

## NCEA Level 3 Calculus (Integration)

### 16. Anti-differentiation

A few weeks ago, it was hinted that finding area was in some way the inverse to finding slope. In last week's questions, the curtain was pulled back a little further when we calculated an area function for a variable endpoint, took the derivative, and got the original function back. Next week, all will be revealed; but first, we need to do a little bit more work. This week, we're going to look at *anti-differentiation*: finding the original function given its derivative.

**Example.** One antiderivative of  $y' = 3x^2 + 4$  is  $x^3 + 4x$ . Another is  $x^3 + 4x + 1$ . A third is  $x^3 + 4x + 7$ . Obviously every function of the form  $y = x^3 + 4x + C$  for some constant  $C$  will differentiate to the given  $y'$ , so we must remember always to make this clear.

We also call antiderivatives **indefinite integrals**, and in this notation the above example is

$$\int 3x^2 + 4 \, dx = x^3 + 4x + C.$$

**Example.** The most general antiderivative of  $\sin x$  is  $-\cos x + C$ .

**Example.** The most general antiderivative of  $\tan x$  is  $-\ln|\cos x| + C$ .

**Example.**  $\int \frac{1}{x+3} \, dx = \ln|x+3| + C$ .

**Example.**  $\int \tan^2 \theta \, d\theta = \int \sec^2 \theta - 1 \, d\theta = \tan \theta - \theta + C$ .

**Example.**  $\int \frac{2x}{x^2+1} \, dx = \ln|x^2+1| + C$ .

**Example.**  $\int K e^{Kx} \, dx = e^{Kx} + C$  for all constants  $K$ .

### Questions

- A** In each case, find the most general antiderivative.
  - $f(x) = 2x$
  - $f(x) = x^{-3}$
  - $f(x) = 0$
  - $f(x) = \sec^2(x) + \sqrt{x}$
  - $f(x) = x\sqrt{x}$
  - $f(t) = \sin t - \cos t$
- A** Verify the above examples by differentiation.
- A** Show that  $\int 3x^2 + 4x + 5 + 2e^{2x} \, dx = x^3 + 2x^2 + 5x + e^{2x} + C$ .
- A** Find  $y'$  when  $y = 3 + \sin(2x + 4)$  and hence find  $\int 2\cos(2x + 4) \, dx$ .
- A** If  $\frac{dy}{dt} = 1.5\sqrt{t}$  and  $y(4) = 10$ , find  $y(t)$  exactly.
- M** Find  $f$  if  $f''(x) = 12x^2 + 6x - 4$ ,  $f(0) = 4$ ,  $f(1) = 1$ .
- A** The velocity of a particle is given by  $v(t) = 2t + 1$ . Find its position at  $t = 4$  if its position at  $t = 0$  is  $x = 0$ .
- M** The acceleration of a particle is given by  $a(t) = 10\sin t + 3\cos t$ . At  $t = 0$ , its position is  $x = 0$ ; at  $t = 2\pi$ , its position is  $x = 12$ . Find its position at  $t = \frac{\pi}{2}$ .
- A** Find all functions  $g$  such that  $g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}$ .

10. M For each function, sketch an antiderivative passing through  $(0,0)$ :

