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## Model Specification

This note seeks to document the model specification we would ideally like to estimate and its key components and reference terms.

$$\begin{aligned}
 p_{ijt} &= \alpha_{ij} + \gamma_t + FB_{ijt} + \varepsilon_{ijt} \\
 \alpha_{ij} &= \alpha_i + \alpha_j + \beta_1 d_{ij} + \beta_2 c_{ij} + u_{ij} \\
 \alpha_i &= \delta_i + \delta_{i1} pop_i + F_i + u_i \\
 \alpha_j &= \delta_j + \delta_{j1} pop_j + F_j + u_j \\
 \gamma_t &= \delta_t + \delta_{t1} day + \delta_{t2} ti + \delta_{t3} w + u_t \\
 FB_{ijt} &= fb_0 + fb_{ijt} z_{ijt} + fb_{ijt} au_{ijt} + u_{ijt}
 \end{aligned} \tag{1}$$

$p_{ijt}$ : proportion of people moving from tile  $i$  to tile  $j$  at time  $t$ ;  
 $\alpha_{ij}$ : component relating to tiles of origin  $i$  and destination  $j$ ;  
 $\gamma_t$ : component relating to temporal variation;  
 $FB_{ijt}$ : component relating to variation in Facebook data quality;  
 $\alpha_i$ : factors relating to tiles of origin  $i$ ;  
 $\alpha_j$ : factors relating to tiles of origin  $j$ ;  
 $\beta_1$ : coefficient capturing the deterring effect of distance  $d_{ij}$ ;  
 $\beta_2$ : coefficient capturing the variation in mobility intensity across tiles in different functional areas  $c_{ij}$ ;  
 $\delta_i$ : average level of mobility related to tiles of origin  $i$ ;  
 $\delta_{i1}$ : coefficient capturing the relationship between mobility intensity and population at origin tile  $pop_i$ ;  
 $F_i$ : coefficient capturing the relationship between mobility intensity and additional attributes at origin  $i$  eg. unemployment, housing affordability, etc.  
 $\delta_j$ : average level of mobility related to tiles of destination  $j$ ;  
 $\delta_{j1}$ : coefficient capturing the relationship between mobility intensity and population at destination tile  $pop_j$ ;  
 $F_j$ : coefficient capturing the relationship between mobility intensity and additional attributes at destination  $j$  eg. unemployment, housing affordability, etc.  
 $\delta_t$ : average level of mobility related to time;  
 $\delta_{t1}$ : coefficient capturing the daily *day* variability in mobility;  
 $\delta_{t2}$ : coefficient capturing the variability in mobility according to the 8 hour time interval  $ti$  in which the Facebook data are organised; that is, 0am, 8am and 4pm;  
 $\delta_{t3}w$ : coefficient capturing the variability in mobility during week days and weekends  $w$ ;  
 $\varepsilon$  and  $u$  correspond to the error terms;

## Estimation

The plan is to estimate Equation 1 in a fractional regression framework (see Papke and Wooldridge, 1996). In this framework errors are assumed to follow a logistic distribution and quasi maximum likelihood procedure is used to estimate the set of regression coefficients. The variable identifying pair combinations of functional areas for tiles of origin and destination (i.e.  $c_{ij}$ ) can enter the model as a dummy variable or as a random effect in a multilevel framework. Time can also be modelled in different ways (i.e. including autoregressive terms of  $p_{ijt}$ , autoregressive terms of errors and splines) and there may be different components which may need to be taken into consideration (i.e. seasonability and autocorrelation)

## References

Papke, L. E. and Wooldridge, J. M. (1996). Econometric methods for fractional response variables with an application to 401 (k) plan participation rates. *Journal of applied econometrics*, 11(6):619–632.