NCEA Level 2 Mathematics (Calculus)

Questions

1. Find an antiderivative of $f(x) = 25x^4 + 12x^3 - x^{-2}$.
2. Evaluate $\int 12z^3 + 18z^{-4}\,\mathrm{d}z$.
3. Find three functions that have derivatives equal to $x^2 - x$.
4. Show that $x^3 + 3x + C$ is an antiderivative of $3x^2 + 3$.
5. (a) Find all possible functions ψ such that
$$\psi'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}.$$
   (b) Suppose that we know that $\psi(0) = -8$. Find ψ.
6. Find g if $g'(x) = x\sqrt{x}$ and $g(1) = 2$.
7. The acceleration of a rocket propelled washing machine is given by $\frac{\mathrm{d}v}{\mathrm{d}t} = 9t^3 - t^4 + t^{-3/2}$, where $0 \leq t \leq 10$. Find the distance which it has travelled after 10 seconds if its initial velocity (at $t = 0$) was $90\,\mathrm{m\,s^{-1}}$.
8. Suppose that θ is a function of x such that
$$\theta'(x) = 8x^3 + 3x^2 + ax,$$
where a is a constant. Given that $\theta(0) = 9$ and $\theta(-1) = 14$, find θ and a.
9. (a) Draw the line $y = 2t + 1$ and use geometry to find the area under this line, above the t-axis, and between the vertical lines $t = 1$ and $t = 3$.
   (b) If $x > 1$, let $A(x)$ be the area of the region that lies under the line $y = 2t + 1$ between $t = 1$ and $t = x$. Sketch this region and use geometry to find an expression for $A(x)$.
   (c) Find $A'(x)$. What do you notice?