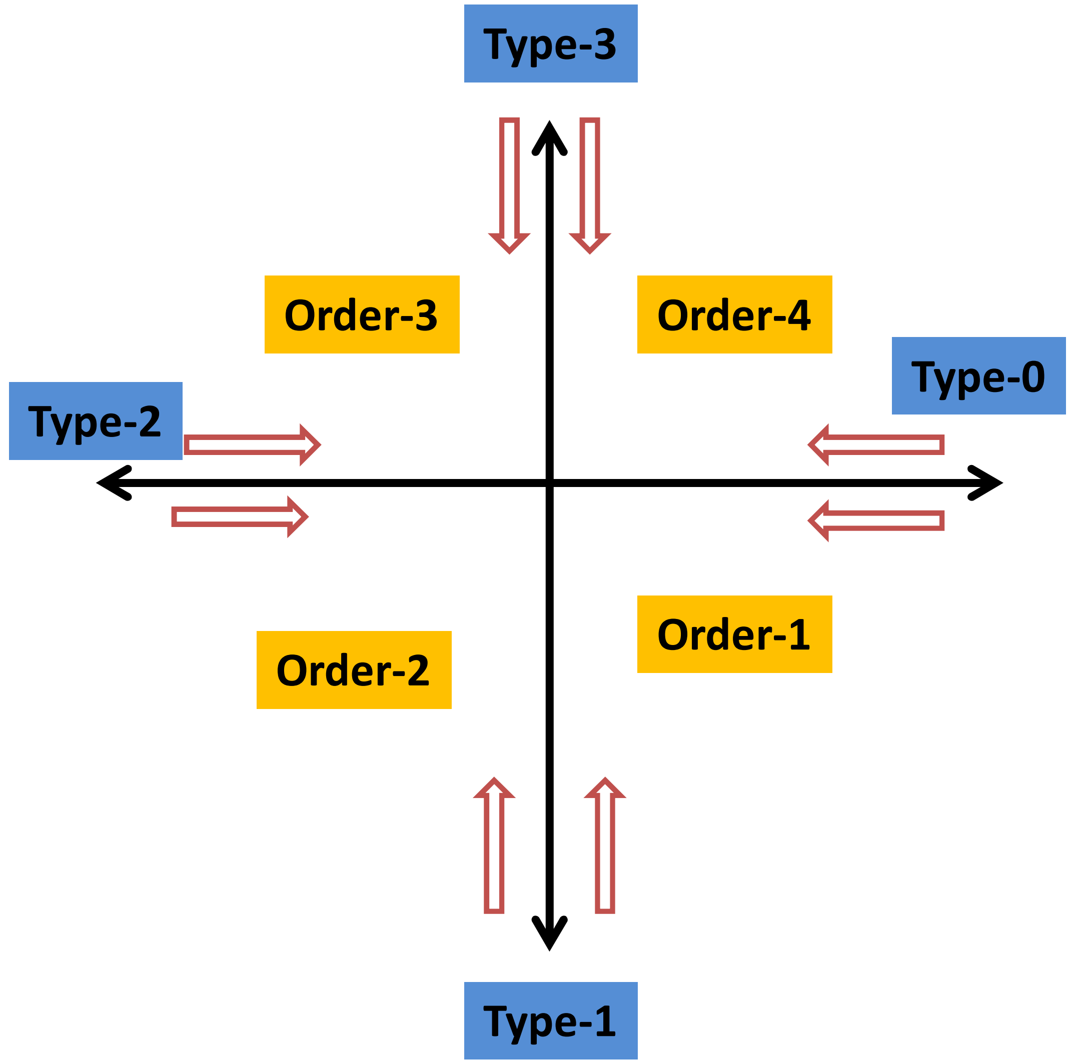
Polar Plot (Nyquist Plot)

The polar plot of a sinusoidal transfer function G(jw) is a plot of the magnitude of G(jw) Vs phase angle of G(jw) on polar coordinates as w is varied from 0 to ∞. The polar plot is also called as Nyquist plot.

