CodingLab7

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Neural Data Science

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LLM Disclaimer: Google Gemini, Google Gemini Diffiusion - Planning, Coding, and Verification.

1 Coding Lab 7: Transcriptomics

```
import numpy as np
import pylab as plt
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

sns.set_style("whitegrid")

# We recommend using openTSNE for experiments with t-SNE
# https://github.com/pavlin-policar/openTSNE
from openTSNE import TSNE

%matplotlib inline

%load_ext jupyter_black

%load_ext watermark
%watermark --time --date --timezone --updated --python --iversions --watermark_
⊶-p sklearn
```

```
The jupyter_black extension is already loaded. To reload it, use: %reload_ext jupyter_black
The watermark extension is already loaded. To reload it, use: %reload_ext watermark
Last updated: 2025-06-08 19:02:52CEST
```

Python implementation: CPython Python version : 3.11.11 IPython version : 9.2.0

sklearn: 1.6.1

numpy : 1.26.4 openTSNE : 1.0.2 matplotlib: 3.9.4 seaborn : 0.13.2 pandas : 2.2.3

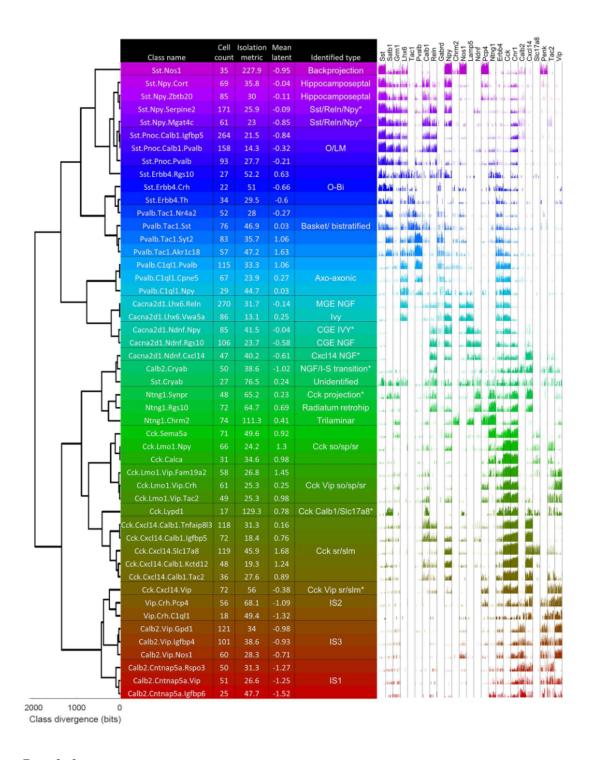
Watermark: 2.5.0

```
[2]: plt.style.use("../matplotlib_style.txt")
```

2 Introduction

In this notebook you are going to work with transcriptomics data, in particular single-cell RNA sequencing (scRNA-seq) data from the paper by Harris et al. (2018). They recorded the transcriptomes of 3,663 inhibitory cells in the hippocampal area CA1. Their analysis divided these cells into 49 fine-scale clusters coresponding to different cell subtypes. They asigned names to these cluster in a hierarchical fashion according to strongly expressed gene in each clusters. The figure below shows the details of their classification.

You will first analyze some of the most relevant statistics of UMI gene counts distributions, and afterwards follow the standard pipeline in the field to produce a visualization of the data.



