

Having a Ball: evaluating game excitement and scoring streaks using in-match trend estimation

Claus Thorn Ekstrøm and Andreas Kryger Jensen
Biostatistics, Institute of Public Health, University of Copenhagen
ekstrom@sund.ku.dk, aeje@sund.ku.dk

05 November, 2020

Abstract

Nu ved jeg godt nok intet om sport, men... ❁!

Keywords: APBRmetrics, Bayesian Statistics, Gaussian Processes, Sports Statistics, Trends

1 Introduction

We introduce the Excitement Trend Index (ETI) as an objective measure of spectator excitement in a given match.

- Excitement defineret som skift af hvem, der er i føring
- Vurdering af om et hold trender lige pt.
- Identificere hvilket “hot periods” et hold har i løbet af en kamp til efterfølgende evaluering

Reference to Quantifying the Trendiness of Trends Jensen and Ekstrøm (2020).

Needs a reference and some kind of comparison to Chen, Dawson, and Müller (2020).

We also need to talk about this one: Chen and Fan (2018)

Vi kan overveje, om vi ikke udelukkende burde fokusere på Eddy i dette manus og foreslå dette som et objektivt excitement index i stedet for at gå ind i en diskussion om TDI også. Jeg tænker også i forhold til at dette værk måske ikke behøver være så voluminøst.

Materials to reproduce this manuscript can be found at Jensen and Ekstrøm (2019).

Philosophical question: What exactly is the sampling model for a single match?

2 Methods

Let m index a given match between teams a and b and let $D_m(t_{mi}) = S_a(t_{mi}) - S_b(t_{mi})$ be the difference in scores at times $t_{m1} < t_{m2} < t_{mi} < \dots < t_{mJ_m}$ being the ordered event times when a score by either team a or b occurs during match m .

We use the model

$$\begin{aligned} (\beta_m, \theta_m, \sigma_m^2) \mid \Psi_m, \mathbf{t}_m &\sim G(\Theta_m \mid \Psi_m, \mathbf{t}_m) \\ f_m(t) \mid \Theta_m &\sim \mathcal{GP}(\mu_{\beta_m}(t), C_{\theta_m}(t, t')) \\ D_m(t_{mi}) \mid f_m(t_{mi}), t_{mi}, \Theta_m &\stackrel{iid}{\sim} N(f_m(t_{mi}), \sigma_m^2) \end{aligned} \quad (1)$$

where f_m models a latent trajectory of score differences. We also have that

$$\begin{bmatrix} f_m(s) \\ df_m(t) \\ d^2f_m(u) \end{bmatrix} \mid \Theta_m \sim \mathcal{GP} \left(\begin{bmatrix} \mu_{\beta_m}(s) \\ d\mu_{\beta_m}(t) \\ d^2\mu_{\beta_m}(u) \end{bmatrix}, \begin{bmatrix} C_{\theta_m}(s, s') & \partial_2 C_{\theta_m}(s, t) & \partial_2^2 C_{\theta_m}(s, u) \\ \partial_1 C_{\theta_m}(t, s) & \partial_1 \partial_2 C_{\theta_m}(t, t') & \partial_1 \partial_2^2 C_{\theta_m}(t, u) \\ \partial_1^2 C_{\theta_m}(u, s) & \partial_1^2 \partial_2 C_{\theta_m}(u, t) & \partial_1^2 \partial_2^2 C_{\theta_m}(u, u') \end{bmatrix} \right) \quad (2)$$

where $d^k \mu_\beta$ is the k 'th derivative of μ_β and ∂_j^k denotes the k 'th order partial derivative with respect to the j 'th variable

We then define the Excitement Trend Index (ETI) as the number of zero-crossings of df_m conditional on the observed score differences. Formally,

$$\text{ETI}_m(\Theta_m) = \mathbb{E}[\#\{t \in \mathcal{I}_m : df_m(t) = 0\} \mid \mathbf{D}_m, \mathbf{t}_m, \Theta_m]$$

where \mathcal{I}_m is the interval of the time duration of a match i.e., $\mathcal{I}_m = [0; 48]$ minutes without overtime. The ETI is given by the integral of the local Excitement Trend Index

$$\text{ETI}_m(\Theta_m) = \int_{\mathcal{I}_m} d\text{ETI}_m(t \mid \Theta_m) dt$$

where $d\text{ETI}$ is the local Excitement Trend Index given by

$$d\text{ETI}_m(t \mid \Theta_m) = \lambda(t \mid \Theta) \phi \left(\frac{\mu_{df}(t \mid \Theta)}{\Sigma_{df}(t, t \mid \Theta)^{1/2}} \right) \left(2\phi(\zeta(t \mid \Theta)) + \zeta(t \mid \Theta) \text{Erf} \left(\frac{\zeta(t \mid \Theta)}{2^{1/2}} \right) \right)$$

