Bode plot of an Integrator

We start with finding the Bode plot asymptotes for a simple system characterized by

$$G(s)=\frac{1}{s} \quad \Rightarrow \quad G(j\omega)=\frac{1}{j\omega}=|G(j\omega)|\angle G(j\omega)=\frac{1}{|\omega|}\angle -90^{\circ}$$

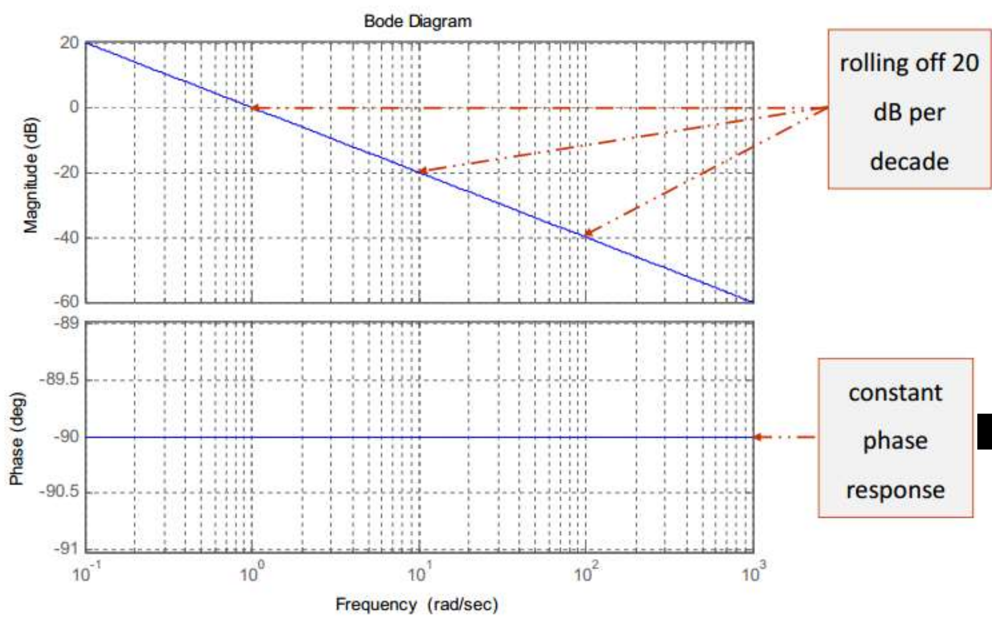
Examining the amplitude in dB scale, i.e.,

$$20\cdot\log_{10}|G(j\omega)|=20\cdot\log_{10}\frac{1}{|\omega|}=-20\cdot\log_{10}|\omega|\ \text{dB}$$

$$\omega=1 \quad \Rightarrow \quad 20\cdot\log_{10}|G(j1)|=-20\cdot\log_{10}1=0\ \text{dB}$$

$$\omega=10 \quad \Rightarrow \quad 20\cdot\log_{10}|G(j10)|=-20\cdot\log_{10}10=-20\ \text{dB}$$

Thus, the above expressions clearly indicate that the magnitude is reduced by -20dB when the frequency is increased by 10 times. It is equivalent to say that the magnitude is rolling off 20dB per decade



The phase response of an integrator is -90 degrees, a constant.

Bode plot of a Diffrentiator

We start with finding the Bode plot asymptotes for a simple system characterized by

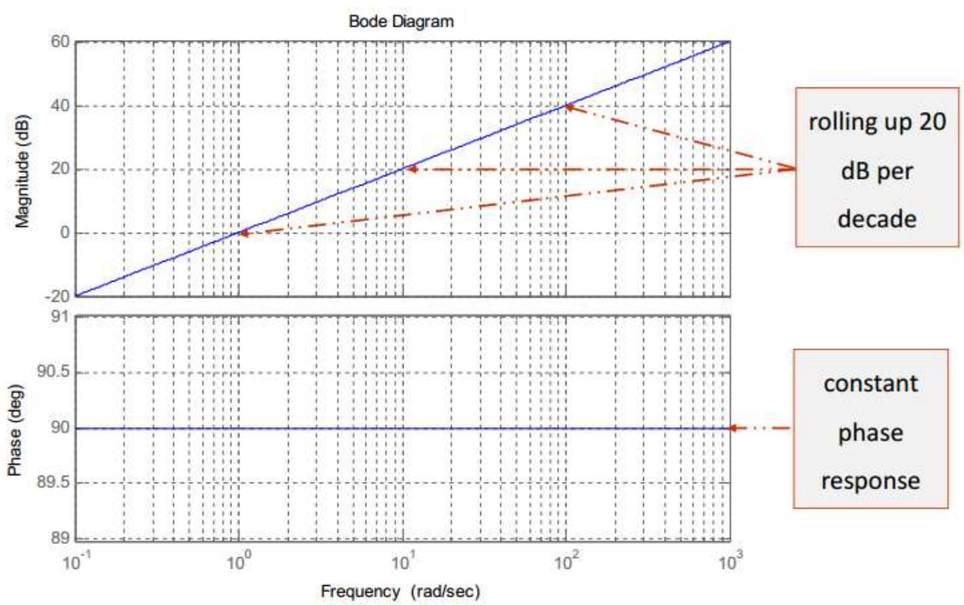
$$G(s) = s \Rightarrow G(j\omega) = j\omega = |G(j\omega)|\angle G(j\omega) = |\omega|\angle 90^0$$

Examining the amplitude in dB scale, i.e.,

$$20\log_{10}|G(j\omega)| = 20.\log_{10}|\omega| = 20\log_{10}|\omega| dB$$

$$\omega = 1 \Rightarrow 20\log_{10}|G(j1)| = 20\log_{10}1 = 0dB$$

$$\omega = 10 \Rightarrow 20\log_{10}|G(j10)| = 20\log_{10}10 = 20dB$$



Thus, the above expressions clearly indicate that the magnitude is increased by 20dB when the frequency is increased by 10 times. It is equivalent to say that the magnitude is rolling up 20dB per decade

The phase response of an integrator is 90 degrees, a constant.