2.1 Load data

Download the data from ILIAS, move it to the data/ directory and unzip it there. The read counts can be found in counts, with rows corresponding to cells and columns to genes. The cluster assignments for every individual cell can be found in clusters, along with the colors used in the publication in clusterColors.

```
[3]: # LOAD HARRIS ET AL DATA
     # Load gene counts
     data = pd.read_csv("../data/nds_cl_7/harris-data/expression.tsv", sep="\t")
     genes = data.values[:, 0]
     cells = data.columns[1:-1]
     counts = data.values[:, 1:-1].transpose().astype("int")
     data = []
     # Kick out all genes with all counts = 0
     genes = genes[counts.sum(axis=0) > 0]
     counts = counts[:, counts.sum(axis=0) > 0]
     print(counts.shape)
     # Load clustering results
     data = pd.read_csv("../data/nds_cl_7/harris-data/analysis_results.tsv", u
      clusterNames, clusters = np.unique(data.values[0, 1:-1], return_inverse=True)
     # Load cluster colors
     data = pd.read_csv("../data/nds_cl_7/harris-data/colormap.txt", sep="\s+",u
      →header=None)
     clusterColors = data.values
     # Note: the color order needs to be reversed to match the publication
     clusterColors = clusterColors[::-1]
     # Taken from Figure 1 - we need cluster order to get correct color order
     clusterOrder = [
         "Sst.No".
         "Sst.Npy.C",
         "Sst.Npy.Z",
         "Sst.Npy.S",
         "Sst.Npy.M",
         "Sst.Pnoc.Calb1.I",
         "Sst.Pnoc.Calb1.P",
         "Sst.Pnoc.P",
         "Sst.Erbb4.R",
         "Sst.Erbb4.C",
         "Sst.Erbb4.T",
         "Pvalb.Tac1.N",
         "Pvalb.Tac1.Ss",
         "Pvalb.Tac1.Sy",
         "Pvalb.Tac1.A",
         "Pvalb.C1ql1.P",
         "Pvalb.C1ql1.C",
         "Pvalb.C1ql1.N",
```

```
"Cacna2d1.Lhx6.R",
    "Cacna2d1.Lhx6.V",
    "Cacna2d1.Ndnf.N",
    "Cacna2d1.Ndnf.R",
    "Cacna2d1.Ndnf.C",
    "Calb2.Cry",
    "Sst.Cry",
    "Ntng1.S",
    "Ntng1.R",
    "Ntng1.C",
    "Cck.Sema",
    "Cck.Lmo1.N",
    "Cck.Calca",
    "Cck.Lmo1.Vip.F",
    "Cck.Lmo1.Vip.C",
    "Cck.Lmo1.Vip.T",
    "Cck.Ly",
    "Cck.Cxcl14.Calb1.Tn",
    "Cck.Cxcl14.Calb1.I",
    "Cck.Cxcl14.S",
    "Cck.Cxcl14.Calb1.K",
    "Cck.Cxcl14.Calb1.Ta",
    "Cck.Cxcl14.V",
    "Vip.Crh.P",
    "Vip.Crh.C1",
    "Calb2.Vip.G",
    "Calb2.Vip.I",
    "Calb2.Vip.Nos1",
    "Calb2.Cntnap5a.R",
    "Calb2.Cntnap5a.V",
    "Calb2.Cntnap5a.I",
]
reorder = np.zeros(clusterNames.size) * np.nan
for i, c in enumerate(clusterNames):
    for j, k in enumerate(clusterOrder):
        if c[: len(k)] == k:
            reorder[i] = j
            break
clusterColors = clusterColors[reorder.astype(int)]
```

(3663, 17965)

3 Task 1: Data inspection

Before we use t-SNE or any other advanced visualization methods on the data, we first want to have a closer look on the data and plot some statistics. For most of the analysis we will compare

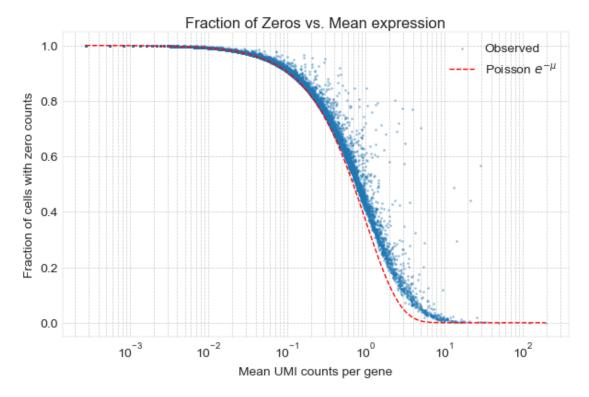
the data to a Poisson distribution.

3.0.1 1.1. Relationship between expression mean and fraction of zeros

Compute actual and predicted gene expression. The higher the average expression of a gene, the smaller fraction of cells will show a 0 count. Plot the data and explain what you see in the plot.

