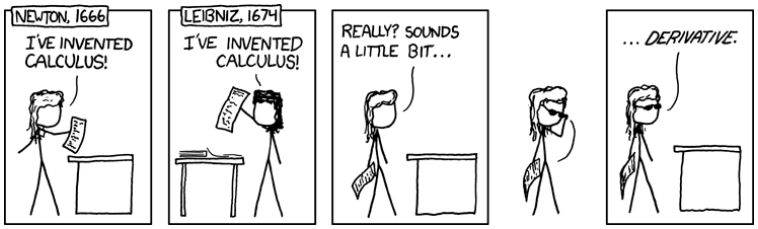
NCEA Level 2 Mathematics (Homework)
15. Kinematics and Rates of Change

Reading

Go and watch...

https://www.youtube.com/watch?v=pI62ANEGK6Q

Sir Isaac Newton (left) and Gottfried Wilhelm von Leibniz (right)



https://xkcd.com/626/

The ancient Greeks used some of the ideas of calculus to calculate areas and volumes of objects: for example, Archimedes (c. 287 – 212 BC) used a limiting process to calculate the area of a circle. The Greeks also considered some of the philosophical problems with these kinds of limiting processes, some of which were not solved until the time of Cauchy in the 18th and 19th centuries.

The modern development of calculus is usually credited to Gottfried Leibniz (1946 – 1716) and Isaac Newton (1642 – 1727), who independently developed coherent theories of calculus at the turn of the 18th century. Both claimed scientific priority: Newton claimed to have begun development of his theory in 1666 as part of his investigations into the laws of motion, but did not publish it until after Leibniz began publishing in 1684.

Some argue that the resulting fallout between the followers of the Englishman, Newton, and the followers of the German, Leibniz, set English mathematics back by decades compared to Europe.

Questions

All distances are given in m, and all times in s, unless otherwise stated.

1. A particle moves in space along a single axis, with velocity function $v(t) = t^2 + t - 12$ (where t is measured from some arbitrary starting point).
   (a) What is the acceleration of the particle at $t = 10\,\text{s}$?
   (b) The particle is closest to the origin at $t = 3\,\text{s}$.
      i. By considering $v(t)$, show that $t = 3$ is indeed a turning point for the graph of the position function $x(t)$ of the particle.
      ii. If the minimum distance between the particle and the origin is $300\,\text{m}$, calculate the distance from the particle to the origin at $t = 10\,\text{s}$.

2. A cubic equation is a polynomial of degree three — that is, a function of the form $f(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$. Recall that a critical point is a point x where $f'(x) = 0$, or $f'(x)$ is undefined.
   i. Sketch examples of a cubic function with zero, one, and two critical points.
   ii. Prove that a cubic function can have a maximum of two critical points.