PH2420/PH3420/MT3240
Electromagnetism
Problem Sheet 5

1. Both electric and magnetic fields may be approximately described by an expansion in terms of monopoles, dipoles, and higher order terms. The potential for a monopole term of an electric multipole expansion is given (fairly obviously) by
$$V_{mon}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$
while the dipole term (derived in lectures) is
$$V_{dip}(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$$
where $\mathbf{p} = q\,\mathbf{d}$, the charge times their vector separation. For present purposes we ignore higher order terms.
   (a) Find the electric field due to a monopole. Center it at the origin and use spherical co-ordinates.
   (b) Find the electric field due to a dipole. Center it at the origin with $\mathbf{d}$ along $\hat{\mathbf{z}}$ and use spherical co-ordinates.

   The elemental magnet is formed of a (vanishingly) small loop of circulating current. There are no magnetic monopoles, therefore the first term in a multipole expansion for the magnetic field is the dipole term. As with electric dipoles, the magnetic vector potential for a small loop may be fairly easily derived,
$$\mathbf{A}_{dip}(r,\theta) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{\mathbf{r}}}{r^2}$$
where
$$\mathbf{m} = I\int d\mathbf{a}.$$
   (c) Use spherical co-ordinates to find the magnetic field due to a magnetic dipole and show that it has a similar structure to the electric field of a dipole.
   (d) Sketch the structure of the (magnetic or electric) field of a dipole.

2. An N-turn toroidal solenoid (with an 'air core') has a mean radius R and a cross-sectional area πr^2, $r << R$. A rising current $I = k't$, where k' is some constant, is turned on at $t = 0$.
   (a) Use a result from a previous problem sheet to find the energy stored in the magnetic field at a time t.
   (b) Find the direction and magnitude of the Poynting vector at a point just inside the windings at time t. Hence find the the rate of change of field energy inside the toroid at time t. Is your answer compatible with part (a)?

   Hint: Think about the electric field induced by the changing magnetic field. Perhaps you need a loop just inside the windings?

3. Use Drude theory to estimate the scattering time of a charge carrier (an electron) in, say, copper. Hence find a typical drift velocity for a 1 m long wire carrying a current driven by a 1 V potential difference. Hence find the current density and the current in a 1 mm^2 cross-section wire. Does this agree with an estimate of the resistance of the wire from its geometry? Use your values to determine how far an electron might travel before a collision. What does this tell you about Drude theory?
   Cu: Atomic Weight 63.5 g, density 8.96 $\times 10^3$ kg m^{-3}, valence? say 1.