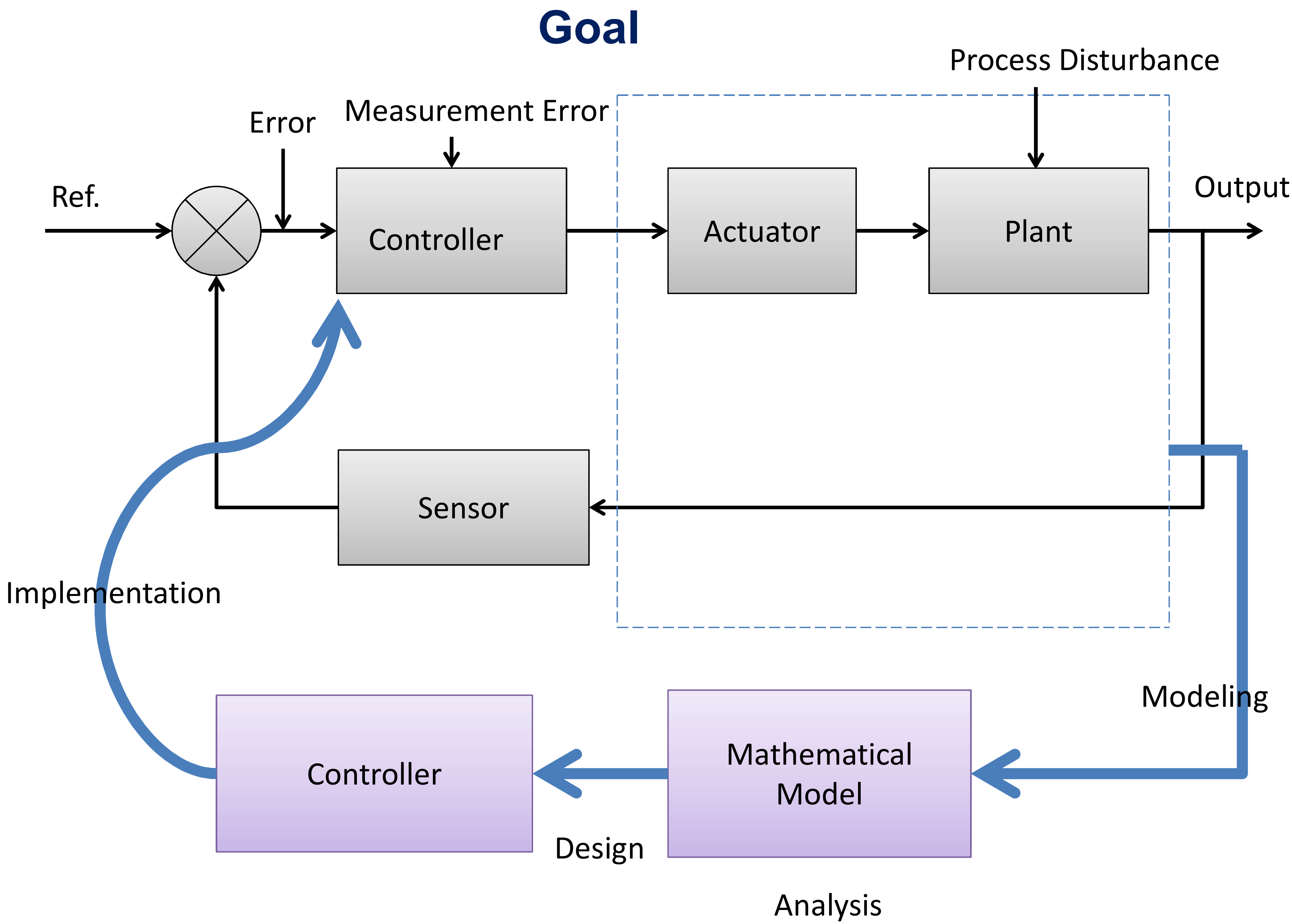
Automatic Control System Design

What is control System

1. A system consisting of the plant and the controller is called a control system.
2. A system controlling the operation of another system.
3. A system can regulate itself and another system.
4. A control system is a device, or set of devices to manage, command, direct or regulate the behaviour of other device(s) or system(s).


Goal
Process Disturbance
Measurement Error
Error
Ref.
Controller
Actuator
Plant
Output
Sensor
Implementation
Modeling
Controller
Mathematical Model
Design
Analysis

Example: Automatic speed control of a car. This is also called ‘cruise control’ works by using the difference or error btween the actual and desired speeds. The aim of this control system is to makes the error to zero, so that the desired speed can be achieved. This design process is called the control system and its implementation is the controller.

