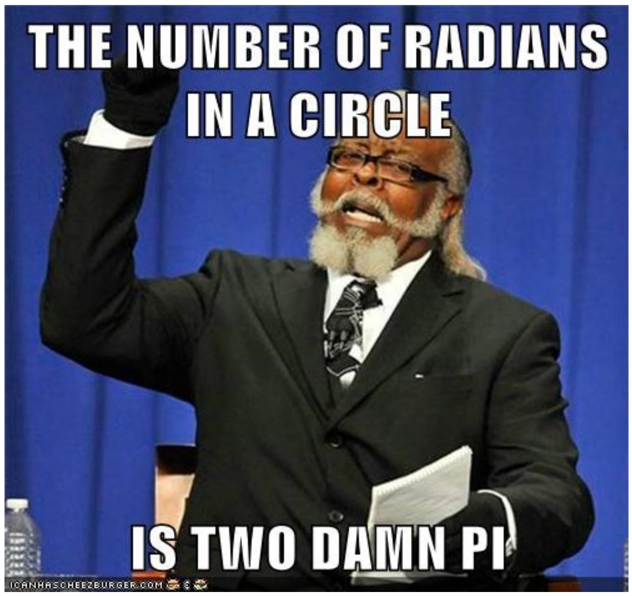

Assignment: Trigonometry

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Useful Formulae

Definitions

$$\sin\theta = \alpha_1(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$
$$\cos\theta = \alpha_2(x,y) = \frac{x}{\sqrt{x^2+y^2}}$$
$$\tan\theta = \alpha_3(x,y) = \frac{y}{x}$$
$$\cot\theta = \alpha_4(x,y) = \frac{x}{y}$$
$$\sec\theta = \alpha_5(x,y) = \frac{\sqrt{x^2+y^2}}{x}$$
$$\csc\theta = \alpha_6(x,y) = \frac{\sqrt{x^2+y^2}}{y}$$

Reciprocals

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta}$$

Double Angles

$$\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ \cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= 2\cos^2\theta - 1\\ &= 1 - 2\sin^2\theta\\ \tan 2\theta &= \frac{2\tan\theta}{1-\tan^2\theta}\end{aligned}$$

Compound Angles

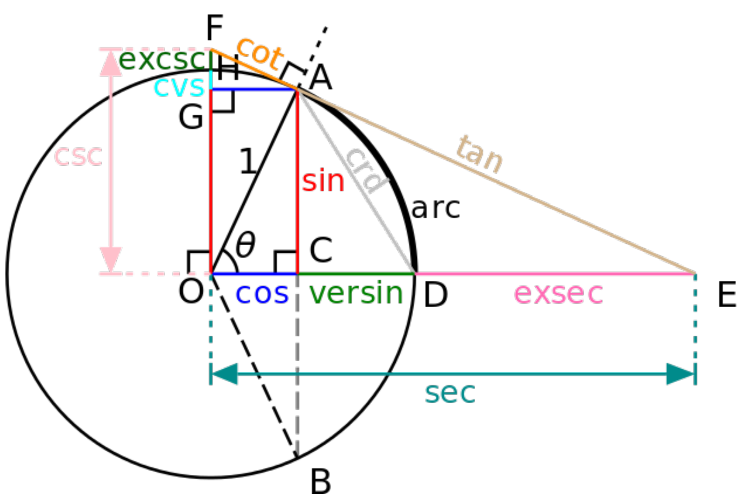
$$\begin{aligned}\sin(x\pm y) &= \sin x\cos y \pm \cos x\sin y\\ \cos(x\pm y) &= \cos x\cos y \mp \sin x\sin y\\ \tan(x\pm y) &= \frac{\tan x \pm \tan y}{1\mp\tan x\tan y}\end{aligned}$$

Products

$$\begin{aligned}2\sin x\cos y &= \sin(x+y) + \sin(x-y)\\ 2\cos x\sin y &= \sin(x+y) - \sin(x-y)\\ 2\cos x\cos y &= \cos(x+y) + \cos(x-y)\\ 2\sin x\sin y &= \cos(x-y) - \cos(x+y)\end{aligned}$$

Cos/Sin Rules

$$C^2 = A^2 + B^2 - 2AB\cos c$$
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



All the trig functions you'll ever need, and many more you won't.

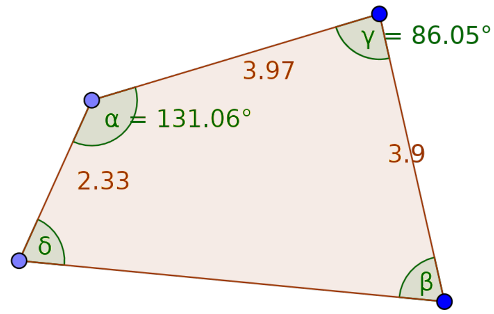


Figure 1: A large field modelled as a polygon.

Part A

1. A beam of gamma rays is to be used to treat a tumor known to be 5.7 cm beneath the patient's skin. To avoid damaging a vital organ, the radiologist moves the source over by 8.3 cm. [29/12]
   (a) At what angle to the patient's skin must the radiologist aim the gamma ray source to hit the tumor?
   (b) How far will the beam have to travel through the patient's body before reaching the tumor?

2. Suppose that one day all 4 million people in NZ climb up on tables. At time $t = 0$, we all jump off. The resulting shock wave as we hit the Earth's surface will start the entire earth vibrating in such a way that its surface first moves *down* from its normal position and then moves an equal distance *up* from its normal position. The displacement y of the surface is a sinusoidal function of time with a period of about 54 min. Assuming that the amplitude is 1 m, answer the following questions: [94/12]
   (a) At what time will the first *maximum* (i.e. the greatest distance *above* the normal position) occur?
   (b) Write an equation expressing displacement in terms of time lapsed since the people jumped.
   (c) Predict the displacement when $t = 21$.
   (d) What are the first *three* times at which the displacement is -0.74 m?

