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Learning Control for Robot Tasks under Geometric Endpoint Constraints

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Abstract

A theory of practices-based "learning control" is developed for a class of robotic tasks under geometric endpoint constraints. A simple algorithm for updating the control input is proposed, which makes the next input be composed of the previous input plus modified terms of previous velocity and force errors at the robot endpoint constrained on a surface. Simulations are presented to demonstrate the convergence of both position and force tracking to a desired path and force specified on the surface. It is then shown that the robot dynamics satisfies the passivity condition regarding the joint torque input vector versus the joint velocity vector even in the case of geometric constraints. With the aid of a slightly relaxed condition of passivity it is shown theoretically that, under the use of a more restricted form of input updating law, the positional trajectories and force signals approach the given desired ones respectively with repetition of practices.

1 INTRODUCTION

"Learning control" is a new approach for the control problem of skill refinement through practices. We humans are able to acquire the skill via repeated practices. Motivated by this observation, much literature concerned with learning control techniques for robotic systems has accumulated very recently[1-8], mainly during the past several years. However, most papers were so far concerned with only tasks described in terms of joint-trajectory tracking. In other words, for a desired motion given to a robot arm, whose endpoint is free to move, the arm repeats exercises to reduce the tracking errors gradually and eventually can learn to exactly trace the desired motion.

However, there is a variety of tasks that must be described in terms of end-point constraint. Writing with a pen is such an example, since the tip of the pen must move in touch with a paper fixed on a table. In such a case, not only a desired end-point trajectory but also a desired time-evolution of contact force acting on the surface along the end-point path must be specified. This is called traditionally a hybrid (position and force) control problem in robotics.

This paper proposes a simple learning control algorithm that is effective for a class of tasks under geometrical end-point constraints. This algorithm is slightly different from those proposed by Aicardi et al[9] and the authors[10]. It updates the command input by adding a correction term consisting of both velocity and force errors to the previous input. Simulations based on the proposed algorithm and the use of a force sensor is carried out, which demonstrates the convergence of both position and force tracking errors with repeating practices. From a theoretical viewpoint, however, it seems difficult to prove the convergence under such a general class of proposed algorithms. To the best of our present ability, it is possible to prove theoretically the convergence under a more restricted class of learning algorithm in which the knowledge on the inertia matrix is used. In the proof a relaxed concept of passivity of error dynamics of robot arms plays a crucial role, differently from the approach by Casalino et al[9]. Hence, we first present some basic results on passivity of robot dynamics under geometric constraints in subsequent three sections. Simulation results will be presented in section 5 and a theoretical proof of the convergence of position and force errors will be given in section 6.

2 PASSIVITY OF ROBOT DYNAMICS

A class of serial-rink manipulators with all revolute-type joints is considered. First we discuss the dynamics of such a manipulator with free end-point, which can be described in terms of joint coordinates vector $q = (q^1, \cdots, q^n)^T$ in the following way:

$$L(q) \triangleq (H_0+H(q))\ddot{q}+(B_0+\frac{1}{2}\dot{H}(q)+S(q,\dot{q}))\dot{q}+g(q) = u \quad (1)$$

where H denotes an inertia matrix, H_0 a positive diagonal matrix representing inertial terms of internal load distribution of actuators, $g(q)$ a vector of gravity terms, u a vector of input torques generated at servo actuators, B_0 a positive definite matrix representing damping factors. It is well known that the inertia matrix $H(q)$ is symmetric and positive definite and, moreover, each entry H_{ij} of H is constant or a trigonometric function of components of joint vector q. Hence, $H(q)$ and any of partial derivatives of $H(q)$ with respect to q^i are uniformly Lipschitz continuous

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in q. The term $S(q,\dot{q})\dot{q}$ expresses

$$S(q,\dot{q})\dot{q} = \frac{1}{2}\dot{H}(q)\dot{q} - \frac{\partial}{\partial q}\left\{\frac{1}{2}\dot{q}^T H(q)\dot{q}\right\}. \qquad (2)$$

and hence the ij-entry of S can be represented by

$$S_{ij} = \frac{\partial}{\partial q_j}\left(\sum_{k=1}^{n}\dot{q}_k H_{ik}\right) - \frac{\partial}{\partial q_i}\left(\sum_{k=1}^{n}\dot{q}_k H_{jk}\right), \qquad (3)$$

which implies the skew-symmetry of $S(q,\dot{q})$, i.e., $r^T S(q,\dot{q})r = 0$, in general. The passivity of robot dynamics follows directly from this property.

