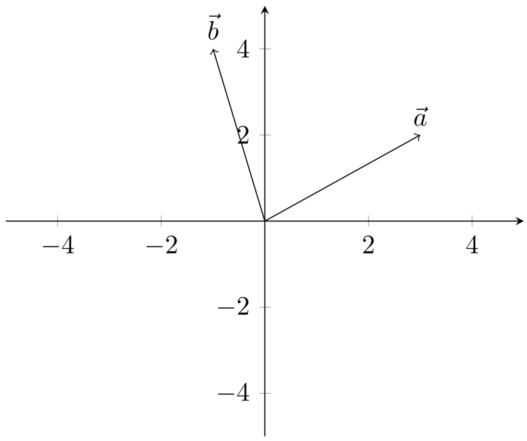
L2 Physics: Problems on mechanics

See the final page for useful formulae and constants.

Sections 2.1-2.2

Main concepts you should understand: vector, displacement, velocity, acceleration

1. Below are drawn vectors $\vec{a}$ and $\vec{b}$. Draw (a) $\vec{a}+\vec{b}$; (b) $\vec{a}-\vec{b}$.



2. A car decelerates from $\vec{v}_0 = 5\,\mathrm{m\,s^{-1}}$ to $\vec{v}_1 = 1\,\mathrm{m\,s^{-1}}$. Draw a vector diagram depicting the two velocity vectors, and the change in velocity.

3. Suppose one starts from point A and walks south 230 m and then walks 350 m west of north to a point B. Find the distance from A to B as the crow flies.

4. A runner runs 3400 m in 10 minutes and 33 seconds; what was her average speed in metres per second?

5. A car moving at $25\,\mathrm{m\,s^{-1}}$ stops suddenly in 14.0 s. What was average acceleration of the car during this time, and how far did the car travel before stopping?

6. A bullet moving at a speed of $150\,\mathrm{m\,s^{-1}}$ travels 3.5 cm into a block of glass before stopping. Find the average acceleration of the bullet, and the time taken for it to stop.

7. A stone is thrown straight upwards from the ground, and goes as high as a nearby building. The stone reaches the ground 3.0 s after it is thrown. How tall is the building?

8. A piano is pushed out of a window that is 8 m above a person sitting below. How fast is it moving when it hits the person, and how long do they have to get out of the way?

9. A train blocks a crossing; it takes 20 s for a carriage to pass through a distance equal to the length of one carriage as the train starts to move. Assuming the length of a carriage is L and the acceleration of the train is constant, how long will it take for the next twenty railway carriages to pass by?

10. A car is moving at $30\,\mathrm{m\,s^{-1}}$. Assuming an average deceleration of $7.0\,\mathrm{m\,s^{-1}\,s^{-1}}$, how long will it take for the car to stop?

11. A ball is thrown straight upwards with a speed v from a point h metres above the ground. Show that the time taken for the ball to strike the ground is

$$\frac{v}{g}\left(1+\sqrt{1+\frac{2hg}{v^2}}\right).$$

Section 2.3

Main concepts you should understand: force, torque, acceleration, equilibrium

1. A 7.0 g bullet is given an acceleration of $40.000\,\mathrm{m\,s^{-1}\,s^{-1}}$ as it is shot from a gun with a 6.0 cm barrel. How large is the average force on the bullet during this process?

2. How much force does an 70 kg person exert on the earth due to gravity?

3. A 1300 kg car must be stpped in a distance of 80 m. The initial speed of the car is $20\,\mathrm{m\,s^{-1}}$. What force needs to be exerted, assuming constant deceleration?

4. A light spring is hung vertically; with no mass attached, it has a length 6.0 cm. As different masses are added, its stretched length is measured; the values are tabulated below.

