8.2 REQUIREMENTS

We are given the job of designing a light, business transport aircraft which will carry five passengers plus the pilot in relative comfort in a pressurized cabin. The specified performance is to be as follows:

1. Maximum level speed at midcruise weight: 250 mi/h.
2. Range: 1,200 mi.
3. Ceiling: 25,000 ft.
4. Rate of climb at sea level: 1,000 ft/min.
5. Stalling speed: 70 mi/h.
6. Landing distance (to clear a 50-ft obstacle): 2,200 ft.
7. Takeoff distance (to clear a 50-ft obstacle): 2,500 ft.

In addition, the airplane should be powered by one (or more) conventional reciprocating engine.

The stipulation of these requirements constitutes an example of the first pivot point in Fig. 7.3.

8.3 THE WEIGHT OF AN AIRPLANE AND ITS FIRST ESTIMATE

As noted in Fig. 7.3, the second pivot point in our conceptual design analysis is the preliminary (almost crude) estimation of the gross weight of the airplane. Let us take this opportunity to discuss the nature of the weight of an airplane in detail.

There are various ways to subdivide and categorize the weight components of an airplane. The following is a common choice.

1. *Crew weight* W_{crew}. The crew comprises the people necessary to operate the airplane in flight. For our airplane, the crew is simply the pilot.
2. *Payload weight* W_{payload}. The payload is what the airplane is intended to transport—passengers, baggage, freight, etc. If the airplane is intended for military combat use, the payload includes bombs, rockets, and other disposable ordnance.
3. *Fuel weight* W_{fuel}. This is the weight of the fuel in the fuel tanks. Since fuel is consumed during the course of the flight, W_{fuel} is a variable, decreasing with time during the flight.
4. *Empty weight* W_{empty}. This is the weight of everything else—the structure, engines (with all accessory equipment), electronic equipment (including radar, computers, communication devices, etc.), landing gear, fixed equipment (seats, galleys, etc.), and anything else that is not crew, payload, or fuel.

The sum of these weights is the total weight of the airplane W. Again, W varies throughout the flight because fuel is being consumed, and for a military combat airplane, ordnance may be dropped or expended, leading to a decrease in the payload weight.

The design takeoff gross weight W_0 is the weight of the airplane at the instant it begins its mission. It includes the weight of all the fuel on board at the beginning of the flight. Hence,

$$W_0 = W_{\text{crew}} + W_{\text{payload}} + W_{\text{fuel}} + W_{\text{empty}} \tag{8.1}$$

In Eq. (8.1), W_{fuel} is the weight of the full fuel load at the beginning of the flight.

In Eq. (8.1), W_0 is the important quantity for which we want a first estimate; W_0 is the desired result from pivot point 2 in Fig. 7.3. To help make this estimate, Eq. (8.1) can be rearranged as follows. If we denote W_{fuel} by W_f and W_{empty} by W_e (for notational simplicity), Eq. (8.1) can be written as

$$W_0 = W_{\text{crew}} + W_{\text{payload}} + W_f + W_e \tag{8.2}$$

or

$$W_0 = W_{\text{crew}} + W_{\text{payload}} + \frac{W_f}{W_0} W_0 + \frac{W_e}{W_0} W_0 \tag{8.3}$$

Solving Eq. (8.3) for W_0, we have

$$\boxed{W_0 = \frac{W_{\text{crew}} + W_{\text{payload}}}{1 - W_f/W_0 - W_e/W_0}} \tag{8.4}$$

The form of Eq. (8.4) is particularly useful. Although at this stage we do not have a value of W_0, we can fairly readily obtain values of the *ratios* W_f/W_0 and W_e/W_0, as we will see next. Then Eq. (8.4) provides a relation from which W_0 can be obtained in an iterative fashion. [The iteration is required because in Eq. (8.4), W_f/W_0 and W_e/W_0 may themselves be functions of W_0.]

8.3.1 Estimation of W_e/W_0

Most airplane designs are evolutionary rather than revolutionary; that is, a new design is usually an evolutionary change from previously existing airplanes. For this reason, historical, statistical data on previous airplanes provide a starting point for the conceptual design of a new airplane. We will use such data here. In particular, Fig. 8.1 is a plot of W_e/W_0 versus W_0 for a number of reciprocating engine, propeller-driven airplanes. Data for 19 airplanes covering the time period from 1930 to the present are shown. The data show a remarkable consistency. The values of W_e/W_0 tend to cluster around a horizontal line at $W_e/W_0 = 0.62$. For gross weights above 10,000 lb, W_e/W_0 tends to be slightly higher for some of the aircraft. However, there is no technical reason for this; rather, the higher values for the heavier airplanes are most likely an historical phenomenon. The P-51, B-10, P-38, DC-3, and B-26 are all examples of 1930's technology. A later airplane, the Lockheed P2V Neptune, is based

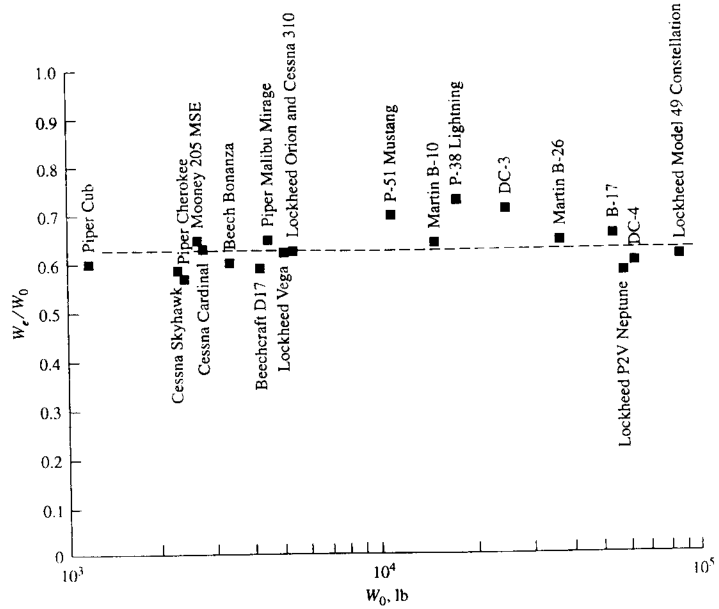

Figure 8.1 Variation of the empty–gross weight ratio W_e/W_0 with gross weight for reciprocating-engine, propeller-dirven airplanes.

on 1940s' technology, and it has a relatively low value of $W_e/W_0 = 0.57$. Eclipsed by jet-propelled airplanes, the design of heavy reciprocating engine/propeller-driven airplanes in the gross weight class above 10,000 lb has virtually ceased since the 1950s. The last major airplanes of this class were the Douglas DC-7 and the Lockheed Super Constellation, both large, relatively luxurious passenger transports. Hence, reflected in Fig. 8.1, no modern airplanes are represented on the right side of the graph. In contrast, the data shown at the left of the graph, for gross weight less than 10,000 lb, are a mixture, representing airplanes from 1930 to the present.

