NCEA Level 3 Trigonometry (exercise set)
3. Taxonomy of Functions

Goal To investigate the relationships between the geometry and algebra of the trig functions.

1. Show that the following hold for all θ:
    (a) $\sec^2\theta + \csc^2\theta = \sec^2\theta\cos^2\theta$
    (b) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \csc\theta)^2 = (1 + \sec\theta\csc\theta)^2$

2. (a) Draw a graph to check the plausibility of the conjecture that

$$\tan\theta = -\cot\left(\theta - \frac{\pi}{2}\right) \tag{*}$$

   holds for all θ.

    (b) Show that (*) holds for all θ.
    (c) Prove that there is **no** ξ such that $\cos\theta = \sec(\theta + \xi)$ holds for all θ.

3. Let O_1 and O_2 be two distinct points, and suppose r_1 and r_2 are positive numbers with $r_1 > r_2$.
    (a) Draw two circles centred at O_1 and O_2 with respective radii r_1 and r_2.
    (b) Suppose L is a tangent line shared by both circles which does not cross the *segment* O_1O_2; show that the angle θ made by L with the *line* O_1O_2 satisfies

$$\sin\theta = \frac{|O_1O_2|}{r_1 - r_2}.$$

    (c) Find the distance between the points of tangency along L.

4. Complete exercise 3.7 in the notes.

5. (a) Illustrate graphically the change in sign and magitude of the functions $3\sin x + 4\cos x$, $e^x\sin x$, and $\sin\left(\frac{\pi}{\sqrt{2}}\sin x\right)$ for all values of x.
    (b) Show that $2x = (2n + 1)\pi\,\mathrm{versin}\,x$ (for n positive) has **exactly** $2n + 3$ possible solutions for x; indicate roughly their locations. [Hint: approach this problem in the same rough direction as (a) — i.e. do not try to compute the values of x directly.]

Additional reading Hobson, chapter III.