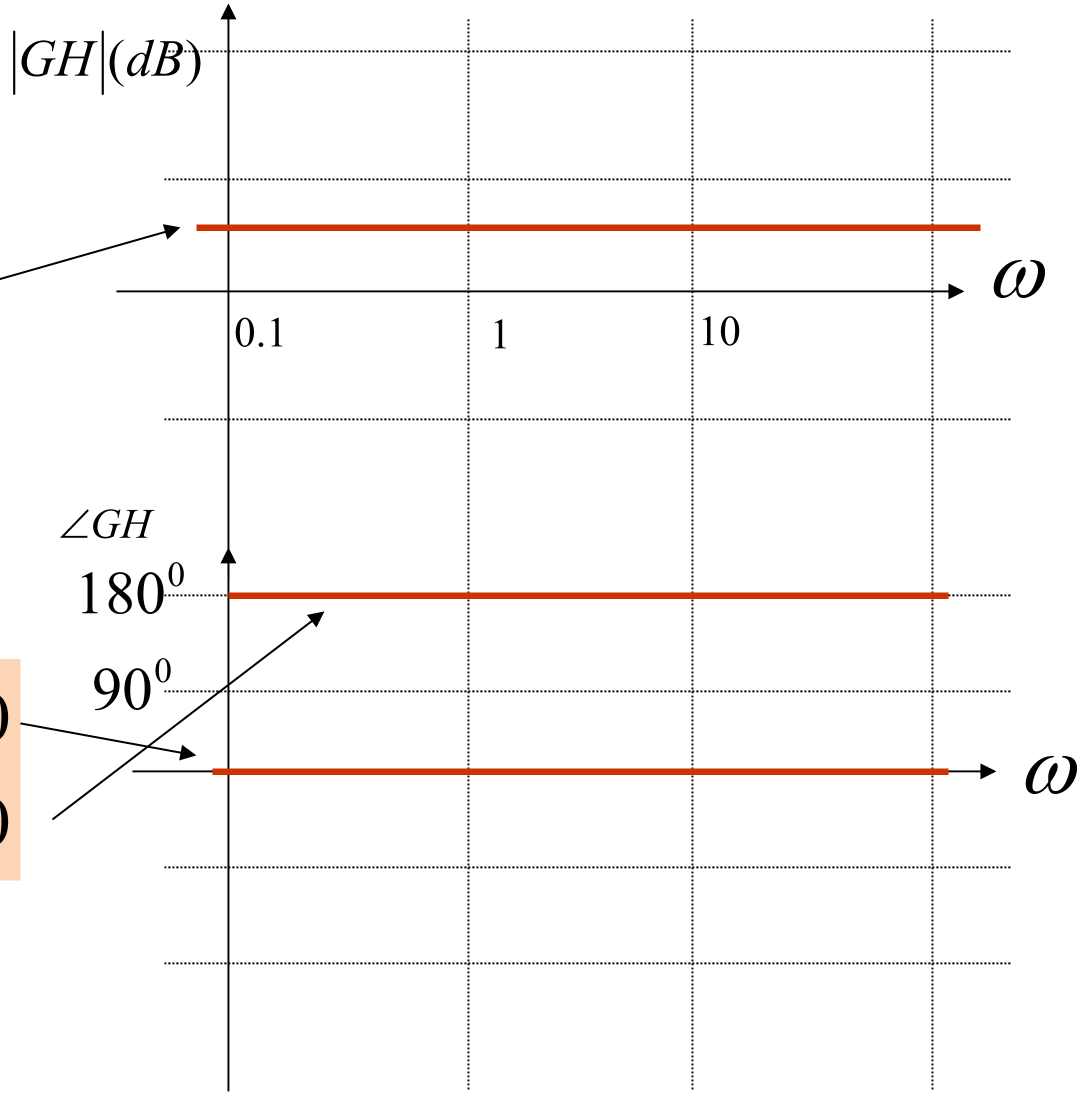
Example : k

Magnitude:

$$|k|_{dB} = 20\log|k|(dB)$$

Phase:

$$\angle k = \begin{cases} 0^o & ,k \succ 0 \\ 180^o & ,k \prec 0 \end{cases}$$



Example:

$$G(s)=\frac{a}{(s+a)}$$

Magnitude:

$$\left|(1+j\frac{\omega}{a})^{-1}\right|_{dB}=-20\log\sqrt{1+(\frac{\omega}{a})^2}$$

$$=-10\log[1+(\frac{\omega}{a})^2]$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a}\approx 0 \Rightarrow dB=-10\log 1=0$$

$$\omega \succ\succ a \Rightarrow 1+j\frac{\omega}{a}\approx\frac{\omega}{a}\Rightarrow dB\approx -20\log\frac{\omega}{a}$$

