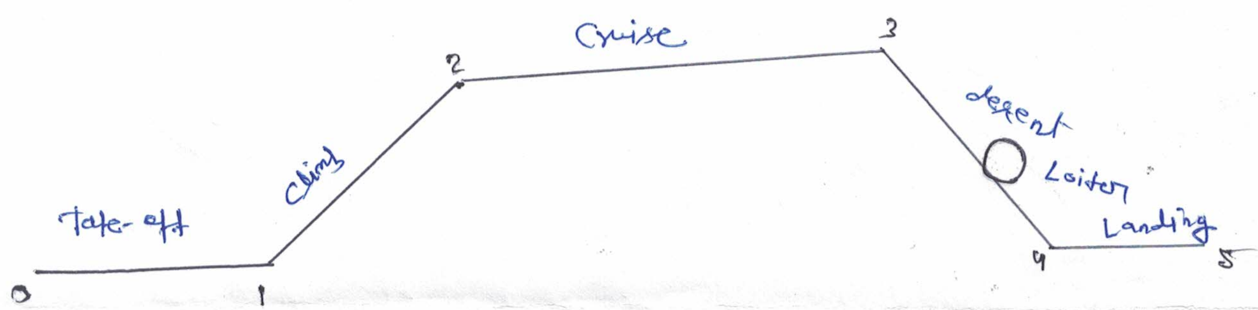
Aircraft Design ! Solved Example

Let us say we have been given the following mission requirement to design the light weight Aircraft.

MISSION REQUIREMENTS

1. Weight of the aircraft : < 3000 kg
2. Four passenger + 1 crew
3. Maximum Range : 1000 km
4. Service Seiling ! 7 km
5. Flight Altitude : mean sea level to 10,000 ft
6. Design cruise speed : 90 to 110 m/s
7. Rate of climb : 5 to 6 m/s (At sea level)
8. take-off distance : 550 to 650 m
9. Landing distance : 500 to 600 m

STEP. 1 Draw the mission profile



Take-off :	0 to 1
Climb :	1 to 2
Cruise :	2 to 3
descent :	3 to 4
Landing :	4 to 5

STEP: 2 ⇒ Weight Estimation of Aircraft

An Aircraft can be have a following types of weight ⇒

$W_c = W_{crew}$ = Weight of the crew

$W_p = W_{payload}$ = weight of the Payload

$W_f = W_{fuel}$ = weight of the fuel

$W_e = W_{empty}$ = Empty weight

If total weight of the Aircraft is 'W_0' then -

$$W_0 = W_c + W_p + W_f + W_e$$

Doing the mathematical manipulation we have,

$$\left[W_0 = \frac{W_c + W_p}{1 - \frac{W_f}{W_0} - \frac{W_e}{W_0}} \right] \quad -(1)$$

It can be noticed that in Eq. (1)

$\frac{W_f}{W_0}$ and $\frac{W_e}{W_0}$ are required to find the total weight of the aircraft.

⇒ Estimation of Empty weight fraction (W_e/W_0)

From the literature survey for weight class 2000 - 3000 kg. Empty weight fraction (W_e/W_0) is 0.62.

$$\boxed{\frac{W_e}{W_0} = 0.62}$$

For more information please refer the Fig. 1

⇒ Estimation of fuel weight fraction (W_f/W_0)

If
R = Range of Aircraft
L/D = Lift to drag ratio
h_{pr} = propeller efficiency
c = specific fuel consumption

then, Range can be written as -

$$R = \frac{h_{pr}}{c}\left(\frac{L}{D}\right) \ln\left(\frac{W_2}{W_3}\right) \quad -(2)$$

for the present case ⇒

$$R_{max} = 1000 \text{ km}$$

$$(L/D)_{max} = 13 \quad \text{(Assumed)}$$

$$\eta_{pr} = 0.85$$

$$C = 0.4 \text{ lb/hp-hr} = \frac{0.4 \times 9.81}{746 \times 3600} \; (1/m)$$

$$= 1.4611 \times 10^{-6} \; (1/m)$$

(reciprocating engine)

