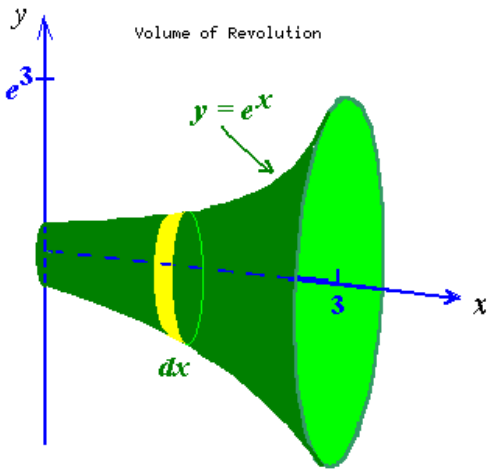
NCEA Level 2 Mathematics (Integration II)

Reading

Recall that definite integrals are simply infinite sums of infinitely small things. While the only application you see in class this year will be finding the area under a curve, there are many others. For example, suppose we wish to find the volume of this figure, obtained by taking $y = e^x$ and rotating it around the x-axis:



We can split it up into segments, each of length $\mathrm{d}x$. Then we can model the volume of that region by a cylinder of length $\mathrm{d}x$ and radius e^x (the height of the function). Using the formula $V = \pi r^2 \ell$, we find the volume of one of the small bits of the figure at $x = x_i$ is $\pi \left(e^{x_i}\right)^2 \mathrm{d}x$, and adding them all up we have the Reimann sum

$$\sum_{i=0}^{3} \pi \left(e^{x_i}\right)^2 \mathrm{d}x.$$

Letting $\mathrm{d}x \to 0$, we obtain the definite integral

$$\int_0^3 \pi \left(e^x\right)^2 \mathrm{d}x = \frac{\pi}{2} e^{6x}.$$

By the same reasoning, we can show that the **volume of revolution** of a curve $y = f(x)$ around the x-axis from $x = a$ to $x = b$ is given by

$$V = \pi \int_a^b y^2 \,\mathrm{d}x.$$

An animation can be found at `https://www.youtube.com/watch?v=i4L5XoUBD_Q` and a Khan Academy video is at `https://www.youtube.com/watch?v=btGaOTXxXs8`.

Questions

1. (a) Evaluate $\int_{-200\pi}^{200\pi} \sin x \,\mathrm{d}x$.

   (b) If $\int_0^{10} f(x)\,\mathrm{d}x = 7$ and $\int_0^6 f(x)\,\mathrm{d}x = 2$, find $\int_6^{10} f(x)\,\mathrm{d}x$.

2. Suppose a car is moving at $80\,\mathrm{km\,h^{-1}}$, and it begins to decelerate at a constant rate of $7\,\mathrm{km/h^2}$. How far does it travel before it stops completely?