# Fourier Transforms

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{in\pi x}{L}}$$

period $2L$

$f(x)$ $\quad -\infty < x < \infty$

$$c_n = \frac{1}{2L} \int_{-L}^{L} e^{-\frac{in\pi x}{L}} f(x)\, dx$$

$$k_n = \frac{n\pi}{L}$$

$$\Delta k_n = k_{n+n} - k_n = \frac{\pi}{L}$$

$$f(x) = \sum_{-\infty}^{\infty} \left(\frac{L}{\pi} c_n\right) e^{ik_n x} \Delta k$$

limiting case

if $f(x)$ not periodic

let $L \to \infty$ $\Delta k \to$ infinitesimally small

$$\sum \to \int$$

$$f(x) = \int_{-\infty}^{\infty} \frac{L c_n}{\pi} e^{iky} dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} e^{-ikx} f(x) dx \right] e^{iky} dx$$

$\int_{-\infty}^{\infty} e^{-ikx} f(x) dx$ : Fourier Transform of function $f(x)$

**Fourier Transform Pair**

F.T. $$\hat{f}(k) = \int_{-\infty}^{\infty} e^{-ikx} f(x) dx$$

inverse transform $$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(k) e^{ikx} dk$$

$$\int_{-\infty}^{+\infty} |f(x)| \, dx < \infty$$