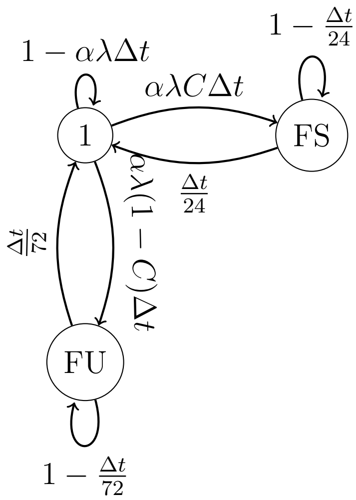
Fault Tolerant Computing and VLSI Testing

Assignment 3

1. The Markov Model for the system with self-diagnostics is:



   - 1 is where the system is working successfully.
   - FS is where a fault has been detected and the system has been safely deactivated.
   - FU is where a fault has not been detected.

   The Mean Time To Failure (MTTF) will be the amount of time we expect to stay in the working state. This is the value of t such that $t\alpha\lambda = 1$. This is equal to $t = \frac{1}{\lambda\alpha}$

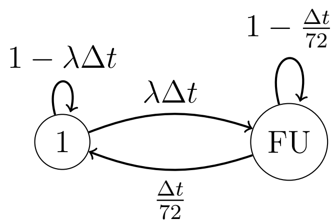
   We detect the failure with probability 0.95, and don't with probability 0.05. This gives us a mean time between failures of:

   $0.95 * 24 + 0.05 * 72 = 26.4$

   Thus, the steady-state availability is:

   $A = \frac{1}{\alpha\lambda(\frac{1}{\alpha\lambda}+26.4)} = \frac{1}{1+26.4\alpha\lambda}$

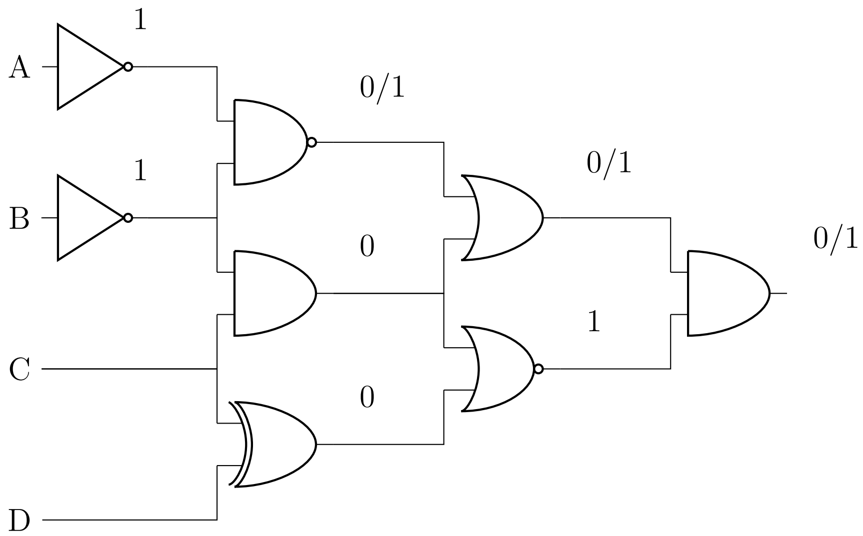
   The Markov Model for the system without self-diagnostics is:



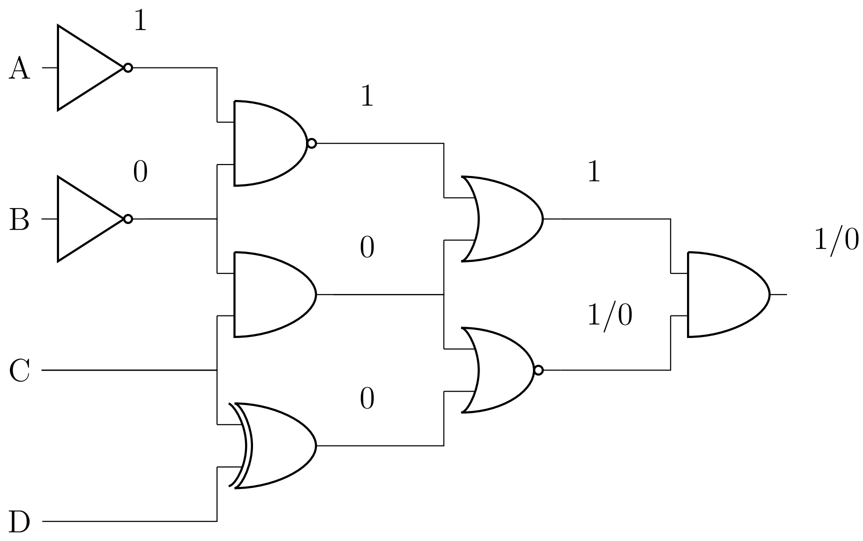
- 1 is where the system is working successfully.
- FU is where a fault has occured.

The MTTF is $\frac{1}{\lambda}$ and the steady-state availability is:
$A = \frac{1}{\lambda(\frac{1}{\lambda}+72)} = \frac{1}{1+72\lambda}$
We can find the value of α for which self-diagnostics begins to degrade the steady-state availability by equating these two equations:
$\frac{1}{1+26.4\alpha\lambda} = \frac{1}{1+72\lambda} \implies 1 + 26.4\alpha\lambda = 1 + 72\lambda \implies 26.4\alpha\lambda = 72\lambda$
$\alpha = \frac{72}{26.4} = 2.727$

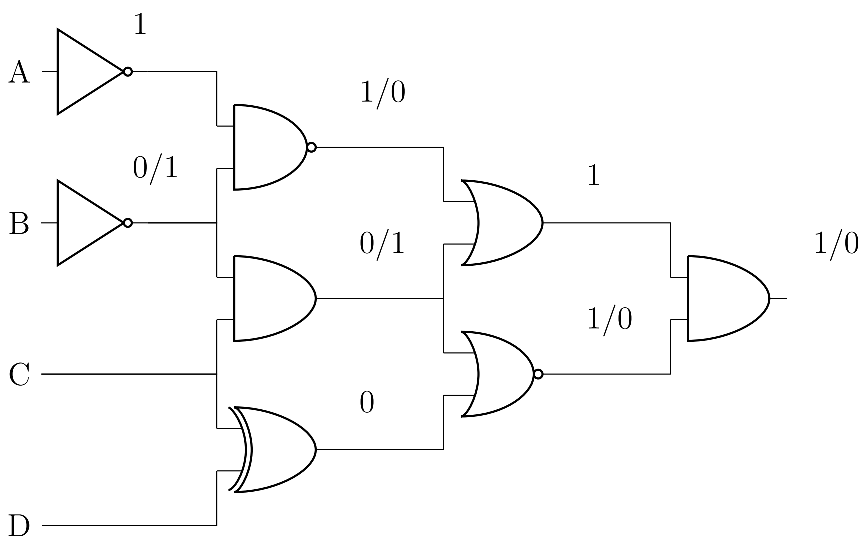
2. **(i)** $ABCD = 0000$ can detect line 4 stuck-at 0:



(ii) $ABCD = 0100$ can detect line 13 stuck-at 1:



(iii) $ABCD = 0111$ can detect input B stuck-at 0:



3. $FC = 80\% \implies DL = 1 - 0.9^{1-0.8} = 0.020852 = 20852PPM$

   $FC = 90\% \implies DL = 1 - 0.9^{1-0.9} = 0.010481 = 10481PPM$

   $FC = 99\% \implies DL = 1 - 0.9^{1-0.99} = 0.001053 = 1053PPM$

   $DL = 20PPM = 2 * 10^{-5} = 1 - 0.6^{1-FC}$

   $0.6^{1-FC} = 1 - 2 * 10^{-5} = 0.99998$

   $1 - FC = \log_{0.6} 0.99998 = 3.915 * 10^{-5}$

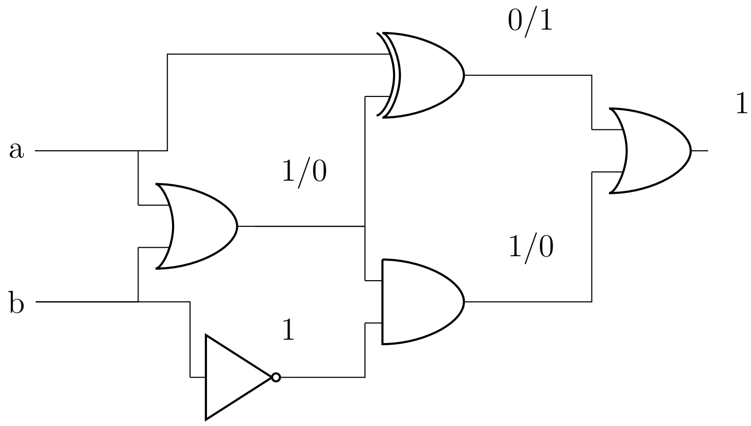
   $FC = 1 - 3.915 * 10^{-5} = 0.99996 = 99.996\%$

4.

AB	00	01	10	11
Z	1	1	1	0
P_0 stuck open	1	LastZ	1	0
P_0 stuck short	1	1	1	I_{DDQ}
P_1 stuck open	1	1	LastZ	0
P_1 stuck short	1	1	1	I_{DDQ}
N_0 stuck open	1	1	1	LastZ
N_0 stuck short	1	I_{DDQ}	1	0
N_1 stuck open	1	1	1	LastZ
N_1 stuck short	1	1	I_{DDQ}	0

5. **(1)** Note that $a = 1$ in order to exercise d sa0, as that is the only input that can set the wire d to 1. Also note that $b = 0$, as otherwise the fault d sa0 will have no effect after the OR-gate.

Therefore the only input that could test this fault is $ab = 10$. Here is what this input produces:



So even with this input, the fault d sa0 cannot be detected. The fault is therefore redundant.

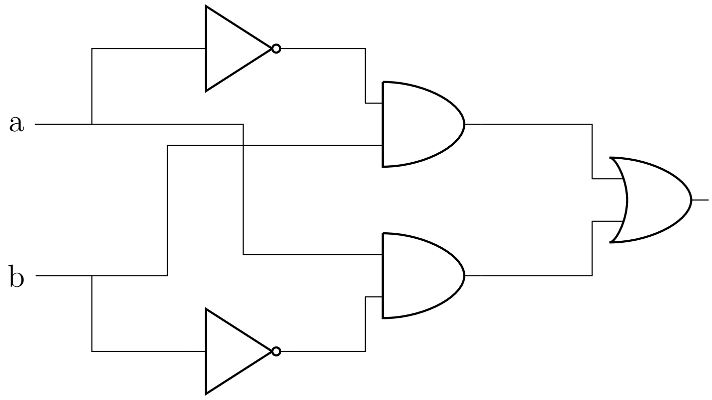
We can remove the redundancy by considering the operation of the circuit:

$a \oplus (a + b) = a\overline{(a + b)} + \overline{a}(a + b) = a\overline{a}\overline{b} + \overline{a}a + \overline{a}b = \overline{a}b$

