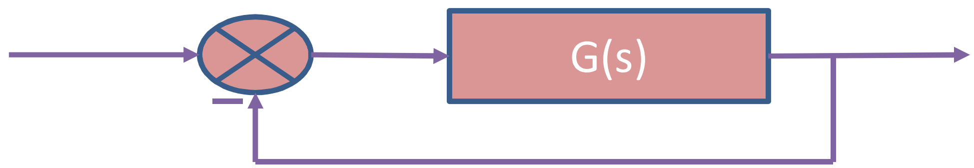
Nyquist Stability Criteria



G(s)=OLTF=$\frac{KN(s)}{D(s)}$

Zeros of OLTF

Poles of OLTF

CLTF=$\frac{KN(s)}{D(s)+K\quad(s)}$

Zeros of CLTF

Poles of CLTF

Characteristic Equation(CE)=1+G(s)=1+$\frac{KN(s)}{D(s)} = \frac{D(s)+KN(s)}{D(s)}$

Zeros of CE

Poles of OLTF

Comparing above equations:

1. Poles of C.E.=Poles of OLTF
2. Zeros of C.E.=Poles of CLTF

These two are very important points for studying Nyquist stability criteria

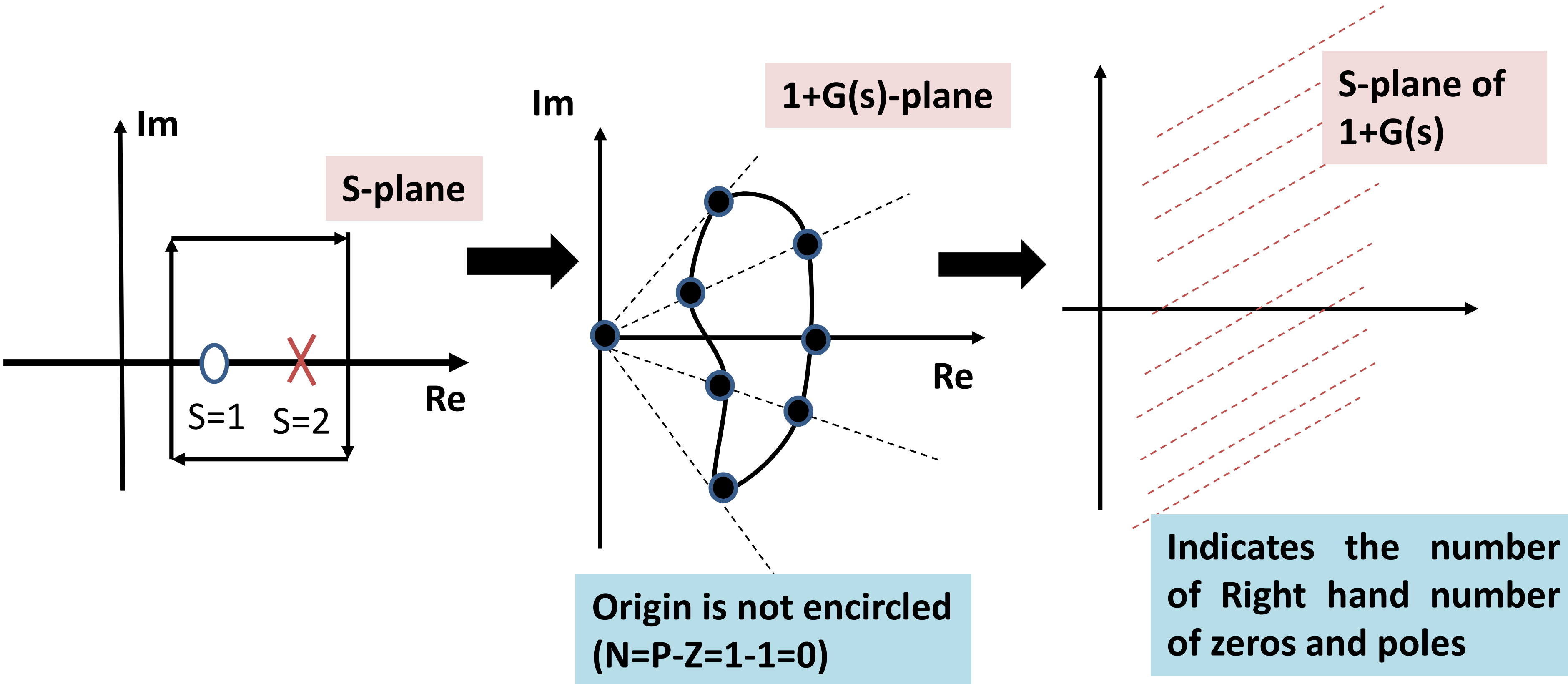
Cauchy's Principle of Argument:

N=P-Z

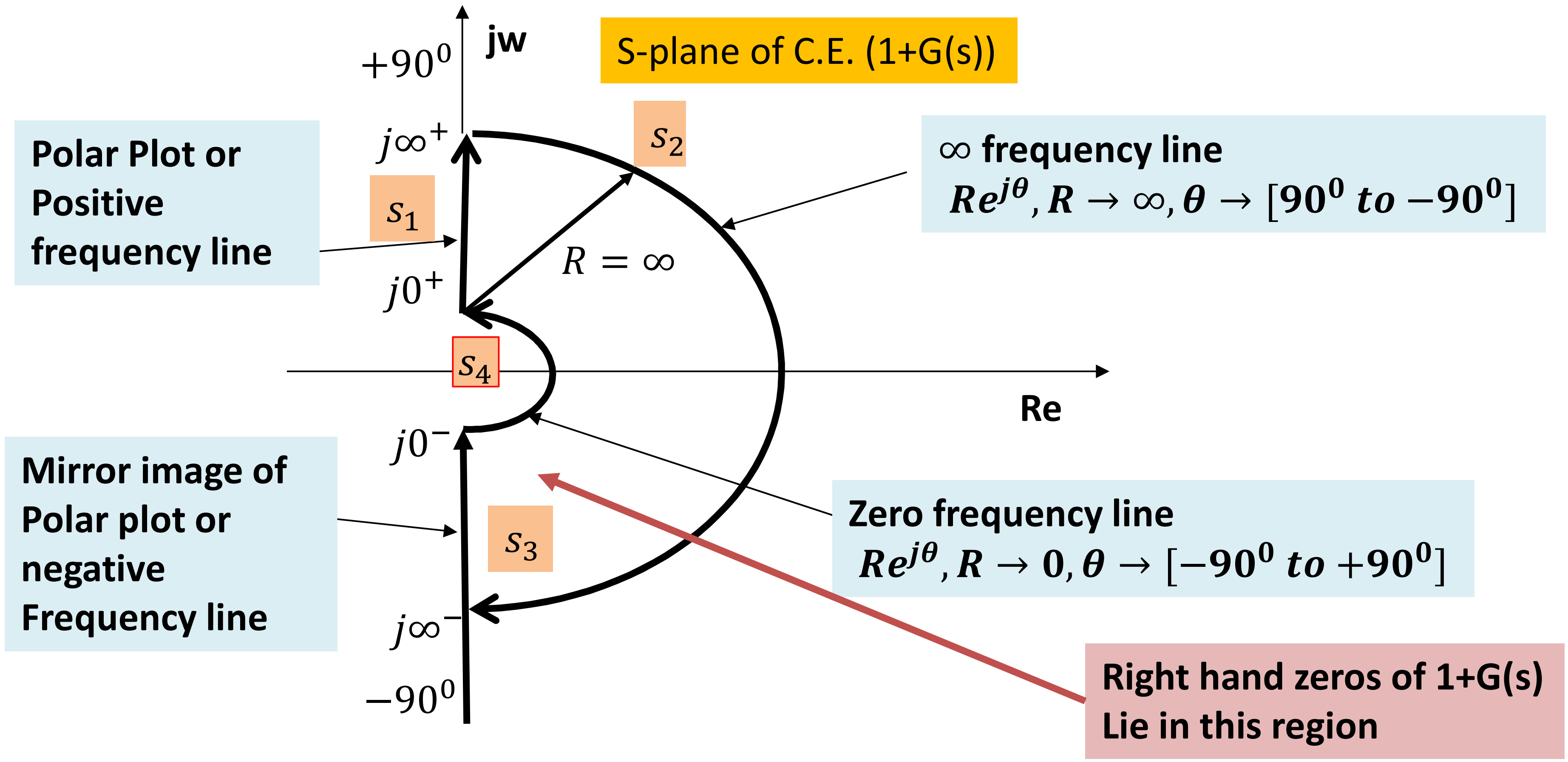
1. P-> No. of poles of OLTF in RHP
2. Z-> No. zeros of OLTF in RHP
3. N-> No. of encirclement about origin ACW/CW