$$dB=-[20\log\omega-20\log a]$$

$$\omega=a\Rightarrow 1+j1\Rightarrow dB=-10\log 2=-3.01$$

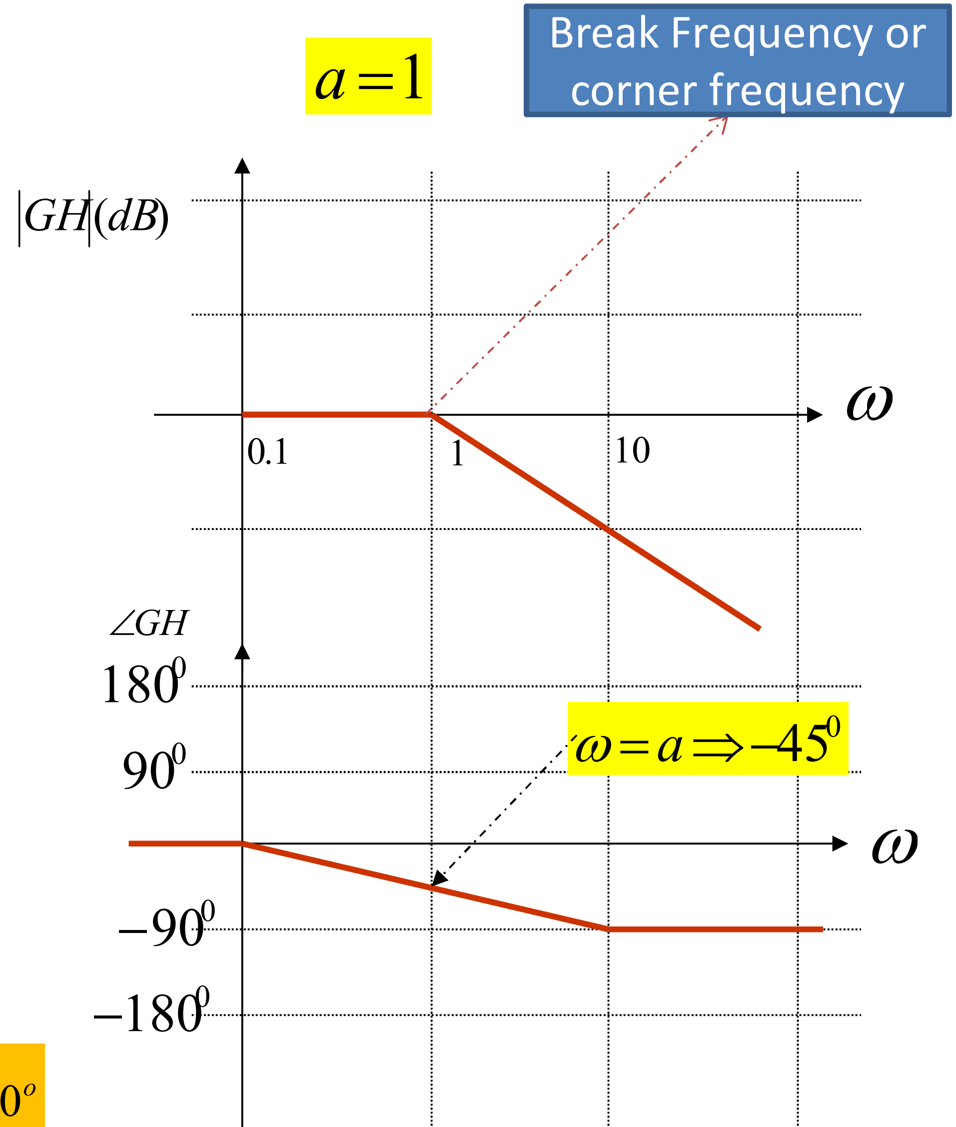
Phase:

$$\angle(1+j\frac{\omega}{a})=0^0-\tan^{-1}\frac{\omega}{a}$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a}\approx 0 \Rightarrow \angle GH\approx \tan^{-1}0=0^o$$

$$\omega \succ\succ a \Rightarrow \frac{\omega}{a}\approx \infty \Rightarrow \angle GH\approx -\tan^{-1}\infty=-90^o$$

$a=1$



Example: $G(s) = \dfrac{10}{(s+10)}$

Magnitude: $\left|(1+j\dfrac{\omega}{10})^{-1}\right|_{dB} = -20\log\sqrt{1+(\dfrac{\omega}{10})^2}$

$= -10\log[1+(\dfrac{\omega}{10})^2]$

$\omega \prec\prec 10 \Rightarrow \dfrac{\omega}{10} \approx 0 \Rightarrow dB = -10\log 1 = 0$

$\omega \succ\succ 10 \Rightarrow 1 + j\dfrac{\omega}{10} \approx \dfrac{\omega}{10} \Rightarrow dB \approx -20\log\dfrac{\omega}{10}$

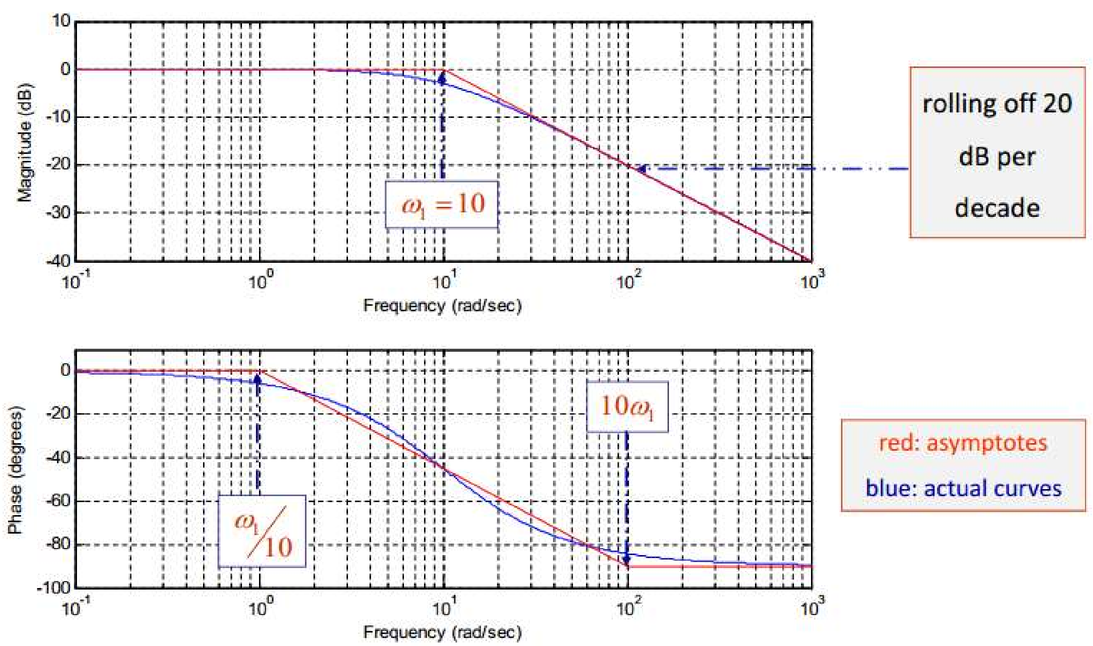
$dB = -[20\log\omega - 20\log 10]$

$\omega = 10 \Rightarrow 1 + j1 \Rightarrow dB = -10\log 2 = -3.01$

Phase: $\angle(1+j\dfrac{\omega}{10}) = 0^0 - \tan^{-1}\dfrac{\omega}{10}$

$\omega \prec\prec 10 \Rightarrow \dfrac{\omega}{10} \approx 0 \Rightarrow \angle GH \approx \tan^{-1} 0 = 0^o$

$\omega \succ\succ 10 \Rightarrow \dfrac{\omega}{10} \approx \infty \Rightarrow \angle GH \approx -\tan^{-1}\infty = -90^o$



Bode Plot- A General First Order Unstable System

$$G(s)=\frac{\omega_1}{s-\omega_1}$$

$$G(s)=\frac{\omega_1}{s-\omega_1}=\frac{-1}{1-{s}/{\omega_1}} \quad \Rightarrow \quad G(j\omega)=\frac{-1}{1-j\left({\omega}/{\omega_1}\right)}=\frac{1}{\sqrt{1+\left({\omega}/{\omega_1}\right)^2}}\angle -180^\circ+\tan^{-1}\left({\omega}/{\omega_1}\right)$$

Let us examine the following situations.

