PH2255 Course:
Introduction to Statistical Methods
Exercise 4

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0.1 Question 1

Using Kirchoff's first law to sum the voltages around the LCR circuit:

$$V = IZ = I\left(R + i\omega L + \frac{1}{i\omega C}\right) \tag{1}$$

We can rewrite this in differential form:

$$L\ddot{q} + r\dot{q} + \frac{1}{C}q = 0 \tag{2}$$

This is in the same form as a damped mechanical oscillator $\ddot{q} + \gamma\dot{q} + \omega_0^2 q = 0$ Therefore $\omega_0 = \sqrt{1/LC}$ From definition $\omega = \frac{2\pi}{T} = 2\pi f$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{3}$$

0.2 Question 2

Using values of $L = 5mH = 5 \times 10^{-3}H$ and $C = 1nF = 1 \times 10^{-9}F$:
$f_0 = 71,176.25Hz$

0.3 Question 3

For $\Delta\phi = 0$, $f = 71.18kHz$
Peak-to-peak voltages:
CH1: $1.463V$
CH2: $1.082V$
$\therefore V_2/V_1 = 0.7395$
This small

$$\sim 10Hz$$

discrepancy comes from additional impedance from the connections/wires in the circuit.

0.4 Question 4

Starting frequency $f_{\min} = 60kHz$, at $V_2/V_1 = 0.1296$
Ending frequency $f_{\max} = 82kHz$, at $V_2/V_1 = 0.1357$

0.5 Question 6

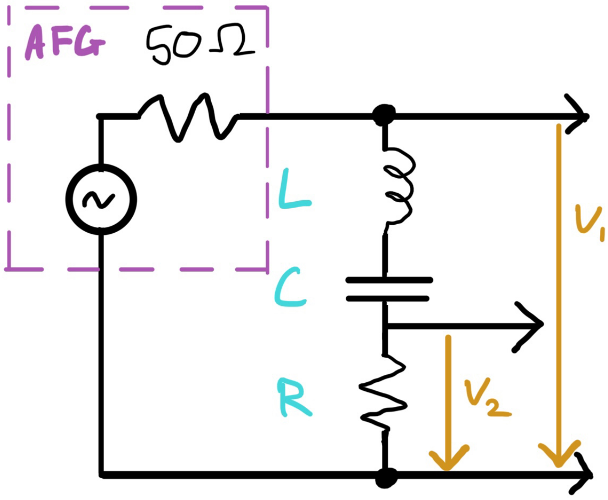
$$|Z| = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \tag{4}$$

At resonance, $\omega L - 1/\omega C = 0$, therefore that term drops out of the equation, giving us a minimum value $|Z| = R$.

At high frequencies, the Inductor term ωL dominates.

At low frequencies, the Capacitor term $1/\omega C$ dominates.

0.6 Question 7



0.7 Question 8

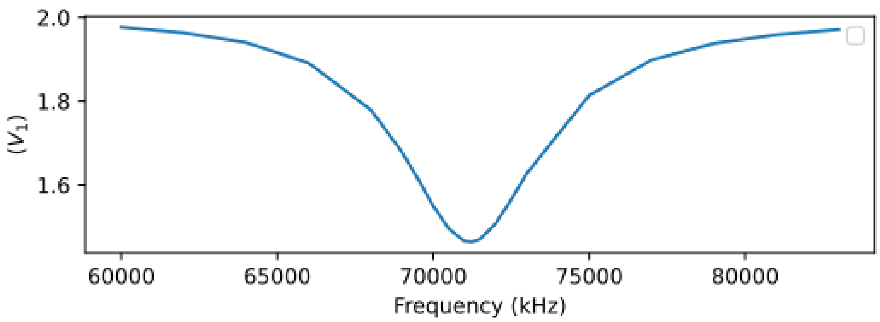


Figure 1: The voltage V_1 plotted against the driving frequency of the AFG

This plot, showing how V_1 changes as we change the AFG frequency, takes the form of a $y = \sqrt{x}$ function. As V_1 takes the potential difference over the entire LCR circuit, we include both the resistance, inductance, and capacitance of the three components into account when using Equation 4.

We need to measure the V_1 across the whole circuit, rather than using the AFG's setting, as there are impedance/resistive effects from the wires connecting the components, and the electronics inside the AFG.

We can apply Ohm's law ($V = IR$) to the resistor, using V_2, to find the current through the resistor. As the circuit is in series, the current through the resistor is the same as the current through each component of the circuit.

