DSP & Digital Filters

Mike Brookes

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- Signals
- Processing
- Syllabus
- Sequences
- Time Scaling
- z-Transform
- Region of Convergence
- z-Transform examples
- Rational z-Transforms
- Rational example
- Inverse z-Transform
- MATLAB routines
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- 18 lectures: feel free to ask questions

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  - quite dense - ensure you understand each line
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- Problems: Mitra textbook contains many problems at the end of each chapter and also MATLAB exercises

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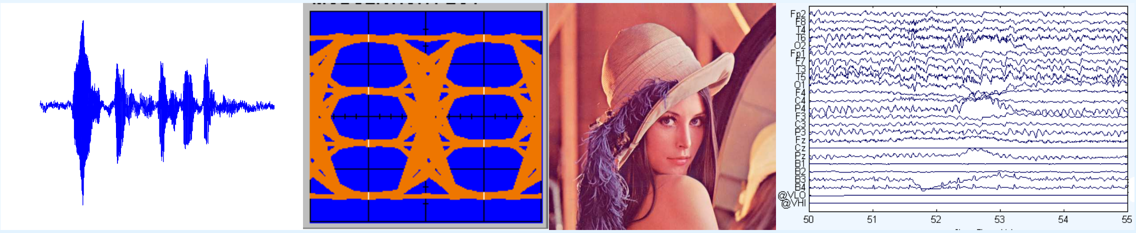
- A signal is a numerical quantity that is a function of one or more independent variables such as time or position.

Examples:



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Signals

- A signal is a numerical quantity that is a function of one or more independent variables such as time or position.
- Real-world signals are analog and vary continuously and take continuous values.

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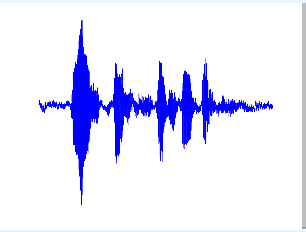
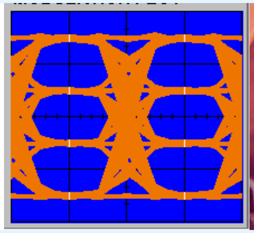

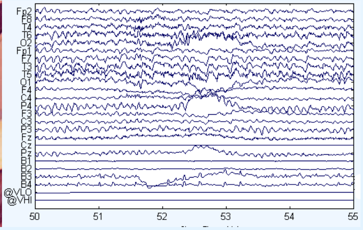

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- A signal is a numerical quantity that is a function of one or more independent variables such as time or position.
- Real-world signals are analog and vary continuously and take continuous values.
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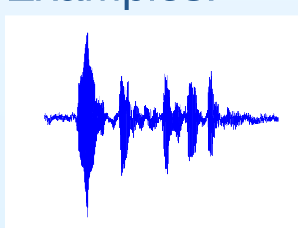
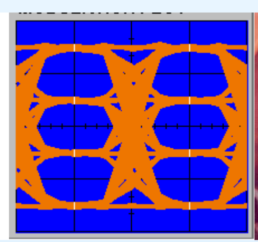
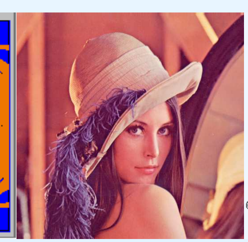
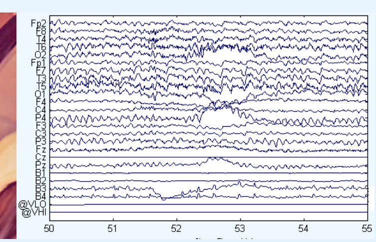