From Eq. (2) ⇒

$$1000 \times 10^3 = \frac{0.85}{1.4611 \times 10^{-6}} \times 13 \ln \frac{W_2}{W_3}$$

$$\frac{W_2}{W_3} = 1.14136$$

$$\boxed{\frac{W_3}{W_2} = 0.87614}$$

and

$$\boxed{\frac{W_4}{W_1} = 1} \rightarrow$$ fuel consumption in descent = 0

$$\boxed{\frac{W_5}{W_4} = 0.995} \rightarrow$$ Amount of fuel used in landing is small

From the mission profile ⇒

$$\frac{W_5}{W_0} = \frac{W_1}{W_0} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \frac{W_4}{W_3} \cdot \frac{W_5}{W_4} \quad - (3)$$

and

$$W_f = W_0 - W_5 \quad \leftarrow (4)$$

or

$$\frac{W_f}{W_0} = \left(1 - \frac{W_5}{W_0}\right)$$

$$\frac{W_f}{W_0} = 1.06\left(1 - \frac{W_5}{W_0}\right) \quad \text{— (5)}$$

6% fuel is reserved

for take-off (0 - 1)

$$\frac{W_1}{W_0} = 0.97$$

climb (1 - 2)

$$\frac{W_2}{W_1} = 0.985$$

So, till Now, we have, $\frac{W_1}{W_0}$, $\frac{W_2}{W_1}$, $\frac{W_3}{W_2}$, $\frac{W_4}{W_1}$ and $\frac{W_5}{W_4}$

inserting the values of these fraction in Eq. (3) we, have,

$$\frac{W_5}{W_0} = \frac{W_1}{W_0} \cdot \frac{W_2}{W_1} \cdot \frac{W_3}{W_2} \cdot \frac{W_4}{W_3} \cdot \frac{W_5}{W_4}$$

$$= 0.97 \times 0.985 \times 0.87617 \times 1 \times 0.995$$

$$\boxed{\frac{W_5}{W_0} = 0.832022}$$

Inserting $\frac{W_f}{W_0}$ in Eq. (5) we get,

$$\frac{W_f}{W_0} = 1.06(1 - 0.8332912)$$

$$\boxed{\frac{W_f}{W_0} = 0.1771}$$

it can be noticed the, to find the total weight of the Aircraft (W_0). We have found out $\frac{W_e}{W_0}$ and $\frac{W_f}{W_0}$. However, W_c and W_p are required to find W_0.

W_c = 80 kg (Average weight of human)

As we know their are total four Passenger

So, passanger weight will be = 4×80
= 320 kg

10 kg extra weight carried by each one

So, total payload weight will be -

W_p = 320 + 5×10

$$\boxed{W_p = 370 \text{ kg}}$$

From Eq. (1) -

$$W_0 = \frac{W_c + W_p}{1 - \frac{W_f}{W_0} - \frac{W_e}{W_0}}$$

$$= \frac{80 + 370}{1 - 0.1771 - 0.62}$$

Total weight of Aircraft.

$$\boxed{W_0 = 2217.84 \text{ kg}}$$

fuel weight Now, can be found as -

$$W_f = \frac{W_f}{W_0} \cdot W_0 = 0.1771 \times 2217.84$$

$$\boxed{W_f = 392.77 \text{ kg}}$$

STEP - 3 Sizing of Wing.

Till now, we have estimated the total weight of the aircraft. This weight will lift by the wing of the Aircraft. In this section, we will ~~the~~ design the wing.

for a steady - straight - level flight

$$\text{weight} = \text{Lift} -$$

$$W = \frac{1}{2}\rho V^2 S C_L$$

or

$$W/S = \frac{1}{2} \rho V^2 C_L \quad - (6)$$

where

W/S = wing loading

ρ = density of air

V = flight velocity

C_L = lift coefficient

further,

$$W/S = \frac{1}{2} \rho V_{stall}^2 \, C_{L_{max}} \quad (7)$$

$$W/S = \frac{1}{2} \times 1.225 \times (30)^2 \times 2.3$$

$$= 1268.4 \cdot 6 \quad N/m^2$$

$$\boxed{W/S = 129.3 \cdot 6 \ kg/m^2}$$

at

$C_L = C_{L_{max}} = 2.3$

$V = V_{stall}$

$= 30 \, m/s$

Note:

