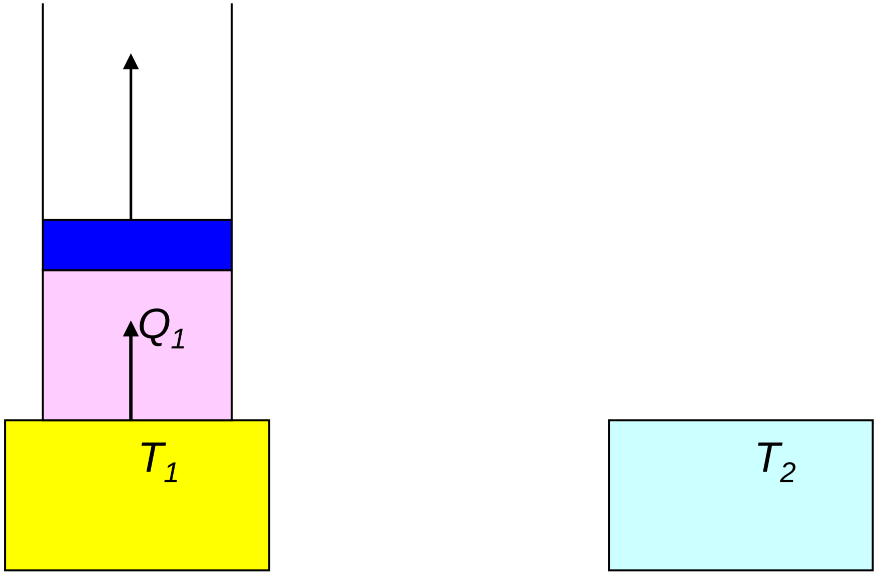
Absolute temperature

Ideal gas Carnot engine

Step 1

Isothermal expansion at T_1, absorb heat Q_1.

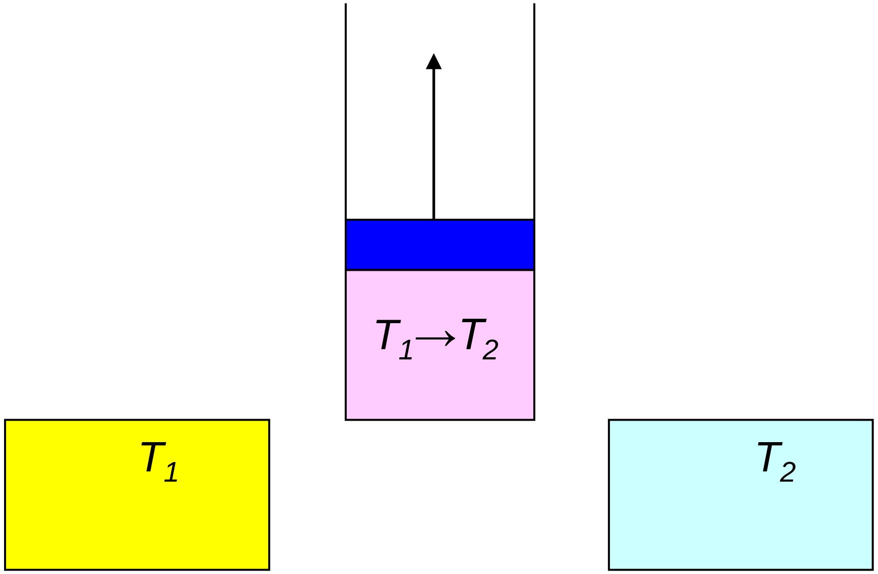


Absolute temperature

Ideal gas Carnot engine

Step 2

Adiabatic expansion,
T falls from T_1 to T_2.

