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$$u_{xx} + u_{yy} = 0$$

$$x, y \geq 0$$

$$u(x, 0) = 0$$

$$u(0, y) = e^{-y}$$

$$F_s \{ u \} = u(x, \omega)$$

حل با تبدیل فوریه سینوسی نسبت به  
متغیر  $y$

$$F_s \{ u_{xx} \} = U_{xx}$$

$$F_s \{ u_{yy} \} = -\omega^2 U$$

$$U_{xx} - \omega^2 U = 0$$

$$U = A(\omega) e^{-\omega x} + B(\omega) e^{\omega x}$$

$$U \ll \infty \quad \underset{x \rightarrow \infty}{\longrightarrow} B(\omega) = 0$$

$$U = A(\omega) e^{-\omega x}$$

$$F_S\{U_\omega(\omega, y)\} = U_\omega(\omega, \omega) = -\omega A(\omega)$$

$$F_S\{e^{-y}\} = \int_{-\infty}^{\infty} \sin \omega y e^{-y} dy = \sqrt{\frac{2}{\pi}} \frac{\omega}{\omega^2 + 1}$$

$$\Rightarrow A(\omega) = -\frac{\sqrt{\frac{2}{\pi}}}{\omega^2 + 1}$$

$$u(\omega, y) = F_S^{-1}\left\{-\sqrt{\frac{2}{\pi}} \frac{1}{\omega^2 + 1} e^{-\omega y}\right\}$$

$$= -\frac{\sqrt{\frac{2}{\pi}}}{\omega^2 + 1} \int_0^\infty \frac{1}{\omega^2 + 1} e^{-\omega w} \sin \omega y dw$$

$U_{(x,y)}$

Wave equation

- $$\bullet F_s \{ u_{xx} \} = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \left( \frac{\partial^2}{\partial x^2} u \right) \sin wy dy$$

$$= \frac{\partial}{\partial x^2} \left( \sqrt{\frac{2}{\pi}} \int_0^{\infty} u \sin wy dy \right), U_{\text{now}}$$
- $$\bullet F_s \{ u_{yy} \}, -w F_c \{ u_y \}$$

$$= -w \left( -\sqrt{\frac{2}{\pi}} \underbrace{U(x,0)}_{\bullet} + w \int_s \{ u \} \right)$$

$$= -w^2 u$$

$$\int \underbrace{\sin wt e^{-st} dt}_w = I$$

$$I = \frac{-1}{s} \left. \sin wt e^{-st} \right|_0^\infty + \frac{w}{s} \int \underbrace{\cos wt e^{-st} dt}_v$$

$$\int \underbrace{\cos wt e^{-st} dt}_v = \frac{-1}{s} \left. \cos wt e^{-st} \right|_0^\infty - \frac{w}{s} \int \underbrace{\sin wt e^{-st} dt}_w$$

$$\frac{I + \frac{1}{s} \left. \sin wt e^{-st} \right|_0^\infty}{\frac{w}{s}} = \frac{-1}{s} \left. \cos wt e^{-st} \right|_0^\infty - \frac{w}{s} I$$

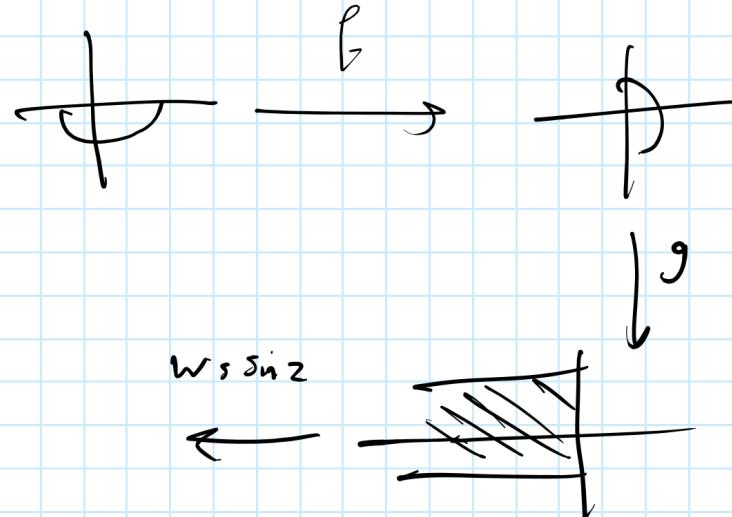
$$\lim_{t \rightarrow \infty} e^{-st} \underset{\text{as}}{\sim} 0 \rightarrow \frac{I}{\frac{w}{s}} = \frac{1}{s} - \frac{w}{s} I$$

