NCEA Level 2 Mathematics (Homework)

9. Exponential and Logarithmic Functions

Reading

Go and watch...

https://www.youtube.com/watch?v=N-7tcTIrers

What's it good for?

People use exponential and logarithmic equations for...

• Chemistry, physics, engineering: whenever the rate of growth or rate of decline of a quantity is proportional to (or inversely proportional to) the amount of quantity present, the quantity is an exponential or logarithmic function of time. (This includes rates of chemical reaction, rates of capacitor charge/discharge, the position of a damped spring over time, and many other examples.)

Questions

1. Thirty minutes after a patient is administered his first dose of a medication, the amount of medication in his bloodstream reaches $224 \,\mathrm{mg}$. The amount of the medication in the bloodstream decreases continuously by 20% each hour. The amount of the medication M mg in the patient's bloodstream after it is administered can be modelled by the function

$$M = 224 \times 0.8^{t-0.5}$$

where t is the time in hours since the drug was first administered.

- (a) Explain what 0.8 means in this function.
- (b) Give the initial amount of medication administered.
- (c) A second dose of the medication can be administered some time later, and again the amount of the medication in the patient's bloodstream from the second dose can be modelled by the same function as that for the first. The total amount of the drug in the blood stream must never exceed 300 mg. How long after administering the first dose can the second dose be administered?
- 2. Here are some revision questions on topics we have already covered.
 - (a) Rearrange the following formula to make x the subject: $\frac{4x}{5} = \frac{y(x+3)}{2}$.
 - (b) Show that the solutions of $x^2 + x 56 = 0$ are four times those of $4x^2 + x 14 = 0$.
 - (c) Find the relationship between the solutions of the equations $dx^2 + ex + f = 0$ and $x^2 + ex + df = 0$ where d, e, and f are real numbers.
 - (d) Consider the equation $(3x+1)^2 = -7$.
 - i. Explain why it has no real solutions; explain what this means graphically.
 - ii. Compute the discriminant of the equation, and explain why this further supports your answer to (i).