L3 Physics: Questions for 2.1-2.4

Majority taken from Knight chap. 8, 12.

1. A 1350 kg car drives around a flat 150 m diameter circular track at $20\,\mathrm{m\,s^{-1}}$. What is the magnitude and direction of the net force on the car? What causes this force?

2. A car drives over the top of a circular-section hill with a radius of 50 m. Draw a force diagram, and calculate the maximum speed the car can drive without taking flight.

3. A new car is tested on a 200 m diameter circular track. If the car speeds up at a steady $1.5\,\mathrm{m\,s^{-1}}$, how long after starting is its centripetal acceleration equal to the tangential acceleration?

4. A 300 g ball and a 600 g ball are connected by a 40 cm rigid massless rod. The structure rotates about its centre of mass at $100\,\mathrm{rev\,min^{-1}}$. What is its rotational kinetic energy?

5. A ball of radius R is placed at aa height of 30 cm on a 15° slope. It is released and rolls without slipping to the bottom. From what height should a circular hoop of radius R be released in order to equal the ball's speed at the bottom?

6. A steel beam, unmelted by jet fuel, is 5 m long and has a mass of 400 kg. It extends horizontally from the point to which it has been bolted to the framework of a building. A construction worker of mass 100 kg, unworried about health and safety regulations, stands on the end of the beam. What is the total torque about the bolt due to the worker and the beam?

7. A 2.0 kg, 30 cm diameter disc is spinning at $300\,\mathrm{rev\,min^{-1}}$. How much friction force must the brake apply to the rim to bring the disc to a halt in 3.0 s?

8. A 200 g toy car is placed on a narrow 60 cm diameter track (mass of track ring 1.0 kg, massless interior) that is free to rotate on a frictionless vertical axis. When the car is started, it soon reaches a steady speed of $0.75\,\mathrm{m\,s^{-1}}$ relative to the track. At this point, what is the track's angular velocity in rpm?

9. [Difficult] If a vertical cylinder of some liquid rotates about its axis, the surface forms a smooth curve. Assuming that the water moves as a unit (i.e. all the water moves with the same angular velocity), show that the shape of the surface is a parabola described by the equation

$$z = \frac{\omega^2}{2g} r^2. \tag{1}$$

[Hint: Each particle of water on the surface is subject to only two forces: gravity, and the normal force due to the water underneath it. The normal force, as always, acts perpendicular to the surface.]