BITTECX LIMITED

Acoustic Emission - Open AE Initiative

https://www.linkedin.com/feed/update/urn:li:activity:6552181436881879041

GENERAL THEORY OF ACOUSTIC EMISSION SIGNALS.

Abstract:

The method of acoustic emission (AE) has been used for a long time since Kaiser’s pioneering work, both in solid state physics to study the properties of materials and in non-destructive testing techniques (Dunegan, Pollock and other authors). But so far in the literature there is no general mathematical theory of the phenomenon of AE, which allows to quantitatively link the sources and the observed form of signals. However, back in the 80s in Kharkov, we developed such a quantitative theory of AE, the main results of which, unfortunately, were not published for various reasons. Some of the materials were published in the research report carried out at Kharkov University on the request of CNIITMASH (the report was deposited in VINITI, Moscow). This document contains a summary of the main points and results of this theory.

Theoretical consideration of the phenomenon of AE is reduced to two problems.

The first is to establish the connection between the signal and the source, as well as the calculation of the shape of the observed signal using the specified parameters of the AE source.

The second is the so-called inverse problem of AE, i.e. determining the coordinates of the source (location), as well as calculating the source parameters from the observed waveform.

Sources of AE signals.

It is known that AE occurs during plastic deformation and destruction of solids.

It was established experimentally that AE is absent under purely elastic deformation, when there are no “ defects”, i.e. small areas of origin and development of plastic slip or destruction.

Sources of AE can have a different physical nature, in fact, as well as the known mechanisms of plastic deformation and destruction. For example, it can be dislocations, twins, cracks and other developing defects of a solid. A detailed consideration of the physical features and internal mechanisms of such defects relates to the field of solid state physics and constitutes a separate and independent area of research.

However, despite the physical diversity of AE sources, they can all be described in a general approximate mathematical model. The size of AE sources, as a rule, is much smaller than the size of the sample or engineering design under investigation . The deformations and stresses in the entire sample or structure can be described with sufficient accuracy using the linear equations of the theory of elasticity, except “defective areas”, small compared with the size of the sample or the construction of areas of plastic deformation or destruction. These are small areas where the linear theory of elasticity is inapplicable and non-linearity is essential and are sources of AE.

Thus, it is reliably established experimental fact that the sources of AE are emerging or developing defects, i.e. small areas where linear theory of elasticity is not applicable. An approximate mathematical model of such behavior has long been developed in the theory of elasticity and is called the Volterra-Somiliana (VS) dislocation theory. The essence of this theory is as follows.

The most common form of a singular solution of linear equations of elasticity theory is the VS dislocation, that is, arbitrary surfaces of discontinuity of the displacement vector. The jumps of the displacement vector during the transition through the VS dislocation surface are called Burgers vector.

A real physical defect that has a small but finite size, is approximated in this theory by the VS dislocation, which is a mathematical idealization of the defect, by the infinitely thin surface of discontinuity of the displacement vector. Such an approximation is suitable for a description of an elastic field at distances large compared with the size of the defect.

Similarly, in electrodynamics, the most general form of a singular solution of field equations is a singularity with a point charge (electron). Therefore, the linear equations of electrodynamics are suitable for describing the field of an electron only at distances of substantially large electron sizes. In the theory of elasticity, the VS dislocation model, like the classical electron model in electrodynamics, is the most general approximate theoretical description of the field at long distances for small physical defects.

The elastic field of a real physical defect, which is a small area of substantially nonlinear behavior (plastic deformation or fracture), can be approximately described as a singular solution of linear field equations at distances significantly larger than the size of the defect. This theoretical model has long been successfully used, for example, in seismology for an approximate description of an earthquake and for calculating the shape of a seismic signal far from the source. A mathematical theory based on the model of VS dislocations was developed by Eshelby, Osho, and other authors. The model of defects in an elastic body as VS dislocations is the most general approximate theoretical model also for AE sources, provided that the signal is observed far from the source.

Thus, there is a mathematical analogy between seismology and the theory of AE signals. However, it should be borne in mind that in theoretical seismology, as a rule, only the propagation of elastic waves in a half-space is considered, whereas in the theory of AE, the shape of a sample or structure may have an arbitrary shape.

Wave solution of the equations of the theory of elasticity for VS dislocations.

In the theory of AE, it is necessary to study the propagation of elastic waves not only in a half-space, but also in bounded and unbounded samples and structures of such ka to the rods, plates and shells. In order to calculate the waveform of an AE, it is necessary to solve a non-stationary boundary value problem for the equations of the theory of elasticity when waves are excited by or arising in a sample or structure by an BC dislocation.

A general solution to this problem can be obtained as follows. (Mathematical details are quite complex and will be published elsewhere. Here it is essential to use functional analysis methods: the theory of generalized functions, spectral theory and calculus of differential operators.) Here we present only a brief summary of the results.

To calculate the waveform of an AE in a body of arbitrary shape, it is necessary to first calculate the dynamic Green's function of the unsteady equations of the theory of elasticity for a given body shape and boundary conditions. The Green function describes the dynamic response of an elastic body to a point force varying in time as a delta function.

