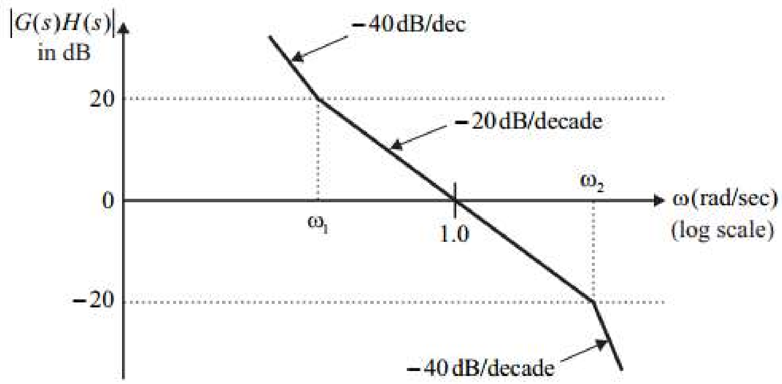
Examples

Prob

The asymptotic Bode plot of the minimum phase open-loop transfer function $G(s)H(s)$ is as shown in the figure. Obtain the transfer function $G(s)H(s)$.



Sol.

For the given Bode magnitude plot, there are two corner frequencies : ω_1 and ω_2 .

The initial slope is -40 dB/dec and this corresponds to a factor s^2 in the denominator of the transfer function.

At ω_1, the slope changes by $+20$ dB/dec so resultant slope will be -20 dB/dec and this is due to the factor $\left(1+\frac{s}{\omega_1}\right)$ in the numerator of the transfer function.

At ω_2, the slope changes by -20 dB/dec so resultant slope will be -40 dB/dec and this is due to the factor $\left(1+\frac{s}{\omega_2}\right)$ in the denominator of the transfer function.

Examples

Calculation of ω_1 :

$$-20 = \frac{0-20}{\log 1 - \log \omega_1}$$

$$-\log \omega_1 = \frac{-20}{-20} = 1 \qquad \Rightarrow \qquad \omega_1 = 10^{-1} = 0.1 \text{ rad/sec}$$

Calculation of ω_2 :

$$-20 = \frac{-20-0}{\log \omega_2 - \log 1} \qquad \Rightarrow \qquad \omega_2 = 10 \text{ rad/sec}$$

Calculation of K :

$$20 = 20 \log K - 40 \log 0.1$$

$$-20 = 20 \log K$$

$$K = 0.1$$

The overall transfer function can be written as,

$$G(s)H(s) = \frac{K\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)}$$

$$G(s)H(s) = \frac{0.1\left(1+\frac{s}{0.1}\right)}{s^2\left(1+\frac{s}{10}\right)} = \frac{10(s+0.1)}{s^2(s+10)}$$

Examples

Problem

$$T(s)=\frac{s}{s^2+2s+100}$$

Find the resonant frequency and bandwidth of the above system

Sol.

Given :

$$T(s)=\frac{s}{s^2+2s+100}$$

$$T(j\omega)=\frac{j\omega}{-\omega^2+j2\omega+100}$$

$$|T(j\omega)|=\frac{\omega}{\sqrt{(100-\omega^2)^2+(2\omega)^2}} \qquad \ldots \text{(i)}$$

$$\frac{\partial|T(j\omega)|}{\partial\omega}=\frac{\sqrt{(100-\omega^2)^2+(2\omega)^2}.1-\omega.\frac{1}{2}\left\{(100-\omega^2)+4\omega^2\right\}^{-1/2}.\left\{2(100-\omega^2)(-2\omega)+8\omega\right\}}{(100-\omega^2)^2+(2\omega)^2}$$

