COMSM0007: Cryptography B 2014-2015

Lecture 3: February 3

Lecturer: Bogdan Warinschi Scribes: Dominic Moylett

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Last week, we introduced the concept of one way functions (OWFs) and showed how we can create a one-time signature scheme given any OWF. This lecture, we are going to prove the security of the scheme.

3.1 Security of the Lamport One-Time Signature Scheme

For simplicity, the Lamport one-time signature scheme for a OWF f will be referred to as π_f .

Theorem 3.1 If f is a OWF then π_f is a secure one-time signature scheme.

We will show this by a reduction: Given an adversary \mathcal{A} against π_f , we can construct another adversary \mathcal{B} against f. We will do this in three parts.

3.1.1 Security Under Two Assumptions

First, we will prove this to be true if the following assumptions hold:

- 1. The adversary A is passive, so it cannot make any signature queries.
- 2. The first bit of the forged message returned by A is 0.

Lemma 3.2 Given an adversary A against π_f and under the above assumptions, there exists an adversary B against f.

Proof: Our adversary against π_f looks like the following black box:

$$\begin{array}{c} & \longleftarrow & \begin{pmatrix} Y_{10}...Y_{l0} \\ Y_{11}...Y_{l1} \end{pmatrix} \\ & \longleftarrow & (0m_2^*...m_l^*, \sigma_1^*...\sigma_l^*) \end{array}$$

Figure 3.1: The first adversary against π_f

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Because the first bit of the forged message $m_1^* = 0$, the start of the forged signature σ_1^* is a preimage of Y_{10} . From this, we can construct an adversary \mathcal{B} for f:

$$\begin{split} & \textbf{input:} \ Y = f(x) \\ & (SK, VK) \leftarrow Kg(n) = \left(\begin{pmatrix} X_{10}...X_{l0} \\ X_{11}...X_{l1} \end{pmatrix}, \begin{pmatrix} Y_{10}Y_{20}...Y_{l0} \\ Y_{11}Y_{21}...Y_{l1} \end{pmatrix} \right) \\ & VK' \leftarrow \begin{pmatrix} YY_{20}...Y_{l0} \\ Y_{11}Y_{21}...Y_{l1} \end{pmatrix} \\ & \mathcal{A} \leftarrow VK' \\ & \mathcal{A} \rightarrow (0m_2^*...m_l^*, \sigma_1^*\sigma_2^*...\sigma_l^*) \\ & \textbf{output} \ \sigma_1^* \end{split}$$

Because $m_1^* = 0$ by assumption, \mathcal{B} is guaranteed to find a preimage of Y as long as \mathcal{A} can forge a signature.

$$\operatorname{Prob}[\mathcal{B} \text{ breaks } f] = \operatorname{Prob}[\mathcal{A} \text{ breaks } \pi_f]$$

If Prob[\mathcal{A} breaks π_f] is non-negligible, then \mathcal{B} succeeds with non-negligible probability.

3.1.2 Removing Assumption 2

Lemma 3.3 Given a passive adversary A against π_f , there exists an adversary B against f.

Proof: Because we can no longer assume $m_1^* = 0$, instead of setting $Y_{10} \leftarrow Y$, we flip a bit to determine the value we set to Y.

$$\begin{split} & \textbf{input:} \ Y = f(x) \\ & (SK, VK) \leftarrow Kg(n) = \left(\begin{pmatrix} X_{10} ... X_{l0} \\ X_{11} ... X_{l1} \end{pmatrix}, \begin{pmatrix} Y_{10} Y_{20} ... Y_{l0} \\ Y_{11} Y_{21} ... Y_{l1} \end{pmatrix} \right) \\ & b \overset{\$}{\leftarrow} \left\{ 0, 1 \right\} \\ & Y'_{1b} \leftarrow Y \\ & Y'_{1\bar{b}} \leftarrow Y_{1\bar{b}} \\ & VK' \leftarrow \begin{pmatrix} Y'_{10} Y_{20} ... Y_{l0} \\ Y'_{11} Y_{21} ... Y_{l1} \end{pmatrix} \\ & \mathcal{A} \leftarrow VK' \\ & \mathcal{A} \rightarrow (m_1^* ... m_l^*, \sigma_1^* ... \sigma_l^*) \\ & \textbf{if} \quad m_1^* \neq b \ \textbf{then abort} \\ & \textbf{output} \ \sigma_1^* \end{split}$$

If \mathcal{A} has succeeded and $m_1^* = b$ then σ_1 is a preimage of Y and \mathcal{B} is therefore successful.

$$\text{Prob}[\mathcal{B} \text{ breaks } f] = \frac{1}{2} \text{Prob}[\mathcal{A} \text{ breaks } \pi_f]$$

Again, \mathcal{B} breaks f with non-negligible probability as long as $\frac{1}{2}\operatorname{Prob}[\mathcal{A} \text{ breaks } \pi_f]$ is non-negligible.

