Entropy

Clausius inequality

Carnot's theorem:

$$\zeta \leq \zeta_{rev}$$

$$\zeta \leq 1 - \frac{T_2}{T_1}$$

$$1 - \frac{Q_2}{Q_1} \leq 1 - \frac{T_2}{T_1}$$

$$\frac{Q_1}{T_1} \leq \frac{Q_2}{T_2}$$

Taking heat into system as positive:

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \leq 0$$

Sum over all processes

$$\sum \frac{Q}{T} \leq 0$$

In the limit

$$\oint \frac{đQ}{T} \leq 0$$

This is known as the **Clausius inequality**

Entropy

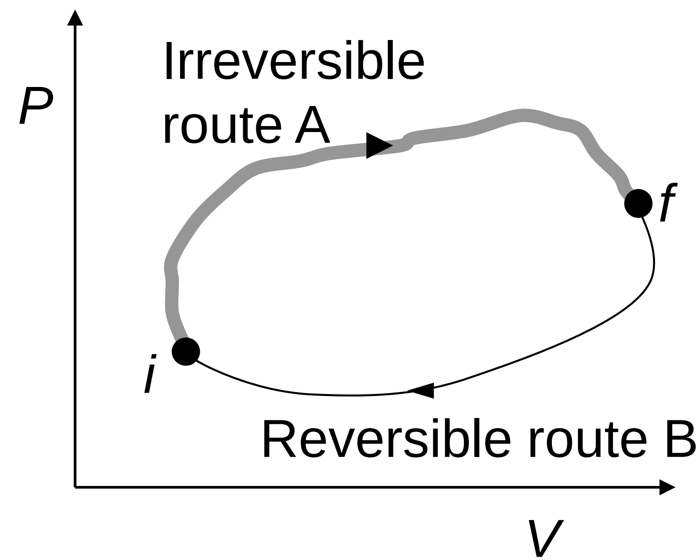
Principle of increasing entropy

Clausius inequality $\oint \frac{đQ}{T} \leq 0$

$$\underbrace{\int_i^f \frac{đQ}{T}}_{A_{irr}} + \underbrace{\int_f^i \frac{dQ_R}{T}}_{B_{rev}} \leq 0$$

$$\therefore \int_i^f \frac{đQ}{T} \leq \int_i^f \frac{dQ_R}{T} = S_f - S_i = \Delta S$$

$$\frac{đQ}{T} \leq dS$$



Entropy

Principle of increasing entropy

Thermally isolated system $dQ = 0$

$$\therefore dS \geq 0$$

The entropy of a thermally isolated system

- stays constant in a reversible process

- increases in an irreversible process

The universe is a thermally isolated system so we have

$$\Delta S(universe) \geq 0$$

which is the most useful expression of the second law of thermodynamics

$\boldsymbol{S \rightarrow}$ **maximum at equilibrium**

Entropy

Central equation of thermodynamics

First law $dU = dQ + dW$

Definition of entropy $dQ_R = TdS$

Work done for a compressible gas $dW = -PdV$

Combining these 3 equations gives **central equation** $dU = TdS - PdV$

The first law holds for both reversible and irreversible processes, but the relation between entropy and heat only holds for reversible processes. However, the central equation contains only functions of state, and the increments are determined uniquely by the initial and final states. Therefore the central equation is **always true**.