NCEA Level 2 Mathematics
Preface

These notes present the NCEA Level 2 mathematics content from a mainly geometric standpoint, and so in some places are a little nonstandard. For example, the study of quadratic equations is approached by analysing quadratic graphs, and many of the exercises ask for intuitive and/or geometric explanations of many of the phenomena we study. This is because I see so many students entering L3 calculus with little to no ability to make use of the interplay between geometric and algebraic views of the same picture.

For example, I can cite the example of finding turning points of graphs: students remember very well that the way to solve this kind of exam problem is to "take the derivative and set it to zero", but most of them cannot explain why this works! This is foreign to me, because I don't remember not understanding this (and in fact I think the technique was taught to me geometrically first, which is really the only way to do it)! I find it hard to believe that teachers of L2 mathematics are so incompetent that they don't mention geometry at all when they introduce calculus, so maybe the problem is that when studying for exams the students just remember how to solve problems and neglect the "big picture": that memorising how to solve "types of problems" for an exam is both counterproductive and damaging because they don't remember the important ideas (about tangent lines, slopes, approximations, geometry) and then wonder why the "skills" they've learned (symbol pushing on an exam paper) are useless in the "real world"!

These notes also have another agenda: to introduce students in a calm way to mathematical proof. There is a definite increase in sophistication required for the later sections, but right from the first problem set students are asked to justify statements mathematically. I make no apology to those who want to use these notes but avoid forcing students to write proofs: it is simply how mathematics is done (and I don't think many of the exercises, if any, are out of the reach of the enthusiastic student).

I have tried to address many of these ideas in my student introduction as well.

Some regrets

It is my feeling that geometry is not given a prominent enough place in the school curriculum; nonetheless, I have (for the most part) resisted the temptation to include more Euclidean geometry beyond the standard coordinate geometry and trigonometry (even when it would make things easier, like problem 10 of section 3 on the centres of the triangle). I have also not included any material on the geometric meaning of the integral, as it is no longer in the Y12 curriculum. There should also be some number theory seen at school: I recommend the book *Elementary number theory* by Dudley Underwood as one book suitable for high-school students.

Another omission is unfortunately shared by many other sets of notes: a dense subset of the results and theories described are not placed in their proper historical context. This is primarily for the sake of space, and the author respectfully submits that he is even less of a historian than a mathematician. A readable (even for high-school students) book which discusses the history of many of the topics is Kline's *Mathematics for the non-mathematician*, though I cannot vouch for its accuracy.

Guide to the bibliography

The bibliography is a mixture of further reading and additional problemsets. I have not included many drill-type problems (like "solve for x given $x^2 + 3x - 20$") because they are easily found for those sections that need them: in particular, Spiegel is a good source of algebra drill problems and Foerster is a good source of trigonometry drill problems. For calculus and coordinate geometry, I have included Thompson — although I know no good source that covers just the material in calculus needed for L2 and so it should be used with caution.

In terms of additional reading, most (all?) of the books are suitable for an enthusiastic Y12 student. I particularly recommend Lauwerier, Bóna, or any books on graph theory and the four-colour theorem for students interested in computer science and/or programming.

The two books by Ben Goldacre are also highly recommended for any students who will be going into the sciences or medicine.

Many of the titles are popular mathematics books (e.g. the two by Bellos) that cover the material we see this year at a slightly lower level, and put it in context (although some of the topics should be taken with a grain of salt: Bellos includes chapters on borderline crank topics like the Golden Ratio).

List of sections with the standards that they cover

Geometry

1. (2.1) Coordinate Geometry
2. (2.4) Arcs and Sectors of Circles
3. (2.4) Trigonometry

Algebra

4. (2.2) Functions
5. (2.2/2.6) Quadratic Modelling
6. (2.6/2.14) Simultaneous Equations
7. (2.6/2.14) Linear Inequations
8. (2.6) The Quadratic Formula
9. (2.2/2.6) Exponential and Logarithmic Functions
10. (2.2/2.6) Negative and Fractional Powers

Calculus

11. (2.7) Slopes and Differentiation
12. (2.7) Tangent Lines and Approximation
13. (2.7) Turning Points Optimisation
14. (2.7) Anti-differentiation
15. (2.7) Kinematics and Rates of Change

Combinatorics

16. (Stats?) Counting and Combinatorics
17. (2.3) Number Sequences and Fractals
18. (2.5) Graphs and Networks

Statistics

19. (2.10/2.11) The Statistical Enquiry Process
20. (2.9/2.11) Sampling
21. (2.9) Statistical Inference
22. (2.12/2.13) Probability and Risk
23. (2.12) Probability Distributions