$C_{L_{max}}$ is at take-off with flap deflection

Wing loading calculation at desired cruise speed and lift coefficient ⇒

Let say our desire cruise speed i.e $70 \, m/s$ and lift coefficient i.e 0.4

From Eq. (6)

$$W/S = \frac{1}{2} \times 1.2256 \times (70)^2 \times 0.4$$

$$\boxed{W/S = 1201.088 = 122.435 \ kg/m^2}$$

Wing Loading calculation when Landing distance is specified ⇒

Using the relation

$$S_g = jN\sqrt{\frac{2}{\rho_\infty}\left(\frac{W}{S}\right)\frac{1}{C_{L_{max}}}} + \frac{j^2 (W/S)}{\rho_\infty g \,(C_{L_{max}})\,\mu_r} \quad -(8)$$

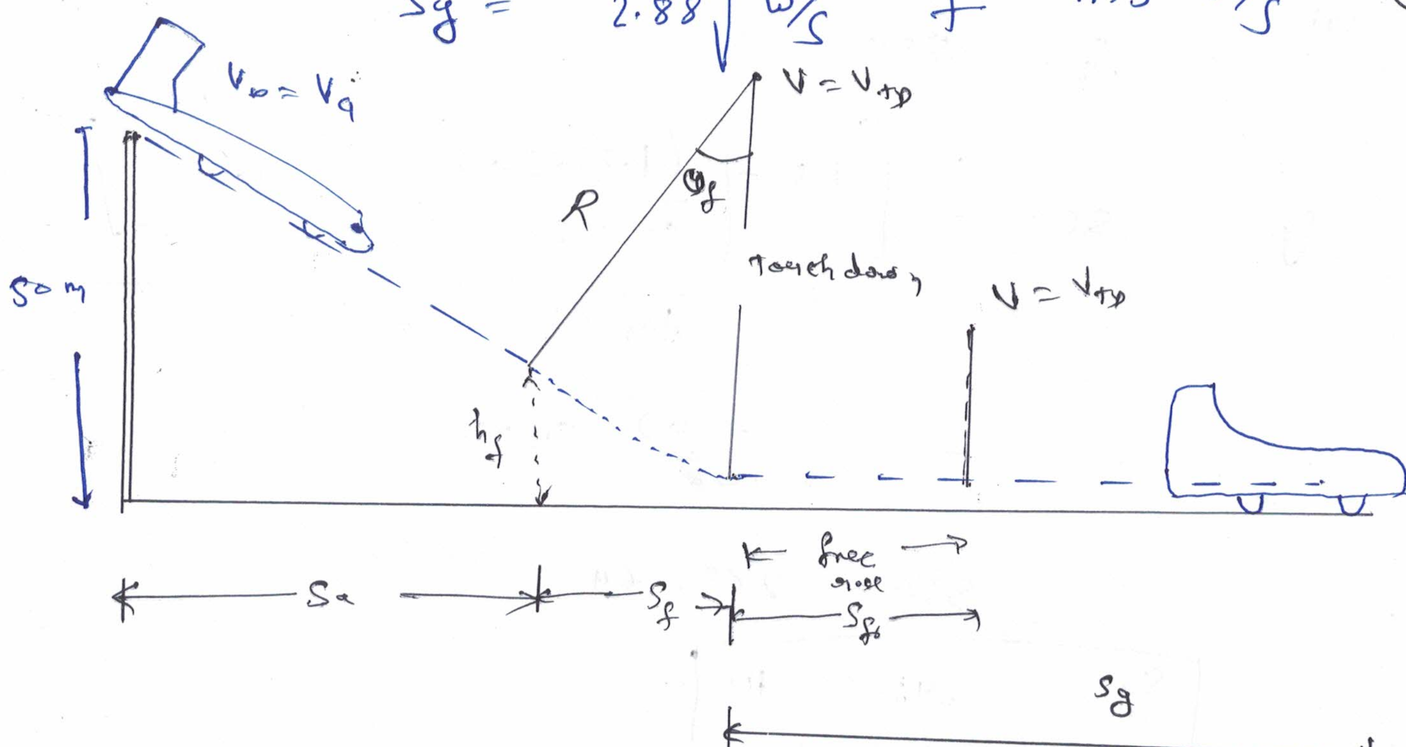
for considered aircraft

$$j = 1.15$$

$$N = 3$$

Eq. (8), becomes

$$S_g = 2.88\sqrt{W/S} + 0.1175\, W/S \quad -(9)$$



S_a = Approach distance, V_a = Approach velocity

S_f = flare distance $\theta_f = \theta_a$ = Approach Angle

S_g = Ground roll distance

distance = $S_a + S_f + S_g$

Here,

$$S_g = S_L - S_a - S_f$$

$$S_g = S_L - \left(\frac{15 - h_f}{\tan\theta_a}\right) - R\sin\theta_a$$

$$= S_L - \left[\frac{15 - \left(R(1-\cos\theta_a)\right)}{\tan\theta_a}\right] - R\sin\theta_a \qquad \Bigg| \; V_f = 1.23 \times V_{steel}$$

$$S_g = S_L - \left\{\frac{15 - \left[\frac{V_f^2}{0.2g}(1-\cos\theta_a)\right]}{\tan\theta_a}\right\} - \left[\frac{V_f^2}{0.2g}\sin\theta_a\right]$$

*

if the total Landing is specified as 500 to 600

Let

$$S_L = 550$$

