NCEA Level 3 Calculus (Differentiation)
3. Derivatives of Common Functions

Now that we understand the purpose and form of the derivative, we can begin to calculate the derivatives of some common functions. We list them here without proving them; all the proofs are reasonably short and can be found in the calculus notes.

Function	**Derivative**
x^n	nx^{n-1}
e^x	e^x
$\ln\lvert x\rvert$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\csc^2 x$

Today, we also introduce several rules for finding the derivative of a more complicated function in terms of the derivatives of component functions. Let f and g be functions, and let λ be a real constant. Then:

$$\begin{aligned}(\lambda)' &= 0\\ (\lambda f)' &= \lambda f'\\ (f+g)' &= f' + g'.\end{aligned}$$

In particular, the derivative is *linear*: that is, for functions f, g and real constants α, β we have $(\alpha f + \beta g)' = \alpha f' + \beta g'$ (formally, the derivative is a linear transformation from the vector space of differentiable functions to the vector space of real-valued functions).

Note that the obvious product rule, $(fg)' = f'g'$ does **not** hold. We will discuss this further soon, although it should be noted that Leibniz initially believed this rule to be true! A counterexample is outlined in the exercises for you to work out.

Questions

1. $\boxed{\texttt{A}}$ Find the derivatives of $3x^3$, $2x^2$, and $6x^5$. Conclude that $(fg)' \neq f'g'$ in general.
2. $\boxed{\texttt{A}}$ Find the derivatives of the following functions with respect to t:
   (a) $y = 2t^3 + 3t^2$
   (b) $y = \sqrt{t}$
   (c) $y = (2t + 1)(t - 4)$
   (d) $g(t) = 4 \sec t + 9 \tan t$
   (e) $h(t) = \sqrt[5]{t} + 2 \csc t - \ln t^3$
   (f) $\phi'(t) = \csc x + 12x^{1273} + 9$
   (g) $y = 2017t^{2016} + (t + 2)^2$
   (h) $y = 940 \sin t + \frac{1}{2}e^{t+2}$
3. $\boxed{\texttt{A}}$ Where is the function $x \mapsto x^3 - 2x^2 - x + 1$ increasing?
4. $\boxed{\texttt{A}}$ Find the slope of the tangent line to $y = e^x - \sin x$ at $x = 2\pi$.
5. $\boxed{\texttt{A}}$ Find the slope of the tangent line to $y = x + \tan x$ at (π, π).
6. $\boxed{\texttt{A}}$ Compute the derivative of $\ln x^2$.
7. $\boxed{\texttt{A}}$ Differentiate $\tan x \cot x \sin x$.
8. $\boxed{\texttt{A}}$ True or false: if $f(x)$ is a cubic polynomial, then $f'(x)$ is a quadratic polynomial.
9. $\boxed{\texttt{A}}$ Suppose the derivative of a function is $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - x - 4$. What could the original function be?
10. $\boxed{\texttt{M}}$ Find the 64th derivative of $\sin x$.
11. $\boxed{\texttt{M}}$ Find the nth derivative of x^n.
12. $\boxed{\texttt{M}}$ If $y = 2 \sin 3x \cos 2x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (Hint: use an identity to rewrite this as a sum of functions)
13. $\boxed{\texttt{M}}$ For which values of x does the graph of $f(x) = x + 2 \sin x$ have a horizontal tangent?
14. $\boxed{\texttt{E}}$ Show that $y = 6x^3 + 5x - 3$ has no tangent line with a slope of 4.
15. $\boxed{\texttt{E}}$ Find real values of α and β such that, if $y = \alpha \sin x + \beta \cos x$, then $y'' + y' - 2y = \sin x$.
16. $\boxed{\texttt{E}}$ Consider a 12 m long ladder leaning against a wall such that the top of the ladder makes an angle θ with the wall. If this angle θ is varied, the distance D between the bottom of the ladder and the wall also changes. If $\theta = \pi/3$, what is the rate of change of D with respect to θ?
17. $\boxed{\texttt{E}}$ Prove that the function φ given by

$$\varphi(x) = \frac{x^{101}}{101} + \frac{x^{51}}{51} + x + 1$$

   has no extreme values.
18. (a) $\boxed{\texttt{A}}$ The area of a circle of radius r is $A = \pi r^2$. Find $\frac{\mathrm{d}A}{\mathrm{d}r}$. What do you notice?
   (b) $\boxed{\texttt{M}}$ Explain part (a) geometrically.
   (c) $\boxed{\texttt{E}}$ The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$. Find an expression for the surface area.

19. $\boxed{\texttt{E}}$ Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \log_{10} x$.