Contour of I+G(s)

$G(s)=\frac{1}{s-2} \Rightarrow 1 + G(s) = \frac{s-1}{s-2}$



From the previous concept, we may have the following **Nyquist Contour**



Nyquist contour:

1. Polar plot
2. Mirror image of Polar plot
3. Zero frequency line
4. Infinite frequency line

Example $G(s)H(s)=\dfrac{1}{s(1+s)(1+2s)}$ → Type-1/order-3

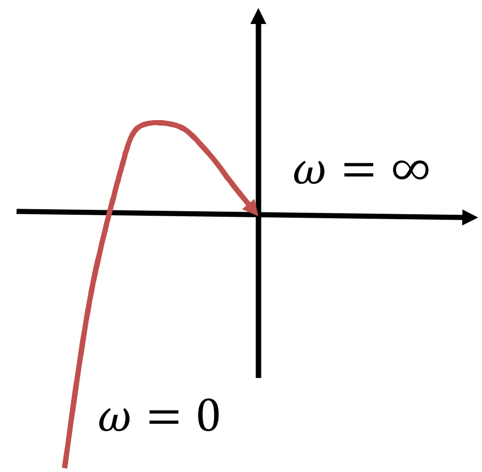
s_1:

$$s=j\omega \quad G(j\omega)H(j\omega)=\frac{1}{j\omega(1+j\omega)(1+2j\omega)}$$

$$M=\frac{1}{\omega\sqrt{(1+\omega^2)}\sqrt{1+4\omega^2}} \qquad \phi=-90^0-\tan^{-1}\omega-\tan^{-1}2\omega$$

$$\omega=0 \qquad M=\infty \qquad \phi=-90\,(tail)$$

$$\omega=\infty \qquad M=0 \qquad \phi=-270\,(head)$$



s_2:

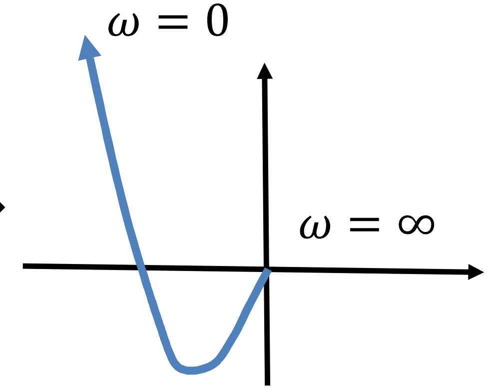
$$\lim_{R\to\infty}\frac{1}{\mathrm{Re}^{j\theta}(1+\mathrm{Re}^{j\theta})(1+2\,\mathrm{Re}^{j\theta})}=\lim_{R\to\infty}\frac{1}{\mathrm{Re}^{j\theta}\,R^3e^{j3\theta}}\approx 0$$

(As long as order of $deno^m > num^n$)

s_3:

$$s=-j\omega \quad G(-j\omega)H(-j\omega)=\frac{1}{-j\omega(1-j\omega)(1-2j\omega)}$$

Hence, Mirror image of s_1



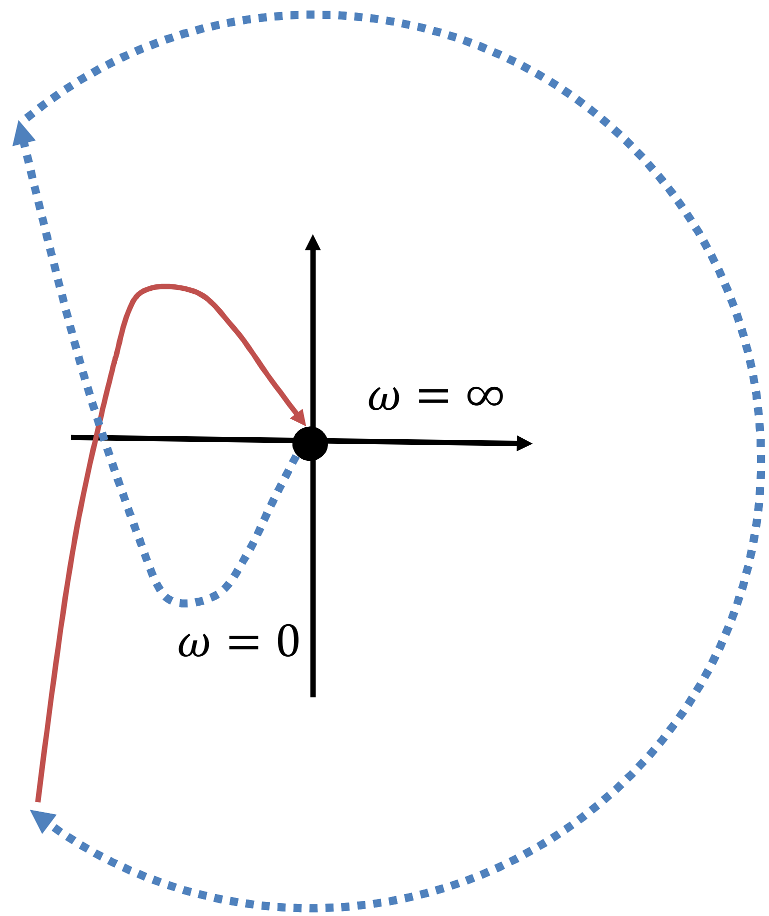
s_3:

$$\lim_{r\to 0}\frac{1}{re^{j\theta}(1+re^{j\theta})(1+2re^{j\theta})}\approx\lim_{r\to 0}\frac{1}{re^{j\theta}}=\lim_{R\to\infty}\mathrm{Re}^{-j\theta}$$

$$[R=1/r;\quad -\theta:\pi/2\ to\ -\pi/2]$$

CW semicircle of infinite radius. No. of infinite SC= type of system

Example $G(s)H(s) = \dfrac{1}{s(1+s)(1+2s)}$ ➡ Nyquist Plot



Nyquist plot without considering so many things

1. Draw Polar Plot.
2. Mirror Image of polar plot.
3. Infinite Semi circle = type of system

Example:

$$G(s)H(s) = \frac{(1+20s)}{s(1+s)(1+10s)}$$

$$G(j\omega)H(j\omega) = \frac{1+20j\omega}{j\omega(1+j\omega)(1+10j\omega)}$$

$$M = \frac{\sqrt{1+400\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{1+100\omega^2}} \qquad \phi = \tan^{-1}20\omega - 90 - \tan^{-1}\omega - \tan^{-1}10\omega$$

Please note that where there is a zero in the transfer function, there may be phase cross over frequency. The phase: $\varphi = (\omega = \omega_{cp}) = -180^0$. The intersection point with the negative real axis in the polar plot.

$$-180 = \tan^{-1}20\omega - 90 - \tan^{-1}\omega - \tan^{-1}10\omega$$

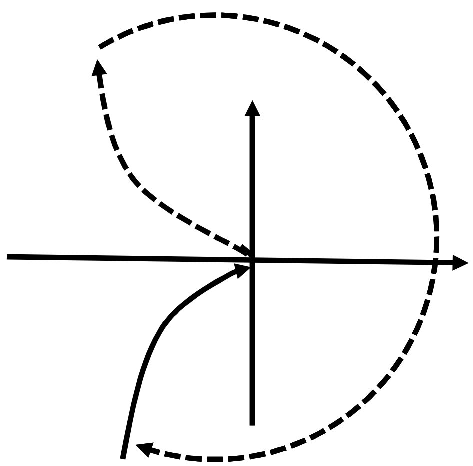
$$\tan^{-1}\frac{11\omega}{1-10\omega^2} = 90 + \tan^{-1}20\omega$$

$$220\omega^2 = 10\omega^2 - 1 \Rightarrow \omega = imag$$

No cross over frequency

$$\omega = 0 \quad M = \infty \quad \phi = -90$$

$$\omega = \infty \quad M = 0 \quad \phi = -180$$



- **A** linear closed-loop ***continuous control system*** is absolutely stable if the roots of the characteristic equation have negative real parts.

- Equivalently, the poles of the closed-loop transfer function, or the zeros of the denominator, ***1 + GH(s),*** of the closed-loop transfer function, must lie in the left-half plane.

- The Nyquist Stability Criterion establishes the number of zeros of ***1 + GH(s)*** in the right-half plane directly from the Nyquist Stability Plot of ***GH(s)***.

