NCEA Level 3 Calculus
Prerequisite Knowledge

I am often asked by Y12 students, 'what should I know for Y13?'. This is a list of the *absolute minimum* knowledge required for L3 calculus: and by 'minimum knowledge', I mean that you should be able to do the following things (a) *without* a calculator, (b) without too much thought (i.e. they should be second nature), and (c) understanding what you are doing at each step.

If you cannot do the following things without a calculator, or if they take you more than about five minutes each, you will struggle significantly with getting a bare pass in NCEA L3 calculus. That being said, I normally give this sheet out at the end of term 2 of Y12, and so you have the time to get up to speed before next year.

1. Write the following expression as a single fraction:

$$\frac{y+x}{3x+2} + \frac{x^2 - xy}{y}$$

2. Fully expand the following expression:

$$(x+2)(x-1)(x+y)(y+3)$$

3. Draw the graph of $(x-3)^2 + 4 = y$.

4. Draw the graph of $-x^2 - 7x - 12 = y$.

5. Draw the graph of $10^{x+1} = y + 2$.

6. Draw the graph of $(x-3)(x-2)(x-1) = y$.

7. Find the vertex of $(x-3)^2 + 1 = y$.

8. Find the vertex of $x^2 + px + q = y$.

9. If $-1 < x < 1$, what values can $x^2 + 2x$ assume?

10. Find all x such that $\frac{x}{x-4} < \frac{-1}{x+2}$.

11. Find $\log_{10}(100000)$ and $\log_{10}(0.00001)$.

12. The following illustrate properties of logarithms; identify the property and the corresponding rule for exponents. As an example the first is done for you.

    (a) $\log_{10} 710 = \log_{10} 7.1 + \log_{10} 100$. *Solution:* This illustrates the property $\log_b xy = \log_b x + \log_b y$, which corresponds to the exponent property $b^{x+y} = b^x b^y$.

    (b) $\log_3 2^{7.1} = 7.1 \log_3 2$

    (c) $\log_7 1 = 0$

    (d) $\log_{37} 37 = 1$

    (e) $\log_{37} \frac{(37)(59)}{67} = \log_{37} 37 + \log_{37} 59 - \log_{37} 67$

    (f) $\log_{18} \sqrt[18]{318} = \frac{1}{18} \log_{18} 318$

13. Solve for x, if $\log_x \frac{1}{x^4} = -4$.

14. Write $\frac{\log_{10} x}{\log_{10} b}$ as a single logarithm.

15. Solve for x if $3^{x^2 - 2x + 1} = 9$.

16. Find the equation of a line through $(3, 2)$ and $(6, 3)$. What is the slope of this line?

17. Prove that there is no line passing through all three points $(0, 1)$, $(1, 0)$, and $(1, 1)$.

18. If a right angled triangle has side lengths 5, 12, and 13:

    (a) State Pythagoras' theorem in general, and check that it 'works' in this specific case.

    (b) What are the sines of the three angles of the triangle?

19. Draw $y = \sin x$, $y = \tan x$, and $y = \cos x$ on the same pair of axes, labelling the important points (maxima, minima, roots, asymptotes).

20. Prove that the distance between (x_0, y_0) and (x_1, y_1) is $\left((x_0 - x_1)^2 + (y_0 - y_1)\right)^{1/2}$.

21. On a number line indicate the values of x which satisfy $(x + 2)^2 \geq 0$.

22. Find explicitly all values of x that satisfy $2x^2 - 13 \leq x^2 + 12$.

23. Find all points (x, y) that lie on both the following lines:

$$y = x - 9$$
$$2x + y = 3$$

24. Solve the following system of equations:

$$x - 2y + 3z = 9$$
$$-x + 3y - z = -6$$
$$2x - 5y + 5z = 17.$$

25. If f is a function defined by $f(x) = (x + 3)(x - 2)(10^{x-3})$, find (a) $f(3)$; (b) $f(z)$; (c) $f(x + 3)$.

References.

- David Crowdis and Brendon Wheeler, *Precalculus Mathematics*. Benziger Bruce and Glencoe (1976). In particular chapters 2, 4, 5, 6, 7.

Where can I go with L3 calculus?

Calculus is *required* for the following:

- Engineering
- Physics
- Pure mathematics
- Applied mathematics
- Economics

Calculus is *strongly recommended* for the following (i.e. you should take it, and if you don't then you will find the first year university calculus paper they make you take very difficult indeed):

- Chemistry
- Medicine
- Computer science

- Biology

- Statistics