Property 1 Motion equation (1) implies the passivity of velocity output $\dot{q}$ with respect to torque input u, i.e.,

$$\int_0^t \dot{q}^T(\tau)u(\tau)d\tau \geq \gamma \qquad (4)$$

for any $t \geq 0$ and a fixed constant γ depending only on the initial state $x(0)(=(q(0),\dot{q}(0)))$.

In fact, we see that

$$\int_0^t \dot{q}^T(\tau)u(\tau)d\tau = \int_0^t \dot{q}^T(\tau)L(q(\tau))d\tau$$
$$= \int_0^t \dot{q}^T B_0\dot{q}d\tau + V(t) - V(0), \qquad (5)$$

where $V(t)$ is defined as

$$V(t) = \frac{1}{2}\dot{q}^T(t)\{H_0 + H(q(t))\}\dot{q}(t) + G(q(t)) \qquad (6)$$

and $G(q)$ denotes the potential function induced by the gravity force, i.e., $g(q) = (\partial G/\partial q^1,\cdots,\partial G/\partial q^n)^T$. Since the constant term of potential is arbitrary, it is reasonable to assume that $\min_q G(q) = 0$. Then, $V(t) \geq 0$ and therefore eq.(5) implies eq.(4).

Now we consider the robot dynamics in the case that the end-effector is in touch with a surface as shown in Fig. 1. Suppose that the surface is described by a scalar equation, $\varphi(x^1,x^2,x^3) = 0$, where $x = (x^1,x^2,x^3)^T$ denotes the cartesian coordinates fixed at the inertial reference frame, and the contact friction arises in the direction of $-\dot{x}$ with the magnitude $\xi(\|\dot{x}\|)$ of frictional coefficient where $\xi(\alpha)$ is a positive function of α. Then the dynamics is expressed by the form

$$L(q) = J_\varphi^T(q)f - \xi(\|\dot{x}\|)J_x^T(q)\dot{x} + v, \qquad (7)$$

where f is the magnitude of the contact force as shown in Fig. 1 and $J_\varphi(q)$ and $J_x(q)$ denote the $1 \times n$ unit normal to the $\varphi(x(q)) = 0$ and the $3 \times n$ Jacobian matrix of x, respectively, with respect to joint vector q, that is,

$$J_x(q) = (\partial x^i(q)/\partial q^j), \qquad (8)$$

$$J_\varphi(q) = \frac{\partial\varphi}{\partial x}J_x \Big/ \left\|\frac{\partial\varphi}{\partial x}J_x\right\|. \qquad (9)$$

Property 2 As long as the end-point of the manipulator is constrained on the surface $\varphi(x) = 0$, the passivity of robot dynamics described by eq.(7) is satisfied, i.e.,

$$\int_0^t \dot{q}^T(\tau)v(\tau)d\tau \geq -V(0) = \gamma. \qquad (10)$$

To see this, first note that the geometrical constraint implies $J_\varphi(q)\dot{q} = 0$, which in fact follows directly from $d\varphi/dt = 0$. Then, by taking the inner product of $\dot{q}(\tau)$ with eq.(7), it is possible to obtain the same conclusion as in Property 1.

3 P-TYPE LEARNING CONTROL

For a desired motion trajectory which is given in terms of joint velocity $\dot{q}_d(t)$ over $t \in [0,T]$, the learning control law is described in the following recursive form:

$$u_{k+1}(t) = u_k(t) - \Phi\dot{r}_k(t) \qquad (11)$$

where r_k denotes the residual error defined by

$$r_k(t) = q_k(t) - q_d(t) \qquad (12)$$

and Φ denotes a positive definite constant gain matrix. The recursive form of eq.(11) is called the P-type algorithm since a Proportional term of the velocity error is used in modification of the input torque. Differently from a D-type algorithm[1-2] in which the derivative of the velocity error is used, a certain extended concept of passivity concerning the residual error dynamics played a crucial role in the proof[5-8] of uniform boundedness and convergence of the motion trajectories during repetitive learning. To gain an insight into this, note that subtraction of the ideal input $u_d(t)$ realizing the desired trajectory $q_d(t)$ from both sides of eq.(11) yields

$$\Delta u_{k+1}(t) = \Delta u_k(t) - \Phi\dot{r}_k(t) \qquad (13)$$

where

$$\Delta u_i = u_i - u_d. \qquad (14)$$

Then, it follows from eq.(13) that

$$\Delta u_{k+1}^T\Phi^{-1}\Delta u_{k+1} = \Delta u_k^T\Phi\Delta u_k - 2\dot{r}_k^T\Delta u_k + \dot{r}_k^T\Phi\dot{r}_k. \qquad (15)$$