$$S_g = 550 - \frac{\left[15 - \left\{\frac{(1.23\times30)^2}{0.2\times9.81}(1-\cos3^\circ)\right\}\right]}{\tan3^\circ}$$

$$- \frac{(1.23\times30)^2}{0.2\times9.81}\sin3^\circ$$

$$= 550 - 268.069 - 36.32$$

$$\boxed{S_g = 245.61 \text{ m}}$$

Inserting S_0 value in Eq. (8) -

$$245.61 = 2.88\sqrt{W/S} + 0.1125\, W/S$$

Let $W/S = x^2$

$\Rightarrow$

$$245.61 = 2.88x + 0.1125x^2$$

$$0.1125x^2 + 2.88x - 245.61 = 0$$

Solving

$$x = \frac{-2.88 \pm \sqrt{(2.88)^2 - 4 \times 0.1125 \times (-245.61)}}{2 \times 0.1125}$$

$$x = \frac{-2.88 \pm 11.123}{2 \times 0.1125}$$

$$x = 35.075$$

$$x^2 = 1230.501$$

$$W/S = 1230.501 \ N/m^2$$

$$\boxed{W/S = 125.433 \ kg/m^2}$$

It can be noticed that -

1. W/S (from Stall velocity approach) (take-off)

$= \underline{127.306}$ kg/m²

2. W/S (from cruise velocity approach)

$= \underline{122.435}$ kg/m²

3. W/S (from landing approach)

$= \underline{125.433}$ kg/m²

All three are very much close to each other.

Now, Area of the wing can be found as-

$$S = \frac{W}{W/S}$$

$$= \frac{2217.84}{122.435}$$

$$\left[S = 18.11 \text{ m}^2\right]$$

Let the Aspect ratio of wing is choosen to be 10

$$AR = 10$$

$$\Rightarrow AR = \frac{b^2}{S}$$

$$b = \sqrt{AR \cdot S}$$

wing span

$$= \sqrt{10 \times 18.11}$$

$$\boxed{b = 13.45 \text{ m}}$$

wing chord (for rectangular wing)

$$\bar{c} = \frac{b}{AR} = \frac{13.45}{10}$$

$$\boxed{\bar{c} = 1.345}$$

Engine Selection ⇒ [Step - 4]