As a result of the data shown in Fig. 8.1, we choose for our first estimate

$$\frac{W_e}{W_0} = 0.62 \tag{8.5}$$

8.3.2 Estimation of W_f/W_0

The amount of fuel required to carry out the mission depends critically on the efficiency of the propulsion device—the engine specific fuel consumption and the

propeller efficiency. It also depends critically on the aerodynamic efficiency—the lift-to-drag ratio. These factors are principal players in the Brequet range equation given by Eq. (5.153), repeated here:

$$R = \frac{\eta_{pr}}{c}\frac{L}{D}\ln\frac{W_0}{W_1} \tag{5.153}$$

Equation (5.153) is very important in our estimation of W_f/W_0, as defined below.

The total fuel consumed during the mission is that consumed from the moment the engines are turned on at the airport to the moment they are shut down at the end of the flight. Between these times, the flight of the airplane can be described by a *mission profile*, a conceptual sketch of altitude versus time such as shown in Fig. 8.2. As stated in the specifications, the mission of our airplane is that of a business light transport, and therefore its mission profile is that for a simple cruise from one location to another. This is the mission profile shown in Fig. 8.2. It starts at the point labeled 0, when the engines are first turned on. The takeoff segment is denoted by the line segment 0–1, which includes warm-up, taxiing, and takeoff. Segment 1–2 denotes the climb to cruise altitude (the use of a straight line here is only schematic and is *not* meant to imply a constant rate of climb to altitude). Segment 2–3 represents the cruise, which is by far the largest segment of the mission. Segment 2–3 shows an increase in altitude during cruise, consistent with an attempt to keep C_L (and hence L/D) constant as the airplane weight decreases because of the consumption of fuel. This is discussed at length in Section 5.13.3. Segment 3–4 denotes the descent, which generally includes loiter time to account for air traffic delays; for design purposes, a loiter time of 20 min is commonly used. Segment 4–5 represents landing.

The mission profile shown in Fig. 8.2 is particularly simple. For other types of missions, especially those associated with military combat aircraft, the mission profiles will include such aspects as combat dogfighting, weapons drop, in-flight refueling, etc. For a discussion of such combat mission profiles, see, for example,

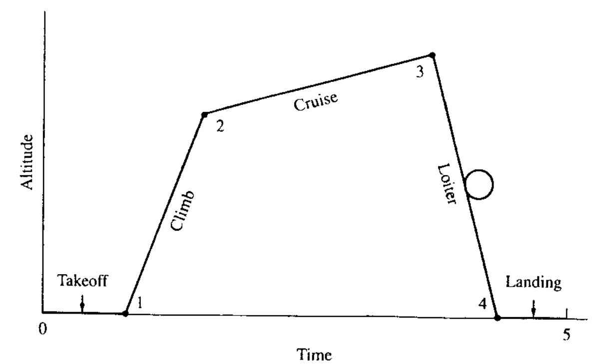

Figure 8.2 Mission profile for a simple cruise.

Ref. 25. For our purposes, we will deal only with the simple cruise mission profile sketched in Fig. 8.2.

The mission profile is a useful bookkeeping tool to help us estimate fuel weight. Each segment of the mission profile is associated with a *weight fraction*, defined as the airplane weight at the end of the segment divided by the weight at the beginning of the segment.

$$\text{Mission segment weight fraction} = \frac{W_i}{W_{i-1}}$$

For example, the cruise weight fraction is W_3/W_2, where W_3 is the airplane weight at the end of the cruise and W_2 is the weight at the beginning of cruise. The fuel weight ratio W_f/W_0, can be obtained from the product of the mission segment weight fractions as follows. Using the mission profile in Fig. 8.2, the ratio of the airplane weight at the end of the mission to the initial gross weight is W_5/W_0. In turn,

$$\frac{W_5}{W_0} = \frac{W_1}{W_0}\frac{W_2}{W_1}\frac{W_3}{W_2}\frac{W_4}{W_3}\frac{W_5}{W_4} \qquad \textbf{[8.6]}$$

The right side of Eq. (8.6) is simply the product of the individual mission segment weight fractions. Also, keep in mind that for the simple cruise mission shown in Fig. 8.2, the change in weight during each segment is due to the consumption of fuel. If, at the end of the flight, the fuel tanks were completely empty, then

$$W_f = W_0 - W_5$$

or

$$\frac{W_f}{W_0} = 1 - \frac{W_5}{W_0} \qquad \textbf{[8.7]}$$

However, at the end of the mission, the fuel tanks are not completely empty—by design. There should be some fuel left in reserve at the end of the mission in case weather conditions or traffic problems require that the pilot of the airplane divert to another airport, or spend a longer-than-normal time in a holding pattern. Also, the geometric design of the fuel tanks and the fuel system leads to some trapped fuel that is unavailable at the end of the flight. Typically, a 6% allowance is made for reserve and trapped fuel. Modifying Eq. (8.7) for this allowance, we have

$$\frac{W_f}{W_0} = 1.06\left(1 - \frac{W_5}{W_0}\right) \qquad \textbf{[8.8]}$$

Hence, the sequence for the calculation of W_f/W_0 that appears in the denominator of Eq. (8.4) is as follows:

1. Calculate each individual mission segment weight fraction W_1/W_0, W_2/W_1, etc., that appears in Eq. (8.6).
2. Calculate W_5/W_0 from Eq. (8.6).
3. Calculate W_f/W_0 from Eq. (8.8).

Let us proceed to make this calculaton for our business transport airplane.

For takeoff, segment 0–1, historical data show that W_1/W_0 is small, on the order of 0.97. Hence, we assume

$$\frac{W_1}{W_0} = 0.97 \qquad \textbf{[8.9]}$$

For climb, segment 1–2, we again rely on historical data for a first estimate, which indicate that W_2/W_1 is also small, on the order of 0.985. Hence, we assume

$$\frac{W_2}{W_1} = 0.985 \qquad \textbf{[8.10]}$$

For cruise, segment 2–3, we make use of the Brequet range equation, Eq. (5.153). This requires an estimate of L/D. At this stage of our design process (pivot point 2 in Fig. 7.3), we cannot carry out a detailed aerodynamic analysis to predict L/D—we have not even laid out the shape of the airplane yet (which comes later in the process, pivot point 4 in Fig. 7.3). Therefore, we can only make a crude approximation, again based on data from existing airplanes. Loftin (Ref. 13) has tabulated the values of $(L/D)_{\max}$ for a number of famous aircraft over the past century. The values for some representative reciprocating engine/propeller-driven airplanes of the size likely to carry four to six people are tabulated below, obtained by Loftin.