Control System and Design Technique

State of the System

The state of a system is defined as a collection of the smallest number of variables necessary to completely specify the system evolution in time. The number of state variables required to represent a system is called order order of the system.

Example:

$$\frac{d^2 y_1}{dt^2} - 2\left(\frac{dy_1}{dt}\right)^3 - 3\sin(y_2) = u$$

$$\frac{dy_2}{dt} + 4y_2^2 = 0$$

State Variables:

$$x_1 = y_1$$

$$x_2 = \frac{dy_1}{dt}$$

$$x_3 = y_2$$

State equations:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = 2x_2^3 + 3\sin(x_3) + u$$

$$\frac{dx_3}{dt} = -4x_3^2$$

$$\frac{dx}{dt} = f(x,u)$$

$$f(x,u) = \begin{Bmatrix} x_2 \\ 2x_2^3 - 3\sin(x_3) + u \\ -4x_3^2 \end{Bmatrix}$$

Output equation:

$$y = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Plant Model

Most plans are governed by the following differential Eq.

$$\frac{dx}{dt} = f[x(t), u(t), v(t), t]$$

The nonlinear function, $f(.)$, is assumed to possess partial derivatives w.r.t. $x(t), u(t)$ and $v(t)$ in the neighborhood of $x_d(t)$ - nominal trajectory. The nominal trajectory of the above Eq. for the unforced:

$$\frac{dx_d}{dt} = f[x_d(t), 0, 0, t], \quad t_i \le t \le t_f$$

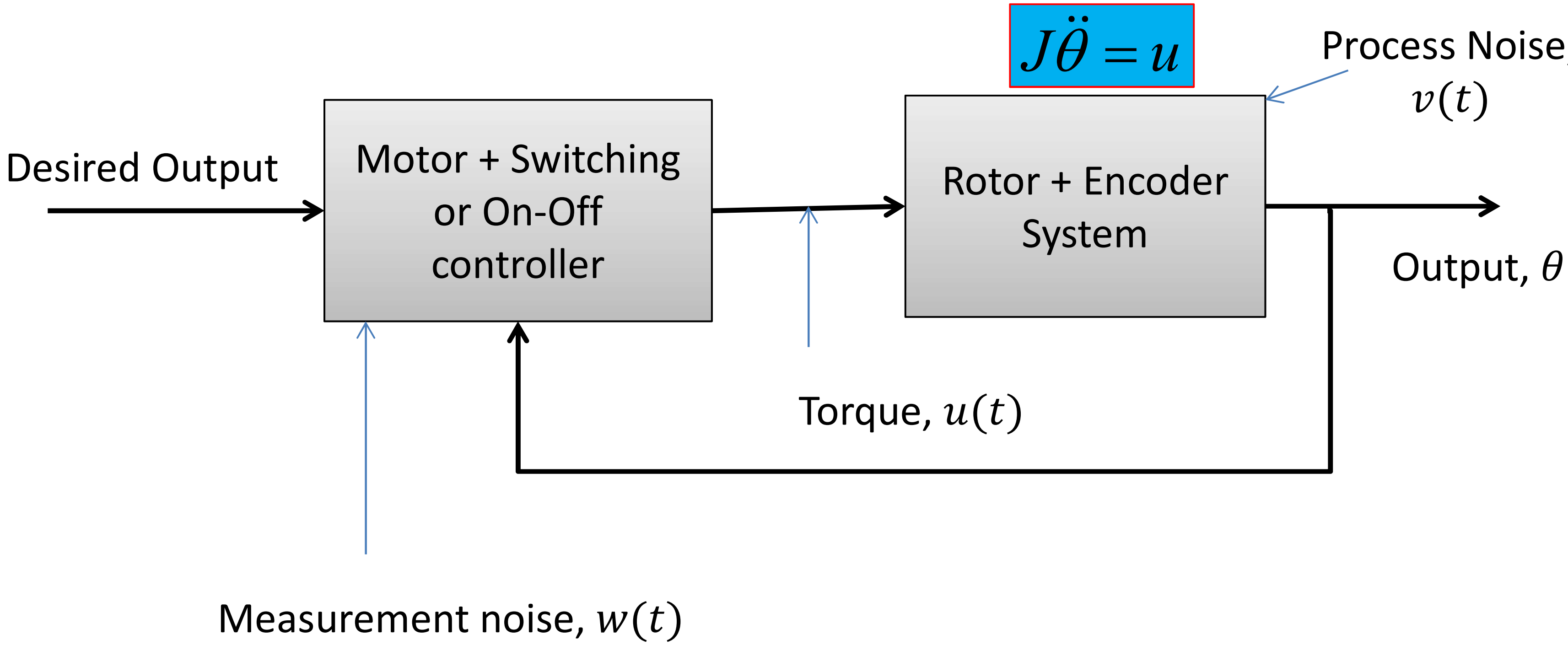
The output equation related state vector $x(t), u(t)$ and $w(t)$ can be written as