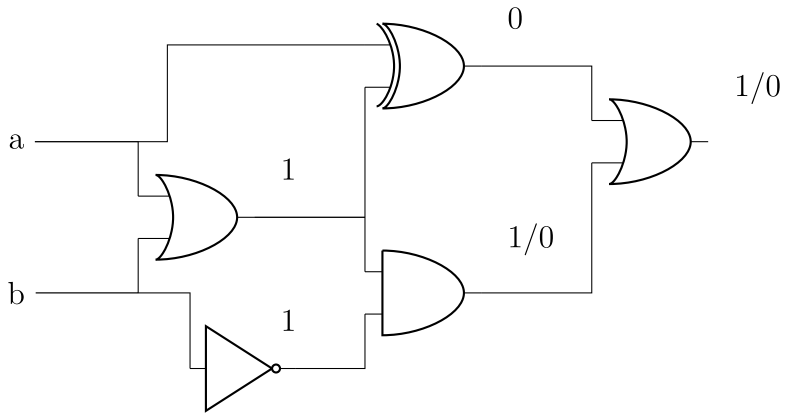
$(a + b)\overline{b} = a\overline{b} + b\overline{b} = a\overline{b}$

$(a \oplus (a + b)) + (a + b)\overline{b} = \overline{a}b + a\overline{b}$

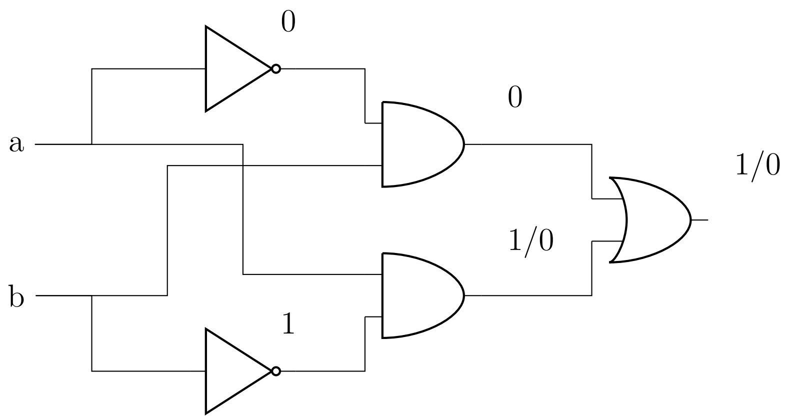
Our redundancy-free circuit is therefore the following:



(2) $ab = 10$ can detect the fault m sa0:



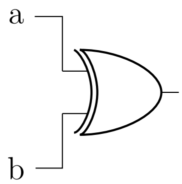
The wire m fed in the original circuit fed into the reduced AND operation $a\overline{b}$. The gate for this operation is still in the redundancy-free circuit, so we can still test for this fault by testing the a input for the AND gate. $ab = 10$ can detect the fault in this wire too:



As stated above, the operation is equivalent to:

$\overline{a}b + a\overline{b} = a \oplus b$

Therefore, the boolean operation this circuit is equivalent to is exclusive-OR. The minimum implementation is:



6. Note that a * symbol is used where appropriate to represent the AND operation for clarity, in order to help distinguish between, for example, $\overline{a * c}$ and $\overline{a} * \overline{c}$

   (i) $\overline{a}\frac{dz}{da} = \overline{a}(z(a=1) \oplus z(a=0)) = \overline{a}((\overline{c} + cb) \oplus cb)$
   $= \overline{a}(\overline{\overline{c} + cb}cb + (\overline{c} + cb)\overline{cb}) = \overline{a}(\overline{c}\overline{cb}cb + \overline{c}\overline{cb} + cb\overline{cb})$
   $= \overline{a} * \overline{c}\overline{cb} = \overline{a} * \overline{c}(\overline{c} + \overline{b}) = \overline{a} * \overline{c} + \overline{a} * \overline{c}\overline{b} = \overline{a} * \overline{c}(1 + \overline{b}) = \overline{a} * \overline{c} = 1$
   The patterns $abc = 000, 010$ can detect the fault line a s-a-1.

