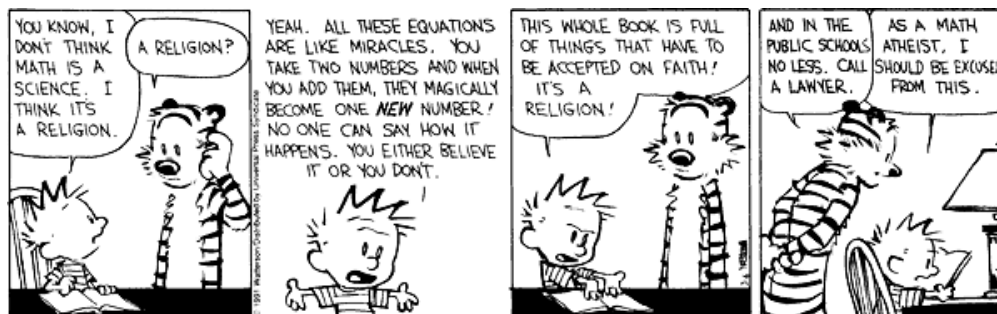


# NCEA Level 3 Calculus

## Prior Revision: Functions



### What is Calculus?

Calculus is the study of:

- Continuous change.
- Slope, area, and volume.
- Functions and relationships.

It has applications in:

- Physics and chemistry.
- Probability theory.
- Population theory.
- Economics (I am assured).

In pure mathematics, calculus can be seen as the computational side of a pretty subject called **real analysis**.

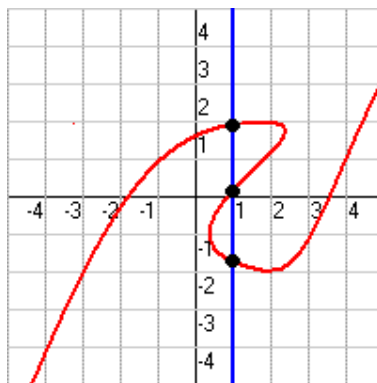
### Revision of Functions

Arguably, the most fundamental concept in calculus is that of a *function*.

**Definition** (Function). A function is a something which takes a set of things (for example, the real numbers  $\mathbb{R}$ ) and assigns to each one exactly one thing (which could be the same or different).

**Example.** The map which takes a number  $x$  and spits out  $x^2$  is a function — for every input, there is exactly one output. If we *graph* this function, we plot its input on the  $x$ -axis and its output on the  $y$ -axis and obtain a parabola.

**Example.** The curve graphed below is *not* a function, since for some inputs (like 1) it has more than one output. We can check this by drawing vertical lines along the function, like that pictured: if a graph is a function, no vertical line can ever cross the curve more than once (this is the *vertical-line test*).



**Example.** The map  $f : x \mapsto \sin x$  is a function. We could also define it by ‘the function  $f$  such that  $f(x) = \sin x$ ’. This function  $f$  can only produce numbers between 1 and  $-1$ ; we say that its *range* is the interval from  $-1$  to  $1$ .

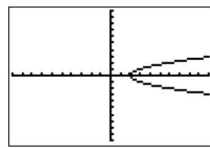
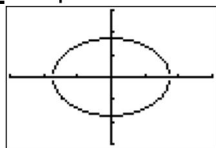
**Example.** The map  $\iota : x \mapsto x$  is a function, called the *identity function*.

**Example.** The map  $\ln x$  is a function, but it is only defined when  $x > 0$ : we say that its *domain* is the positive real numbers.

**Example.** Some more non-examples:

### Non – Examples of a Function

#### #1: Graphs



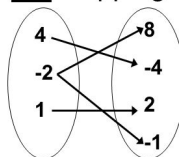
#### #2: Table

x	-1	2	1	0	-1
y	-5	3	2	-1	4

#### #3: Set

$\{(-1,2), (1,3), (-3,-1), (1,4), (-4,-2), (2,0)\}$

#### #4: Mapping



## Questions

1. Which of the following are functions?

- (a)  $E(x) = 2^x$
- (b)  $\phi : x \mapsto \frac{2}{x}$
- (c) The thing which maps every person to their youngest sibling.
- (d) The thing which sends every person to their youngest sibling that isn't themselves.
- (e)  $x \mapsto \lfloor x \rfloor$  (the floor map).
- (f) The relation that sends every person to their age.

2. I will define two functions,  $\varphi$  and  $\vartheta$ , as follows:

$$\varphi(x) = 2x - 7, \quad \vartheta(\zeta) = \frac{1}{7}(14\zeta - 49).$$

Explain why these functions are equal.

3. If  $f(x) = x^2 + x$ , find:
- (a)  $f(1)$
  - (b)  $f(y)$
  - (c)  $f(x + h)$
4. Find the distance between  $(-3, 4)$  and  $(2, 1)$ .
5. Three sides of a triangle are have lengths 8, 15, and 17.
- (a) Show that the triangle is right-angled.
  - (b) Find the other two angles.
6. Factorise and solve  $x^2 - 3x + 2 = 0$ .
7. How many **lines** are there through the point  $(2, 3)$  and the origin? Give the equations of all such lines.
8. Find the slope of the line  $4x + 3y + 2 = 0$ .
9. Find the solution to the following linear system:
- $$\begin{aligned} 2x + y &= 7 \\ 3x - y &= 8 \end{aligned}$$
10. How many (real) solutions does  $x^2 + 4x + 1$  have?
11. Draw  $\sin(x)$ ,  $\cos(x)$ ,  $\tan(x)$ ,  $\exp(x)$ , and  $\ln(x)$ .
12. How many solutions does  $\cos(3\pi x + 1) = 2$  have?
13. How many solutions does  $\sin(3x) = \frac{1}{3}$  have?