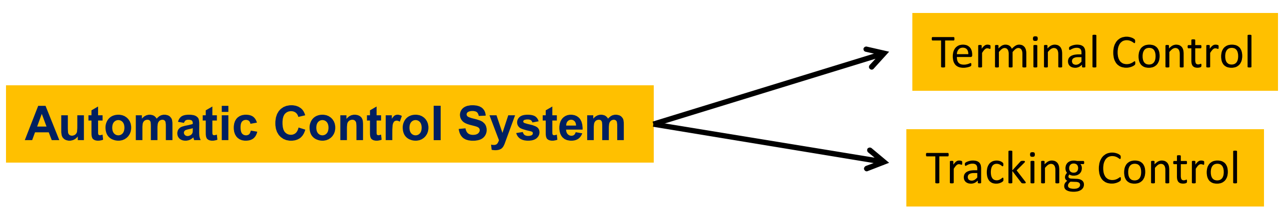
$$y(t) = h[x(t), u(t), w(t), t]$$

The output variables of a plant result from either direct or indirect measurements related to the state variables and control inputs through sensors.

Automatic Control System

A controller that performs its duty without human intervention is termed as automatic controller and obeys well defined mathematical relationships between plant state variables and control inputs, called control laws.

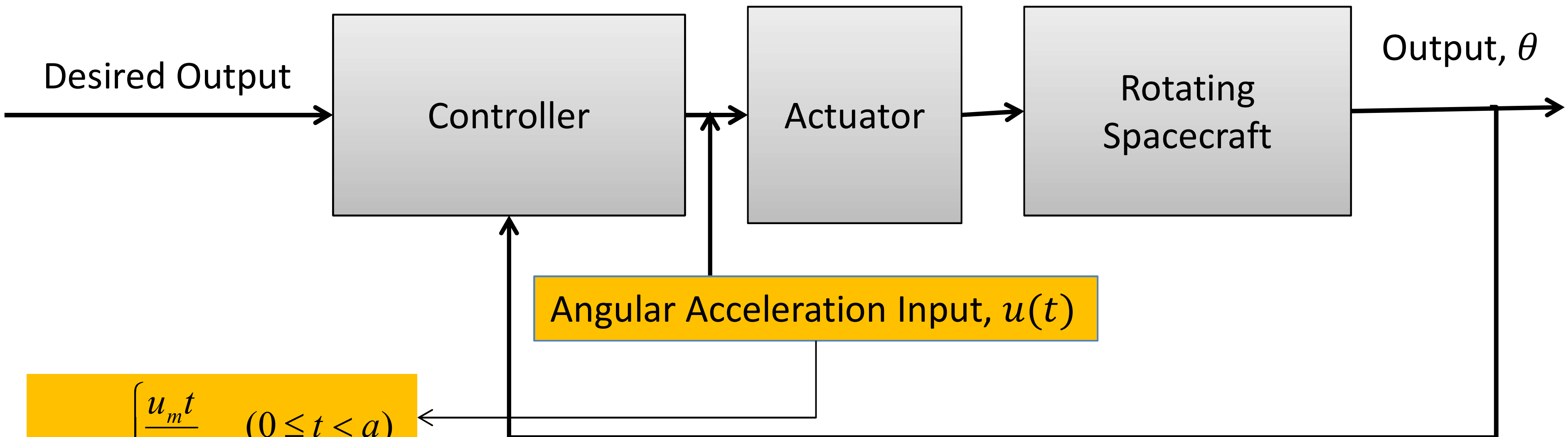




Terminal Control

A terminal control system aims at changing the plant's state from an initial state, $x(t_i)$, to a terminal state, $x(t_f)$, in a specified time, t_f, by applying a control input, $u(t)$, in the fixed control interval, $(t_i \leq t \leq t_f)$.