Airplane	$(L/D)_{\max}$
Cessna 310	13.0
Beach Bonanza	13.8
Cessna Cardinal	14.2

Hence, a reasonable first approximation for our airplane is

$$(L/D)_{\max} = 14 \qquad \textbf{[8.11]}$$

Also needed in the range equation, Eq. (5.153), are the specific fuel consumption c and propeller efficiency η. As stated in Section 3.3.1, a typical value of specific fuel consumption for current aircraft reciprocating engines is 0.4 lb of fuel consumed per horsepower per hour. In consistent units, noting that 1 hp = 550 ft·lb/s, we have

$$c = 0.4\frac{\text{lb}}{\text{hp·h}}\frac{1\ \text{hp}}{550\ \text{ft·lb/s}}\frac{1\ \text{h}}{3{,}600\ \text{s}}$$

$$c = 2.02 \times 10^{-7}\frac{\text{lb}}{(\text{ft·lb/s})}(\text{s}) \qquad \textbf{[8.12]}$$

A reasonable value for the propeller efficiency, assuming a variable-pitch propeller (we will make this choice now—for a business aircraft, the cost of a variable-pitch propeller over that for a cheaper, fixed-pitch propeller is justified) is, from Section 3.3.2,

$$\eta_{pr} = 0.85 \qquad \textbf{[8.13]}$$

Returning to Eq. (5.153), the ratio W_0/W_1 in that equation is replaced for the mission segment 2–3 by W_2/W_3. Hence, from Eq. (5.153),

$$R = \frac{\eta_{pr}}{c}\frac{L}{D}\ln\frac{W_2}{W_3} \tag{8.14}$$

Solving Eq. (8.14) for W_2/W_3, we have

$$\ln\frac{W_2}{W_3} = \frac{c}{\eta_{pr}}\frac{R}{L/D} \tag{8.15}$$

In Eq. (8.15), the range is stipulated in the requirements as $R = 1{,}200$ mi $= 6.64 \times 10^6$ ft. Also inserting the values given by Eqs. (8.11) to (8.13) into Eq. (8.15), we have

$$\ln\frac{W_2}{W_3} = \frac{2.02 \times 10^{-7}}{0.85}\frac{6.64 \times 10^6}{14} = 0.1127$$

Hence,

$$\frac{W_2}{W_3} = e^{0.1127} = 1.119$$

or

$$\frac{W_3}{W_2} = \frac{1}{1.119} = 0.893 \tag{8.16}$$

The loiter segment 3–4 in Fig. 8.2 is essentially the descent from cruise altitude to the landing approach. For our approximate calculations here, we will ignore the details of fuel consumption during descent, and just assume that the horizontal distance covered during descent is part of the required 1,200-mi range. Hence, for this assumption

$$\frac{W_4}{W_3} = 1 \tag{8.17}$$

Finally, the fuel consumed during the landing process, segment 4–5, is also based on historical data. The amount of fuel used for landing is small, and based on previous airplanes, the value of W_5/W_4 is approximately 0.995. Hence, we assume for our airplane that

$$\frac{W_5}{W_4} = 0.995 \tag{8.18}$$

Collecting the various segment weight fractions from Eqs. (8.9), (8.10), (8.16), (8.17), and (8.18), we have from Eq. (8.6)

$$\frac{W_5}{W_0} = \frac{W_1}{W_0}\frac{W_2}{W_1}\frac{W_3}{W_2}\frac{W_4}{W_3}\frac{W_5}{W_4} = (0.97)(0.985)(0.893)(1)(0.995) = 0.85 \tag{8.19}$$

Inserting the value of W_5/W_0 from Eq. (8.19) into Eq. (8.8), we have

$$\frac{W_f}{W_0} = 1.06\left(1 - \frac{W_5}{W_0}\right) = 1.06(1 - 0.85)$$

or

$$\frac{W_f}{W_0} = 0.159 \tag{8.20}$$

8.3.3 Calculation of W_0

Return to Eq. (8.4) for the design takeoff gross weight W_0. We have obtained a value for W_e/W_0 given by Eq. (8.5). We have also obtained a value for W_f/W_0 given by Eq. (8.20). All we need to obtain W_0 from Eq. (8.4) are values for the crew and payload weights W_{crew} and $W_{payload}$, respectively.

Corning (Ref. 55) suggests the average passenger weight of 160 lb, plus 40 lb of baggage per passenger. A more recent source is Raymer (Ref. 25) who suggests an average passenger weight of 180 lb (dressed and with carry-on bags), plus 40 to 60 lb of baggage per person in the cargo hold. For our airplane, there are five passengers and one pilot, six people in total. Let us assume the average weight per person is 170 lb. Hence, since the only crew is the pilot, we assume

$$W_{crew} = 170 \text{ lb} \tag{8.21}$$

The payload is the five passengers, plus the baggage for all six people. The type of short business trip for which this airplane will most likely be used would require less baggage than a longer, intercontinental trip. Hence, it is reasonable to assume 20 lb of baggage per person rather than the 40 lb mentioned above. Thus, including the pilot's baggage, we have

$$W_{payload} = 5(170) + 6(20) = 970 \text{ lb} \tag{8.22}$$

Inserting the values from Eqs. (8.5) and (8.20) to (8.22) into Eq. (8.4), we have

$$\begin{aligned} W_0 &= \frac{W_{crew} + W_{payload}}{1 - W_f/W_0 - W_e/W_0} = \frac{170 + 970}{1 - 0.159 - 0.62} \\ &= \frac{1{,}140}{0.221} = 1{,}140(4.525) = \boxed{5{,}158 \text{ lb}} \end{aligned} \tag{8.23}$$

This is our first estimate of the gross weight of the airplane. We have now completed pivot point 2 in Fig. 7.3.

Important comment. The calculation in Eq. (8.23) clearly shows the amplified impact of crew and payload weight on the gross weight of the airplane. The amplification factor is 4.525; that is, for every increase of 1 lb of payload weight, the airplane's gross weight increases by 4.525 lb. For example, if we had allowed each person 40 lb of baggage rather than the 20 lb we chose, the design gross weight of the airplane would have increased by $(6)(20)(4.525) = 543$ lb, that is, more than a

10% increase in the gross weight. This is a clear demonstration of the importance of weight in airplane design. For our example, 1 lb saved in any manner—payload reduction, reduced structural weight, reduced fuel weight, etc.—results in a 4.525-lb reduction in overall gross weight. It is easy to see why aeronautical engineers are so weight-conscious.