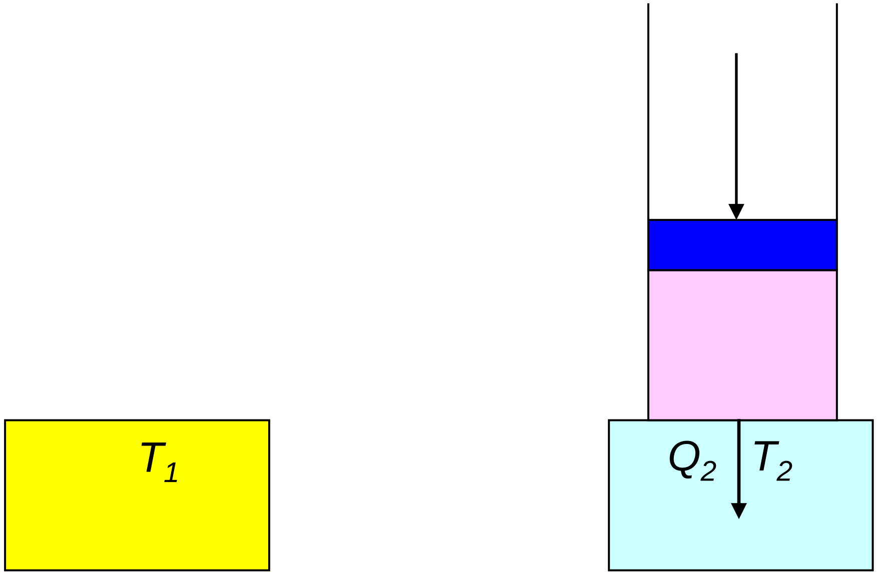


Absolute temperature

Ideal gas Carnot engine

Step 3

Isothermal compression at T_2, deliver heat Q_2.

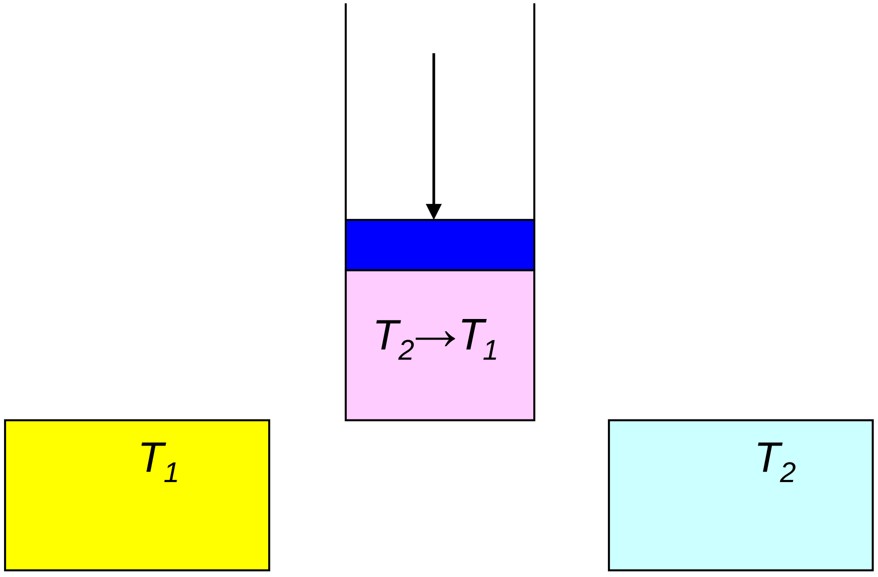


Absolute temperature

Ideal gas Carnot engine

Step 4

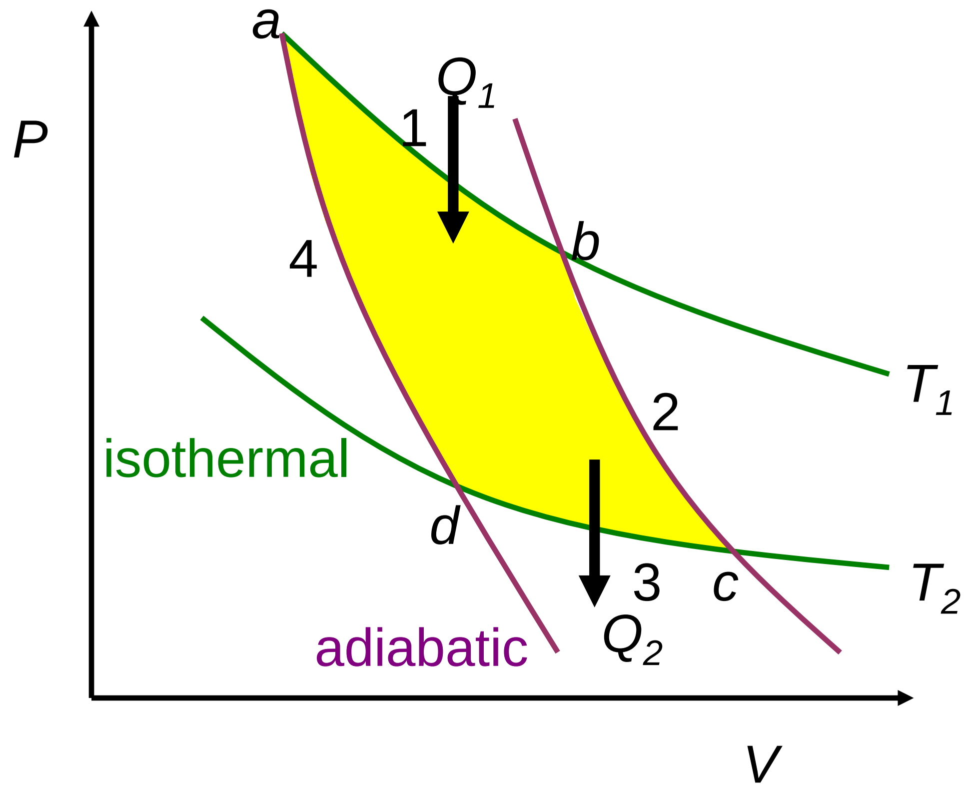
Adiabatic compression,
T rises from T_2 to T_1.