(3 pts)

```
[9]: array([0.91784928, 0.99945415, 0.99972704, ..., 0.4146063, 0.92640843, 0.99809082])
```



Explanation (1 pt)

3.0.2 Gene mean vs zero-fraction plot

For each gene we plot its mean UMI count (μ) against the fraction of cells with zero counts. The red curve shows the Poisson expectation

$$P(X=0) = e^{-\mu}$$
.

- Low expression $(\mu \ll 1)$ points trace the curve, indicating sampling noise alone.
- Higher expression $(\mu \gtrsim 1)$ points lie above $e^{-\mu}$; the excess zeros arise from technical drop-outs and true biological heterogeneity, producing overdispersion / zero-inflation beyond Poisson.

Hence single-cell RNA-seq counts are better modelled with a negative-binomial or zero-inflated

negative-binomial distribution that captures both the mean–variance dependence and the elevated zero rate.

3.0.3 1.2. Mean-variance relationship

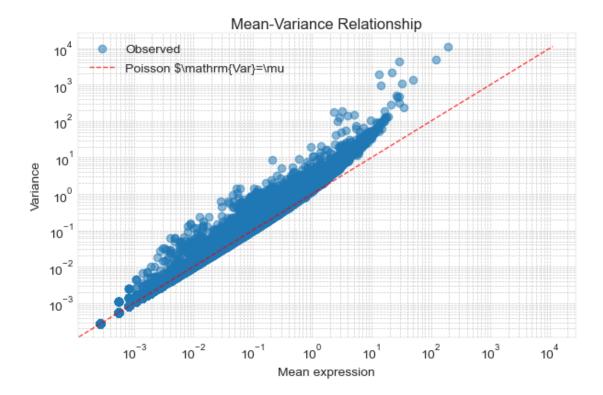
If the expression follows Poisson distribution, then the mean should be equal to the variance. Plot the mean-variance relationship and interpret the plot.

(2.5 pts)

```
[17]: # ------
# Compute the variance of the expression counts of each gene (0.5 pt)
# ------
gene_vars = np.var(counts, axis=0)
gene_vars
```

```
[17]: array([1.49899881e-01, 5.45702429e-04, 2.72925744e-04, ..., 1.24532529e+00, 8.20630026e-02, 1.90734998e-03])
```

```
[23]: # ------
     # Plot the mean-variance relationship on a log-log plot (1 pt)
     # Plot the Poisson prediction as a line
     fig, ax = plt.subplots(figsize=(6, 4))
     plt.plot(gene_means, gene_vars, "o", alpha=0.5, label="Observed")
     lims = [
         np.min([plt.xlim(), plt.ylim()]), # find the min of the axes
         np.max([plt.xlim(), plt.ylim()]), # find the max of the axes
     # Plot the y=x line
     plt.plot(lims, lims, "r--", alpha=0.75, label="Poisson $\mathrm{Var}=\mu")
     plt.xscale("log")
     plt.yscale("log")
     plt.xlabel("Mean expression")
     plt.ylabel("Variance")
     plt.title("Mean-Variance Relationship")
     plt.grid(True, which="both", linestyle="--", linewidth=0.5)
     plt.legend()
     plt.savefig("../images/lab7-mean_vs_variance.png", dpi=300, bbox_inches="tight")
     plt.show()
```



Explanation (1 pt) ...

3.0.4 Gene mean vs variance plot (Task 1.2)

For each gene we plot its mean UMI count (μ) against its variance on a log-log scale. The red line shows the Poisson expectation

$$Var(X) = \mu$$
.

- Low expression $(\mu \ll 1)$ points lie on the line, indicating only sampling noise.
- **Higher expression** $(\mu \gtrsim 1)$ points rise **above** the line; variance exceeds the mean, revealing **overdispersion** from technical noise (drop-outs, amplification bias) and true cell-to-cell heterogeneity.

Thus single-cell UMI counts are better modelled by a (possibly zero-inflated) **negative-binomial** distribution, which allows

$$Var(X) = \mu + \alpha \mu^2, \qquad \alpha > 0,$$

so variance can grow faster than the mean.