We also note that in our calculation of W_0 we have assumed that W_e/W_0 is independent of W_0, that is, independent of the gross weight of the airplane. This assumption was based on previous piston-engine airplanes, as shown in Fig. 8.1, where we chose $W_e/W_0 = 0.62$, independent of W_0. This is not usually the case for most classes of aircraft; in general, W_e/W_0 is a function of W_0. Indeed, Raymer (Ref. 25) gives empirical equations for this function for 13 different classes of aircraft. When W_e/W_0 is treated as a function of W_0, then the calculation of W_0 from Eq. (8.4) becomes an iteration. First, W_0 has to be assumed. Then W_e/W_0 is obtained for this assumed W_0. Next, a new value of W_0 is calculated from Eq. (8.4). This new value of W_0 is then used to estimate a new value of W_e/W_0, and the calculation of W_0 from Eq. (8.4) is repeated. This iteration is continued until convergence is obtained. In our calculation above, an iterative process was not required because we assumed that W_e/W_0 was a fixed value.

Finally, let us calculate the fuel weight; this will become important later in sizing the fuel tanks. From Eq. (8.20), $W_f/W_0 = 0.159$. Hence,

$$W_f = \frac{W_f}{W_0} W_0 = (0.159)(5{,}158) = 820 \text{ lb}$$

The weight of aviation gasoline is 5.64 lb/gal. Hence, the capacity of the fuel tank (or tanks) should be

$$\text{Tank capacity} = \frac{820}{5.64} = 145.4 \text{ gal}$$

8.4 ESTIMATION OF THE CRITICAL PERFORMANCE PARAMETERS

We now move to pivot point 3 in Fig. 7.3, namely, an estimation of the critical performance parameters $(C_L)_{\max}$, L/D, W/S, and T/W. These parameters are dictated by the requirements given in Section 8.2; that is, they will be determined by such aspects as maximum speed, range, ceiling, rate of climb, stalling speed, landing distance, and takeoff distance.

8.4.1 Maximum Lift Coefficient

This is the stage in the design process where we make an initial choice for the airfoil shape for the wing. Historically, general aviation airplanes have employed the NACA four-digit, five-digit, and 6-series airfoil sections—the laminar-flow series.

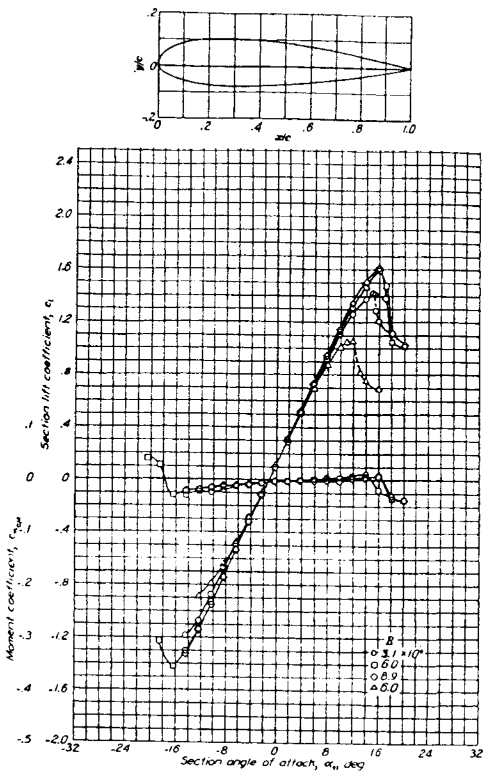

Figure 8.3 Lift coefficient, moment coefficient, and airfoil shape for the NACA 23018 airfoil.

The NACA five-digit airfoils have been particularly favored by the general aviation industry in the United States. These airfoils, such as the NACA 23018 and the NACA 23012 shown in Figs. 8.3 and 8.4, respectively, were designed in the middle 1930s. The maximum camber was placed closer to the leading edge (at $0.15c$ for the two airfoils shown) than was the case for the earlier NACA four-digit airfoils. A benefit of this design is a higher $(c_l)_{\max}$ compared to the earlier airfoils. A disadvantage is the very sharp stalling behavior, as seen in Figs. 8.3 and 8.4.

For many airplanes, including some general aviation aircraft, one airfoil section is used at the wing root, and another airfoil shape is used at the wing tip, with the airfoil

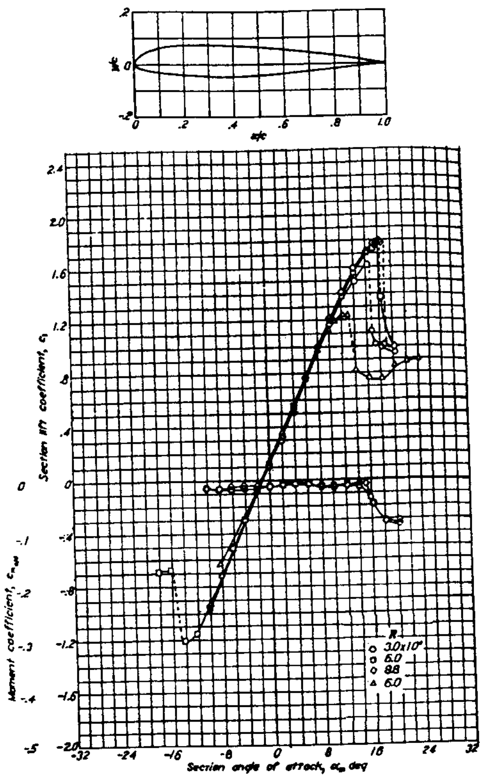

Figure 8.4 Lift coefficient, moment coefficient, and airfoil shape for the NACA 23012 airfoil.

sections between the root and tip being a linear interpolation between the root and tip sections. Several examples from existing general aviation airplanes are tabulated below.

Airplane	Root Section	Tip Section
Beechcraft Bonanza	NACA 23016.5	NACA 23012
Beechcraft Baron	NACA 23015.5	NACA 23010.5
Cessna Caravan	NACA 23017.4	NACA 23012
Piper Cheyene	NACA 63A415	NACA 63A212

In these examples, the root airfoil section is relatively thick (about 15% to 17%), and the wing airfoil shape tapers to a thinner section at the tip (about 12%). There are good reasons for this. Structurally, the wing bending moment is greatest at the root; a thicker airfoil readily allows the design for greater structural strength at the root. Aerodynamically, an 18% airfoil will stall at a lower angle of attack than a 12% airfoil. Hence, a wing which has airfoil sections which taper from 18% thick at the root to 12% thick at the tip will tend to stall first at the wing root, with attached flow still at the tip. The resulting buffeting that occurs at stall at the root is a warning to the pilot, while at the same time the ailerons remain effective because flow is still attached at the tip—both distinct advantages. Finally, a thicker wing section at the root allows additional volume for the storage of fuel in the wing.

