Problem Statement 1: [50 marks]

The marks awarded for an assignment set for a Year 8 class of 20 students were as follows:

6 7 5 7 7 8 7 6 9 7 4 10 6 8 8 9 5 6 4 8

Problem Statement 2: [50 marks]

The number of calls from motorists per day for roadside service was recorded for a particular month:

28, 122, 217, 130, 120, 86, 80, 90, 140, 120, 70, 40, 145, 113, 90, 68, 174, 194, 170, 100, 75, 104, 97, 75,

123, 100, 75, 104, 97, 75, 123, 100, 89, 120, 109

Calculate the mean, median, mode and standard deviation for the problem statements 1&2

Solution 1:

1. Mean:

Mean = (Σ xi) / n

xi = Each value in the data set

n = Total number of values in the data set

Σ of xi = 6+7+5+7+7+8+7+6+9+7+4+10+6+8+8+9+5+6+4+8

(Σ xi) / n=137/20=6.85

The Mean of the given data is 6.85

2. Median:

The Median is the middle value of a sorted list of numbers

If the no. of values are odd, we will have one median, else we will have 2 values and median would be average of it

s.no	values
1	4
2	4
3	5
4	5
5	6
6	6
7	6
8	6
9	7
10	7
11	7
12	7
13	7
14	8
15	8
16	8
17	8
18	9
19	9
20	10

Median of the given data set = Sum of middle values / 2

=(7 + 7) / 2 = 6.5

3. Mode:

Mode is the most repeated value in the data set.

Mode of the given data set = 7 (most repeated value)

4. Standard Deviation:

SD = √∑(x-$\bar{x}$)^2/n

Where :

SD = Standard Deviation

x= each value in the data set

$\bar{x}$= Mean is the data set

n = number of values in the data set

s.no	values	$\bar{x}$	(x-$\bar{x}$)^2
1	4	6.85	8.1225
2	4	6.85	8.1225
3	5	6.85	3.4225
4	5	6.85	3.4225
5	6	6.85	0.7225
6	6	6.85	0.7225
7	6	6.85	0.7225
8	6	6.85	0.7225
9	7	6.85	0.0225
10	7	6.85	0.0225
11	7	6.85	0.0225
12	7	6.85	0.0225
13	7	6.85	0.0225
14	8	6.85	1.3225
15	8	6.85	1.3225
16	8	6.85	1.3225
17	8	6.85	1.3225
18	9	6.85	4.6225
19	9	6.85	4.6225
20	10	6.85	9.9225

Standard Deviation = 50.55 /20 = √2.5275 = 1.5898

Solution 2:

∑xi = 28+122+217+130+ 120+86+80+90+ 140+120+70+40+145+113+90+68+174+194+ 170+100+ 75+104+97+75+ 123+ 100+ 75+ 104+97+ 75+123+100+ 89+ 120+ 109

n = 35
∴ Mean=3763 / 35 = 107. 51

Median:

{(n + 1) ÷ 2}th element

n = 35

∴ n+1 = 35+1 = 36 ∴ {(n + 1) ÷ 2}th element = 36/ 2 = 18 = 100 from the below table

s.no	x
1	28
2	40
3	68
4	70
5	75
6	75
7	75
8	75
9	80
10	86
11	89
12	90
13	90
14	97
15	97
16	100
17	100

18	100
19	104
20	104
21	109
22	113
23	120
24	120
25	120
26	122
27	123
28	123
29	130
30	140
31	145
32	170
33	174
34	194
35	217

3. Mode:

Mode of the data is the most repeated value in the data set.

