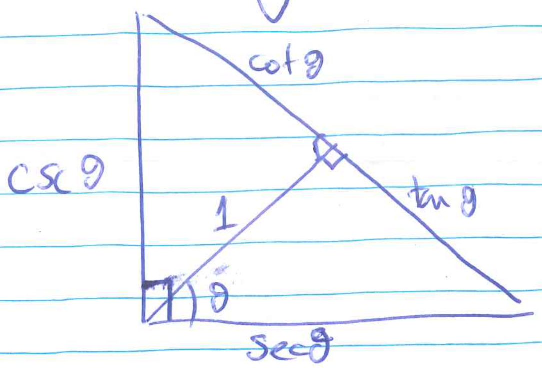
Schol Trig prob set 3.

1 a)

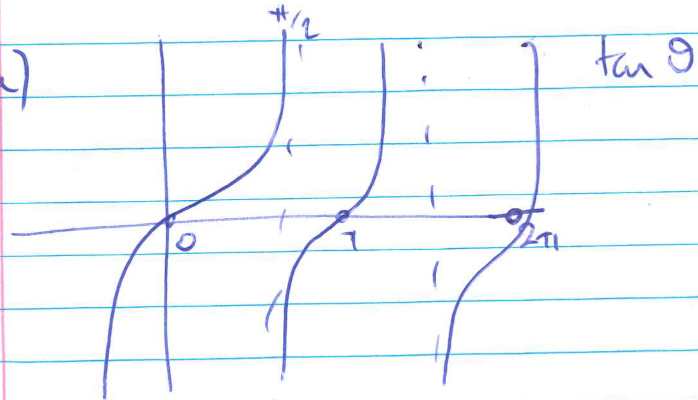


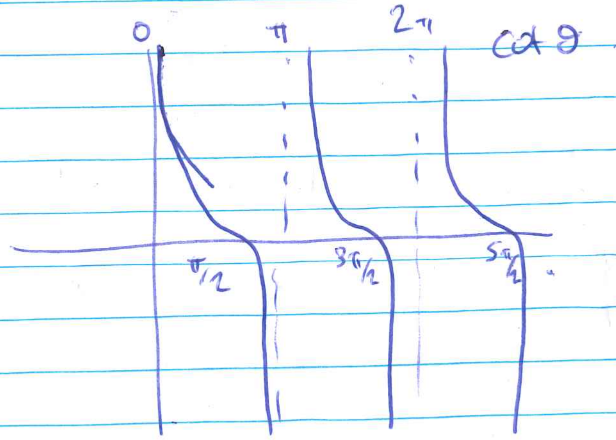
$$\csc^2\theta + \sec^2\theta$$
$$= (\cot\theta + \tan\theta)^2$$
$$= \left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)^2$$
$$= \left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)^2$$
$$= \frac{1}{\sin^2\theta\cos^2\theta} = \sec^2\theta\csc^2\theta.$$

b) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \csc\theta)^2$

$$= \left(\sin\theta + \frac{1}{\cos\theta}\right)^2 + \left(\cos\theta + \frac{1}{\sin\theta}\right)^2$$
$$= \sin^2\theta + 2\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos^2\theta} + \cos^2\theta + 2\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin^2\theta}$$
$$= 1 + 2\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right) + \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$$
$$= 1 + 2\frac{1}{\sin\theta\cos\theta} + \frac{1}{\sin^2\theta\cos^2\theta} = 1 + 2\sec\theta\csc\theta + \sec^2\theta\csc^2\theta$$
$$= (1 + \sec\theta\csc\theta)^2.$$

2a)





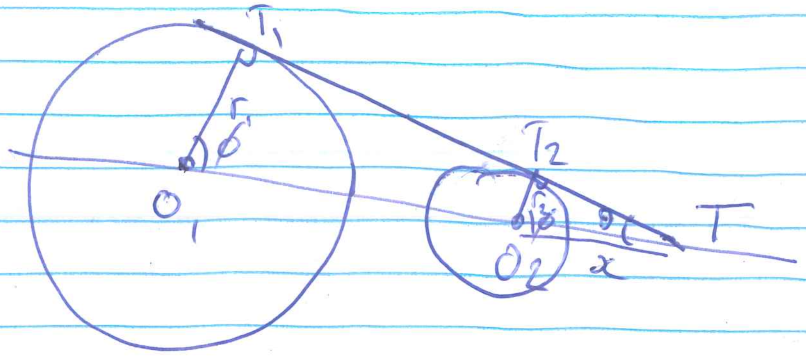
b) $\cancel{\tan}\ -\cot(\theta - \pi/2) = -\frac{\sin(\theta - \pi/2)}{\cos(\theta - \pi/2)} = +\frac{\sin(\pi/2 - \theta)}{\cos(\theta - \pi/2)}$

$$= \frac{\cos\theta}{\sin\theta} = \tan\theta.$$

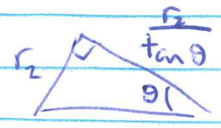
c) Suppose the ~~ws~~; $|\sec x| = |1/\cos x| \geq 1$ for all x, and $\cos y = 0$ for $y = \pi/2$.