Examples

To find maximum value of $|T(j\omega)|$

$$\frac{\partial |T(j\omega)|}{\partial \omega} = 0$$

$$\Rightarrow \quad \left[(100-\omega^2)^2+4\omega^2\right]^{1/2} = \frac{\omega}{2}\left[(100-\omega^2)^2+4\omega^2\right]^{-1/2}\left\{-4\omega(100-\omega^2)+8\omega\right\}$$

$$\Rightarrow \quad \left[(100-\omega^2)^2+4\omega^2\right] = -2\omega^2(100-\omega^2)+4\omega^2$$

$$\Rightarrow \quad 10^4-200\omega^2+\omega^4 = -200\omega^2+2\omega^4$$

$$\Rightarrow \quad 10^4 = \omega^4$$

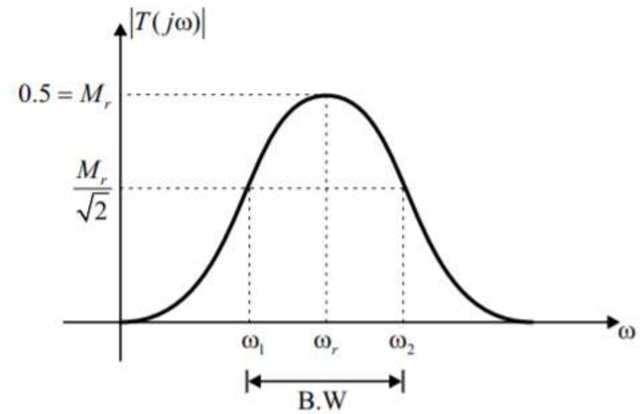
$$\therefore \quad \omega = \omega_r = 10 \text{ r/s}$$

$$M_r = |T(j\omega)|_{\omega=\omega_r}$$

From equation (i)

$$M_r = \frac{10}{\sqrt{(100-100)^2+(20)^2}}$$

$$M_r = \frac{10}{20} = 0.5$$



At $\omega = \omega_1$ & $\omega = \omega_2$

$$|T(j\omega)| = \frac{1}{\sqrt{2}} \cdot M_r$$

Examples

$$\frac{0.5}{\sqrt{2}} = \frac{\omega}{\sqrt{(100-\omega^2)^2 + 4\omega^2}}$$

$$\Rightarrow \quad \frac{1}{2\sqrt{2}} = \frac{\omega}{\sqrt{(100-\omega^2)^2 + 4\omega^2}}$$

$$\Rightarrow \quad 8\omega^2 = (100-\omega^2)^2 + 4\omega^2$$

$$\Rightarrow \quad 8\omega^2 = 10^4 - 200\omega^2 + \omega^4 + 4\omega^2$$

$$\Rightarrow \quad \omega^4 - 204\omega^2 + 10^4 = 0$$

Let $\omega^2 = x$

$$x^2 - 204x + 10^4 = 0$$

$$\Rightarrow \quad x = 122.1 \ \& \ 81.90$$

When $\omega^2 = 122.1$ When $\omega^2 = 81.9$

$$\omega = \sqrt{122.1} \qquad\qquad \omega = \sqrt{81.9}$$

$$= \pm 11.04 \qquad\qquad = \pm 9.04$$

$$\therefore \quad \omega = 11.04 \text{ r/s} \qquad \therefore \quad \omega = 9.04 \text{ r/s}$$

$\therefore$ Lower cut-off frequency $\omega_1 = 9.04$ r/s

Upper cut-off frequency $\omega_2 = 11.04$ r/s

$\therefore$ Bandwidth $= \omega_2 - \omega_1 = 2$ r/s

Examples

Problem

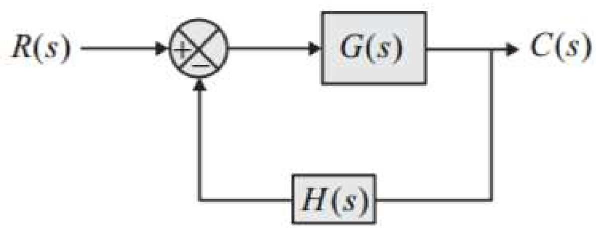
A double integrator plant.

$$G(s) = \frac{K}{s^2}, H(s) = 1$$

is to be compensated to achieve the damping ratio $\xi = 0.5$, and an undamped natural frequency, $\omega_n = 5\ rad/s$. What should be the expression of the compensator

Sol.

