Assignment: Mathematical Writing Practice II

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1 Task

Suppose that y is a function of x given by

$$y = x^3 + Bx^2 + Cx + 1,$$

and that the graph of y has a minimum at $(3, 0)$.

Find B and C.

Ensure that you write 'properly'. That means using complete sentences, justifying all logic, and aiming for clarity!

2 Hints

A list of things to think about:

- What information are you given?
- What information do you need to solve the problem?

3 Example Answer

This one is more difficult, as we don't get given numbers we can substitute in straight away to find one of the coefficients. However, we do know two bits of information: the graph of the function passes through $(3, 0)$, and the function has a critical point at $x = 3$. The derivative of the function is $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2Bx + C$, and so we have the following set of simultaneous equations:

$$\begin{aligned} 0 = 3^3 + B \cdot 3^2 + C \cdot 3 + 1 \qquad &\Rightarrow 0 = 28 + 9B + 3C, \\ 0 = 3 \cdot 3^2 + 2B \cdot 3 + C \qquad &\Rightarrow 0 = 27 + 6B + C. \end{aligned}$$

Substituting $C = -6B - 27$ into the first equation, we obtain $B = \frac{53}{9}$. Then $C = -6 \cdot \frac{53}{9} - 27 = -\frac{25}{3}$..