NCEA Level 3 Calculus (Differentiation)
5. The Product and Quotient Rules (Homework)

Reading

Even next year, when you will be expected to know the derivative rules, it is not necessary to remember the quotient rule because it is just a special case of the product rule. However, here is a little rhyme:

If it's the quotient rule you wish to know,
It's low-de-high less high-de-low.
Then draw the line and down below,
Denominator squared will go.[*]

Questions

1. Find the derivatives:
   (a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \sin x \ln x$.
   (b) $\frac{\mathrm{d}y}{\mathrm{d}x}$ if $y = x \sec kx$ (k constant).
   (c) $\frac{\mathrm{d}f}{\mathrm{d}\theta}$ if $f(\theta) = \frac{\cos \pi\theta}{\sin \pi\theta + \cos \pi\theta}$.
   (d) $\frac{\mathrm{d}y}{\mathrm{d}t}$ if $y = \cos^4(\sin^3 t)$.
2. The force F acting on a body with mass m and velocity v is the rate of change of momentum, $F = \frac{\mathrm{d}}{\mathrm{d}t}[mv]$. If m is constant, this becomes $F = ma$, where $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ is the acceleration of the body. However, due to relativistic effects, the mass of a particle varies with v as
   $$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$
   where m_0 is the rest mass of the body and c is the speed of light. Show that
   $$F = \frac{m_0 a}{\left(1 - \dfrac{v^2}{c^2}\right)^{\frac{3}{2}}}.$$
3. Recall that if θ is given in degrees, then $\frac{\pi\theta}{180}$ is the equivalent angle in radians. Find the derivative of $\sin\theta$ if θ is given in degrees.

[*]Quoted in *Mathematical Apocrypha* by Steven G. Krantz (p.36).