Physics Department

Year 2 Laboratory Handbook for PH2250, PH2260 and PH2270

2020-2021

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OPT-03: Fraunhofer and Fresnel Diffraction

1.1 Aims and Objectives

- To investigate the phenomenon of diffraction of light.
- To observe different types of diffraction patterns for a monochromatic light (He-Ne laser) using slits, diffraction gratings and circular apertures.
- To model these patterns and evaluate the geometry of the apertures in Fraunhofer regime using Fourier transform.
- To observe and qualitatively explain Fresnel diffraction on a circular aperture.

1.2 Introduction

Fraunhofer and Fresnel diffraction are examples of interference by division of wavefront. These phenomena are of great importance in the field of optics and are presented in lectures 12 to 18 of the PH2310 Optics course and in chapter 10 of "Optics and Photonics" by F. Graham-Smith, T.A. King and D. Wilkins. In this experiment we will investigate phenomena that make it evident that light does not always travel in straight lines. Francesco Maria Grimaldi (Italy, 1618-1663) coined the term "diffraction" to describe the apparent bending of the rays when light encounters an obstacle and the spreading out of waves past small apertures. In the wave theory of light, the observed diffraction patterns can be explained by considering the Huygens-Fresnel secondary wavelets generated along the wavefront, together with the principle of superposition.

1.3 Experimental setup

In the remote learning circumstances of 2021 you will be provided with photographs of the diffraction patterns, obtained in the leaching lab. The apparatus, Fig. 1.1, includes a He-Ne laser (wavelength $\lambda = 623.8\,\text{nm}$), a Gambrell viewing screen, an array of apertures and a beam expansion lens. All components can be mounted along an optical rail. The viewing screen includes a ruler, graduated with unnumbered millimetre and numbered centimetre marks, that can be used to measure distances between intensity minima in the patterns.

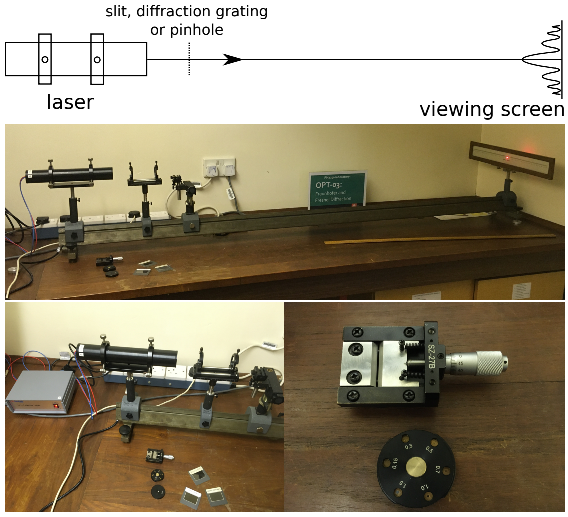

Figure 1.1: Schematic diagram and photographs of the set-up for observing Fraunhofer diffraction patterns. The laser and viewing screen are mounted at either end of an optical rail. An adjustable slit, a double slit, a diffraction grating, or a pinhole can be placed between the laser and the viewing screen giving rise to a diffraction pattern that can be observed on the screen. Fresnel diffraction is observed in far field by adding a beam expander lens between the laser and the pinhole. Photos: the entire setup; laser, various apertures and lens; adjustable slit and array of pinholes.

Your tasks will involve interpreting the photographed patterns qualitatively and quantitatively. The photographs are available on Moodle. Several versions of the photographs have been provided For each exercise, except Q6. Please select one photo from each category and include your choice in the report. The images can be studied, annotated and cropped in a graphical editor and then imported into the reports.

To support the investigation, a simulator of intensity $I(\theta)$ for one-dimensional Fraunhofer diffraction patterns is available on Moodle and CoCalc. Plots produced by the simulator may be used in the report to illustrate the observed intensities.