Hence, if there are positive constants $\lambda > 0$ (not so large) and $\beta > 0$ such that

$$\int_0^t e^{-\lambda\tau}\dot{r}_k^T(\tau)\Delta u_k(\tau)d\tau$$
$$\geq \frac{1+\beta}{2}\int_0^t e^{-\lambda\tau}\dot{r}_k^T(\tau)\Phi\dot{r}_k(\tau)d\tau, \qquad (16)$$

then it follows from eq.(15) that

$$\|\Delta u_{k+1}\|_\lambda^2 \leq \|\Delta u_k\|_\lambda^2 - \beta\|\Phi\dot{r}_k\|_\lambda^2 \qquad (17)$$

where

$$\|\Delta u_k\|_\lambda^2 = \int_0^T e^{-\lambda t}\Delta u_k^T(t)\Phi^{-1}\Delta u_k(t)dt.$$

This means that the squared the input error norm decreases with repetition of exercises as long as the squared integral of velocity error does not vanish.

As in the previous literature[5-8], the inequality

$$\int_0^t e^{-\lambda\tau} r^T(\tau)\Delta u(\tau)d\tau \geq \gamma \tag{18}$$

is called the exponential passivity concerning the residual dynamics. Clearly this is weaker and more relaxed than the ordinary passivity discussed previously. However, inequality (16) is stronger and hence more restricted than the exponential passivity. Therefor, it is reasonable to call inequality (16) with $\beta > 0$ and $\lambda > 0$ the exponential passivity with a specified quadratic margin. We will next show that such a stronger condition is valid for the residual error dynamics of the manipulator under the end-point constraint provided the inner servo loop is properly composed of a force feedback in addition to the ordinary position and velocity feedback.

4 PASSIVITY OF ERROR DYNAMICS

Now suppose that a desired end-point path $x_d(t)$ and a desired time-evolution $f_d(t)$ of the magnitude of the contact force are defined on $t \in [0, T]$ and given to the manipulator. We reasonably assume that $x_d(t)$ satisfies $\varphi(x_d(t)) = 0$ and the contact force directs inside the contact surface, that is, $f_d(t) > 0$, for all $t \in [0, T]$. If the degree of freedom of the manipulator is greater than three ($n > 3$), then there is a possibility of existence of many $q_d(t)$ that may satisfy $\varphi(x(q_d)) = 0$ for all $t \in [0, T]$, where $x(q) = (x^1(q), x^2(q), x^3(q))$ denotes the transformation from joint coordinates q to cartesian coordinates x. We assume that in that case ($n > 3$) the posture of ($n-3$) components of q is specified and hence there exists a unique $q_d(t)$ satisfying $x(q_d) = x_d$. However, we neither need the computation of q_d on the basis of inverse kinematics nor use it in the closed-loop servo. We need on-line measurement data on the reaction force $f(t)$ caused by the contact between the end-point and the surface. Thus we consider a servo-loop as follows:

$$v = -Aq - B_1\dot{q} - J_\varphi^T(q)(f - f_d) + u. \tag{19}$$

The third term of the right hand side refers to the force error feedback and the fourth u refers to the feedforward input that must be determined through learning. Substitution of eq.(19) into eq.(7) yields

$$L(q) + B_1\dot{q} + Aq = J_\varphi^T(q)f_d - \xi(\|\dot{x}\|)J_x^T(q)\dot{x} + u \tag{20}$$

where $\dot{x}_d$ (and hence $\dot{q}_d$) is assumed to be differentiable and both $\ddot{x}_d$ (and hence $\ddot{q}_d$) and f_d are piecewise continuous. Since eq.(20) is invertible from output $\dot{q}$ to input u, it is possible to assume the existence of an ideal input u_d that realizes $\dot{q}_d$, i.e.,

$$u_d = L(q_d) + B_1\dot{q}_d + Aq_d - J_\varphi^T(q_d)f_d + \xi(\|\dot{x}_d\|)J_x^T(q_d)\dot{x}_d. \tag{21}$$

