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PH2610: Classical & Statistical Thermodynamics
Question Sheet 2

Entropy

1. One mole of monatomic ideal gas is subject to the following sequence of processes between equilibrium states:

   (a) an isobaric expansion from (P_1,V_1,T_1) to (P_1,V_2,T_2);

   (b) an isochoric change from (P_1,V_2,T_2) to (P_2,V_2,T_1);

   (c) an isothermal change from (P_2,V_2,T_1) to (P_1,V_1,T_1).

   Find expressions in terms of T_1, T_2 and the gas constant R for the entropy change of the gas in each case.
   Verify that the algebraic sum of the three entropy changes is zero.

2. An electric current of 10A flows for one minute through a 20Ω resistor which is maintained at 10°C by being immersed in running water. What are the entropy changes in the resistor, the water and the universe?

3. Two identical bodies of heat capacity C are initially at temperatures T_1 and T_2.

   (a) They are brought into thermal contact so that their common final temperature is $T_a = (T_1+T_2)/2$.
   Show that the overall entropy change in this case is

   $$\Delta S = 2C\ln\left\{\frac{T_1+T_2}{2\sqrt{T_1T_2}}\right\}$$

   and that this expression is necessarily positive.

   (b) They are brought to a common final temperature T_b by a Carnot engine operating between them. The temperatures of the bodies do not change appreciably during one cycle and the bodies may be regarded as reservoirs in the cycle.
   Show that

   $$T_b = \sqrt{T_1T_2}\ .$$

   What is the overall entropy change in this case?

General applications

4. We proved in the notes that, in general, for any system

$$C_P - C_V = \frac{TV\beta_P^2}{\kappa_T}.$$

where β_P is the isobaric expansion coefficient and κ_T is the isothermal compressibility.

By carefully considering the definition of these two quantities, use the above result to show that for one mole of ideal gas

$$C_P - C_V = R$$

5. Show that the Joule-Kelvin coefficient is given by

$$\left(\frac{\partial T}{\partial P}\right)_H = \frac{1}{C_P}\left\{T\left(\frac{\partial V}{\partial T}\right)_P - V\right\}.$$

One mole of a real gas may be described approximately by the van der Waal's equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

Find an expression for the inversion temperature of such a gas in terms of its volume, and given that $b \ll V$ at low pressure, show that the maximum inversion temperature is

$$T_I^{MAX} = \frac{2a}{Rb}.$$