Schol trig probset 2.

1a) $\cos\left(\frac{\pi}{2}+\theta\right)+\cos\left(\pi/2-\theta\right)+\sin(\pi+\theta)+\sin\theta = ?$

- $\cos\left(\pi/2-\theta\right) = \cos\left(\theta-\pi/2\right)$ (2.7.4)
  $= \sin\left(\left(\theta-\pi/2\right)+\pi/2\right)$ (2.7.5)
  $= \sin\theta$.

- $\sin(\pi+\theta) = \sin(\pi-(-\theta))$
  $= \sin(-\theta)$ (2.7.6)
  $= -\sin(\theta)$ (2.7.4).

- $\cos\left(\pi/2+\theta\right) = \sin\left(\left(\pi/2+\theta\right)+\pi/2\right)$ (2.7.5)
  $= \sin(\pi+\theta)$
  $= -\sin(\theta)$ ~~(2.7.6)~~ (by previous result).

$\therefore \cos\left(\pi/2+\theta\right)+\cos\left(\pi/2-\theta\right)+\sin(\pi+\theta)+\sin\theta =$
$= -\sin\theta + \sin\theta - \sin\theta + \sin\theta = 0.$

b) Note $\alpha+\beta+\gamma=\pi$. So $\sin\left(\frac{\gamma}{2}\right) = \sin\left(\frac{\pi-\alpha-\beta}{2}\right)$

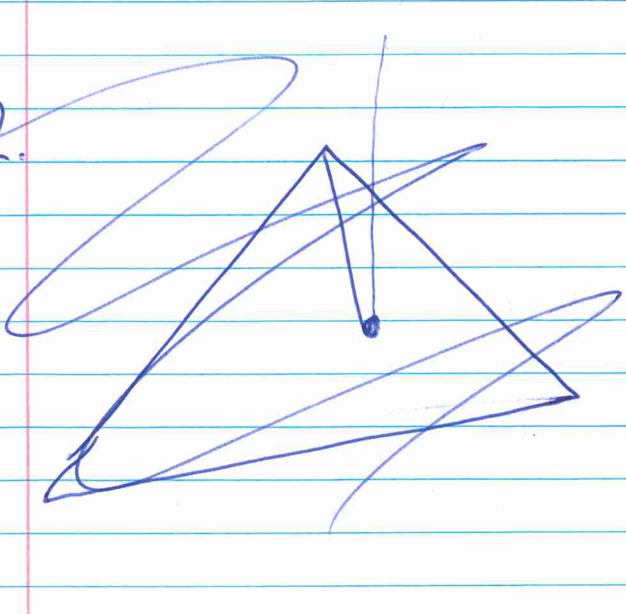
$= \sin\left(\frac{\pi}{2}-\frac{\alpha+\beta}{2}\right)$