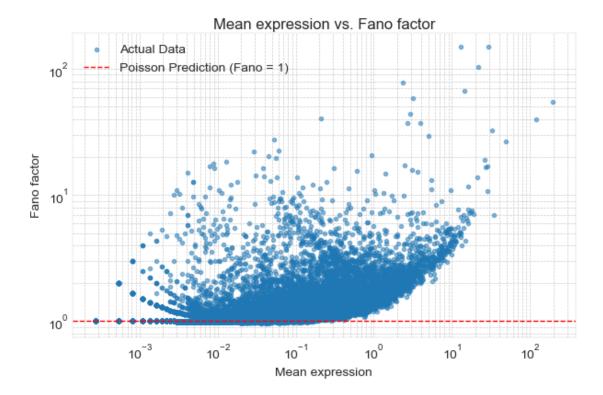
3.0.5 1.3. Relationship between the mean and the Fano factor

Compute the Fano factor for each gene and make a scatter plot of expression mean vs. Fano factor in log-log coordinates, and interpret what you see in the plot. If the expression follows the Poisson distribution, then the Fano factor (variance/mean) should be equal to 1 for all genes.

```
(2.5 pts)
```

[24]: array([1.74867282, 0.999454 , 0.999727 , ..., 1.41445784, 1.07355992, 0.998089])

```
[ ]:  # -----
    # plot fano-factor vs mean (1 pt)
    # incl. fano factor
    # -----
    # Plot a Poisson prediction as line
    # Use the same style of plot as above.
    fig, ax = plt.subplots(figsize=(6, 4))
    plt.scatter(gene means, fano, alpha=0.5, label="Observed")
    plt.xscale("log")
    plt.yscale("log")
    plt.xlabel("Mean expression")
    plt.ylabel("Fano factor")
    plt.title("Mean expression vs. Fano factor")
    plt.grid(True, which="both", linestyle="--", linewidth=0.5)
    plt.axhline(y=1, color="red", linestyle="--", label="Poisson Prediction (Fano =__
     plt.legend()
    plt.savefig("../images/lab7-mean_vs_fano.png", dpi=300, bbox_inches="tight")
    plt.show()
```



Explanation (1 pt) ...

3.0.6 Fano factor vs mean expression (Task 1.3)

For each gene we plot its Fano factor

$$F = \frac{\operatorname{Var}(X)}{\mu}$$

against the mean UMI count μ .

- Poisson baseline The red horizontal line at F = 1 marks the Poisson expectation where variance equals the mean.
- Low expression $(\mu \ll 1)$ Genes cluster near $F \approx 1$, consistent with pure sampling noise.
- **Higher expression** $(\mu \gtrsim 1)$ F rises well above 1, signalling **overdispersion** caused by technical drop-outs, amplification bias, and genuine cell-to-cell heterogeneity.

A (possibly zero-inflated) negative-binomial model accommodates this pattern because it allows

$$Var(X) = \mu + \alpha \mu^2 \implies F = 1 + \alpha \mu > 1 \quad (\alpha > 0),$$

so the Fano factor grows with the mean, matching the empirical trend.

3.0.7 1.4. Histogram of sequencing depths

choose a reasonable number of bins, e.g. 50

ax.set_ylabel("Number of cells")

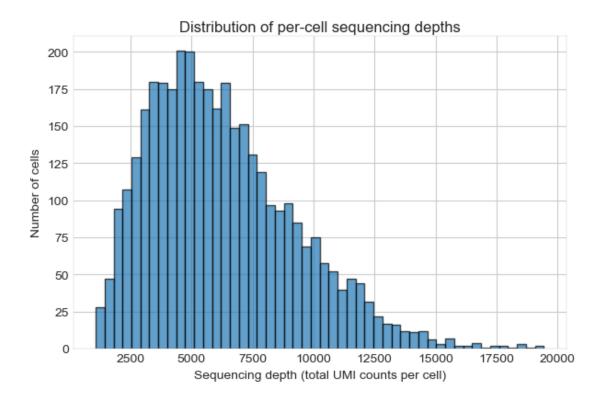
plt.show()

ax.hist(depths, bins=50, edgecolor="black", alpha=0.7)

ax.set_xlabel("Sequencing depth (total UMI counts per cell)")

ax.set title("Distribution of per-cell sequencing depths")

Different cells have different sequencing depths (sum of counts across all genes) because the efficiency can change from droplet to droplet due to some random expreimental factors. Make a histogram of sequencing depths.



