Level Two Calculus

There are three questions, worth a total of 24 marks.
Attempt ALL questions, showing all working.
Read questions carefully before attempting them.
Marks are available for partial answers.
The amount of time expected to be spent per question may not necessarily correlate "nicely" to the number of marks.
Diagrams may be used to support answers.
Candidates who do not provide diagrams for some questions may be disadvantaged.
Some marks are given for clarity and neatness of solutions or proofs.

Time Allowed: One Hour
Achieved: 8 marks
Merit: 14 marks
Excellence: 20 marks

Question:	Differentiation	Integration	Kinematics	Total
Points:	8	8	8	24
Score:				

Available Grades: *Not Achieved* *Achieved* *Merit* *Excellence*

Question 1: Differentiation

Consider the function $y = 4px^3 - 6x^2 - qx - 2$, where p and q are non-zero real constants.

(a) Find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the function. (2)

(b) Suppose that the function has only one turning point.

i. Find p in terms of q. (3)

ii. If $q = -1$, find the y-ordinate of the turning point. You do not need to classify it. (3)

Question 2: Integration

(a) Find the most general antiderivative of $f''(x) = 15x^4 + 2x^3 + 7$. (2)

(b) Let $g''(x) = 7x^5 + 4x^3 + 7x + H$, where H is a real constant.

i. Suppose $g'(x)$ has a local extrema at $x = 1$. Find H. (3)

ii. If $g'(0) = g(0) = 2$, find an exact formula for $g(x)$. (3)

Question 3: Kinematics

Consider a particle with velocity in the x-direction $v_x(t)$ graphed in figure 1.

(a) What is the value of $a_x(t)$, the acceleration of the particle in the x-direction, at $t = 4$? (2)

(b) Suppose that the particle has an x-ordinate of $x = 100$ when $t = 10$. Sketch the graph of the particle's displacement from the origin in the x-direction, $x(t)$. (3)

(c) The velocity of the particle in the y-direction is given by $v_y(t) = 3t^3 + 9t + 2$. If the y-ordinate of the particle is $y = 0$ when $t = 0$, give the position of the particle (both x and y coordinates) at $t = 10$. (3)

Diagrams and Useful Formulae

- If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
- If $y = ax^n$, then $\frac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$.

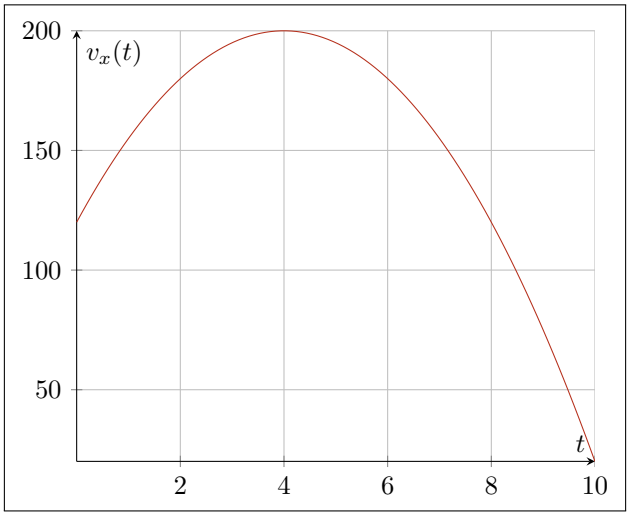

Figure 1: The x-velocity of the particle modelled in question 3.

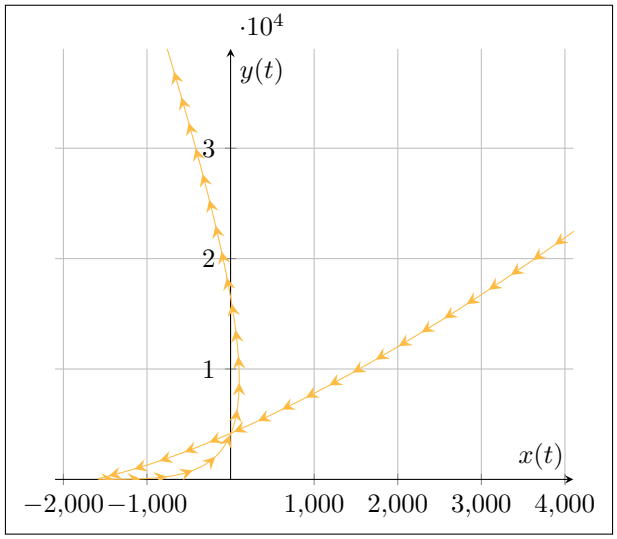

Figure 2: A graph of the trajectory (x, y) followed by the particle modelled in question 3, over the time period $-15 \leq t \leq 15$. Arrows denote the direction of time.