0.8 Question 9

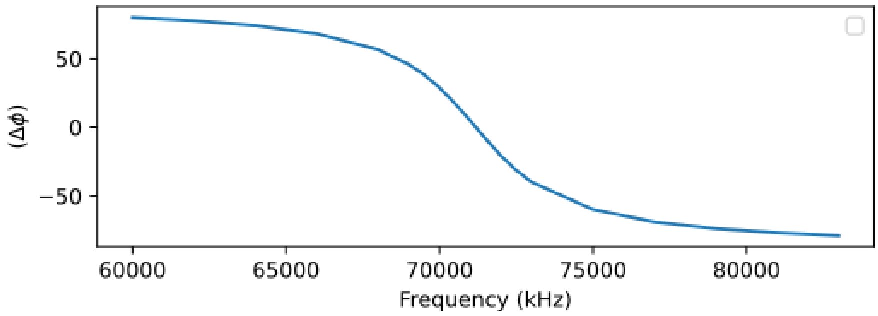


Figure 2: The phase different $\Delta\phi$ plotted against the driving frequency of the AFG

From the equation given in the lab script:

$$\Delta\phi = \tan^{-1}[(\omega L - 1/\omega C)/R] \tag{5}$$

We can see that as $f \to 0$, the arctan will approach 90 degrees, and as $f \to \infty$, the arctan approaches -90 degrees.

0.9 Question 10

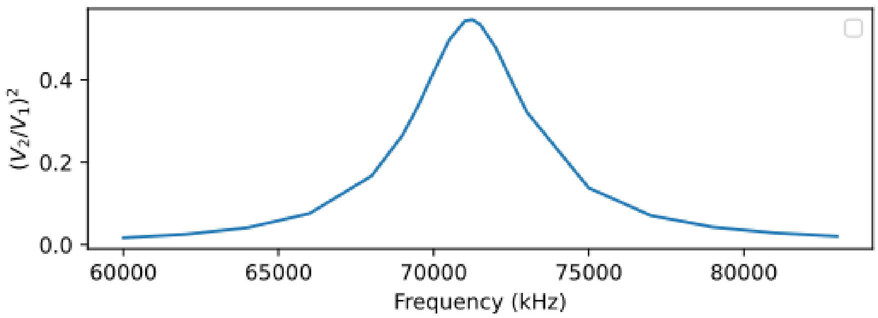


Figure 3: $(V_2/V_1)^2$ plotted against the driving frequency of the AFG

Estimating values either side of resonance where $(V_2/V_1)^2$ are at half maximum:

$f_1 \approx 69050Hz$

$f_2 \approx 74500 Hz$

Using these, we can estimate the quality factor of the resonance using the equation provided in the lab script:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \tag{6}$$

Where $\Delta f = f_2 - f_1$

Using this, we obtain a value of $Q = 13.06$

However, if we use the equation involving L and R:

$$Q = \frac{\omega_1 L}{R} \tag{7}$$

We obtain a value of $Q = 3.56$. While these differ, both are above the critical $Q = \frac{1}{2}$ value, indicating that this is an underdamped system.

0.10 Question 11

Using Equation 3 to find an estimate of f_0 for a system with $L = 5mH$, $C = 1nF$, and $R{-}200\Omega$, we obtain $f_0 = 71,176.25 Hz$, the same value as before. This is because we are only changing the resistor, which has no effect using Equation 3. Using the AFG simulator, we do in fact obtain the same experimental value of f_0. Because of this, we can use the same f_{max} and f_{min} as before.

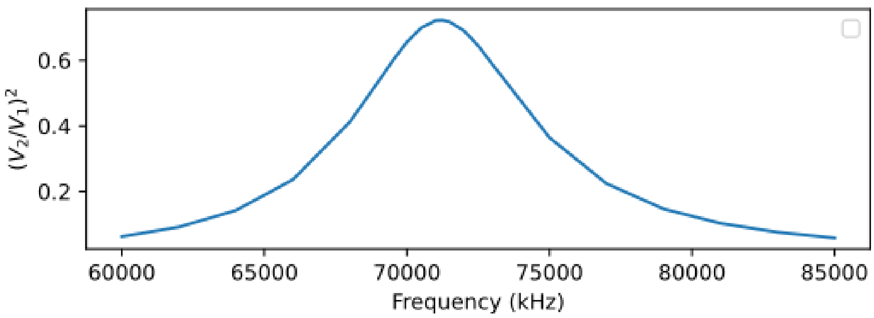


Figure 4: $(V_2/V_1)^2$ plotted against the driving frequency of the AFG

If we again estimate f_1 and f_2 at half-maximum height:

$f_1 = 66500 Hz$

$f_2 = 75050 Hz$

From these values, we obtain a value of $Q = 8.32$, again an underdamped system. If we use Equation 7, we obtain a value of $Q = 1.78$

0.11 Question 12

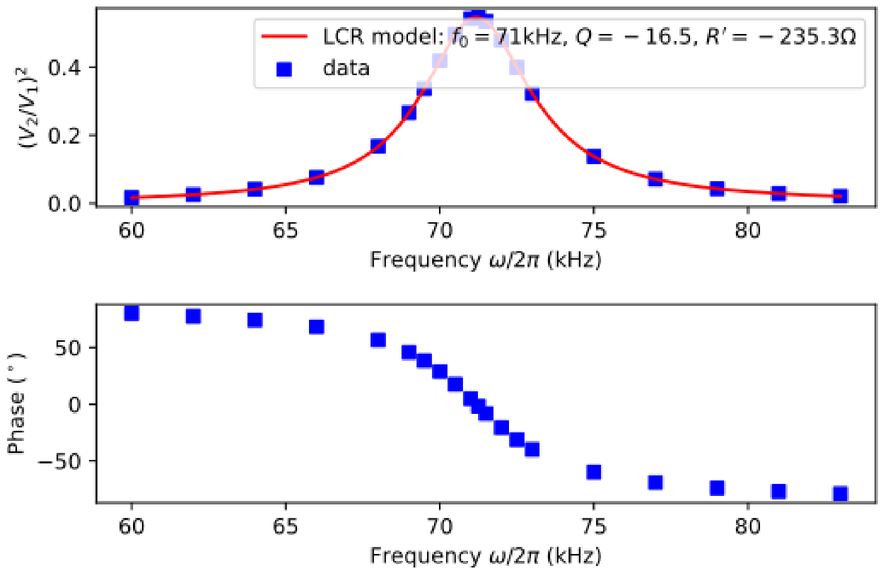


Figure 5: $(V_2/V_1)^2$ plotted against the driving frequency of the AFG for $L = 5 \times 10^{-3}H$, $C = 1 \times 10^{-9}$, and $R = 100\Omega$, and the fitted curve

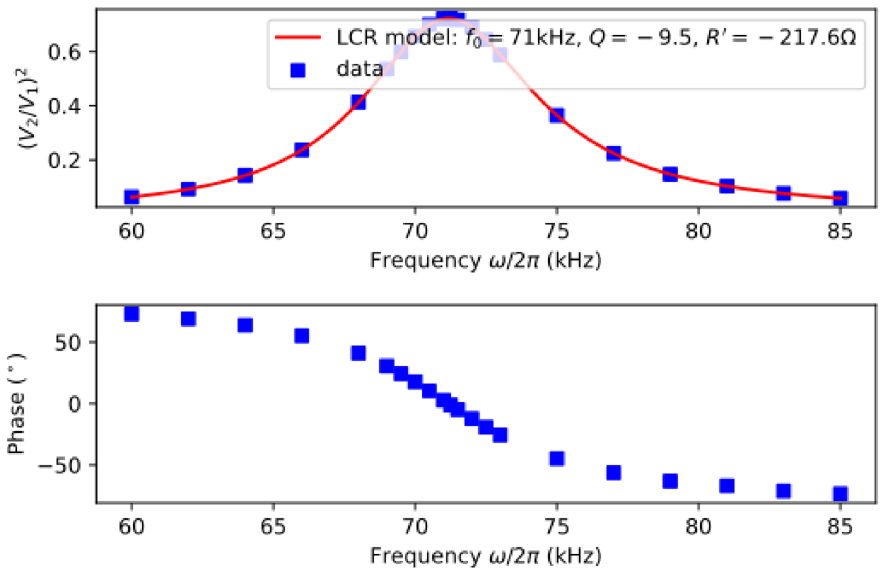


Figure 6: $(V_2/V_1)^2$ plotted against the driving frequency of the AFG for $L = 5 \times 10^{-3}H$, $C = 1 \times 10^{-9}$, and $R = 200\Omega$, and the fitted curve

0.12 Question 13

For the first fit, the values obtained were $f_0 = 71.18kHz$, $Q = -16.49$, and $R' = -235.27$.

For the second fit, these values were $f_0 = 71.18kHz$, $Q = -9.48$, and $R' = -217.56$.

These values can be related to R, L, and C using the following equations:

$$Q = \omega_0 L/(R + R') \tag{8}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \tag{9}$$