

# Numeracy

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**1. Fractions**

(1) Simplify:

(a)  $\frac{6}{36}$

(b)  $\frac{3}{72}$

(c)  $\frac{51}{21}$

(d)  $\frac{96}{44}$

(e)  $\frac{42}{105}$

(f)  $\frac{2}{16}$

(g)  $\frac{4}{16}$

(h)  $\frac{6}{4}$

(2) Calculate, simplifying your answer:

(a)  $\frac{3}{7} + \frac{1}{4}$

(b)  $\frac{12}{3} + \frac{2}{6}$

(c)  $\frac{4}{15} - \frac{2}{25}$

(d)  $1\frac{1}{4} + \frac{7}{12}$

(e)  $\frac{23}{3} + \frac{3}{23}$

(f)  $\frac{2}{7} - \frac{2}{17}$

(g)  $\frac{2}{3} - \frac{1}{2}$

(3) Calculate, simplifying your answer:

(a)  $\frac{2}{3} \times \frac{3}{4}$

(b)  $\frac{9}{7} \times \frac{3}{2}$

(c)  $\frac{2}{3} \div \frac{3}{4}$

(d)  $\frac{12}{5} \div \frac{5}{12}$

(e)  $\frac{\left(\frac{12}{4}\right)}{2}$

$$(f) \left( \frac{3}{9} \div \frac{2}{3} \right) \times \frac{5}{7}$$

$$(g) \frac{3}{9} \div \left( \frac{2}{3} \times \frac{5}{7} \right)$$

$$(h) \frac{6}{\left( \frac{9}{7} \right)}$$

$$(i) \frac{\left( \frac{3}{5} \right)}{\left( \frac{7}{11} \right)}$$

- (4) (a) 15 bags of potatoes cost \$20. How much will 25 bags of potatoes cost?
- (b) 18 out of 540 fish caught by fisherman are undersized and need to be thrown back. What percentage is this?
- (c) Write  $\frac{3}{5}$  as a percentage.
- (d) Write 35% as a fraction.
- (e) What percentage of 105 is 15 (to 3dp)?
- (f) Find 30% of 70.
- (g) A car is purchased for 3.5 thousand dollars. When it is sold, the seller makes a 20% profit. What is the selling price?
- (h) 360 is 135% of what number?
- (i) Find 95% of 6.

## 2. Arithmetic

- (1) Calculate:
- (a)  $7 - 9$
- (b)  $-7 + 2$
- (c)  $-9 + 9$
- (d)  $9 - 18 + 7$
- (e)  $16 + 2 - 19$
- (f)  $\frac{2}{3} - 1$
- (g)  $0.5 - 16$
- (h)  $1.2 - 1.3$
- (i)  $-2 + (-2)$
- (j)  $2 - (-4)$
- (k)  $-6 - (-3)$

- (l)  $\frac{6}{7} - \frac{8}{3}$
- (m)  $-\frac{2}{3} - \frac{-7}{3}$
- (n)  $\frac{a}{3} + b$ , where  $a = -2$  and  $b = 6$ .
- (2) Calculate:
- (a)  $-1 \times 2$
- (b)  $-3 \times -9$
- (c)  $-3 \div 9$
- (d)  $6 \div -2$
- (e)  $-2 + (-6 \times -2)$
- (f)  $-5 \div \frac{5}{7}$
- (g)  $\frac{m}{-n}$  if  $m = -2$  and  $n = -3$ .
- (h)  $-\frac{3}{4} \times \frac{7}{5}$
- (i)  $-\frac{3}{7} \div -\frac{2}{9}$
- (j)  $a(b - c)$  where  $a = 2.5$ ,  $b = 3$ , and  $c = 3.3$ .
- (3) Calculate:
- (a)  $2 \times 7 + 12 - 15 \div 3$
- (b)  $3(5 - 7)$
- (c)  $\frac{4(2 + 6)}{2 \times 3} - 6$
- (d)  $\frac{ab}{c - d} + 2e$  where  $a = e = 4$ ,  $b = d = 3$ , and  $c = 9$ .
- (e)  $\frac{2(-3)}{6 + 9 \times -3}$
- (f)  $\left(2 \div \frac{1}{4}\right) \times \frac{3}{7}$

