NCEA Level 2 Mathematics (Homework)
13. Turning Points and Optimisation

Reading

Go and watch...

https://www.youtube.com/watch?v=F5RyVWI4Onk

Polya's four-step approach to problem solving

1. *Preparation:* Understand the problem
   - Learn the necessary underlying mathematical concepts
   - Consider the terminology and notation used in the problem:
     (a) What sort of a problem is it?
     (b) What is being asked?
     (c) What do the terms mean?
     (d) Is there enough information or is more information needed?
     (e) What is known or unknown?
   - Rephrase the problem in your own words.
   - Write down specific examples of the conditions given in the problem.
2. *Thinking Time:* Devise a plan
   - You must start somewhere so try something.
   - How are you going to attack the problem?
   - Possible strategies: (i.e. reach into your bag of tricks.)
     (a) Draw pictures
     (b) Use a variable and choose helpful names for variables or unknowns.
     (c) Be systematic.
     (d) Solve a simpler version of the problem.
     (e) Guess and check. Trial and error. Guess and test. (Guessing is OK as long as you can back it up later.)
     (f) Look for a pattern or patterns.
     (g) Make a list.
   - Once you understand what the problem is, if you are stumped or stuck, set the problem aside for a while. Your subconscious mind may keep working on it.
   - Moving on to think about other things may help you stay relaxed, flexible, and creative rather than becoming tense, frustrated, and forced in your efforts to solve the problem.
3. *Insight:* Carry out the plan
   - Once you have an idea for a new approach, jot it down immediately. When you have time, try it out and see if it leads to a solution.
   - If the plan does not seem to be working, then start over and try another approach. Often the first approach does not work. Do not worry, just because an approach does not work, it does not mean you did it wrong. You actually accomplished something, knowing a way does not work is part of the process of elimination.

- Once you have thought about a problem or returned to it enough times, you will often have a flash of insight: a new idea to try or a new perspective on how to approach solving the problem.
- The key is to ***keep trying until something works***.

4. *Verification:* Look back

   - Once you have a potential solution, check to see if it works.
     (a) Dd you answer the question?
     (b) Is your result reasonable?
     (c) Double check to make sure that all of the conditions related to the problem are satisfied.
     (d) Double check any computations involved in finding your solution.
   - If you find that your solution does not work, there may only be a simple mistake. Try to fix or modify your current attempt before scrapping it. Remember what you tried — it is likely that at least part of it will end up being useful.
   - Is there another way of doing the problem which may be simpler? (You need to become flexible in your thinking. There usually is not one right way.)
   - Can the problem or method be generalized so as to be useful for future problems?

Questions

1. A cylindrical tube (with open ends) is to be made from a sheet of paper with area $25\,\text{cm}^2$. What should the dimensions of the tube be in order to maximise the volume of the tube? Justify that you have found a maximum.

2. A function f is given by $f(x) = 2x^3 - 3ax^2 + 6bx - 2$. The function has two turning points, at $x = 2$ and at $x = 3$. What are the values of a and b?

3. Suppose a wire of length ℓ is cut into two pieces, one of length x and one of length $\ell - x$. One piece is used to form the circumference of a circle, and the other is used to form the perimeter of a square. How long should the length x be in order to ensure that the total area of the circle and the square is minimised? Carefully justify that you have found a minimum.