AE673 - HomeWork-1

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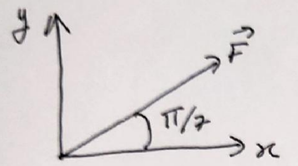
Q.1. -



$$\vec{F} = a\hat{i} + b\hat{j}$$

$$\vec{F}\cdot\hat{i} = a = |\vec{F}|\cos\pi/7 = (19)\cos(\pi/7) = 17.118$$

$$\vec{F}\cdot\hat{j} = b = |\vec{F}|\sin\pi/7 = (19)\sin(\pi/7) = 8.243$$

$$\therefore \boxed{\vec{F} = 17.118\,\hat{i} + 8.243\,\hat{j}}$$

Q.2. -

$$\vec{F} = -\hat{i} + 3\hat{j} + \hat{k} \quad ; \quad \vec{G} = 2\hat{j} - 4k$$

$$|\vec{F}| = \sqrt{11} \qquad |\vec{G}| = \sqrt{20}$$

$$\vec{F}\cdot\vec{G} = 2 = |\vec{F}|\,|\vec{G}|\cos\theta = \sqrt{11}\sqrt{20}\cos\theta.$$

$$\therefore \cos\theta = 0.1348. \; \cancel{= 82.25}$$

$$\therefore \boxed{\theta = 82.25^\circ}$$

Q.3. -

$$\left\{[\vec{a}\times(\vec{b}\times\vec{c})] = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}\right\}$$

$$(\vec{a}\times(\vec{b}\times\vec{c}))_i = a_j c_j b_i + a_k c_k b_i - a_j c_i b_j - a_k c_i b_k + \underline{a_i b_i c_i - a_i b_i c_i}$$

$$(\vec{a}\times(\vec{b}\times\vec{c}))_j = a_k b_j c_k + a_i c_i b_j - a_k b_k c_j - a_i b_i c_j + \underline{a_j b_j c_j - a_j b_j c_j}$$

$$(\vec{a}\times(\vec{b}\times\vec{c}))_k = a_i c_i b_k + a_j c_j b_k - a_i b_i c_k - a_j b_j c_k + \underline{a_k b_k c_k - a_k b_k c_k}$$

$$\therefore (\vec{a}\times(\vec{b}\times\vec{c}))_i = (a_i c_i + a_j c_j + a_k c_k) b_i - (a_i b_i + a_j b_j + a_k b_k) c_i = (\vec{a}\cdot\vec{c}) b_i - (\vec{a}\cdot\vec{b}) c_i$$

similarly

$$(\vec{a}\times(\vec{b}\times\vec{c}))_j = (\vec{a}\cdot\vec{c}) b_j - (\vec{a}\cdot\vec{b}) c_j$$

$$(\vec{a}\times(\vec{b}\times\vec{c}))_k = (\vec{a}\cdot\vec{c}) b_k - (\vec{a}\cdot\vec{b}) c_k.$$

$$\therefore (\vec{a}\times(\vec{b}\times\vec{c})) = (\vec{a}\cdot\vec{c})(b_i + b_j + b_k) - (\vec{a}\cdot\vec{b})(c_i + c_j + c_k)$$

$$= (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$$

$\therefore$ Proved.

Q.4.(a) - (Volume of Parallelopiped)

consider $\vec{a} = \{0, 4, 0\}$
$\vec{b} = \{-2, 3, 1\}$
$\vec{c} = \{-1, 3, 3\}$ forming incident sides of ~~Fa~~ a Parallelopiped.

→ volume of Parallelopiped is given by Scalar Product $(\vec{a} \times \vec{b}) \cdot \vec{c}$

$\therefore (\vec{a} \times \vec{b}) = \{0, 4, 0\} \times \{-2, 3, 1\} = \{4, 0, 8\}$

$(\vec{a} \times \vec{b}) \cdot (\vec{c}) = \{4, 0, 8\} \cdot \{-1, 3, 3\} = 20.$

$\therefore$ Volume is 20 units.

Q.5. curvature -

$x(t) = \cos(t) + t\sin(t)$
$y(t) = \sin(t) - t\cos(t)$
$z(t) = t^2$
$t > 0$

or consider

$$\vec{s}(t) = \begin{Bmatrix} \cos(t) + t\sin(t) \\ \sin(t) - t\cos(t) \\ t^2 \end{Bmatrix}$$

let a unit tangent vector to this curve be $\vec{T}(t)$. $\vec{T}(t) = \dfrac{\vec{s}'(t)}{|\vec{s}'(t)|}$

$$\vec{s}'(t) = \begin{Bmatrix} -\sin(t) + t\cos(t) + \sin(t) \\ \cos(t) + t\sin(t) - \cos(t) \\ 2t \end{Bmatrix} = \begin{Bmatrix} t\cos t \\ t\sin t \\ 2t \end{Bmatrix}$$

$$|\vec{s}'(t)| = \sqrt{t^2(\sin^2 t + \cos^2 t) + 4t^2}$$
$$= \sqrt{5}\, t$$

$$\therefore \vec{T}(t) = \frac{1}{\sqrt{5}} \begin{Bmatrix} \cos t \\ \sin t \\ 2 \end{Bmatrix}$$

