# Laplace's Equation

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# This week

## Laplace's Equation

- *What* is Laplace's Equation?
- *Solve* : by Separation of Variables

  : Different coordinate systems give different O.D.E
  - Cartesian Coordinates (Lecture)
  - Polar (Spherical) Coordinates (Lecture)
  - Cylindrical Coordinates (Lecture)

Reading: Chapter 6 of lecture notes

# Laplace's Equation!

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

In the vector calculus course, this appears as $\nabla^2 \phi = 0$ where $\nabla = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix}$

- no dependence on time, just on the spatial variables x,y.

Laplace's Equation describes steady state situations such as:

- steady state temperature distributions
- steady state potential distributions
- steady state flows, for example in a cylinder

# Examples of Laplace's Equation



## Heat Equation in 3D

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u + q(x, y, z, t)$$

in steady state and with no source becomes

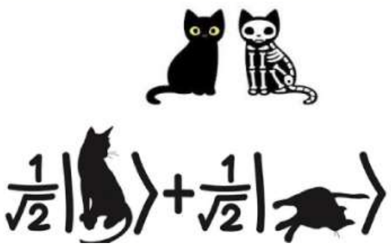
$$\nabla^2 u = 0$$

## Wave Equation

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0$$

## Schrodinger Eq

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$



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