Homework Assignments 1 & 2 (Combined)
AE-649A

Due date- 10^{th} November 2019

1. Derive an expression for the transfer function of the following system

$$\dot{x} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u; \quad y = [1 \;\; 2 \;\; 3]x$$

what will be the relation between the eigenvalues of A and the poles of the transfer function? State the similarity transformation that will convert a general state variable form into diagonal form. {Hint: use the eigenvector based transformation matrix}.

2. The state variable model of a dynamical system is given by

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -0.0071 & -0.111 & 0.12 \\ 0 & 0.07 & -0.3 \end{bmatrix} x + \begin{bmatrix} 0 \\ -0.095 \\ 0.072 \end{bmatrix} u; \quad y = [1 \;\; 0 \;\; 0]x$$

a) Test whether the system is stable. b) Obtain unit impulse response of the system with zero initial condition. c) Test the controllability and observability of the system.

3. Transfer the following system into controllable and observable canonical forms

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ 0 & -3 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u; \quad y = [1 \;\; 1]x$$

Design linear state feedback controller for the above system such that the closed loop poles are at a) $(-3 \pm i4)$ b) $(-2, -2)$.

4. Design full order observer for the following continous time systems with observer poles located ar $-5, -5$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad y = [0 \;\; 1]x$$

5. Design a reduced order observer for observing the second state variable for the following continuous time system so that the observer pole is located at -8.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0.01 \end{bmatrix} u; \quad y = [1 \;\; 0]x$$

6. The dynamical model of a non-linear system for the cruise control system is given by

$$\ddot{x}_1 + \frac{b}{m}\dot{x}_1 = \frac{u}{m}$$

where x_1 is the displacement of the vehicle and u is the force input. For the simplicity but no loss of generality, we assume that $m = 1$ and $b = 1$. However, in practical situations, there are always disturbances present in the system. Thus, a more realistic model should be the following

$$\ddot{x}_1 + \dot{x}_1 = u + \textit{some noise}$$

Assume that only the displacement of the vehicle can be meaured, *i.e.*, the measurment output is

$$y = x_1 + w(t)$$

where $w(t)$ is the measurement noise and is assumed to be white and independent of the system noise in the ODE.

- Convert the ODE model of the system into a state space form $\dot{x} = Ax + Bu + v(t)$.
- Assume that $v(t)$ is nonexistent and all states of the plant are available for feedback. Find an LQR control law, which minimizes the following performance index:

$$J(x, u, Q, R) = \int_0^{\infty} (x^T Q x + u^T R u) dt, \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad R = 0.01$$

  what are the gain and phase margins resulting from your LQR design?
- Design a Kalman filter for the plant. Assume that both $v(t)$ and $w(t)$ have zero means and

$$E[v(t)v^T(\tau)] = Q_e \delta(t - \tau), \;\; Q_e = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \;\; E[w(t)w^T(\tau)] = R_e \delta(t - \tau), \;\; R_e = 0.2$$

- Design an LQG control law, which minimizes the following performance index:

$$J(x, u, Q, R) = \underset{T \to \infty}{Lim} \frac{1}{T} E\left[\int_0^T (x^T Q x + u^T R u) dt\right], \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad R = 0.1$$

  What are the closed-loop eigenvalues? Simulate your design using matlab with

$$r = 0\,(ref.), \;\; x(0) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}, \;\; \hat{x}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$