$$I \left( \frac{s}{w} + \frac{w}{s} \right) = \frac{1}{s} \Rightarrow I = \frac{w}{s^2 + w^2}$$

$$\bullet \frac{d\theta}{dt} = \omega^2 \theta \text{ s.}$$

$$r^2 = \omega^2 \text{ s.} \quad r = I \omega$$

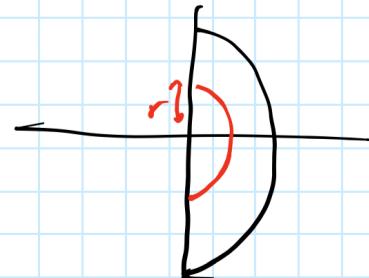
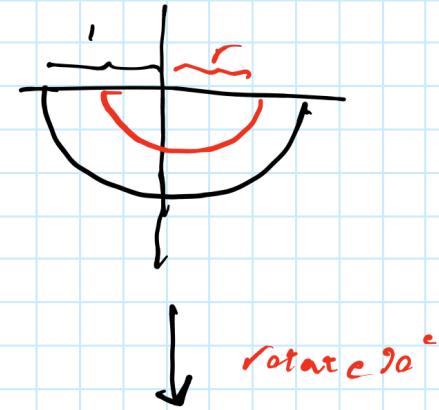
$$\theta \text{ s.} A \cos \theta + B \sin \theta$$



$$f(z = re^{i\theta}) = re^{i(\theta + \frac{\pi}{2})}$$

Now  $f(z) = az \rightarrow re^{i(\theta + \frac{\pi}{2})} = arc^{i\theta}$   
 $\Rightarrow a \cdot e^{i\frac{\pi}{2}} = i$

$$f(z) = iz$$



أو:  $g = \log z$

$$\log R e^{i\theta} \rightarrow \log R + i\theta$$

$$\xrightarrow{R=1} -\frac{\pi}{2} \leq \log 1 + i\theta \leq \frac{\pi}{2}$$

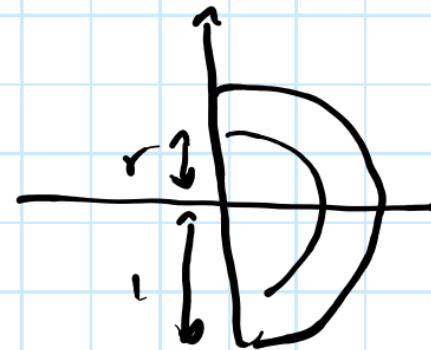
خط عمودي  
 $(u=0)$  في  $\frac{\pi}{2} < \theta < \frac{3\pi}{2}$ .

مكتوب

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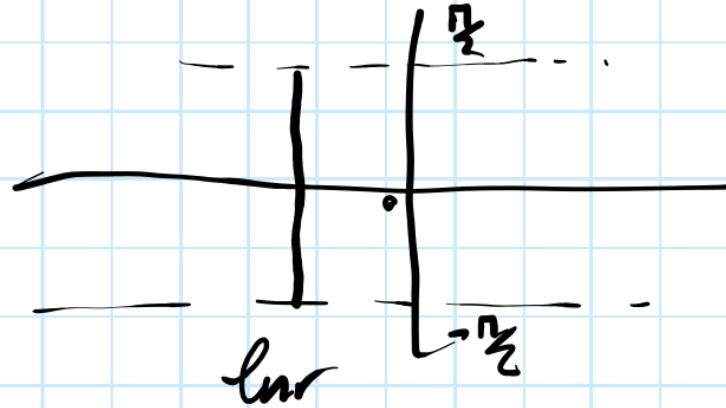
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$$0 \leq r \leq 1$$

$$-\infty < \operatorname{Im} r \leq 0$$

$$-\infty < u(x, y) \leq 0$$



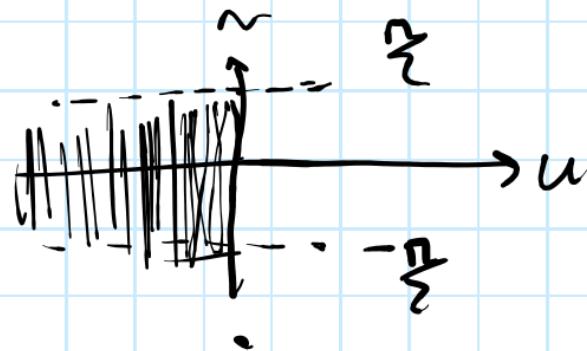
$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$\rightarrow -\frac{\pi i}{2} \leq i\theta \leq \frac{\pi i}{2}$$

$$-\frac{\pi}{2} i \leq iV(x, y) \leq \frac{\pi}{2} i$$

$$U \in (-\infty, 0]$$

$$V \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$



خط انتقالی:  $z = a + bi \rightarrow \sin(z) = \sin(a) \cosh(b) + i \cos(a) \sinh(b)$