For all these reasons, we make an initial choice of the airfoil section for our airplane design as follows: at the root, an NACA 23018 section (Fig. 8.3); at the tip, an NACA 23012 section (Fig. 8.4). We will assume that a linear interpolation between the root and tip defines the local airfoil sections elsewhere along the wing. The resulting $(C_L)_{\max}$ for the wing will be an average of the root and tip section values, depending on the planform taper ratio and the degree of geometric twist of the wing (if there is any). Also $(C_L)_{\max}$ for the finite wing is less than that for the airfoil due to three-dimensional flow effects. Since we have not laid out the planform shape or twist distribution yet, we will assume that $(C_L)_{\max}$ is a simple average of those for the airfoil sections at the root and tip, reduced by 10% for the effect of a finite aspect ratio. For the NACA 23018, from Fig. 8.3, $(c_l)_{\max} = 1.6$; for the NACA 23012, from Fig. 8.4, $(c_l)_{\max} = 1.8$. Taking the average, we have for the averge airfoil maximum lift coefficient for our wing

$$\text{Average } (c_l)_{\max} = \frac{1.6 + 1.8}{2} = 1.7$$

To aid in the takeoff and landing performance, we will design the wing with trailing-edge flaps. For simplicity (and hence production cost savings), we choose a simple plain flap. From Fig. 5.28, such a flap deflected 45° will yield an increase in the airfoil maximum lift coefficient $\Delta(c_l)_{\max} = 0.9$. Hence, for our average airfoil maximum lift coefficient, we have

$$\text{Average } (c_l)_{\max} \text{ with } 45^\circ \text{ flap deflection } = 1.7 + 0.9 = 2.6$$

Finally, to account for the three-dimensional effect of the finite aspect ratio, Raymer (Ref. 25) suggests that, for finite wings with aspect ratio greater than 5,

$$(C_L)_{\max} = 0.9(c_l)_{\max} \qquad \textbf{[8.24]}$$

Since we are designing a low-speed business, general aviation airplane, where efficient cruise is important, we most certainly will have a wing with an asepct ratio greater than 5. Hence, we use, as a preliminary estimate of maximum lift coefficient, from

Eq. (8.24)

$$(C_L)_{\max} = 0.9(2.6)$$

$$\boxed{(C_L)_{\max} = 2.34} \qquad [8.25]$$

We will treat this as $(C_L)_{\max}$ for the complete airplane, ignoring for the time being the effect of the fuselage, tail, and other parts of the configuration.

8.4.2 Wing Loading *WS*

In most airplane designs, wing loading is determined by considerations of V_{stall} and landing distance. However, W/S also plays a role in the maximum velocity of the airplane [see Eq. (5.50)]; $V_{\max}$ increases as W/S increases. For our current airplane design, which is a low-speed aircraft, the primary constraints on W/S will be V_{stall} and landing, and we will take that approach. From Eq. (5.67), repeated here:

$$V_{\text{stall}} = \sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{(C_L)_{\max}}} \qquad [5.67]$$

solving Eq. (5.67) for W/S, we have

$$\frac{W}{S} = \frac{1}{2}\rho_\infty V_{\text{stall}}^2 (C_L)_{\max} \qquad [8.26]$$

The requirements specify $V_{\text{stall}} \leq 70\text{m}i/h = 102.7$ ft/s. Using $(C_L)_{\max}$ from Eq. (8.25) and making the calculation at sea level, where $\rho_\infty = 0.002377$ slug/ft^3, we have from Eq. (8.26)

$$\frac{W}{S} = \frac{1}{2}(0.002377)(102.7)^2(2.34) = 29.3 \text{ lb/ft}^2 \qquad [8.27]$$

Equation (8.27) gives us the value of W/S constrained by the stalling velocity.

Let us examine the constraint imposed by the specified landing distance. In Fig. 6.17, the landing distance is the sum of the approach distance s_a, the flare distance s_f, and the ground roll s_g. The approach angle θ_a is given by Eq. (6.104), which requires knowledge of L/D and T/W. Since we have not made estimates of either quantity yet, we assume, based on the rule of thumb that $\theta_a \leq 3°$ for transport aircraft, that $\theta_a = 3°$. Following the discussion of approach distance in Section 6.8.1, we have, from Eq. (6.107) for the flight path radius during flare,

$$R = \frac{V_f^2}{0.2g} \qquad [6.107]$$

In Eq. (6.107), V_f is the average velocity during flare, given by $V_f = 1.23V_{\text{stall}}$. From our design, $V_f = 1.23(102.7) = 126.3$ ft/s. From Eq. (6.107)

$$R = \frac{(126.3)^2}{0.2(32.2)} = 2{,}477 \text{ ft}$$

From Eq. (6.106), the flare height h_f is given by

$$h_f = R(1 - \cos\theta_a) = 2{,}477(1 - \cos 3°) = 3.4 \text{ ft}$$

Finally, from Eq. (6.108), the approach distance required to clear a 50-ft obstacle is given by

$$s_a = \frac{50 - h_f}{\text{Tan}\,\theta_a} = \frac{50 - 3.4}{\text{Tan}\,3°} = 889 \text{ ft}$$

The flare distance s_f is given by Eq. (6.109):

$$s_f = R\sin\theta_a = 2{,}477 \sin 3° = 130 \text{ ft}$$

An approximation for ground roll s_g is given by Eq. (6.123). In that equation, let us assume that lift has been intentionally made small by retracting the flaps combined with a small angle of attack due to the rather level orientation of the airplane relative to the ground. (We are assuming that we will use tricycle landing gear for the airplane.) Furthermore, assuming no provision for thrust reversal, and ignoring the drag compared to the friction force between the tires and the ground, Eq. (6.123) simplifies further to

$$s_g = jN\sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{(C_L)_{\max}}} + \frac{j^2(W/S)}{g\rho_\infty(C_L)_{\max}\mu_r} \qquad [8.28]$$

As stated in Section 6.8.3, $j = 1.15$ for commercial airplanes. Also, N is the time increment for free roll immediately after touchdown, before the brakes are applied. By assuming $N = 3$ s and $\mu = 0.4$, Eq. (8.28) becomes

$$s_g = (1.15)(3)\sqrt{\frac{2}{0.002377}\frac{W}{S}\frac{1}{2.34}} + \frac{(1.15)^2(W/S)}{(32.2)(0.002377)(2.34)(0.4)}$$

or

$$s_g = 65.4\sqrt{\frac{W}{S}} + 18.46\frac{W}{S} \qquad [8.29]$$

Since the allowable landing distance is specified in the requirements as 2,200 ft, and we have previously estimated that $s_a = 889$ ft and $s_f = 130$ ft, the allowable value for s_g is

$$s_g = 2{,}200 - 889 - 130 = 1{,}181 \text{ ft}$$

Inserting this value for s_g into Eq. (8.29), we have

$$1{,}181 = 65.4\sqrt{\frac{W}{S}} + 18.46\frac{W}{S} \qquad [8.30]$$

Equation (8.30) is a quadratic equation for $\sqrt{W/S}$. Using the quadratic formula, we obtain

$$\frac{W}{S} = 41.5 \text{ lb/ft}^2 \qquad [8.31]$$

Compare the value of $W/S = 41.5$ lb/ft^2 obtained from the landing distance constraint, Eq. (8.31), with the value of $W/S = 29.3$ lb/ft^2 obtained from the stall constraint, Eq. (8.27). Clearly, if $W/S < 41.5$ lb/ft^2, the landing distance will be shorter than 2,200 ft, clearly satisfying the requirements. Hence, for our airplane design, W/S is determined from the specified stall velocity, namely,

$$\boxed{\frac{W}{S} = 29.3 \text{ lb/ft}^2} \qquad \textbf{[8.32]}$$

The value of W/S from Eq. (8.32) along with that for W_0 from Eq. (8.23) allows us to obtain the wing area.

