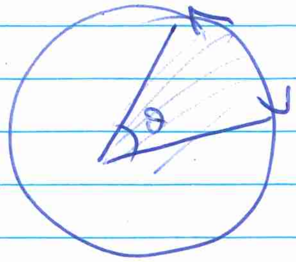
School trig probs 1.

1) a. δ° is $\frac{\delta}{360}$ fraction of a circle, i.e. $\frac{\delta}{360} \cdot 2\pi$ rad
$= \frac{\delta\pi}{180}$ rad.

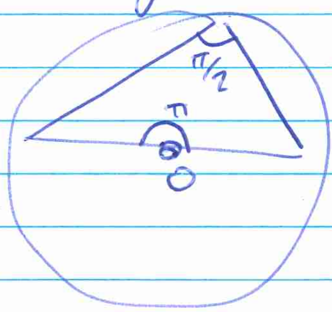
b.



$$\frac{\theta}{2\pi} \cdot A_{total} = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{\theta r^2}{2}.$$

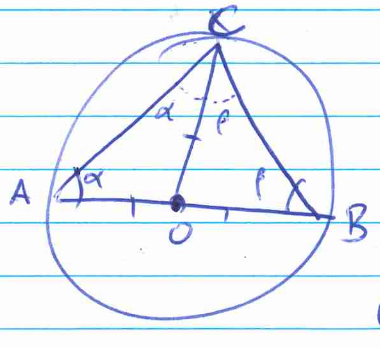
$$\frac{\theta}{2\pi} \cdot C_{total} = \frac{\theta}{2\pi} \cdot 2\pi r = \theta r.$$

2) a. Via inscribed angle thm:



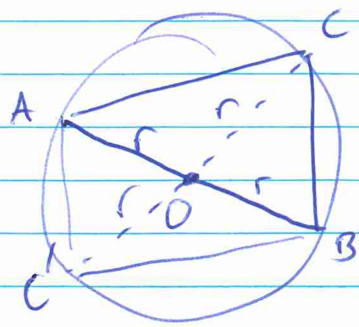
Angle on circumfn is ½ angle at centre; angle at centre is π, so angle at circ. is $\pi/2$.

b. Via angle pushing.



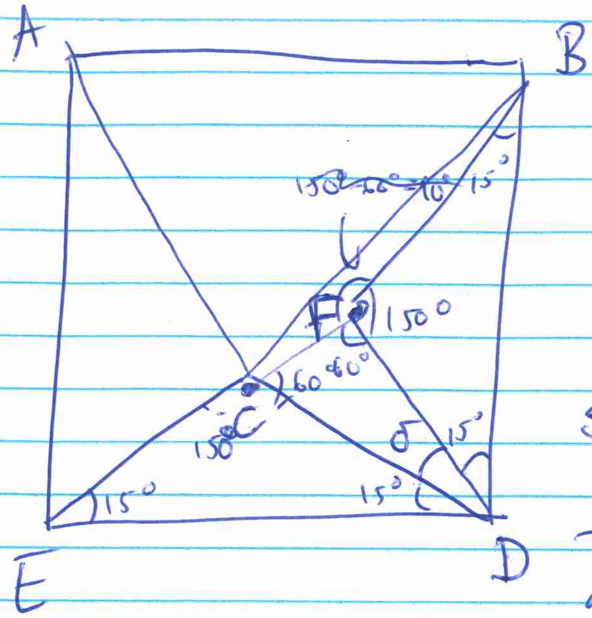
Let C be the point on the circumfn as given in the problem, and let O be the centre. Then $|OA| = |OB| = |OC|$ (= radius). $\therefore$ angle at C is $\alpha+\beta$ (since AOC and AOB isos.)
$\therefore \pi = \alpha+\beta+(\alpha+\beta)$
$\therefore \underline{\alpha+\beta = \pi/2}$.

c.



Let C' be the other end of the diagonal through C. Then ACBC' is a quadrilateral with equal diagonals (= 2r) and so is a rectangle.

