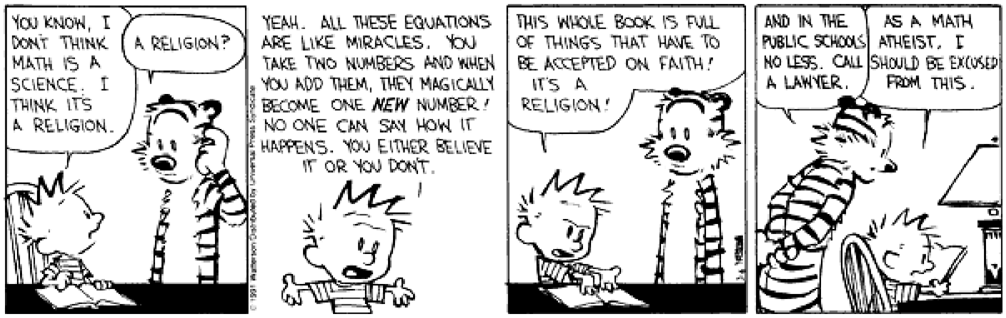
NCEA Level 3 Calculus
Prior Revision: Functions



What is Calculus?

Calculus is the study of:

- Continuous change.
- Slope, area, and volume.
- Functions and relationships.

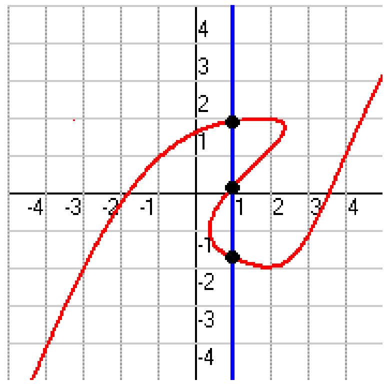
It has applications in:

- Physics and chemistry.
- Probability theory.
- Population theory.
- Economics (I am assured).

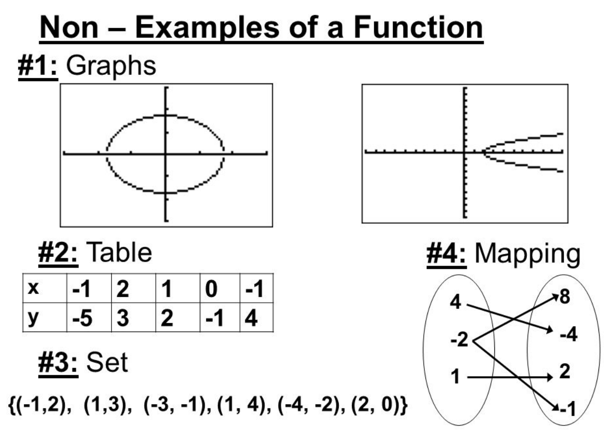
In pure mathematics, calculus can be seen as the computational side of a pretty subject called **real analysis**.

Revision of Functions

The most fundamental concept in calculus is that of a *function*.
Definition (Function)**.** A function is a something which takes a set of things (for example, the real numbers $\mathbb{R}$) and assigns to each one exactly one thing (which could be the same or different).
Example. The map which takes a number x and spits out x^2 is a function — for every input, there is exactly one output. If we *graph* this function, we plot its input on the $x-$axis and its output on the $y-$axis and obtain a parabola.
Example. The curve graphed below is *not* a function, since for some inputs (like 1) it has more than one output. We can check this by drawing vertical lines along the function, like that pictured: if a graph is a function, no vertical line can ever cross the curve more than once (this is the *vertical-line test*).



Example. The map $f : x \mapsto \sin x$ is a function. We could also define it by 'the function f such that $f(x) = \sin x$'. This function f can only produce numbers between 1 and -1; we say that its *range* is the interval from -1 to 1.
Example. The map $\iota : x \mapsto x$ is a function, called the *identity function*.
Example. The map $\ln x$ is a function, but it is only defined when $x > 0$: we say that its *domain* is the positive real numbers.
Example. Some more non-examples:



Questions

1. Which of the following are functions?
   (a) $E(x) = 2^x$
   (b) $\phi : x \mapsto \frac{2}{x}$
   (c) The thing which maps every person to their youngest sibling.
   (d) The thing which sends every person to their youngest sibling that isn't themself.
   (e) $x \mapsto \lfloor x \rfloor$ (the floor map).
   (f) The relation that sends every person to their age.
2. I will define two functions, φ and ϑ, as follows:
$$\varphi(x) = 2x - 7, \qquad \vartheta(\zeta) = \frac{1}{7}(14\zeta - 49).$$
   Explain why these functions are equal.

3. If $f(x) = x^2 + x$, find:
   (a) $f(1)$
   (b) $f(y)$
   (c) $f(x+h)$
4. Find the distance between $(-3, 4)$ and $(2, 1)$.
5. Three sides of a triangle are have lengths 8, 15, and 17.
   (a) Show that the triangle is right-angled.
   (b) Find the other two angles.
6. Factorise and solve $x^2 - 3x + 2 = 0$.
7. How many **lines** are there through the point $(2, 3)$ and the origin? Give the equations of all such lines.
8. Find the slope of the line $4x + 3y + 2 = 0$.
9. Find the solution to the following linear system:

$$\begin{aligned} 2x + y &= 7 \\ 3x - y &= 8 \end{aligned}$$

10. How many (real) solutions does $x^2 + 4x + 1$ have?
11. Draw $\sin(x)$, $\cos(x)$, $\tan(x)$, $\exp(x)$, and $\ln(x)$.
12. How many solutions does $\cos(3\pi x + 1) = 2$ have?
13. How many solutions does $\sin(3x) = \frac{1}{3}$ have?