Given $\quad G(s) = \frac{K}{s^2}, H(s) = 1$



Transfer function of system is given by,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K / s^2}{1 + K / s^2}$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + K}$$

Characteristic equation of uncompensated system,

$$s^2 + K = 0$$

Standard characteristic equation of second order system,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

Comparing above two equation,

$$\omega_n^2 = K$$

$$\omega_n = \sqrt{K}$$

and $\quad 2\xi\omega_n = 0$

$$\xi = 0$$

Since $\xi = 0$, so undamped system.

Examples

Problem

A unity feedback control system has the open loop transfer function.

$$G(s) = \frac{4(1+2s)}{s^2(s+2)}$$

If the input to the system is a unit ramp, what will be the steady state error

Sol.

Given : $G(s) = \dfrac{4(1+2s)}{s^2(s+2)}$, and $r(t) = t\,u(t)$

Taking Laplace transform of $r(t)$, we get

$$R(s) = \frac{1}{s^2}$$

Steady state error is given by,

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\times\frac{1}{s^2}}{1+\frac{4(1+2s)}{s^2(s+2)}} = \lim_{s\to 0} \frac{1}{s+\frac{4(1+2s)}{s(s+2)}} = 0$$

Hence, the correct option is (A).

Alternatively,

Velocity error coefficient is given by,

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{s\times 4(1+2s)}{s^2(s+2)} = \infty$$

Steady state error for ramp input is given by,

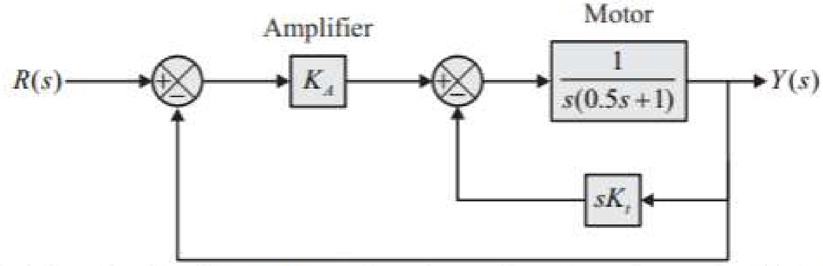
$$e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0$$

For type - 2 system steady state error due to ramp input will be zero.

Examples

Problem

Block diagram model of a position control system is shown in figure.



(a) In absence of derivative feedback $(K_t = 0)$, determine damping ratio of the system for amplifier gain $K_A = 5$. Also find the steady state error to unit ramp input.

(b) Find suitable values of the parameters K_A and K_t so that damping ratio of the system is increased to 0.7 without affecting the steady-state error as obtained in part (a).

Sol.

(a) Given : $K_t = 0$

$$G(s) = \frac{5}{s(0.5s+1)} = \frac{10}{s(s+2)} \text{ and } H(s) = 1$$

Closed-loop transfer function for negative feedback is given by,

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s^2+2s+10} \quad \text{..... (i)}$$

Transfer function for second-order system with unit step input is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad \text{.....(ii)}$$

where, ξ = damping ratio, ω_n = natural angular frequency

On comparing equation(i) and (ii), we get

$$\omega_n = \sqrt{10} \text{ rad/sec. and } 2\xi\omega_n = 2$$

$$\xi = \frac{1}{\sqrt{10}}$$

Steady state error is given by,

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

$$e_{ss} = \lim_{s\to 0} \frac{s\times\frac{1}{s^2}}{1+\frac{10}{s(s+2)}} = \lim_{s\to 0} \frac{1}{s+\frac{10}{(s+2)}} = 0.2$$