$$\therefore\ |0| = |\cos\pi/2| \stackrel{?}{=} |\sec(\pi/2 + \xi)| \geq 1 \ \lightning.$$

3. a)



b) $T_1O_1T \sim T_2O_2T$. Here



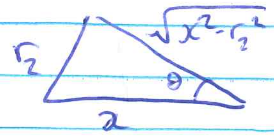
$$\frac{r_2}{x} = \frac{r_1}{|O_1O_2|+x}$$

$$r_2(|O_1O_2|+x) = r_1 x$$

$$r_2|O_1O_2| = (r_1-r_2)x$$

$$r_2\frac{|O_1O_2|}{(r_1-r_2)} = x.$$

Thus $\sin\theta = r_2 \Big/ r_2\frac{|O_1O_2|}{r_1-r_2} = \frac{r_1-r_2}{|O_1O_2|}$.



c) ~~On~~ $|T_2T| = \frac{r_2}{\tan\theta}$ $\quad |T_1T| = \frac{r_1}{\tan\theta}$

$\therefore |T_1T_2| = \frac{r_1-r_2}{\tan\theta}$

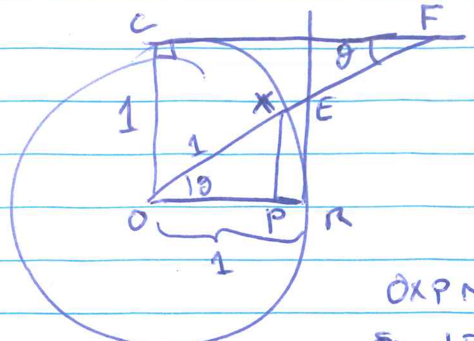
$\cancel{\tan\theta = \frac{\sin\theta}{\cos\theta} = \left(\frac{r_1-r_2}{|O_1O_2|}\right)}$

$$\tan\theta = r_2 \Big/ \sqrt{r_2^2\frac{|O_1O_2|^2}{(r_1-r_2)^2} - r_2^2} = \frac{1}{\sqrt{\frac{|O_1O_2|^2}{(r_1-r_2)^2}-1}}$$

$$\therefore |T_1T_2| = (r_1-r_2)\sqrt{\frac{|O_1O_2|^2}{(r_1-r_2)^2}-1}$$

$$= \sqrt{|O_1O_2|^2-(r_1-r_2)^2}.$$

4.



$OXP \sim OER$

1. $|ER| = \frac{|ER|}{|OR|} = \frac{|XP|}{|OP|} = \frac{\sin\theta}{\cos\theta} = \tan\theta.$

2. From triangle OCF, $|FC| = \frac{1}{\tan\theta} = \cot\theta.$

3. From triangle EOR, $|EO| = \sqrt{1^2 + |ER|^2}$
$= \sqrt{1+\tan^2\theta}$
$= \sec\theta$ (3.3)

4. From triangle FOC, $|FO| = \sqrt{1+\cot^2\theta}$ $|CF|$
$= \csc\theta$ (3.6.1).

5a. See over page.

b. $\text{Versin}\, x = 1-\cos x$. So claim is that
$2x = (2n+1)\pi(1-\cos x) = (2n+1)\pi - (2n+1)\pi\cos x$

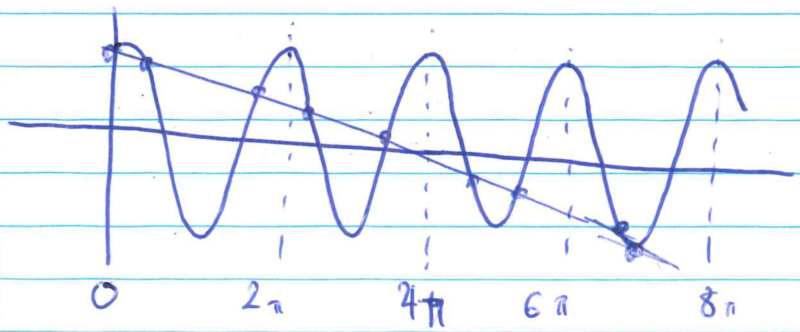
has $2n+3$ solutions.