3. A surveyor measures the three sides of a triangular field and finds that they are 117, 165, and 257 metres long. [261/24]
   (a) What is the measure of the largest angle of the field?
   (b) What is the area of the field?

4. A field has the shape of a quadrilateral (four-sided shape) that is *not* a rectangle. Three sides measure 2.33, 3.97, and 3.9 kilometres, and two angles measure $\alpha = 131.06°$ and $\gamma = 86.05°$ (as in figure 1). [261/25, ggb file]
   (a) By dividing the quadrilateral into two triangles, find its area.
   (b) Find the length of the fourth side.
   (c) Find the measures of the other two angles, β and δ.

5. Suppose that the country of Parah has just launched two satellites. The government of Noya sends aloft its most reliable astronaut, Ivan Advantage, to observe the satellites. [262/27,29,30]
   (a) As Ivan approaches the two satellites, he finds that one of them is 8 km away from him and the other is at 11 km. He measures the angle between them to be 120°. How far apart are the satellites?
   (b) Three ships are assigned to rescue Ivan as his spacecraft plunges into the ocean. The ships are at the vertices of a triangle with sides of 5, 7, and 10 kilometres.
      i. Find the measure of the largest angle of this triangle.
      ii. Find the area of ocean in the triangular region bounded by the three ships.
   (c) To welcome their returning hero, the Noyans give Ivan a parade. The parade goes between the three cities of Triy, Kwe, and Stin. These cities are at the vertices of an equilateral triangle. The roads connecting them are straight, level, and direct, and the parade goes at a constant speed with no stops. From Triy to Kwe takes 80 min and from Kwe to Stin takes 80 min, but from Stin back to Triy along the third side of the triangle takes one hour and 20 minutes. How do you explain this discrepancy in times?

Part B

6. Use the definitions of the trig functions to prove the following identities:
   (a) $\sin^2\theta + \cos^2\theta = 1$;
   (b) $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$; and
   (c) $\sec\theta\cot\theta = \csc\theta$.

7. In which quadrant(s) of the Cartesian plane does the function $\tan x$ take a positive value while the function $\csc x$ take a negative value?

8. Let i be a constant. Show that $\cos(-x) + i\sin(-x) = \cos x - i\sin x$.

9. This question requires you to find exact values for trig functions *without* using a calculator. [Schol 1999 Q1(a)]
   (a) Sketch an equilateral triangle of side 2 units. Hence, or otherwise, show that
      - $\cos\frac{\pi}{3} = \frac{1}{2}$,
      - $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and
      - $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$.

   (b) Sketch a right angled isosceles triangle whose equal sides enclose the right angle and are of length 1. Hence, or otherwise, show that $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

10. Find all solutions for x of $\frac{2\tan x}{1-\tan^2 x} = -1$.

11. Suppose $2\sin\phi\cos\phi = \sqrt{2}\cos\pi$. Find (exactly) all possible values of ϕ within the interval $-\pi \leq \phi \leq \pi$.

12. Find the *global minimum* of $y = x^2 - \cos(10x)$.

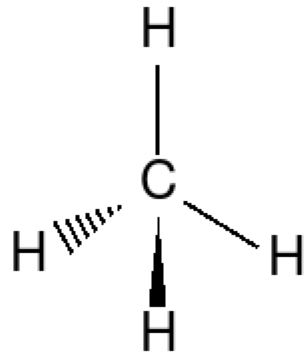


Figure 2: A molecule of methane.

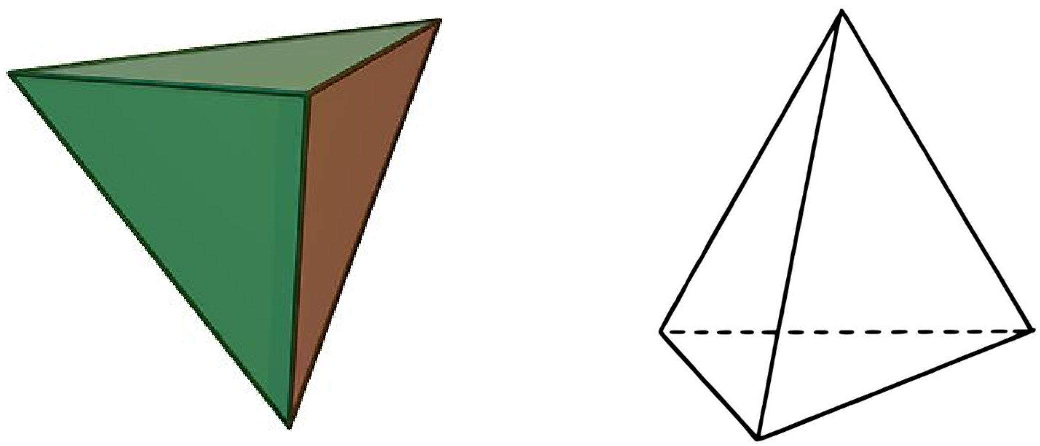


Figure 3: Different views of a tetrahedron.

Part C

13. The methane molecule (pictured in figure 2) is completely symmetrical, and the hydrogen atoms can be joined with lines to form a regular tetrahedron (see figure 3). Due to this symmetry, the angle between adjacent bonds is constant for every methane molecule. What is the measure of this angle?