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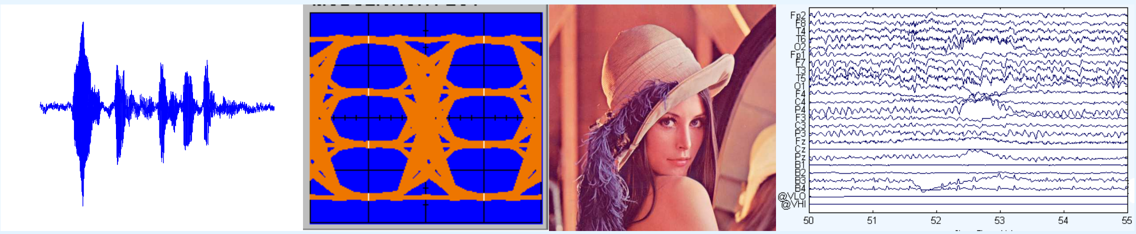

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- Digital signals are sampled at discrete times and are quantized to a finite number of discrete values
- We will mostly consider one-dimensionsal real-valued signals with regular sample instants; except in a few places, we will ignore the quantization.
    - Extension to multiple dimensions and complex-valued signals is mostly straighforward.

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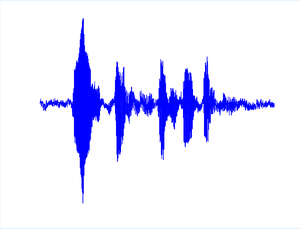
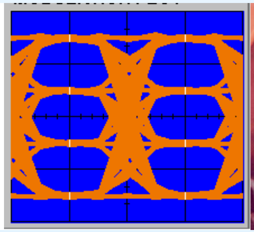

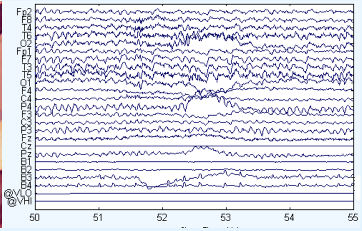

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Processing

- Aims to “improve” a signal in some way or extract some information from it
- Examples:
  - Modulation/demodulation
  - Coding and decoding
  - Interference rejection and noise suppression
  - Signal detection, feature extraction
- We are concerned with linear, time-invariant processing

Syllabus

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Main topics:

- Introduction/Revision
- Transforms
- Discrete Time Systems
- Filter Design
  - FIR Filter Design
  - IIR Filter Design
- Multirate systems
  - Multirate Fundamentals
  - Multirate Filters
  - Subband processing

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- Finite Energy: $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$ (e.g. $x[n] = n^{-1}u[n-1]$)
- Absolutely Summable: $\sum_{n=-\infty}^{\infty} |x[n]| < \infty \Rightarrow$ Finite energy

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For sampled signals, the n^{th} sample is at time $t = nT = \frac{n}{f_s}$ where $f_s = \frac{1}{T}$ is the sample frequency.

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For sampled signals, the n^{th} sample is at time $t = nT = \frac{n}{f_s}$ where $f_s = \frac{1}{T}$ is the sample frequency.

Often easiest to scale time so that $f_s = 1\,\mathrm{Hz}$. E.g. to design a $1\,\mathrm{kHz}$ low-pass filter for $f_s = 44.1\,\mathrm{kHz}$ we can design a $0.0227\,\mathrm{Hz}$ filter for $f_s = 1\,\mathrm{Hz}$.

Time Scaling

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To scale back to real-world values: *Every quantity* of dimension $(\text{Time})^n$ is multiplied by T^n (or equivalently by f_s^{-n}). Thus all *times* are multiplied by T and all *frequencies* and *angular frequencies* by T^{-1}.

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We use Ω for "real" angular frequencies and ω for normalized angular frequency. The units of ω are "radians per sample".

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Warning: Several MATLAB routines scale time so that $f_s = 2$ Hz. Weird, non-standard and irritating.

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The z-transform converts a sequence, $\{x[n]\}$, into a function, $X(z)$, of an arbitrary complex-valued variable z.