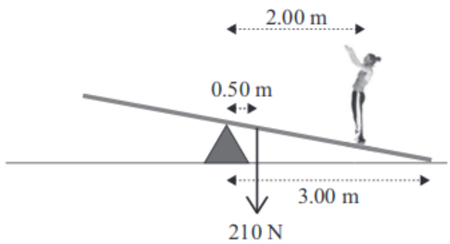
Mass (g)	**Spring length (cm)**
0	6.00
50	6.63
100	7.26
200	8.52
300	9.77

   (a) Graph the mass added to the spring against the change in length of the spring. Use the x–axis to denote the added mass in g, and use the y–axis to denote the change in length in cm.

   (b) Explain why the slope of your graph is $\frac{g}{10k}$, where g is the acceleration due to gravity and k is the spring constant of the spring. (Hint: the units you are using are *not* kg and m.)

   (c) Using your graph, or otherwise, calculate the spring constant k of the spring.

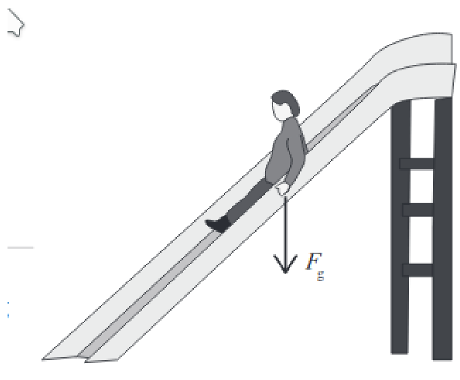
5. (NZQA 2012) Hannah (55 kg) stands on a see-saw. The see-saw has a weight of 210 N. Calculate the size and direction of the force that the floor exerts on the right hand end of the see-saw.



6. (NZQA 2013) Jason is at a fair of some description.

   (a) Jason goes for a ride on a go-kart. Towards the end of the ride, he decelerates at $2.5\,\mathrm{m\,s^{-1}\,s^{-1}}$ and comes to a stop in 4.2 seconds. By calculating Jasons initial velocity, determine the distance he travels while coming to a stop.

   (b) Jason sits on a slide. He is sliding down at constant speed.

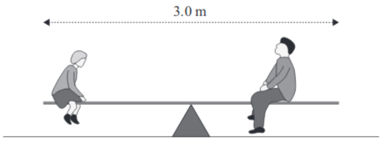


      i. State the size of the net force on Jason. Explain your answer.

ii. On the diagram on the right, draw the remaining forces (as labelled vectors) acting on Jason.

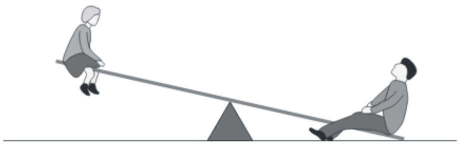
iii. Complete and label the vector addition diagram of the forces acting on Jason.

7. (NZQA 2013) The diagram below represents a see-saw on a pivot at its centre with Jane and her dad sitting on opposite sides such that the see-saw is in equilibrium. The mass of the see-saw itself is 60 kg.



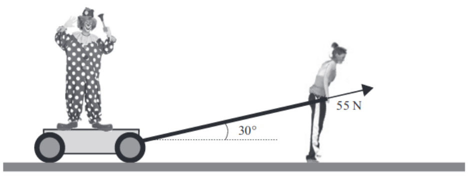
(a) On the diagram above, draw labelled vectors to show all the forces acting on the see-saw.

(b) Jane and her dad move to opposite ends of the see-saw. The diagram below shows what happens when Jane sits at one end of the see-saw while her dad sits at the other end.



If Jane's mass is 30 kg and Jane's dad's mass is 72 kg, calculate the size of the support force from the ground at the end where Jane's dad is sitting. Round your answer to the correct number of significant figures.

8. (NZQA 2012) The Clown makes his entrance riding on a cart pulled by Hannah. The clown and cart have a combined mass of 85 kg. The handle of the cart makes an angle of 30° to the horizontal as shown in the diagram below. Hannah applies a force of 55 N to the handle.



(a) Calculate the size of the horizontal component of the force on the handle.