### 3. Expanding and Factorising

- (1) Simplify:
- (a)  $3(-4m)$
- (b)  $5 + r - 2$
- (c)  $4r + 4$
- (d)  $\frac{3}{4} \times \frac{4}{3}r$
- (e)  $\frac{1}{3}r + \frac{4}{3}r$

(f)  $9x + 2y + x$

(g)  $-3p + 4q + 6p + 2p - q$

(2) Expand and simplify:

(a)  $6(3 + 2\ell)$

(b)  $x - (y + (4 - y))$

(c)  $\ell + (y + \ell + y + \ell - (y - \ell))$

(d)  $3xy - 5x(z - y)$

(e)  $2t(3r + 5s + 7t) - 11r(13r - 17s \times 3t + 2t)$

(3) Expand and simplify:

(a)  $(x + y)(x + y)$

(b)  $(3 + x)(4 + y)$

(c)  $3r - 2(3t(1 + 2r))$

(d)  $(\ell + m + n)(x - z)$

(e)  $\alpha(x + 2)(x - 2)$

(f)  $2m^2 + 3m + 5m^2$

(g)  $2p + (3q - (4p + q - (p + 1)))$

(h)  $-a(-a(-a(-a) + b(6 + c)))$

(i)  $\alpha + a - \alpha^2 + (\alpha + a)(\alpha - a)$

(j)  $(x - \alpha)(x - \beta)(x - \gamma)$

(k)  $-x(3 - y(2x + 1)) - 5(x - 3(x - y))$

(4) Factorise:

(a)  $pq + pr$

(b)  $ux - 3ux^2 + 3u^2x$

(c)  $(2x + 1)(3 + 2x) + (2x + 1)(5 + x^2)$

(d)  $x^2 + 5x + 6$

(e)  $y^2 + 12x + 35$

(f)  $x^2 - 5x - 14$

(g)  $(x^2 + 4x + 3)(x - 2)$

(5) Simplify:

(a)  $\frac{7ab + 14ac}{35a}$

(b)  $\frac{6xy}{2xz}$

(c)  $\frac{a - b}{2} + \frac{b - a}{3a}$

(d)  $\frac{3}{a} \div ab \times a^3$

- (e)  $\frac{ab}{c} \times \frac{a^3b^3c^3}{abc}$
- (f)  $\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b}$
- (g)  $\frac{4}{ab} \div \left( \frac{a+2}{2b} - \frac{1}{b^2} \right)$
- (h)  $\frac{1}{7x} - \frac{x^2}{7}$
- (i)  $\frac{1}{a} \times \frac{1}{3} \left( \frac{1}{a} + \frac{1}{b} \right)$

#### 4. Simple Equations

- (1) Solve the following for  $x$ :
- (a)  $x - 3 = 2$
- (b)  $2x + 1 = 5$
- (c)  $x + b = 4$
- (d)  $4x - 5 = 31$
- (e)  $3x = 4x + 1$
- (f)  $\frac{3}{4}x = 15$
- (g)  $2x - 7 = 5x - 13$
- (h)  $3(x + 2) - 2 = 16$ .
- (i)  $\frac{2}{x} = \frac{1}{24}$
- (j)  $\frac{1}{x+1} = \frac{3}{6}$
- (k)  $\frac{y}{x} = \frac{17}{y}$
- (l)  $\frac{2x-3}{x} = \frac{7}{4} - \frac{2}{x}$
- (2) Maria purchased 10 notebooks and 5 pens. A pen cost \$2, and she paid \$40 altogether. How much did a notebook cost?
- (3) Solve  $\frac{1}{f} = \frac{1}{D_i} + \frac{1}{D_o}$ :
- (a) For  $f$ .
- (b) For  $D_i$ .
- (4) Solve  $\frac{H_i}{H_o} = \frac{D_i}{D_o}$ :
- (a) For  $H_i$ .
- (b) For  $D_o$ .
- (5) Half a number plus 5 is 11. What is the number?



