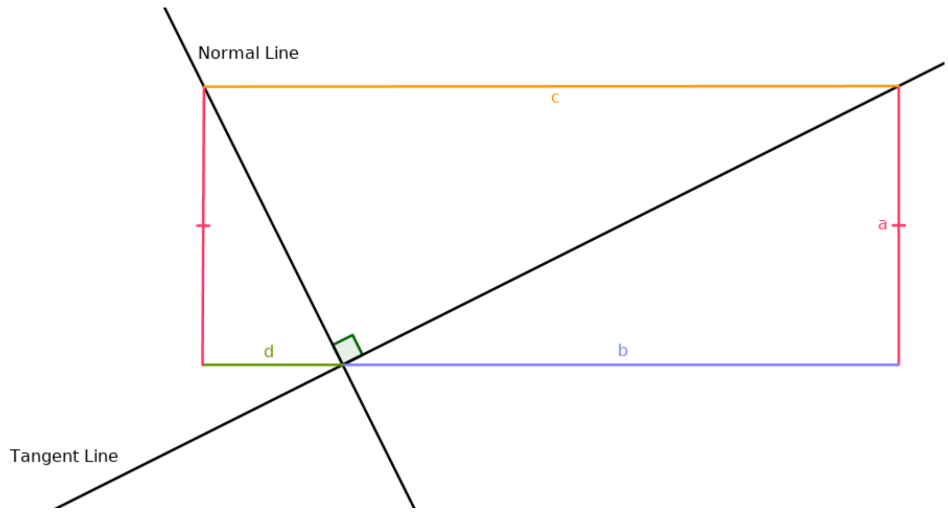

Figure 1. Diagram showing the various triangles used.

Theorem. *If a tangent line has slope m, then the normal line to it has slope $-1/m$.*

Proof. Consider the line shown in figure 1, and let $m = \frac{a}{b}$ be the slope of the tangent line. So the length of the hypotenuse of the ab triangle is $\sqrt{a^2+b^2}$. But $c = d+b$. Hence the length of the hypotenuse of the ad triangle can be found in two ways:

$$\sqrt{(d+b)^2 - \left(\sqrt{a^2+b^2}\right)^2} = \sqrt{a^2+d^2}.$$

Since all the lengths are positive, we have

$$d^2 + 2db + b^2 - a^2 - b^2 = a^2 + d^2$$

and therefore

$$db = a^2 \Rightarrow \frac{d}{a} = \frac{a}{b} = m$$

So the slope of the normal line is simply $-\frac{a}{d} = -\frac{1}{m}$. □