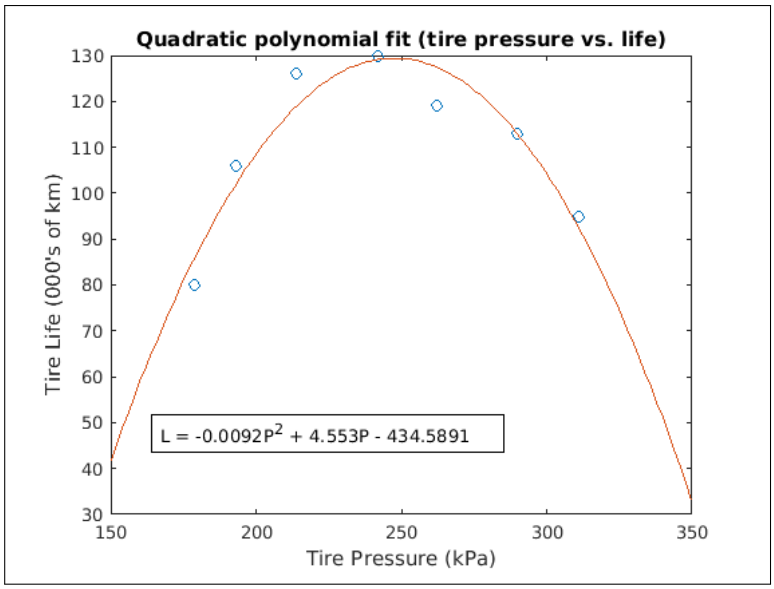
NCEA Level 2 Mathematics (Calculus)

1. Differentiate the function. [Ste 2.3.1-10]
   (a) $f(x) = 186.5$
   (b) $f(x) = \sqrt{30}$
   (c) $f(x) = 5x - 1$
   (d) $F(x) = -4x^{10}$
   (e) $f(x) = x^3 - 4x + 6$
   (f) $f(t) = \frac{1}{2}t^6 - 3t^4 + t$
   (g) $g(x) = x^2(1 - 2x)$
   (h) $h(x) = (x - 2)(2x + 3)$
   (i) $y = x^{-2/5}$
   (j) $B(y) = cy^{-6}$

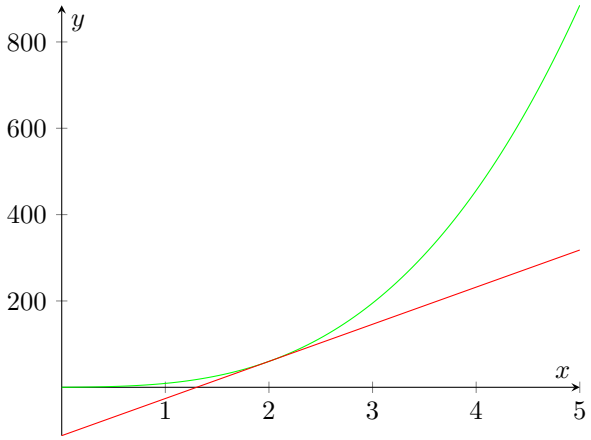
2. Car tires need to be inflated properly because overinflation or underinflatioon can cause premature treadware. The graph shows tire life L (in thousands of kilometres) for a certain type of tire at various pressures P (in kPa), as well as a quadratic function that models the tire life. [Ste 2.3.66(b)]



   Use the model to estimate $\frac{dL}{dP}$ when $P = 200$ and when $P = 300$. What is the meaning of the derivative? What is the significance of the sign of the derivatives?

3. If f is a function, and $f'(x)$ is negative, what is the behaviour of f like around x?

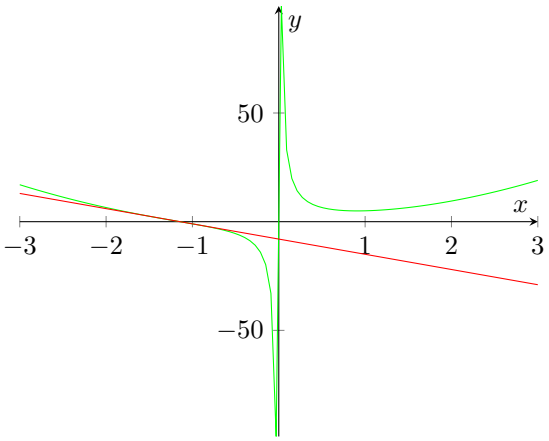
4. Consider the function $f(x) = 7x^3 + 2x$.



   (a) What is the slope of the graph of $y = f(x)$ around $x = 2$?

   (b) Give the equation of the tangent line to the graph at $x = 2$. (The tangent line to a graph at a point is the line through that point with the same slope as the graph.)

5. Consider the function $g(x) = 2x^2 + \frac{3}{x}$.



   (a) What is the slope of the graph of $y = g(x)$ around $x = -1$?

   (b) Give the equation of the tangent line to the graph at $x = -1$.

   (c) The normal line to a graph at a point is the line going through that point that lies at right angles to the graph (and hence to the tangent line to the graph).

      i. Consider the line with slope m going through $(x_0, y_0$; it has equation $(y - y_0) = m(x - x_0)$. What is the slope of the line at right angles to it going through the same point?

      ii. Give the equation of the normal line to the graph of $y = g(x)$ at $x = -1$.

6. Graph $y = x^2$, and the tangent and normal lines to the graph at $(2, 4)$.

7. Find the nth derivative of each function by calculating the first few derivatives and observing the pattern that occurs. [Ste 2.3.86]

   (a) $f(x) = x^n$

   (b) $f(x) = \frac{1}{x}$