Examples

(b) The open loop transfer function can be written as,

$$\text{OLTF} = \frac{2K_A}{s^2 + 2s + 2sK_t} = \frac{2K_A}{s[s + (2 + 2K_t)]}$$

The closed loop transfer function can be written as,

$$\text{CLTF} = \frac{Y(s)}{R(s)} = \frac{2K_A}{s^2 + s(2 + 2K_t) + 2K_A} \quad \text{.....(i)}$$

Transfer function for second-order system with unit step input is given by,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad \text{..... (ii)}$$

where, ξ = damping ratio, ω_n = natural angular frequency

Velocity error coefficient is given by,

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

$$K_v = \lim_{s\to 0} s\frac{2K_A}{s[s + (2 + 2K_t)]} = \frac{2K_A}{2 + 2K_t}$$

Steady state error is given by,

$$e_{ss} = \frac{1}{K_v}$$

$$0.2 = \frac{1}{\frac{2K_A}{2 + 2K_t}}$$

$$0.2K_A = 1 + K_t \quad \text{..... (iii)}$$

On comparing equation (i) and (ii), we get

$$\omega_n = \sqrt{2K_A}$$

$$2 + 2K_t = 2\xi\omega_n$$

$$\xi = \frac{2 + 2K_t}{2\omega_n} = \frac{1 + K_t}{\sqrt{2K_A}}$$

$$0.7 = \frac{1 + K_t}{\sqrt{2K_A}}$$

$$1 + K_t = 0.9899\sqrt{K_A} \approx \sqrt{K_A} \quad \text{..... (iv)}$$

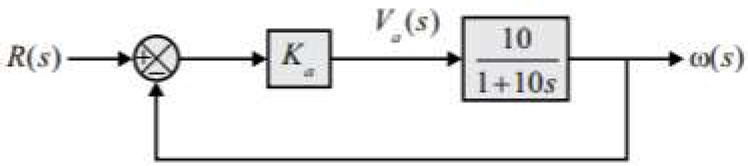
From equation (iii) and equation (iv), we get

$K_A = 25$ and $K_t = 4$.

Problem

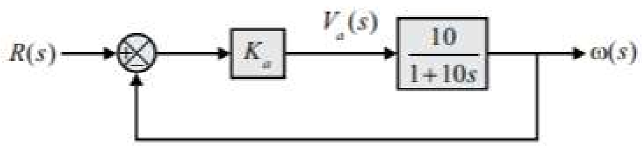
Examples

The open-loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$. When connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is



Sol.

Given : $\tau_{\text{closed loop}} = \frac{1}{100}\tau_{\text{open loop}}$ where τ represents time constant and $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$



For first order system loop transfer function is $\frac{C(s)}{R(s)} = \frac{K}{1+s\tau}$ comparing with $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$ $\tau_{\text{open loop}} = 10$

Closed-loop transfer function for negative unity feedback is given by,

$$T(s) = \frac{G(s)}{1+G(s)}$$

Here $G(s) = K_a\left(\frac{10}{1+10s}\right)$

$$\frac{\omega(s)}{R(s)} = \frac{K_a\left(\frac{10}{1+10s}\right)}{1+K_a\left(\frac{10}{1+10s}\right)} = \frac{K_a 10}{1+10s+K_a 10} = \frac{10K_a}{10s+(10K_a+1)}$$

Examples

Dividing numerator and denominator by $10K_a + 1$

$$\frac{\omega(s)}{R(s)} = \frac{\dfrac{10K_a}{10K_a + 1}}{1 + \left(\dfrac{10}{10K_a + 1}\right)s}$$

For first order system loop transfer function is $\dfrac{C(s)}{R(s)} = \dfrac{K}{1 + s\tau}$. On comparing with $\dfrac{\omega(s)}{R(s)}$ we get

$$\tau_{\text{closed loop}} = \frac{10}{10K_a + 1}$$

We have

$$\tau_{\text{closed loop}} = \frac{1}{100}\tau_{\text{open loop}}$$

$$\frac{10}{10K_a + 1} = \frac{1}{100}10$$

$$10K_a + 1 = 100$$

$$10K_a = 99$$

$$K_a = 9.9 \simeq 10$$