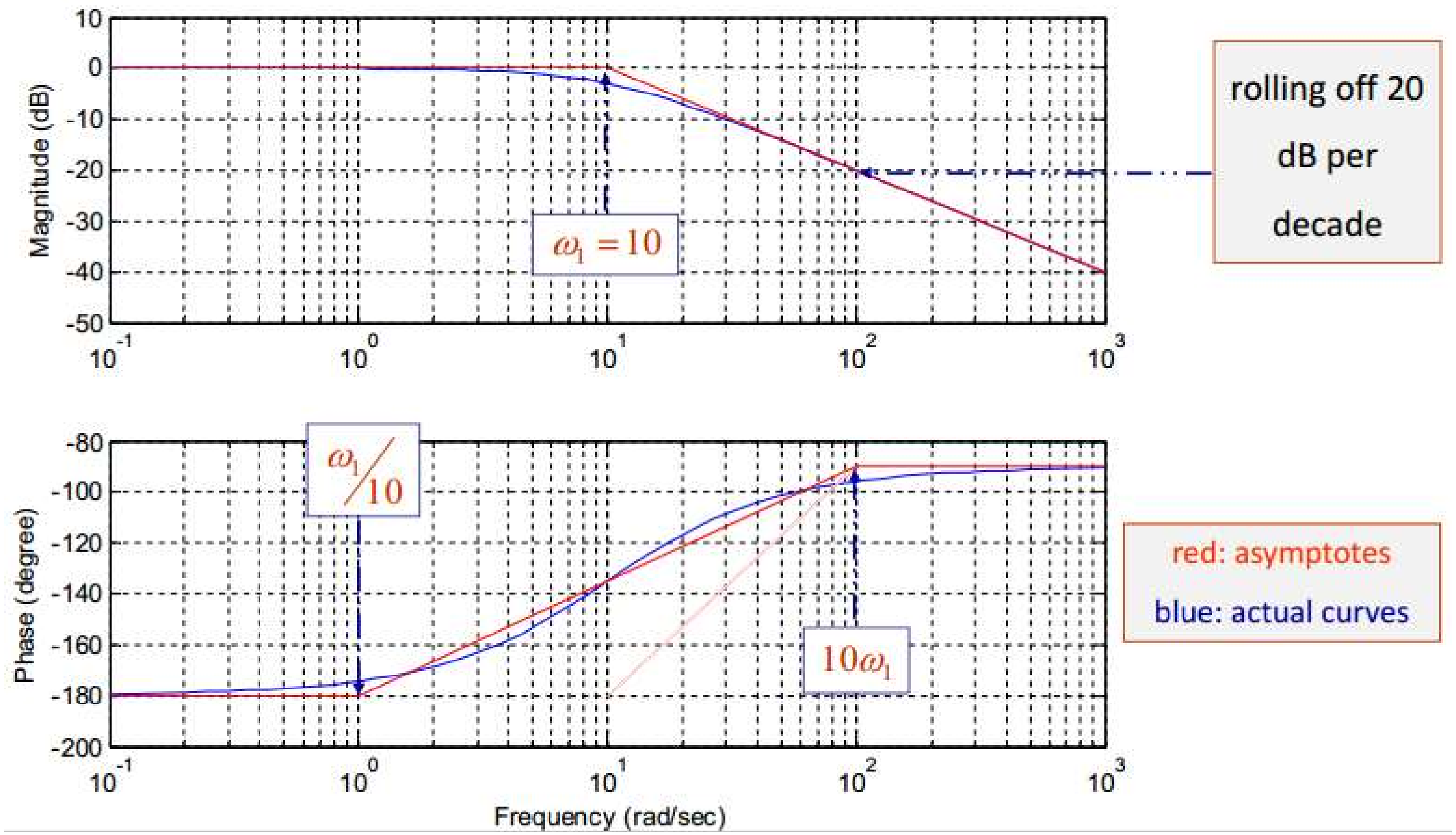
$$\omega=\omega_1 \quad \Rightarrow \quad G(j\omega_1)=\frac{1}{\sqrt{1+\left({\omega_1}/{\omega_1}\right)^2}}\angle \tan^{-1}\left({\omega_1}/{\omega_1}\right)=0.707\angle -135^\circ \quad (0.707=-3\text{ dB})$$

$$\omega \ll \omega_1 \quad \Rightarrow \quad G(j\omega)=\frac{1}{\sqrt{1+\left({\omega}/{\omega_1}\right)^2}}\angle -180^\circ+\tan^{-1}\left({\omega}/{\omega_1}\right)\approx 1\angle -180^\circ$$

$$\omega \gg \omega_1 \quad \Rightarrow \quad G(j\omega)=\frac{1}{\sqrt{1+\left({\omega}/{\omega_1}\right)^2}}\angle -180^\circ+\tan^{-1}\left({\omega}/{\omega_1}\right)\approx \frac{\omega_1}{\omega}\angle -90^\circ$$

Example: Consider the following system where $\omega_1 = 10$ rad/s is the corner frequency

