NCEA Level 2 Mathematics (Calculus)

Questions

1. Sketch graphs with the following properties:
   (a) A maximum at $x = 3$ and a minimum at $x = 6$.
   (b) No maximums or minimums, but critical points at $x = 2$ and $x = 4$.
   (c) Maximums at $x = 2$ and $x = 4$ but no minimums.

2. Find the maximum and minimum points of the function g defined by
$$g(x) = 2x^3 + x^2 + 2x. \tag{1}$$

3. Draw the derivative of $\tan x$.

4. The function F, where
$$F(x) = x^{\frac{4}{5}}(x-4)^2, \tag{2}$$
has critical points at $x \in \left\{0, \frac{8}{7}, 4\right\}$. Classify each one as a maximum, a minimum, or neither.

5. Show that
$$\int 470x^4 + 2x + 1 + 6x^{-3}\,\mathrm{d}x = 94x^5 + x^2 + x - 3x^{-2} + C. \tag{3}$$

6. Prove that the function φ given by
$$\varphi(x) = x^{101} + x^{51} + x + 1 \tag{4}$$
has no extreme values.

7. A function f is given by
$$f(x) = 2 - 4x + 5x^2 + ax^3. \tag{5}$$
The gradient of the graph at the point where $x = 1$ is 3. Find the value of a.

8. Suppose $a < b$. Show that $\frac{a+b}{2} < b$. Hint: try adding something onto both sides.