$$S = \frac{W_0}{W/S} = \frac{5{,}158}{29.3} = \boxed{176 \text{ ft}^2} \qquad \textbf{[8.33]}$$

8.4.3 Thrust-to-Weight Ratio

The value of T/W determines in part the takeoff distance, rate of climb, and maximum velocity. To obtain the design value of T/W, we have to examine each of these three constraints.

First, let us consider the takeoff distance, which is specified as 2,500 ft to clear a 50-ft obstacle. Using Eq. (6.95) to estimate the ground roll, we have

$$s_g = \frac{1.21(W/S)}{g\rho_\infty (C_L)_{\max}(T/W)} \qquad \textbf{[6.95]}$$

where $(C_L)_{\max}$ in Eq. (6.95) is that value with the flaps only partially extended, consistent with their takeoff setting. Hence, we need to recalculate $(C_L)_{\max}$ for this case. Following the guidance provided in Table 5.3, we assume a flap deflection of 20° for takeoff. To return to Fig. 5.28, the $\Delta(c_l)_{\max}$ for a 45° flap deflection is 0.9. Assuming a linear variation of $\Delta(c_l)_{\max}$ with flap deflection angle, we have for takeoff $\Delta(c_l)_{\max} = 0.9(25/45) = 0.5$. Hence, for the wing, the average $(c_l)_{\max}$ with a 20° flap deflection is $1.7 + 0.5 = 2.2$. Taking into account the finite aspect ratio, as discussed in Section 8.4.1, we have for the wing

$$(C_L)_{\max} = 0.9(c_l)_{\max} = 0.9(2.2) = 1.98$$

This is the takeoff value of $(C_L)_{\max}$ that will be used in Eq. (6.95). Returning to Eq. (6.95), we have

$$s_g = \frac{1.21(W/S)}{g\rho_\infty (C_L)_{\max}(T/W)} = \frac{(1.21)(29.3)}{(32.2)(0.002377)(1.98)(T/W)} = \frac{233.9}{T/W} \qquad \textbf{[8.34]}$$

Recall from our discussion in Section 6.7.1 that when T varies with velocity, as it does for a propeller-driven airplane, the value of T/W in Eq. (6.95) is assumed to be that for a velocity $V_\infty = 0.7V_{LO}$, where V_{LO} is the liftoff velocity, taken as $V_{LO} = 1.1V_{\text{stall}}$.

To calculate the distance while airborne to clear an obstacle (Section 6.7.2), we need the value of V_{stall} corresponding to the $(C_L)_{\max}$ with flaps in the takeoff position, that is, corresponding for our case to $(C_L)_{\max} = 1.98$. From Eq. (5.67)

$$V_{\text{stall}} = \sqrt{\frac{2}{\rho_\infty}\frac{W}{S}\frac{1}{(C_L)_{\max}}} = \sqrt{\frac{2(29.3)}{(0.002377)(1.98)}} = 111.6 \text{ ft/s}$$

From Eq. (6.98), the flight path radius is

$$R = \frac{6.96(V_{\text{stall}})^2}{g} = \frac{6.96(111.6)^2}{32.2} = 2{,}692 \text{ ft}$$

From Eq. (6.99), the included flight path angle is

$$\theta_{OB} = \text{Cos}^{-1}\left(1 - \frac{h_{OB}}{R}\right) \qquad \textbf{[6.99]}$$

where h_{OB} is the obstacle height, $h_{OB} = 50$ ft, so

$$\theta_{OB} = \text{Cos}^{-1}\left(1 - \frac{50}{2{,}692}\right) = 11.06°$$

From Eq. (6.100), the airborne distance is

$$s_a = R\sin\theta_{OB} = 2{,}692 \sin 11.06° = 516.4 \text{ ft} \qquad \textbf{[8.35]}$$

Combining Eqs. (8.34) and (8.35), we have

$$s_g + s_a = 2{,}500 = \frac{233.9}{T/W} + 516.4$$

or

$$\left(\frac{T}{W}\right)_{0.7V_{LO}} = \frac{233.9}{2{,}500 - 516.4} = 0.118 \qquad \textbf{[8.36]}$$

This is the value of required T/W at a velocity

$$V_\infty = 0.7V_{LO} = 0.7(1.1V_{\text{stall}}) = 0.7(1.1)(111.6) = 85.9 \text{ ft/s}$$

At this velocity, the power required to take off at the gross weight $W_0 = 5{,}158$ lb [see Eq. (8.23)] is

$$P_R = TV_\infty = \frac{T}{W}W_0V_\infty = (0.118)(5{,}158)(85.9) = 5.228 \times 10^4 \text{ ft·lb/s} \qquad \textbf{[8.37]}$$

This power required must equal the power available P_A, obtained from Eq. (3.13).

$$P_A = \eta_{\text{pr}}P \qquad \textbf{[3.13]}$$

Solving Eq. (3.13) for the shaft brake power P, we have

$$P = \frac{P_A}{\eta_{\text{pr}}} \qquad \textbf{[8.38]}$$

Typical propeller efficiencies are shown in Fig. 3.7. In our design we choose to use a constant-speed propeller. Hence, from Fig. 3.7, it appears reasonable to assume

$\eta_{pr} = 0.8$. Hence, the shaft brake power from the engine should be at least [from Eq. (8.38)]

$$P = \frac{P_A}{\eta_{pr}} = \frac{5.228 \times 10^4}{0.8} = 6.535 \times 10^4 \text{ ft·lb/s}$$

