MTH 1020 Tutorial 1 (Wk 2)

1 Agenda

1. **As you come in:** Take a question sheet corresponding to the email you got last week. If you didn't get an email come and see me.

Mathematician	Question number
Galois	Q1
Mirzakhani	Q2
Noether	Q4
Russell	Q5

2.
   - Working in small groups improves your understanding and grades. There is significant evidence for this. For example, a 1999 meta-analysis of 37 different studies shows that if students who are in the 50th percentile in a standardised test are exposed to small-group collaborative/cooperative teaching, then they move on average up to the 70th percentile.[1]
   - This depends on you all working together respectfully. You are all adults.

3. **Split into groups of four**, made up of one person from each of the four mathematicians/colours/assigned questions. Find a whiteboard space around the room and write your preferred names (forename + surname) and student ID numbers at the top of the board. This week only, please also write your major/specialisation (e.g. mathematics, chemistry, etc.) up. These will hopefully be your permanent groups for the rest of the semester.

4. I will go through question 6 on the sheet with a model answer to show you the level of writing/explanation that I expect.

5. **From next week**, in the first 20 minutes I will ask you to take turns presenting your preprepared solution to the rest of your group, taking about 5 minutes each for this. I thought it would be a bit awkward to start with it on the first day before you got to know each other so today we will be a bit more informal about things.

6. Work in your groups on the assigned problems on the whiteboards. I will wander around and give feedback. You should all have prepared an answer to the highlighted problem on your sheet beforehand—everyone should have a go at leading your group through the problem you prepared. To get full marks for the attendance I need to see you talking about your assigned problem on the whiteboard!

[1] Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of Small-Group Learning on Undergraduates in Science, Mathematics, Engineering, and Technology: A Meta-Analysis. *Review of Educational Research* **69**(1), 21-51. https://doi.org/10.3102/00346543069001021

2 Model answer to Q6

We are asked to show that, if a and b are real numbers such that ab is irrational, then either a is irrational or b is irrational.

The theorem we are to prove is an implication of the form $P \implies Q$, where P is the statement 'ab is irrational' and Q is the statement 'a is irrational or b is irrational'. This implication is logically equivalent to its contrapositive $\neg Q \implies \neg P$, so it suffices to prove this instead.

Substituting the statements P and Q into the contrapositive and using the logical laws to simplify, we find that our original statement is equivalent to the statement 'if a and b are rational, then ab is rational'.

So suppose that a and b are rational. Then there exist integers p, q, r, s so that $a = p/q$ and $b = r/s$. Hence $ab = (p/q)(r/s) = (pr)/(qs)$. But pr and qs are products of integers and hence are integers. Therefore ab is rational, and we have proved the contrapositive of the desired theorem.