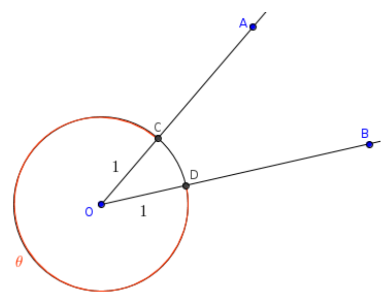
NCEA Level 3 Trigonometry (exercise set)
1. The Pythagorean Theorem

Goal To familiarise ourselves with some of the tools of plane geometry that we will be utilising this year.

A gentle reminder My problem sets are intended to be difficult. Thus one should not expect to be able to write down complete solutions immediately (or even within the week allotted). *This is not a reflection of your mathematical ability.* Even the best students should have at least one problem that they simply cannot 'get', and this is by design. (If I end up with a student that does write a set of perfect solutions, then I will simply add another more difficult problem!) My advice would be to make a number of passes: first read all the questions, draw any diagrams that have not been supplied, and understand in each case what is being asked. Then attempt each problem, allowing yourself only five or so minutes on each and moving on if you don't get it. Return every day or so to attack the problems you have not yet completed; and note the attempts you have made, even if they didn't work. (The problems are not necessarily in order of difficulty, especially since no such ordering exists anyway.)

1. Let O be a point, and let A and B be points distinct from O (but A is allowed to be the same point as B). Draw the circle of radius 1 centred at O, and let θ be the **anti-clockwise** length[1] of the circumference of the circle that is cut off by the rays $\vec{OA}$ and $\vec{OB}$. Then θ is said to be the measure of the angle between OA and OB (in radians).



   Define the number π to be the measure of a right angle (i.e. one quarter of the unit circle).

   (a) Show that if an angle has measure δ in degrees, then it has measure $\frac{\pi\delta}{180}$.

   (b) Show that if an angle θ is cut through a circle of radius r, then the area of the sector formed between the two rays of the angle is $(\theta r^2)/2$ and the arc length cut off by the angle is θr. [You may take for granted that the area of a circle is πr^2 and the length of the circumference is $2\pi r$. It is not worth being too formal here.]

2. Prove *Thale's theorem*: if AB is the diameter of a circle, and if C is any other point on the circle, then ABC is a right triangle with the right angle at C. I can think of three ways right now; try all three:

   (a) Use the inscribed angle theorem.

   (b) Draw in the angles and push them around (in the same way we proved the inscribed angle theorem).

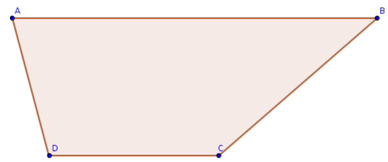
   (c) I'm not telling, try to find another way yourself without getting your hands dirty dealing with angles!

3. Let $ABDE$ be a square. Pick a point C inside the square such that the triangle CDE is isosceles, with angles at D and E that measure $15° = \pi/12$. What kind of triangle is ABC? (Hint: draw another point F in the square such that FBD is congruent to CED; what can you say about the triangle DCF?)

(cont'd)

[1] I will **always** take angles to be in the anti-clockwise direction.

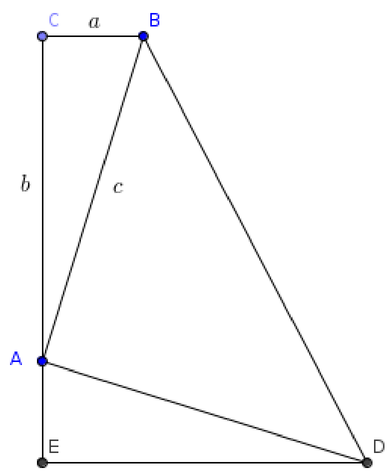
4. Recall that a *trapezoid* is a quadrilateral with two opposite sides parallel. For example, in the following diagram $ABCD$ is a trapezoid with parallel sides AB and CD.



Suppose $ABCD$ is a trapezoid, with parallel sides AB and CD; suppose that $|AB| = a$ and $|CD| = b$, and suppose that the distance between the two parallel sides is h (the height). Show that $\mathcal{A}(ABCD) = h\frac{a+b}{2}$.

5. We have already seen several proofs of the Pythagorean theorem; here is another. This proof is due to a former president of the United States of America (James A. Garfield), and is usually accompanied by the observation that "mathematical ability, while likely a hinderance, is thus not a complete disqualification from American politics".

   Suppose a right triangle ABC has leg lengths a and b and hypotenuse length c. Consider the following figure:



   The triangle ADE is congruent to (has the same lengths and angles as) ABC, so $|AE| = a$ and $|ED| = b$.

   (a) Show that the angle DAB is a right angle.

   (b) Calculate the area of the trapezoid $BCED$ in two ways: using the formula from question (4), and by adding the areas of the three right triangles ABC, ADE, and ABD.

   (c) Deduce that $a^2 + b^2 = c^2$.

Additional reading Coxeter, sections 1.1 to 1.3 inclusive.