

NCEA Level 2 Mathematics (Homework)

5. Quadratic Modelling

Reading

Go and watch...

<https://www.youtube.com/watch?v=hoh4TmPzu1w>

What's it good for?

People use parabola for...

- Engineering, and the sciences: modelling situations where a quantity is roughly proportional to the square of another. In physics, a perfect projectile follows the path of a parabola.
- Mathematics: a parabola is a specific case of a conic section (the others are the hyperbola, the circle, and the ellipse). Conic sections are geometric figures that were systematically studied by the Greeks.

Questions

1. Show that if $y = ax^2 + bx + c$, then

$$y = a \left(x + \frac{b}{2a} \right)^2 + \left(c - \frac{b^2}{4a} \right).$$

(You might want to start with the complicated equation and simplify it, rather than going the other way.)

2. Explain why this shows that changing b and c only shifts the parabola around the plane rather than changing the size. In other words, it is possible for us to shift any parabola anywhere we want without affecting the size.
3. In fact, we have shown that every quadratic can be written in the form $y = a(x - \alpha)^2 + \beta$; since changing α and β only shifts the parabola around, the only parameter that can change the size is a . Therefore, if we have a parabola $y = ax^2 + bx + c$ then it has the same shape as $y = ax^2$. Show that if (x_1, y_1) is on the parabola $y = ax^2$, then $(\frac{b}{a}x_1, \frac{b}{a}y_1)$ is on the parabola $y = bx^2$ (where a and b are any nonzero numbers).
4. Conclude that any parabola can be mapped onto any other parabola by shifting it around the plane (changing the location of the vertex) and scaling it in both directions by an appropriate constant.