PH2420/PH3420/MT3240
Electromagnetism
Problem Sheet 4

1. (a) In Tutorial Sheet 1, you calculated the capacitance per unit length of a cylindrical capacitor (or coaxial cable). First you used Gauss' law to find $\mathbf{E}$, then you integrated $\mathbf{E}$ to get V, finally you used $Q = CV$ to find an expression for C. Now that you know an expression for the energy density of the electric field, you can instead use the work done in charging a capacitor to some voltage V, (that is, $\frac{1}{2}CV^2$), to find C. Integrate the energy density of the electric field in the coaxial cable to find the total energy, use the voltage diffference found earlier, and hence find the capacitance per unit length.

   (b) The same energetics method may be used to find the inductance per unit length of the coaxial cable. Assume that a current I runs down the surface of the core (so, in a thin cylindrical surface of radius a) and back in the shield (another cylindrical sheet, radius $b > a$).

      i. Derive an expression for the magnetic field. It will have a radial dependence but will be contained in the gap between the core and the shield, why?

      ii. Next find the energy density of this magnetic field.

      iii. Next, integrate to find the magnetic field energy per unit length.

      iv. Use the expression for the work done in creating a magnetic field in an inductor to express the inductance per unit length.

      v. Finally, estimate the inductance of a meter of RG58 coax.

   [Note, the expressions for the Work Done to charge capacitors and inductors should be known, but their proof comes later in the course. Hint: Consider integrating over cylindrical shells.]

2. (a) A length of coax cable has a potential difference V between the core and the shield, each of which carries a current I in opposite directions along the length of the cable. Find $\mathbf{E}$, $\mathbf{B}$ and hence the energy per unit time (that is, the power), transported down the cable via the fields.

   (b) Show that the power transported, P, is equivalent to IV. How is the power transported, via charge motion, or via the fields?

3. A parallel plate capacitor with circular plates of radius r is being charged, from zero at $t = 0$. Ignore the fringing fields. The charging current at any moment is $I(t)$.

   (a) Find the electric field between the plates as a function of time and the magnetic field between the plates as a function of the radial distance s from the axis. You will need a 'sort of' circular Amperian loop of radius s. Why 'sort of'?

   (b) Hence find the rate of change of energy density after some time t in the volume between the plates.

   (c) Find the Poynting vector.

   (d) Integrate the Poynting vector over some convenient surface to find the rate of increase of energy in the capacitor. Hence show that the power flowing into the capacitor is equivalent to the rate of change of total field energy.