(b) The cart is in equilibrium.
   - State what "equilibrium" means in terms of the forces acting on the cart.
   - Describe what it tells you about the velocity of the cart.
   - On the diagram above, draw labelled arrows showing the direction of any non-vertical forces acting on Hannah.

(c) Explain how Hannah can make the cart and clown accelerate without changing the size of the force she exerts on the handle. (Reducing friction is not a possibility.)

9. A box of mass m slides down an incline that makes an angle θ with the vertical, with an acceleration a. A friction force f impedes its motion. (a) Draw a free body (force) diagram; (b) find the acceleration a in terms of θ, m, and f.

10. The weight W of an object is defined to be the force exerted on it by gravity; so $W = mg$, where m is the mass of the object. If a three ton (3000 kg) car is travelling at $40\,\mathrm{km\,h^{-1}}$ and the brakes are suddenly applied, and it skids to rest, how far does the car skid given that the friction force experienced by the tires is around 0.7 times the weight of the car?

11. Newton's law of gravity tells us that if we have two objects A and B, with masses m_A and m_b, such that the distance between them is r, then the force that A exerts on B due to gravity is

$$F = \frac{Gm_Am_B}{r^2}.$$

Given that $F = ma$, the units of force are $\mathrm{N} = \mathrm{kg\,m\,s^{-1}\,s^{-1}}$. What are the units of the gravitational constant G?

12. Suppose a horizontal spring is fixed at one end, and a mass is attached to the other end in such a way that it is free to move back and forth along some surface, stretching and compressing the string. Suppose the mass is gently pulled away from the fixed end, and let go.
    (a) Draw a series of diagrams, each depicting one particular instant of motion of the mass, and showing the force(s) acting on the mass at each instant.
    (b) Where is the mass positioned when the following quantities are maximised or minimised:
        i. The net force felt by the mass.
        ii. The acceleration of the mass.
        iii. The velocity of the mass.

Sections 2.4-2.5

Main concepts you should understand: momentum, energy (kinetic, gravitational potential, elastic potential), conservation laws of momentum and energy, impulse, elastic/inelastic collision.

1. A 1500 kg car, travelling at 20 metres per second, reduces its speed over 3.0 s to $15\,\mathrm{m\,s^{-1}}$. Calculate the impulse felt by the car. What was the average force causing the deceleration?
2. Find the momentum of an 800 g object after it falls freely from rest a height of 60 cm.
3. A 120 g ball moving at $18\,\mathrm{m\,s^{-1}}$ hits a wall perpendicularly and rebounds with the same speed. After intially touching the wall, the centre of the ball moves an extra 0.27 cm towards it before rebounding. (a) Assuming constant deceleration, show that the total time the ball is in contact with the wall is 2×0.00030s. (b) What is the average force the ball exerts on the wall?
4. A 60 kg astronaut becomes seperated from her ship; she is 15.0 m away and at rest relative to it. She throws a 500 g spanner at a speed of $8.0\,\mathrm{m\,s^{-1}}$ in a direction away from the ship; how long does it take her to get back to the ship?
5. An object of mass 2.0 kg falls, from rest, through a height h. When it reaches the bottom, its speed is measured as $4\,\mathrm{m\,s^{-1}}$. Calculate h. How long did the object take to fall through the height?
6. A 2000 kg car is travelling at $20\,\mathrm{m\,s^{-1}}$ up a frictionless hill when the motor stops. (a) If the car is a vertical distance of 8 m from the top of the hill at that point, will it be able to reach the top? (b) How far below the top of the hill could the car be to still reach the top?
7. What power does a 60 kg person develop when they lift themselves 12.0 m in 20.0 s using a flight of stairs?
8. A 20.0 kg crate is pushed 6.0 m along the floor at a constant speed by a force inclined 30° below the horizontal. (a) Describe the changes of energy occuring as the crate moves along the floor. (b) If the friction force retarding the motion is 140 N, draw a free body diagram and calculate the net force acting on the crate. (c) How much work is done by the pushing force?
9. (NZQA 2012) Jess is a trapeze artist at the circus. As part of her act she hangs on a long rope and swings downwards. When she gets to the lowest point she grabs onto Hannah (another performer) and they keep moving together. Jess has a mass of 65 kg; Hannah has a mass of 55 kg.
    (a) Name the quantity that is conserved as Jess swings down.
    (b) Name the quantity that is conserved as Jess graps Hannah and they swing together.
    (c) Immediately after Jess grabs Hannah, they move together at a speed of $5.5\,\mathrm{m\,s^{-1}}$. Calculate the vertical height that Jess dropped down.
10. (NZQA 2012) Hannah flies through the air and lands on an elastic rope, which is held under tension between two supports.
    (a) Name the main energy changes that occur as Hannah is falling AND as she is coming to a stop.