To sketch the polar plot of $G(j\omega)$ for the entire range of frequency ω, i.e., from 0 to infinity, there are four key points that usually need to be known:

1) The start of plot where $\omega = 0$,
2) The end of plot where $\omega = \infty$,
3) Where the plot crosses the real axis, i.e., $\mathrm{Im}(G(j\omega)) = 0$, and
4) Where the plot crosses the imaginary axis, i.e., $\mathrm{Re}(G(j\omega)) = 0$.



Polar Plot of first order system: $G(j\omega) = \dfrac{1}{1 + j\omega T}$

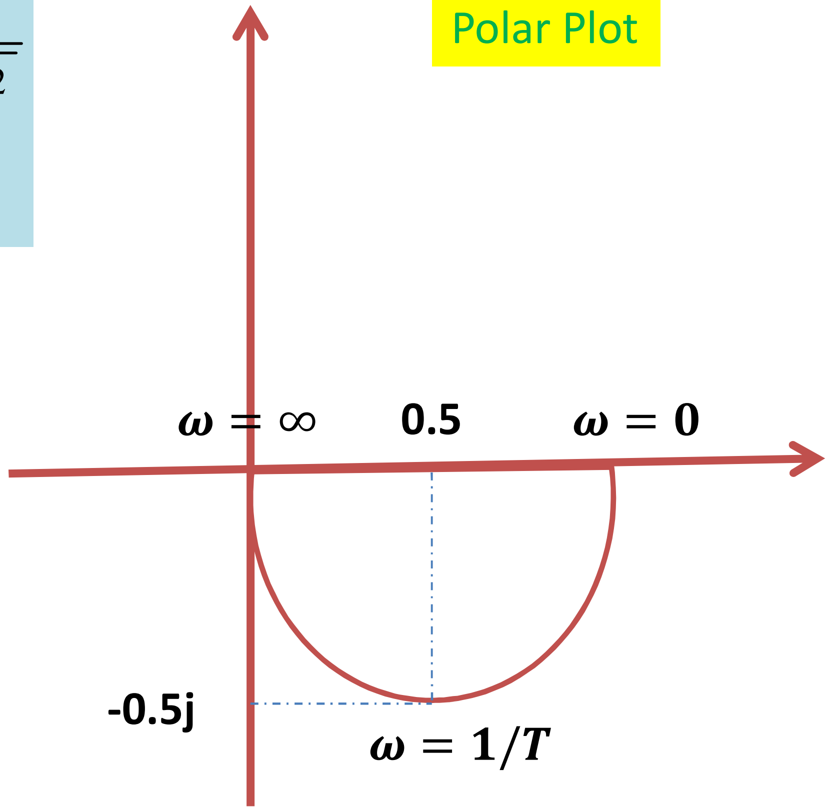
Magnitude and phase of G(jw) are:

$$M = |G(j\omega)| = \left|\frac{1}{1 + j\omega T}\right| = \frac{1}{\sqrt{1 + \omega^2 T^2}}$$

$$\varphi = -\tan^{-1}(\omega T)$$

	$\lvert G(j\omega)\rvert$	φ
$\omega = 0$	0	0
$\omega = \dfrac{1}{T}$	$\dfrac{1}{\sqrt{2}}$	-45^0
$\omega = \propto$	0	-90^0

Polar Plot



Polar Plot of second order syste $G(j\omega) = \dfrac{1}{(1 + j\omega T_1)(1 + j\omega T_1)}$

Magnetude and phase of G(jw) are:

$$|G(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 T_1^2}\sqrt{1 + \omega^2 T_2^2}}$$