Since 550 ft·lb/s = 1 hp, we have

$$P = \frac{6.535 \times 10^4}{550} = 118.8 \text{ hp}$$

As stated in Section 3.3.1, for a reciprocating engine P is reasonably constant with V_∞. Hence, to satisfy the takeoff constraint, the total power must be at least

$$P \geq 118.8 \text{ hp} \qquad \textbf{[8.39]}$$

Next, let us consider the constraint due to the specified rate of climb of 1,000 ft/min at sea level. Here, we need to make an estimate of the zero-lift drag coefficient $C_{D,0}$. We will use the same approach as illustrated in Example 2.4. From Fig. 2.54, for single-engine general aviation airplanes, the ratio of wetted area to the wing reference area is approximately $S_{wet}/S_{ref} = 4$. The skin-friction coefficient C_{fe} is shown as a function of Reynolds number in Fig. 2.55, where some data points for various jet airplanes are also plotted. Our airplane design will probably be about the same size as that of some early jet fighters, but with about one-third the speed. Hence, based on mean length, a relevant Reynolds number for us is about 10^7. For this case, Fig. 2.55 yields

$$C_{fe} = 0.0043$$

Hence, from Eq. (2.37)

$$C_{D,0} = \frac{S_{wet}}{S} C_{fe} = (4)(0.0043)$$

or

$$\boxed{C_{D,0} = 0.017} \qquad \textbf{[8.40]}$$

We also need an estimate for the coefficient K that appears in the drag polar, Eq. (2.47), repeated here:

$$C_D = C_{D,0} + KC_L^2 \qquad \textbf{[2.47]}$$

where, from Eqs. (2.43) to (2.46),

$$K = k_1 + k_2 + k_3 = k_1 + k_2 + \frac{C_L^2}{\pi e \text{AR}} \qquad \textbf{[8.41]}$$

In Eq. (8.41), e is the span efficiency factor to account for a nonelliptical lift distribution along the span of the wing, and $C_L^2/(\pi e\text{AR})$ is the induced drag coefficient.

Let us estimate the value of K to be consistent with the earlier assumed value of $(L/D)_{max} = 14$ [see Eq. (8.11)]. From Eq. (5.30),

$$\left(\frac{L}{D}\right)_{max} = \sqrt{\frac{1}{4C_{D,0}K}} \qquad \textbf{[5.30]}$$

we have

$$K = \frac{1}{4C_{D,0}(L/D)^2_{max}} = \frac{1}{4(0.017)(14)^2}$$

or

$$\boxed{K = 0.075} \qquad \textbf{[8.42]}$$

This estimate for K also allows an estimate of the aspect ratio, as follows. It is conventional to define another efficiency factor, the Oswald efficiency e_o, as

$$\frac{C_L^2}{\pi e_o \text{AR}} \equiv k_1 + k_2 + \frac{C_L^2}{\pi e \text{AR}} \equiv KC_L^2 \qquad \textbf{[8.43]}$$

A reasonable estimate for e_o for a low-wing general aviation airplane is 0.6 (see McCormick, Ref. 50). From Eq. (8.42),

$$\text{AR} = \frac{1}{\pi e_o K} = \frac{1}{\pi(0.6)(0.075)}$$

or

$$\boxed{\text{AR} = 7.07} \qquad \textbf{[8.44]}$$

Finally, to return to the consideration of rate of climb, Eq. (5.122) gives an expression for maximum rate of climb for a propeller-driven airplane as

$$(R/C)_{max} = \frac{\eta_{pr}P}{W} - \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}\frac{W}{S}}\right)^{1/2} \frac{1.155}{(L/D)_{max}} \qquad \textbf{[5.122]}$$

Solving for the power term, we have

$$\frac{\eta_{pr}P}{W} = (R/C)_{max} + \left(\frac{2}{\rho_\infty}\sqrt{\frac{K}{3C_{D,0}}\frac{W}{S}}\right)^{1/2} \frac{1.155}{(L/D)_{max}} \qquad \textbf{[8.45]}$$

Everything on the right side of Eq. (8.45) is known, including $(R/C)_{max}$ which from the specifications is 1,000 ft/min = 16.67 ft/s at sea level. Hence, from Eq. (8.45),

$$\frac{\eta_{pr}P}{W} = 16.67 + \left[\frac{2}{0.002377}\sqrt{\frac{0.075}{3(0.017)}}(29.3)\right]^{1/2} \frac{1.155}{14} \qquad \textbf{[8.45a]}$$

$$\frac{\eta_{pr}P}{W} = 16.67 + 14.26 = 30.93 \text{ ft/s}$$

Assuming W is equal to the takeoff gross weight $W_0 = 5{,}158$ lb (ignoring the small amount of fuel burned during the takeoff run), and recalling our estimate of $\eta_{pr} = 0.8$, we have from Eq. (8.45)

$$P = \frac{30.93 W_0}{\eta_{pr}} = \frac{(30.93)(5{,}158)}{0.8} = 1.994 \times 10^5 \text{ ft·lb/s}$$

In terms of horsepower,

$$P = \frac{1.994 \times 10^5}{550} = 362.5 \text{ hp} \qquad \textbf{[8.46]}$$

Thus, to satisfy the constraint on rate of climb, the power must be

$$P \geq 362.5 \text{ hp} \qquad \textbf{[8.47]}$$

The third constraint on T/W (or P/W) is the maximum velocity V_{max}. The requirements stipulate $V_{max} = 250$ mi/h $= 366.7$ ft/s at midcruise weight. The altitude for the specified V_{max} is not stated. However, the requirements call for a pressurized cabin, and we can safely assume that an altitude of 20,000 ft would be comfortable for the pilot and passengers. Therefore, we assume that the specified V_{max} is associated with level flight at 20,000 ft. In level flight, $T = D$, and the drag D is given by Eq. (5.12)

$$T = D = \frac{1}{2}\rho_\infty V_\infty^2 S C_{D,0} + \frac{2KS}{\rho_\infty V_\infty^2}\left(\frac{W}{S}\right)^2 \qquad \textbf{[5.12]}$$

Couching Eq. (5.12) in terms of the thrust-to-weight ratio, we have

$$\frac{T}{W} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D,0}}{W/S} + \frac{2K}{\rho_\infty V_\infty^2}\frac{W}{S} \qquad \textbf{[8.48]}$$

Since the requirements stipulate V_{max} *at midcruise weight*, the value of W that appears in Eq. (8.48) is less than the gross takeoff weight. To return to our weight estimates in Section 8.3, W_2 and W_3 are the weights at the beginning and end of cruise, respectively. We have, from Section 8.3.2,

$$\frac{W_2}{W_0} = \frac{W_1}{W_0}\frac{W_2}{W_1} = (0.97)(0.985) = 0.955$$

Hence,

$$W_2 = 0.955 W_0 = 0.955(5{,}158) = 4{,}926 \text{ lb}$$

At midcruise (defined here as when one-half of the fuel needed to cover the full cruise range is consumed), we have for the midcruise weight W_{MC}

$$W_{MC} = W_2 - \tfrac{1}{2}(W_2 - W_3)$$

or

$$\frac{W_{MC}}{W_2} = \frac{1}{2}\left(1 + \frac{W_3}{W_2}\right) \qquad \textbf{[8.49]}$$