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- Useful operations on sequences correspond to simple operations on the z-transform:
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- Definition: $X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$

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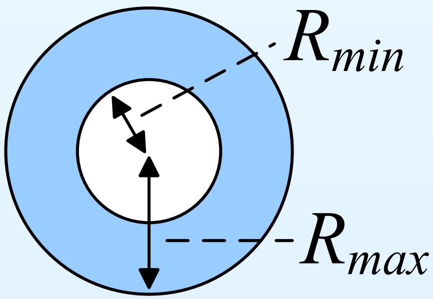
Complex analysis $\Rightarrow$: the ROC of a power series (if it exists at all) is always an annular region of the form $0 \leq R_{min} < |z| < R_{max} \leq \infty$.



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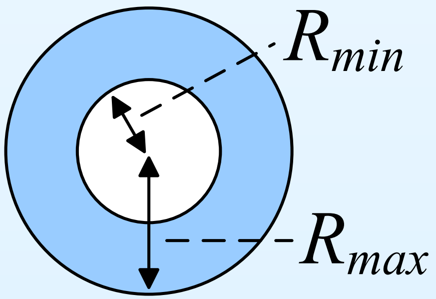

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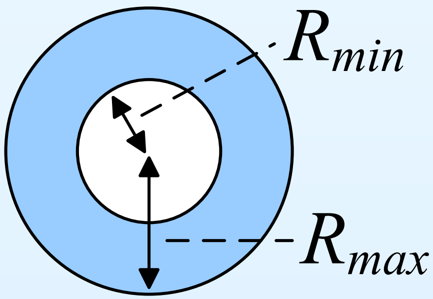

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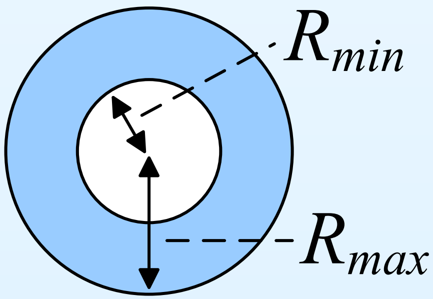

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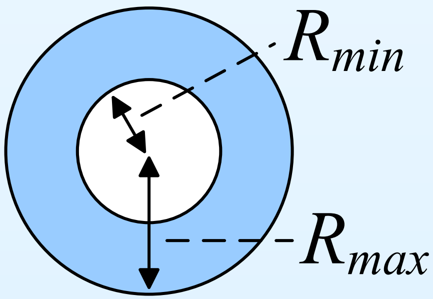

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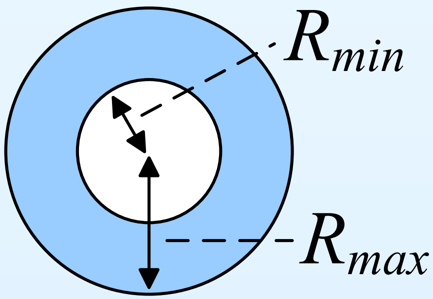

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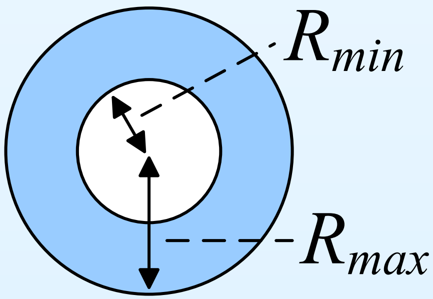

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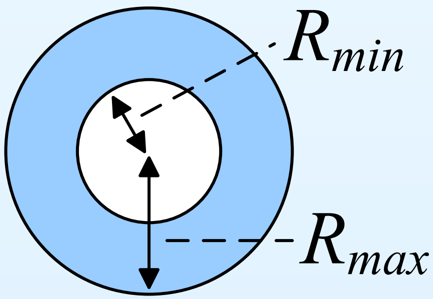

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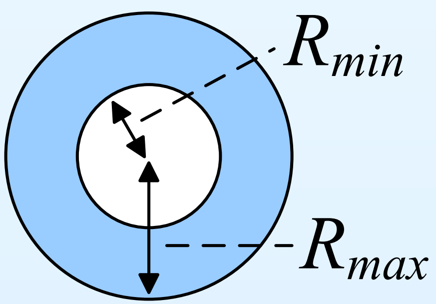