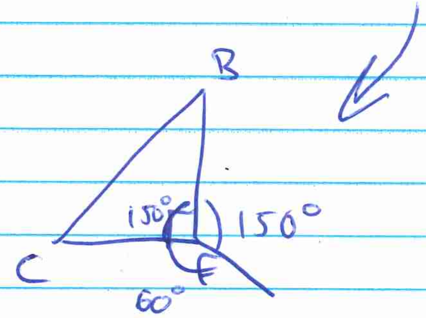
3.



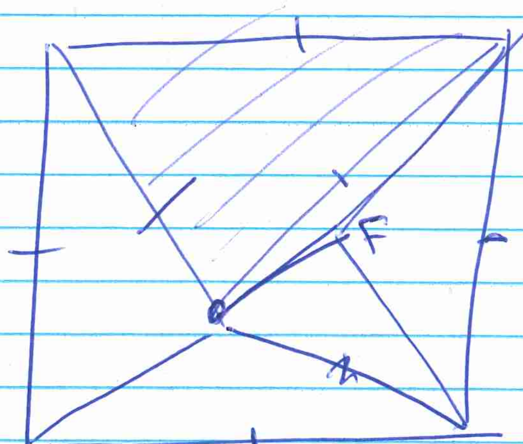
$\delta = 90° - 15° - 15°$
$= 60°$.

Since $ECD \cong DFB$,
$|DC| = |FD|$.
Thus CDF has two $=$ sides and an angle between them of $60° \Rightarrow \triangle CDF$ is equilateral.
$\therefore |CF| = |CD|$.

But $|FD| = |FB|$; so CFB is of the form



Which is $\cong$ to ECD.



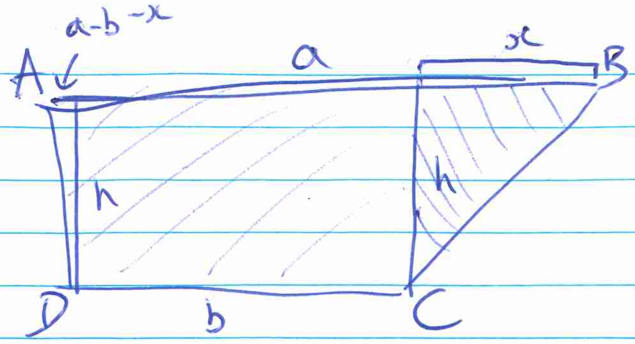
$\therefore |BC| = |ED|$.

By same argument, $|AC| = |ED|$.

Hence ABC is equilateral.

(Do not trust my pictures).

4)

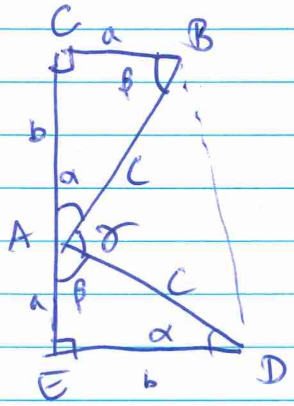


$$A(ABCD) = \underbrace{hb}_{\text{section}} + \underbrace{\tfrac{1}{2}xh}_{\text{right hand } \Delta} + \underbrace{\tfrac{1}{2}(a-b-x)h}_{\text{left-hand } \Delta}$$

$$= hb + \frac{1}{2}(a-b)h$$

$$= \frac{1}{2}(a+b)h. \quad \square.$$

5 a)



Angle sum of ABC: $\cancel{\pi} = \pi/2 + \alpha + \beta$.

But at A, $\pi = \alpha + \beta + \gamma$.
Hence $\gamma = \cancel{\alpha +}\ \pi/2$.

b) $A = \frac{1}{2}\underbrace{(a+b)}_{\text{height}}\underbrace{(a+b)}_{\text{sum of bases}} = \frac{1}{2}(a+b)^2$

$A = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2.$

c) $\therefore \underset{\substack{\parallel \\ a^2+2ab+b^2}}{(a+b)^2} = 2ab + c^2$

$\therefore a^2 + b^2 = c^2$ ✓