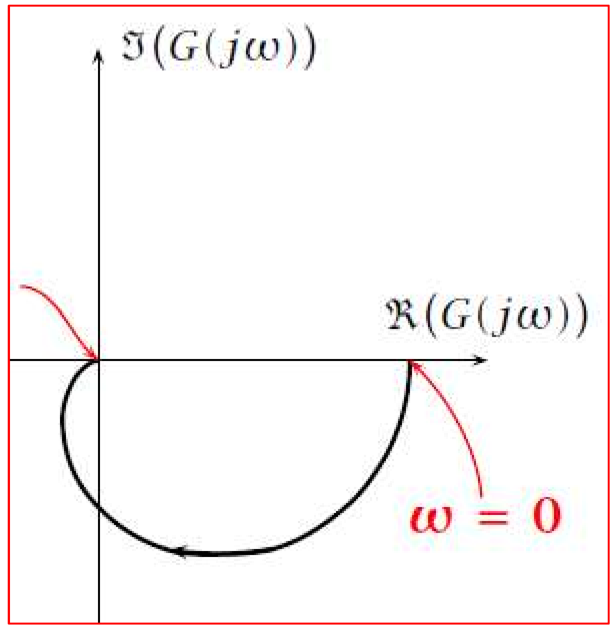
$$\angle G(j\omega) = -\arctan(\omega T_1) - \arctan(\omega T_2)$$

The start of plot where ω = 0

$$|G(j\omega)| = \frac{1}{\sqrt{1+0}\sqrt{1+0}} = 1$$

$$\angle G(j\omega) = -\tan^{-1}(0) - \tan^{-1}(0) = 0°$$

	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
$\omega = 0$	1	0
$\omega = \rightarrow \infty$	0	$-180°$



Polar Plot of third order system: $G(j\omega) = \dfrac{500}{(1+j\omega)(3+j\omega)(10+j\omega)}$

Compact form: $G(j\omega) = 500\dfrac{(-14\omega^2+30) - j(43\omega - \omega^3)}{(-14\omega^2+30)^2 + (43\omega-\omega^3)^2}$

Magnitude response:

$$\left|G(j\omega)H(j\omega)\right| = \sqrt{\mathrm{Re}^2 + \mathrm{Im}^2} = \frac{500}{\sqrt{(-14\omega^2+30)^2 + (43\omega-\omega^3)^2}}$$

Phase response:

$$\angle G(j\omega)H(j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right) = \tan^{-1}\left(\frac{-\left(43\omega-\omega^3\right)}{-14\omega^2+30}\right)$$

Point 1: The start of plot where ω = 0

$$|G(0)H(0)| = \frac{500}{\sqrt{(30)^2}} = 16.67$$

$$\angle G(0)H(0) = \tan^{-1}\frac{0}{30} = 0^\circ$$

Point 2: The end of plot where ω = ∞

$$|G(\infty)H(\infty)| = \frac{500}{\sqrt{\infty}} = 0$$

$$\angle G(\infty)H(\infty) = \tan^{-1}\frac{\infty^3}{30} = -3 \times 90° = -270°$$

Point 3: Where the plot crosses the real axis, i.e., **Im($G(j\omega)$) = 0**

Take the imaginary part of equation (a), and put equal to zero, to get the value of frequency ω at the interception of real axis.

$$\frac{-(43\omega-\omega^3)}{(-14\omega^2+30)^2+(43\omega-\omega^3)^2} = 0 \Rightarrow \begin{cases} 43\omega-\omega^3 = 0 \quad \Rightarrow \quad \omega = 0 \quad \text{and} \quad \omega = 6.56 \text{ rad/s} \\ \omega = \infty \end{cases}$$

