NCEA Level 3 Calculus (Integration)
24. Kinematics (Homework)

Reading

Kinematics may seem like an odd topic to end with, especially as it is more of a physics topic than a mathematics topic. The reason for its inclusion in these notes is by means of revision from level 2; and the reason it is in level 2, is so that you can apply calculus to L3 physics. If you are not studying physics, you may wonder why you should bother learning this particular application of calculus; the answer is that, historically, calculus began as an attempt to formalise mechanics and so a physical intuition can often be useful when solving problems that are not at first glance physical.

Beyond this, there is only one fundamental concept in this topic that you must remember: the derivative is just a rate of change. Velocity is rate of change of position, and acceleration is rate of change of velocity. If you slow down faster, your acceleration is more negative.

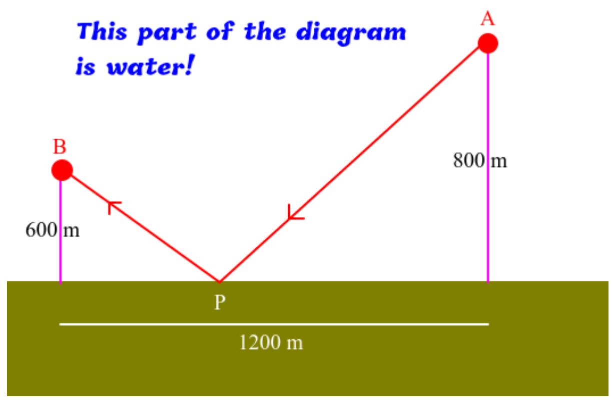
In terms of integration, if you have a certain velocity at a given point then that implies that over a given period of time, you travel a certain distance. It follows that if you add up all these instantaneous velocities, multiplying each by the infinitesimal time that you are travelling for each one, then you obtain the total distance you travel; that is, you see that $x = \int \frac{\mathrm{d}x}{\mathrm{d}t} \,\mathrm{d}t$.

Questions

All distances are given in m, and all times in s, unless otherwise stated.

1. A distress flare is fired vertically into the air from a boat at sea. The height in metres of the flare t seconds after firing is given by
$$h = 122.5t - 4.9t^2.$$
   (a) What is the initial velocity of the flare?
   (b) At the peak of its flight, what is the vertical velocity of the flare?
   (c) What is the maximum height reached by the flare?

2. Part of the course for an ocean swim runs from bouy A to bouy B. Swimmers must come ashore on the at some point P along a long straight beach on the way. Bouy A is 800 m away from the beach, and bouy B is 600 m away from the beach. What is the least distance that a swimmer must swim? (Hint: minimise $PA + PB$.)



3. The following graph shows the acceleration of a rocket from launch until it reaches orbit. Given that the initial velocity of the rocket was $0\,\mathrm{ft\,s^{-1}}$, find the final velocity of the rocket.