

# NCEA Level 3 Calculus (Differentiation)

## 2. Limits (Homework)

### Reading

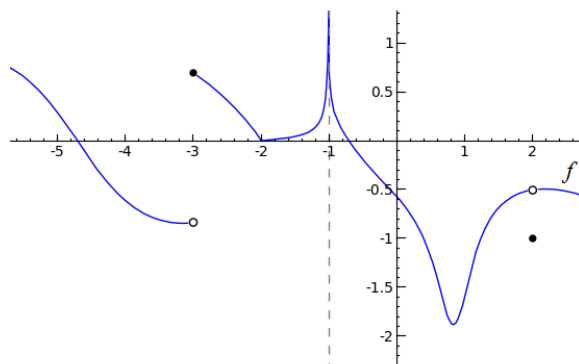
Derivatives and limits allow us to classify functions and their behaviour. Consider the following:

### Properties of Functions

- A function is **increasing** if its derivative is positive.
- A function is **decreasing** if its derivative is negative.
- A function is **concave down** if its derivative is decreasing.
- A function is **concave up** if its derivative is increasing.
- A function  $f$  is **continuous** at a point  $a$  if  $\lim_{x \rightarrow a} f(x) = f(a)$ .

### Questions

1. Describe all the function properties given above geometrically, and give an example of each.
2. Consider the function graphed below.



- (a) Find  $\lim_{x \rightarrow -2} f(x)$  and  $\lim_{x \rightarrow 2} f(x)$ .
  - (b) Does  $\lim_{x \rightarrow -3} f(x)$  exist? Why/why not?
  - (c) Does  $\lim_{x \rightarrow 0} f(x)$  exist? Why/why not?
  - (d) On what intervals is  $f(x)$  continuous?
  - (e) At what points is  $f(x)$  not differentiable?
3. On an axis, sketch a graph of some function  $f$  that has the following features:
    - Is continuous for  $0 < x < 5$  and  $5 < x < 9$  and is discontinuous when  $x = 5$
    - Is concave down ( $f''(x) < 0$ ) for  $0 < x < 5$
    - Has  $f'(x) = 0$  at  $(3, 8)$
    - Has  $\lim_{x \rightarrow 5} f(x) = 6$ .
    - Is not differentiable at  $(7, 3)$ .