Subtracting this equation from eq.(20) yields

$$\begin{aligned}(H_0 + H(q_d + r))\ddot{r} + (B + \frac{1}{2}\dot{H}(q_d + r))\dot{r} \\ +S(q_d + r, \dot{q}_d + \dot{r})\dot{r} + Ar + h = \Delta u\end{aligned} \tag{22}$$

where

$$\begin{aligned}&\Delta u = u - u_d, \qquad r = q - q_d, \qquad B = B_0 + B_1, \\ &h = h(r, \dot{r}) = \{H(q_d + r) - H(q_d)\}\ddot{q}_d \\ &\quad + \frac{1}{2}\left\{\dot{H}(q_d + r) - \dot{H}(q_d)\right\}\dot{q}_d \\ &\quad + \{S(q_d + r, \dot{q}_d + \dot{r}) - S(q_d, \dot{q}_d)\}\dot{q}_d \\ &\quad + g(q_d + r) - g(q_d) \\ &\quad - \{J_\varphi(q_d + r) - J_\varphi(q_d)\}^T f_d \\ &\quad + \xi(\|\dot{x}\|)J_x^T(q_d + r)J_x(q_d + r)(\dot{q}_d + \dot{r}) \\ &\quad - \xi(\|\dot{x}_d\|)J_x^T(q_d)J_x(q_d)\dot{q}_d.\end{aligned} \tag{23}$$

Note that every entry of $H(q)$ is constant or a sinusoidal function of components of q and every entry of $S(q, \dot{q})$ is linear in $\dot{q}$. Therefore, h is linear in $\dot{r}$ and hence can be rewritten into the following form:

$$h = E(f_d, q_d, \dot{q}_d, \ddot{q}_d)r + F(q_d, \dot{q}_d, r)\dot{r} + \bar{h}(f_d, q_d, \dot{q}_d, \ddot{q}_d, r, \dot{r}) \tag{24}$$

where all linear terms of $\dot{r}$ in h in eq.(23) are firstly recast into the second term of the right hand side and hence the remaining terms become irrelevant to $\dot{r}$. In detail,

$$\begin{aligned}F(q_d, \dot{q}_d, r)\dot{r} = &\frac{1}{2}\left\{\sum_{i=1}^{n} H^i(q_d + r)\dot{r}^i\right\}\dot{q}_d \\ &+\xi(\|\dot{x}\|)J_x^T(q_d + r)J_x(q_d + r)\dot{r} \\ &+ \{S(q_d + r, \dot{q}_d + \dot{r}) - S(q_d + r, \dot{q}_d)\}\dot{q}_d\end{aligned} \tag{25}$$

where $H^i = \partial H / \partial r^i$, and

$$\begin{aligned}&E(f_d, q_d, \dot{q}_d, \ddot{q}_d)r + \bar{h}(f_d, q_d, \dot{q}_d, \ddot{q}_d, r, \dot{r}) \\ &\quad = g(q_d + r) - g(q_d) \\ &\quad + \frac{1}{2}\left[\sum_{i=1}^{n}\{H^i(q_d + r) - H^i(q_d)\}\dot{q}_d^i\right]\dot{q}_d \\ &\quad + \{H(q_d + r) - H(q_d)\}\ddot{q}_d \\ &\quad + \{S(q_d + r, \dot{q}_d) - S(q_d, \dot{q}_d)\}\dot{q}_d \\ &\quad - \{J_\varphi(q_d + r) - J_\varphi(q_d)\}^T f_d \\ &\quad + \{\xi(\|\dot{x}\|)J_x^T(q_d + r)J_x(q_d + r) \\ &\qquad - \xi(\|\dot{x}_d\|)J_x^T(q_d)J_x(q_d)\}\dot{q}_d.\end{aligned} \tag{26}$$

Note again that all H, S, g, and J_φ are periodic in r and thereby all entries and components of F and $\bar{h}$ are bounded provided all components of $\ddot{q}_d$ and f_d are piecewise continuous and hence bounded. In addition, we assume that there exists a constant $\bar{\rho} > 0$ such that the friction in the direction $-\dot{x}$ satisfies

$|\xi(\|\dot{x}\|) - \xi(\|\dot{x}_d\|)| < \bar{\rho}\,\|\dot{x}\|$. According to these observations, we see that there exist constants $\rho_0 > 0$ and $\rho_1 > 0$ such that for any r and $\dot{r}$

$$|\dot{r}^T h| \le |\dot{r}^T E r| + |\dot{r}^T F \dot{r}| + |\dot{r}^T \bar{h}| \le \rho_0 r^T r + \rho_1 \dot{r}^T \dot{r}. \tag{27}$$

We now show the exponential passivity with a quadratic margin for the residual dynamics of eq.(22).

