

NCEA Level 3 Calculus

Differentiation Assignment

1. (a) (2 points) If $y = 3x^2 + 2x - \frac{1}{\sqrt{x+1}} + \frac{e^x}{\sin x}$, find $\frac{dy}{dx}$.
- (b) An important mathematical skill is the ability to write down examples of objects satisfying certain properties.
 - i. (2 points) Draw the graph of a function f passing through $(0, 1)$ such that $f'(x) < 0$ for all x , but $f''(x) > 0$ for all x .
 - ii. (1 point) Give an explicit, simple example of such a function.
- (c) (3 points) Show that a solution to the differential equation

$$\frac{dx}{dt} = rx(1 - x)$$

is given by

$$x(t) = \frac{1}{1 + \left(\frac{1}{x_0} - 1\right)e^{-rt}}.$$

2. Let f be the function defined by

$$f(x) = \sin(\tan(x + \pi/6) + \cos(4\pi)) \ln(1/x).$$

- (a) (2 points) Compute the derivative of $\tan(x + \pi/6) + \cos(4\pi)$ with respect to x .
 - (b) (3 points) Write down explicitly $f'(x)$.
 - (c) (3 points) Give the equation of the best linear approximation to $f(x)$ at $\left(\pi, -\ln(\pi) \sin\left(1 + \frac{1}{\sqrt{3}}\right)\right)$, giving constants to three decimal places.
3. (a) (2 points) Compute the derivative of $(x + 1)^3$ using the definition of the derivative.
 - (b) (6 points) You may use the following lemma in answering this question. **Do not attempt to prove this lemma.**

Squeeze Lemma

If f , g , and h are all functions such that $f(x) \leq g(x) \leq h(x)$ for all x in the domain of definition of the three functions, and if $\lim_{x \rightarrow h} f(x) = \lim_{x \rightarrow h} h(x) = L$, then $\lim_{x \rightarrow h} g(x) = L$.

- i. Draw a sketch that displays the intuition behind the squeeze lemma.
- ii. Show that $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$. [Hint: use the fact that $-1 \leq \sin x \leq 1$.]