

## NCEA Level 3 Trigonometry (exercise set)

### 3. Taxonomy of Functions

**Goal** To investigate the relationships between the geometry and algebra of the trig functions.

1. Show that the following hold for all  $\theta$ :

(a)  $\sec^2 \theta + \csc^2 \theta = \sec^2 \theta \cos^2 \theta$

(b)  $(\sin \theta + \sec \theta)^2 + (\cos \theta + \csc \theta)^2 = (1 + \sec \theta \csc \theta)^2$

2. (a) Draw a graph to check the plausibility of the conjecture that

$$\tan \theta = -\cot \left( \theta - \frac{\pi}{2} \right) \quad (*)$$

holds for all  $\theta$ .

- (b) Show that (\*) holds for all  $\theta$ .

- (c) Prove that there is **no**  $\xi$  such that  $\cos \theta = \sec(\theta + \xi)$  holds for all  $\theta$ .

3. Let  $O_1$  and  $O_2$  be two distinct points, and suppose  $r_1$  and  $r_2$  are positive numbers with  $r_1 > r_2$ .

- (a) Draw two circles centred at  $O_1$  and  $O_2$  with respective radii  $r_1$  and  $r_2$ .

- (b) Suppose  $L$  is a tangent line shared by both circles which does not cross the *segment*  $O_1O_2$ ; show that the angle  $\theta$  made by  $L$  with the *line*  $O_1O_2$  satisfies

$$\sin \theta = \frac{r_1 - r_2}{|O_1O_2|}.$$

- (c) Find the distance between the points of tangency along  $L$ .

4. Complete exercise 3.7 in the notes.

5. (a) Illustrate graphically the change in sign and magnitude of the functions  $3 \sin x + 4 \cos x$ ,  $e^x \sin x$ , and  $\sin \left( \frac{\pi}{\sqrt{2}} \sin x \right)$  for all values of  $x$ .

- (b) Show that  $2x = (2n + 1)\pi \operatorname{versin} x$  (for  $n$  positive) has **exactly**  $2n + 3$  possible solutions for  $x$ ; indicate roughly their locations. [Hint: approach this problem in the same rough direction as (a) — i.e. do not try to compute the values of  $x$  directly.]

**Additional reading** Hobson, chapter III.