

NCEA Level 3 Trigonometry (exercise set)

4. Identity Fishing

Goal To practice proving additional identities relating the trigonometric functions.

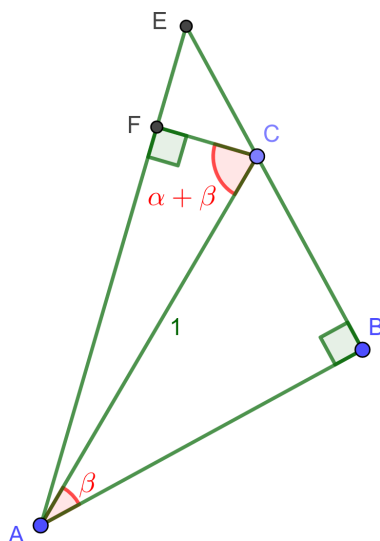
1. Prove the following identities true for all angles x .

(a) $\sec 2x = \frac{\sec^2 x}{2 - \sec^2 x}$

(b) $\cos x + \sin x = \frac{\cos 2x}{\cos x - \sin x}$

2. Write an expression for $\cos 4x$ in terms of only $\cos x$ (and powers and multiples of $\cos x$).

3. Fix two numbers $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$. Consider the following diagram.



We have two right triangles, where ABC has an angle β and hypotenuse $|AC| = 1$. From C we measure an angle $\alpha + \beta$, and extend this line to meet a line from C at a right angle (so both interior angles at F are right). Hence $|AB| = \cos \beta$, $|BC| = \sin \beta$, and $|FC| = \cos(\alpha + \beta)$.

(a) Show that the interior angle at E is α .

(b) Show that $|EC| = \frac{\cos(\alpha + \beta)}{\sin \alpha}$.

(c) Use (b) and the discussion below the diagram to show that

$$\frac{|AB|}{|BE|} = \frac{\cos \beta}{\sin \beta + \frac{\cos(\alpha + \beta)}{\sin \alpha}}.$$

(d) Use (a) to show that $\frac{|AB|}{|BE|} = \frac{\sin \alpha}{\cos \alpha}$.

(e) Prove that $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$.

4. Find all the points x such that $\sin^2 x > \cos^2 x$.

5. If a , b , c , and d are the internal angles of a quadrilateral, show that

$$\cos(a + b + c) + \cos(b + c + d) + \cos(c + d + a) + \cos(d + a + b) = -4 \cos \frac{a + b}{2} \cos \frac{a + b}{2} \cos \frac{a + d}{2}.$$

Additional reading Hobson, chapter IV, V.