$= \sin\left(\frac{\pi}{2}+\left(-\frac{\alpha+\beta}{2}\right)\right)$

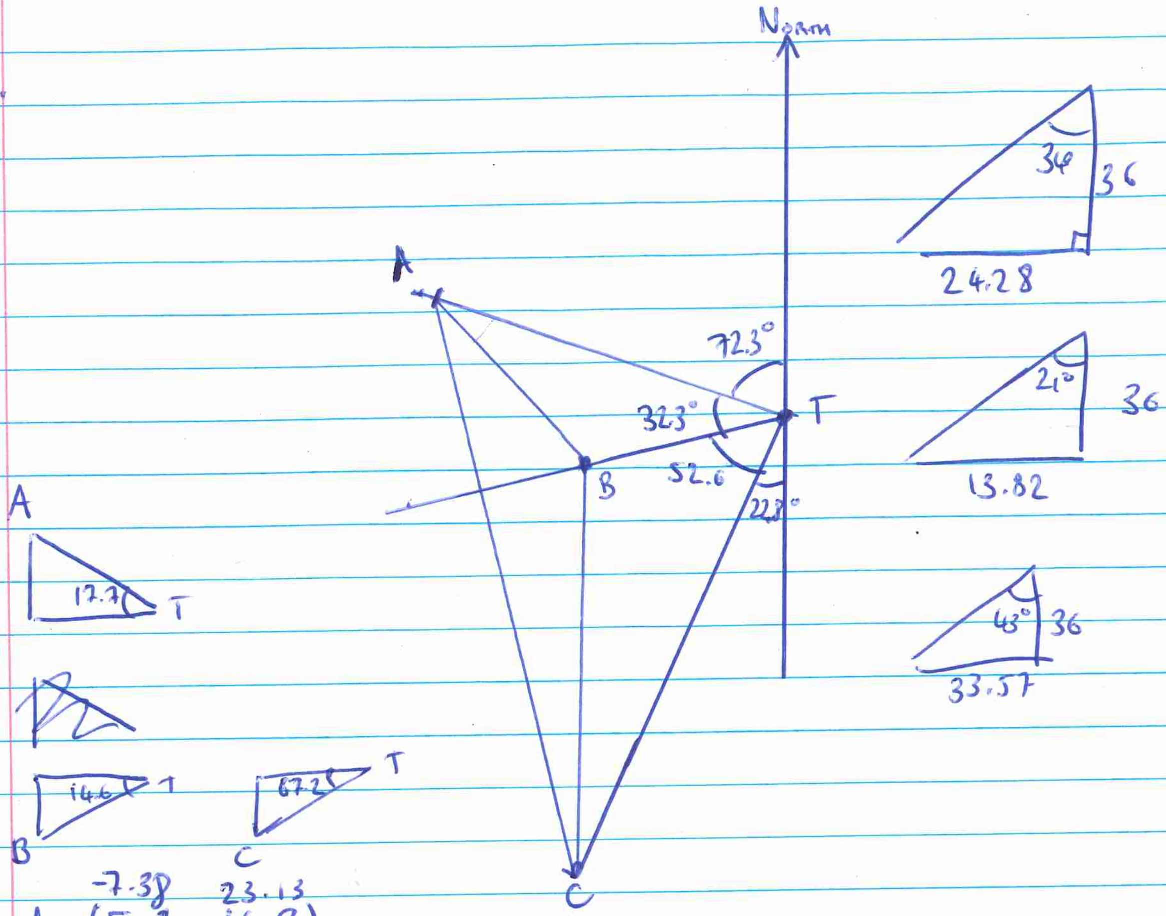
$= \cos\left(-\frac{\alpha+\beta}{2}\right)$ (2.7.5)

$= \cos\left(\frac{\alpha+\beta}{2}\right)$. (2.7.4).

2.



2.



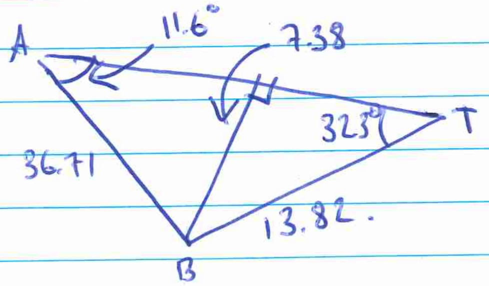
$A = (\cancel{5.4}\ {-7.38},\ \cancel{16.9}\ 23.13)$

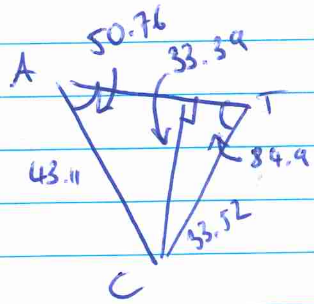
$B = (-3.48, -13.37)$

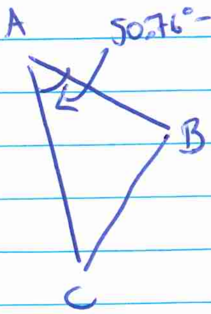
$C = (\cancel{22.39}, \cancel{24.55})$
$(-30.9, -13)$