3.0.8 1.5. Fano factors after normalization

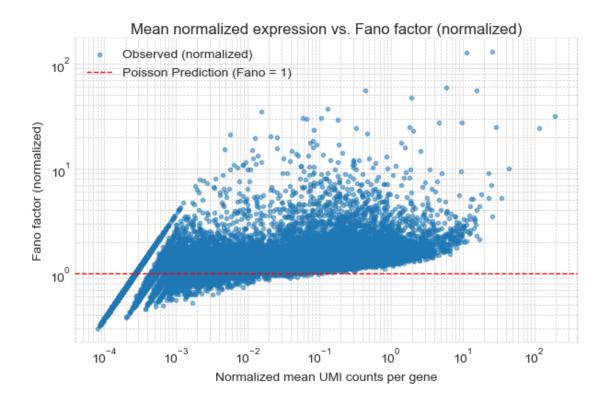
Normalize counts by the sequencing depth of each cell and multiply by the median sequencing depth. Then make the same expression vs Fano factor plot as above. After normalization by sequencing depth, Fano factor should be closer to 1 (i.e. variance even more closely following the mean). This can be used for feature selection.

(2.5 pts)

```
fano_norm
```

[30]: array([1.4743044 , 1.64107479, 1.01368967, ..., 1.81740881, 1.18732466, 0.78672188])

```
[32]: # -----
      # plot normalized counts and find the top 10 genes (1 pt)
     # hint: keep appropriate axis scaling in mind
     fig, ax = plt.subplots(figsize=(6, 4))
     # add plot
     plt.scatter(gene_means_norm, fano_norm, alpha=0.5, label="Observedu
      plt.xscale("log")
     plt.yscale("log")
     plt.xlabel("Normalized mean UMI counts per gene")
     plt.ylabel("Fano factor (normalized)")
     plt.title("Mean normalized expression vs. Fano factor (normalized)")
     plt.grid(True, which="both", linestyle="--", linewidth=0.5)
     plt.axhline(y=1, color="red", linestyle="--", label="Poisson Prediction (Fano =_
      <1)")
     plt.legend()
     plt.savefig("../images/lab7-mean_vs_fano_normalized.png", dpi=300, u
       ⇔bbox_inches="tight")
     plt.show()
```



```
[36]: #
      # Find top-10 genes with the highest normalized Fano factor (0.5 pts)
      # Print them sorted by the Fano factor starting from the highest
      # Gene names are stored in the `genes` array
      top10_idx = np.argsort(fano_norm)[::-1][:10]
      # print gene names and their normalized Fano factors
      print("Top 10 genes with the highest normalized Fano factor:")
      for i in top10_idx:
          print(f"{genes[i]:<15} Fano={fano_norm[i]:.2f}")</pre>
```

Sst Fano=131.14 Fano=128.40 Npy Fano=59.11 Vip Cck Fano=55.65 Cpne2 Fano=55.22

Fano=47.65

Fano=37.25

Top 10 genes with the highest normalized Fano factor:

Ptpn23 Pdzd9 Fano=35.06 Malat1 Fano=31.42

Pcp4

4 Task 2: Low dimensional visualization

In this task we will construct a two dimensional visualization of the data. First we will normalize the data with some variance stabilizing transformation and study the effect that different approaches have on the data. Second, we will reduce the dimensionality of the data to a more feasible number of dimensions (e.g. d = 50) using PCA. And last, we will project the PCA-reduced data to two dimensions using t-SNE.

4.0.1 2.1. PCA with and without transformations

Here we look at the influence of variance-stabilizing transformations on PCA. We will focus on the following transformations: - Square root (sqrt(X)): it is a variance-stabilizing transformation for the Poisson data. - Log-transform (log2(X+1)): it is also often used in the transcriptomic community.