→ curvature $k = \left\| \dfrac{d\vec{T}}{d\vec{s}} \right\| = \dfrac{\left\| \frac{d\vec{T}}{dt} \right\|}{\left\| \frac{d\vec{s}}{dt} \right\|} = \dfrac{\left\| \frac{1}{\sqrt{5}} \begin{Bmatrix} -\sin t \\ \cos t \\ 0 \end{Bmatrix} \right\|}{\sqrt{5}\, t} = \dfrac{1}{5t}$

$$\therefore \boxed{K = \frac{1}{5t}}$$

Q.4.(b) — (find length of curve)

$x(t) = \cos(t)$
$y(t) = \sin(t)$
$z(t) = t/3.$
$\underline{-4\pi \le t \le 4\pi}$

$$L = \int_{-4\pi}^{4\pi} \sqrt{\left(\frac{dx(t)}{dt}\right)^2 + \left(\frac{dy(t)}{dt}\right)^2 + \left(\frac{dz(t)}{dt}\right)^2}\, dt$$

$$\therefore\ L = \int_{-4\pi}^{4\pi} \sqrt{\sin^2 t + \cos^2 t + \frac{1}{9}}\ dt$$

$$L = \int_{-4\pi}^{4\pi} \sqrt{10/9}\ dt. = \boxed{\frac{8\sqrt{10}}{3}\pi}$$

Q.6. —

To Prove: $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [(\vec{a}\times\vec{b})\cdot\vec{d}]\,\vec{c} - [(\vec{a}\times\vec{b})\cdot\vec{c}]\,\vec{d}$
$= [(\vec{c}\times\vec{d})\cdot\vec{a}]\,\vec{b} - [(\vec{c}\times\vec{d})\cdot\vec{b}]\,\vec{a}$

→ we know: (Q.3) $\underline{(\vec{u}\times(\vec{v}\times\vec{w})) = (\vec{u}\cdot\vec{w})\vec{v} - (\vec{u}\cdot\vec{v})\vec{w}}$

consider $\vec{a}\times\vec{b}$ ~~as~~ as a vector. $\vec{t}$

$\therefore\ \vec{t}\times(\vec{c}\times\vec{d}) = (\vec{t}\cdot\vec{d})\vec{c} - (\vec{t}\cdot\vec{c})\vec{d}$
$= [(\vec{a}\times\vec{b})\cdot\vec{d}]\,\vec{c} - [(\vec{a}\times\vec{b})\cdot\vec{c}]\,\vec{d}$ — ①

now consider $\vec{c}\times\vec{d}$ as a vector. $\vec{t}$

$(\vec{a}\times\vec{b})\times\vec{t} = -[\vec{t}\times(\vec{a}\times\vec{b})] = -[(\vec{t}\cdot\vec{b})\vec{a} - (\vec{t}\cdot\vec{a})\vec{b}]$
$= (\vec{t}\cdot\vec{a})\vec{b} - (\vec{t}\cdot\vec{b})\vec{a}$
$= [(\vec{c}\times\vec{d})\cdot\vec{a}]\,\vec{b} - [(\vec{c}\times\vec{d})\cdot\vec{b}]\,\vec{a}$ — ②

$\therefore$ Proved.

~~Q~~

Q.7. –

$$\vec{d} = \frac{[(b\times \vec{c})\cdot\vec{d}]\vec{a} + [(\vec{c}\times\vec{a})\cdot\vec{d}]\vec{b} + [(\vec{a}\times\vec{b})\cdot\vec{d}]\vec{c}}{[(\vec{a}\times\vec{b})\cdot\vec{c}]}$$

→ from last Q.6.

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = ((\vec{a}\times\vec{b})\cdot\vec{d})\,\vec{c} - ((\vec{a}\times\vec{b})\cdot\vec{c})\,\vec{d}$$
$$= ((\vec{c}\times\vec{d})\cdot\vec{a})\,\vec{b} - [(\vec{c}\times\vec{d})\cdot\vec{b}]\,\vec{a}$$

∴ rearranging them.

$$\frac{[(\vec{a}\times\vec{b})\cdot\vec{d}]\vec{c} + [(\vec{c}\times\vec{d})\cdot\vec{b}]\,\vec{a} - [(\vec{c}\times\vec{d})\cdot\vec{a}]\,\vec{b}}{((\vec{a}\times\vec{b})\cdot\vec{c})} = \vec{d}$$

→ using Properties of ~~...~~ circular arrangement of scalar triple product.

$$\vec{d} = \frac{[(\vec{b}\times\vec{c})\cdot\vec{d}]\vec{a} + [(\vec{c}\times\vec{a})\cdot\vec{d}]\vec{b} + [(\vec{a}\times\vec{b})\cdot\vec{d}]\vec{c}}{[(\vec{a}\times\vec{b})\cdot\vec{c}]}$$

∴ Proved.

~~cont~~ → express $\vec{d} = [3,2,1]$

in terms of $\vec{a} = [1,2,3]$ $\vec{b} = [2,3,1]$ $\vec{c} = [3,1,2]$

calculating gives.

$(\vec{a}\times\vec{b})\cdot\vec{c} = -18$.

$(\vec{b}\times\vec{c})\cdot\vec{d} = 6$

$(\vec{c}\times\vec{a})\cdot\vec{d} = -12$

$\vec{a}\times\vec{b}\cdot\vec{d} = -12$

$$\therefore \vec{d} = \frac{1}{-18}(6\vec{a} - 12\vec{b} - 12\vec{c})$$

or $\boxed{\vec{d} = \frac{1}{3}(-\vec{a} + 2\vec{b} + 2\vec{c})}$