L3 Physics: Revision questions for part 1 (Mechanics)

Questions 1 and 2 are from Howison, chapter 5. Questions 4, 5, 6, and 7 are from past exams (2014, 2018, 2014 and 2014, respectively).

1. John is a trampoline artist. He warms up by working his arms and legs until he is jumping high and vertically above the trampoline mat.

   On one occasion as he leaves the mat, John's total kinetic energy (both linear and rotational) is 2200 J. At the top of the jump his linear speed is momentarily zero and he has gained 1400 J of gravitational potential energy.

   (a) At the top of the jump, assuming mechanical energy is conserved, calculate (i) John's height above the mat, and (ii) John's rotational kinetic energy.

   (b) At the top of the jump, the angular speed of John's sumersault is $18\,\mathrm{rad\,s^{-1}}$. Calculate John's rotational inertia about his centre of mass.

   (c) To complete the sumersault while in the air, John folds his body into a tucked position, causing it to rotate at a faster rate. At the end of the rotation he straightens his body and lands back on the mat in an upright position. Explain why this tucking causes John's body to rotate faster than if he remains in the untucked position.

2. A solid, uniform disc rolls from rest down a ramp of height h. At the bottom of the ramp the disc has translational speed v. Show that $v = \sqrt{\frac{4}{3}gh}$.

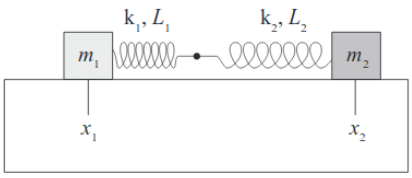
3. A space station is shaped like a long, hollow cylinder spinning around its long axis. The radius of the cylinder is 1 km, and the cylinder is rotating with an angular speed ω. How large must ω be for a person standing on the inside of the cylinder to feel a centripetal acceleration equal to the usual pull of gravity ($9.81\,\mathrm{m\,s^{-1}}$)?

4. The mass of Mercury is 3.30×10^{23} kg. Mercury has a period of rotation of 5.067×10^{6} s. Show that a satellite needs to be positioned 2.43×10^{8} m from the centre of Mercury so that it remains stationary from the point-of-view of an observer on that planet.

5. (a) A spring of length L and spring constant k is cut into two parts of lengths L_1 and L_2 with spring constants of k_1 and k_2 respectively. Show that

   $$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}.$$

   (b) Consider the situation shown below with two springs of spring constants k_1 and k_2 connected together and linking masses m_1 and m_2, which sit on a frictionless surface. The equilibrium lengths of the springs are L_1 and L_2, respectively, and the centre of mass of m_1 lies at x_1 and the centre of mass of x_2 lies at x_2.

   Mass m_1 is held fixed while a force F_0 is applied to mass m_2 in the direction of mass m_1. Using the result of part (a), show that the change in position of mass m_2 is

   $$\Delta x_2 = F_0 \frac{k_1 + k_2}{k_1 k_2}.$$

   (c) Both masses are now released simultaneously. The values for the masses and spring constants are $m_1 = 2.0$ kg, $m_2 = 3.0$ kg, $k_1 = 5.0\,\mathrm{N\,m^{-1}}$, and $k_2 = 10\,\mathrm{N\,m^{-1}}$, and the force $F_0 = 2.0$ N. Assume the mass of the springs is negligible compared to m_1 and m_2.

   Describe in detail the resulting motion of the two masses.

(d) Show that the maximum velocity reached by mass m_2 is $0.40\,\mathrm{m\,s^{-1}}$.

(e) Explain how the motion of the system would be altered if the mass of the springs was not negligible compared to m_1 and m_2.

6. A vertical pendulum is set up, hanging from a ceiling. The length of the cord (of negligible mass) is 1.55 m. The bob has a mass of 1.80 kg.

   (a) Calculate the length of time it takes for the bob to swing from one side to the other.

   (b) Explain how the forces acting on the bob change the bob's speed as it travels from the point of release to the centre.

   (c) The bob is released again in such a way that it swings in a horizontal circular path, with radius 0.290 m, as a conical pendulum.

      i. By first calculating the size of the angle that the cord makes with the vertical, show that the tension force in the cord is 18.0 N.

      ii. Calculate the speed that the mass must have been given when released, in order to attain a horizontal circular path at a radius of 0.290 m.

7. A system consists of two discs, A and B, attached together with a light cord. The discs slide across a frictionless surface. Disc A has mass 0.517 kg and disc B has mass 0.684 kg. Disc B is stationary, and disc A is moving towards disc B with a speed of $1.21\,\mathrm{m\,s^{-1}}$.

   (a) Show that the speed of the centre of mass of the system is $0.521\,\mathrm{m\,s^{-1}}$.

   (b) The discs collide and after the collision they are moving at right angles to each other. Disc A receives an impulse of 0.250 N s.

      i. Show that the speed of disc B after the collision is $0.365\,\mathrm{m\,s^{-1}}$.

      ii. Determine the size of the momentum of disc A after the collision.

   (c) The discs continue to slide until the cord is fully extended. When this happens, both discs change their speed and direction. By considering the force(s) that act on the discs, explain why the momentum of the system must be conserved.

8. What is the escape velocity of a human from Earth? State any assumptions you make.

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