Property 3. As long as the end-point of the manipulator is constrained on the surface $\varphi(x) = 0$, the exponential passivity of the residual robot dynamics of eq.(22) is satisfied, i.e.,

$$\int_0^t e^{-\lambda\tau} \dot{r}^T(\tau)\Delta u(\tau)d\tau \ge \frac{1+\beta}{2}\int_0^t e^{-\lambda\tau}\dot{r}^T(\tau)\Phi\dot{r}(\tau)d\tau + \gamma \tag{28}$$

with $\lambda > 0$ (not so large) and $\beta > 1$, where γ depends on only the initial state $(r(0), \dot{r}(0))$.

To prove this, we observe that

$$\begin{aligned}\int_0^t & e^{-\lambda\tau}\dot{r}^T(\tau)\Delta u(\tau)d\tau \\ &\ge e^{-\lambda t}U(r(t),\dot{r}(t)) - U(r(0),\dot{r}(0)) \\ &\quad - \int_0^t e^{-\lambda\tau}\left[\lambda U(r(\tau),\dot{r}(\tau)) + \dot{r}^T(\tau)B\dot{r}(\tau)\right]d\tau \\ &\quad - \int_0^t e^{-\lambda\tau}\left\{\rho_0 r^T(\tau)r(\tau) + \rho_1\dot{r}^T(\tau)\dot{r}(\tau)\right\}d\tau\end{aligned} \tag{29}$$

where

$$U(r,\dot{r}) = \frac{1}{2}\left\{\dot{r}^T(H_0 + H(q_d + r))\dot{r} + r^T A r\right\}. \tag{30}$$

Next define a scalar function

$$\begin{aligned}W(\lambda; r, \dot{r}) &= \lambda U(r,\dot{r}) + \dot{r}^T B\dot{r} \\ &\quad -\rho_0 r^T r - \rho_1\dot{r}^T\dot{r} - \frac{1+\beta}{2}\dot{r}^T\Phi\dot{r}\end{aligned} \tag{31}$$

which becomes positive definite in r and $\dot{r}$ with an appropriate choice for $\lambda > 0$. From this it follows that

$$\begin{aligned}\int_0^t e^{-\lambda\tau}\dot{r}^T\Delta u d\tau &\ge -U(r(0),\dot{r}(0)) \\ &\quad +\frac{1+\beta}{2}\int_0^t e^{-\lambda\tau}\dot{r}^T(\tau)\Phi\dot{r}(\tau)d\tau\end{aligned} \tag{32}$$

which proves Property 3.

It is important to remark that if B is large enough so that $2B > 2\rho_1 I + (1+\beta)\Phi$ for small $\beta > 0$ then λ can be chosen small enough so that $\lambda A > \rho_0 I$.

5 SIMULATION RESULTS

To show the effectiveness of proposed learning algorithm under geometric constraints, computer simulation is carried out by using a model of a 3 d.o.f. manipulator illustrated in Fig.2. Link parameters of the manipulator are specified in Table 1. The recursive learning law used in the simulation is as follows:

$$u_{k+1} = u_k - \Phi\dot{r}_k + \psi J_\varphi^T(q_k)\Delta f_k \tag{33}$$

where servo-loop gains and learning control gains are specified as follows:

$$A = diag(10.0, 10.0, 10.0), B = diag(4.0, 4.0, 4.0)$$
$$\Phi = diag(0.5, 0.5, 0.5), \psi = 0.8$$

where all numerals stand for appropriate physical values based on MKS units.

The end-point of the manipulator is constrained on a vertical plain. The desired task is to draw a circle with radius 15 cm on that plain within just one second under a specified speed profile. The equation of motion with an algebraic constraint is solved numerically by a Runge-Kutta method modified so as to solve such a DAE (Differential and Algebraic Equation). Sampling time of the simulation is 0.001 second.

