Level Three Calculus
Differentiation

There are three questions, worth a total of 30 marks.
Attempt ALL questions, showing all working.
Read questions carefully before attempting them.
Marks are available for partial answers.
The amount of time expected to be spent per question may not necessarily correlate "nicely" to the number of marks.
Diagrams may be used to support answers.
Candidates who do not provide diagrams for some questions may be disadvantaged.
Some marks are given for clarity and neatness of solutions or proofs.

Time Allowed: One Hour
Achieved: 11 marks
Merit: 19 marks
Excellence: 27 marks

Question:	1	2	3	Total
Points:	10	10	10	30
Score:				

Available Grades: *Not Achieved* *Achieved* *Merit* *Excellence*

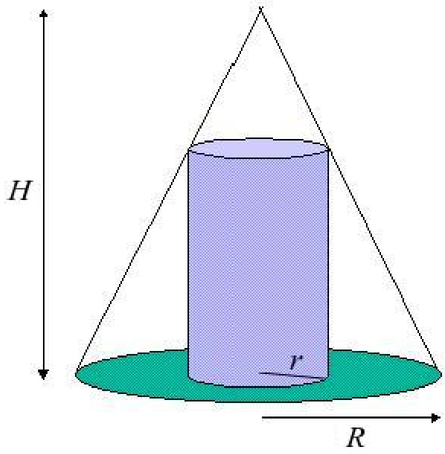

Figure 1: A cylinder inscribed in a cone.

Question 1

(a) Find $\frac{\mathrm{d}D}{\mathrm{d}t}$ and $\frac{\mathrm{d}D}{\mathrm{d}x}$ if $D = A\sin(kx - \omega t + \phi_0)$. (2)

(b) Find the rate of change of y with respect to x at the point $(0, \frac{\pi}{4})$ if (3)

$$\tan y = e^x$$

(c) Consider a cylinder of radius r within a right-angled circular cone of radius R and height H (see figure 1). Show that the volume of the cylinder is maximised when $r = \frac{2}{3}R$. *Show any derivatives you require, and carefully justify that the volume is a maximum.* (5)

Question 2

(a) Compute the derivatives of the following functions.

i. (2)

$$y = \frac{e^{-x} + \sin x}{x}$$

ii. (1)

$$y = \sqrt{x}\left(x^2 + 5x + \frac{1}{x}\right)$$

(b) i. Write down the expression for the derivative of x^3 from first principles. *You need not evaluate the limit.* (1)

ii. Hence, or otherwise, compute the following limit: (2)

$$\lim_{h \to 0} \frac{(2+h)^3 - 8}{h}$$

(c) Find the interval(s) on which the function f defined below is concave up. (4)

$$f(x) = \frac{x^5}{20} - \frac{2x^3}{3} + 16x + 9$$

Question 3

(a) Show that the rate of change of g with respect to t at $t = 2$ is $\frac{3\pi}{2}$ if (2)

$$g(t) = \sin(\pi\sqrt{3t^2 + 4}).$$

(b) Find the equation to the normal line of the curve $y = x^4 - 3x^3 - 2x^2 + 2x - 1$ at the point $(0, -1)$. (3)

(c) Sand is being poured into a hole at a rate of $\frac{\mathrm{d}S}{\mathrm{d}t} = 3t + 4$, and the depth of the hole is given by $h = 12 - \sqrt{S}$. Find $\frac{\mathrm{d}h}{\mathrm{d}t}$ in terms of t and S, and show that h has no maxima or minima at any time after $t = 0$. (2)

(d) Show that $y = 2x^3 - 18x^2 + 90x + 3$ has no tangent line with a slope of 3. (3)