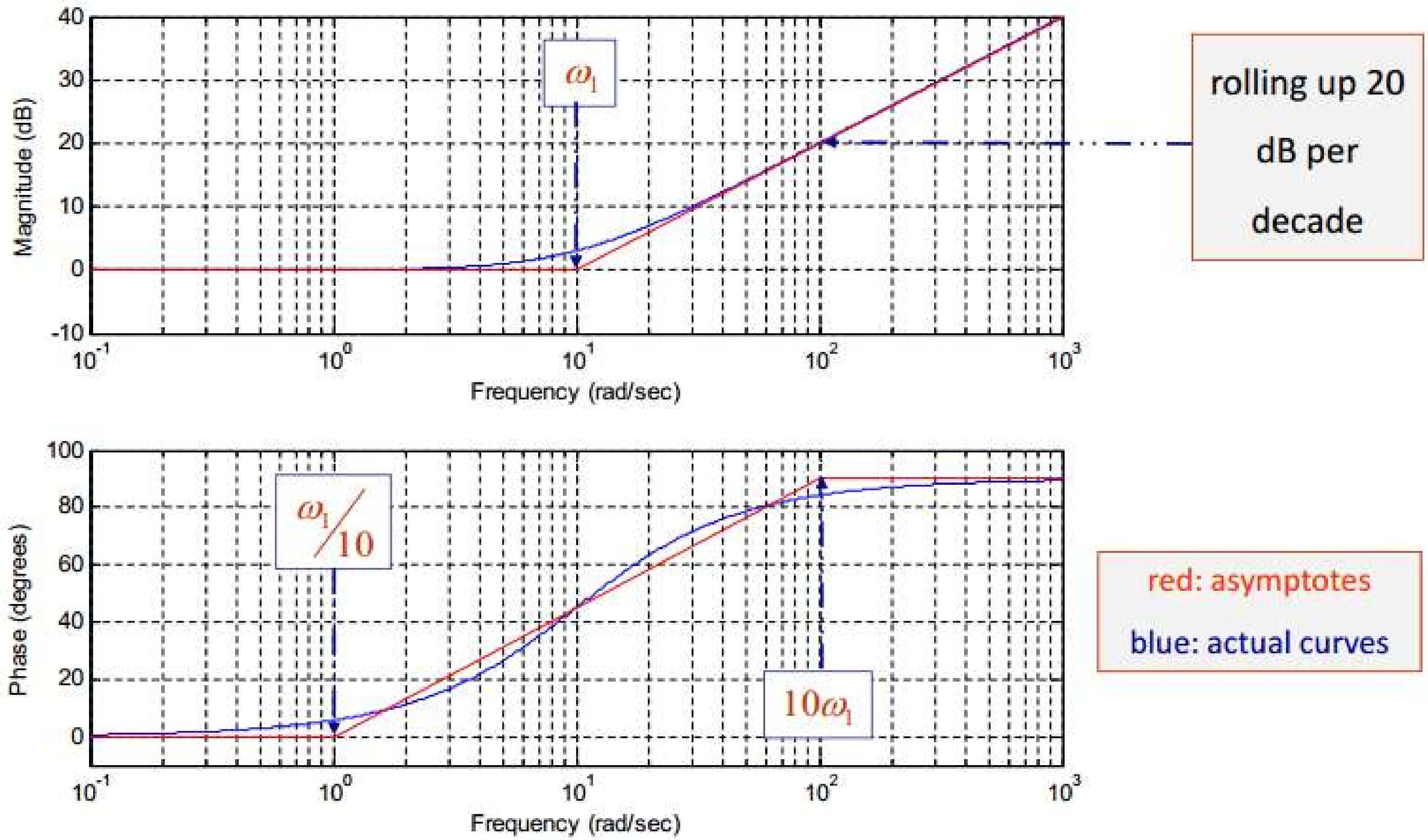
$$G(s) = \frac{-1}{1 - s/10}$$



Bode plot- a simple zero factor

$G(s) = 1 + s/\omega_1$

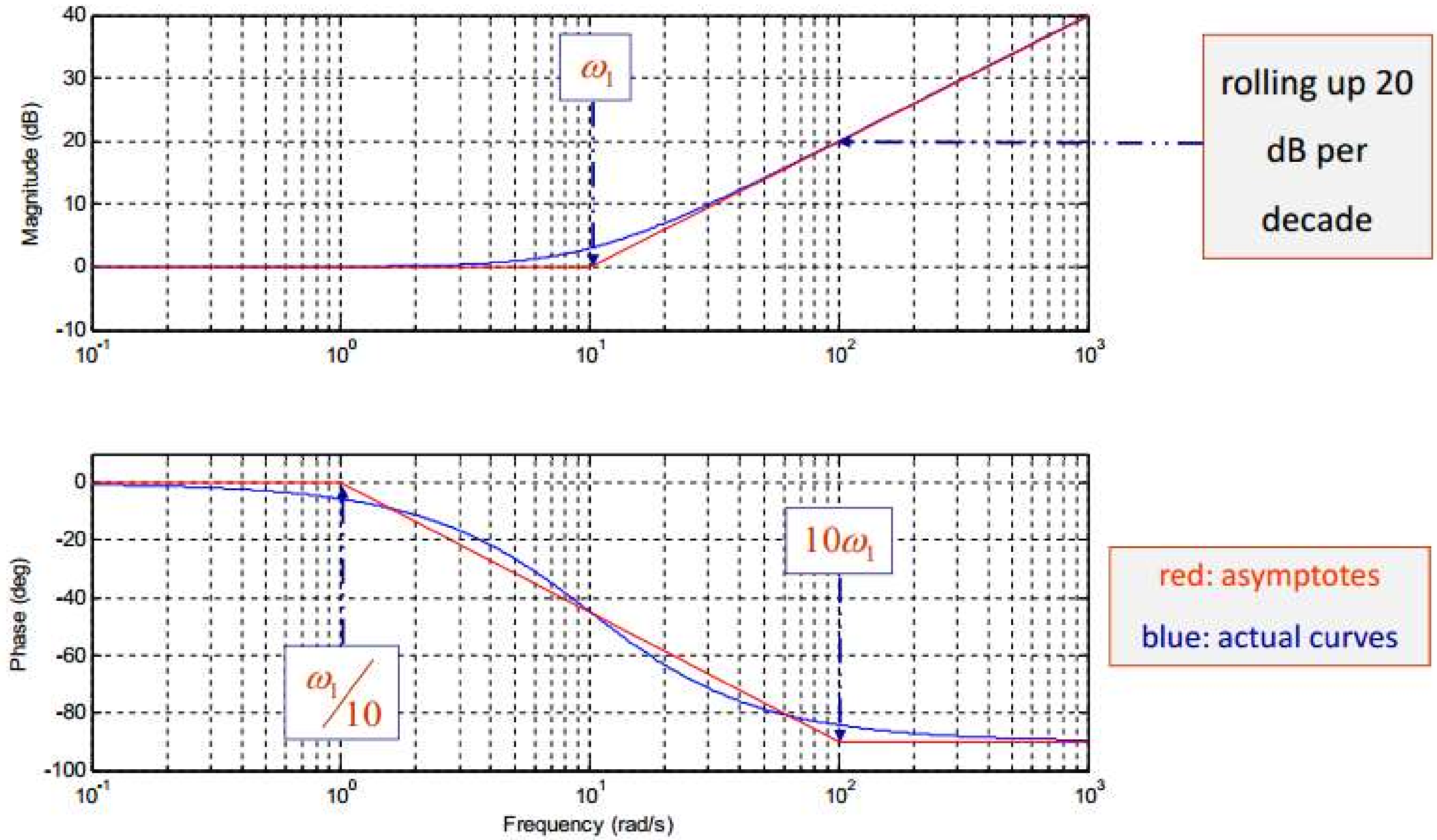
The Bode plot of $G(s) = 1 + s/\omega_1$ can be done similarly….