- (6) The sum of two consecutive numbers is 25. What are the two numbers?
- (7) Henry the octopus has twice as many shoes as he does socks, and three times as many socks as he has apples. All together, he has 18 socks and shoes. How many apples does he have?
- (8) Solve  $A = \frac{1}{3}ah + \frac{1}{3}h$  for  $h$ .

### 5. Trigonometry

- (1) The lengths of the two shorter sides of several right angled triangles are given here. Find the third side's length.
  - (a) 3 and 4
  - (b) 1 and 1
  - (c) 6 and 3
  - (d) 4 and 25
  - (e)  $\sqrt{2}$  and  $\sqrt{3}$
- (2) Two angles in a triangle are 40 and 67 degrees. What is the measure of the third angle?
- (3) The length of the hypotenuse and the measure of one of the acute angles of several right angled triangles are given here. Find the lengths of the other two sides.
  - (a) 3 and 45 degrees
  - (b) 2 and 16 degrees
  - (c) 9 and 60 degrees
- (4) A right angled triangle has side lengths of 7, 24, and 25. Find the measures of all three angles.

### 6. Exponents

- (1) Compute:
  - (a)  $25^0$
  - (b)  $2^{-3}$
  - (c)  $8^2 \div 2$
  - (d)  $5^{-2} \times 125$
- (2) Simplify:
  - (a)  $3x^2 \times 9x^{-3} \div -27x^{-13}$
  - (b)  $(3x^2y)^2(-2xy)^3$
  - (c)  $\left(\frac{3}{4b}\right)^5$
  - (d)  $(2x^{-2})^4 + x^2$
- (3) Simplify and express using only positive powers:
  - (a)  $(-3pq^{-2})^3$

$$(b) \frac{3xy^{-2}}{x^{-3}y^2x^2}$$

$$(c) \frac{3x}{9yz^{-2}} + \frac{(yz)^2}{x}$$

$$(d) e^{2x} \times 10^{3x} \div e$$

$$(e) 2^x \times 4^x \times 8^x$$

$$(f) 64 \div 8^{-y}$$

$$(g) \frac{(-x^2)^3}{2^{2^2} \div x^4}$$

(4) Replace the question marks with appropriate numerals or expressions:

$$(a) u^5 u^3 = u^?$$

$$(b) x^{33} \div x^{? \times 2} = x^{27}$$

$$(c) \left(\frac{a}{b}\right)^5 = \frac{a^5}{b^?}$$

$$(d) x^3 = \frac{x^?}{x^4}$$

(5) Simplify the following:

$$(a) x^{\frac{2}{3}}$$

$$(b) \left(2x^{\frac{1}{3}}y^{-\frac{2}{3}}\right)^3$$

$$(c) \left(\frac{4x^{\frac{1}{3}}}{x^{\frac{1}{2}}}\right)^{\frac{1}{2}}$$

(6) Without a calculator, calculate:

$$(a) 16^{\frac{1}{2}}$$

$$(b) 8^{\frac{1}{3}}$$

$$(c) -9^{\frac{1}{2}}$$

$$(d) 32^{\frac{2}{5}}$$

$$(e) \left(\frac{4}{25}\right)^{\frac{3}{2}}$$

## 7. Quadratics

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(1) Factorise and solve the following quadratics for  $x$  or  $t$  exactly.

$$(a) x^2 = 1$$

$$(b) x^2 - 25 = 0$$

$$(c) x^2 + 2x + 3 = 0$$

$$(d) x^2 - t^4 = 0$$

(e)  $x^2 - yx + y^2 = 0$

(f)  $4x^2 + 5 = 4x$

(g)  $\pi x^2 + \pi x = 3\pi$

(h)  $x^2 + ax = 9$

(i)  $x^2 - x - 1 = 0$

(j)  $3x^2 + 2x - 7 = 0$

(k)  $e^{2x} + e^x - 72 = 0$

(l)  $4 \times 10^{2x} - 3 \times 10^x - 10^2 = 10$

(m)  $t^4 - t^2 - 3 = 0$

(n)  $2t^6 + 3t^3 = 9000$

- (2) Bill the farmer has a rectangular field. One side of the field has a length of  $3 + x$ , and the adjoining side is twice as long. The total area of Bill's field is 450 square metres. Find  $x$ .