1.4 Fraunhofer Diffraction

When considering the form of a diffraction pattern the amplitude and phase at a point P on the detection screen is determined by splitting the signal source into elements and summing the phasor contributions of all the elements. In Fraunhofer limit the phase of the contributions of various points of the source at P varies linearly across the source. As discussed in Lectures 12 and 18 of the PH2310 Optics course, the amplitude of the Fraunhofer diffraction pattern is the Fourier transform of the transmission function of the diffracting aperture.

1.4.1 Diffraction on a single slit

The aperture function for a slit of width a extended along the y axis

$$A(x) = \begin{cases} A_0, & \text{for } |x| \le a/2, \\ 0, & \text{for } |x| > a/2. \end{cases} \tag{1.1}$$

In this one-dimensional problem the aperture function only depends on x, therefore the diffraction is observed along x. The intensity of light $I(\theta)$ observed at angle θ away from the forward direction is

$$\frac{I(\theta)}{I(0)} = \frac{\sin^2\alpha}{\alpha^2}, \quad \text{where } \alpha = \frac{\pi}{\lambda} a \sin\theta. \tag{1.2}$$

Q1 Illustrate Eq. (1.2). You can use `Diffraction.py` on Moodle or `Diffraction` Jupyter notebook on CoCalc. Plot $I(\theta)$ for several slit widths between 50-200 μm.

Next you will observe the diffraction pattern in photographs `Q2_slitA#.jpg` and `Q2_slitB#.jpg` where # is a number of your choice. The forward direction $\theta = 0$ is roughly aligned with '0' mark on the viewing screen. Allowing for a small offset $\Delta\theta$, the coordinate X on the screen can be related to the angle θ via

$$\frac{X}{L} = \tan(\theta + \Delta\theta), \tag{1.3}$$

where L is the distance between the slit and the screen. According to Eq. (1.2) $I(\theta)$ has minima (dark spots in the diffraction pattern) at $a\sin\theta = n\lambda$, where n is a non-zero integer. Using Eq. (1.3) and small angle approximation $\theta \approx \sin\theta \approx \tan\theta$ we obtain

$$\frac{X}{L} = \frac{\lambda}{a} n + \Delta\theta \tag{1.4}$$

for the positions of the intensity minima.

Q2 Analyse photographs of diffraction patterns `Q2_slitA#.jpg` and `Q2_slitB#.jpg`. Find the locations $X(n)$ of the dark spots on both sides of the central maximum up to order n = 3-8. Plot $X(n)$, fit to Eq. (1.4) and obtain slit width a. The distance L between the slit and the screen is specified in the photographs.

1.4.2 Diffraction on Young's double slit

The intensity of Fraunhofer diffraction on two parallel slits of width a separated by d (measured between the middles of the slits) is

$$\frac{I(\theta)}{I(0)} = \left(\frac{\sin^2\alpha}{\alpha^2}\right)\cos^2\beta, \quad \text{where } \beta = \frac{\pi}{\lambda} d \sin\theta. \tag{1.5}$$

The r.h.s. of Eq. (1.5) is a product of the single slit pattern Eq. (1.1) and $\cos^2\beta$ term that represents the interference between the slits, a consequence of the convolution theorem. Zeros in the first term suppress some of the peaks in the second term leading to *missing orders*.

Q3 Illustrate Eq. (1.5) showing the dependence of the diffraction pattern on a and d. Select $a = 20$ mu and several d in 50-500 μm range. Identify missing orders.

Q4 Analyse `Q4_double_slit#.jpg` photograph taken using a grating with $d = 250\,\mu$m and $a \approx 60\,\mu$m. Analyse Eq. (1.5) for these parameters and find the conditions of intensity minima. Measure the positions of two well-defined minima of the same order and infer distance L between the slits and the screen.

1.4.3 Diffraction on a multi-slit grating

For a grating composed of N slits the Fraunhofer intensity pattern is given by

$$\frac{I(\theta)}{I(0)} = \left(\frac{\sin^2\alpha}{\alpha^2}\right)\frac{\sin^2(N\beta)}{N^2\sin^2\beta}, \tag{1.6}$$

again a product of the single slit pattern and a term due to the interference between the N slits.

