1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Schol Trig prob set 4.

1a.

$$\sec 2x = \frac{1}{\cos 2x} = \frac{1}{2\cos^2 x - 1} = \frac{\sec^2 x}{\sec^2 x} \cdot \frac{1}{2\cos^2 x - 1}$$

$$= \frac{\sec^2 x}{2 - \sec^2 x}.$$

b. $$\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$$

$$\Rightarrow \frac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x.$$

2. $$\cos 4x = \cos(2x + 2x)$$

$$= \cos 2x \cos 2x - \sin 2x \sin 2x$$

$$= (2\cos^2 x - 1)^2 - (2\sin 2x \cos 2x)^2$$

$$= 2\cos^4 x - 4\cos^2 x + 1 - 4(1 - \cos^2 2x)\cos^2 2x$$

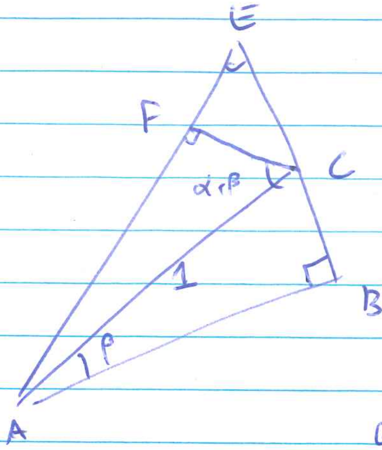
$$= 2\cos^4 x - 4\cos^2 x + 1 - 4(1 - (2\cos^2 x - 1)^2)(2\cos^2 x - 1)^2$$

$$= 2\cos^4 x - 4\cos^2 x + 1 \underbrace{- 4(2\cos^2 x - 1)}_{-8\cos^2 x + 4} + 4(2\cos^2 x - 1)^4$$

$$= 2\cos^4 x - 12\cos^2 x + 5 + 4(16\cos^8 x - 32\cos^6 x + 24\cos^4 x - 8\cos^2 x + 1)$$

$$= 64\cos^8 x - 128\cos^6 x + 98\cos^4 x - 60\cos^2 x + 9.$$

3.



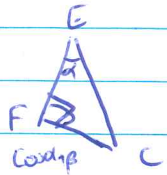
a) $\angle A = \beta + \left(\frac{\pi}{2} - \alpha - \beta\right)$

$= \frac{\pi}{2} - \alpha.$

$\therefore \angle E = \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right) = \alpha.$

b) $|FC| = \cos(\alpha+\beta)$, so

$|EC| = \dfrac{\cos(\alpha+\beta)}{\sin\alpha}.$

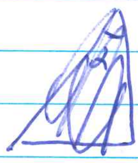


c) $|EB| = |EC| + |CB|$

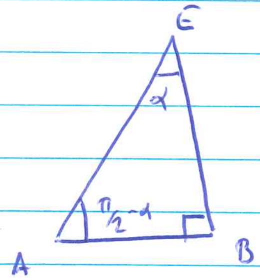
$= \dfrac{\cos(\alpha+\beta)}{\sin\alpha} + \sin\beta$

$|AB| = \cos\beta$

$\therefore \dfrac{|AB|}{|BE|} = \dfrac{\cos\beta}{\frac{\cos\alpha+\beta}{\sin\alpha} + \sin\beta}.$



d)



$\dfrac{|AB|}{|EB|} = \tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}.$

e) $\dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\cos\beta}{\frac{\cos\alpha+\beta}{\sin\alpha} + \sin\beta}$

$\Downarrow$

$\cos(\alpha+\beta) + \sin\beta\sin\alpha = \cos\beta\cos\alpha$

$\Downarrow$

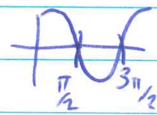
$\cos(\alpha+\beta) = \cos\beta\cos\alpha - \sin\beta\sin\alpha$

$\square$.

4. $\sin^2 x > \cos^2 x \Rightarrow \sin^2 x - \cos^2 x > 0$

$\Rightarrow -\cos(2x) > 0$

$\Rightarrow \cos(2x) < 0$



$\cos x < 0$ when $\frac{\pi}{2} + 2\pi n < x < \frac{3\pi}{2} + 2\pi n \quad \forall n$

so $\cos 2x < 0$ when $\frac{\pi}{4} + \pi n < x < \frac{3\pi}{4} + \pi n.$

5. Note $a+b+c+d=2\pi$. Then:-

$$\cos(a+b+c)+\cos(b+c+d)+\cos(c+d+a)+\cos(d+a+b)$$

$$= 2\cos\frac{a+2b+2c+d}{2}\cos\frac{a-d}{2}+2\cos\frac{c+2d+2a+b}{2}\cos\frac{c-b}{2}$$

$$= 2\cos\frac{2\pi+b+c}{2}\cos\frac{a-d}{2}+2\cos\frac{2\pi+a+d}{2}\cos\frac{c-b}{2}$$

$$= -2\cos\frac{b+c}{2}\cos\frac{a-d}{2}-2\cos\frac{a+d}{2}\cos\frac{c-b}{2}$$

$\ldots \frac{b+c}{2} = (2\pi-a-d+c)/2 = \pi - \frac{a+d}{2}$ etc...

$$= 2\cos\frac{a+d}{2}\cos\frac{a-d}{2}-2\cos\frac{a+d}{2}\cos\frac{c-b}{2}$$

$$= 2\cos\frac{a+d}{2}\left(\cos\frac{a-d}{2}-\cos\frac{c-b}{2}\right)$$

$$= 2\cos\frac{a+d}{2}\left(-2\sin\frac{a+c-b-d}{4}\sin\frac{a-d+b-c}{4}\right)$$

$$= -4\cos\frac{a+d}{2}\sin\frac{a+c-2\pi+a+c}{4}\sin\frac{a+b-2\pi+a+b}{4}$$

$$= -4\cos\frac{a+d}{2}\sin\frac{a+c-\pi}{2}\sin\frac{a+b-\pi}{2}$$

$$= -4\cos\frac{a+d}{2}\cos\frac{a+c}{2}\cos\frac{a+b}{2}.$$