Problem Statement 1:
A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Problem Statement1 :

Solution : $P = 0.30$

This is a binomial distribution where $n = 6$ $r = 2$

$$p(x_i = 2) = 6_{C_2} * (0.3)^2 * (1-0.3)^{6-2}$$

$$= \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{2}! \times \cancel{4!}} \times (0.3)^2 \times (0.7)^4$$

$$= 15 \times 0.09 \times 0.2401$$

$$= 0.3241$$

Problem Statement 2:

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Problem Statement 2 :

$P(G) = 0.75$

$n = 8$

(i) Probability of Gaurav Solving 5 correctly

$P(G)(x_i = 5)$

$$= 8_{C_5} \times (0.75)^5 \times (0.25)^3$$

$$= \frac{8 \times 7 \times \cancel{6} \times \cancel{5!}}{\cancel{3 \times 2 \times 1} \times \cancel{5!}} \times 0.2373 \times 0.0156$$

$$= 56 \times 0.2373 \times 0.0156$$

$$= 0.2076$$

Probability of Gaurav Solving 4 correctly

$P(G)(x_i = 4)$

$$= 8_{C_4} \times (0.75)^4 \times (0.25)^4$$

$$= \frac{\overset{2}{\cancel{8}} \times 7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4 \times 3 \times 2} \times 1 \times \cancel{4!}} \times (0.75)^4 \times (0.25)^4$$

$$= 70 \times 0.31640 \times 0.0039$$

$$= 0.0865$$

Probability of Barakha Solving 5 correctly

$PB = 0.45$

$n = 12$

$(B)(x = 5)$

$$= 12_{C_5} \times (0.45)^5 \times (0.55)^7$$

$$= 792 \times (0.45)^5 \times (0.55)^7$$

$$= 0.22249$$

Prob. of Barakha Solving 6 Correctly

$P(B)(x = 6)$

$$\Rightarrow 12_{C_6} \times (0.45)^6 \times (0.55)^6$$

$$= 924 \times (0.45)^6 \times (0.55)^6$$

$$= 0.21237$$

Problem Statement 3:
Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

Problem Stat 3 :-

$\Rightarrow$ 72 Persons/hr $= \frac{72}{60} = 1.2$ Persons/min

i) 4 minutes $= 1.2 \times 4 = 4.8$ customers.

ⓐ 5 customers in 4 min

$$\Rightarrow \frac{e^{(-4.8)} \times (4.8)^5}{5!}$$

$$= \frac{0.0082297 \times (4.8)^5}{5!}$$

$$= 0.1747$$

ⓑ not more than 3 Customers

i) $x = 0$

$$\frac{e^{-(4.8)} \times (4.8)^0}{0!}$$

$$= 0.008229$$

$x = 1$

$$\frac{e^{(4.8)} \times (4.8)^1}{1!}$$

$$= 0.0394$$

$x = 2$

$$\frac{e^{-4.8} \times (4.8)^2}{2!}$$

$$= 0.09479$$

$x = 3$

$$\frac{e^{-4.8} \times (4.8)^3}{3!}$$

$$= 0.151690$$

$$\Rightarrow (0.1516 + 0.09479 + 0.0394 + 0.00822)$$

$$= 0.2941$$

ⓒ more than 3 custome

1 − P(less than 3 customers)

1 − 0.2941

$\Rightarrow$ .705891

$\Rightarrow$

Problem Statement 4:
I work as a data analyst in Aeon Learning Pvt. Ltd. After analyzing data, I make reports, where I have the efficiency of entering 77 words per minute with 6 errors per hour. What is the probability that I will commit 2 errors in a 455-word financial report?
What happens when the no. of words increases (in case of 1000 words) or decreases (255 words)? How is the λ affected? How does it influence the PMF? Give a pictorial representation of the same to validate your answer.

Prob. Stmt 4:

Errors per hr = 6

words per hour = 77×60 = 4,620 words/hr

Error per word = $\frac{6}{4620} = \frac{1}{770}$

(i) Prob. of 2 errors in 455 words.

$\lambda = \frac{455}{770} = 0.591$

$P(x=2) = \frac{e^{-(0.591)} \times (0.591)^2}{2!}$

$= \cancel{0.09671}$

0.09671

(ii) (a) words increased to 1000

$\lambda = \frac{1000}{770} = 1.2987$

$P(x=2) = \frac{e^{-(1.2987)} \times (1.2987)^2}{2!}$

$= 0.2303$

(b) words decreased to 255

$\lambda = \frac{255}{770} = 0.3311$

$p(x!=2) = \frac{e^{-(0.3311)} \times (0.3311)^2}{2!}$

$= 0.0393$

Problem Statement 5:

The current measured in a copper wire is modelled by a continuous random variable X. X is in milliamperes. Assume that the range of X is [0, 20 mA]. The probability density function is given by, $f(x) = 0.05$ for $0 \le x \le 20$. What is the probability that a current measurement is less than 10 milliamperes? Draw the PDF and the CDF diagrams as well.

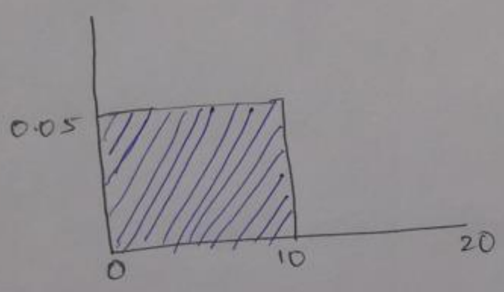
Probl. Stemt 5:-

$f(x) = 0.05 \quad \text{for } 0 \le x \le 20$

$$\int_{i=0}^{10} f(x).dx = \int_{0}^{10} 0.05$$

$$[0.05]_0^{10} = 0.5 - 0 = 0.5$$

PDF



CDF