3.1.3 Removing Assumption 1

To complete our proof of Theorem 3.1, we need to allow our adversary \mathcal{A} to make one signature query. Note that if we allow \mathcal{A} to make more than one signature query, \mathcal{A} can just query the messages 0^l and 1^l to acquire the entire signing key.

Proof: Our final adversary \mathcal{A} against π_f is this black box:

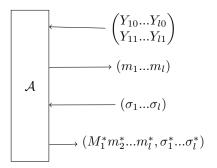


Figure 3.2: The final adversary against π_f

Recall from the definition of EUF-CMA as presented in Lecture 2 that the message forged by the adversary cannot match the message the adversary queried. This means that there exists some index i^* such that $m_{i^*} \neq m_{i^*}^*$ and thus σ_{i^*} is a preimage computed by \mathcal{A} .

Therefore, our final adversary \mathcal{B} against f is as follows:

$$\begin{split} & \text{input: } Y = f(x) \\ & (SK, VK) \leftarrow Kg(n) = \left(\begin{pmatrix} X_{10} ... X_{l0} \\ X_{11} ... X_{l1} \end{pmatrix}, \begin{pmatrix} Y_{10} ... Y_{l0} \\ Y_{11} ... Y_{l1} \end{pmatrix} \right) \\ & i^* \overset{\$}{\leftarrow} \left\{ 1, ..., l \right\} \\ & b \overset{\$}{\leftarrow} \left\{ 0, 1 \right\} \\ & \text{for } i \in \left\{ 1, ..., l \right\} \\ & \text{if } i = i^* \text{ then } \\ & Y'_{i^*b} \leftarrow Y \\ & Y'_{i^*\bar{b}} \leftarrow Y'_{i^*\bar{b}} \\ & \text{otherwise } \\ & Y'_{i0} \leftarrow Y_{i0} \\ & Y'_{i1} \leftarrow Y_{i1} \\ VK' \leftarrow \begin{pmatrix} Y'_{10} ... Y'_{l0} \\ Y'_{11} ... Y'_{l1} \end{pmatrix} \\ & \mathcal{A} \leftarrow VK' \\ & \mathcal{A} \rightarrow (m_1 ... m_l) \\ & \text{if } m^*_{i^*} = b \text{ then abort } \\ & \mathcal{A} \leftarrow (X_{1m_1} ... X_{lm_l}) \\ & \mathcal{A} \rightarrow (m^*_1 ... m^*_l, \sigma^*_1 ... \sigma^*_l) \\ & \text{if } m^*_{i^*} \neq b \text{ then abort } \\ & \text{output } \sigma^*_{i^*} \end{split}$$

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When \mathcal{A} makes its oracle query, \mathcal{B} still has access to the signing key and can therefore compute a signature. The only exception is if $m_{i^*}^* = b$; \mathcal{B} is unable to provide a response in this case as $Y_{i^*b} = Y$, so computing a signature would require finding a preimage of Y.

As long as these three conditions hold:

- 1. \mathcal{A} successfully breaks π_f
- 2. $m_{i^*} \neq m_{i^*}^*$
- 3. and $m_{i^*}^* = b^{-1}$

Then σ_{i^*} is a preimage of Y. So in order for \mathcal{B} to succeed, \mathcal{A} needs to succeed and we need to select suitable values for the tuple $(i^*, b) \in \{1, ..., l\} \times \{0, 1\}$. There are 2l possible values for the tuple and at least one satisfies the above condition.

$$\text{Prob}[\mathcal{B} \text{ breaks } f] \geq \frac{1}{2l} \text{Prob}[\mathcal{A} \text{ breaks } \pi_f]$$

If $\mathcal A$ succeeds with non-negligible probability then so does $\mathcal B$ and thus our proof is complete.

¹Note that if conditions 1 and 2 hold, then $m_{i^*} \neq b$ and thus both abort cases are covered.