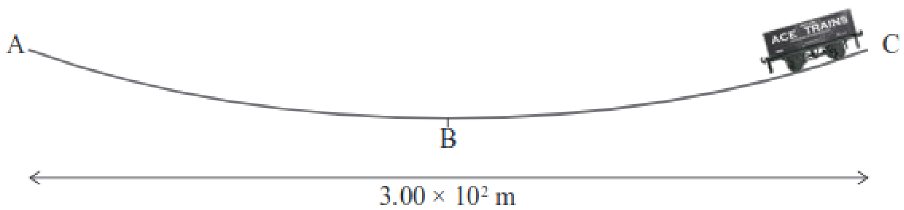
L3 Physics: Questions for 1.3 (Mechanics, oscillation)

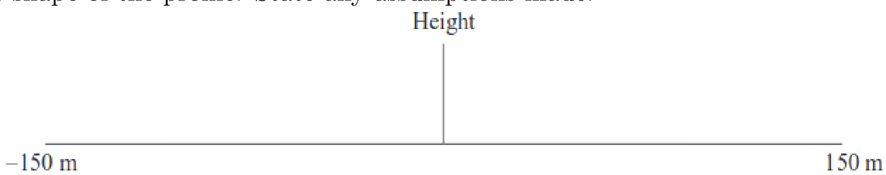
The first two questions are past exam questions, from 2013 and 2018 respectively. Remainder from Knight, chapter 15 and Bendall, chapter 10.

1. A ball, attached to a cord of length 1.20 m, is set in motion so that it is swinging backwards and forwards like a pendulum.
   (a) Show that the period of a pendulum of length 1.20 m that is oscillating in simple harmonic motion is 2.20 s.
   (b) Explain what must be done to ensure that the motion of the ball approximates simple harmonic motion.
   (c) Sketch a graph to show what happens to the ball's **total** energy over time until it stops swinging.
   (d) It is possible to get the ball swinging by holding the top end of the cord and gently shaking it backwards and forwards.
   Explain how shaking the top end of the cord can make the ball on the bottom of the cord oscillate in simple harmonic motion.
   In your answer, you should consider resonance and energy transfer.
   (e) Simple harmonic motion requires a restoring force that changes in proportion to the size of the displacement.
   Discuss what provides the restoring force when the ball is swinging in simple harmonic motion.
   In your answer, you should:
   - describe what forces act on the ball
   - explain how these forces change as the ball swings
   - draw vectors to show how a restoring force is produced

2. A railway wagon full of sand is released from rest at point C. The wagon oscillates on a vertically curved track between A and C with simple harmonic motion of period 60.0 s. The effects of friction can be ignored.



   (a) i. State the conditions that must apply for the motion to be simple harmonic motion.
   ii. Show that the maximum speed attained is 15.7 m s^{-1}.
   (b) When the wagon is halfway between B and C, calculate its approximate height above B.
   (c) i. The wagon has a small hole from which sand leaks onto the flat ground beneath the rail track at a steady rate. Sketch a height profile of the sand on the graph below and explain the shape of the profile. State any assumptions made.



   ii. When the wagon arrives back at C, the remaining sand is suddenly dumped from the wagon. Explain what effect this removal of mass will have on the physical parameters of the wagon's motion.
   (d) The track A to C is an arc of a circle.
   By first calculating the radius of the circle, discuss whether the original assumption that this motion is simple harmonic motion is valid.

3. Suppose a large spherical object (e.g. a planet), of mass M and radius R, has a narrow tunnel passing diametrically through it. A particle of mass m is inside the tunnel at a distance $x \leq R$ frokm the centre. It can be shown that the net force on the particle is due entirely to the sphere of mass with radius x; there is no net gravitational force from the spherical shell outside this sphere.

   (a) Find an expression for the gravitational force on the particle, assuming the large object has uniform density. Your expression should be in terms of x, m, M, R, and any necessary constants.

   (b) In (a) you should have found that the gravitational force is a net restoring force. Consequently, in the absence of air resistance, objects in the tunnel will oscillate with SHM. Suppose an astronaut exploring a 150 km diameter, 3.5×10^{18} asteroid discovers a tunnel through the centre. If she jumps into the hole, how long will it take her to fall through the asteroid and out the other side?

4. For a particle in simple harmonic motion, show that $v_{\max} = (\pi/2)v_{\text{avg}}$ (where v_{avg} is the average speed during one cycle of the motion).

5. A grandfather clock ticks each time the pendulum passes the lowest point. Roughly how long must the pendulum be for the ticks to occur once a second?

6. A spring is hung from the ceiling. When a block is attached to its end, it stretches 2.0 cm before reaching its new equilibrium point. The block is then pulled slightly down and released. What is the frequency of the resulting oscillation?

7. Sarah bounces on a trampoline. As long as the amplitude of her motion is small, she stays in contact with the mat throughout each oscillation and her motion is approximately modelled by SHM. She bounces at a frequency of 1.1 Hz, and an initial amplitude of 0.15 m.

   (a) Show that it takes Sarah 0.12 s to drop 0.11 m below her rest position.

   (b) If Sarah is 0.11 m below her rest position:

      i. Calculate her speed;

      ii. State and explain the direction of her acceleration; and

      iii. Calculate the magnitude of her acceleration.

   (c) Calculate Sarah's maximum speed, and explain when she will be travelling at this speed.

   (d) Calculate Sarah's maximum acceleration, and explain when she will reach this acceleration.

8. The bob of a simple pendulum has mass 0.25 kg and maximum speed $1.8\,\text{m}\,\text{s}^{-1}$.

   (a) Calculate the vertical height of the bob above its equilibrium point when it was released.

   (b) Explain why, over time, the amplitude of oscillation of the pendulum will decrease.

   (c) Discuss the effect of an oscillating driving force on the pendulum.

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