Universal quantifier

$$\dfrac{\begin{array}{c}\overline{\mathtt{x} : A}\\ \vdots \\ \mathtt{E} : B(x)\end{array}}{\mathtt{fun\ x\ =>\ E} : \forall x : A, B(x)}\ \forall I \qquad \dfrac{\mathtt{E}_1 : \forall a : A, B(x) \qquad \mathtt{E}_2 : A}{\mathtt{E}_1\ \mathtt{E}_2 : B[x \mapsto E_2]}\ \forall E$$

Implication

$$\dfrac{\begin{array}{c}\overline{\mathtt{x} : A}\\ \vdots \\ \mathtt{E} : B\end{array}}{\mathtt{fun\ x\ =>\ E} : A \to B}\ \to I \qquad \dfrac{\mathtt{E}_1 : A \to B \qquad \mathtt{E}_2 : A}{\mathtt{E}_1\ \mathtt{E}_2 : B}\ \to E$$

Conjunction

$$\dfrac{\mathtt{E}_1 : A \qquad \mathtt{E}_2 : B}{(\mathtt{E}_1,\ \mathtt{E}_2) : A \wedge B}\ \wedge I \qquad \dfrac{\mathtt{E} : A \wedge B}{\mathtt{fst\ E} : A}\ \wedge E_1 \qquad \dfrac{\mathtt{E} : A \wedge B}{\mathtt{snd\ E} : B}\ \wedge E_2$$

Disjunction

$$\dfrac{\mathtt{E} : A}{\mathtt{inl\ E} : A \vee B}\ \vee I_1 \qquad \dfrac{\mathtt{E} : B}{\mathtt{inr\ E} : A \vee B}\ \vee I_2$$

$$\dfrac{\mathtt{E} : A \vee B \qquad \begin{array}{c}\overline{\mathtt{x} : A}\\ \vdots \\ \mathtt{E}_1 : C\end{array} \qquad \begin{array}{c}\overline{\mathtt{y} : B}\\ \vdots \\ \mathtt{E}_2 : C\end{array}}{\mathtt{match\ E\ with\ |\ inl\ x\ =>\ E_1\ |\ inr\ y\ =>\ E_2\ end}\ : C}\ \vee \mathrm{E}$$

Existential quantifier

$$\dfrac{\mathtt{E}_1 : A \qquad \mathtt{E}_2 : B[x \mapsto E_1]}{\{\mathtt{E}_1,\ \mathtt{E}_2\} : \exists x : A,\ B(x)}\ \exists I \qquad \dfrac{\mathtt{E} : \exists x : A,\ B(x) \qquad \begin{array}{c}\overline{\mathtt{x} : A} \quad \overline{\mathtt{y} : B(x)}\\ \vdots \\ \mathtt{E}' : C\end{array}}{\mathtt{let\ \{x,\ y\}\ :=\ E\ in\ E'} : C}\ \exists E$$