PH2255 Week 19. Statistical Data Analysis 3.

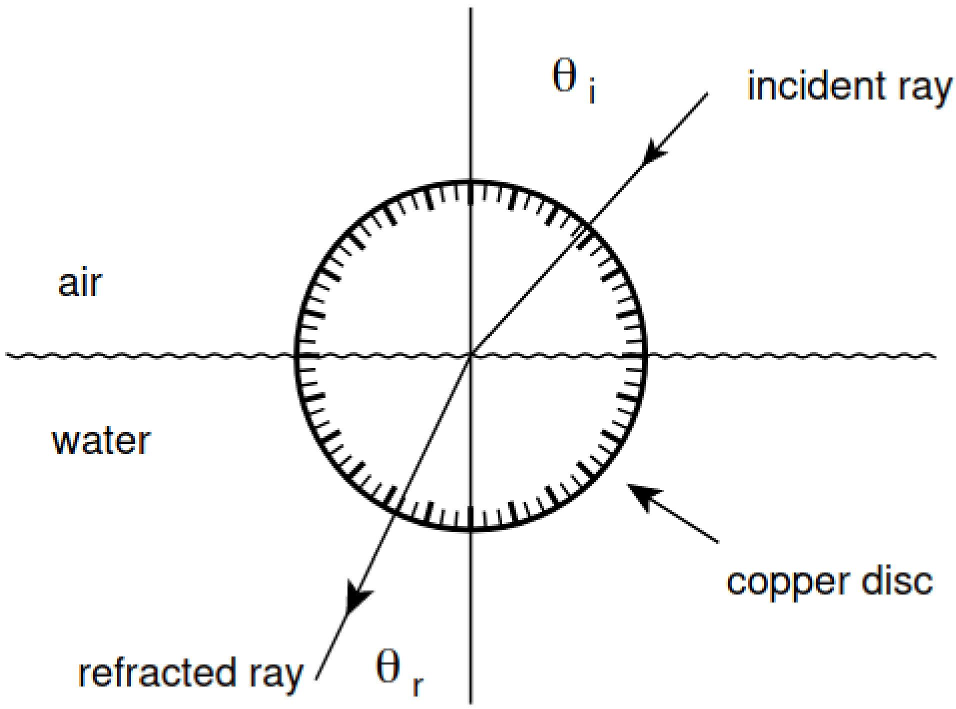
Generic Python header: here the interactive/inline plotting style is selected, relevant libraries are imported, python2/3 compatibility is addressed.

In [1]:

```
# %matplotlib inline
# this line is required for the plots to appear in the Jupyter cells,
# rather than launching the matplotlib GUI
%matplotlib widget
# this allows interactive view but you need to be in classic
# rather than CoCalc Jupyter notebook for this to work
from __future__ import division,print_function
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
from scipy.optimize import curve_fit
import scipy.stats
```

Exercise 3. Analysis of refraction data from Ptolemy

The astronomer Claudius Ptolemy performed experiments on the refraction of light using a circular copper disc submerged to its centre in water:



Angles of refraction θ_r for 8 values of the angle of incidence θ_i obtained by Ptolemy around 140 A.D. are given in the Table 2 (reproduced from Glen Cowan's "Introduction to Statistic Methods"):

Table 2: Angles of incidence and refraction (in degrees).

$\theta_{\rm i}$	$\theta_{\rm r}$
10	8
20	$15\frac{1}{2}$
30	$22\frac{1}{2}$
40	29
50	35
60	$40\frac{1}{2}$
70	$45\frac{1}{2}$
80	50

For purposes of this exercise we will take the angles of incidence to be known with negligible error and treat the angles of reflection as independent Gaussian-distributed measurements with standard deviations of $\sigma = \frac{1}{2}^{\circ}$. (This is a reasonable guess given that the angles are reported to the nearest half degree. Note that we can absorb an error in θ_i into an effective error in θ_r.)

In this exercise you will need to input the data into Python yourself.

In [2]:

```
# Define the data set
```

Report the parameter estimates, their standard deviations, and where relevant the covariance matrix for all of the fits below. Make plots of the fitted curves and the data.

Exercise 3(a).

Until the discovery of the correct law of refraction (see below), a commonly used hypothesis was

$$\theta_r = \alpha\theta_i,$$

although it is reported that Ptolemy preferred the formula

$$\theta_r = \alpha\theta_i - \beta\theta_i^2.$$

Find the least-squares estimates of the parameters for both hypotheses and determine the minimized χ^2. Comment on the goodness-of-fit for both hypotheses. Is it plausible that all of the data values are based on actual measurements?

In [3]:

```
# Exercise 3a
```

Exercise 3(b).

The law of refraction discovered by the Persian mathematician and physicist Ibn Sahl in the 10th century and rediscovered by others including Snell in 1621 is

$$\theta_r = \sin^{-1}\left(\frac{\sin\theta_i}{r}\right),$$

where $r = n_r/n_i$ is the ratio of indices of refraction of the two media.

Determine the least-squares estimate for r and find value of the minimized χ^2. Comment on the validity of assumption that $\sigma = \frac{1}{2}^\circ$.

Suggestions: `np.sin(x)` returns $\sin x$ for angle x measured in radians; `np.arcsin(y)` returns $\sin^{-1} y$ in radians; `np.deg2rad(x)` and `np.rad2deg(x)` convert between degrees and radians. All these functions correctly handle `np.array` arguments.

In [4]:

```
# Exercise 3b
```

26/01/2021. This Jupyter notebook was assembled by Lev Levitin based on the course written by Glen Cowan.