In the case of a space-limited body, this Green's function can be computed explicitly by expanding it into a series of eigenfunctions (the body's normal modes). The eigenfunctions and eigenvalues (forms and eigenfrequencies of oscillations) should be obtained from the solution of the boundary value problem for the theory of elasticity operator and the corresponding boundary conditions. In many cases of good symmetry, these eigenvalues and functions can be accurately expressed through elementary or special functions. For samples of more complex shape that do not have special symmetry, calculation is required using suitable numerical or approximate analytical methods. It is possible to significantly simplify calculations using the available approximate theories of wave propagation for special cases that are important in engineering, such as rods, plates and shells.

For a space-limited body, the attenuation of signals is also essential. If you do not take into account the damping, then the oscillations of a perfectly elastic body will last indefinitely. Attenuation of oscillations can occur due to two reasons: viscous dissipation in the volume or radiation through the boundary of the body into the environment.

Therefore, in the theory of AE signals, it is necessary, generally speaking, to consider the boundary problem for a non-self-adjoint operator whose eigenfunctions are not orthogonal. The theory of such non-self-adjoint dissipative operators was studied in 1965 by the mathematicians Gohberg and Krein in Odessa, as well as other authors. However, because the viscous damping of sound in solids is small, we can confine ourselves to the low attenuation using the perturbation method for dissipative operators, who also considered the first Gohberg and Krein.

A specific example of such a calculation for the cylindrical specimen will be discussed below.

In another case, when the body in which the propagation of waves is considered to be space-unlimited, besides the natural oscillations, the waves are also transmitted to infinity. Then, to calculate the dynamic Green's function, besides the discrete one, it is also necessary to take into account the continuous spectrum of the boundary value problem, in other words, besides the natural modes of oscillations, it is also necessary to take into account the radiation of the waves.

A simple example of such a case is a cylindrical specimen clamped in the grips of a tensile machine, as in the Kaiser experiments. In this case, not only the oscillations of the sample are excited at eigenfrequencies, but also an attenuation of the AE signal arises due to the transmission of waves through the grips of a tensile machine.

Let us return to the calculation of the shape and spectrum of the AE signal itself in a body of arbitrary shape. The mathematical technique to be used here is as follows (the details of the calculations are omitted).

It is known that for differential operators, in particular for the differential operator of the elasticity theory, there is a calculus. This calculus allows us to deal with differential operators in much the same way as with numbers in ordinary arithmetic and, in particular, to calculate the inverse operator.

The calculus of operators is generated by duality. In the case of differential operators, this is the duality between the exterior derivative and the boundary of the manifold, as expressed by the Stokes' theorem. It is also necessary to use the differential reciprocity relation for the equations of the theory of elasticity, which is simply a generalization of the Leibniz formula to differential forms and the operator of the theory of elasticity. Transforming the reciprocity relation using the Stokes' theorem (integration by parts) and taking into account the boundary conditions, we determine the adjoint operator. In the absence of dissipation, the elastic theory operator is self-adjoint.

Further, substituting the definition of the Green function, we obtain an explicit integral expression for the inverse operator. In the non-stationary case, it is also necessary to perform the direct and inverse Laplace transform, therefore a time convolution also occurs.

An explicit expression for the inverse operator of the non-stationary problem of elasticity theory is obtained in the form of a time convolution integral operator, using the dynamic Green function corresponding to a given body geometry and boundary conditions. The resulting solution is suitable for the case of natural oscillations of a limited body, and in the unbounded case of a continuous spectrum, when it is necessary to take into account the radiation of waves.

In the case of dissipation, the operator becomes non-self-adjoint, but the method described continues to work. It is only necessary to use in the case of a bounded body the expansion of the Green's function in a bi-orthogonal system of functions, since the eigenfunctions become non-orthogonal. Simply, however, take into account the low attenuation by the perturbation method or an explicit calculation of losses by radiation. (This will not stop here.)

A simplified example of such a calculation is shown below for a cylindrical rod.

The solution for a source in the form of a VS dislocation is obtained if we consider that the entire surface of the body consists of two pieces: the outer surface and VS dislocation surface. The boundary conditions on the part of the surface of the corresponding VS dislocation are reduced simply to the conditions for the jump of the displacement vector (Burgers vector).

The solution has the form of an integral over the VS dislocation surface and the time convolution of the Burgers vector (understood as a generalized function) and the derivatives (gradient) of the Green function for the corresponding body shape and boundary conditions. This solution can be used to explicitly calculate the AE waveform. If the source parameters are known, ie. that is, the shape of the BC dislocation surface, as well as the time dependence and the distribution over the surface of the Burgers vector, then the AE signal waveform can be calculated. Thus, in general, the first problem of the theory of AE signals is solved on the calculation of the source of a waveform for an arbitrary waveform, bounded or unlimited, by given parameters.

The foregoing is enough to restore all the calculations. But for those who do not want to dive into mathematical theory, we will give a simple explanation.

The propagation of the AE signal in a sample or engineering design can be considered as a blackbox model. The input is the AE signal source, and the output is the signal observed on the oscilloscope screen, and blackbox itself is the inverse integral operator of the non-stationary theory of elasticity, a time convolution whose kernel is determined by the dynamic Green function, depending on the geometry of the body and boundary conditions.