Rearrange: $\cos x = 1 - \frac{2}{(2n+1)\pi}x.$

Only intersections will be when $-1 \le 1 - \frac{2}{(2n+1)\pi}x \le 1$

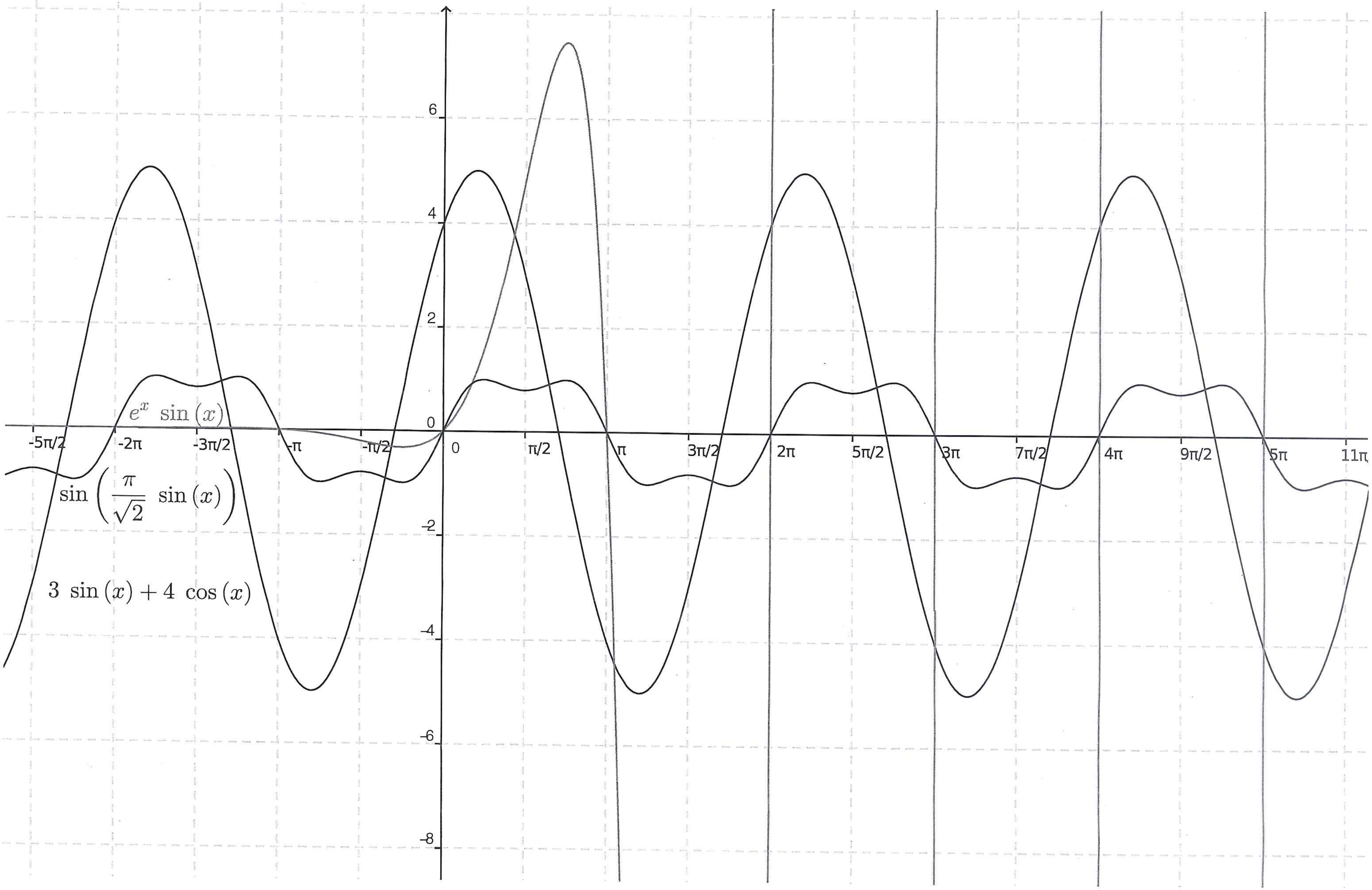
i.e. $0 \le \frac{2}{(2n+1)\pi}x \le 2$

$0 \le x \le (2n+1)\pi.$



At $x=0$, $1-\frac{2}{(2n+1)\pi}\cdot 0 = 1$;
and at $x = (2n+1)\pi$,
$1 - \frac{2}{(2n+1)\pi}\cdot(2n+1)\pi = -1.$
So we have a line, which clearly crosses $\cos x$ twice for each oscillation $(2\cdot(n+1)) + 1$ extra.

$n=3$, $9 = 2\cdot3+3$ intersection points.


6
4
2
0
-2
-4
-6
-8
-5π/2
-2π
-3π/2
-π
-π/2
0
π/2
π
3π/2
2π
5π/2
3π
7π/2
4π
9π/2
5π
11π
$e^x \sin(x)$
$\sin\left(\frac{\pi}{\sqrt{2}} \sin(x)\right)$
$3 \sin(x) + 4 \cos(x)$