We will only work with the most important genes. For that, transform the counts into normalized counts (as above) and select all genes with normalized Fano factor above 3 and remove the rest. We will look at the effect that both transformations have in the PCA-projected data by visualizing the first two components. Interpret qualitatively what you see in the plot and compare the different embeddings making use of the ground truth clusters.

(3.5 pts)

```
# ------
# Select important genes (0.5 pts)
# -------

depths = counts.sum(axis=1)
counts_norm = counts / depths[:, None] * np.median(depths)

fano_norm = counts_norm.var(axis=0) / counts_norm.mean(axis=0)
hv_mask = fano_norm > 3
counts_sel = counts_norm[:, hv_mask]
genes_sel = genes[hv_mask]
print("Selected", counts_sel.shape[1], "highly variable genes")
```

Selected 707 highly variable genes

```
[50]: # -------
# transform data and apply PCA (1 pt)
# ------
from sklearn.decomposition import PCA

# perform PCA

raw_data = counts_sel
```

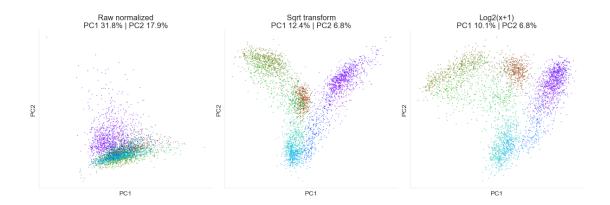
```
sqrt_data = np.sqrt(counts_sel)
log_data = np.log2(counts_sel + 1)

# 2. Fit PCA *and* keep both the model and the scores
pca_raw = PCA(n_components=50)
pc_raw = pca_raw.fit_transform(raw_data)

pca_sqrt = PCA(n_components=50)
pc_sqrt = pca_sqrt.fit_transform(sqrt_data)

pca_log = PCA(n_components=50)
pc_log = pca_log.fit_transform(log_data)
```

```
[51]: # -----
      # plot first 2 PCs for each transformation (1 pt)
      fig, axs = plt.subplots(1, 3, figsize=(12, 4), facecolor="white")
      for ax, pca, pc, title in zip(
          axs,
          [pca_raw, pca_sqrt, pca_log],
          [pc_raw, pc_sqrt, pc_log],
          ["Raw normalized", "Sqrt transform", "Log2(x+1)"],
      ):
          v1, v2 = pca.explained_variance_ratio_[:2]
          ax.scatter(
              pc[:, 0], pc[:, 1], s=6, alpha=0.7, c=clusterColors[clusters],
       ⇔edgecolors="none"
          ax.set_title(f"{title}\nPC1 {v1*100:.1f}% | PC2 {v2*100:.1f}%", fontsize=12)
          ax.set xlabel("PC1")
          ax.set ylabel("PC2")
          ax.set_xticks([])
          ax.set_yticks([])
          for side in ["top", "right"]:
              ax.spines[side].set_visible(False)
      plt.show()
```



Explanation (1 pt) ...

The three panels show PCA on the top Fano > 3 genes after different variance-stabilizing transforms.

- Raw normalized: PC1 = 31.8% and PC2 = 17.9% of total variance. The embedding is stretched along PC1, reflecting a few highly expressed genes dominating the variance. Clusters are partially separable but elongated.
- Sqrt transform: PC1 = 12.4% and PC2 = 6.8%. The square-root stabilizes Poisson noise, producing three "arms" that better resolve major cell-type groups (e.g. Sst, Pvalb, Vip).
- Log (x+1) transform: PC1 = 10.1% and PC2 = 6.8%. This also compresses extreme values and separates clusters, though with slightly less dynamic range than the sqrt.

Overall, variance stabilization ($\sqrt{}$ or log) before PCA yields cleaner, more balanced embeddings by down-weighting outlier genes and highlighting biological heterogeneity.

