NCEA Level 3 Calculus
Differentiation Assignment

1. (a) (2 points) If $y = 3x^2 + 2x - \frac{1}{\sqrt{x+1}} + \frac{e^x}{\sin x}$, find $\frac{dy}{dx}$.

   (b) An important mathematical skill is the ability to write down examples of objects satisfying certain properties.

      i. (2 points) Draw the graph of a function f passing through $(0,1)$ such that $f'(x) < 0$ for all x, but $f''(x) > 0$ for all x.

      ii. (1 point) Give an explicit, simple example of such a function.

   (c) (3 points) Show that a solution to the differential equation

   $$\frac{dx}{dt} = rx(1-x)$$

   is given by

   $$x(t) = \frac{1}{1 + \left(\frac{1}{x_0} - 1\right)e^{-rt}}.$$

2. Let f be the function defined by

   $$f(x) = \sin(\tan(x + \pi/6) + \cos(4\pi))\ln(1/x).$$

   (a) (2 points) Compute the derivative of $\tan(x + \pi/6) + \cos(4\pi)$ with respect to x.

   (b) (3 points) Write down explicitly $f'(x)$.

   (c) (3 points) Give the equation of the best linear approximation to $f(x)$ at $\left(\pi, -\ln(\pi)\sin\left(1 + \frac{1}{\sqrt{3}}\right)\right)$, giving constants to three decimal places.

3. (a) (2 points) Compute the derivative of $(x+1)^3$ using the definition of the derivative.

   (b) (3 points) Show that $\lim_{x \to 0} \ln x$ does not exist, by showing that for all L there exists some $\delta > 0$ such that if $x < \delta$ then $\ln x < L$.

   (c) (3 points) Show that $\lim_{x \to \infty} \frac{\sin x}{x} = 0$. [Hint: use the fact that $-1 \leq \sin x \leq 1$, and write $A(x) \leq \frac{\sin x}{x} \leq B(x)$ for two functions A and B that both go to zero as x grows].