Rewriting the Cauchy’s Principle of Argument According to Nyquist Contour

$$N=P-Z$$

1. P=No. of R.H.P of CE
2. Z=Right hand zeros of characteristic equation (CE) (indirectly R.H.P of CLTF)
3. No. of encirclement about origin in ACW/CW in 1+G(s) plane

Nyquist Contour of $1 + G(s) = \frac{s-1}{s-2}$

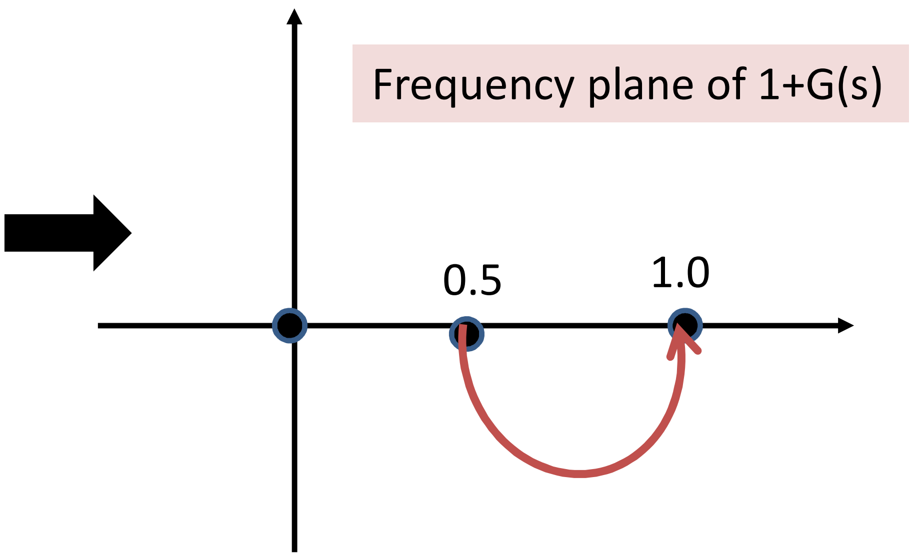
$$\left(1+G(s)\right)\Big|_{s=j\omega} = \left(1+G(j\omega)\right) = \frac{j\omega-1}{j\omega-2}$$

$$\left|1+G(j\omega)\right|_{\omega=0} = \frac{1}{\sqrt{2}}; \left|1+G(j\omega)\right|_{\omega=\infty} = 1$$

$$\angle 1+G(j\omega) = 180^0 - \tan^{-1}\omega - 180^0 + \tan^{-1}\frac{\omega}{2}$$

$$= -\tan^{-1}\omega + \tan^{-1}\frac{\omega}{2}$$

$$\angle 1+G(j\omega)\Big|_{\omega=0} = 0^0; \angle 1+G(j\omega)\Big|_{\omega=\infty} = -90^0 + 90^0$$



Nyquist Plot

Nyquist plot is the frequency response of the open loop transfer function (OLTF) and to check the Stability of the closed loop system (CTF). When we talk about frequency response of OLTF or Nyquist the, critical point (-1+j0) is the referred point for encirclement. The origin of 1+G(s) to -1+j0 (C.P.) for Nyquist plot.

N=P-Z

where

1. P=No. of R.H.P of CE=No. of R.H.P. of OLTF.
2. Z=Right hand zeros of characteristic equation (CE) (indirectly R.H.P of CLTF)
3. No. of encirclement about critical point (-1+j0) in ACW.

$G(s)=\frac{1}{s-2}$

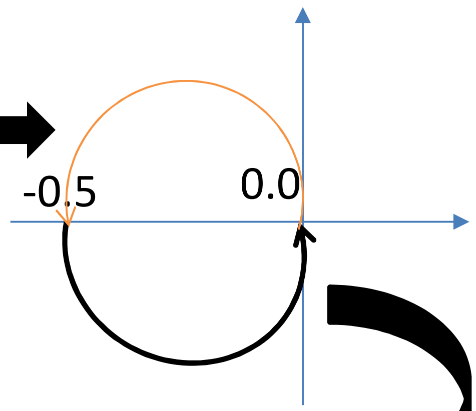
1. P=1
2. Z=?
3. N=?

$$G(j\omega)=\frac{1}{j\omega-2} \Rightarrow |G(j\omega)|_{\omega=0}=\frac{1}{2}$$

$$|G(j\omega)|_{\omega=\infty}=0$$

$$\angle G(j\omega)=-180^0+\tan^{-1}\frac{\omega}{2}$$

$$\angle G(j\omega)|_{\omega=0}=-180^0 \ \& \ \angle G(j\omega)|_{\omega=\infty}=-90^0$$



N=0;P=1 and Z=1 It means there is a one closed loop Poles in the right hand side