$\therefore\ |AB| = 36.71 \qquad |AC| = 43.11, \qquad |BC| = 27.42.$

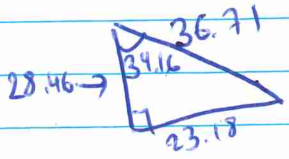
$|BT| = 13.82. \qquad |CT| = 33.52.$



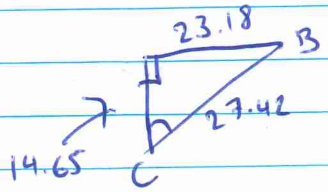




$50.76° - 11.6° = 39.16°.$



$A_1 = \frac{1}{2} \times 28.46 \times 23.18 = 329.85$

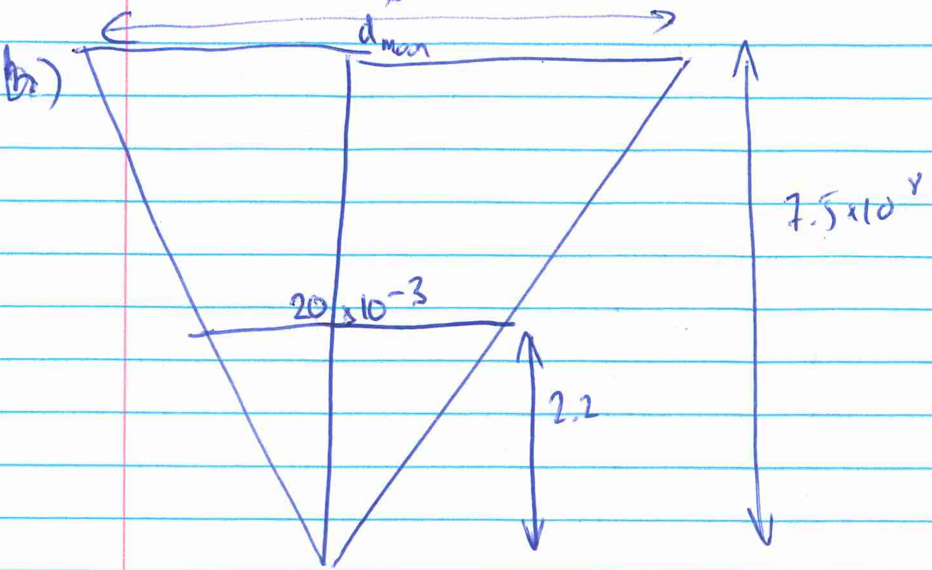


$A_2 = \frac{1}{2} \times 23.18 \times 14.65$
$= 169.79$

$\therefore\ A(ABC) = 329.85 + 169.79 = \underline{500\,m^2}.$

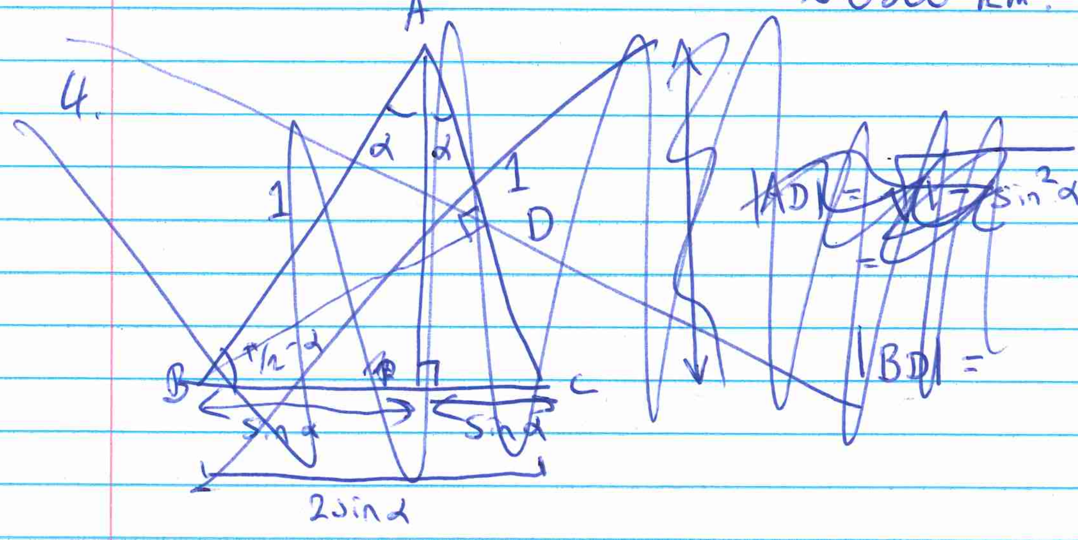
3. a) $3\times10^{8}\,\mathrm{m/s} \times 2.5\,\mathrm{s} = 7.5\times10^{8}\,\mathrm{m}$.

b)

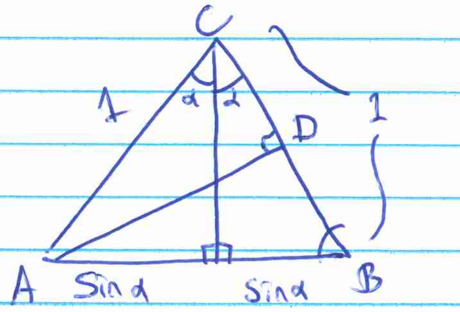


$$\frac{d_{moon}}{7.5\times10^{8}} = \frac{20\times10^{-3}}{2.2} \Rightarrow d_{moon} = 7.5\times10^{8}\times\frac{20\times10^{-3}}{2.2}$$
$$= 6.8\times10^{6}\,\mathrm{m}$$
$$\approx 6800\ \mathrm{km}.$$

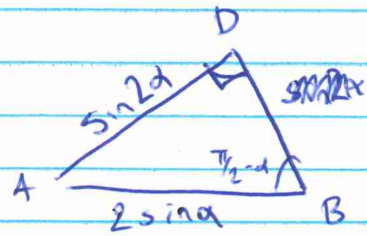
4.



