gamma function

$x > 0$

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt$$

diverges for $x \leq 0$

$x = 1$

$$\Gamma(1) = \int_0^\infty e^{-t} \cancel{t^{1-1}} \, dt = \left[-e^{-t}\right]_0^\infty$$

$$= 0 - \left[-e^{0}\right]$$

$= 0 - [-1] = 1$

consider

$$\Gamma(x+1) = \int_0^\infty e^{-t} t^x \, dt$$

integrate by parts.

$u = t^x \qquad dv = e^{-t} \, dt$

$du = x t^{x-1} \, dt \qquad v = -e^{-t}$

$$\Gamma(x+1) = t^x(-e^{-t})\Big|_0^\infty + \int e^{-t} x t^{x-1} \, dt$$

$$= [0 - 0] + x \int^\infty e^{-t} t^{x-1} \, dt$$

$$\Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma(1) = 1$$

$$\Gamma(2) = 1\,\Gamma(1) = 1$$

$$\Gamma(3) = 2\,\Gamma(2) = 2$$

$$\Gamma(4) = 3\,\Gamma(3) = 3 \times 2 \times 1$$

$\Rightarrow$ For integer x

$$\Gamma(x) = (x-1)!$$

$$\Gamma(x+1) = x!$$

eg non-integer

$x = 1/2$

$$\Gamma(1/2) = \int_0^\infty e^{-t} t^{-1/2} \, dt = \sqrt{\pi}$$

define for $x < 0$

$$\Gamma(x) = \frac{\Gamma(x+1)}{x}$$

For $x \geq 1$

$\Gamma(x)$ connects the dots between $(x-1)!$,

$x < 0$ Γ diverges
for $x =$ integer
$x \leq 0$

$$\boxed{\Gamma(x) = \int_0^\infty e^{-t} t^{x-1}\, dt \qquad x > 0}$$

Summary

gamma function