

# Having a Ball: evaluating game excitement and scoring streaks using in-match trend estimation

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## Abstract

Nu ved jeg godt nok intet om sport, men... ❁!

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## 1 Introduction

We introduce the Excitement Trend Index (ETI) as an objective measure of spectator excitement in a given match.

- Excitement defineret som skift af hvem, der er i føring
- Vurdering af om et hold trender lige pt.
- Identificere hvilket “hot periods” et hold har i løbet af en kamp til efterfølgende evaluering

Reference to Quantifying the Trendiness of Trends Jensen and Ekstrøm (2020b).

We need to reference and discuss the following papers

- Chen, Dawson, and Müller (2020)
- Chen and Fan (2018)
- Gabel and Redner (2012)

Vi kan overveje, om vi ikke udelukkende burde fokusere på Eddy i dette manus og foreslå dette som et objektivt excitement index i stedet for at gå ind i en diskussion om TDI også. Jeg tænker også i forhold til at dette værk måske ikke behøver være så voluminøst.

Materials to reproduce this manuscript can be found at Jensen and Ekstrøm (2020a).

Philosophical question: What exactly is the sampling model for a single match?

## 2 Methods

Let  $m$  index a given match between teams  $a$  and  $b$  and let  $D_m(t_{mi}) = S_a(t_{mi}) - S_b(t_{mi})$  be the difference in scores at times  $t_{m1} < t_{m2} < t_{mi} < \dots < t_{mJ_m}$  being the ordered event times when a score by either team  $a$  or  $b$  occurs during match  $m$ .

We use the model

$$\begin{aligned} (\beta_m, \theta_m, \sigma_m^2) \mid \Psi_m, \mathbf{t}_m &\sim G(\Theta_m \mid \Psi_m, \mathbf{t}_m) \\ f_m(t) \mid \Theta_m &\sim \mathcal{GP}(\mu_{\beta_m}(t), C_{\theta_m}(t, t')) \\ D_m(t_{mi}) \mid f_m(t_{mi}), t_{mi}, \Theta_m &\stackrel{iid}{\sim} N(f_m(t_{mi}), \sigma_m^2) \end{aligned} \quad (1)$$

where  $f_m$  models a latent trajectory of score differences. We also have that

$$\begin{bmatrix} f_m(s) \\ df_m(t) \\ d^2f_m(u) \end{bmatrix} \mid \Theta_m \sim \mathcal{GP} \left( \begin{bmatrix} \mu_{\beta_m}(s) \\ d\mu_{\beta_m}(t) \\ d^2\mu_{\beta_m}(u) \end{bmatrix}, \begin{bmatrix} C_{\theta_m}(s, s') & \partial_2 C_{\theta_m}(s, t) & \partial_2^2 C_{\theta_m}(s, u) \\ \partial_1 C_{\theta_m}(t, s) & \partial_1 \partial_2 C_{\theta_m}(t, t') & \partial_1 \partial_2^2 C_{\theta_m}(t, u) \\ \partial_1^2 C_{\theta_m}(u, s) & \partial_1^2 \partial_2 C_{\theta_m}(u, t) & \partial_1^2 \partial_2^2 C_{\theta_m}(u, u') \end{bmatrix} \right) \quad (2)$$

where  $d^k \mu_\beta$  is the  $k$ 'th derivative of  $\mu_\beta$  and  $\partial_j^k$  denotes the  $k$ 'th order partial derivative with respect to the  $j$ 'th variable

We then define the Excitement Trend Index (ETI) as the number of zero-crossings of  $df_m$  conditional on the observed score differences. Formally,

$$\text{ETI}_m(\Theta_m) = \mathbb{E}[\#\{t \in \mathcal{I}_m : df_m(t) = 0\} \mid \mathbf{D}_m, \mathbf{t}_m, \Theta_m]$$

where  $\mathcal{I}_m$  is the interval of the time duration of a match i.e.,  $\mathcal{I}_m = [0; 48]$  minutes without overtime. The ETI is given by the integral of the local Excitement Trend Index

$$\text{ETI}_m(\Theta_m) = \int_{\mathcal{I}_m} d\text{ETI}_m(t \mid \Theta_m) dt$$

where  $d\text{ETI}$  is the local Excitement Trend Index given by

$$d\text{ETI}_m(t \mid \Theta_m) = \lambda(t \mid \Theta) \phi \left( \frac{\mu_{df}(t \mid \Theta)}{\Sigma_{df}(t, t \mid \Theta)^{1/2}} \right) \left( 2\phi(\zeta(t \mid \Theta)) + \zeta(t \mid \Theta) \text{Erf} \left( \frac{\zeta(t \mid \Theta)}{2^{1/2}} \right) \right)$$

