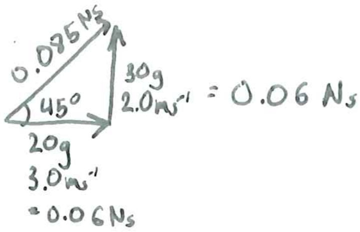
L3 Physics, Mechanics §§ 1.1 - 1.5.

1. 

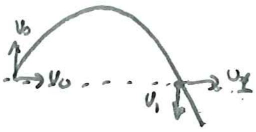


So $v_{final} = \dfrac{0.085}{50 \times 10^{-3}} = 1.70 \text{ ms}^{-1}$ due north-east (i.e. 45°).

2. Initial relative velocity is 0 ms^{-1}; final rel. velocity is $2.5 - 0.4 = 2.10 \text{ ms}^{-1}$.
" mass is $1.5 - 0.1 = 1.40$ kg.
∴ final momentum of squid is 2.94 Ns
∴ " " " water is "
∴ final velocity of water is $2.94/0.10 = \underline{29.4 \text{ ms}^{-1}}$.

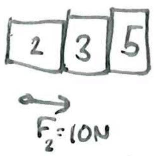
3. 

Initial sideways velocity: $64.95 \text{ ms}^{-1} = v_0^{horiz}$
" vertical " : $37.50 \text{ ms}^{-1} = v_0^{vert}$



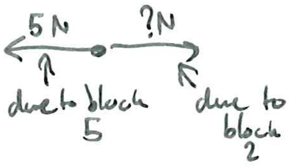
By cons. of K.E. we must have
$v_1^{vert} = -v_0^{vert}$, $v_1^{horiz} = -v_0^{horiz}$.
Hence KE ~~due to~~ at time 1 is $\frac{1}{2} m \cdot 75^2$;
need to add gained GPE of $U = 12 \times m \times 9.81$,
so final energy is $m\left(\frac{1}{2} 75^2 + 12 \times 9.81\right) = m(2930.22)$

$\therefore v_{final} = \sqrt{2 \times 2930.22}$
$= 76.55 \text{ ms}^{-1}$.

4. 

So $a = \frac{10}{2+3+5} = 1 \text{ ms}^{-2}$.

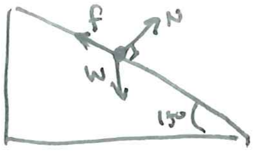
Block 5 is feeling a force 5N, this must come entirely from block 3. Hence (a) is 5N.
For block 3, we have the following force diagram:



and total force is $3\text{kg} \times 1 \text{ ms}^{-2}$,
so $5\text{N} - ? = 3\text{N}$
$\Rightarrow ? = 2\text{N}$.
So block 2 is providing a force of $\underline{2\text{N}}$.

5a. Unitless (since $[N] = \mu[N]$.)

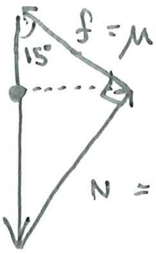
b.i.



$W = 4000 \times 9.81 = 39240\,N$

$N = \frac{W}{\sin 15°} = 151611\,N$

thus $f = \mu N = 136450\,N$.

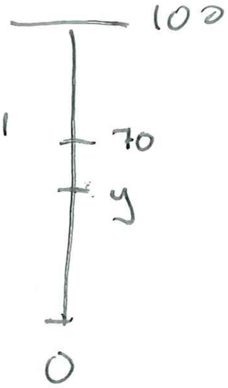


$f = \mu N = W\cos 15° = 37900\,N$

$39240 = W$

$N = W \sin 15° = 10156\,N$

$\therefore \mu = f/N =$



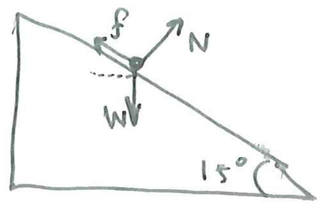
loss in GPE: $(100-y)\cdot 80\cdot 9.81$

Gain in EPE: $\frac{1}{2}\cdot 40\cdot (70-y)^2$.

