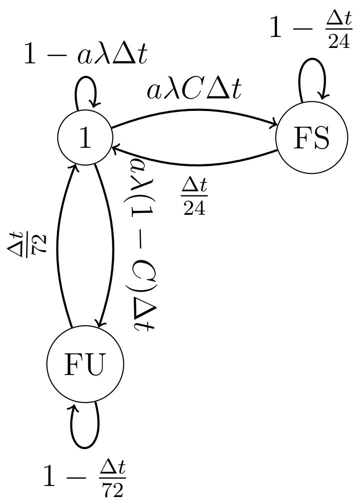
# Fault Tolerant Computing and VLSI Testing

## Assignment 3

1. The Markov Model for the system with self-diagnostics is:




- 1 is where the system is working successfully.
- FS is where a fault has been detected and the system has been safely deactivated.
- FU is where a fault has not been detected.

$$P(t) = \begin{bmatrix} P_1(t) \\ P_{FS}(t) \\ P_{FU}(t) \end{bmatrix}, P_1(t) + P_{FS}(t) + P_{FU}(t) = 1, P(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 - a\lambda\Delta t & \frac{\Delta t}{24} & \frac{\Delta t}{72} \\ a\lambda C\Delta t & 1 - \frac{\Delta t}{24} & 0 \\ a\lambda(1-C)\Delta t & 0 & 1 - \frac{\Delta t}{72} \end{bmatrix}$$

$$P(t+\Delta t) = AP(t) = \begin{bmatrix} P_1(t)(1 - a\lambda\Delta t) + P_{FS}(t)\frac{\Delta t}{24} + P_{FU}(t)\frac{\Delta t}{72} \\ P_1(t)a\lambda C\Delta t + P_{FS}(t)(1 - \frac{\Delta t}{24}) \\ P_1(t)a\lambda(1-C)\Delta t + P_{FU}(t)(1 - \frac{\Delta t}{72}) \end{bmatrix}$$

$$P_1(t+\Delta t) = P_1(t)(1 - a\lambda\Delta t) + P_{FS}(t)\frac{\Delta t}{24} + P_{FU}(t)\frac{\Delta t}{72}$$

$$P_{FS}(t+\Delta t) = P_1(t)a\lambda C\Delta t + P_{FS}(t)(1 - \frac{\Delta t}{24})$$

$$P_{FU}(t+\Delta t) = P_1(t)a\lambda(1-C)\Delta t + P_{FU}(t)(1 - \frac{\Delta t}{72})$$

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = -P_1(t)a\lambda + \frac{P_{FS}(t)}{24} + \frac{P_{FU}(t)}{72}$$

$\frac{P_{FS}(t+\Delta t)-P_{FS}(t)}{\Delta t} = P_1(t)a\lambda C - \frac{P_{FS}(t)}{24}$

$\frac{P_{FU}(t+\Delta t)-P_{FU}(t)}{\Delta t} = P_1(t)a\lambda(1-C) - \frac{P_{FU}(t)}{72}$

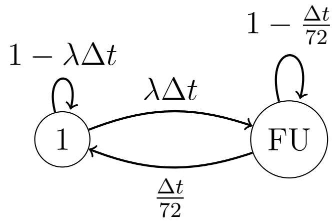
As $\Delta t \to 0$:

$\frac{dP_1(t)}{dt} = -P_1(t)a\lambda + \frac{P_{FS}(t)}{24} + \frac{P_{FU}(t)}{72}$

$\frac{dP_{FS}(t)}{dt} = P_1(t)a\lambda C - \frac{P_{FS}(t)}{24}$

$\frac{dP_{FU}(t)}{dt} = P_1(t)a\lambda(1-C) - \frac{P_{FU}(t)}{72}$

The Markov Model for the system without self-diagnostics is:




- 1 is where the system is working successfully.
- FU is where a fault has occured.

$P(t) = \begin{bmatrix} P_1(t) \\ P_{FU}(t) \end{bmatrix}, P_1(t)+P_{FU}(t) = 1, P(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 1-\lambda\Delta t & \frac{\Delta t}{72} \\ \lambda\Delta t & 1-\frac{\Delta t}{72} \end{bmatrix}$

$P(t+\Delta t) = AP(t) = \begin{bmatrix} P_1(t)(1-\lambda\Delta t) + P_{FU}(t)\frac{\Delta t}{72} \\ P_1(t)\lambda\Delta t + P_{FU}(t)(1-\frac{\Delta t}{72}) \end{bmatrix}$

$\frac{P_1(t+\Delta t)-P_1(t)}{\Delta t} = -P_1(t)\lambda + \frac{P_{FU}(t)}{72}, \frac{P_{FU}(t+\Delta t)-P_FU(t)}{\Delta t} = P_1(t)\lambda - \frac{P_{FU}(t)}{72}$

As $\Delta t \to 0$:

$\frac{dP_1(t)}{dt} = -P_1(t)\lambda + \frac{P_{FU}(t)}{72}, \frac{dP_{FU}(t)}{dt} = P_1(t)\lambda - \frac{P_{FU}(t)}{72}$

