frac{C_L}{C_D} \simeq 15.5$

$\frac{\text{Thickness}}{\text{Chord}} = 12\%$

$e = 0.77$

We choose NACA - 4412 airfoil

$C_{Lmax} = 1.25$

$\alpha_{stall} = 13$ degree.

$C_{M0} = -0.098$

$C_l = 0.4$

$$C_{L\alpha} = \frac{dC_L}{d\alpha} = 1.8\pi\left(1 + 0.8\frac{t_{max}}{c}\right)$$

$$C_{L\alpha_{2d}} = 6.198$$

$$C_{L\alpha} = \frac{C_{L\alpha 2D}}{1 + \frac{C_{L\alpha 2D}}{\pi . AR}}$$

$$= \frac{6.18}{1 + \frac{6.18}{\pi \times 10}}$$

$\boxed{C_{L\alpha} = 5.164}$ per radian

$\Sigma F_n = W - L$

$L = C_{L0} + C_{L\alpha}\alpha + C_{L\delta e}\delta e.$

$C_{L\delta e} = \frac{S_c}{S} C_{L\delta e}.$

Mean value

$$\overline{\frac{dC_L}{d\alpha}} = \frac{S_f}{S}\left(\frac{dC_l}{d\alpha}\right)_f + \left(1 - \frac{S_f}{S}\right)\frac{dC_l}{d\alpha}$$

$f \rightarrow$ denotes the flap.

$C_{D\alpha}$

$$C_L = C_{L\alpha} \cdot \alpha$$

$$C_D = C_{D0} + KC_L^2 \qquad K = \frac{1}{\pi A R e}$$

$$C_{D\alpha} = \left(\frac{dC_D}{dC_L}\right) \cdot \left(\frac{dC_L}{d\alpha}\right)$$

$$C_{D\alpha} = 2KC_L \cdot C_{L\alpha}$$

$$C_{D\alpha} = 2 \times 0.042 \times 0.45 \times 5.164$$

$$\boxed{C_{D\alpha} = 0.1952}$$

$C_{M\alpha}$

$$\Sigma M = C_{M0} + C_{M\alpha} \cdot \alpha + C_{M\delta e} \delta e,$$

$$C_{M\alpha} = C_{L\alpha} \left(\bar{x}_{cg} - \bar{x}_{NP}\right)$$

$$C_{M\alpha} = -5.1 \times 0.15$$

Static margin $= 0.15$

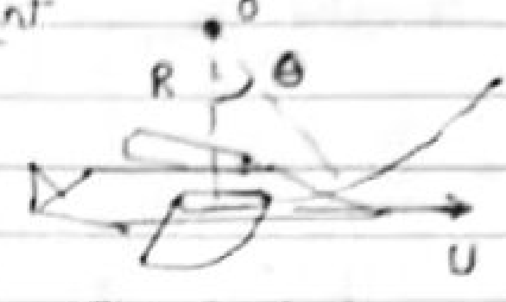
$$\boxed{C_{M\alpha} = -0.765}$$

q = pitch rate

Stability Derivatives

C_{L_q} → Represents change in airplane lift with varying pitching velocity, with α constant

→ Wing Contribution to C_{L_q}



Pitching at Constant α

$$\Delta\alpha = \frac{q\,(x_{cg} - x_{ac})}{U}$$

x_{cg} = dist. to c.g.

x_{ac} = dist. to a.c (aerodynamic center)

* If a.c. is very close to c.g. Contribution from wing is negligible.

* For light airplanes, fuselage Contribution to C_{L_q} is smaller than wing, So can be ignored.

Wing Contribution → $$\boxed{\frac{\partial C_L}{\partial\left(\frac{cq}{2U}\right)} = \frac{2x'}{c}\,C_{L\alpha}}$$

[$\frac{cq}{2U}$ is dimensionless form]

where → x' = distance from C.g. to wing quarter chord

$C_{L\alpha}$ = wing lift slope curve

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$\underline{C_{D_q}}$ → Represents change in drag with varying pitch velocity at constant 'α'.

* For subsonic flight, C_{D_q} is very small and ignored.

$\underline{C_{M_q}}$ → Change in pitching moment coefficient due to change in pitching velocity.

* Wing contribution to C_{M_q} either opposes or increases the pitching motion.

* Fuselage contribution is ignored.

$$\to C_{m_q}\Big|_{tail} = \frac{\partial C_m}{\partial\left(\frac{cq}{2U}\right)}\Bigg|_{tail} \xrightarrow{\left(\frac{\partial C_m}{\partial C_L}\right)\left(\frac{\partial C_L}{\partial q}\right)} = \frac{-l_t}{c}\,\frac{\partial C_L}{\partial\left(\frac{cq}{2U}\right)}\Bigg|_{tail}$$

* C_{m_q} is always −ve, C_{m_q} is +ve.

$$\to C_{m_q}\Big|_{wing} = \frac{\partial C_m}{\partial\left(\frac{cq}{2U}\right)} = \frac{-|x'|}{c}\,\frac{\partial C_L}{\partial\left(\frac{cq}{2U}\right)}\Bigg|_{wing}$$

$$\therefore\ C_{m_q} = \frac{-2x'}{c^2}|x'|\,C_{L\alpha} - \frac{2l_t^2}{c^2}\,C_{L\alpha_t}\,\frac{S_t}{S_w}\,\eta_t$$

→ Typical value of $C_{L\dot{\alpha}}$ for light airplane falls in range 4.0 - 7.0 depending on the type of wing.

→ The value of C_{L_q} for Cessna 182 → 3.9

→ C_{D_q} is chosen to be 0.

→ C_{m_q} for Cessna 182 = -12.43