# NCEA Level 2 Mathematics (Homework)
# 18. Graphs and Networks

## Reading

**Go and watch...**

https://www.youtube.com/watch?v=CruQylWSfoU

**What's it good for?**

People use graphs and networks for...

- Computer science: modelling computer networks, modelling connections between objects (Google's search algorithm is based on weighted graphs).
- Engineering: electrical circuits can be modelled with weighted graphs.
- Linguistics: syntax trees and semantic networks are used when modelling language and the changes of languages over time.
- Physics and chemistry: graphs can be used to model crystal structures or the structures of molecules (especially in organic chemistry).
- Mathematics: graph theory has tight links with topology and knot theory. The four-colour theorem (the vertices of any graph that can be drawn with no edge crossings can be coloured with at most four colours so no adjacent vertices have the same colour) was only proved in the latter half of the 20th century, and was the first major proof to involve considerable work by computers.

## Questions

[This is a sample internal assessment task for this standard, based on those provided by the Ministry of Education.]

This assessment activity requires you to apply graphs and networks to a real-world situation. Read the entire activity before beginning work.

The Lower North Island Logistics Company (LNILC) wants to streamline its business by redesigning its logistics network. In Resource A, a list of town/city pairs in the current network and travel times by truck between them are given.

1. Draw a weighted graph modelling the information in Resource A.
2. Find the shortest route (in terms of travel time) between Carterton and Marton.
3. The cost of providing a truck between two of the city pairs is directly proportional to the travel time.
   (a) Determine any links that will (i) be in *every* minimal spanning tree, and (ii) will be in *no* minimal spanning tree.
   (b) Determine the smallest possible network that covers all the destinations, and costs the least to maintain.
4. LNILC is considering signing a deal with KiwiRail that provides rail services between the following city pairs:
   - Wellington to Palmerston North: 135 minutes
   - Palmerston North to Whanganui: 50 minutes
   - Whanganui to New Plymouth: 50 minutes
   - Palmerston North to Hastings: 110 minutes
   - Wellington to Carterton: 80 minutes
   - Carterton to Masterton: 18 minutes
   - Masterton to Hastings: 140 minutes
   - Masterton to Palmerston North: 65 minutes

   How do your answers above change, incorporating these new links?
5. Produce a final report, incorporating your findings above, that could recommend:
   - If any truck links should not be maintained;
   - Which, if any, rail links should be included in the future network;
   - Any other changes that could be made to improve the cost-effectiveness of the entire network, and minimise the time taken for packages to travel between particular major towns and cities. (For example, you may recommend that a particular link be kept despite not appearing in your minimum spanning tree if it significantly reduces the distance between two key destinations.) Resource B may be useful for this.

The quality of your reasoning and how well you link this context to graph and network methods will determine the overall grade. Include calculations, diagrams or formulae, as appropriate. Clearly communicate your method using correct mathematical statements where appropriate.

## Resource A: town-city pairs with times

All links are bi-directional and all travel times are in minutes.

| City 1 | City 2 | Travel time by road |
|---|---|---|
| Wellington | Porirua | 27 |
| Wellington | Lower Hutt | 22 |
| Lower Hutt | Wainuiomata | 12 |
| Lower Hutt | Upper Hutt | 22 |
| Porirua | Upper Hutt | 28 |
| Porirua | Kapiti | 30 |
| Kapiti | Upper Hutt | 68 |
| Upper Hutt | Martinborough | 49 |
| Upper Hutt | Carterton | 51 |
| Carterton | Masterton | 15 |
| Martinborough | Masterton | 43 |
| Masterton | Dannevirke | 77 |
| Masterton | Palmerston North | 79 |
| Kapiti | Otaki | 20 |
| Otaki | Levin | 18 |
| Levin | Palmerston North | 40 |
| Levin | Whanganui | 74 |
| Palmerston North | Whanganui | 56 |
| Palmerston North | Fielding | 16 |
| Fielding | Marton | 25 |
| Marton | Whanganui | 29 |
| Palmerston North | Dannevirke | 54 |
| Dannevirke | Napier | 89 |
| Dannevirke | Hastings | 74 |
| Napier | Hastings | 24 |
| Whanganui | Hawera | 68 |
| Hawera | Stratford | 26 |
| Stratford | New Plymouth | 33 |
| Hawera | New Plymouth | 90 |

## Resource B: major towns and cities in the Lower North Island

The following urban areas served by LNILC have populations over 10,000:

- Wellington (207,900)*
- Napier-Hastings (133,000)*
- Lower Hutt (104,700)
- Palmerston North (85,300)
- New Plymouth (57,500)*
- Porirua (55,900)
- Upper Hutt (43,200)
- Kapiti (42,300)
- Whanganui (40,300)*
- Masterton (21,800)
- Levin (20,900)
- Fielding (16,550)

(*Urban area served by a port.)