$\therefore\ 78480 - 784.8y = 98000 - \underset{3500}{\cancel{40}}y + 20y^2$.

5. a. Unitless. (since $F = \mu W \Rightarrow [N] = \mu [N]$).

b.



$W = 4000 \times 9.81 = 39240\ N$.

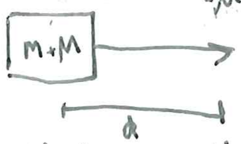
$\rightarrow$ $N = W \sin 15°$

$= 10156\ N$

i.e. $f = \mu N = 9140\ N$.

c.

$\leftarrow F = \mu N = -\mu (M+m) g$



$\Rightarrow \vec{a} = F/m = -\mu g$.

$V_0 = ?$ $V_1 = 0\ ms^{-1}$

Know that $V_1^2 = V_0^2 + 2ad$

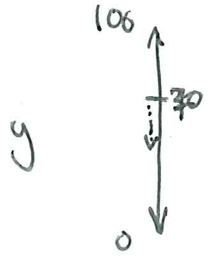
$\Rightarrow V_0 = \sqrt{2d\mu g}$.

But $V_0 (m+M) = V_{bullet}\, m$ (cons. of momentum);

So $V_{bullet} = \frac{m+M}{m}\sqrt{2d\mu g}$

$= \left(1 + \frac{M}{m}\right)\sqrt{2d\mu g}$.

6. At max elongation, we have the following:



$F_1 = 40(70 - y)$

$F_g = 80 \cdot 9.81$

$\therefore$ $80 \cdot 9.81 = 40(70 - y)$

so $y = 50.4$ m.

7. Speed of orbit is $v = \frac{2\pi R}{T}$, so $\frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$

$\Rightarrow \frac{GM}{R} = \frac{4\pi^2 R^2}{T^2}$

$\Rightarrow \frac{GM}{4\pi^2} T^2 = R^3$.

8. $\Delta p = \int_{t_0}^{t_1} F\,dt = \frac{1}{2} \times 5 \times F_{max}.$

But $\Delta p = mv_i - mv_f$. $m = 200g$.

Drop from a height of 2.0m $\rightarrow$ $\Delta U = 2 \times 0.2 \times 9.81$
$= 3.92\ J$
i.e. $v = \sqrt{\frac{2\Delta U}{0.2}} = 6.26\ ms^{-1}$.

Gain a height of 1.5m $\rightarrow$ $\Delta u = 1.5 \times 0.2 \times 9.81$
$= 2.94\ J$
i.e. $v = \sqrt{\frac{2\Delta u}{0.2}} = 5.42\ ms^{-1}$.

$\therefore$ $\Delta p = 0.2\,(6.26 + 5.42) = 2.336\ Ns$.

$\therefore$ $F_{max} = \frac{2.336 \times 2}{5} = \underline{0.93\ N}$.

9a. $U = \int_{x_0}^{x_1} F\,dx$. The total PE is simply the amount of work we need to put into move one planet from far away to the distance r:

$$U = \int_{\infty}^{r} \frac{GMm}{r^2}\,dr$$

$$= -\left[\frac{GMm}{r}\right]_{\infty}^{r}$$

$$= -\frac{GMm}{r}.$$

b. Need to give it enough energy to escape gravity well of earth:-

$$\Delta U = \frac{GMm}{r_{earth}} = \frac{6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1000}{6.37 \times 10^{6}} = 6.26 \times 10^{10}\ J$$

$\therefore$ $v = \sqrt{\frac{2\Delta U}{1000}} = \underline{11,000\ ms^{-1}}$.

10. Initial energy: $E = -\frac{GMm}{r_{initial}} = -2.41 \times 10^{33}$ J

final energy: $E = -\frac{GMm}{2r_{jupiter}} = -1.722 \times 10^{36}$ J.

so $\Delta E = 1.72 \times 10^{36}$ J

and so each planet has

$K = \frac{\Delta E}{2} = 8.5\cancel{0}0$ J

$v = \sqrt{\frac{2K}{1.9 \times 10^{24}}} = \underline{30000 \text{ ms}^{-1}}$.