

KLEINIAN GROUPS AND HYPERBOLIC GEOMETRY

A short lecture series on Kleinian groups and hyperbolic geometry. We will touch on knot theory, 3-manifold geometry, Teichmüller theory, and discrete groups. The goal will be to discuss the ‘big ideas’ of this area of mathematics, not to learn technical definitions and proofs; this is firstly because (for instance) a course in Teichmüller theory would usually be a semester-long second year graduate course, and secondly because I would like to emphasise the geometry rather than the underlying analysis.

1. EXPECTED KNOWLEDGE

Working knowledge of complex analysis, linear algebra, and topology, though the talks will be pitched at the most elementary level possible.

2. FORMAT

Five hour-long talks, all likely to be in 303.610 at 2pm on the respective days

- (1) Tilings, geometric manifolds, and the Poincaré polyhedron theorem (Mon 7/2)
- (2) Uniformisation of Riemann surfaces, Kleinian groups (Tue 8/2)
- (3) Fundamental domains, the Maskit combination theorems, and examples (Wed 9/2)
- (4) Hyperbolic knots and links (Thu 10/2)
- (5) Teichmüller theory, moduli spaces, and braids (Fri 11/2)

We will unfortunately not have time to discuss many other aspects of the theory, for instance the very nice connections to dynamics, computation aspects, fractal geometry...

3. RELEVANT VIDEOS

- Curtis T. McMullen, Mathematics as Metaphor, <https://www.youtube.com/watch?v=tzSCDaONrPQ>
- The Geometry Center, Not Knot, <https://www.youtube.com/watch?v=4aN6vX7qXPQ>
- The Geometry Center, The Shape of Space, https://www.youtube.com/watch?v=-gLN1C_hQ3M
- Bill Thurston, Knots to Narnia, <https://www.youtube.com/watch?v=IKSrBt2kFD4>
- Bill Thurston, The Mystery of 3-Manifolds, <https://www.youtube.com/watch?v=4jdmkUQDWtQ>
- Vi Hart, Andrea Hawksley, Sabetta Matsumoto, and Henry Segerman, Non-euclidean virtual reality, <https://www.youtube.com/watch?v=ztsi0CLxmJw>

4. RELEVANT BOOKS

Roughly in increasing order of difficulty

- Francis Bonahon, *Low-Dimensional Geometry: From Euclidean Surfaces to Hyperbolic Knots*.
- David Mumford, Caroline Series, and David Wright, *Indra's Pearls: The vision of Felix Klein*.
- Alan F. Beardon, *The Geometry of Discrete Groups*.
- Curtis T. McMullen, *Riemann surfaces, dynamics and geometry*, <https://people.math.harvard.edu/~ctm/papers/home/text/class/notes/rs/course.pdf>
- Bernard Maskit, *Kleinian Groups*.
- Benson Farb and Dan Margalit, *A Primer on Mapping Class Groups*.

- Hershel M. Farkas and Irwin Kra, *Riemann surfaces*.
- Albert Marden, *Outer Circles*.
- Jessica Purcell, *Hyperbolic Knot Theory*.
- John G. Ratcliffe, *Foundations of Hyperbolic Manifolds*.
- William Thurston, *Three-Dimensional Geometry and Topology (volume 1)* and *Geometry and topology of three-manifolds* (unpublished lecture notes).