Bode plot- a simple unstable zero factor

$G(s) = 1 - s/\omega_1$

The Bode plot of $G(s) = 1 - s/\omega_1$ can be done similarly….



Bode plot- Putting all together

Assume a given system has only simple poles and zeros

$$G(s)=\frac{b_m s^m + b_{m-1}s^{m-1}+\cdots+b_1 s+b_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_1 s+a_0}=\frac{b_m(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_m)}$$

or

$$G(s)=\frac{b_m(s+z_1)\cdots(s+z_m)}{(s+p_1)\cdots(s+p_m)}=\frac{k(1+s/z_1)\cdots(1+s/p_{m_1})}{s^q(1+s/p_1)\cdots(1+s/p_{n_1})}$$

In dB scale, we have

$$|G(j\omega)|\,\text{dB}=|k|\text{ in dB}+\left|1+j\omega/z_1\right|\text{ in dB}+\cdots+\left|1+j\omega/z_{m_1}\right|\text{ in dB}-|\omega|\text{ in dB}\times q-\left|1+j\omega/p_1\right|\text{ in dB}-\cdots-\left|1+j\omega/p_{n_1}\right|\text{ in dB}$$

Similarly,

$$\angle G(j\omega)=\angle k+\angle\left(1+j\omega/z_1\right)+\cdots+\angle\left(1+j\omega/z_{m_1}\right)-90^\circ\times q-\angle\left(1+j\omega/p_1\right)-\cdots-\angle\left(1+j\omega/p_{n_1}\right)$$

Thus, the Bode plot of a complex system can be broken down to the additions and subtractions of some simple systems…

