Problem Set #2 for G6010: Physical Cosmology

Fall Semester 2014

Problem #1

Reproduce the plots in Figure 1. These plots show the proper distance as a function of redshift and the look-back time. Note that there are three curves – two single-component universes and the multi-component universe we worked with in Problem Set #1. Assume spacetime is flat.

Problem #2

Determine R_{00}, R_{11}, R_{22} and R_{33} assuming the following:

$$g_{\mu\nu} = \begin{bmatrix} -c^2 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^2 r^2 & 0 \\ 0 & 0 & 0 & a^2 r^2 \sin^2\theta \end{bmatrix} \quad \text{and} \quad T_{\mu\nu} = \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & pa^2 & 0 & 0 \\ 0 & 0 & pa^2 r^2 & 0 \\ 0 & 0 & 0 & pa^2 r^2 \sin^2\theta \end{bmatrix}. \tag{1}$$

Then use the Einstein field equations in the following form

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{2}$$

to show that the Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho + \frac{c^2\Lambda}{3} \tag{3}$$

and the acceleration equation is

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3P) + \frac{c^2\Lambda}{3}. \tag{4}$$

Problem #3

Compute the covariant derivative of the energy-momentum tensor in Equation 1 and derive the fluid equation.

Problem #4

Reproduce the plot in Figure 2 and use it to explain when recombintation, decoupling and last scattering occurred.

Problem #5

Use the scale factor we computed previously for the multi-component universe to compute the angular size of the horizon on the surface of last scattering as observed today.

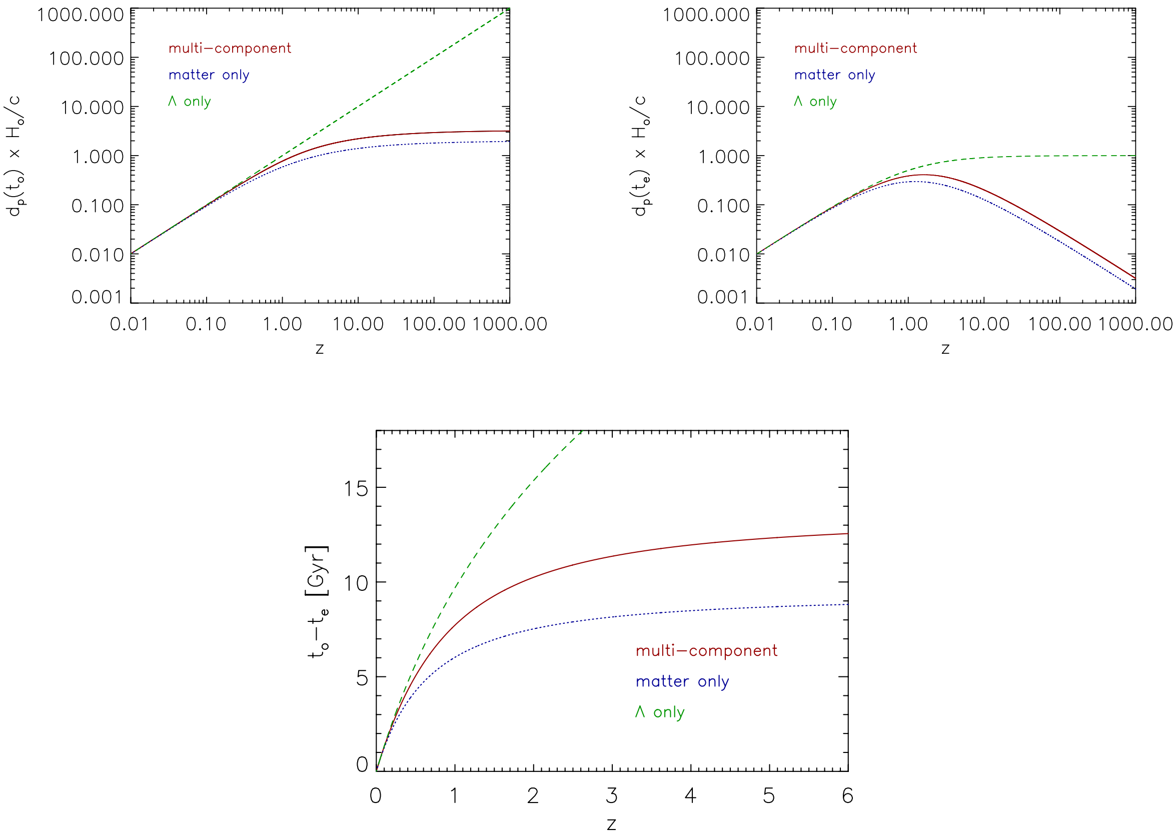

Figure 1: Proper distance vs. redshift and look-back time vs. redshift.

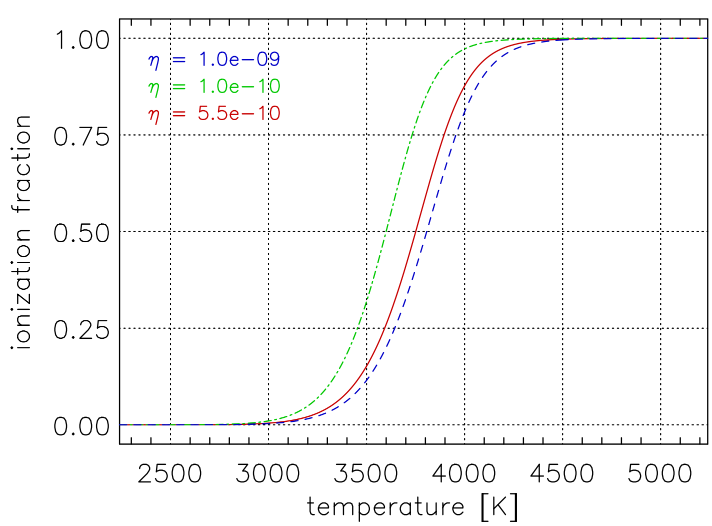

Figure 2: Ionization fraction vs. temperature.