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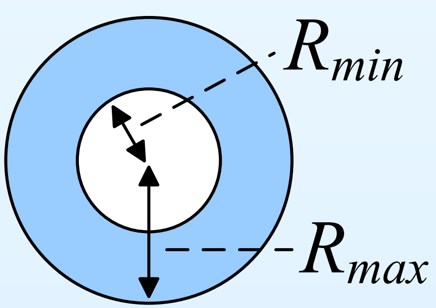
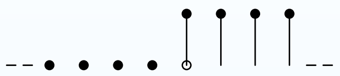


z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

$u[n]$



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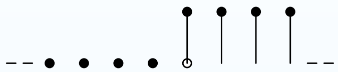
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \leq \infty$

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

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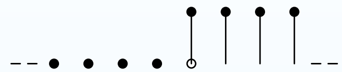
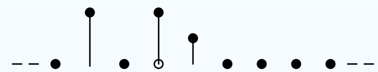
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$			

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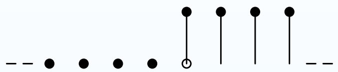
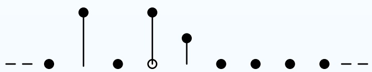
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$

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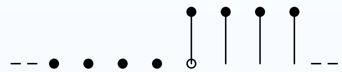
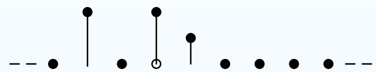
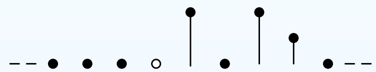
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$			

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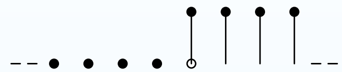
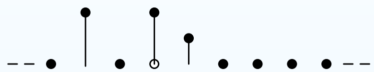
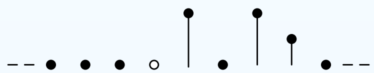
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \|z\| \leq \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \|z\| < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \|z\| \leq \infty$

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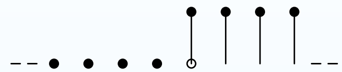
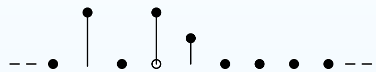
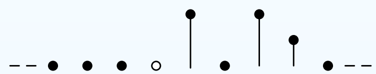
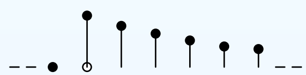
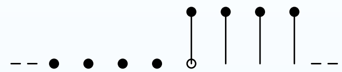
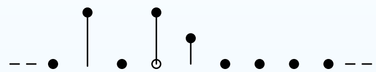
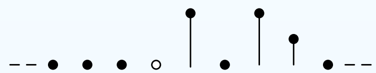
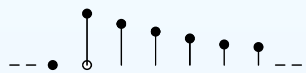
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$			

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \le \infty$

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

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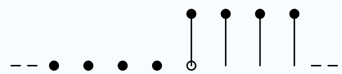
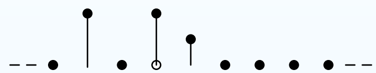
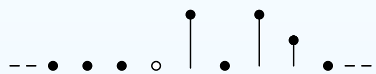
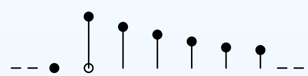
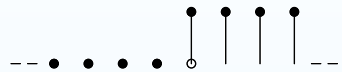
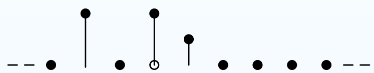
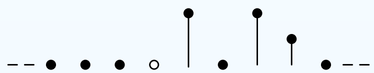
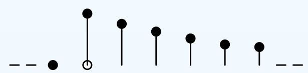
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \le \infty$
$-\alpha^n u[-n-1]$			

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

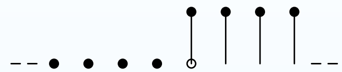
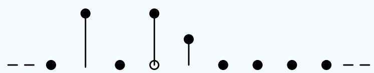
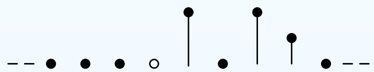
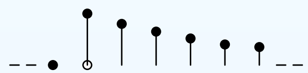
The sample at $n = 0$ is indicated by an open circle.