Figure 3 shows the convergence of both PD-tracking errors and force errors, where the PD and force errors are evaluated on the following form:

$$PD - error = \frac{1}{1000}\sum_{i=1}^{1000}\frac{1}{2}\left\{\dot{r}^T(i)M\dot{r}(i) + r^T(i)Ar(i)\right\},$$

$$Force - error = \frac{1}{1000}\sum_{i=1}^{1000}\left\{\Delta f(i)\right\}^2$$

where M stands for an averaged value of inertia matrix $M(q)(= H_0 + H(q))$ and A does for the position feedback gain matrix. The speed of convergence can be improved by increasing learning control gains (diagonal elements of Φ).

6 THEORETICAL PROOF

We now discuss the problem of theoretical proof for the convergence of both position and force trajectories through repeated trainings. At the present stage of theoretical development, however, it seems difficult in a general setup of such learning algorithms as eq.(33). To the best of our ability, we have succeeded only in proving the convergence for a specially restricted class of learning law described in the following form:

$$u_{k+1} = u_k - \phi\dot{r}_k + \psi M^{-1}(q_k)J_\varphi^T(q_k)\Delta f_k \tag{34}$$

with a condition $\psi > 0$ such that

$$\psi M^{-1}(q) < (1-\delta)I \tag{35}$$

for a $1 > \delta > 0$, where

$$M(q) = H_0 + H(q), \quad \Delta f = f - f_d. \tag{36}$$

The algorithm of eq.(34) differs from that of eq. (33) in that the force correction term (the last term in eq.(34)) must be multiplied by the inverse of $M(q_k)$ at each trial. This assumes the knowledge of inertia matrix and hence imposes a burden of much complication on implementation of the algorithm. However, by the use of this restricted algorithm it is possible to prove the convergence directly from the exponential passivity of error dynamics of robot arms under geometric constraints.

Now, we assume the existence of a unique ideal input $u_d(t)$ that realizes the desired trajectories of position $q_d(t)$ and force $f_d(t)$. Subtracting u_d from both sides of eq.(34) yields

$$\Delta u_{k+q} = \Delta u_k - \phi \dot{r}_k + \psi M^{-1}(q_k) J_\varphi^T(q_k) \Delta f_k. \quad (37)$$

Similarly to eq.(15), we obtain

$$\begin{aligned}\Delta u_{k+1}^T \Delta u_{k+1} &= \Delta u_k^T \Delta u_k - 2\phi \dot{r}_k^T \Delta u_k + \phi^2 \dot{r}_k^T \dot{r}_k \\ &+\psi^2 \Delta f_k^2 J_\varphi(q_k) M^{-1}(q_k) M^{-1}(q_k) J_\varphi^T(q_k) \\ &-2\phi\psi \Delta f_k \dot{r}_k^T M^{-1}(q_k) J_\varphi^T(q_k) \\ &+2\psi \Delta f_k \Delta u_k^T M^{-1}(q_k) J_\varphi^T(q_k). \quad (38)\end{aligned}$$

On the other hand, we consider the case that the servo-loop is composed of only a PD feedback (without use of the force feedback, differently from the case of eq.(19)), and a feedforward u to be learned (in what follows we omit suffix k), i.e.,

$$v = -A(q - q_d) - B_1 \dot{q} + u \quad (39)$$

Substituting this into eq.(7) and subtracting the corresponding equation where $q = q_d$, we obtain

$$\begin{aligned}(H_0 + H(q_d + r))\ddot{r} &+ (B + \frac{1}{2}\dot{H}(q_d + r))\dot{r} \\ &+S(q_d + r, \dot{q}_d + \dot{r})\dot{r} + Ar + h \\ &= J_\varphi^T(q_d + r)\Delta f + \Delta u \quad (40)\end{aligned}$$

where we use the relation

$$J_\varphi^T(q) f - J_\varphi^T(q_d) f_d = J_\varphi^T(q)\Delta f + (J_\varphi^T(q) - J_\varphi^T(q_d)) f_d \quad (41)$$

and h is the same as in eq.(23).

