NCEA Level 3 Calculus (Differentiation)
9. Implicit Differentiation (Homework)

Reading

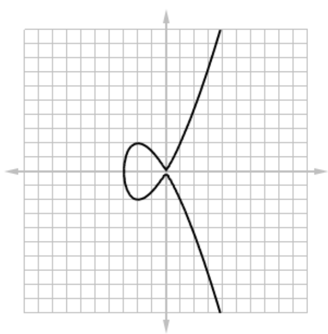
Underpinning all our work this week was the idea that if an implicit formula in x and y is 'nice enough' then there is a way to find a 'function' from x to y that we can differentiate. This rather vague notion is formalised by the rather important *implicit function theorem*, which states that if an equation $F(x, y) = 0$ has solution $(x, y) = (a, b)$ then, under certain conditions*, the equation implicitly defines in some region around x a function with a continuous derivative that takes the value b at $x = a$ — in other words, there is some function whose graph is the graph of $F(x, y) = 0$ for all of the points around the point (a, b) that we care about.

The proof of the theorem is long and we won't attempt it here; the important thing to take away is that this notion of taking a graph and then looking at the function which it describes in a small region is well-defined, and well-defined in such a way that we can do calculus on it.

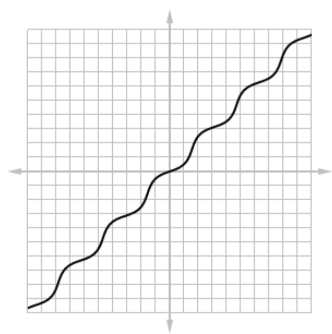
Questions

1. Find y' in each case:

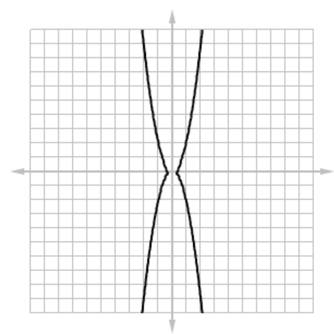
   (a) $y^2 = x^3 + 3x^2$ (Tschirnhausen cubic)

   (b) $\sin(x + y) = 2x - 2y$

   (c) $y^2 = 5x^4 - x^2$ (kampyle of Eudoxus)

2. Find the equation of the normal line to the curve $x^2 + 2xy - y^2 + x = 2$ at the point $(1, 2)$.

3. Show that the sum of the x and y intercepts of any tangent line to the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$ is just c.

* Essentially, that the graph of the function is not 'vertical' at (a, b).