The source of the AE signal at the input of the blackbox is the distribution of the Burgers vector over VS dislocation surface.

Particular cases and simplified calculation of the waveform AE.

There are important for engineering special cases of simple geometry, when the theory allows us to calculate the shape of the AE signal explicitly exactly or approximately using some reasonable simplifying assumptions.

Annihilation of rectilinear dislocations in an unbounded medium. The problem of calculating the waveform of AEs for rectilinear dislocations in an unbounded medium is more likely to be of theoretical interest, since it allows us to write out the exact solution.

The solution to this problem is published in the journal FTT.

Calculation of screw dislocations is Busy is performed by calculating the convolution integral that is evaluated exactly in terms of elementary functions. For edge dislocations can be Utilized method Cagniard de Hoop.

The problem of the excitation of Rayleigh waves by an edge dislocation reaching the surface was also solved.

Calculation of radiation of a dislocation loop (dislocation dipole).

It is also rather easy to solve the problem of the radiation field of a dislocation loop at large distances from the loop. Any AE source in an unlimited medium can be approximated at large distances as a dislocation loop (dipole).

Cylindrical specimen. This is the most important form of the sample for physical and strength research. Starting with Kaiser's pioneering work, the plastic deformation of a cylindrical specimen in a tensile machine is used as a standard experimental model for AE research.

Again omitting the calculations we give the results.

The simplest approximate rod theory gives the result with acceptable accuracy for engineering applications. The AE signal in a cylindrical specimen in the grips of a tensile machine is calculated as the superposition of the own longitudinal vibrations of the rod excited by the slip strip in the sample.

Found that the signal spectrum has peaks at the natural frequencies of the longitudinal vibrations of the rod. Radiation into the seizures of a tensile machine is also taken into account. Attenuation of the natural oscillations and the AE signal occurs mainly due to radiation into the grips of the tensile machine. The obtained approximate result is refined by using a more complex theory of longitudinal waves in the rods of Mindlin–Herrmann model.

Plates and shells. For plates and shells, the propagation of acoustic waves is complex. However, given the well-known approximate theory of wave propagation in such structures solve e It has the form of the convolution integral, which in many cases simple symmetry is calculated. On the details of the calculations (rather cumbersome) we will not stop here.

AE in piezoelectrics. The problem of the emergence of a rectilinear screw dislocation from a piezoelectric crystal was also considered. The results are published in JETP Letters. It turns out that the exit of the screw dislocation is accompanied by the excitation of the surface Gulyayev-Blustein waves. The waveform is calculated using the Cagniard - de Hoop method.

The inverse problem of the theory of AE signals .

The second or partial problem of the theory of AE signals is to determine the parameters of the source from the observed signal. A similar task has long been successfully considered in seismology.

In the theory of the inverse problem of AE, the fundamental fact should be clearly understood: this problem belongs to the class of the so-called incorrect problems of mathematical physics. The definition of the correctness of the problem of mathematical physics was first considered by Hadamard. But the solution to this inverse problem of AE theory does exist, provided that the concept of correctness according to Tikhonov is used.

For the simplest case of the cylindrical specimen, a complete analysis of the inverse problem of the theory of AE was performed. Formulas are obtained that connect the duration of the AE source with the shape and spectrum of signals. A summary can be found in a message published in the Letters of the JTP.

The limits of applicability of the theory.

The considered theory of AE signals is applicable only at large distances from the source when the dimensions of the defect - the source of AE are substantially smaller than the dimensions of the sample or structure. This is similar to classical electrodynamics, where solutions for a point source - an electron are applicable only at large distances (at low energies) compared with the electron radius.

This is characteristic of the most important case in engineering when the initial stage of the development of plastic deformation and the formation of microcracks is studied. As for the final stage of destruction, when the dimensions of the plastic deformation or crack region can be comparable to the body size, the theory presented here is not applicable, since such a process is essentially non-linear and its description cannot be reduced to solving linear equations of elasticity theory.

Experimental verification of the theory.

We have not experimentally verified the stated theory. It is important to conduct such a check for the simplest standard sample forms.

First of all, it is necessary to experimentally study the conformity of the theory and form under consideration, as well as the spectrum of AE signals for the simplest case of a core sample.

It is also possible to simply test the theory for a half-space in the form of a massive polycrystalline or single-crystal sample, in which AE signals are excited by indentation of a diamond indenter. Such an experimental scheme is standard for studying the dynamics of dislocation semi-loops emerging on the surface. On the other hand, the shape of the excited AE signal in this case is easily calculated using the stated theory.

A more complex, but it is also possible to test the theory for plates and shells. A diamond indenter or a broken graphite rod proposed by Dunegan can also be used as a test source of AE.

Application in engineering.

The proposed solution to the inverse problem of the theory of AE signals allows us to develop equipment of a new generation, so to speak, an “acoustic emission microscope”, which not only determines the coordinates of the defect - the source of AE, but also calculates its parameters, in particular, its energy characteristics and danger to the tested structure . Such equipment will also significantly clarify the location accuracy of the AE source.