$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \le \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \le \lvert z \rvert < \alpha$

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

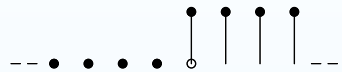
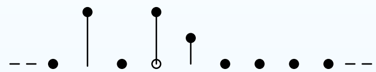
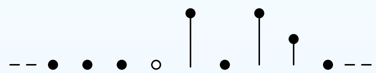
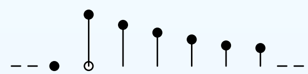
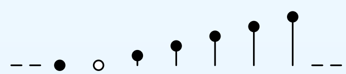
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \leq \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \leq \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \leq \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \leq \lvert z \rvert < \alpha$

Note: Examples 4 and 5 have the same z-transform but different ROCs.

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \leq \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \leq \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \leq \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \leq \lvert z \rvert < \alpha$
$nu[n]$			

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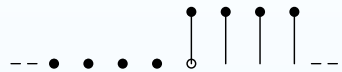
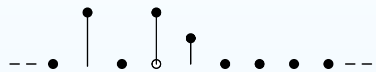
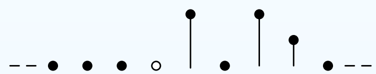
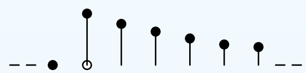
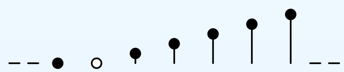
z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \le \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \le \lvert z \rvert < \alpha$
$nu[n]$		$\frac{z^{-1}}{1-2z^{-1}+z^{-2}}$	$1 < \lvert z \rvert \le \infty$

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z-Transform examples

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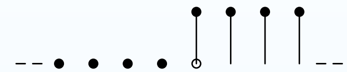
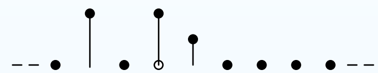
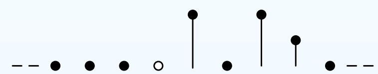
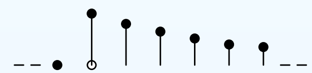
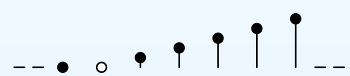
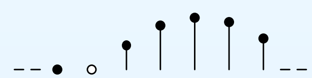
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \|z\| \leq \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \|z\| < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \|z\| \leq \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \|z\| \leq \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \leq \|z\| < \alpha$
$nu[n]$		$\frac{z^{-1}}{1-2z^{-1}+z^{-2}}$	$1 < \|z\| \leq \infty$
$\sin(\omega n)u[n]_{\omega=0.5}$			

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Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

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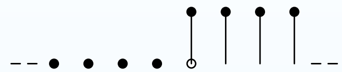
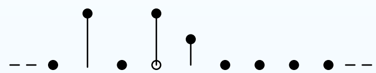
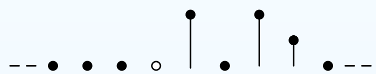
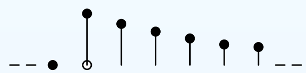
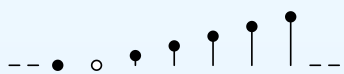
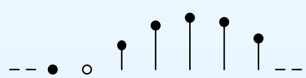
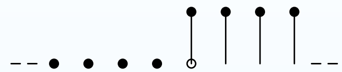
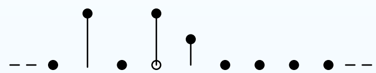
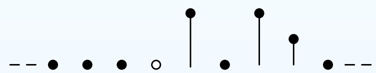
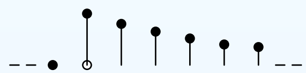
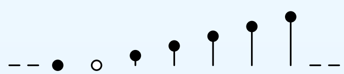
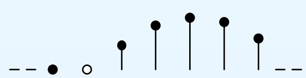
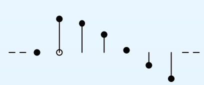
$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \|z\| \le \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \|z\| < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \|z\| \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \|z\| \le \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \le \|z\| < \alpha$
$nu[n]$		$\frac{z^{-1}}{1-2z^{-1}+z^{-2}}$	$1 < \|z\| \le \infty$
$\sin(\omega n)u[n]_{\omega=0.5}$		$\frac{z^{-1}\sin(\omega)}{1-2z^{-1}\cos(\omega)+z^{-2}}$	$1 < \|z\| \le \infty$

Note: Examples 4 and 5 have the same z-transform but different ROCs.