The weight fraction W_3/W_2 has been estimated in Eq. (8.16) as $W_3/W_2 = 0.893$. Hence, from Eq. (8.49)

$$\frac{W_{MC}}{W_2} = \frac{1}{2}(1 + 0.893) = 0.9465$$

Since $W_2 = 4{,}926$ as obtained above, we have

$$W_{MC} = (0.9465)(4{,}926) = 4{,}662 \text{ lb} \qquad \textbf{[8.50]}$$

This weight is used to define the new wing loading that goes into Eq. (8.48). This value is [recalling from Eq. (8.33) that $S = 176$ ft^2]

$$\frac{W_{MC}}{S} = \frac{4{,}662}{176} = 26.5 \text{ lb/ft}^2$$

Returning to Eq. (8.48), written in terms of the midcruise weight, we have

$$\frac{T}{W_{MC}} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{D,0}}{W_{MC}/S} + \frac{2K}{\rho_\infty V_\infty^2}\frac{W_{MC}}{S} \qquad \textbf{[8.51]}$$

From Appendix B, at 20,000 ft, $\rho_\infty = 0.0012673$ slug/ft^3. Also, inserting $V_{max} = 366.7$ ft/s for V_∞ in Eq. (8.51), we have

$$\frac{T}{W_{MC}} = \frac{1}{2}(0.0012673)(366.7)^2\frac{0.017}{26.5} + \frac{2(0.075)(26.5)}{(0.0012673)(366.7)^2} \qquad \textbf{[8.52]}$$

$$= 0.0547 + 0.0233 = 0.0780$$

Comment: It is interesting to note that the two terms on the right side of Eq. (8.51) represent the effects of zero-lift drag and drag due to lift, respectively. In the above calculation, the zero-lift drag is about 2.3 times larger than the drag due to lift. This is consistent with the usual situation that as speed increases, the drag due to lift becomes a smaller percentage of the total drag. In this case, the drag due to lift is $0.0233/0.0780 = 0.3$ of the total drag, or less than one-third of the total drag.

The shaft power required P is given by

$$\eta_{pr} P = T V_\infty \qquad \textbf{[8.53]}$$

At V_{max} at midcruise weight, Eq. (8.53) is written as

$$P = \frac{1}{\eta_{pr}}\frac{T}{W_{MC}} W_{MC} V_{max} = \frac{1}{0.8}(0.0780)(4{,}662)(366.7) = 1.667 \times 10^5 \text{ ft·lb/s}$$

In terms of horsepower,

$$P = \frac{1.667 \times 10^5}{550} = 303.1 \text{ hp} \qquad \textbf{[8.54]}$$

To summarize the results from this section, the three constraints on power required for our airplane design have led to the following:

Takeoff	$P \geq 118.8$ hp
Rate of climb	$P \geq 362.5$ hp
Maximum velocity	$P \geq 303.1$ hp

Clearly, the specification of the maximum rate of climb at sea level of 1,000 ft/min is the determining factor of the required power from the engine. For our airplane design, the engine should be capable of producing a maximum power of 362.5 hp or greater.

We can couch this result in terms of the more relevant performance parameters T/W or P/W. When these parameters are quoted for a given airplane, the weight is usually taken as the gross takeoff weight W_0. Hence, for our design

$$\text{Power-to-weight ratio} = \frac{362.5 \text{ hp}}{5{,}158 \text{ lb}} = 0.07 \text{ hp/lb}$$

For a propeller-driven airplane, the power-to-weight ratio is more relevant than the thrust-to-weight ratio, which makes more sense to quote for jet airplanes. For a reciprocating engine/propeller-driven airplane, the shaft power is essentially constant with velocity, whereas the thrust decreases with velocity, as discussed in Chapter 3. Hence, for a reciprocating engine/propeller-driven airplane, to quote the power-to-weight ratio makes more sense. In the aeronautical literature, historically the *power loading*, which is the reciprocal of the power-to-weight ratio, is frequently given.

$$\text{Power loading} \equiv \frac{W}{P}$$

The definition of the power loading is semantically analogous to that for the wing loading W/S. The wing loading is the weight divided by wing area; the power loading is the weight divided by the power. For our airplane, we have estimated that

$$\text{Power loading } \frac{W}{P} = \frac{1}{0.07} = 14.3 \text{ lb/hp}$$

[We note that Raymer (Ref. 25) quotes a typical value of 14 lb/hp for general aviation single-engine airplanes—so our estimation appears to be very reasonable.]

There is something important that is implicit in our discussion of the engine characteristics. Although the engine is sized at 362.5 hp to meet the rate-of-climb specification at sea level, it must also produce 303.1 hp at 20,000 ft to achieve the specified maximum velocity. Since the power of a conventional reciprocating engine is proportional to the air density [see Eq. (3.11)], such a conventional engine sized at 362.5 hp at sea level will produce only 193 hp at 20,000 ft—clearly unacceptable for meeting our specifications. Hence, the engine for our airplane must be *supercharged* to maintain sea-level power to an altitude of 20,000 ft.

8.5 SUMMARY OF THE CRITICAL PERFORMANCE PARAMETERS

We have now completed pivot point 3 in Fig. 7.3, namely, the first estimate of the critical performance parameters from airplane design. They are summarized as follows:

Maximum lift coefficient	$(C_L)_{\max} = 2.34$
Maximum lift-to-drag ratio	$\left(\dfrac{L}{D}\right)_{\max} = 14$
Wing loading	$\dfrac{W}{S} = 29.3 \text{ lb/ft}^2$
Power loading	$\dfrac{W}{P} = 14.3 \text{ lb/hp}$

In the process of estimating these performance parameters, we have found other characteristics of our airplane:

Takeoff gross weight	$W_0 = 5{,}158$ lb
Fuel weight	$W_f = 820$ lb
Fuel tank capacity	145.4 gal
Wing area	$S = 176 \text{ ft}^2$
High-lift device	Single-slotted trailing-edge flaps
Zero-lift drag coefficient	$C_{D,0} = 0.017$
Drag-due-to-lift coefficient	$K = 0.075$
Aspect ratio	AR = 7.07
Propeller efficiency	0.8
Engine power, supercharged to 20,000 ft	362.5 hp

We are now ready to draw a picture of our airplane design, that is, to construct a configuration layout. This is the subject of the next section.

8.6 CONFIGURATION LAYOUT

We now move to pivot point 4 in Fig. 7.3—the configuration layout. Based on the information we have calculated so far in this chapter, we are ready to draw a picture, with dimensions, of our airplane. Even though the data summarized in Section 8.5 clearly define a certain type of airplane, there are still an infinite number of different sizes and shapes that could satisfy these data. There are no specific laws or rules that