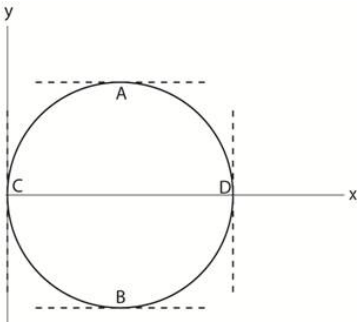
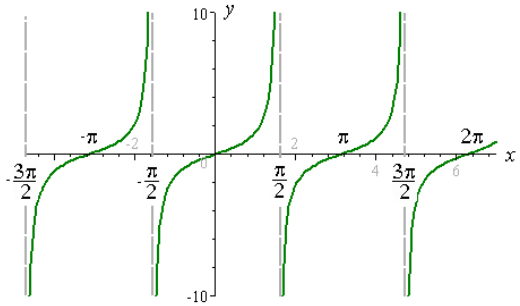
NCEA Level 2 Mathematics (Calculus)

Reading

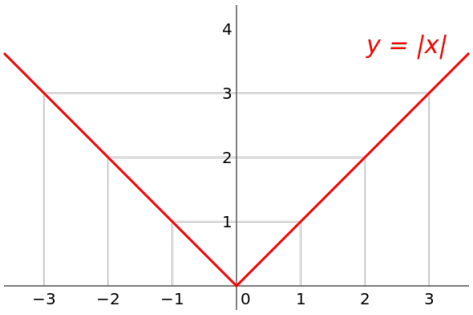
So far, we have only looked at 'nice' functions. However, it is possible to find functions which are not so nice, in the sense that the derivative does not exist at some point. What does this mean? It means that at some point, the graph of the function doesn't have a well-defined slope. For example, the function could become vertical (what is the slope of a vertical line?), or it could jump from one place to another without passing any of the points in between. Note that a function can be differentiable everywhere except one point, for example the absolute value function! Here are some examples of some non-differentiable functions.



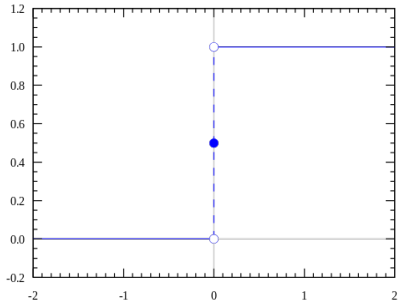
The circle is not differentiable at C or D because the tangent lines are vertical.



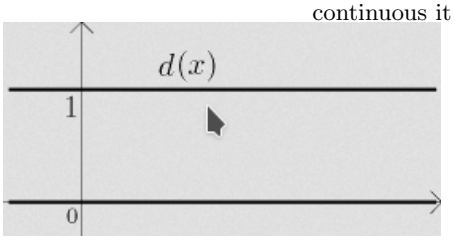
The tan function is not differentiable when the angle is an odd multiple of $\frac{\pi}{2}$ because the function is undefined there.



The absolute value function is not differentiable at $x = 0$ because it has no tangent line (try to draw one in!).



The Heaviside step function is the derivative of the absolute value (can you see this geometrically?) with $H(0) = \frac{1}{2}$, plugging the hole. However, because it is not continuous it is not differentiable at $x = 0$.

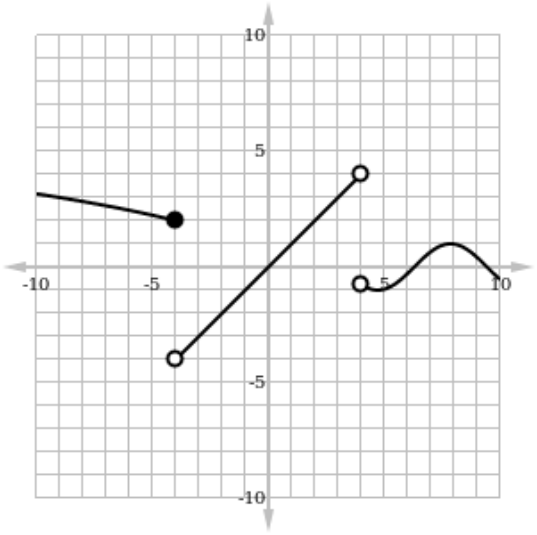


The Dirichlet function $d(x)$ takes the value 1 when x is irrational and 0 when x is rational, and so is continuous nowhere (there is a rational number between any two irrationals and vice versa, so the function jumps between 0 and 1 infinitely often). As you might expect, it is differentiable nowhere.

Questions

The questions this week focus on the visual geometry of the derivative.

1. Answer the following questions about this graph. Open circles denote locations where the function *is not* defined, while filled circles denote locations at the end of a segment where the function *is* defined.



   (a) What is the slope of the graph at $x = 8$?

   (b) Does the function have a derivative everywhere (i.e. can you draw a tangent line onto the graph at every point)? If not, where does it fail to be differentiable?

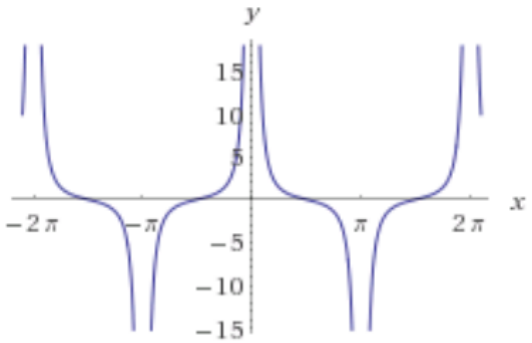
   (c) At $x = -5$, is the derivative positive or negative?

   (d) What is the slope of the curve at $x = 0$?

   (e) The function has three local extrema. Two of them can be found where the derivative is zero (the tangent line is flat). Find all three.

2. The following graph shows $f'(x)$, the derivative of a function f. Use the graph of the derivative to recreate the graph of the original function.

   Hint: Where must the original function be decreasing or increasing? Where will it have maximums and minimums? How fast does it change?



(x from -6.6 to 6.6)