(b) Hannah doesnt like the rope to be too tight when she lands on it. State the direction of the force on her from the rope. Explain, in terms of the force acting on Hannah, why the rope should not be too tight when she lands on it.

(c) An elastic rope is suspended from a beam so that it is hanging vertically down. Hannah hangs vertically down on the elastic rope. The rope is stretched 0.60 m below its normal position when Hannah hangs from it. Calculate the elastic potential energy stored in the elastic rope. (Hannah has a mass of 55 kg.)

11. (NZQA 2013) Jason is still at some kind of fair. Each bumper car in the fair has a rubber bumper all round it.

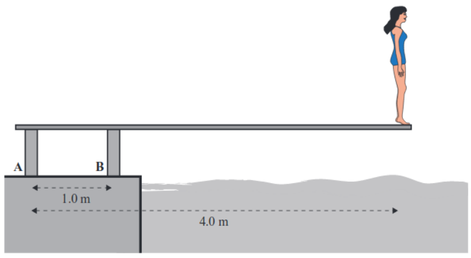
(a) The mass of a bumper car is 240 kg. Jason has a mass of 65 kg and is travelling at a speed of $2.4\,\mathrm{m\,s^{-1}}$. Calculate the size of the momentum of Jason and his bumper car.

(b) The bumper cars are designed to minimise injury. Discuss the reasons for the bumper cars having rubber bumpers all round them. Assume cars with and without bumpers have the same mass. Assume change in velocity is the same with and without bumpers.

(c) Jason collides head-on with Janet who is in another bumper car. The bumpers dont work properly and after collision both cars lock together. The mass of each bumper car is 240 kg. Jason has a mass of 65 kg and Janet has a mass of 58 kg. They are travelling towards each other in opposite directions, Jason with a speed of $2.4\,\mathrm{m\,s^{-1}}$ to the right and Janet with a speed of $2.7\,\mathrm{m\,s^{-1}}$ to the left. Calculate their combined velocity after collision, as a vector.

(d) The rubber bumper in Jasons bumper car has a spring constant of $78\,000\,\mathrm{N\,m^{-1}}$. On one occasion he collides with the wall, causing a compression of 15 cm. Calculate the elastic potential energy stored in the rubber bumper, and determine the impulse if the collision lasted for 0.80 s (making sure you include a unit with your answer).

12. (NZQA 2016) Sarah stands at the end of a diving board of total length 4.0 The diving board is fixed to two supports, A and B, which are 1.0 m apart. The mass of the board is 10 kg and Sarahs mass is 50 kg. Assume the mass of the board is evenly distributed.



(a) Calculate the torque exerted by Sarah about support B.

(b) What is the direction of the force supplied by support A? Explain your answer.

(c) The diving board sags 0.050 m when Sarah stands still on the end of the board. Assuming the board acts like a spring, calculate the spring constant of the board.

(d) Sarah then jumps up and lands on the board, depressing it by a *further* 0.20 m, before she dives into the water. Calculate Sarahs speed when she lands on the board.

13. (NZQA 2016) Sarah releases a red car, from rest, down a slope of length 0.50 m. The red car accelerates steadily and reaches a speed of $1.5\,\mathrm{m\,s^{-1}}$ when it gets to the bottom of the slope.

