NCEA Level 2 Mathematics (Calculus)

Reading

The undisputed father of mathematical problem solving is George Pólya (December 13, 1887 – September 7, 1985), one of the giants of classical analysis in the 20th century.

Pólya was born in Budapest, Hungary and died in Palo Alto, California, almost 98 years later. Both of his parents were born Jewish but converted to Catholicism. His father was born Jakab Pollák, a surname suggesting Polish origins. Jakab changed his name to be more Hungarian, believing this would help him obtain his goal of a university position. He was a talented solicitor, but because he often accepted cases without fees, he was not a financial success. George, who was originally called György, attended Dániel Berzsenyi Gymnasium, where he earned a fine academic reputation but did not shine in mathematics. Initially he resisted the career that fate had in store for him, because as he later recalled his mathematics instructors who should have been his models were "despicable teachers."

Even at an early age George had great skill for analyzing and solving problems. His uncle encouraged him to pursue a mathematical career but Pólya wanted to become a lawyer like his father. He entered the University of Budapest, but became bored with all the legal technicalities he was required to memorize. After reading Charles Darwin's The Descent of Man, Pólya briefly took up the study of biology, but when his brother insisted there was no money to be made in the subject George shifted to languages and literature. Next he turned to philosophy but to better understand it he had to learn mathematics and he was hooked. He was awarded a PhD in mathematics from the University of Budapest (1912) for an essentially unsupervised thesis in geometric probability. He spent the following year in Göttingen.

From `http://yurizkamelia.blogspot.com/2012/01/problem-solving-in-mathematics-by-polya.html`.

Polya's First Principle: Understand the Problem

This seems so obvious that it is often not even mentioned, yet students are often stymied in their efforts to solve problems simply because they don't understand it fully, or even in part. Polya taught teachers to ask students questions such as:

1. Do you understand all the words used in stating the problem?
2. What are you asked to find or show?
3. Can you restate the problem in your own words?
4. Can you think of a picture or a diagram that might help you understand the problem?
5. Is there enough information to enable you to find a solution?

Polya's Second Principle: Devise a plan

Polya mentions (1957) that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

1. Guess and check
2. Make and orderly list
3. Eliminate possibilities
4. Use symmetry
5. Consider special cases
6. Use direct reasoning
7. Solve an equation
8. Look for a pattern
9. Draw a picture
10. Solve a simpler problem
11. Use a model
12. Work backward
13. Use a formula
14. Be ingenious

Polya's Third Principle: Carry out the plan

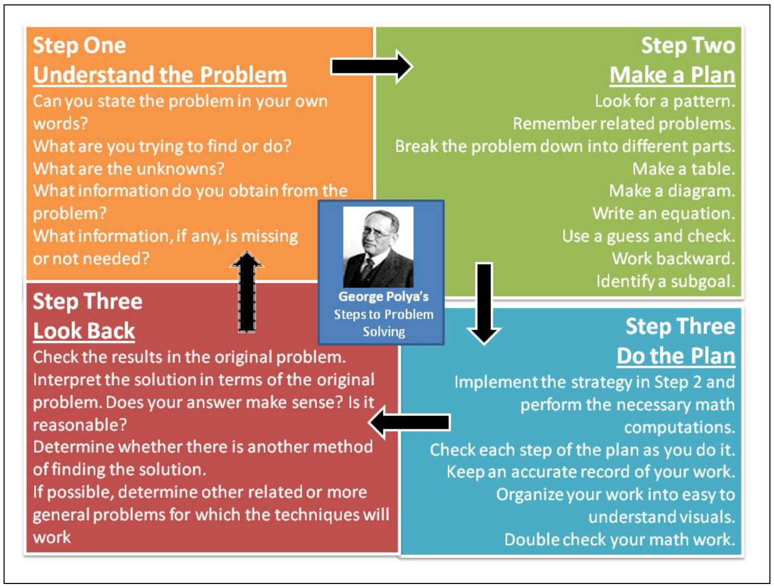
This step is usually easier than devising the plan. In general (1957), all you need is care and patience, given that you have the necessary skills. Persistent with the plan that you have chosen. If it continues not to work discard it and choose another. Don't be misled, this is how mathematics is done, even by professionals.

Polya's Fourth Principle: Look back

Polya mentions (1957) that much can be gained by taking the time to reflect and look back at what you have done, what worked and what didn't. Doing this will enable you to predict what strategy to use to solve future problems.

George Polya went on to publish a two-volume set, Mathematics and Plausible Reasoning (1954) and Mathematical Discovery (1962). These texts form the basis for the current thinking in mathematics education and are as timely and important today as when they were written. Polya has become known as the father of problem solving.

From `http://www.math.wichita.edu/history/men/polya.html`.



Questions

1. Look at the following table of trigonometric derivatives.

$f(x)$	$f'(x)$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)}$

   A spring oscillates such that the position of its end after a length of time t is given by $x = 2\sin(t)$. What is the approximate acceleration of its end at $t = 5$?

2. A cubic equation is a polynomial of degree three — that is, a function of the form $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$. Recall that a critical point is a point x where $f'(x) = 0$, $f'(x)$ is undefined, or x is at the end of the domain of the function.
   i. Sketch examples of a cubic function with zero, one, and two critical points.
   ii. Prove that a cubic function can have a maximum of two critical points.