

Fault Tolerant Computing and VLSI Testing

Assignment 3

1.

2.

$$3. FC = 80\% \implies DL = 1 - 0.9^{1-0.8} = 0.020852 = 20852PPM$$

$$FC = 90\% \implies DL = 1 - 0.9^{1-0.9} = 0.010481 = 10481PPM$$

$$FC = 99\% \implies DL = 1 - 0.9^{1-0.99} = 0.001053 = 1053PPM$$

$$DL = 20PPM = 2 * 10^{-5} = 1 - 0.7^{1-FC}$$

$$0.6^{1-FC} = 1 - 2 * 10^{-5} = 0.99998$$

$$1 - FC = \log_{0.6} 0.99998 = 3.915 * 10^{-5}$$

$$FC = 1 - 3.915 * 10^{-5} = 0.99996 = 99.996\%$$

4.

AB	00	01	10	11
Z	1	1	1	0
P_0 stuck open	1	LastZ	1	0
P_0 stuck short	1	1	1	I_{DDQ}
P_1 stuck open	1	1	LastZ	0
P_1 stuck short	1	1	1	I_{DDQ}
N_0 stuck open	1	1	1	LastZ
N_0 stuck short	1	I_{DDQ}	1	0
N_1 stuck open	1	1	1	LastZ
N_1 stuck short	1	1	I_{DDQ}	0

5. TODO: Add tests here!

$$a \oplus (a + b) = a(\overline{a + b}) + \overline{a}(a + b) = a\overline{a}\overline{b} + \overline{a}a + \overline{a}b = \overline{a}b$$

$$(a + b)\overline{b} = a\overline{b} + b\overline{b} = a\overline{b}$$

$$(a \oplus (a + b)) + (a + b)\overline{b} = \overline{a}b + a\overline{b} = a \oplus b$$

Therefore, the boolean operation this circuit is equivalent to is exclusive-OR.

6. Test patterns are written as abc .

Not that a * symbol is used where appropriate to represent the AND operation for clarity, in order to help distinguish between, for example, $\overline{a} * \overline{c}$ and $\overline{a} * \overline{c}$

$$\begin{aligned}
 \text{(i)} \quad \bar{a} \frac{dz}{da} &= \bar{a}(z(a=1) \oplus z(a=0)) = \bar{a}((\bar{c} + cb) \oplus cb) \\
 &= \bar{a}(\overline{\bar{c} + cb}cb + (\bar{c} + cb)\bar{c}\bar{b}) = \bar{a}(\bar{c}\bar{c}\bar{b}cb + \bar{c}\bar{c}\bar{b} + cb\bar{c}\bar{b}) \\
 &= \bar{a} * \bar{c}\bar{c}\bar{b} = \bar{a} * \bar{c}(\bar{c} + \bar{b}) = \bar{a} * \bar{c} + \bar{a} * \bar{c}\bar{b} = \bar{a} * \bar{c}(1 + \bar{b}) = \bar{a} * \bar{c} = 1
 \end{aligned}$$

The patterns $\{000, 010\}$ can detect the fault.

$$\begin{aligned}
 \text{(ii)} \quad h \frac{dz}{dh} &= h(z(h=1) \oplus z(h=0)) = h(1 \oplus a\bar{c}) = h\bar{a}\bar{c} = h(\bar{a} + c) \\
 &= h\bar{a} + hc
 \end{aligned}$$

Note that $h = cb$.

$$h \frac{dz}{dh} = \bar{a}bc + bc = bc(a + 1) = bc = 1$$

The patterns $\{011, 111\}$ can detect the fault.

$$\begin{aligned}
 \text{(iii)} \quad \bar{h} \frac{dz}{dh} &= \bar{h}\bar{a} + \bar{h}c = \bar{c}\bar{b}\bar{a} + \bar{c}\bar{b}c = (\bar{c} + \bar{b})\bar{a} + (\bar{c} + \bar{b})c \\
 &= \bar{a}\bar{b} + \bar{a}\bar{c} + \bar{b}c + \bar{c}c = \bar{a}\bar{c} + \bar{a}\bar{c} + \bar{b}c = 1
 \end{aligned}$$

The patterns $\{000, 001, 010, 101\}$ can detect the fault.

$$\begin{aligned}
 \text{(iv)} \quad e \frac{dz}{de} &= e(z(e=1) \oplus z(e=0)) = e((a\bar{c} + b) \oplus a\bar{c}) \\
 &= e(\overline{a\bar{c} + b}a\bar{c} + (a\bar{c} + b)\bar{a}\bar{c}) = e(\bar{b} * \bar{a}\bar{c}a\bar{c} + a\bar{c}\bar{a}\bar{c} + b\bar{a}\bar{c}) = e\bar{b}a\bar{c} \\
 &= be(\bar{a} + c) = \bar{a}be + bec
 \end{aligned}$$

Note that $e = c$.

$$e \frac{dz}{de} = \bar{a}bc + bc = bc(\bar{a} + 1) = bc = 1$$

The patterns $\{011, 111\}$ can detect the failure.

$$\text{(v)} \quad \bar{e} \frac{dz}{de} = \bar{a}\bar{b}\bar{e} + \bar{b}\bar{e}c = \bar{a}\bar{b}\bar{c} + \bar{b}c\bar{c} = \bar{a}\bar{b}\bar{c}$$

The pattern $\{010\}$ can detect the failure.

$$\begin{aligned}
 \text{(vi)} \quad c \frac{dz}{dc} &= c(z(c=1) \oplus z(c=0)) = c(b \oplus a) = c(\bar{a}b + a\bar{b}) = \bar{a}bc + a\bar{b}c = 1 \\
 &\text{The patterns } \{011, 101\} \text{ can detect the fault.}
 \end{aligned}$$

7. Test patterns are written as ab .

$$\text{(a)} \quad \bar{a} \frac{di}{da} = \bar{a}(i(a=1) \oplus i(a=0)) = \bar{a}(b \oplus 0) = \bar{a}b = 1$$

The pattern $\{01\}$ can detect the fault.

$$\text{(b)} \quad \bar{d} \frac{di}{da} = \bar{d}(i(d=1) \oplus i(d=0)) = \bar{d}(ab \oplus 0) = \bar{d}ab$$

Note that $d = a$.

$$\bar{d} \frac{di}{da} = \bar{a}ab = 0 = 1$$

We have reached a contradiction, therefore this fault cannot be detected.

$$\text{(c)} \quad \bar{g} \frac{di}{dg} = \bar{g}(i(g=1) \oplus i(g=0)) = \bar{g}(ab \oplus 0) = \bar{g}ab$$

Note that $g = ab$.

$$\bar{g} \frac{di}{dg} = \bar{a}bab = 0 = 1$$

We have reached a contradiction, therefore this fault cannot be detected.

8. Below are the signatures calculated for the sequence $M = 10011011$ (fault-free) and the faulty sequence $M' = 11111111$

M	1	x	x^2	x^3	M'	1	x	x^2	x^3
1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	1	1	0	0	0
0	0	1	0	0	1	1	1	0	0
1	0	0	1	0	1	1	1	1	0
1	1	0	0	1	1	1	1	1	1
0	0	0	0	0	1	0	0	1	1
1	0	0	0	0	1	0	1	0	1
1	1	0	0	0	1	0	1	1	0
R	1	1	0	0	R	1	0	1	1

The signatures calculated for the faulty sequence M' is different from the fault-free sequence M , so this fault can be detected.