NCEA Level 2 Mathematics (Homework)
14. Anti-differentiation

Reading

Go and watch...

`https://www.youtube.com/watch?v=j4hW7AwETZA`

What's it good for?

As I kind of hinted in the notes and in the final problem in the problemset, the geometric meaning of integration is to do with area. In fact, integration is the general way to find volumes, lengths, areas, and in general any kind of extent in space. People use integration for...

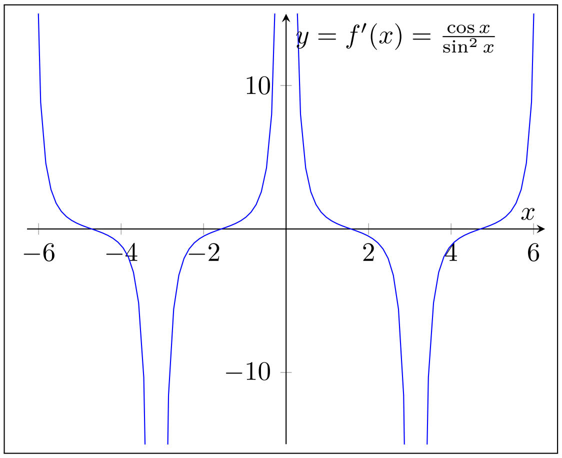
- The sciences and engineering: integration, both in its guise as "undoing slope-finding" and in its guise as an area-finding device, is heavily used in all the sciences (either explicitly, as in physics, or implicitly, as in chemistry and biology). Differential equations, equations involving functions and their derivatives, are also often found in these subjects as rates of change play an important role in engineering and in science; integrals are used to solve differential equations. In physics especially, multi-dimensional integrals and derivatives play an important role in the theories of the universe.

- Mathematics: the study of integrals can be done on two levels — as "recreational mathematics", where people try to solve hard integrals for fun (like solving a sudoku puzzle or a crossword), or as a deep subject known as measure theory — the theory of functions which measure things, and the different varieties of integrals that apply those functions onto space.

Questions

1. The following graph shows $f'(x)$, the derivative of a function f. Use the graph of the derivative to recreate the graph of the original function.

   Hint: Where must the original function be decreasing or increasing? Where will it have maximums and minimums? How fast does it change?

2. It is known that Φ is a function such that:

   - Φ has a stationary point at $x = 0$.
   - $\frac{\mathrm{d}^2\Phi}{\mathrm{d}x^2} = 60x^4 - 180x^2 + 48$.
   - $\Phi(2) = 0$.

   (a) Describe the nature of the stationary point at $x = 0$.

   (b) Find the locations of the other stationary points.

   (c) Find an exact expression for $\Phi(x)$.