and  $\phi: x \mapsto 2^{-1/2} \pi^{-1/2} \exp(-\frac{1}{2}x^2)$  is the standard normal density function,  $\text{Erf}: x \mapsto 2\pi^{-1/2} \int_0^x \exp(-u^2) du$  is the error function, and  $\lambda, \omega$  and  $\zeta$  are functions defined as

$$\begin{aligned} \lambda(t \mid \Theta) &= \frac{\Sigma_{d^2f}(t, t \mid \Theta)^{1/2}}{\Sigma_{df}(t, t \mid \Theta)^{1/2}} (1 - \omega(t \mid \Theta)^2)^{1/2} \\ \omega(t \mid \Theta) &= \frac{\Sigma_{df, d^2f}(t, t \mid \Theta)}{\Sigma_{df}(t, t \mid \Theta)^{1/2} \Sigma_{d^2f}(t, t \mid \Theta)^{1/2}} \\ \zeta(t \mid \Theta) &= \frac{\mu_{df}(t \mid \Theta) \Sigma_{d^2f}(t, t \mid \Theta)^{1/2} \omega(t \mid \Theta) \Sigma_{df}(t, t \mid \Theta)^{-1/2} - \mu_{d^2f}(t \mid \Theta)}{\Sigma_{d^2f}(t, t \mid \Theta)^{1/2} (1 - \omega(t \mid \Theta)^2)^{1/2}} \end{aligned}$$

A derivation of this expression can be found in the supplementary material to Jensen and Ekström (2020b).

The posterior distribution of the hyper-parameters given the observed data is then. We define  $\tilde{\Theta}_m \sim P(\Theta_m \mid \mathbf{D}_m, \Psi_m, \mathbf{t}_m)$  hence

$$\tilde{\Theta}_m \sim \frac{G(\Theta_m \mid \Psi_m, \mathbf{t}_m) \int P(\mathbf{D}_m \mid f(\mathbf{t}_m), \Theta_m, \Psi_m, \mathbf{t}_m) dP(f_m(\mathbf{t}_m) \mid \Theta_m, \Psi_m, \mathbf{t}_m)}{\iint P(\mathbf{D}_m \mid f_m(\mathbf{t}_m), \Theta_m, \Psi_m, \mathbf{t}_m) dP(f_m(\mathbf{t}_m) \mid \Theta_m, \Psi_m, \mathbf{t}_m) dG(\Theta_m \mid \Psi_m, \mathbf{t}_m)}$$

What we estimate is then the random variable  $\widehat{\text{ETI}}_m = \text{ETI}_m(\tilde{\Theta}_m)$  which can be summarized by its moments or quantiles.

We need to argue that ETI for  $S_a(t_{mi}) - S_b(t_{mi})$  is symmetric in  $a$  and  $b$  so that our choice of “reference group” in  $D_m$  is not important. The reason is that we look at both up- and down-crossings at 0 of  $df_m$  so the choice of sign in  $D_m$  is not relevant.

## 2.1 Estimation

We have implemented the model described in the previous section in Stan (Carpenter et al. 2017).

Prior mean and covariance:

$$\mu_{\beta_m}(t) = \beta_m, \quad C_{\theta_m}(t, t') = \alpha_m^2 \exp\left(-\frac{(t - t')^2}{2\rho_m^2}\right)$$

with  $\alpha_m, \rho_m > 0$ .

Hyper-parameters: We used independent priors on  $\Theta_m = (\beta_m, \alpha_m, \rho_m, \sigma_m)$  of the form

$$G(\Theta_m \mid \Psi_m, \mathbf{t}_m) = G(\beta_m \mid \Psi_{\beta_m})G(\alpha_m \mid \Psi_{\alpha_m})G(\rho_m \mid \Psi_{\rho_m})G(\sigma_m \mid \Psi_{\sigma_m})$$

where each prior is a heavy-tailed distribution with a moderate variance centered at the marginal maximum likelihood estimates. We used the following distributions

$$\beta_m \sim T(\widehat{\beta}_m^{\text{ML}}, 3, 3), \quad \alpha_m \sim T^+(\widehat{\alpha}_m^{\text{ML}}, 3, 3), \quad \rho_m \sim N^+(\widehat{\rho}_m^{\text{ML}}, 1), \quad \sigma_m \sim T^+(\widehat{\sigma}_m^{\text{ML}}, 3, 3)$$

where  $T^+(\cdot, \cdot, \text{df})$  and  $N^+(\cdot, \cdot)$  denotes the location-scale half T- and normal distribution functions with df degrees of freedom. For each match we ran four independent Markov chains for 25,000 iterations each with half of the iterations used for warm-up. Convergence was assessed by trace plots of the MCMC draws and the potential scale reduction factor,  $\hat{R}$ , of Gelman and Rubin (1992).

