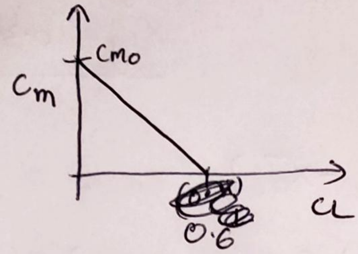
⇒ Tail Calculations

⇒ Goal: To achieve Static Margin of 15%.

assuming $(C_L)_{eq\ by\ design} = 0.6$ $\left[C_L = \frac{(W/S)}{\frac{1}{2}\rho V^2} \right]$ based on mission altitude and velocity.

→



we know that

$$SM = -\frac{\partial C_m}{\partial C_L}$$

[with assumption of $\left(\frac{\partial C_L}{\partial \alpha}\right)_{a/c} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{wing}$]

→ $C_{mo} = C_m$ at $\alpha=0$ about CG

$$\frac{\partial C_m}{\partial C_L} = -0.15$$

~~$C_m = C_{mo} + C_{m_\alpha} C_L$~~

$$C_m = C_{mo} + \frac{\partial C_m}{\partial C_L} C_L$$

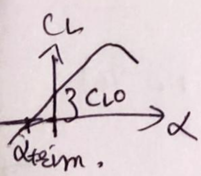
$$\therefore 0 = C_{mo} - 0.15(0.6)$$

$$\boxed{C_{mo} = 0.09}$$ ← This is the required C_{mo} given SM.

→ consider only wing

$$(C_m) = C_{mo} + (C_{m_\alpha})\alpha$$

where $C_{mo} = C_{mac,w} + C_{Lo,w}[\bar{X}_{cg} - \bar{X}_{ac}]$



let $\alpha_{trim} = -2.5°$

$(C_{l\alpha})_{airfoil} = 5.9$/rad

$AR = 10$

$e = 0.7$

$C_{mac,airfoil} = -0.05$

} for a cambered flat bottom airfoil. ⇒

3D

$$(C_{L\alpha})_w = \frac{5.9}{1+(5.9)K}$$

$$= 4.65/\text{rad}$$

$$C_{Lo} = (4.65)(2.5)\pi/180$$

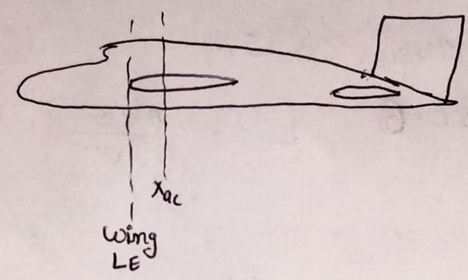
$$C_{Lo} = 0.2029$$

$$C_{mac,w} = C_{mac(airfoil)} \times \left(\frac{AR}{AR+2}\right)$$

$$= -0.05 \times \left(\frac{10}{12}\right) = -0.041$$

→ A.C lies at $c/4$ $\therefore X_{ac,w} = \frac{c}{4} = \frac{1.1}{4} = 0.275$

→ **locating CG**



(initial guess)
→ assume x_{cg} at 35% of leading edge.

$\therefore \quad \bar{x}_{cg} = 0.35$

$x_{cg} = 0.35 \times 1.1 = 0.385$

$$C_{m_{0_{wing}}} = C_{mac} + C_{L_0}\left[\bar{x}_{cg} - \bar{x}_{ac}\right]$$

$$= -0.041 + (0.2029)\left[0.35 - 0.25\right]$$

$$\boxed{C_{m_{0_{wing}}} = -0.020}$$

Since $C_{m_{0_{wing}}} \neq C_{m_{0_{eq}}}$) a tail is required.

→ **including Tail**

$C_{m_{0_{eq}}} = 0.09$ | $(C_{m_0})_{a/c \text{ with tail}} = C_{m_{0w}} + \eta V_H C_{L\alpha_t}(\epsilon_0 + i_w - i_t) + \cancel{C_{m_{0_{fusl}}}}$

$\therefore \quad (C_{m_0})_{a/c} = C_{mac} + C_{L_{0,w}}\left[\bar{x}_{cg} - \bar{x}_{ac_w}\right] + \eta V_H C_{L\alpha t}(\epsilon_0 + i_w - i_t)$

[assume $\eta = \frac{q_{tail}}{q_{wing}} = 0.9$ $\quad C_{L\alpha t} = 3.8/\text{rad}$

$V_H = \frac{l_t S_t}{S\bar{c}} = 0.6$ $\quad \epsilon_0 = \frac{2C_{L0}}{\pi AR} = 0.012 \text{ rad} = 0.74°$]

$$0.09 = -0.041 + 0.2029\left[0.35 - 0.25\right] + (0.9)(0.6)(3.8)\left[0.74 + 0 - i_t\right]\left(\frac{\pi}{180}\right)$$

$$\Rightarrow \boxed{i_t = -2.33°}$$

→ now, we shall find SM from this value and iterate x_{cg} till we converge the SM

→ To find SM ; we need NP ;

$X_{cg} = X_{NP}$ at $C_{m\alpha} = 0$.

$$(C_{m\alpha})_{a/c} = C_{L\alpha,w}(\bar{x}_{cg} - \bar{x}_{ac}) - \eta V_H C_{L\alpha t}\left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) + \cancel{(C_{m\alpha})_{fuselage}}^{0}$$

$$\frac{\partial \epsilon}{\partial \alpha} = \frac{2 C_{L\alpha w}}{\pi AR} \approx 0.2961$$

$$\therefore 0 = \left(4.65 \cdot \frac{\pi}{180}\right)(\bar{x}_{NP} - 0.25) - (0.9)(0.6)(3.8)\left(\frac{\pi}{180}\right)(1 - 0.2961)$$

$$\boxed{\bar{x}_{NP} = 0.5606}$$

$SM = \bar{x}_{NP} - \bar{x}_{cg} = 0.2106 \neq 0.15$

∴ change $\bar{x}_{cg}$;

→ let $\bar{x}_{cg} = 0.45$.

$SM = 0.5606 - 0.45 = 0.11 \neq 0.15$

for approx. $\bar{x}_{cg}$; let $SM = 0.15$

$\therefore 0.15 = 0.5606 - \bar{x}_{cg}$

$\therefore \bar{x}_{cg} = 0.41$

→ for $\bar{x}_{cg} = 0.41$

$$(C_{m0})_{a/c} = C_{mac} + C_{L0,w}(\bar{x}_{cg} - \bar{x}_{ac}) + \eta V_H C_{L\alpha t}(\epsilon_0 + i_w - i_t)$$

$$0.09 = -0.041 + 0.2029(0.41 - 0.25) + (0.9)(0.6)(3.8)\left(\frac{\pi}{180}\right)(0.74 - i_t)$$

$$\therefore \boxed{i_t = -2.01^\circ}$$

∴ $[i_t = -2.01^\circ]$; $[\bar{x}_{cg} = 0.41]$ to get 15% SM and 0.09 C_{m_0}