Q5 Illustrate Eq. (1.6) for $N = 1, 2, 3, 10$. Choose one set of a and d used in Q3.

Q6 Diffraction on two gratings is presented in photos `Q6_gratingA.jpg` and `Q6_gratingB.jpg`. Which photo corresponds to a grating with larger spacing d? Explain qualitatively how the width of the intensity maxima differs between single slit, double slit and gratings.

Q7 So far you studied *horizontal* diffraction patterns produced by slits extended *vertically*. The `Q7_perp_gratings#.jpg` photograph was obtained with the two gratings superimposed. Grating A was rotated to an arbitrary orientation, grating B was then positioned perpendicular to A. Identify the directions of the slits in each grating and explain your choice.

1.4.4 Circular aperture

The intensity pattern for Fraunhofer diffraction from a uniformly circular aperture of diameter d is determined by the first-order Bessel function $J_1(z)$

$$\frac{I(\theta)}{I(0)} = \left(\frac{2J_1(z)}{z}\right)^2, \quad \text{where } z = \frac{\pi}{\lambda} d \sin\theta. \qquad (1.7)$$

and is known as an Airy pattern (see "Optics and Photonics" Chapter 10, to be covered in PH2310 in the coming weeks). The dark rings correspond to the Bessel function's zeros. The first few[1] zeros, $J_1(z_i) = 0$, are $z_1 = 3.832, z_2 = 7.016, z_3 = 10.173, z_4 = 13.324, z_5 = 16.471, z_6 = 19.616$. Notice the unequal spacing between the z_i. The condition for i-th dark ring is therefore $\sin\theta_i = z_i\lambda/(\pi d)$. If D_i is the diameter of the i-th dark ring on the viewing screen and L is the distance between the aperture and screen, then $2\theta_i = D_i/L$. Note that the angular offset error $\Delta\phi$ in Eq. (1.3) is eliminated by measuring the diameter on both sides of the ruler. Thus

$$D_i = \frac{2L\lambda}{\pi d} z_i, \qquad (1.8)$$

Q8 Analyse the photo `Q8_pinhole#.jpg` of diffraction on $d = 0.3$ mm circular aperture. Measure the diameters of the first few dark rings D_i, plot $D_i(z_i)$, fit to Eq. (1.8) and determine the distance L between the aperture and screen. The $[z_1, z_2, \ldots, z_n]$ array can be obtained by calling `scipy.special.jn_zeros(1, n)` Python function.

[1] $z = 0$ is also a zero of $J_1(z)$, but $J_1(z)/z$ remains finite.

1.5 Fresnel Diffraction

Fresnel diffraction occurs when the viewing screen is closer to the source than in the Fraunhofer case or when the wavefronts of the light incident at the aperture have significant curvature. When considering diffraction the source is again split up into elements, however in this case the phase of the contributions at P no longer varies linearly across the source and we need to include quadratic terms. See Optics lecture 14 for the discussion on the Fraunhofer and Fresnel limits.

The condition for an aperture of size a illuminated by flat wavefronts to produce Fraunhofer diffraction at distance L is

$$\frac{a^2}{L\lambda} \ll 1. \qquad (1.9)$$

Q9 Check if Eq. (1.9) is valid for the geometry of Q2 and Q4.

As you have observed in the Fraunhofer limit for both slits and circular apertures there is a bright central maximum in the forward direction of the diffraction pattern ($\theta = 0$). In Fresnel diffraction under certain circumstances you can observe the seemingly 'bizarre' effect of a minimum in intensity in the forward direction.

Q10 Study the photos `Q10_fresnelA#.jpg` and `Q10_fresnelB#.jpg`, in which laser light is first passed through a lens, making the wavefront spherical, and then diffracted by a circular aperture. Describe the observed features qualitatively in the context of Fresnel diffraction.

Note that Eq. (1.9) is not valid for curved incident wavefronts, allowing Fresnel diffraction to be observed in this experimental setup.