

# Homework 2: Parallel IRL and the Helium Atom

Andrew Emerick

December 14, 2013

## 1 Parallel IRL Algorithm

In `irl_p.cpp` (with the accompanying `irl_p.h` header file), I employ a parallel IRL algorithm computed utilizing the GSL library to serially solve for the eigenvalues of  $H$ , and to perform the QR decomposition of  $H_{k+p}$ . The serial version of this code (not included) is as close to a translation of the matlab `irl.m` to C++ as possible. The primary differences between the matlab version and the C++ version were the many extra loops needed to properly multiply, add, or subtract vectors. In addition, function calls became slightly more complex (needing to pass more parameters, as I tried to avoid using global variables). For the GSL eigensolver and QR decomposition, I wrote two functions ('`eigensolve`' and '`qrdecomp`') that are essentially wrappers on the GSL functions. These wrappers allow me to pass matrices constructed with `std::vector`, and be returned matrices as `std::vector` in the same format as the matlab code. This limits the use of GSL specific variable types/classes to these functions.

The parallelization comes by spreading the  $v$  matrix (and thus  $vp$  and  $evector$ ) over the number of processors (essentially dividing the total volume amongst the number of processors). The code as written cannot take an arbitrary processor number. It requires that the number of processors be a perfect cube (e.g. 1, 8, 27), and that the number of global grid points in each dimension is evenly divisible by the number of processors in each dimension (e.g. if 27 processors are used, global grid points in each dimension must be divisible by 3). Once the volume is constructed (more on this later), and proper accounting is employed for proper sharing across processors, the process of solving for the eigenvalues occurs simultaneously on each processor, rather than having only the rank 0 processor do the computation, and then having to distribute the information across all processors for the next iteration.

For proper accounting, I construct three matrices: `p_xyz`, `nbr`, and `nbrfc`. `p_xyz` has a number of rows equal to the number of processors, so row  $n$  corresponds to the  $n-1$  lexical order processor number (the rank). The first three columns contain the 3 dimensional processor coordinate of the rank  $n$  processor (called `px`, `py`, `pz`), and the following 6 contain the lexical order processor number (rank) of the adjacent processors in  $x+$ ,  $x-$ ,  $y+$ ,  $y-$ ,  $z+$ , and  $z-$ . This matrix is used to properly set with processors should be sending and receiving data from any given processor.

The  $v$  matrix contains buffers at its end to take in values from the adjacent processors. The total length of the buffer is 6 times the number of grid points on each given face (or  $6.0 * L * L$ ). The first  $L * L$  indices are set to contain the values of  $v$  for the processor adjacent towards the  $x-$  direction. Each subsequent set of  $L * L$  elements correspond to the  $x+$ ,  $y-$ ,  $y+$ ,  $z-$ , and  $z+$  values. The `nbrfc` matrix is used to record the lexical order values in  $v$  of all grid points sitting on a face. For a given processor, `nbrfc[0]` contains a list of all indices in  $v$  corresponding to grid points sitting on the  $x+$  side of the cube. Each row after this (1-5) contains  $x-$ ,  $y+$ ,  $y-$ ,  $z+$ , and  $z-$  respectively. This matrix is necessary to properly send information in the `MP_Isend` calls.

The `nbr` and `nbrfc` matrices are set in the `set_bc_nbr_nbrfc` function (similar to the serial function `set_bc_nbr` from the matlab code), while the `p_xyz` matrix is set in the `p_xyz` function. The sharing of information across processors occurs immediately before any call of the  $Av$  matrix times vector function, and occurs in the `share_v` function.  $v$  is the only matrix that needs to be shared and contains buffers. Any other sharing that occurs in the algorithm occurs as `MPI_Allreduce`'s for when a dot product or norm is taken over the entire volume.

An example output for `irl_p.cpp` is included (`answer_p8_8.txt`) for the algorithm run on 8 processors solving for 8 eigenvalues using `kmax = 12` and `pmax = 22`. The global number of grid points in each dimension is 40. This took 8.2 minutes, compared to 13.3 minutes on 1 processor.

## 2 Helium Atom

Utilizing the IRL and CG solvers, I compute the ground state energy of the Helium Atom following the matlab code from Mikael Kuisma. I will give a summary of results first, then go into how the code is structured. After our discussion on Thursday, I successfully figured out the normalizations and have produced the correct ground state energy of Helium. I have included an output run on 1 processor with  $L=40$  in each dimension, run with 10 DFT iterations. As you can see, it converges to -2.678 after only 7 iterations. Unfortunately, however, I have somehow introduced a bug in making these changes. Previously my single and multiple processor runs converged together, but now they are converging to different values. I have included an output for  $L=40$  run on 8 processors over 10 DFT iterations. Unfortunately, I have to table finding the bug until later. If I do, I will send you an update.

The code is structured as follows. The `irl_func.cpp` code contains the irl solver function and associated functions needed. `cg_func.cpp` contains the conjugate gradient solver and associated functions. `parallel_functions.cpp` contains functions needed to set up the volume, including setting boundary conditions, setting up the local neighbor matrix, and setting up the global processor neighbor matrix. These three are employed in the `helium.cpp` file, which computes the energies, following the matlab function. The exchange, hartree, and external energies are calculated within the main function, with  $V_h$  originating from the conjugate gradient solver and  $\psi$  from the IRL algorithm. The compensation (`ncomp`) is employed directly from the matlab code. The external potential is set initially in a separate function, along with `ncomp`. The loop calculates kinetic energy with a special function, since it involves matrix times vector operations.