For convenience, we rewrite eq.(40) in the following short form:

$$M(q)\ddot{r} + g(r, \dot{r}) = J_\varphi^T(q)\Delta f + \Delta u \quad (42)$$

Now, we note that $J_\varphi(q)\dot{q} = 0$ and $J_\varphi(q)\ddot{q} + \dot{J}_\varphi(q)\dot{q} = 0$ imply

$$J_\varphi(q)\dot{r} = (J_\varphi(q_d) - J_\varphi(q))\dot{q}_d = \alpha(r), \quad \alpha(0) = 0, \quad (43)$$

$$J_\varphi(q)\ddot{r} = \gamma(r, \dot{r}), \quad \gamma(0,0) = 0 \quad (44)$$

where $\alpha(r)$ and $\gamma(r, \dot{r})$ are periodic in r and linear in $\dot{r}$. From eq.(42) it follows that

$$\begin{aligned}&J_\varphi^T(q) M^{-1}(q) \Delta u \\ &\quad = -J_\varphi(q) M^{-1} J_\varphi^T(q) \{\Delta f - g(r, \dot{r})\} - \gamma(r, \dot{r}). \quad (45)\end{aligned}$$

In addition, it is easy to see that

$$\begin{aligned}&-2\phi\psi \Delta f \dot{r}^T M^{-1}(q) J_\varphi(q) \\ &\quad \le \psi^2 \Delta f^2 J_\varphi(q) M^{-2}(q) J_\varphi^T(q) + \phi^2 \dot{r}^T \dot{r} \\ &\quad \le \psi(1-\delta)\Delta f^2 J_\varphi(q) M^{-1}(q) J_\varphi^T(q) + \phi^2 \dot{r}^T \dot{r} \quad (46)\end{aligned}$$

Substituting eqs.(45) and (46) into eq.(34) yields

$$\begin{aligned}\Delta u_{k+1}^T \Delta u_{k+1} &\le \Delta u_k^T \Delta u_k - 2\phi \dot{r}_k^T \Delta u_k \\ &+2\phi^2 \dot{r}_k^T \dot{r}_k - 2\psi\delta \Delta f_k^2 J_\varphi(q_k) M^{-1}(q_k) J_\varphi(q_k) \\ &+2\psi \Delta f_k \bar{\gamma}(r_k, \dot{r}_k) \quad (47)\end{aligned}$$

where $\bar{\gamma}(r, \dot{r})$ has the same property as $\gamma(r, \dot{r})$. On the other hand, it follows from the same argument as in previous section that

$$\begin{aligned}\int_0^T e^{-\lambda t} \dot{r}^T \Delta u dt &\ge -U(r(0), \dot{r}(0)) \\ &+ \frac{(1+\beta)\phi}{2} \int_0^T e^{-\lambda t} \dot{r}^T \dot{r} dt \\ &+ \int_0^T e^{-\lambda t} \{W(\lambda; r, \dot{r}) - J_\varphi \dot{r} \Delta f\} dt. \quad (48)\end{aligned}$$

Multiplying eq.(47) by $\exp(-\lambda t)$, integrating it over $[0, T]$, and substituting eq.(48) into this resulting form, we obtain

$$\begin{aligned}\|\Delta u_{k+1}\|_\lambda^2 &\le \|\Delta u_k\|_\lambda^2 - (\beta - 1)\|\phi \dot{r}_k\|_\lambda^2 \\ &- U(r_k(0), \dot{r}_k(0)) - 2\psi\delta\phi \|\Delta f_k\|_\lambda^2 / m \\ &- \int_0^T e^{-\lambda t} [W(\lambda; r_k, \dot{r}_k) - 2\psi \Delta f_k \bar{\bar{\gamma}}(r_k, \dot{r}_k)] dt, \quad (49)\end{aligned}$$

where $\bar{\bar{\gamma}}(r_k, \dot{r}_k)$ has the same property as $\gamma(r_k, \dot{r}_k)$. Finally, note that

$$2\psi \Delta f \bar{\bar{\gamma}}(r, \dot{r}) \le \frac{\psi\delta\phi}{m}\Delta f^2 + \frac{\psi m}{\delta\phi}(\bar{\rho}_0 r^T r + \bar{\rho}_1 \dot{r}^T \dot{r}). \quad (50)$$

Substituting this into eq.(49), we conclude that

$$\begin{aligned}\|\Delta u_{k+1}\|_\lambda^2 &\le \|\Delta u_k\|_\lambda^2 - (\beta - 1)\|\phi \dot{r}_k\|_\lambda^2 \\ &- \frac{\psi\delta\phi}{m}\|\Delta f_k\|_\lambda^2 - \int_0^T e^{-\lambda t} [W(\lambda; r_k, \dot{r}_k) \\ &- \frac{\psi m}{\delta\phi}(\bar{\rho}_0 r_k^T r_k + \bar{\rho}_1 \dot{r}_k^T \dot{r}_k)] dt. \quad (51)\end{aligned}$$

