NCEA Level 3 Physics (Modern Physics)

A nucleus is what is known as a *bound system*. In other words, energy must be supplied in order to break the bonds between the nucleons. This is similar to the way that electrons orbiting the nucleus are in negative energy levels — we supply the ionization energy to free the electrons from the bound system.

The energy which must be supplied to the nucleus to break it apart is known as the *binding energy*, and is on the order of tens or hundreds of MeV. This amount is high enough that its mass equivalence is non-negligible.

Consider some nucleus ${}^{w}_{n}X$ with a mass M. Experimentally, it is found that $nm_{\text{proton}} + (w-n)m_{\text{neutron}} > M$; in other words, the energy stored in the mass of the whole is less than the sum of the parts! The binding energy of the nucleus in this case is

$$E_B = (nm_{\text{proton}} + (w-n)m_{\text{neutron}} - M)c^2.$$

This is the energy which must be supplied to break apart the nucleus into its components. *

Note that this binding energy will increase as n increases, simply because there are more bonds in the nucleus. In order to more effectively compare different atoms, we use the *binding energy per nucleon*, $E_\beta = E_B/w$.

Example calculations

Consider lead, ${}^{207}_{82}\text{Pb}$. The atomic rest mass of lead is

$$M = 206.975\,871\,\text{u} \times 1.661 \times 10^{-27}\,\text{kg}\,\text{u}^{-1} = 3.437\,869 \times 10^{-25}\,\text{kg},$$

but the sum of the masses of the individual nucleons is just

$$wm_{\text{proton}} = 207 \times 1.67 \times 10^{-27}\,\text{kg} = 3.4569 \times 10^{-25}\,\text{kg}.$$

Hence the mass deficit is

$$3.4569 \times 10^{-25}\,\text{kg} - 3.437\,869 \times 10^{-25}\,\text{kg} = 1.9031 \times 10^{-27}\,\text{kg},$$

and the binding energy of lead is

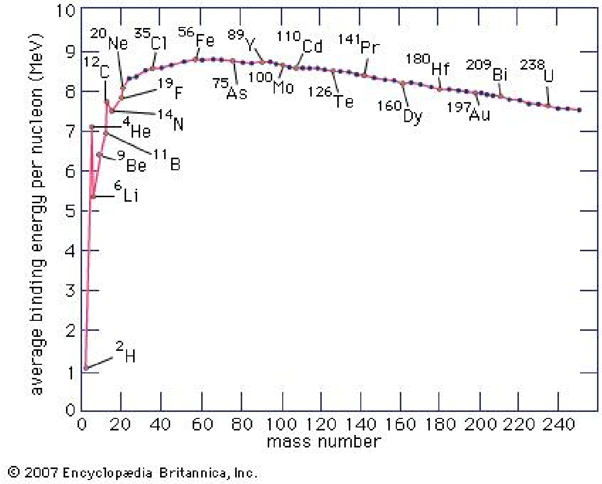
$$1.9031 \times 10^{-27}\,\text{kg} \times \left(2.99 \times 10^{8}\,\text{m}\,\text{s}^{-1}\right)^2 = 1.701\,390 \times 10^{-10}\,\text{J} = 1.063\,369\,\text{GeV}.$$

The binding energy per nucleon is 5.137 MeV.

* You may note that this energy does not take into account the mass of the orbiting electrons; we do not consider this here since $m_{\text{electron}} << m_{\text{proton}}$. See pp.1237-8 of Knight.

Fission and Fusion

Consider the following graph, which plots binding energy per nucleon against atomic mass number.



This curve has a number of interesting features.

- There are pronounced peaks at $w = 4, 12,$ and 16. These represent more tightly bound nuclei, and are due to filled *nuclear shells* (similar to electron shells).
- The binding energy per nucleon becomes roughly constant at around 8 MeV, which suggests the binding force is only short-range (adding more nucleons around the outside affects the ones inside only slightly).
- There is a global maximum at around $w = 60$, which shows that larger nuclei become more stable by breaking up, while smaller nuclei become more stable by fusing together.