4.0.2 2.2. tSNE with and without transformations

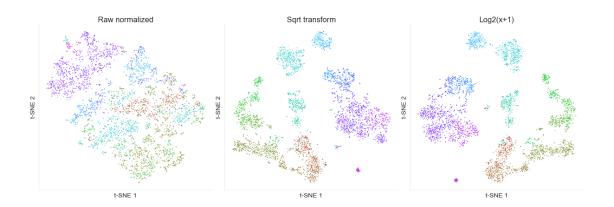
Now, we will reduce the dimensionality of the PCA-reduced data further to two dimensions using t-SNE. We will use only n=50 components of the PCA-projected data. Plot the t-SNE embedding for the three versions of the data and interpret the plots. Do the different transformations have any effect on t-SNE?

(1.5 pts)

```
[]: import numpy as np

# write all six to one file
```

```
np.savez_compressed(
         "embeddings_task2.npz",
          pca50_raw=pc_raw,
          pca50_sqrt=pc_sqrt,
          pca50_log=pc_log,
          tsne_raw=tsne_raw,
          tsne_sqrt=tsne_sqrt,
         tsne_log=tsne_log,
      )
      print("Saved PCA+TSNE embeddings to embeddings task2.npz")
[53]: data = np.load("embeddings_task2.npz")
      pc_raw = data["pca50_raw"]
      pc_sqrt = data["pca50_sqrt"]
      pc_log = data["pca50_log"]
      tsne_raw = data["tsne_raw"]
      tsne_sqrt = data["tsne_sqrt"]
      tsne_log = data["tsne_log"]
[55]: # -----
      # plot t-SNE embedding for each dataset (1 pt)
      fig, axs = plt.subplots(1, 3, figsize=(12, 4), facecolor="white")
      for ax, emb, title in zip(
          axs,
          [tsne_raw, tsne_sqrt, tsne_log],
          ["Raw normalized", "Sqrt transform", "Log2(x+1)"],
      ):
          ax.scatter(
             emb[:, 0],
             emb[:, 1],
             s=6,
             alpha=0.7,
              c=clusterColors[clusters],
             edgecolors="none",
          ax.set_title(title)
          ax.set_xlabel("t-SNE 1")
          ax.set_ylabel("t-SNE 2")
          ax.set_xticks([])
          ax.set_yticks([])
          for side in ["top", "right"]:
              ax.spines[side].set_visible(False)
      plt.show()
```



4.0.3 2.3. Leiden clustering

Now we will play around with some clustering and see whether the clustering methods can produce similar results to the original clusters from the publication. We will apply Leiden clustering (closely related to the Louvain clustering), which is standard in the field and works well even for very large datasets.

Choose one representation of the data (best transformation based in your results from the previous task) to use further in this task and justify your choice. Think about which level of dimensionality would be sensible to use to perform clustering. Visualize in the two-dimensional embedding the resulting clusters and compare to the original clusters.

(1.5 pts)

```
[56]: # To run this code you need to install leidenalg and igraph
# conda install -c conda-forge python-igraph leidenalg

import igraph as ig
from sklearn.neighbors import NearestNeighbors, kneighbors_graph
import leidenalg as la
```

```
[57]: # Define some contrast colors

clusterCols = [
    "#FFFF00",
    "#1CE6FF",
    "#FF34FF",
    "#FF4A46",
    "#008941",
    "#006FA6",
    "#A30059",
    "#FFDBE5",
    "#7A4900",
    "#0000A6",
    "#63FFAC",
```

```
"#B79762",
"#004D43",
"#8FB0FF",
"#997D87",
"#5A0007",
"#809693",
"#FEFFE6",
"#1B4400",
"#4FC601",
"#3B5DFF",
"#4A3B53",
"#FF2F80",
"#61615A",
"#BA0900",
"#6B7900",
"#00C2A0",
"#FFAA92",
"#FF90C9",
"#B903AA",
"#D16100",
"#DDEFFF",
"#000035",
"#7B4F4B",
"#A1C299",
"#300018",
"#0AA6D8".
"#013349",
"#00846F",
"#372101",
"#FFB500".
"#C2FFED",
"#A079BF",
"#CC0744",
"#COB9B2",
"#C2FF99",
"#001E09",
"#00489C",
"#6F0062",
"#0CBD66",
"#EEC3FF",
"#456D75",
"#B77B68",
"#7A87A1",
"#788D66",
"#885578",
"#FAD09F",
"#FF8A9A",
```