Since by definition of W in eq.(32) the content of bracket [ ] in eq.(51) becomes positive definite in r and $\dot{r}$ for an appropriate choice for $\lambda > 0$. Hence,

$$\|\Delta u_{k+1}\|_\lambda^2 \le \|\Delta u_k\|_\lambda^2 - (\beta - 1)\|\phi \dot{r}_k\|_\lambda^2 - \frac{\psi\delta\phi}{m}\|\Delta f_k\|_\lambda^2, \quad (52)$$

provided that

$$q_k(0) = q_d(0), \dot{q}_k(0) = \dot{q}_d(0) \quad \text{for} \quad k = 1, 2, \cdots. \tag{53}$$

Thus, it is possible to conclude:

Theorem 1 Under the learning law of eq.(34), both position and force trajectories $q_k(t)$ and $f_k(t)$ converge to given desired ones $q_d(t)$ and $f_d(t)$ as $k \to \infty$ in the sense of L^2-norm provided that the perfect initialization (eq.(53)) is satisfied.

7 CONCLUDING REMARKS

A recursive algorithm of P-type learning control is proposed for bettering robot tasks with geometric end-point constraints. A new concept of exponential passivity with a quadratic margin is introduced for a class of residual error dynamics of robots with or without geometric constraints. Simulations based on the proposed algorithm are given, which show the convergence of both position and force errors. However, at the present stage it is possible to the best of our ability to prove the convergence theoretically under a more specialized class of learning control algorithms that employ the knowledge of inertia matrix.

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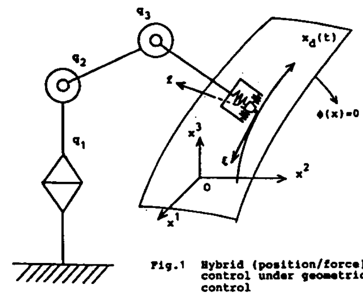

Fig.1 Hybrid (position/force) control under geometric control

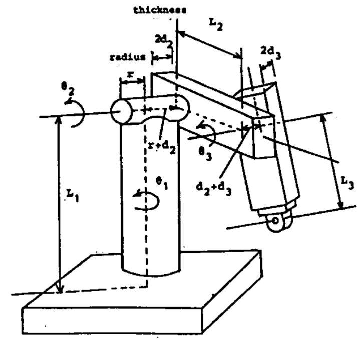

Fig.2 Robot arm with three degree of freedom

link number	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	$-\pi/2$	L	θ_2
3	0	0	L_2	θ_3

1a Link parameters

L	132.0mm		L_1	390.0mm
L_2	205.0mm		L_3	205.0mm
l_1	440.0mm		l_2	409.5mm
l_3	391.0mm		r	40.5mm
d_2	35.0mm		d_3	21.5mm
w_2	86.0mm		w_3	70.0mm
m_1	3.0Kg		m_2	1.0Kg
m_3	1.0Kg			

1b Physical sizes

link number	figure	mass	size
1	cylinder	m_1	radius=r, hight=l_1
2	rectangle solid	m_2	length=l_2,thickness=$2\times d_2$,width=$2\times w_2$
3	rectangle solid	m_3	length=l_3,thickness=$2\times d_3$,width=$2\times w_3$

1c Shapes of Links

Table 1 Physical parameters of the robot arm described in Fig. 2

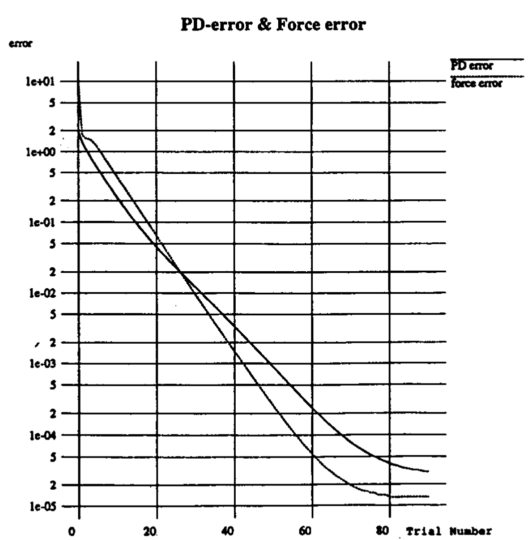

Fig.3 Error performance of the learning control law described by eq.(33)