WHAT IS A TRUTH FUNCTIONAL COMPONENT ?

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Discussions with James Martin have convinced me that many published definitions of truth functionality are defective. Someone might insist that because the definitions are stipulative they cannot possibly be defective. Since the meaning of 'truth functional component' is *given* by the definitions, there cannot be a counterexample which shows that the definitions fail. I could retreat a bit and say I want to suggest an alternative stipulation, but this sort of response would concede too much. A man might be content with an inadequate definition of 'courageous act' even though he can tell when an act is courageous and when it is not. No one who has read a Socratic dialogue needs to be told that a person can use a word correctly without being able to give a correct account of its use. This ancient moral applies to many technical terms as well as it applies to more important words and phrases used in everyday life. It applies in particular, I think, to 'truth functional component' We are clearer about what is or is not a truth functional component of a given sentence than we are about how to say what it is for one sentence to be a truth functional component of another. I shall try to provide a definition which is free of the defects that Martin and I have noticed.

Here is a definition which is clearer and more precise than most.

A compound of given components is a truth function of them if its truth value remains unchanged under all changes of the components so long as the truth values of the components remain unchanged ([1]).

Let us suppose that we know what it is for one sentence to be a component of another. And let us restrict our attention to declarative sentences. The definition can be rewritten as follows: P is a truth functional component of Q if and only if

([1]) W. V. QUINE, *Methods of Logic*, Revised Edition (New York, 1959), p. 8.

P is a component of Q, and

(S) (if S has the same truth value as P, then the sentence which results from replacing P in Q by S has the same truth value as Q).

This will not do. Consider

(1) Henry knows that beavers lay eggs.

According to the definition, 'Beavers lay eggs' is a truth functional component of (1). (1) must be false because 'Beavers lay eggs' is false. And the truth value of (1) remains unchanged no matter what sentence is substituted for 'Beavers lay eggs' so long as the truth value of the sentence remains unchanged, that is, so long as the substituted sentence is false.

Although the sentence that results from replacing 'Beavers lay eggs' in (1) by one false sentence has the same truth value as the sentence that results from replacing 'Beavers lay eggs' in (1) by any other false sentence, sentences which result from replacement by different *true* sentences need not have the same truth value. This is why we want not to regard 'Beavers lay eggs' as a truth functional component of (1). The following is a necessary condition of 'P is a truth functional component of Q):

P is a component of Q, and

(S)(T) (if S and T have the same truth value, then the sentence which results from replacing P in Q by S has the same truth value as the sentence which results from replacing P in Q by T).

'Beavers lay eggs' is not a truth functional component of (1) because it fails to satisfy this condition.

Unfortunately, the condition is not sufficient. Consider

(2) Henry believes that beavers build dams $\supset$ Henry believes something.

'Beavers build dams' is a component of (2). Since (2) is true no matter what sentence is substituted for 'Beavers build dams', when any two sentences with the same truth value are respectively substitued for it, the resulting sentences are both true and thus have the same truth value.

The following principle seems to be implicit in our understanding of truth functionality:

If P is a component of R which is in turn a component of Q, then P is a truth functional component of Q only if P is also a truth functional component of R.

Example (2) shows that satisfaction of our necessary condition does not insure that this principle is satisfied. We want not to count 'Beavers build dams' as a truth functional component of (2) because it is not a truth functional component of 'Henry believes that beavers build dams'. Let us emend our necessary condition accordingly. The task will be somewhat simpler if we stipulate that every sentence is a component of itself. This stipulation will lead to the conclusion, which I find neither welcome nor unwelcome, that every (declarative) sentence is a truth functional component of itself. If the conclusion seems anomalous, it can easily be avoided by an increase of complexity. P, then, is a truth functional component of Q only if

P is a component of Q, and
(R) (if P is a component of R and R is a component of Q, then
(S)(T) (if S and T have the same truth value, then the sentence which results from replacing P in R by S has the same truth value as the sentence which results from replacing P in R by T)).

Since we count sentences as components of themselves, this condition is satisfied only if the first condition is satisfied. 'Beavers build dams' is not a truth functional component of (2) because it fails to satisfy this new condition.

Unfortunately, this also is not sufficient. Consider

(3) Henry knows that (dogs live in trees and beavers chew wood).

'Beavers chew wood' is a component of (3). Since (3) is false no matter what sentence is substituted for 'Beavers chew wood', when any two sentences with the same truth value are respectively substituted for it, the resulting sentences are both false and thus have the same truth value. Furthermore, the sentence 'Dogs live in trees and beavers chew wood' satisfies, as it should, all our conditions for containing 'Beavers chew wood' as a truth functional component.

There seems to be still another principle implicit in our understanding of truth functionality:

> If P is a component of R which is in turn a component of Q, then P is a truth functional component of Q only if R is also a truth functional component of Q.

Example (3) shows that satisfaction of our revised necessary condition does not insure that this second principle is satisfied. We want not to count 'Beavers chew wood' as a truth functional component of (3) because it is a component of 'Dogs live in trees and beavers chew wood' which is not itself a truth functional component of (3). Also, example (2) shows that satisfaction of the second principle does not insure satisfaction of the first. Further revision and complication of our condition are called for.

I believe that the following is both a necessary and a sufficient condition of 'P is a truth functional component of Q':

> P is a component of Q, and
> (R)(if P is a component of R and R is a component of Q, then
> (S)(T)(if S and T have the same truth value, then
> ((the sentence which results from replacing P in R by S has the same truth value as the sentence which results from replacing P in R by T) and
> (the sentence which results from replacing R in Q by S has the same truth value as the sentence which results from replacing R in Q by T)))).

I would be pleased, but not surprised, if someone could provide a simpler formulation which does the same job. I would be rather less pleased, but still not surprised, if someone could think of an example which shows that the job cannot be done adequately without further complication.

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Note added in proof: I neglect to make explicit that I want to say when a particular occurrence of a sentence is truth functional. Separate occurrences must be handled separately. Thus the second occurrence of 'Dogs bury bones' in '(Henry knows that dogs bury bones) ⊃ dogs bury bones" is truth functional, but the first occurrence is not.