and $\phi: x \mapsto 2^{-1/2} \pi^{-1/2} \exp(-\frac{1}{2}x^2)$ is the standard normal density function, $\text{Erf}: x \mapsto 2\pi^{-1/2} \int_0^x \exp(-u^2) du$ is the error function, and λ, ω and ζ are functions defined as

$$\begin{aligned} \lambda(t \mid \Theta) &= \frac{\Sigma_{d^2f}(t, t \mid \Theta)^{1/2}}{\Sigma_{df}(t, t \mid \Theta)^{1/2}} (1 - \omega(t \mid \Theta)^2)^{1/2} \\ \omega(t \mid \Theta) &= \frac{\Sigma_{df, d^2f}(t, t \mid \Theta)}{\Sigma_{df}(t, t \mid \Theta)^{1/2} \Sigma_{d^2f}(t, t \mid \Theta)^{1/2}} \\ \zeta(t \mid \Theta) &= \frac{\mu_{df}(t \mid \Theta) \Sigma_{d^2f}(t, t \mid \Theta)^{1/2} \omega(t \mid \Theta) \Sigma_{df}(t, t \mid \Theta)^{-1/2} - \mu_{d^2f}(t \mid \Theta)}{\Sigma_{d^2f}(t, t \mid \Theta)^{1/2} (1 - \omega(t \mid \Theta)^2)^{1/2}} \end{aligned}$$

A derivation of this expression can be found in the supplementary material to Jensen and Ekström (2020).

The posterior distribution of the hyper-parameters given the observed data is then. We define $\tilde{\Theta}_m \sim P(\Theta_m \mid \mathbf{D}_m, \Psi_m, \mathbf{t}_m)$ hence

$$\tilde{\Theta}_m \sim \frac{G(\Theta_m \mid \Psi_m, \mathbf{t}_m) \int P(\mathbf{D}_m \mid f(\mathbf{t}_m), \Theta_m, \Psi_m, \mathbf{t}_m) dP(f_m(\mathbf{t}_m) \mid \Theta_m, \Psi_m, \mathbf{t}_m)}{\iint P(\mathbf{D}_m \mid f_m(\mathbf{t}_m), \Theta_m, \Psi_m, \mathbf{t}_m) dP(f_m(\mathbf{t}_m) \mid \Theta_m, \Psi_m, \mathbf{t}_m) dG(\Theta_m \mid \Psi_m, \mathbf{t}_m)}$$

What we estimate is then the random variable $\widehat{\text{ETI}}_m = \text{ETI}_m(\tilde{\Theta}_m)$ which can be summarized by its moments or quantiles.

We need to argue that ETI for $S_a(t_{mi}) - S_b(t_{mi})$ is symmetric in a and b so that our choice of “reference group” in D_m is not important. The reason is that we look at both up- and down-crossings at 0 of df_m so the choice of sign in D_m is not relevant.

2.1 Estimation

We have implemented the model described in the previous section in Stan (Carpenter et al. 2017).

Prior mean and covariance:

$$\mu_{\beta_m}(t) = \beta_m, \quad C_{\theta_m}(t, t') = \alpha_m^2 \exp\left(-\frac{(t - t')^2}{2\rho_m^2}\right)$$

with $\alpha_m, \rho_m > 0$.

Hyper-parameters: We used independent priors on $\Theta_m = (\beta_m, \alpha_m, \rho_m, \sigma_m)$ of the form

$$G(\Theta_m \mid \Psi_m, \mathbf{t}_m) = G(\beta_m \mid \Psi_{\beta_m})G(\alpha_m \mid \Psi_{\alpha_m})G(\rho_m \mid \Psi_{\rho_m})G(\sigma_m \mid \Psi_{\sigma_m})$$

where each prior is a heavy-tailed distribution with a moderate variance centered at the marginal maximum likelihood estimates. We used the following distributions

$$\beta_m \sim T\left(\widehat{\beta}_m^{\text{ML}}, 3, 3\right), \quad \alpha_m \sim T^+\left(\widehat{\alpha}_m^{\text{ML}}, 3, 3\right), \quad \rho_m \sim N^+\left(\widehat{\rho}_m^{\text{ML}}, 1\right), \quad \sigma_m \sim T^+\left(\widehat{\sigma}_m^{\text{ML}}, 3, 3\right)$$

where $T^+(\cdot, \cdot, \text{df})$ and $N^+(\cdot, \cdot)$ denotes the location-scale half T- and normal distribution functions with df degrees of freedom. For each match we ran four independent Markov chains for 25,000 iterations each with half of the iterations used for warm-up. Convergence was assessed by trace plots of the MCMC draws and the potential scale reduction factor, \hat{R} , of Gelman and Rubin (1992).