(a) Calculate the acceleration of the red car as it moves down the slope.

(b) At the bottom of the slope, the track is flat. The red car, moving with the speed of $1.5\,\mathrm{m\,s^{-1}}$, collides with a stationary blue car. The mass of the red car is 0.050 kg, and the mass of the blue car is 0.040 kg.

i. If the velocity of the blue car after the collision is $1.2\,\mathrm{m\,s^{-1}}$, calculate the velocity of the red car after the collision.

ii. If the duration of the collision was 0.08 seconds, calculate the average force that the red car exerts on the blue car.

Section 2.6

Main concepts you should understand: vector components, projectiles.

1. Define 'projectile motion'. (Describe the situations for which it is a reasonable model, and explain which simplifying assumptions are made.)

2. (a) A ball is shot vertically upwards with a velocity of $30.0\,\mathrm{m\,s^{-1}}$.
   i. What maximum height does it reach?
   ii. How long does it take for the ball to fall back to its initial height?

   (b) The ball is shot at $30°$ to the horizontal, with the same initial speed.
   i. Draw a free body diagram showing the forces acting on the ball at the following three points: (A) when it has just left the ground; (B) when it is at its peak; (C) the instant before it hits the ground at the end of its flight.
   ii. What maximum height does it reach?
   iii. How long does it take for the ball to fall back to its initial height?

   (c) If the ball is shot at the same speed, but at $45°$, does its range increase when compared with part (b)?

3. (NZQA 2016) During a cricket game a batsman hits the ball at an angle of $40.0°$ with the ground at a velocity of $20.0\,\mathrm{m\,s^{-1}}$. Give a comprehensive explanation of the effect of the force(s) acting on the ball during its flight. Assume air resistance is negligible.

   In your answer you should:

   - describe the horizontal motion
   - discuss the effect of force(s) on horizontal motion
   - describe the vertical motion
   - discuss the effect of force(s) on vertical motion.

4. (NZQA 2012) Jess drops vertically onto the end of a see-saw, causing Hannah (who is standing at the other end) to be thrown into the air. When Jess lands on the see-saw, Hannah is thrown into the air at a speed of $15.0\,\mathrm{m\,s^{-1}}$, at an angle of $70°$ to the horizontal.

   (a) Calculate the time that Hannah takes to reach the highest point of her trajectory.

   (b) When Hannah takes off, the horizontal component of her velocity is $5.1\,\mathrm{m\,s^{-1}}$. State the size and direction of her velocity at the highest point. Explain your answer.

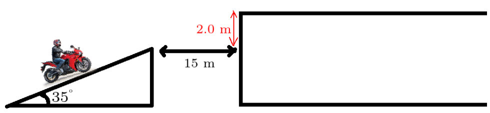
5. (NZQA 2013) Hillary attempts to throw a basketball into a hoop.

   (a) Explain the effect of the force(s) acting on the ball, once it has left Hilarys hand until it reaches maximum height. You may ignore the effects of air resistance.

   (b) On another occasion, Hillary stands 3.0 metres from the hoop. She throws a ball with an initial velocity of $6.5\,\mathrm{m\,s^{-1}}$ at an angle of $60°$ to the horizontal. The hoop is $1.35\,\mathrm{m}$ above the bottom of the ball when it is thrown initially. Carry out calculations to determine whether or not the ball will go through the hoop. Begin your answer by calculating the horizontal and vertical components of the initial velocity of the ball.

6. Jeremy throws a ball horizontally at a speed of $15.0\,\mathrm{m\,s^{-1}}$. It is initially at a height of $2.00\,\mathrm{m}$. Draw a free-body diagram of force(s) acting on the ball at the instant it is thrown. How far does it travel?

7. In the situation pictured below, a stunt driver wishes to shoot off the incline and land on the platform. What is the minimum speed the driver needs to move at in order to succeed?



