NCEA Level 2 Mathematics (Calculus)

Reading

Some of the most perplexing problems in life today are those involving superlatives. We seek the *biggest* profit, the *most efficient* method, the *smallest* friction, the *shortest* time, the *least* wear. In short, we often seek the biggest or the smallest. Calculus may assist in solving such problems since many of them reduce to finding the highest (maximum) and the lowest (minimum) points on the graph of a function.

Let us consider the function f, determined by $f(x) = x^4 + 5$. Clearly there is no largest value of the function since $x^4 + 5$ can be made as large as desired by taking x sufficiently large. On the other hand, since x^4 is never negative, the function $x^4 + 5$ will always be at least 5. The value of $f(0)$ is 5. Hence, 5 is the minimum value that the function $f(x) = x^4 + 5$ obtains, and it obtains this minimum when $x = 0$.

In many cases, it is not as simple as this to determine the maximum and minimum values of a function. Consider the function $y = x^4 - 32x + 6$. It is fairly obvious that y may be made as large as you wish by taking x sufficiently large; hence the function has no biggest value. However, it is not immediately obvious that the value -42, which occurs when $x = 2$, is the smallest or minimum value of the function determined by $x^4 - 32x + 6$.

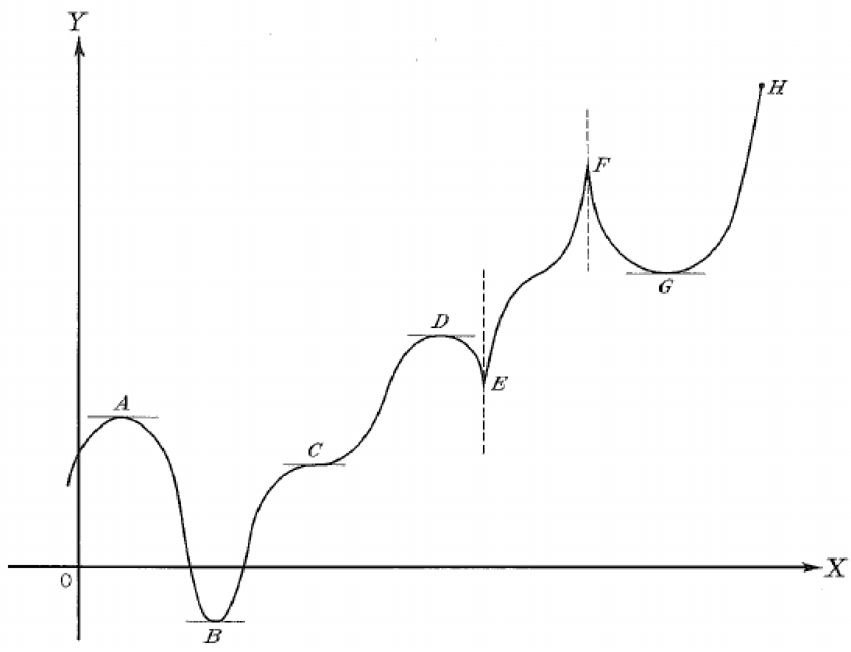

Figure 4-5

In general, relative maximum and relative minimum values of a function will only occur at points where one of the following conditions holds:

1. The derivative $f'(x_1) = 0$ (points A, B, D, and G of Fig. 4-5).
2. The derivative $f'(x_1)$ does not exist (points E and F of Fig. 4-5).
3. x_1 occurs at the end point of the domain of the function (point H of Fig. 4-5).

These conditions are *necessary* but not *sufficient* — take $y = x^3$ at $x = 0$.

Adapted from Calc. w/ An. Ge.

Questions

1. Show that 5 is a critical value of the function $f(x) = 2 + (x-5)^3$, but that f does not have a local extreme value at 5.

2. (a) Find the extreme value(s) of $f(x) = x^2 - 4x + 3$.

   (b) Suppose $g(x) = \frac{x^3-6x^2+11x-6}{x-2}$.

      i. The polynomial $x^3 - 6x^2 + 11x - 6$ has roots at $x = 1$, $x = 2$, and $x = 3$. Use this information to factorise the numerator of g, and simplify the fraction.

      ii. Find the extreme value(s) of g.