3 Results

We use data from Sports Reference LLC (2020).

General idea: We estimate ETI for all matches in a given season and make a nice plot of the distribution of ETI_m . Then we can rank the matches according to increasing ETI and show the running score difference for e.g., the lowest, median and highest ranked matches. Maybe a large forest plot of ETI_m would look impressive.

Given the posteriors ETI_m we can summarize them by a posterior mean and variance and then do a **meta analysis** where we adjust for game-specific fixed effects such as number of spectators, location, stratify by season and so on.

4 Discussion

We could define a weighted Excitement Trend Index, ETI_m^W , so that zero crossings of the derivative of score differences are weighted higher towards the end of the game as in

$$\text{ETI}_m^W = \int_{\mathcal{I}_m} \text{ETI}_m(t) w_m(t) dt$$

where w_m is an increasing weight function.

Another approach would be to define team-specific excitement index nested with a match. Here we would only look at the **up**-crossings at zero of df_m and we would get two excitement indices for each match ($\text{ETI}_{am}, \text{ETI}_{bm}$). for teams a and b . This would somehow reflect how exciting each team were in match m with respect to chancing the sign of the score differences in their favor.

Acknowledgements

Bibliography

Carpenter, Bob, Andrew Gelman, Matthew D Hoffman, Daniel Lee, Ben Goodrich, Michael Betancourt, Marcus Brubaker, Jiqiang Guo, Peter Li, and Allen Riddell. 2017. “Stan: A Probabilistic Programming Language.” *Journal of Statistical Software* 76 (1).

Chen, Tao, and Qingliang Fan. 2018. “A Functional Data Approach to Model Score Difference Process in Professional Basketball Games.” *Journal of Applied Statistics* 45 (1): 112–27.

Chen, Yaqing, Matthew Dawson, and Hans-Georg Müller. 2020. “Rank Dynamics for Functional Data.” *Computational Statistics & Data Analysis*, 106963.

Gelman, Andrew, and Donald B. Rubin. 1992. “Inference from Iterative Simulation Using Multiple Sequences.” *Statistical Science* 7 (4): 457–72.

Jensen, Andreas Kryger, and Claus Thorn Ekstrøm. 2019. “GitHub Repository for the Trendiness of Trends.” 2019. <https://github.com/aejensen/TrendinessOfTrends>.

———. 2020. “Quantifying the Trendiness of Trends.” *Journal of the Royal Statistical Society: Series C*.

Sports Reference LLC. 2020. “Basketball Reference.” 2020. <https://www.basketball-reference.com/>.

Appendix (say what?)

the joint distribution of (f, df, d^2f) conditional on \mathbf{Y} , \mathbf{t} and the hyper-parameters Θ evaluated at any finite vector \mathbf{t}^* of p time points is

$$\begin{bmatrix} f(\mathbf{t}^*) \\ df(\mathbf{t}^*) \\ d^2f(\mathbf{t}^*) \end{bmatrix} \mid \mathbf{Y}, \mathbf{t}, \Theta \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where $\boldsymbol{\mu} \in \mathbb{R}^{3p}$ is the column vector of posterior expectations and $\boldsymbol{\Sigma} \in \mathbb{R}^{3p \times 3p}$ is the joint posterior covariance matrix. Partitioning these as

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_f(\mathbf{t}^* \mid \Theta) \\ \mu_{df}(\mathbf{t}^* \mid \Theta) \\ \mu_{d^2f}(\mathbf{t}^* \mid \Theta) \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_f(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \\ \Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \\ \Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \end{bmatrix}$$

the individual components are given by

$$\begin{aligned}
\mu_f(\mathbf{t}^* \mid \boldsymbol{\Theta}) &= \mu_\beta(\mathbf{t}^*) + C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\mu_{df}(\mathbf{t}^* \mid \boldsymbol{\Theta}) &= d\mu_\beta(\mathbf{t}^*) + \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\mu_{d^2f}(\mathbf{t}^* \mid \boldsymbol{\Theta}) &= d^2\mu_\beta(\mathbf{t}^*) + \partial_1^2 C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\Sigma_f(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{df}(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= \partial_1 \partial_2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= \partial_1^2 \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1^2 C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= \partial_2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \boldsymbol{\Theta}) &= \partial_1 \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left(C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*)
\end{aligned}$$