Example: Guidance of Spacecrafts and Rockets $\ddot{\theta} = u$

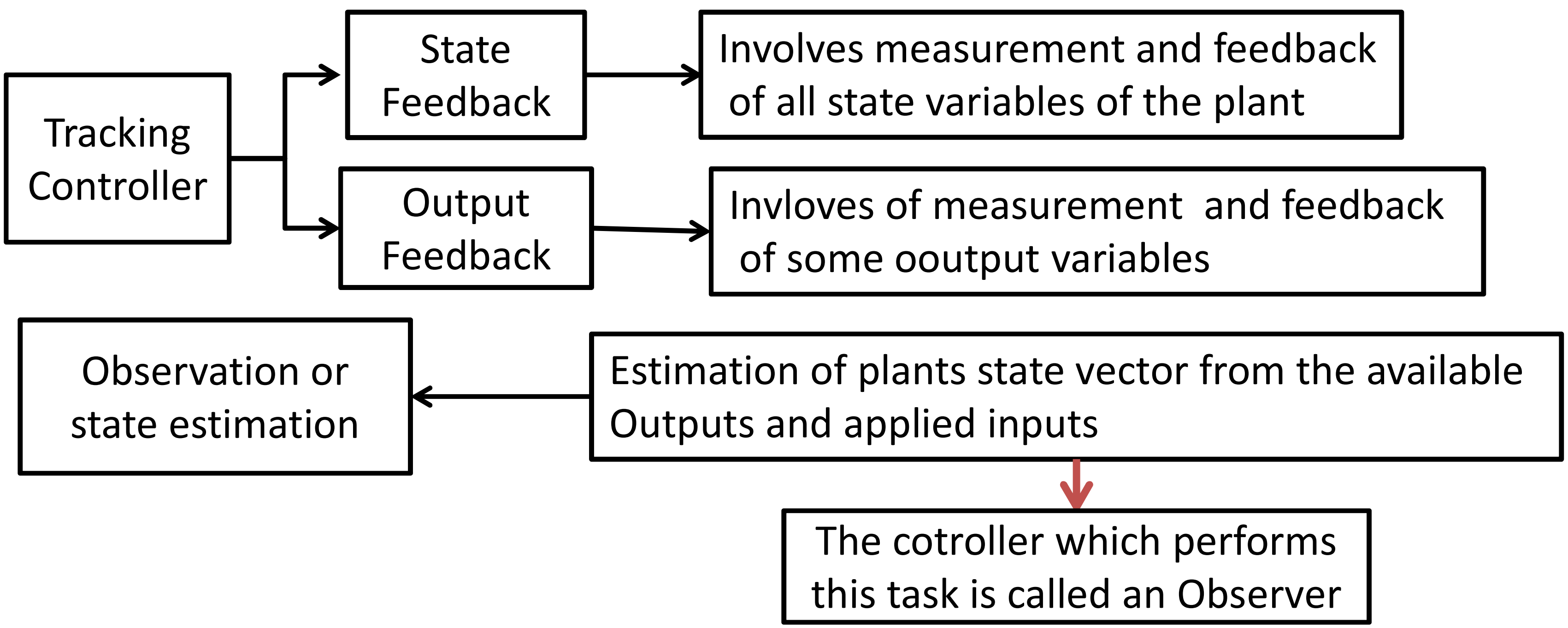


$$u(t) = \begin{cases} \dfrac{u_m t}{a} & (0 \leq t < a) \\ u_m & (a \leq t \leq t_f) \\ 0 & (t > t_f) \end{cases}$$

$$u(t) = \begin{cases} \theta(0) + \dot{\theta}(0)t + \dfrac{u_m t^3}{6a} & (0 \leq t < a) \\ \theta(0) + \dot{\theta}(0)t + u_m \dfrac{a^2}{6} + \dfrac{u_m}{2} t(t-a) & (a \leq t \leq t_f) \\ \theta(0) + \dot{\theta}(0)t_f + u_m \dfrac{a^2}{6} + \dfrac{u_m}{2} t_f(t_f - a) + u_m (t_f - \dfrac{a}{2})(t - t_f) & (t > t_f) \end{cases}$$