Point 4: Where the plot crosses the imaginary axis, Re($G(j\omega)$) = 0

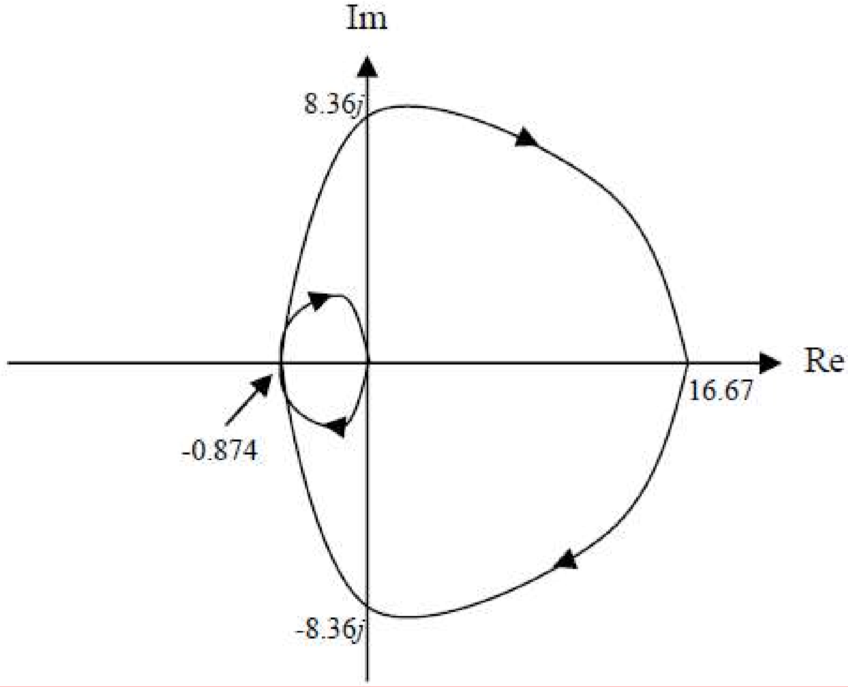
Take the real part of equation (a), and put equal to zero, to get the value of frequency ω at the interception of imaginary axis.

$$\frac{-14\omega^2+30}{(-14\omega^2+30)^2+(43\omega-\omega^3)^2} = 0 \Rightarrow \begin{cases} -14\omega^2+30 = 0 \quad \Rightarrow \quad \omega = 1.46 \text{ rad/s} \\ \omega = \infty \end{cases}$$

Key Points of the polar plot:

	$\lvert GH \rvert$	$\angle GH$
$\omega = 0$	16.67	0
$\omega = \infty$	0	-270°
Cross Re: $\omega = 0$ $\omega = \infty$ $\omega = 6.56$ rad/s	See above 0.874	See above -180°
Cross Re: $\omega = 0$ $\omega = \infty$ $\omega = 1.46$ rad/s	See above 8.36	See above -90°

Nyquist diagram



Sketch the polar plot

$$G(s)=\frac{10}{s(s+1)(s+5)}$$

$$G(j\omega)=\frac{-60\omega^2-j10\omega(5-\omega^2)}{36\omega^4-(5\omega-\omega^3)^2}$$

Point 1: The start of plot where ω = 0

$$G(j\omega)\big|_{\omega=0}=\frac{10}{5j\omega}\bigg|_{\omega=0}=\frac{2}{j\omega}\bigg|_{\omega=0}=\infty$$

$$\angle G(j\omega)\big|_{\omega=0}=\lim_{\omega\to 0}\angle\frac{2}{j\omega}=-90^\circ$$

Point 2: The end of plot where ω = ∞

$$\left|G(j\omega)\right|_{\omega\to\infty}=\lim_{\omega\to\infty}\left|\frac{10}{(j\omega^3)}\right|=\lim_{\omega\to\infty}\frac{10}{\omega^3}=0$$

$$\angle G(j\omega)\,|_{\omega\to\infty}=\angle\lim_{\omega\to\infty}\left[\frac{10}{(j\omega)^3}\right]=-270^\circ$$

Point 3: Where the plot crosses the real axis, i.e., **Im($G(j\omega)$) = 0**

Take the imaginary part of equation (a), and put equal to zero, to get the value of frequency ω at the interception of real axis.

$$\Rightarrow -\frac{10\omega(5-\omega^2)}{36\omega^4+(5\omega-\omega^3)^2}=0$$

$$\Rightarrow 10(5-\omega^2)=0$$

$$\Rightarrow \omega^2=5$$

$$\Rightarrow \omega=\sqrt{5}$$

Therefore, the intersection point between the polar plot and the real axis, when is located at;

$$\boxed{G(j\omega)|_{\omega=\sqrt{5}}=-\frac{1}{3}}$$

Point 4: Where the plot crosses the imaginary axis, Re($G(j\omega)$) = 0

Take the real part of equation (a), and put equal to zero, to get the value of frequency ω at the interception of imaginary axis.

