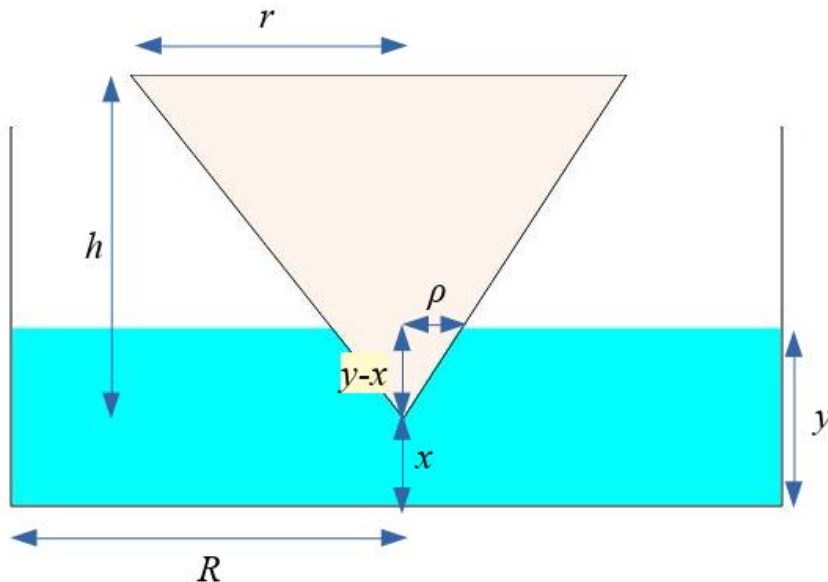


WORKED SOLUTION FOR PROBLEM:

(Difficult) A cone of radius r centimetres and height h centimetres is lowered point first at a rate of 1 cm^2 into a cylinder of radius R centimetres that is partially filled with water. How fast is the water level rising at the instant that the cone is fully submerged?



Let V be the volume of water in the tank, which we know to be constant. We are given that $\frac{dx}{dt} = -1$. Using trigonometry, we find $\rho = \frac{r}{h}(y-x)$ so we have

$$\begin{aligned} V &= x\pi R^2 + (y-x)\pi R^2 - \frac{1}{3}\pi \rho^2(y-x) \\ &= \pi R^2 y - \frac{1}{3}\pi \frac{r^2}{h^2}(y-x)^3 \end{aligned}$$

Hence $\frac{dV}{dx} = \pi R^2 - \frac{\pi r^2}{h^2}(y-x)^2(1 - \frac{dx}{dy})$; but V is constant, so any derivative of V is zero and

$$0 = \pi R^2 - \frac{\pi r^2}{h^2}(y-x)^2(1 - \frac{dx}{dy}).$$

When the cone is exactly submerged, $y-x = h$. So $0 = R^2 - r^2(1 - \frac{dx}{dy})$ and $\frac{dx}{dy} = \frac{r^2 - R^2}{r^2}$; hence

$$\frac{dy}{dt} = \frac{dx}{dt} \frac{dy}{dx} = -1 \times \frac{r^2}{r^2 - R^2} = \frac{r^2}{R^2 - r^2}.$$