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

z-Transform examples

The sample at $n = 0$ is indicated by an open circle.

$u[n]$		$\frac{1}{1-z^{-1}}$	$1 < \lvert z \rvert \leq \infty$
$x[n]$		$2z^2 + 2 + z^{-1}$	$0 < \lvert z \rvert < \infty$
$x[n-3]$		$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z \rvert \leq \infty$
$\alpha^n u[n]_{\alpha=0.8}$		$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z \rvert \leq \infty$
$-\alpha^n u[-n-1]$		$\frac{1}{1-\alpha z^{-1}}$	$0 \leq \lvert z \rvert < \alpha$
$nu[n]$		$\frac{z^{-1}}{1-2z^{-1}+z^{-2}}$	$1 < \lvert z \rvert \leq \infty$
$\sin(\omega n)u[n]_{\omega=0.5}$		$\frac{z^{-1}\sin(\omega)}{1-2z^{-1}\cos(\omega)+z^{-2}}$	$1 < \lvert z \rvert \leq \infty$
$\cos(\omega n)u[n]_{\omega=0.5}$			

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z-Transform examples

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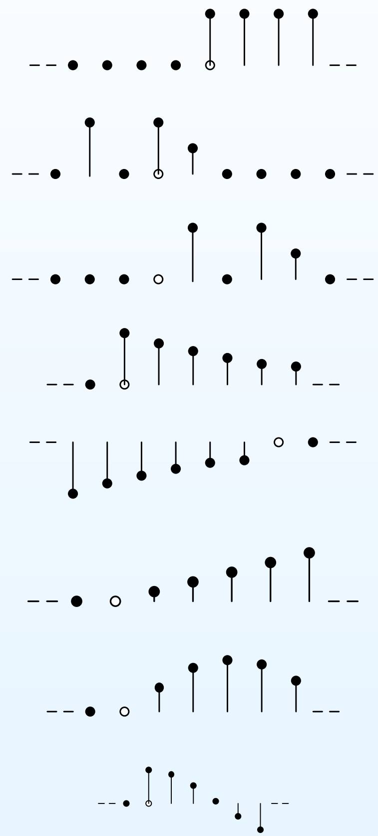

$u[n]$	$\frac{1}{1-z^{-1}}$	$1 < \lvert z\rvert \le \infty$
$x[n]$	$2z^2 + 2 + z^{-1}$	$0 < \lvert z\rvert < \infty$
$x[n-3]$	$z^{-3}\left(2z^2 + 2 + z^{-1}\right)$	$0 < \lvert z\rvert \le \infty$
$\alpha^n u[n]_{\alpha=0.8}$	$\frac{1}{1-\alpha z^{-1}}$	$\alpha < \lvert z\rvert \le \infty$
$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$0 \le \lvert z\rvert < \alpha$
$nu[n]$	$\frac{z^{-1}}{1-2z^{-1}+z^{-2}}$	$1 < \lvert z\rvert \le \infty$
$\sin(\omega n)u[n]_{\omega=0.5}$	$\frac{z^{-1}\sin(\omega)}{1-2z^{-1}\cos(\omega)+z^{-2}}$	$1 < \lvert z\rvert \le \infty$
$\cos(\omega n)u[n]_{\omega=0.5}$	$\frac{1-z^{-1}\cos(\omega)}{1-2z^{-1}\cos(\omega)+z^{-2}}$	$1 < \lvert z\rvert \le \infty$

Note: Examples 4 and 5 have the same z-transform but different ROCs.

