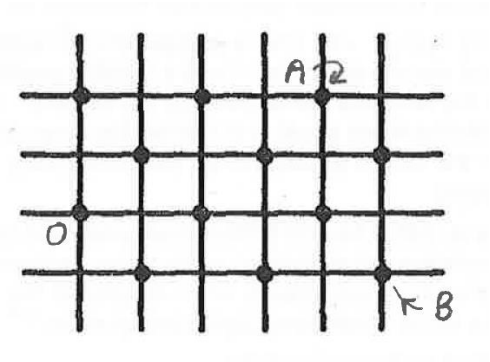
PH2710 Solid state physics
Problem set 1: Topic 1

Question 1

The figure below shows a hypothetical two-dimensional crystal. The atoms (black blobs) are arranged on a square grid as shown. The squares are of side *a*. [Note, to avoid confusion: *the grid is not the lattice*].



This crystal can be generated by a Bravais lattice, with one atom on each lattice point.

(i) Write down a possible pair of primitive lattice vectors ($\mathbf{a}_1$, $\mathbf{a}_2$).

(ii) Sketch the primitive unit cell, and evaluate its area.

(iii) With the origin at O, specify the coordinates of atom A and atom B, in the form $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2$, where n_1 and n_2 are integers.

Alternatively, this crystal could also be described as a square lattice with a two atom basis.

(iv) Write down the lattice vectors, and basis vectors in this case.

(v) Evaluate the area of the unit cell and compare with (ii) above.

Question 2

Consider the 2D hexagonal lattice [refer to chapter 4 of notes, where the primitive lattice vectors are specified].

(i) Determine the area of the primitive unit cell A_p.

In 2D the primitive vectors of the reciprocal lattice are given by:

$$\mathbf{b}_1 = 2\pi\frac{\mathbf{a}_2\times\hat{\mathbf{z}}}{A_p}\ ;\ \mathbf{b}_2 = 2\pi\frac{\hat{\mathbf{z}}\times\mathbf{a}_1}{A_p},$$

where $\hat{\mathbf{z}}$ is the unit vector normal to the plane of the 2D lattice.

(ii) Confirm that the condition $e^{i\mathbf{K}.\mathbf{R}} = 1$ is indeed satisfied for any choice of lattice vector and reciprocal lattice vector in 2D.

(iii) Calculate the primitive vectors of the reciprocal lattice for the 2D hexagonal lattice.

(iv) Show that this reciprocal lattice is also hexagonal.

(v) Sketch it. Construct the Wigner-Seitz cell in the reciprocal lattice (first Brillouin zone), and determine the area of the Wigner-Seitz cell.

Question 3

A simple cubic crystal with lattice parameter 0.4 nm is mounted with its [001] axis (the z-axis) perpendicular to a monochromatic x-ray beam of wavelength 0.1 nm (i.e the beam lies in the xy plane).

The crystal is initially set so as to produce the (020) diffraction, i.e. the m=2 order diffraction from the (010) plane.

(i) What is the direction of the beam?

(ii) Through what angle must the crystal be turned to produce the (030) diffraction?

(ii) Through what angle must the crystal be turned to produce the (130) diffraction?