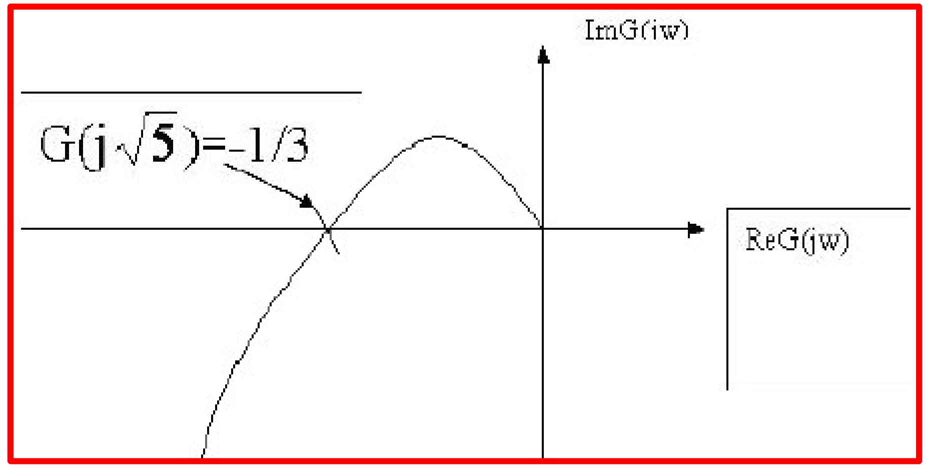
$$\Rightarrow \quad -\frac{60\omega^2}{36\omega^4+(5\omega-\omega^3)^2}=0$$

$$\Rightarrow \quad \omega=\infty$$

Therefore, the intersection point between the polar plot and the imaginary axis is when is located at;

$$\boxed{G(j\infty)=0}$$

	$\lvert G(j\omega) \rvert$	$\angle G(j\omega)$
$\omega = 0$	∞	-90^{o}
$\omega = \infty$	0	-270^{o}



The Concept of Mapping

- If we take a complex number $s = x+jy$ on the s-plane and substitute it into a function, F(s), another complex number results. This process is called mapping.