Geometric Progression: $\sum_{n=q}^{r} \alpha^n z^{-n} = \frac{\alpha^q z^{-q} - \alpha^{r+1} z^{-r-1}}{1-\alpha z^{-1}}$

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Most z-transforms that we will meet are rational polynomials with real coefficients, usually one polynomial in z^{-1} divided by another.

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$$G(z) = g \frac{\prod_{m=1}^{M} \left(1 - z_m z^{-1}\right)}{\prod_{k=1}^{K} \left(1 - p_k z^{-1}\right)}$$

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Completely defined by the poles, zeros and gain.

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$$G(z) = g\frac{\prod_{m=1}^{M}\left(1-z_m z^{-1}\right)}{\prod_{k=1}^{K}\left(1-p_k z^{-1}\right)} = gz^{K-M}\frac{\prod_{m=1}^{M}\left(z-z_m\right)}{\prod_{k-1}^{K}\left(z-p_k\right)}$$

Completely defined by the poles, zeros and gain.

The absolute values of the poles define the ROCs:

$\exists R+1$ different ROCs

where R is the number of distinct pole magnitudes.

Note: There are $K-M$ zeros or $M-K$ poles at $z=0$ (easy to overlook)

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$$G(z) = \frac{8-2z^{-1}}{4-4z^{-1}-3z^{-2}}$$

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Poles/Zeros: $G(z) = \frac{2z(z-0.25))}{(z+0.5)(z-1.5)}$

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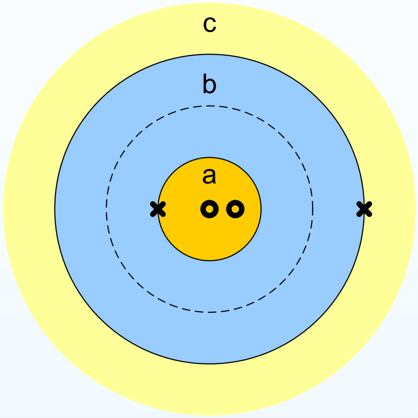

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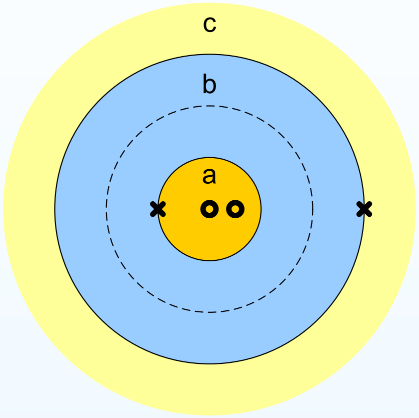

Partial Fractions: $G(z) = \frac{0.75}{1+0.5z^{-1}} + \frac{1.25}{1-1.5z^{-1}}$

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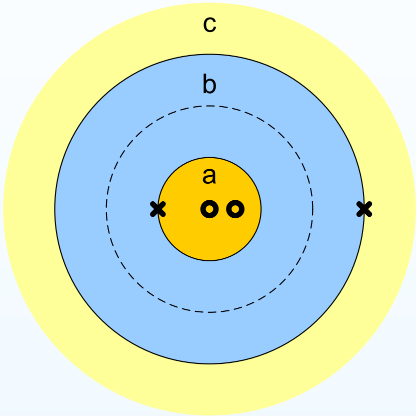