- $U(a, b) = \sin a \cosh b$

$$\rightarrow \frac{U^2}{\cosh^2 b} + \frac{V^2}{\sinh^2 b} = 1$$

- $V(a, b) = \cos a \sinh b$

- $\sin^2 a + \cos^2 a = 1$

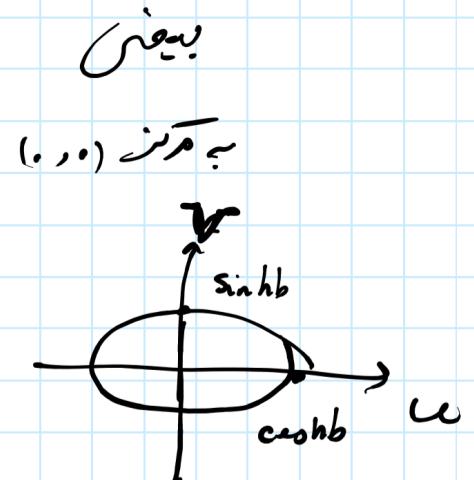
$$U = 0 \rightarrow V = \pm \sinh b$$

$$V = 0 \rightarrow U = \pm \cosh b$$

جزء طرفی:  $\cosh^2 b = 1 + \sinh^2 b$

$$\rightarrow |\cosh b| > |\sinh b|$$

بعض افقط



$$b > \frac{\pi}{2} \rightarrow U = \sin a \cosh \frac{b-\pi}{2}, V = \cos a \sinh \frac{b-\pi}{2}$$

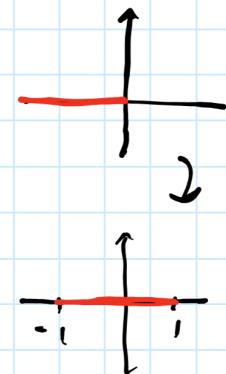
$$b > \frac{\pi}{2} \rightarrow U = (\sin a) \cosh \frac{b-\pi}{2}, V = (\cos a) \sinh \frac{b-\pi}{2}$$

دایره بین سروتی که از  
ویرانی

$$a \in (-\infty, 0] \rightarrow \sin a, \cos a \in [-1, 1]$$

$$-\frac{\pi}{2} < b < \frac{\pi}{2} \quad \text{if } b = 0 \rightarrow U = \sin a \in [-1, 1]$$

$$V = 0$$



$$\sinh x = \frac{e^x - e^{-x}}{2} \rightarrow (\sinh)'(x) = \frac{e^x + e^{-x}}{2} \geq 1 \quad \text{محدود آکیده}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \rightarrow (\cosh)'(x) = \frac{e^x - e^{-x}}{2} = 0 \rightarrow x=0 \quad \begin{matrix} \text{محدود} \\ \text{برابر} \end{matrix} \quad \begin{matrix} \text{نیز} \\ \text{برابر} \end{matrix}$$

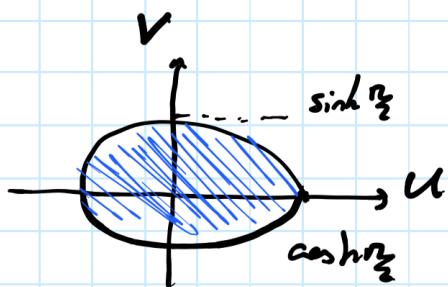
$$-\frac{\pi}{2} \leq b \leq \frac{\pi}{2} \rightarrow \sinh \frac{\pi}{2} \leq \sinh b \leq \sinh \frac{\pi}{2}$$

$$1 = \cosh 0 \leq \cosh b \leq \cosh \frac{\pi}{2}$$



بینوں میں سے ہر ہی طور پر متغیر بین اور  $\cosh \frac{\pi}{2}$  میں محدود ہے:

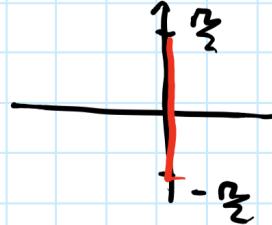
لے  
مان پہنچ کر مختصر کر دیں  
و پہنچ کر تابع کے محدود