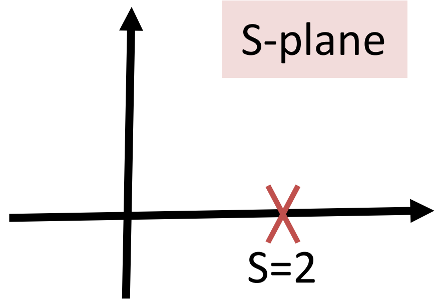
OR

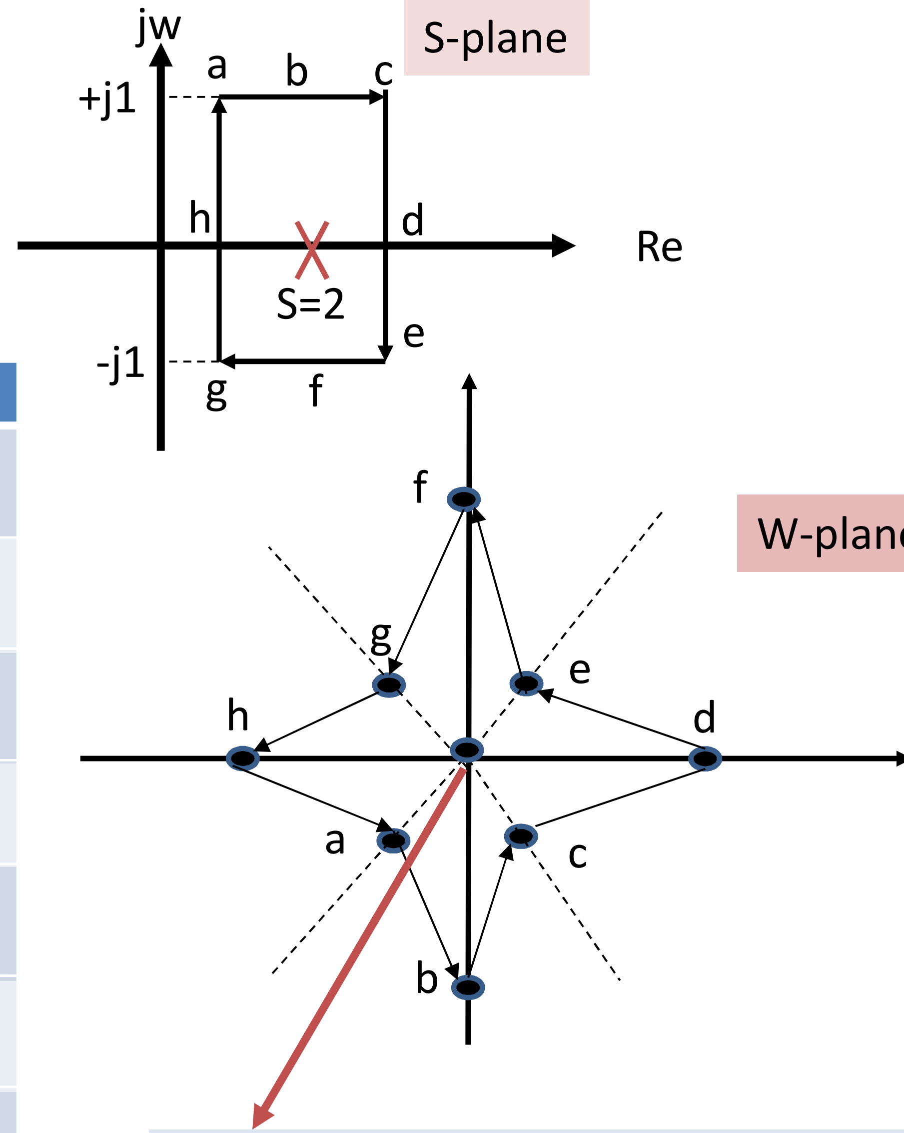
- The term mapping is defined as the substitution of a complex number into a function, F(s), to get another complex number.

- **For example**, substituting $\mathbf{s = 4 + j3}$ into the function $F(s) = (s^2 + 2s + 1)$ yields **16+j30**. We say that 4 + j3 maps into 16+j30 through the function $s^2 + 2s + 1$.

Mapping

Example: $G(s) = \frac{1}{s-2}$



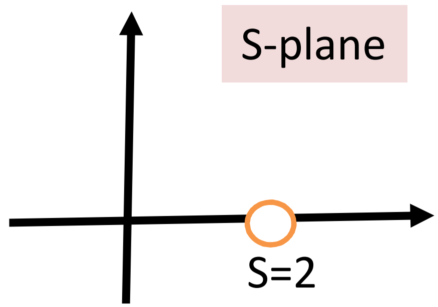


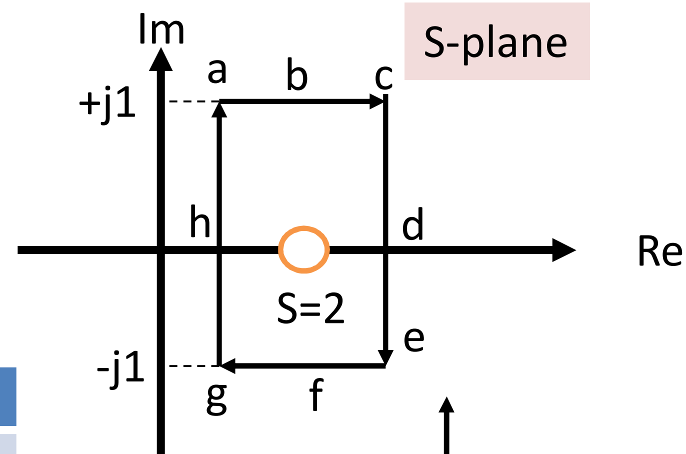
Points	G(s)	$\lvert G(s)\rvert$	$\angle G(s)$
S=a=1+j1	$\frac{1}{-1+j1}$	$\frac{1}{\sqrt{2}}$	-135^0
S=b=2+j1	$\frac{1}{j1}$	1	-90^0
S=c=3+j1	$\frac{1}{1+j1}$	$\frac{1}{\sqrt{2}}$	-45^0
S=d=3+j0	1	1	0^0
S=e=3-j1	$\frac{1}{1-j1}$	$\frac{1}{\sqrt{2}}$	45^0
S=f=2-j1	$\frac{1}{-j1}$	1	90^0
S=g=1-j1	$\frac{1}{-1-j1}$	$\frac{1}{\sqrt{2}}$	135^0
S=h=1+jo	$\frac{1}{-1}$	1	180^0