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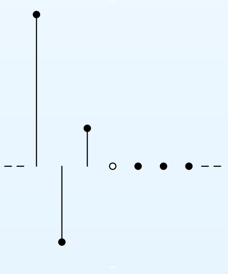
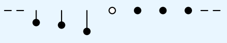
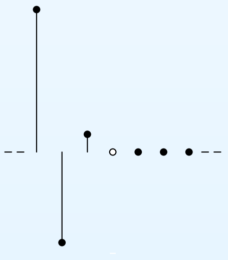
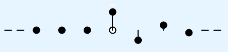
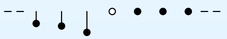
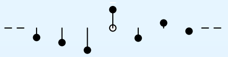
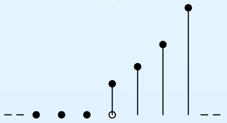
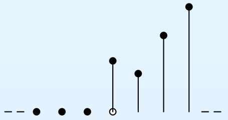
ROC	ROC	$\frac{0.75}{1+0.5z^{-1}}$	$\frac{1.25}{1-1.5z^{-1}}$	$G(z)$
a	$0 \le \lvert z \rvert < 0.5$			
b	$0.5 < \lvert z \rvert < 1.5$			
c	$1.5 < \lvert z \rvert \le \infty$			

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Inverse z-Transform

$g[n] = \frac{1}{2\pi j} \oint G(z) z^{n-1} dz$ where the integral is anti-clockwise around a circle within the ROC, $z = Re^{j\theta}$.

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$$\stackrel{\text{(i)}}{=} \sum_{m=-\infty}^{\infty} g[m] \frac{1}{2\pi j} \oint z^{n-m-1} dz$$

(i) depends on the circle with radius R lying within the ROC

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(i) depends on the circle with radius R lying within the ROC

(ii) Cauchy's theorem: $\frac{1}{2\pi j} \oint z^{k-1} dz = \delta[k]$ for $z = Re^{j\theta}$ anti-clockwise.

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$$\frac{dz}{d\theta} = jRe^{j\theta} \Rightarrow \frac{1}{2\pi j}\oint z^{k-1} dz = \frac{1}{2\pi j}\int_{\theta=0}^{2\pi} R^{k-1} e^{j(k-1)\theta} \times jRe^{j\theta} d\theta$$

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(i) depends on the circle with radius R lying within the ROC

(ii) Cauchy's theorem: $\frac{1}{2\pi j}\oint z^{k-1}dz = \delta[k]$ for $z = Re^{j\theta}$ anti-clockwise.

$$\frac{dz}{d\theta} = jRe^{j\theta} \Rightarrow \frac{1}{2\pi j}\oint z^{k-1}dz = \frac{1}{2\pi j}\int_{\theta=0}^{2\pi} R^{k-1}e^{j(k-1)\theta}\times jRe^{j\theta}d\theta$$

$$= \frac{R^k}{2\pi}\int_{\theta=0}^{2\pi} e^{jk\theta}d\theta$$

$$= R^k\delta(k) = \delta(k) \qquad [R^0 = 1]$$

In practice use a combination of partial fractions and table of z-transforms.

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MATLAB routines

tf2zp,zp2tf	$\frac{b(z^{-1})}{a(z^{-1})} \leftrightarrow \{z_m, p_k, g\}$
residuez	$\frac{b(z^{-1})}{a(z^{-1})} \rightarrow \sum_k \frac{r_k}{1-p_k z^{-1}}$
tf2sos,sos2tf	$\frac{b(z^{-1})}{a(z^{-1})} \leftrightarrow \prod_l \frac{b_{0,l}+b_{1,l}z^{-1}+b_{2,l}z^{-2}}{1+a_{1,l}z^{-1}+a_{2,l}z^{-2}}$
zp2sos,sos2zp	$\{z_m, p_k, g\} \leftrightarrow \prod_l \frac{b_{0,l}+b_{1,l}z^{-1}+b_{2,l}z^{-2}}{1+a_{\in 1,l}z^{-1}+a_{2,l}z^{-2}}$
zp2ss,ss2zp	$\{z_m, p_k, g\} \leftrightarrow \begin{cases} x' = Ax + Bu \\ y = Cx + Du \end{cases}$
tf2ss,ss2tf	$\frac{b(z^{-1})}{a(z^{-1})} \leftrightarrow \begin{cases} x' = Ax + Bu \\ y = Cx + Du \end{cases}$

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- Time scaling: assume $f_s = 1$ Hz so $-\pi < \omega \le \pi$

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  - Causal: $\infty \in$ ROC

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For further details see Mitra:1 & 6.