Decay Revision

Alpha decay:

$$^{w}_{n}X \rightarrow^{w-4}_{n-2} Y + \alpha + \text{ energy}$$

where $\alpha =^4_2$ He.

Beta-minus (electron) decay:

$$^{w}_{n}X \rightarrow^{w}_{n+1} Y + \beta^- + \text{ energy}$$

where $\beta^- = e^-$.

Beta-plus (positron) decay:

$$^{w}_{n}X \rightarrow^{w}_{n-1} Y + \beta^+ + \text{ energy}$$

where $\beta^- = e^+$.

Gamma decay: the release of a photon (γ) when a nucleon jumps from a higher to a lower energy state.

Questions

Useful data: $c \approx 2.99 \times 10^{8}\,\mathrm{m\,s^{-1}}$, $h \approx 6.63 \times 10^{-34}\,\mathrm{J\,s}$, $e \approx 1.6 \times 10^{-19}\,\mathrm{C}$, $1\,\mathrm{eV} \approx 1.6 \times 10^{-19}\,\mathrm{J}$, $1\,\mathrm{u} \approx 1.661 \times 10^{-27}\,\mathrm{kg}$, $m_{\mathrm{proton}} = 1.007\,283\,\mathrm{u}$, $m_{\mathrm{neutron}} = 1.008\,665\,\mathrm{u}$.

1. Show that $1\,\mathrm{u} \equiv 931.49\,\mathrm{MeV}$.
2. Consider two nuclei, one of weight 200 u and one of weight 60 u.
   (a) Which has the higher binding energy? Explain.
   (b) Which is more tightly bound? Explain.
3. Calculate the binding energy of ${}^{56}_{30}\mathrm{Fe}$, if $m({}^{56}_{30}\mathrm{Fe}) = 55.9349\,\mathrm{u}$.
4. Calculate, in MeV, the binding energy per nucleon of ${}^{3}\mathrm{H}$ and of ${}^{2}\mathrm{H}$. Which is more tightly bound?
5. (a) Younger stars contain a lot of hydrogen gas. How do they generate so much energy?
   (b) Older stars are richer in helium than hydrogen. Why do they produce less energy than younger stars?
6. Consider ${}^{15}_{8}\mathrm{O}$. If this particle decays via positron emission, what daughter particle will be produced and what will the maximum kinetic energy of the released positron be?
7. The uranium isotope ${}^{238}\mathrm{U}$ undergoes α-decay to ${}^{234}\mathrm{Th}$. What is the kinetic energy of the released alpha particle, in MeV?
8. Hayden Leete is trying to convert a wooden table into electricity to fuel a nuclear missile. Given that the table is 50 kg, and that wood is approximately 50% water and 50% carbon by mass, how much energy would be released if the table's nuclei were totally split?
9. The following reaction takes place within a conventional nuclear weapon.

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{89}_{36}\mathrm{Kr} + {}^{144}_{56}\mathrm{Ba} + 2\left({}^{1}_{0}\mathrm{n}\right) $$

   How much energy is released in this reaction?

The following two questions are taken from Q25 of the 2016 VUW Scholarship Physics booklet.

10. A ${}^{66}_{28}\mathrm{Ni}$ nucleus with a mass of 65.9291 u decays by β^{-} emission.
    (a) Identify the nucleus that results from this decay.
    (b) If the daughter nucleus has a mass of 65.9289 u, what is the maximum kinetic energy of the emitted β^{-} particle?
    (c) Why would the emitted particle have less than this kinetic energy?
11. (a) Consider the following nuclear process, in which a proton is removed from an oxygen nucleus.

$$ {}^{16}_{8}\mathrm{O} + \text{ energy } \rightarrow {}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{p} $$

    Find the energy required for this process to occur.

    (b) Now, consider a process in which a neutron is removed.

$$ {}^{16}_{8}\mathrm{O} + \text{ energy } \rightarrow {}^{15}_{8}\mathrm{O} + {}^{1}_{0}\mathrm{n} $$

    Find the energy required for this process to occur.

    (c) Which particle is more tightly bound to the oxygen nucleus? Explain your answer.