8. A projectile is shot from the ground with a velocity u at an angle α with the horizontal. It returns to the ground at a distance R from the initial point. Show that, if friction is negligible, then

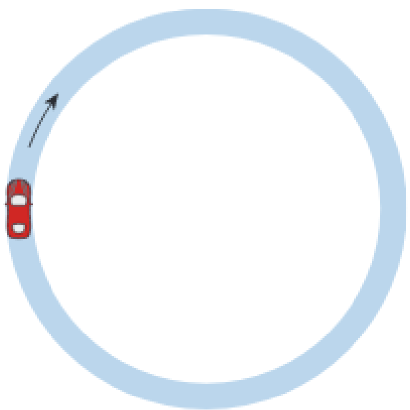
$$R = \frac{2u^2 \sin\alpha\cos\alpha}{g};$$

given that $2\sin\alpha\cos\alpha = \sin 2\alpha$, show that the range is maximised when $\alpha = 45°$.

Section 2.7

1. In the Bohr model of the hydrogen atom, an electron is modelled as rotating in a circle (with radius 0.5×10^{-10} m) about the positively charged nucleus of the atom.
   (a) What force furnishes the centripetal force which causes the electron to be trapped?
   (b) The mass of an electron is 9×10^{-31} kg, and the speed of an electron is modelled as $2.3 \times 10^{6}\,\mathrm{m\,s^{-1}}$. How strong is the centripetal force?
   (c) There is a significant issue with this model. What is it? [Hint 1: there is a conservation law being violated. Which, and why?]
   (d) There is a subtle issue with the argument in (c). What is it? [Hint 2: Unpack $\Delta E = F\Delta x$.] (Note: nonetheless, this argument — when sufficiently patched up — does disprove the Bohr model of the atom.)[1]

2. (NZQA 2016) A toy red car was going round a circular part of the track at a constant speed.



   (a) Name the force acting on the car, and draw a labelled vector on the diagram above to show the direction of the force acting on the car at the instant shown.
   (b) Discuss the effect of the force on the size and direction of the velocity of the red car.

3. (NZQA 2012) Jess is moving in a circular path on a trapeze. When she gets to the lowest point in her swing, the tension force in the rope is greater than the gravity force acting on her. Draw a diagram of the forces acting on Jess at this time, and explain why the tension force is greater than the gravity force.

4. (NZQA 2017) During one of her dance routines, Sally is spinning a ball above her head in a horizontal circle. The ball of mass 0.050 kg makes 5 rotations in 4.0 s. The length of the string from the ball to Sallys hand is 0.60 Calculate the size of the force experienced by the ball during these rotations.

5. A ball tied to the end of a string is swing in a vertical circle of radius r under the action of gravity.
   (a) The ball is rotating at a constant speed. Describe the acceleration felt by the ball at the top of the circle.
   (b) Draw a free body diagram showing the forces acting on the ball at the top of the circle.

[1]See Griffiths, *Intro. to Electrodynamics*, §11.

(c) Calculate the tension T in the string at that point.

6. The moon orbits the Earth in an approximately circular path of radius 3.8×10^8 m. It takes about 27 days to complete one circuit. What is the mass of the Earth? (Hint: draw a free body diagram.)

Formulae

91171 Demonstrate understanding of mechanics

$$v = \frac{\Delta d}{\Delta t} \qquad a = \frac{\Delta v}{\Delta t} \qquad v_f = v_i + at$$

$$d = v_i t + \frac{1}{2}at^2 \qquad d = \frac{v_i + v_f}{2}t \qquad v_f^{\,2} = v_i^{\,2} + 2ad$$

$$a_c = \frac{v^2}{r}$$

$$F = ma \qquad \tau = Fd \qquad F = -kx$$

$$F_c = \frac{mv^2}{r} \qquad p = mv \qquad \Delta p = F\Delta t$$

$$E_p = \frac{1}{2}kx^2 \qquad E_k = \frac{1}{2}mv^2 \qquad \Delta E_p = mg\Delta h$$

$$W = Fd \qquad P = \frac{W}{t}$$

where needed, use $g = 9.8 \text{ m s}^{-2}$