4.



$|AB| = 2\sin\alpha$.
$|AD| = \sin 2\alpha$.
$\angle B = \pi - \alpha - \pi/2 = \pi/2 - \alpha$.



$\therefore \sin 2\alpha = 2\sin\alpha \sin(\pi/2 - \alpha)$
$= 2\sin\alpha\cos\alpha$ □.

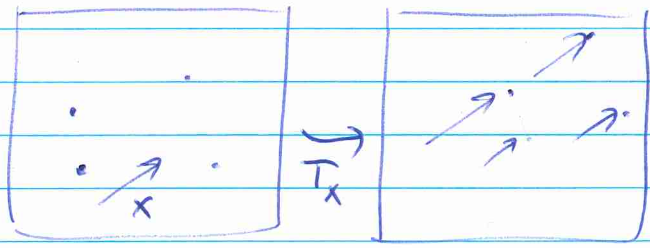
5. a. Let P, Q be points.

$$|T_x(P)\,T_x(Q)| = \sqrt{(p_1+x_1-q_1-x_1)^2 + (p_2+x_2-q_2-x_2)^2}$$
$$= \sqrt{(p_1-q_1)^2 + (p_2-q_2)^2} = |PQ|.$$ So T_x is an isometry.

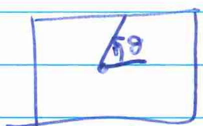
Suppose $F = (f_1, f_2)$ is a fixed point. So:

$$T_x(F) = F$$
$$\Rightarrow (f_1 + x_1, f_2 + x_2) = (f_1, f_2) \Rightarrow \begin{matrix} f_1 = f_1 + x_1 \\ f_2 = f_2 + x_2 \end{matrix} \Rightarrow \begin{matrix} x_1 = 0 \\ x_2 = 0 \end{matrix}.$$

$\therefore\ \cancel{x} = (0,0)$ contradiction.



b. $$|R_\theta(P)\,R_\theta(Q)|^2 = \left[(p_1\cos\theta - p_2\sin\theta) - (q_1\cos\theta - q_2\sin\theta)\right]^2$$
$$+ \left[(p_1\sin\theta + p_2\cos\theta) - (q_1\sin\theta + q_2\cos\theta)\right]^2$$
$$= \left[(p_1-q_1)\cos\theta - (p_2-q_2)\sin\theta\right]^2 + \left[(p_1-q_1)\sin\theta + (p_2-q_2)\cos\theta\right]^2$$
$$= (p_1-q_1)^2\cos^2\theta - 2(p_1-q_1)(p_2-q_2)\cos\theta\sin\theta + (p_2-q_2)^2\sin^2\theta$$
$$+ (p_1-q_1)\sin^2\theta + 2(p_1-q_1)(p_2-q_2)\cos\theta\sin\theta + (p_2-q_2)\cos^2\theta$$
$$= (\cos^2\theta + \sin^2\theta)(p_1-q_1)^2 + (\cos^2\theta + \sin^2\theta)(p_2-q_2)^2$$
$$= (p_1-q_1)^2 + (p_2-q_2)^2 = |PQ|^2. \checkmark$$



Rg action.

Suppose (x,y) is fixed. So $x = x\cos\theta + y\sin\theta$
$y = x\sin\theta + y\cos\theta.$

Clearly $x=y=0$ works. Claim: no other fixed points. Assume $x, y \neq 0$. Then

$x(\cos\theta - 1) = y\sin\theta \longrightarrow y = x\dfrac{\cos\theta - 1}{\sin\theta}$

$y(1-\cos\theta) = x\sin\theta \longrightarrow x\sin\theta = x\dfrac{\cos\theta-1}{\sin\theta} \Rightarrow \sin^2\theta = \cos\theta - 1$

$\Rightarrow \theta = 0.$

$\therefore\ y = 0$ contradiction.

5c $R_\phi(R_\theta(1,0)) = R_\phi(\cos\theta, \sin\theta)$

$= (\cos\theta\cos\phi - \sin\theta\sin\phi, \cos\theta\sin\phi + \sin\theta\cos\phi)$

But $R_\phi(R_\theta(1,0))$ is a rotn thru θ then ϕ;

i.e. $R_\phi(R_\theta(1,0)) = R_{\phi+\theta}(1,0)$.

$R_{\phi+\theta}(1,0) = (\cos(\theta+\phi), \sin(\theta+\phi))$.

$\therefore \cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

and $\sin(\theta+\phi) = \cos\theta\sin\phi + \sin\theta\cos\phi$.