Example

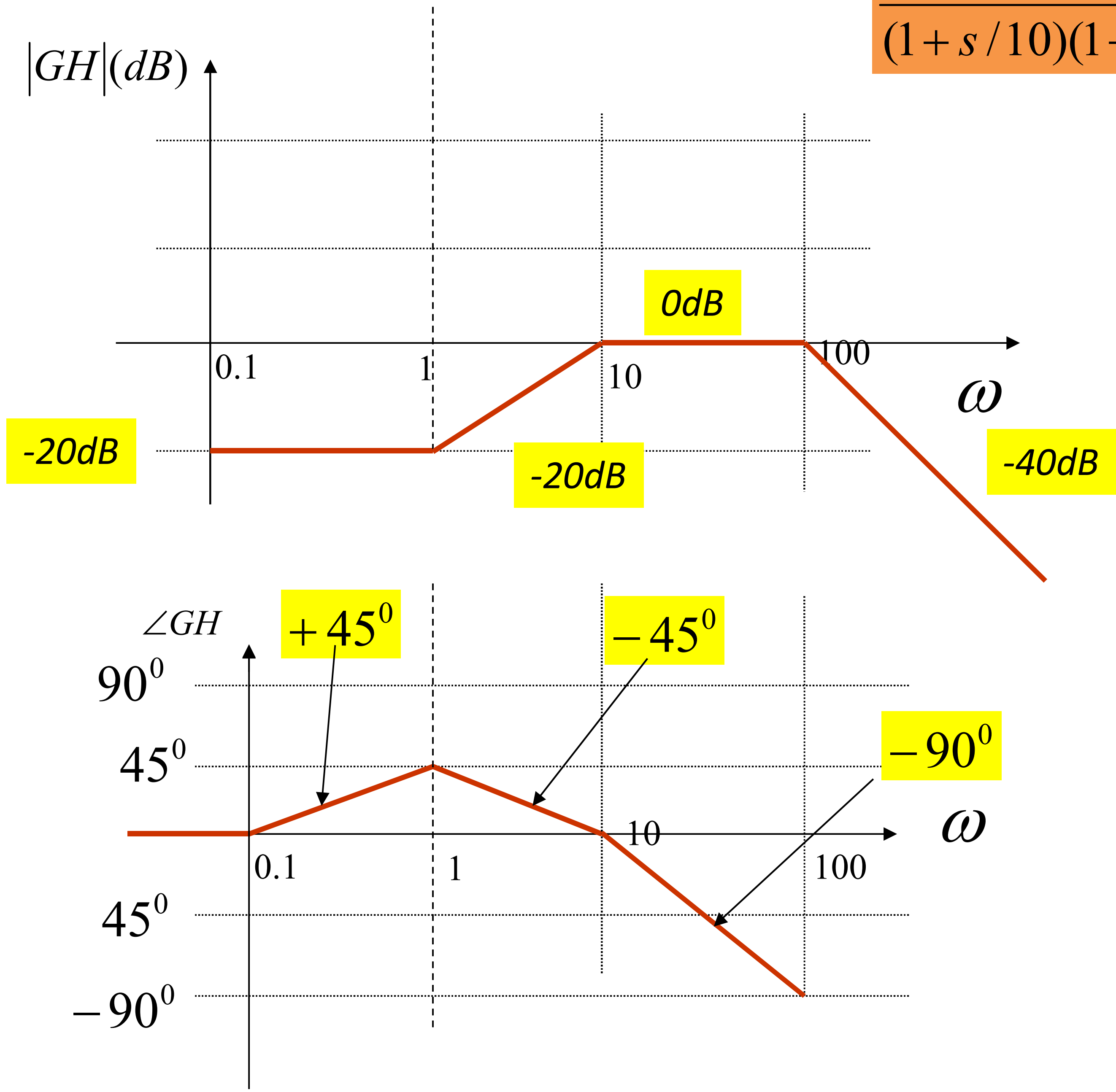
sketch bode plot of

$$G(s) = \frac{10^4(1+s)}{(10+s)(100+s)^2}$$

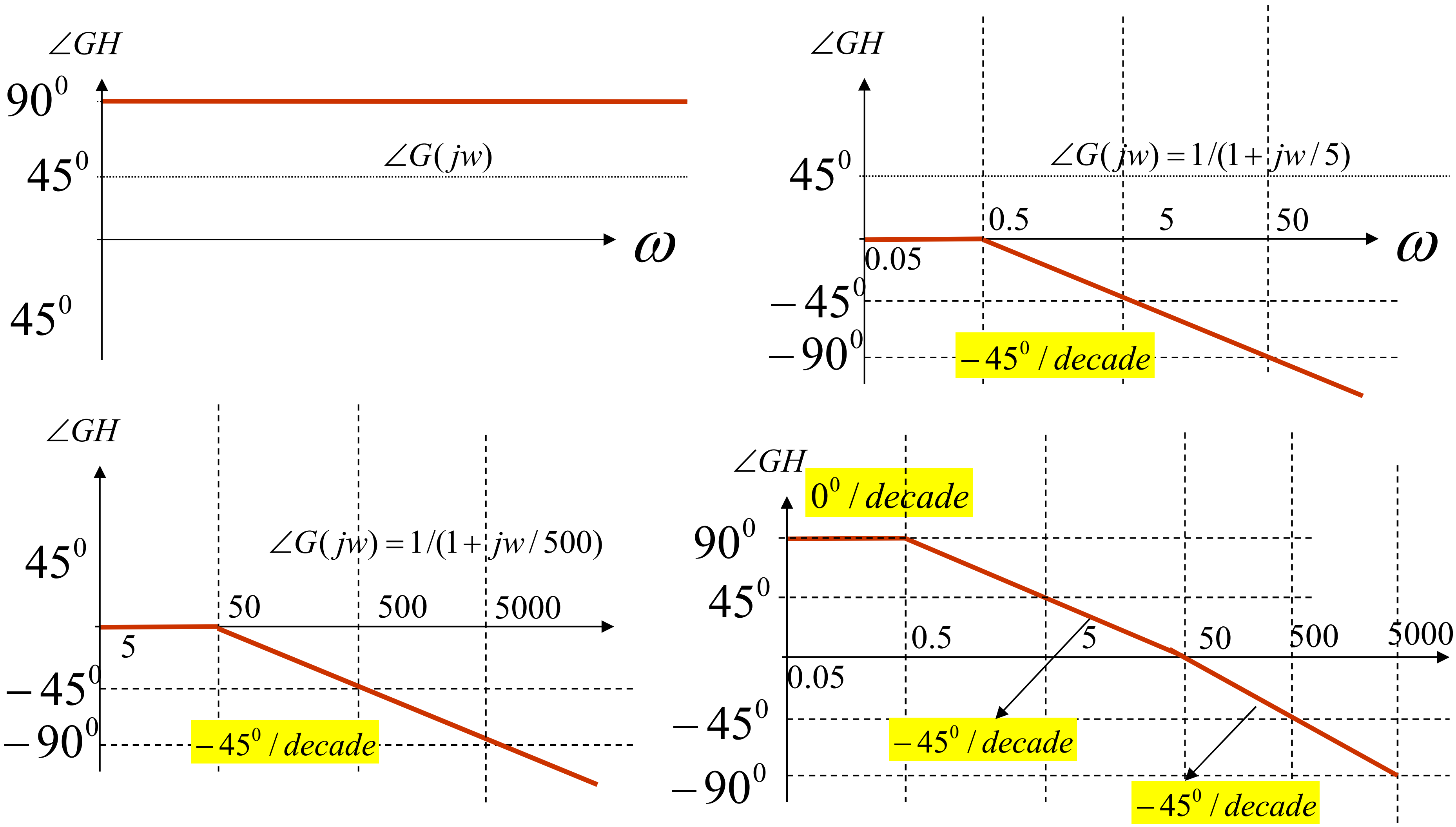
$$= \frac{0.1(1+s)}{(1+s/10)(1+s/100)^2}$$