Estimation of thrust to weight ratio:

let us consider the take-off distance: [Ground roll distance]

$$S_g = \frac{1.21 (W/S)}{g \rho_0 C_{L_{max}} (T/W)} \quad — (10)$$

$$= \frac{1.21 \times 122.435 \times 9.81}{9.81 \times 1.225 \times 2 \times (T/W)}$$

$$S_g = \frac{60.43}{(T/W)} = \frac{60.43}{T/W} \quad — (11)$$

for $C_{L_{max}} = 2$

$$V_{stall} = \sqrt{\frac{2 W/S}{\rho S C_{L_{max}}}}$$

$$= 31.30 \text{ m/s}$$

[take-off velocity] $V_{LO} = 1.1 V_{stall} = 34.435$

$$R = \frac{6.98 V_s^2}{g} = \frac{6.98 \times (31.30)^2}{9.81}$$

$$\boxed{R = 697.06 \text{ m}}$$

Induced flight path angle i.e. $\theta_{OB} = \cos^{-1}\left(1 - \frac{h_{OB}}{R}\right)$

$$= \cos^{-1}\left(1 - \frac{15}{\quad}\right) = 11.9^{\circ}$$

Airborn distance

$$S_a = R \sin \theta_{OB} = 697.06 \times \sin(11.9)$$

$$= 143.82 \text{ m}$$

If take-off distance is specified as

600 m

then

$$600 = S_g + S_a \qquad — (12)$$

From Eq. (11)

$$600 = \frac{60.43}{(T/W)} + 143.82$$

$$\frac{1}{T/W} = \frac{600 - 143.82}{60.43}$$

$$\frac{1}{(T/W)} = 7.5488$$

$$\boxed{T/W = 0.13246}$$

or

$$\boxed{\left(T/W\right)_{0.7 V_{LO}} = 0.13246}$$

$$V_{LO} = 0.7 V_{LO} = 0.7 \times 34.435$$

$$\boxed{V_{LO} = 24.1045 \text{ m/s}}$$

V_{LO} = lift-off velocity

Power required ⇒

$$P_R = T V_\infty$$

$$= (T/W)_0 \, W_0 \cdot V_\infty$$

$$= 0.1324 \times 2217.84 \times 24.10 \times 9.81$$

$$= 69424.139 \text{ (Watt)}$$

$$P_R = \cancel{82.954} \text{ (hp)}$$
$$93.06$$

Power Available ⇒

$$P_A = \frac{P_R}{\eta} \qquad \eta = 0.8$$

$$= \frac{82.954}{0.8}$$

$$\boxed{P_A = 116.3965 \text{ hp}}$$

$$\boxed{P_A = 116.4 \text{ hp}}$$

to satisfy the take-off constraint, the total power must be at least

$$\boxed{P \geq 116.4 \text{ hp}}$$

Next,

Power available calculation to perform the 5 m/s (rate of climb)

Using

$$\frac{h_{pr} P}{W} = (R/C)_{max} + \left[\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D_0}}}\left(\frac{W}{S}\right)\right]^{1/2} \times \frac{1.155}{(L/D)_{max}}$$

$$= 5 + \left[\frac{2}{1.2256}\sqrt{\frac{0.0422}{3\times 0.035}}\ 122.435\times 9.81\right]^{1/2} \times \frac{1.155}{13}$$

$$= 5 + 3.133$$

$$h_{pr} P/W = 8.133$$

$$P = \frac{8.133\times W_0}{h_{pr}} = \frac{8.133\times 2217.84\times 9.81}{0.8}$$

$$= 221188.2165 \text{ watt}$$

$$\boxed{P = 296.5\,hp}$$

Induced drag factor

$$K = \frac{1}{4C_{D_0}(L/D)^2_{max}}$$

Assume

$C_{D_0} = 0.035$ (zero lift drag coeff.)

To statisfy the constraint on rate of climb, the power must be

$$\boxed{P \geq 296.5 \text{ hp}}$$

Let us summarize the findings,

Total weight, W_0 = 2217.84 kg

Fuel weight, W_f = 392.77 kg

Wing Area, S = 18.11 m^2

Span, b = 13.45 m

Wing chord, $\bar{c}$ = 1.345

AR (Aspect ratio) = 10

Maximum lift to drag ratio $(L/D)_{max}$ = 13

Wing loading (W/S) = 122.435 kg/m^2

Power loading (W/P) = 7.48 (kg/hp)

Induced drag coefficient factor, K = 0.0422

Zero-lift drag coefficient, C_{D0} = 0.035