## 3 Results

We use data from Sports Reference LLC (2020).

General idea: We estimate ETI for all matches in a given season and make a nice plot of the distribution of  $\text{ETI}_m$ . Then we can rank the matches according to increasing ETI and show the running score difference for e.g., the lowest, median and highest ranked matches. Maybe a large forest plot of  $\text{ETI}_m$  would look impressive.

Given the posteriors  $\text{ETI}_m$  we can summarize them by a posterior mean and variance and then do a **meta analysis** where we adjust for game-specific fixed effects such as number of spectators, location, stratify by season and so on.

## 4 Discussion

We could define a weighted Excitement Trend Index,  $\text{ETI}_m^W$ , so that zero crossings of the derivative of score differences are weighted higher towards the end of the game as in

$$\text{ETI}_m^W = \int_{\mathcal{I}_m} d\text{ETI}_m(t)w(t)dt$$

where  $w$  is an increasing weight function.

Maybe changes in monotonicity of the score differences are not so important if one team is already far ahead of the other team. This would motivate a weighted ETI of the form

$$\text{ETI}_m^W = \int_{\mathcal{I}_m} d\text{ETI}_m(t)w(t, |f_m(t)|)dt$$

where  $w(\cdot, \cdot)$  is a decreasing function in its second variable.

Another approach would be to define team-specific excitement index nested with a match. Here we would only look at the **up**-crossings at zero of  $df_m$  and we would get two excitement indices for each match  $(ETI_{am}, ETI_{bm})$ . for teams  $a$  and  $b$ . This would somehow reflect how exciting each team were in match  $m$  with respect to chancing the sign of the score differences in their favor.

## Acknowledgements

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## Appendix (say what?)

the joint distribution of  $(f, df, d^2f)$  conditional on  $\mathbf{Y}, \mathbf{t}$  and the hyper-parameters  $\Theta$  evaluated at any finite vector  $\mathbf{t}^*$  of  $p$  time points is

$$\begin{bmatrix} f(\mathbf{t}^*) \\ df(\mathbf{t}^*) \\ d^2f(\mathbf{t}^*) \end{bmatrix} \mid \mathbf{Y}, \mathbf{t}, \Theta \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where  $\boldsymbol{\mu} \in \mathbb{R}^{3p}$  is the column vector of posterior expectations and  $\boldsymbol{\Sigma} \in \mathbb{R}^{3p \times 3p}$  is the joint posterior covariance matrix. Partitioning these as

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_f(\mathbf{t}^* \mid \Theta) \\ \mu_{df}(\mathbf{t}^* \mid \Theta) \\ \mu_{d^2f}(\mathbf{t}^* \mid \Theta) \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_f(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \\ \Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{df}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) & \Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \\ \Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta)^T & \Sigma_{d^2f}(\mathbf{t}^*, \mathbf{t}^* \mid \Theta) \end{bmatrix}$$

the individual components are given by

$$\begin{aligned}
\mu_f(\mathbf{t}^* | \boldsymbol{\Theta}) &= \mu_\beta(\mathbf{t}^*) + C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\mu_{df}(\mathbf{t}^* | \boldsymbol{\Theta}) &= d\mu_\beta(\mathbf{t}^*) + \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\mu_{d^2f}(\mathbf{t}^* | \boldsymbol{\Theta}) &= d^2\mu_\beta(\mathbf{t}^*) + \partial_1^2 C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} (\mathbf{Y} - \mu_\beta(\mathbf{t})) \\
\Sigma_f(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{df}(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= \partial_1 \partial_2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{d^2f}(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= \partial_1^2 \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1^2 C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{f,df}(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= \partial_2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{f,d^2f}(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*) \\
\Sigma_{df,d^2f}(\mathbf{t}^*, \mathbf{t}^* | \boldsymbol{\Theta}) &= \partial_1 \partial_2^2 C_\theta(\mathbf{t}^*, \mathbf{t}^*) - \partial_1 C_\theta(\mathbf{t}^*, \mathbf{t}) \left( C_\theta(\mathbf{t}, \mathbf{t}) + \sigma^2 I \right)^{-1} \partial_2^2 C_\theta(\mathbf{t}, \mathbf{t}^*)
\end{aligned}$$