Corner frequency at 1,10,100

$$\frac{0.1(1+s)}{(1+s/10)(1+s/100)^2}$$



Sketch only phase plot of $G(s) = \dfrac{20s}{(1+s/5)(1+s/500)}$



Example

sketch bode plot of

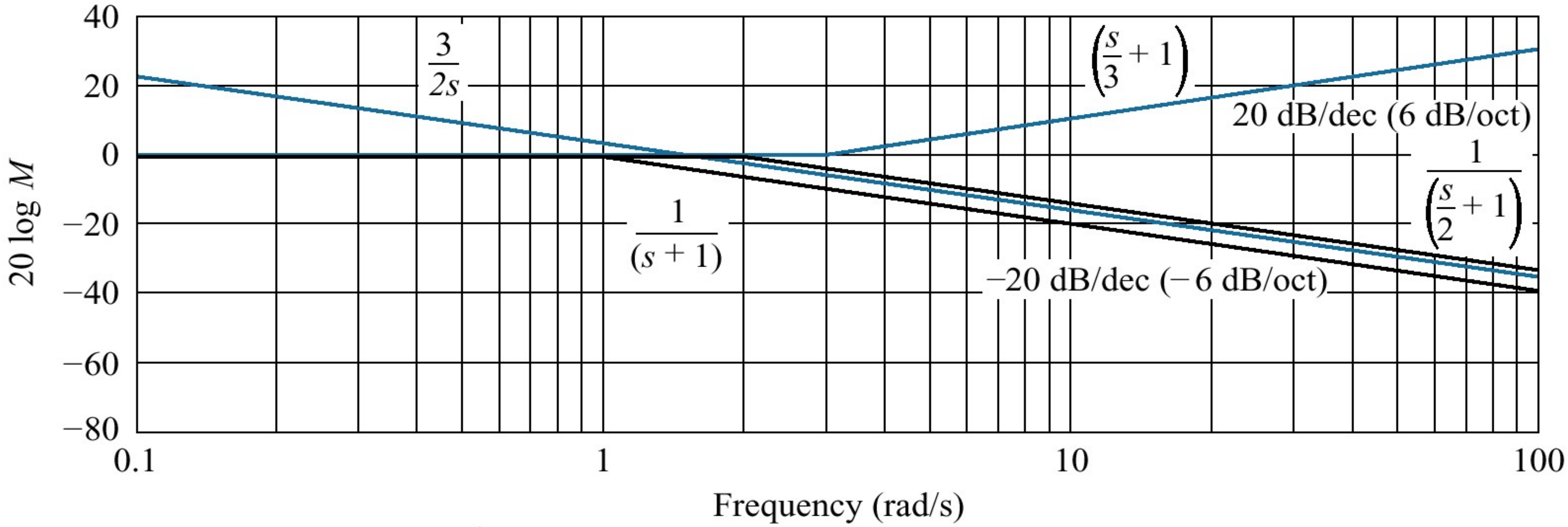
$$G(s) = \frac{(s+3)}{s(s+1)(s+2)}$$

$$= \frac{(3/2)(1+s/3)}{s(1+s)(1+s/2)}$$

Corner frequency at 1,2,3

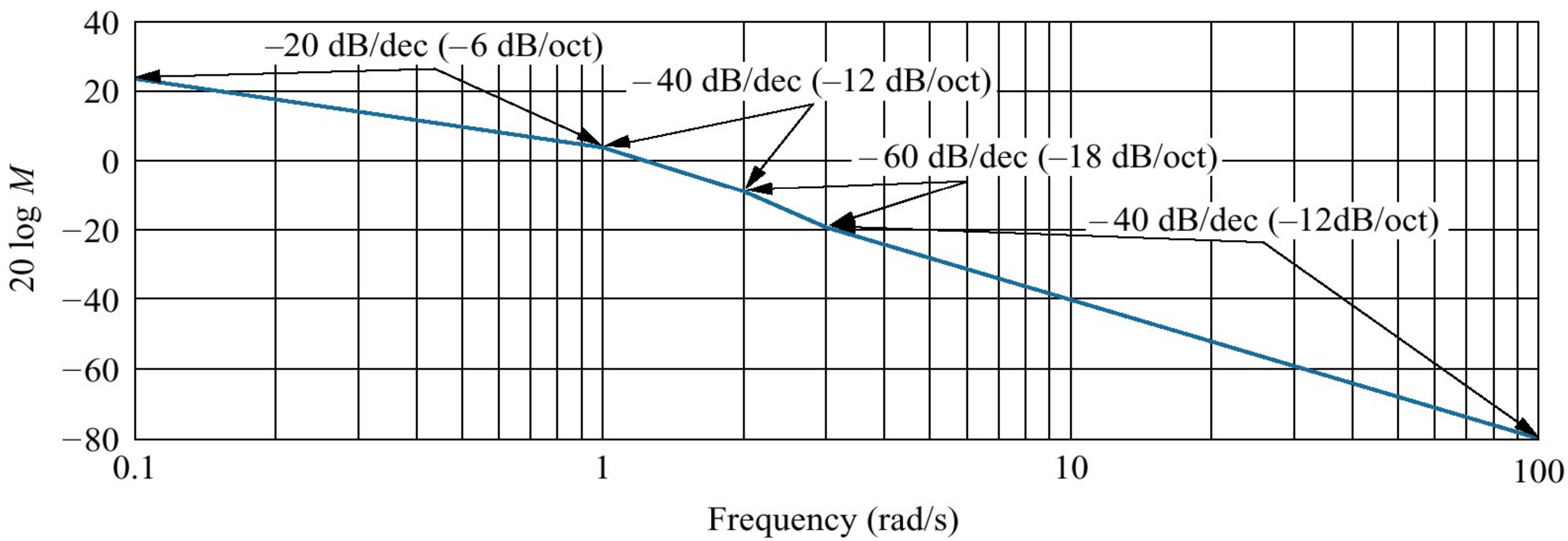
Slope at each break frequency for magnitude plot

Frequency	**small**	1	2	3
s	-20	-20	-20	-20
1/(s+1)	0	-20	-20	-20
1/(s+2)	0	0	-20	-20
(s+3)	0	0	0	20
Total Slope	-20	-40	-60	-40



Magnitude Plot

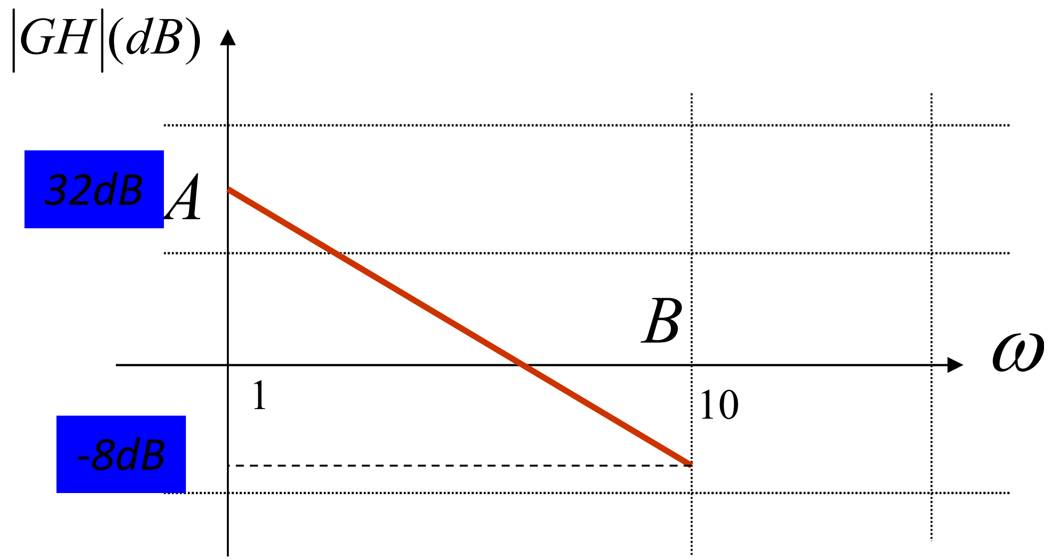
(*a*)



(*b*)

Bode Plots to Transfer Function

1. We can also take the bode plot and extact the transfer fucntion from it.
2. First, determine the constant gain factor, k
3. Next, move from lowest to highest frequency noting the appearance and order of the poles and zeros.



$$20\log|G(jw)| = 32dB \ \ at \ \ w = 1r/s$$
$$20\log|G(jw)| = -8dB \ \ at \ \ w = 10r/s$$

The slope can be defined as

$$\frac{-8-32}{\log(10)-\log(1)} = -40dB$$

The transfer function can written as G(s)=K/s^2

$$32 = 20\log K - 40\log 1$$
$$K = 10^{\frac{32}{20}} = 39.8$$
$$G(s) = 39.8/s^2$$

Find the transfer function- Try at Home