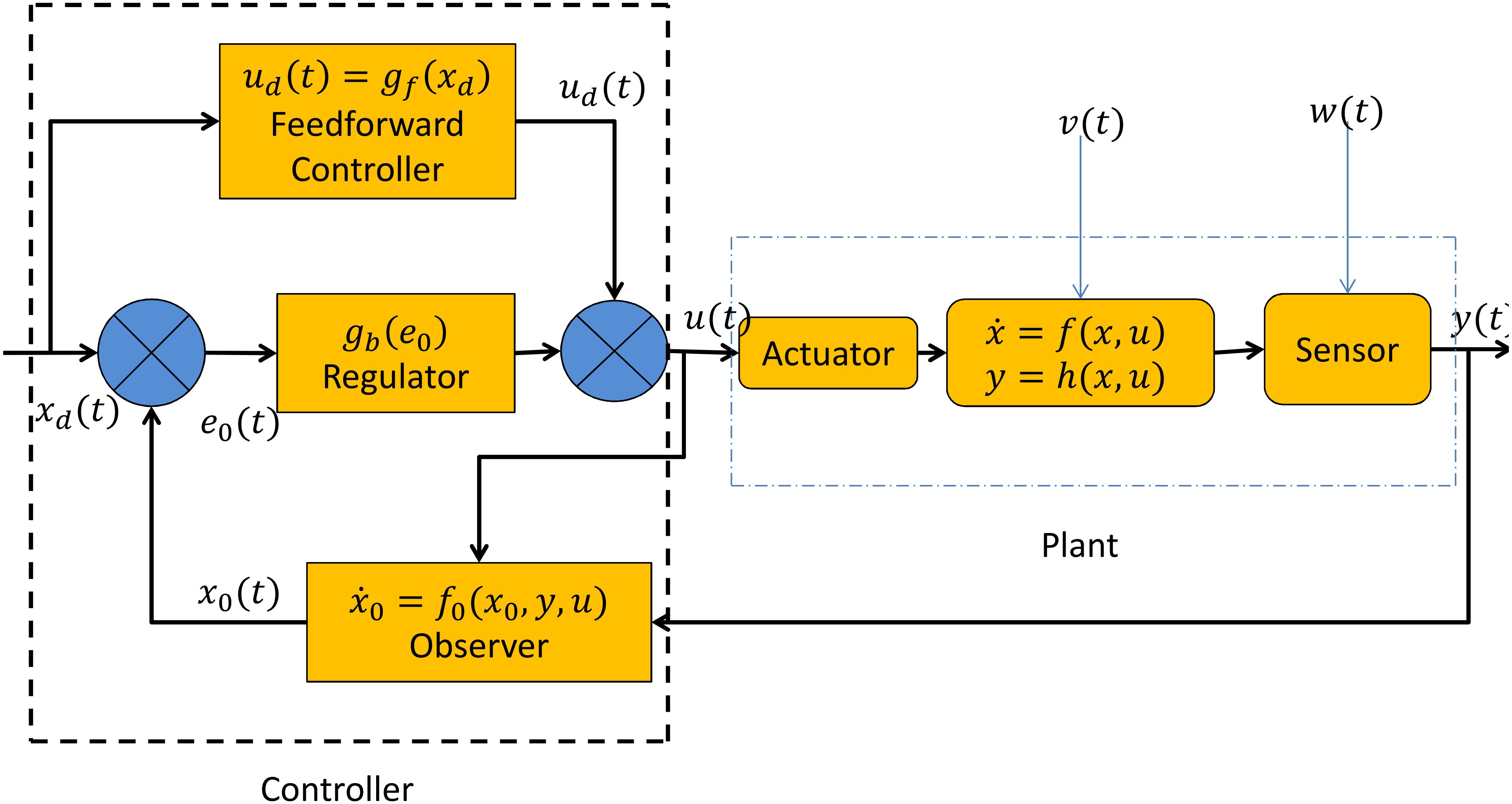
Tracking Control

The objective of the tracking controller is to maintain the plant's state, $x(t)$, close to nominal trajectory, $x_d(t)$, by the control input, $u(t)$. The tracking controller can be designed through linearizing the plant about the nominal trajectory, $x_d(t)$. The tracking controller compares the plant's state with the nominal state and generate the control signal based on error vector: $e(t) = x_d(t) - x(t)$.



Example: Most of the flight control problems-such as aircraft guidance, orbital control of Spacecraft, and attitude control of all aerospace vehicles

Tracking Control System with an Observer



Tracking control for Spacecraft Rotation