Absolute temperature

Ideal gas Carnot engine

Step 1
Isothermal expansion $a \rightarrow b$
Step 2
Adiabatic expansion $b \rightarrow c$
Step 3
Isothermal compression $c \rightarrow d$
Step 4
Adiabatic compression $d \rightarrow a$



Absolute temperature

Ideal gas Carnot engine

For an isothermal expansion of an ideal gas $\Delta U = 0$ and, therefore $Q = - W$. Thus for $a \rightarrow b$ the heat entering the engine is

$$Q_1 = \int_a^b PdV = nRT_1 \int_a^b \frac{dV}{V} = nRT_1 \ln\left(\frac{V_b}{V_a}\right)$$

Similarly for $c \rightarrow d$ the heat entering the engine is

$$Q = nRT_2 \ln\left(\frac{V_d}{V_c}\right)$$

Q_2 is the heat leaving the engine so $Q_2 = - Q$ and

$$Q_2 = nRT_2 \ln\left(\frac{V_c}{V_d}\right)$$

Absolute temperature

Ideal gas Carnot engine

For an isothermal expansion of an ideal gas *ΔU = 0* and, therefore $Q = -W$. Thus for $a \rightarrow b$ the heat entering the engine is

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Q_2 is the heat leaving the engine so $Q_2 = -Q$ and

$$Q_2 = nRT_2 \ln\left(\frac{V_c}{V_d}\right)$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \frac{\ln(V_b / V_a)}{\ln(V_c / V_d)}$$

Absolute temperature

Ideal gas Carnot engine

Now consider the adiabatic lines

$b \rightarrow c$

$$T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1}$$

$d \rightarrow a$

$$T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1}$$

so that (taking the ratio of these two equations)

$$\frac{V_b}{V_a} = \frac{V_c}{V_d}$$

Hence we have the simple and important result

$$\boxed{\frac{Q_1}{Q_2} = \frac{T_1}{T_2}}$$

Absolute temperature

Ideal gas Carnot engine

The efficiency of any engine is given by

$$\zeta = 1 - \frac{Q_2}{Q_1}$$

For an ideal gas Carnot engine

$$\zeta_C = 1 - \frac{T_2}{T_1}$$

But all Carnot engines operating between the same two reservoirs have same efficiency

Therefore

$$\zeta_C = 1 - \frac{T_2}{T_1} \quad \text{and} \quad \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \textbf{for any Carnot engine}$$

Absolute temperature

Carnot engine

We are able to define a theoretical temperature scale in terms of heat flows to, and from, a Carnot engine, since this is **independent of any particular working substance**.

The **absolute thermodynamic temperature** is defined

$$\frac{T_1^{Thermodynamic}}{T_2^{Thermodynamic}} = \frac{Q_1}{Q_2}$$

and we have shown that, with the same fixed point,

$$T^{Thermodynamic} = T^{IdealGas}$$

Absolute temperature

Absolute thermodynamic temperature can therefore be related to real working thermometers via the constant-volume ideal-gas thermometer.