NCEA Level 3 Calculus (Differentiation)
3. Derivatives of Common Functions (Homework)

Reading

Theorem. *Suppose f and g are functions which are differentiable at some point x, and suppose that λ is a real constant. Then:*

1. $(\lambda)'(x) = 0$,
2. $(f+g)'(x) = f'(x) + g'(x)$, *and*
3. $(\lambda f)'(x) = \lambda f'(x)$.

Proof. We prove these using the properties of the limits.

1.
$$(\lambda)'(x) = \lim_{h\to 0} \frac{\lambda - \lambda}{h} = 0.$$

2.
$$\begin{aligned}(f+g)'(x) &= \lim_{h\to 0} \frac{(f+g)(x+h) - (f+g)(x)}{h} \\ &= \lim_{h\to 0} \frac{f(x+h) - f(x) + g(x+h) - g(x)}{h} \\ &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\ &= f'(x) + g'(x).\end{aligned}$$

3.
$$\begin{aligned}(\lambda f)'(x) &= \lim_{h\to 0} \frac{(\lambda f)(x+h) - (\lambda f)(x)}{h} \\ &= \lim_{h\to 0} \frac{\lambda(f(x+h) - f(x))}{h} \\ &= \lambda \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lambda f'(x).\end{aligned}$$

□

Questions

1. Differentiate with respect to x:
   (a) $x^2 + \ln x$
   (b) tx^t
   (c) $\sin x - \cos x$
   (d) $\sqrt[5]{x^4}$
2. Explain why you cannot use the power rule to find the derivative of x^x.
3. Find the nth derivative of $\frac{1}{x^n}$.
4. (More difficult!) Suppose a population grows exponentially with time, such that after t years the population $P = P_0 + 10^t$.
   (a) Given that the derivative of e^{cx} is ce^{cx} when c is constant, find the rate of change of the population at $t = 100$.
   (b) Explain why this population model is unrealistic.