Encirclement about origin is
one but opposite in direction (ACW)

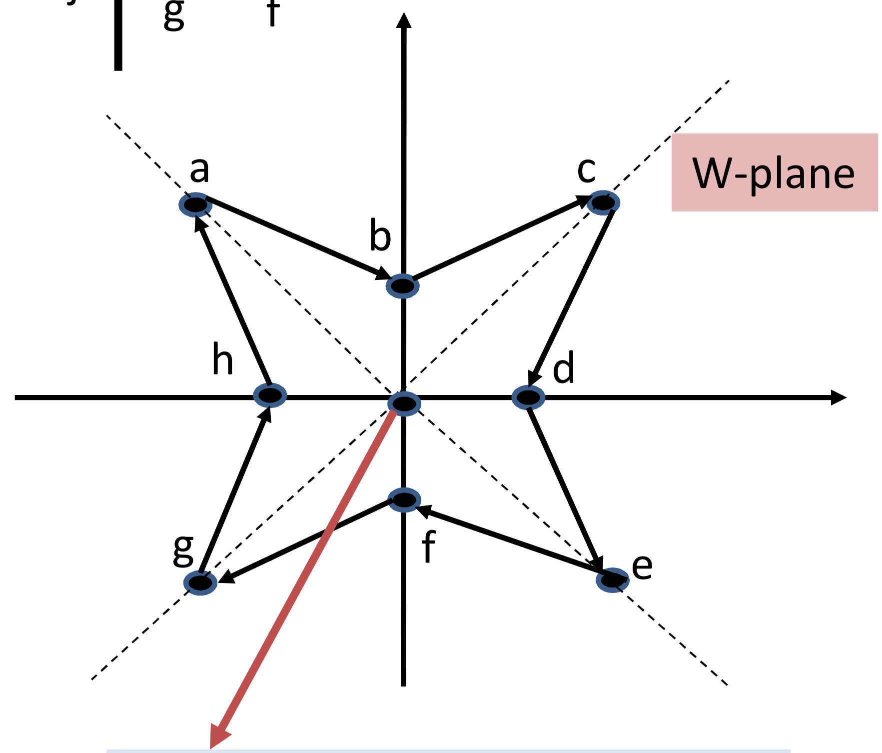
Mapping

Example: $G(s) = s - 2$





Points	G(s)	$\lvert G(s)\rvert$	$\angle G(s)$
S=a=1+j1	$-1 + j1$	$\sqrt{2}$	135^0
S=b=2+j1	$j1$	1	90^0
S=c=3+j1	$1 + j1$	$\sqrt{2}$	45^0
S=d=3+j0	1	1	0^0
S=e=3-j1	$1 - j1$	$\sqrt{2}$	-45^0
S=f=2-j1	-$j1$	1	-90^0
S=g=1-j1	$-1 - j1$	$\sqrt{2}$	-135^0
S=h=1+jo	-1	1	-180^0



Origin is enclosed but in same direction (CW)

Examples of Contour Mapping

The **contour B** maps in a counter clockwise direction if **F(s)** has just poles that are encircled by the contour, Also, you should verify that, if the pole or zero of **F(s)** is enclosed by **contour A**, the mapping encircles the origin .