   (ii) $h\frac{dz}{dh} = h(z(h=1) \oplus z(h=0)) = h(1 \oplus a\overline{c}) = h\overline{a\overline{c}} = h(\overline{a} + c)$
   $= h\overline{a} + hc$
   Note that $h = cb$.
   $h\frac{dz}{dh} = \overline{a}bc + bc = bc(a+1) = bc = 1$
   The patterns $abc = 011, 111$ can detect the fault line h s-a-0.

   (iii) $\overline{h}\frac{dz}{dh} = \overline{h}\overline{a} + \overline{h}c = \overline{cb}\overline{a} + \overline{cb}c = (\overline{c} + \overline{b})\overline{a} + (\overline{c} + \overline{b})c$
   $= \overline{a}\overline{b} + \overline{a}\overline{c} + \overline{b}c + \overline{c}c = \overline{a}\overline{b} + \overline{a} * \overline{c} + \overline{b}c = 1$
   The patterns $abc = 000, 001, 010, 101$ can detect the fault line h s-a-1.

   (iv) $e\frac{dz}{de} = e(z(e=1) \oplus z(e=0)) = e((a\overline{c} + b) \oplus a\overline{c})$
   $= e(\overline{a\overline{c} + b}a\overline{c} + (a\overline{c} + b)\overline{a\overline{c}}) = e(\overline{b} * \overline{a\overline{c}}a\overline{c} + a\overline{c}\overline{a\overline{c}} + b\overline{a\overline{c}}) = eb\overline{a\overline{c}}$
   $= be(\overline{a} + c) = \overline{a}be + bec$
   Note that $e = c$.
   $e\frac{dz}{de} = \overline{a}bc + bc = bc(\overline{a} + 1) = bc = 1$
   The patterns $abc = 011, 111$ can detect the fault line e s-a-0.

   (v) $\overline{e}\frac{dz}{de} = \overline{a}b\overline{e} + b\overline{e}c = \overline{a}b\overline{c} + bc\overline{c} = \overline{a}b\overline{c}$
   The pattern $abc = 010$ can detect the fault line e s-a-1.

   (vi) $c\frac{dz}{dc} = c(z(c=1) \oplus z(c=0)) = c(b \oplus a) = c(\overline{a}b + a\overline{b}) = \overline{a}bc + a\overline{b}c = 1$
   The patterns $abc = 011, 101$ can detect the fault line c s-a-0.

7. (a) $\overline{a}\frac{di}{da} = \overline{a}(i(a=1) \oplus i(a=0)) = \overline{a}(b \oplus 0) = \overline{a}b = 1$
   The pattern $ab = 01$ can detect the fault a/1.
   (b) $\overline{d}\frac{di}{da} = \overline{d}(i(d=1) \oplus i(d=0)) = \overline{d}(ab \oplus 0) = \overline{d}ab$
   Note that $d = a$.
   $\overline{d}\frac{di}{da} = \overline{a}ab = 0 = 1$
   We have reached a contradiction, therefore the fault d/1 cannot be detected.
   (c) $\overline{g}\frac{di}{dg} = \overline{g}(i(g=1) \oplus i(g=0)) = \overline{g}(ab \oplus 0) = \overline{g}ab$
   Note that $g = ab$.
   $\overline{g}\frac{di}{dg} = \overline{ab}ab = 0 = 1$
   We have reached a contradiction, therefore the fault g/1 cannot be detected.

8. Below are the signatures calculated for the sequence $M = 10011011$ (fault-free) and the faulty sequence $M' = 11111111$

M	R_1	R_2	R_3	R_4
1	0	0	0	0
1	1	0	0	0
0	1	1	0	0
1	0	1	1	0
1	1	0	1	1
0	0	0	0	1
0	1	1	0	0
1	0	1	1	0
R	1	0	1	1

M'	R_1	R_2	R_3	R_4
1	0	0	0	0
1	1	0	0	0
1	1	1	0	0
1	1	1	1	0
1	1	1	1	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
R	1	0	1	1

The signature for the faulty sequence M' matches the signature for the fault-free sequence M, so this fault is not detected.