لـ  $z = a + yi$

$$\Rightarrow \sin z = \underbrace{\sin a \cdot \cosh y}_{U(a,y)} + \underbrace{\cos a \cdot \sinh y}_i i$$

أمثلة:

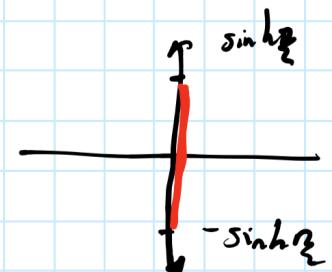


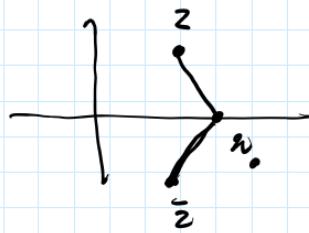
$$z = yi \quad y \in [-\frac{\pi}{2}, \frac{\pi}{2}]$$

$$\hookrightarrow \sin(z) \in [-\sinh \frac{\pi}{2}, \sinh \frac{\pi}{2}]$$

$$U = \dots$$

$$V = \sinh y$$





$$\left| \frac{z - z_0}{\bar{z} - \bar{z}_0} \right| = 1$$

$\text{لـ} \rightarrow$   
 $\text{أـ} \rightarrow$

$$\text{درس: } g(z) = \frac{z-a}{\bar{z}-\bar{a}}$$

$$z = x + bi \quad \rightarrow \quad g(z) = \frac{(x-a) + bi}{(x-a) - bi}$$

$$\xrightarrow{\substack{\text{مخرج} \\ \text{غير مزدوج}}} \frac{1}{(x-a)^2 + b^2} \left( (x-a)^2 + 2(x-a)bi - b^2 \right)$$

...  $\infty$

$$\text{درس: } g(z) = \sin(z)$$

$$\begin{aligned} \text{إنتقال خطوط} \\ \text{معزى: } z = x + bi \rightarrow g(z) = g(x+bi) &= \sin(x) \cosh(b) \\ &\quad + i \cos(x) \sinh(b) \end{aligned}$$

$$\begin{aligned} u(x, b) &= \sin x \cosh b \\ V(x, b) &= \cos x \sinh b \quad \rightarrow \quad \frac{u^2}{\cosh^2 b} + \frac{V^2}{\sinh^2 b} = 1 \end{aligned}$$

$$\frac{-\pi}{2} < b < \frac{\pi}{2} \quad \rightarrow \quad \cosh b = \frac{1}{2} (e^b + e^{-b})$$

$$\text{درس: } g(z) = \frac{z}{\bar{z}}$$

$$\frac{x+yi}{x-yi} = \frac{1}{x^2+y^2} (x^2 + 2xyi - y^2)$$

3)  
الف

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z - z_0)^n}$$

لـ  $f(z)$  حول  
 $z_0$  ممتلئ

تحليل دوال

$$\Rightarrow f(z) = g(z) + \frac{b_1}{z - z_0}$$

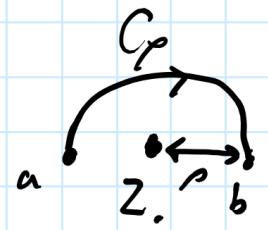
قطب در  $z_0$

تفصيل  $f$   
هي دوبيس تحليل  
و غيره تحليل  
 $z_0$  در

$$\int_{C_P} f(z) dz = \int_{C_P} g(z) dz + \int_{C_P} \frac{b_1}{z - z_0} dz$$

①

②



$$\int_{C_p} g(z) dz \leq M L = M \pi \rho$$

$\downarrow$

$$\int_{C_p} dz$$

$\downarrow$

$z^{1,0} \tilde{F}^i dz$   
 $\rho e^{i\theta} \sim$

$$\Rightarrow \lim_{\rho \rightarrow 0} \int_{C_p} g(z) dz \leq \lim_{\rho \rightarrow 0} M \pi \rho = 0 \Rightarrow \textcircled{1} = 0$$

$$\textcircled{2} = \int_{C_p} \frac{b_1}{z-z_0} dz, \quad b_1 \int_{C_p} \frac{1}{z-z_0} dz$$

$$z(t) = z_0 + \rho e^{it} \Rightarrow z'(t) = \rho i e^{it}$$

مما يُaskell  
في مسار خارجي  
 $z'$  تساوي

$$\int_{C_p} \frac{1}{z-z_0} dz = \int_0^\pi \frac{1}{\rho e^{it} + z_0 - z_0} (\rho i e^{it}) dt$$

$$= \int_0^\pi i dt = i\pi$$

$$\Rightarrow \textcircled{2} + \pi i b_1 \xrightarrow{\hspace{1cm}} \pi i \operatorname{Res}_{z=z_0} f(z)$$