∴ Mode of the given data set = 75 (most repeated value)

4. Standard Deviation:

SD = $\sqrt{\sum(x-\bar{x})^2/n}$

Where :

SD = Standard Deviation

x= each value in the data set

$\bar{x}$= Mean is the data set

n = number of values in the data set

s.no	x	$\bar{x}$	$(x-\bar{x})^2$
1	28	107.51	6322.52
2	40	107.51	4558.18
3	68	107.51	1561.38
4	70	107.51	1407.32
5	75	107.51	1057.18
6	75	107.51	1057.18
7	75	107.51	1057.18
8	75	107.51	1057.18
9	80	107.51	757.04
10	86	107.51	462.86
11	89	107.51	342.78
12	90	107.51	306.75
13	90	107.51	306.75
14	97	107.51	110.55
15	97	107.51	110.55
16	100	107.51	56.46
17	100	107.51	56.46
18	100	107.51	56.46
19	104	107.51	12.35
20	104	107.51	12.35
21	109	107.51	2.21
22	113	107.51	30.09
23	120	107.51	155.89
24	120	107.51	155.89
25	120	107.51	155.89
26	122	107.51	209.84
27	123	107.51	239.81
28	123	107.51	239.81

29	130	107.51	505.61
30	140	107.51	1055.32
31	145	107.51	1405.18
32	170	107.51	3904.46
33	174	107.51	4420.35
34	194	107.51	7479.78
35	217	107.51	11987.12

Standard deviation = sqrt(52616.74/35)= 38.77

Problem Statement 3: [100 marks]

The number of times I go to the gym in weekdays, are given below along with its associated probability:

x = 0, 1, 2, 3, 4, 5

f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01

Calculate the mean no. of workouts in a week. Also evaluate the variance involved in it.

Expected Mean of the Work outs:

Let us call x as No. of Work outs in a week

P(x) is given as - f(x) = 0.09, 0.15, 0.40, 0.25, 0.10, 0.01

Calculation of Expected mean number of workouts in a week = Weighted average of the workouts and the probability.

x	Probability f(x)	Weighted sum of probability
0	0.09	0.00
1	0.15	0.15
2	0.40	0.80
3	0.25	0.75
4	0.10	0.40
5	0.01	0.05
Mean work out E(x)		**2.15**

Variance:

x	Probability f(x)	Weighted sum of probability	x-μ	$(x-\mu)2$	$(x-\mu)2 * p(x)$
0	0.09	0.00	-2.15	4.6225	0.416025
1	0.15	0.15	-1.15	1.3225	0.198375
2	0.40	0.80	-0.15	0.0225	0.009
3	0.25	0.75	0.85	0.7225	0.180625
4	0.10	0.40	1.85	3.4225	0.34225
5	0.01	0.05	2.85	8.1225	0.081225
Mean work out		2.15			1.2275
Variance value					1.10792599

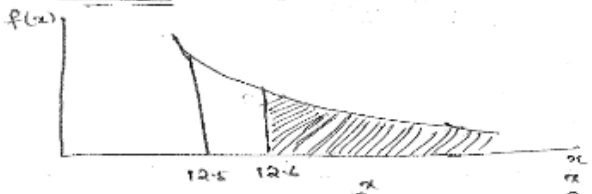
Problem Statement 4: [100 marks]

Let the continuous random variable D denote the diameter of the hole drilled in an aluminum sheet. The target diameter to be achieved is 12.5mm. Random disturbances in the process often result in inaccuracy.

Historical data shows that the distribution of D can be modelled by the PDF f(d) = 20e−20(d−12.5), d ≥12.5. If a part with diameter > 12.6 mm needs to be scrapped, what is the proportion of those parts? What is the CDF when the diameter is of 11 mm? What is your conclusion regarding the proportion of

scraps?

Solution :-



$$P(x \geq 12.60) = \int_{12.6}^{\infty} f(x)\,dx = \int_{12.6}^{\infty} 20\,e^{-20(x-12.5)}\,dx = -e^{-20(x-12.5)}\Big|_{12.6}^{\infty}$$

$$= 0.135$$

∴ Proportion of parts between 12.5 and 12.6 mm

$$= P(12.5 < x < 12.6) = \int_{12.5}^{12.6} f(x)\,dx = -e^{-20(x-12.5)}\Big|_{12.5}^{12.6}$$

$$= 0.865$$

[OR]

$$P(12.5 < x < 12.6) = 1 - P(x > 12.6)$$

$$= 1 - 0.135$$

$$= 0.865.$$

Total units to be scrapped = 0.865