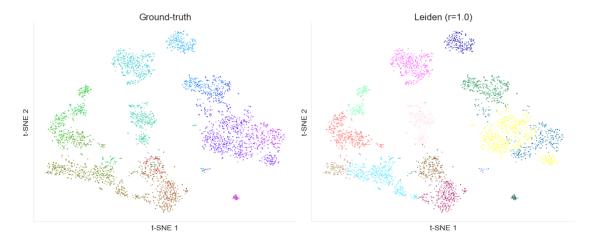
```
"#D157A0",
"#BEC459",
"#456648",
"#0086ED",
"#886F4C",
"#34362D",
"#B4A8BD",
"#00A6AA",
"#452C2C",
"#636375",
"#A3C8C9",
"#FF913F",
"#938A81",
"#575329",
"#00FECF",
"#B05B6F",
"#8CDOFF",
"#3B9700",
"#04F757",
"#C8A1A1",
"#1E6E00",
"#7900D7",
"#A77500",
"#6367A9",
"#A05837",
"#6B002C".
"#772600",
"#D790FF",
"#9B9700",
"#549E79",
"#FFF69F",
"#201625",
"#72418F",
"#BC23FF",
"#99ADCO",
"#3A2465",
"#922329",
"#5B4534",
"#FDE8DC",
"#404E55",
"#0089A3",
"#CB7E98",
"#A4E804",
"#324E72",
"#6A3A4C",
"#83AB58",
"#001C1E",
```

```
"#D1F7CE",
    "#004B28",
    "#C8D0F6",
    "#A3A489",
    "#806C66",
    "#222800",
    "#BF5650",
    "#E83000",
    "#66796D",
    "#DA007C",
    "#FF1A59".
    "#8ADBB4",
    "#1E0200",
    "#5B4E51",
    "#C895C5",
    "#320033",
    "#FF6832",
    "#66E1D3",
    "#CFCDAC",
    "#DOAC94",
    "#7ED379",
    "#012C58",
]
clusterCols = np.array(clusterCols)
```

Leiden r=1.0 \rightarrow 14 clusters

```
[62]: # --
      # Plot the results (1 pt)
      from sklearn.metrics import adjusted_rand_score
      fig, axs = plt.subplots(1, 2, figsize=(10, 4), facecolor="white")
      for ax, labs, title in zip(
          axs, [clusters, labels1], ["Ground-truth", "Leiden (r=1.0)"]
      ):
          ax.scatter(
              tsne_sqrt[:, 0],
              tsne_sqrt[:, 1],
              s=6,
              alpha=0.7,
              c=(clusterColors if title == "Ground-truth" else clusterCols)[labs],
              edgecolors="none",
          ax.set_title(title)
          ax.set_xlabel("t-SNE 1")
          ax.set_ylabel("t-SNE 2")
          ax.set_xticks([])
          ax.set_yticks([])
          for side in ["top", "right"]:
              ax.spines[side].set_visible(False)
      ari = adjusted_rand_score(clusters, labels1)
      print(f"ARI = {ari:.2f}")
```

ARI = 0.40



4.0.4 2.4. Change the clustering resolution

The number of clusters can be changed by modifying the resolution parameter. How many clusters did we get with the default value? Change the resolution parameter to yield 2x more and 2x fewer clusters Plot all three results as t-SNE overlays (same as above).

(1.5 pts)

{1.0: 13, 0.5: 10, 2.0: 21}

```
[66]: # -----
      # Plot the results (1 pt)
     fig, axs = plt.subplots(1, 3, figsize=(12, 4), facecolor="white")
     # add plot
     for ax, r, labs, cnt in zip(axs, res, labels, counts):
         ax.scatter(
             tsne_sqrt[:, 0],
             tsne_sqrt[:, 1],
             s=6,
             alpha=0.7,
             c=clusterCols[labs],
             edgecolors="none",
         )
         ax.set_title(f"res={r}
         ax.set_xlabel("t-SNE 1")
         ax.set_ylabel("t-SNE 2")
         ax.set_xticks([])
         ax.set_yticks([])
         for side in ["top", "right"]:
             ax.spines[side].set_visible(False)
     plt.show()
```