$b_1$  is  $\operatorname{Res}_{z=z_0} f(z)$

$$\int_{C_\ell} f(z) dz + \textcircled{1} + \textcircled{2} = \pi i \operatorname{Res}_{z=z_0} f(z)$$

$\tilde{z}_1$ 

$$\int_C f(z) dz = \int_{C_\rho} f(z) dz + \int_{C_R} f(z) dz + \int_{-R}^{-2-\rho} f(z) dz + \int_{-2+\rho}^R f(z) dz$$

$\hookrightarrow$   $\text{contour integral}$   
 $z^s dz \rightarrow dz^s dz$

$$f(z) = \frac{1}{z^3 + 8} = \frac{1}{(z+2)(z^2 - 2z + 4)}$$

$$\hookrightarrow z = -2$$

$$\hookrightarrow z = \frac{2 \pm \sqrt{-12}}{2} = 1 \pm i\sqrt{3}$$

اگر  $R$  به اندازه کافی بزرگ باشد  $C$  نقطه  $1+i\sqrt{3}$  را نزدیکی دارد  
زیرینج این قسم در روند خمنه افتاده و کاملاً در محاسبه ندارد.

$$\oint_C f(z) dz = 2\pi i \sum \operatorname{Res} f(z) + 2\pi i \operatorname{Res}_{z=1+i\sqrt{3}} f(z) = \frac{-1}{24} (1+i\sqrt{3})(2\pi i) = \frac{-1}{12} \pi (-\sqrt{3}+i)$$

$$\operatorname{Res}_{z=1+i\sqrt{3}} f(z) = \left( \frac{1}{z-i\sqrt{3}} - (z-(1+i\sqrt{3})) \right)^{(1)} \Big|_{z=1+i\sqrt{3}} = \frac{1}{(z+i\sqrt{3})(z-(1-i\sqrt{3}))} \Big|_{z=1+i\sqrt{3}} = \frac{1}{(3+i\sqrt{3})(2i\sqrt{3})} = \frac{1}{6} \times \frac{1}{i\sqrt{3}-1} = \frac{-1}{24} (1+i\sqrt{3})$$

(٢)

$$\lim_{\rho \rightarrow 0} \int_{C_\rho} f(z) dz = \lim_{\rho \rightarrow 0} - \int_{-C_\rho} f(z) dz = -\frac{\pi}{12} i$$

الآن  $\lim_{\rho \rightarrow 0} \int_{-C_\rho} f(z) dz + \pi i \operatorname{Res}_{z=2} f(z) = \frac{\pi}{12} i$

$$\operatorname{Res}_{z=2} f(z) = \frac{1}{(z+2)(z^2-2z+4)} (z+2) \Big|_{z=-2} = \frac{1}{12}$$

$$\bullet \lim_{R \rightarrow \infty} \int_{C_R} f(z) dz = \lim_{R \rightarrow \infty} \frac{\pi R}{R^3 - 8} = 0$$

$$\int_{C_R} f(z) dz \leq M L = \frac{\pi R}{R^3 - 8}$$

$\downarrow$

$$\max_{z \in C_R} \{f(z)\} \int_{C_R} dz$$

$$|z^3 + 8 + (-8)| \leq |z^3 + 8| + |-8|$$

$$\rightarrow |z^3 + 8| \geq \|z^3\| - 8 = R^3 - 8 \rightarrow \frac{1}{\|z^3 + 8\|} \leq \frac{1}{R^3 - 8} \quad (R > 2)$$

$$\Rightarrow \max\{f(z)\} \leq \frac{1}{R^3 - 8}$$

$$\bullet \lim_{\rho \rightarrow 0} \left( \int_{-R}^{-2-\rho} f(z) dz + \int_{-2+\rho}^R f(z) dz \right) = \int_{-R}^{-2} f(z) dz + \int_{-2}^R f(z) dz = I$$

$$\Rightarrow I = \lim_{\substack{R \rightarrow \infty \\ \rho \rightarrow 0}} \left( \int_C f(z) dz - \int_{C_R} f(z) dz - \int_{C_\rho} f(z) dz \right)$$

$$\Rightarrow I = \frac{-1}{12}\pi(-\sqrt{3}+i) - \left(-\frac{\pi}{12}i\right) - 0 = \frac{\sqrt{3}}{12}\pi$$