# NCEA Level 2 Mathematics (Calculus Revision)

1. Given each function y of x, write down an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. (A/M)

   (a) $y = x^2$

   (b) $y = x^5 + x + 1$

   (c) $y = 3x^2 + 5$

   (d) $y = 4$

   (e) $y = \frac{1}{7}x^9 + 16x$

   (f) $y = 4.6x^{10} + 20x^9 + 3x^2 + 9.32$

   (g) $y = 6x^2 + 3x^{-3}$

   (h) $y = \sqrt{x} + \sqrt[3]{x}$

   (i) $y = \frac{1}{x^2} + 2x^{-1}$

   (j) $y = \frac{1}{\sqrt[4]{x^3}}$

   (k) $y = \frac{1}{x^{\frac{3}{4}}}$

   (l) $y = \frac{1}{2x^{-1}} + x^2 + \frac{2}{\sqrt{x}}x^3$

2. The following part-questions require you to find higher derivatives of functions. (M)

   (a) Find $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$ if $x = 3t^2 + 4$.

   (b) Suppose $f(x) = 9x^2 + \frac{1}{x^3} + \sqrt{x}$. Find $f''(x)$.

   (c) Find $\frac{\mathrm{d}^3 y}{\mathrm{d}t^3}$ if $y = 3x^{-1}$.

   (d) Find $f''(x)$ if $f'(x) = 9x^2 + 3x^{-2}$.

   (e) Find $a(t) = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$ if $s(t) = 3t - 4t^{-1}$.

   (f) Find $\frac{\mathrm{d}^n f}{\mathrm{d}x^n}$ if $f(x) = \frac{1}{x}$. (E+)

3. Find the extreme values (if any) of the following functions of x: (M/E)

   (a) $y = x^5$

   (b) $y = \frac{1}{x}$

   (c) $y = x^2 - 1$

   (d) $y = 2x^3 - 21x^2 + 72x + 18$

   (e) $f(x) = x^{10} - 4$

   (f) $y = \frac{1}{\sqrt{x}} + x^2$

   (g) $y = x^3 - x - 1$

   (h) $y = x^3 - x^2 + x - 1$

   (i) $f(x) = \frac{1}{x} + x - x^2$

   (j) $y = 16$

   (k) $y = \frac{x^{-2} + x^2}{2x}$

   (l) $x = \frac{y-2}{x+3}$

4. Maximise the derivative of $2x^2 - x^3$. (M/E)

5. A projectile follows a path through space modelled by $y = 4x - x^2$. At what distance along the ground is it at its maximum height, and what is that height? (M)

6. A child moves a Buzzy Bee$^{\text{TM}}$ toy forwards and backwards along a straight line. At time t, where $0 \le t \le 10$, the toy's position is modelled by $x = 3t - 1.3t^2 + 0.1t^3$.

    (a) At which time(s) is the toy stationary?   (A/M)

    (b) What is the acceleration of the toy at $t = 3$?   (M)

    (c) What displacement is the toy from the origin when the velocity of the toy is most negative?   (E)

7. A particle is moving through space along an axis. Its displacement from the origin at any time $t > 0$ is given by $s(t) = t^5 - 38t^4 + 560t^3 - 3982t^2 + 13599t - 17820$.

    (a) Find an expression for the velocity of the particle at time t, $v(t)$.   (A)

    (b) At what time is the particle moving with the most speed towards the origin, and how fast will it be moving at that time?   (M/E)

    (c) What is the acceleration of the particle at that time?   (M)

    (d) How many times does the particle change direction after $t = 0$?   (E+)

8. Show that $y = x^4 + 3x^3 - x^2 - x + 20$ does not pass through the x-axis.   (M/E)

9. Show that $x^3 - 5x + 100 = 0$ has only one real solution.   (M/E)

10. Show that $x^3 - x^2 - x + 1$ has exactly two roots.   (M/E)

11. Show that, as we increase the radius of a circle, the area increases at an instantaneous rate equal to the circumference.   (M)

12. Draw the derivative of the function graphed below.   (A/M)
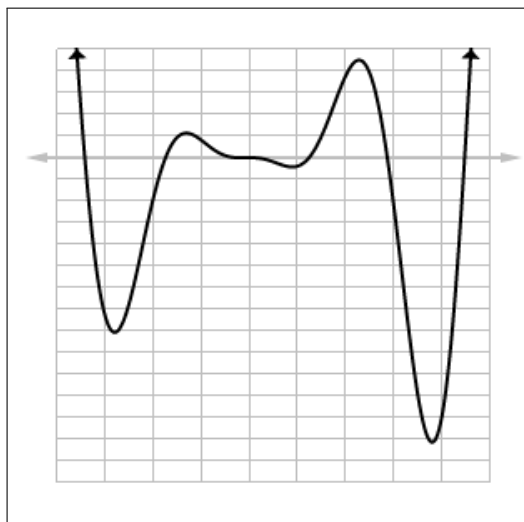
13. A farmer needs to create a rectangular field with a fence. He has $500\,\text{m}$ of fencing available, and a building is on one side of the field (so that side does not need fencing). Determine the dimensions of the field to enclose the greatest area.   (M/E)

14. Suppose that a rectangle has perimeter p. Show that if the rectangle is to have the greatest possible area then it must be a square.   (E)

15. Where is the graph of $y = \frac{x^2}{2} - \frac{x^3}{3}$ increasing?   (M/E)

16. Suppose that f is a function of x given by $f(x) = 12x^5 - 195x^4 - 1340x^3 - 390x^2 - 4080x + 3$. Find the value of $f'(17)$, and describe the behaviour of the graph $y = f(x)$ around that point.   (A/M)

17. Suppose that $\frac{\mathrm{d}x}{\mathrm{d}t} = 2t^3 + 6t + t$. Find an explicit expression for x in terms of t. (M)

18. Let f be a function of x defined by $f(x) = 3x^2 + 6x + 6$. Show that f is a solution of the *differential equation* $f(x) - f'(x) = 3x^2$. (M)

19. Show that the function f of y defined by $f(y) = \frac{3}{y^2} + 2x$ is not differentiable at $x = 0$. (E+)

20. The *product rule* for differentiation is $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$. (E++)

    (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = (x^3 + 3x + 9)(x^{30} + \pi x^3 + 9x + 7x^{-2})$.

    (b) It so happens that $\frac{\mathrm{d}(\sin x)}{\mathrm{d}x} = \cos x$ and $\frac{\mathrm{d}(\cos x)}{\mathrm{d}x} = -\sin x$. Show that $(\sin x)^2 + (\cos x)^2 = k$, where k is a constant. Find k.