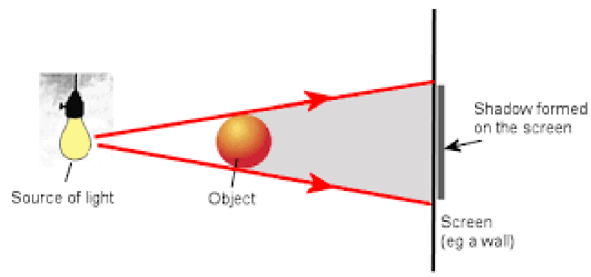
NCEA Level 1 Mathematics (Trigonometry)

1. A person 100 m from the base of a tree estimates that the angle between his eye level (around 1.5 m above the ground) and the top of the tree is $18°$. Compute the height of the tree.

2. The angle of elevation of a balloon moving straight up changes from $25°$ at 10:00 am to $60°$ at 10:02 am. The point of observation of the angle of elevation is situated 200 m away from the take off point. If the balloon is moving upwards at a constant speed v, compute v to a reasonable accuracy.

3. Consider the diagram below.



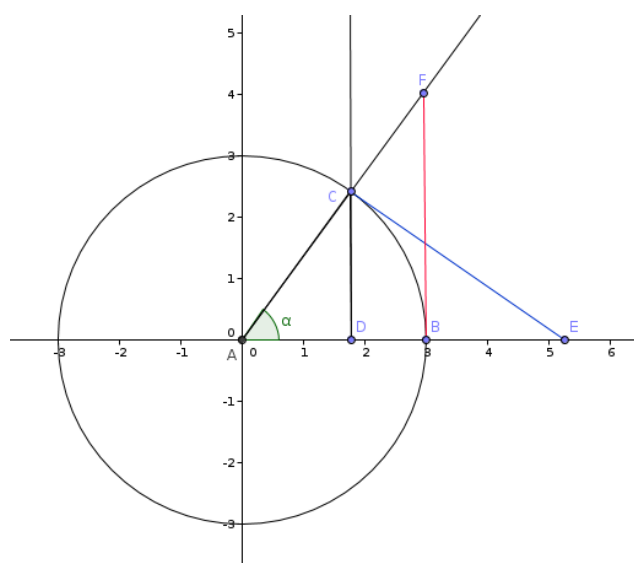
The radius of the ball is sixteen centimetres, the distance from the screen to the light source is two metres, and the centre of the ball is exactly halfway between the light source and the screen, find the height of the shadow formed.

4. Prove that $\sin\theta = \cos(90° - \theta)$.

5. Show that $\sin 45° = \frac{1}{\sqrt{2}}$ exactly.

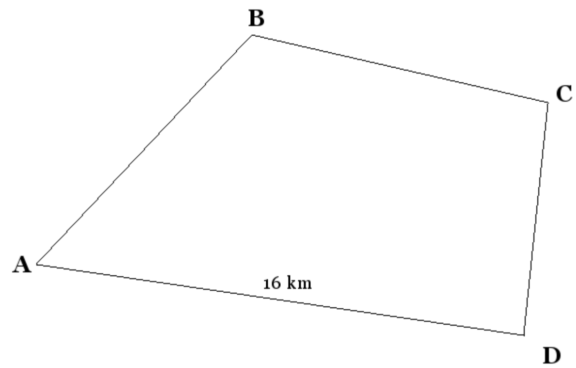
6. Suppose a triangle has two sides, measuring 4 m and 5 m respectively. If the angle between these sides is $40°$, what is the angle of the triangle?

7. Consider a circle, along with a ray AC from the centre and tangents BF and CE. Show that $\ell(CE) = \ell(BF) = \tan\alpha$.



8. A **regular n-gon** ($n \geq 3$) is a shape with exactly n sides, each with exactly the same length. For example, a regular 3-gon is an equilateral triangle. Find the internal angle between the sides of (a) a regular 3-gon, (b) a regular 4-gon, and (c) a regular n-gon (for n any natural number).

9. In Ye Olden Days(TM), long distances were measured and maps drawn using triangulation. Consider the following map, where each point A, B, C, and D is a mountain,

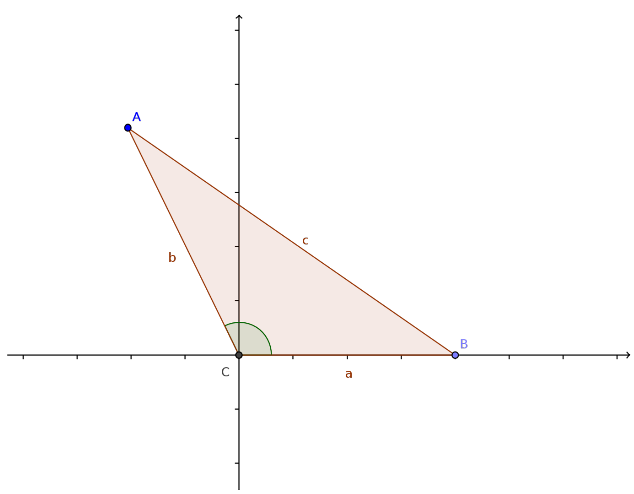


The distance $\ell(AD)$ is known to be 16 km, and the angles $\angle BAD$, $\angle ABC$, and $\angle ADC$ are measured to be $30°$, $110°$, and $100°$ respectively. In terms of height, D is measured to be $10°$ below A and C is measured to be $14°$ above D. The height of peak A is 1100 m. What is the straight-line distance between the tops of peaks A and D, and what is the height of the peak at D? Is it possible to calculate the height of peak B?

10. Take a regular tetrahedron (triangular pyramid) and drop a perpendicular line from the incentre of each face; at the point in the centre of the solid where the three lines meet, what angle does each line form with the others?

11. You know that for a right-angled triangle, the Pythagorean Theorem can be used to calculate the length of any side given the other two. In this question, we will derive a more general side-length formula that works for arbitrary triangles.

    Let us consider an arbitrary triangle ABC, and move it around so that the point C is at $(0, 0)$ and the point B is at $(b, 0)$; see the following diagram.



    (a) Let θ be the measure of the angle $\angle ACB$. Show that the coordinates of the point A are $(b\cos\theta, b\sin\theta)$.
    (b) Use the Pythagorean Theorem to calculate the distance c using the coordinates of A and B.
    (c) Hence show that
    $$c^2 = a^2 + b^2 - 2ab\cos\theta. \qquad \text{(Cosine rule.)}$$

12. Calculate the bond angle of a methane molecule (CH_4).