Assignment: Mathematical Writing Practice III

Alexander Elzenaar
Upper Hutt College

14th June 2017

1 Task

A wire L metres long is cut into two pieces. One of the pieces is bent into the shape of a square, the other into an equilateral triangle. Where should the cut be made to minimise the total area of the square and the triangle?

Ensure that you write 'properly'. That means using complete sentences, justifying all logic, and aiming for clarity!

2 Hints

A list of things to think about:

- Draw a diagram.

3 Example Answer

Let x be the distance from the end of the wire to the cut. So the perimeter of the square is x, and the perimeter of the triangle is $L - x$. The area of the square is $A_s = \left(\frac{x}{4}\right)^2$, and the area of the triangle is $A_t = \frac{\sqrt{3}}{4}(L-x)^2$ (prove this yourself).

So we are trying to maximise

$$A = A_t + A_s = \frac{\sqrt{3}}{4}(L-x)^2 + \left(\frac{x}{4}\right)^2.$$

Expanding this out, we find that

$$A = \left(\frac{4\sqrt{3}+1}{16}\right)x^2 - \frac{L\sqrt{3}}{2}x + \frac{L^2\sqrt{3}}{4}.$$

Taking the derivative, we find that

$$\frac{\mathrm{d}A}{\mathrm{d}x} = \left(\frac{4\sqrt{3}+1}{8}\right)x - \frac{L\sqrt{3}}{2},$$

and when $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ then $x = \frac{4L\sqrt{3}}{4\sqrt{3}+1}$.

This is obviously a minimum, since the area function was an upwards-opening parabola and so the only critical point is a minimum.