$sP_1(s) + sP_{FU}(s) = 1 \implies P_{FU}(s) = \frac{1-sP_1(s)}{s}$

$sP_1(s) - P_1(0) = sP_1(s) - 1 = -P_1(s)\lambda + \frac{P_{FU}(s)}{72} = -P_1(s)\lambda + \frac{1-sP_1(s)}{72s}$

$s^2P_1(s) - s = -sP_1(s)\lambda + \frac{1-sP_1(s)}{72}$

$s^2P_1(s) + sP_1(s)\lambda + \frac{sP_1(s)}{72} = sP_1(s)(s+\lambda+\frac{1}{72}) = s + \frac{1}{72}$

$P_1(s) = \frac{s+\frac{1}{72}}{(s)(s+\lambda+\frac{1}{72})}$

We can solve this equation via partial fractions:

$P_1(s) = \frac{s+\frac{1}{72}}{(s)(s+\lambda+\frac{1}{72})} = \frac{A}{s} + \frac{B}{s+\lambda+\frac{1}{72}} \implies A(s+\lambda+\frac{1}{72}) + Bs = s + \frac{1}{72}$

$s = 0 \implies A(\lambda+\frac{1}{72}) = \frac{1}{72} \implies A = \frac{1}{72(\lambda+\frac{1}{72})}$

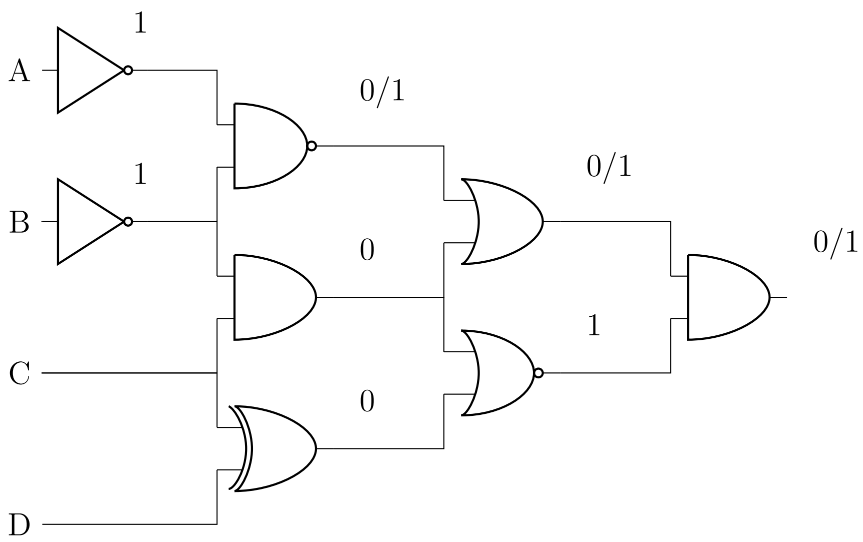
$s = -(\lambda + \frac{1}{72}) \implies -B(\frac{1}{72} + \lambda) = -\lambda \implies B = \frac{\lambda}{\frac{1}{72}+\lambda}$

$P_1(s) = \frac{1}{72(\lambda+\frac{1}{72})s} + \frac{\lambda}{(\frac{1}{72}+\lambda)(s+\lambda+\frac{1}{72})}$

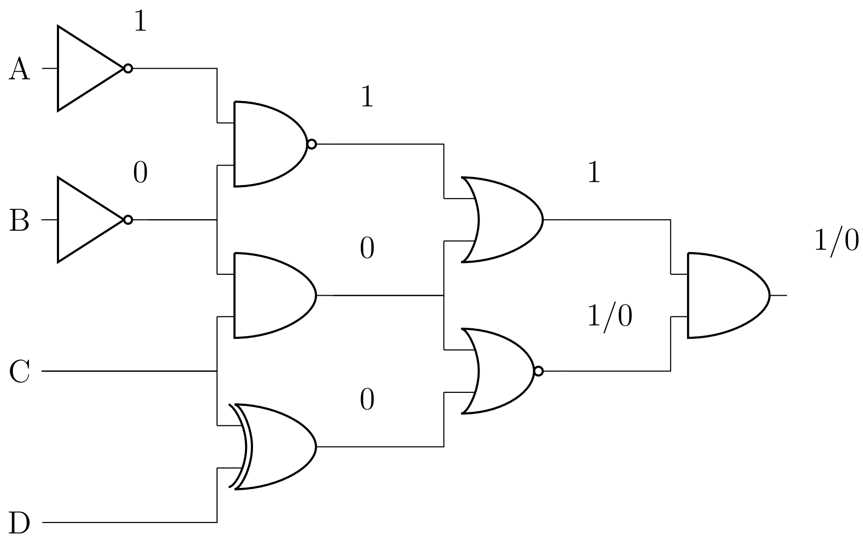
$R(t) = P_1(t) = \frac{1}{72(\lambda+\frac{1}{72})} + \frac{\lambda}{\frac{1}{72}+\lambda}e^{-(\lambda+\frac{1}{72})} = \frac{1}{72\lambda+1} + \frac{\lambda}{\frac{1}{72}+\lambda}e^{-(\lambda+\frac{1}{72})}$

2. Test patterns are written as $ABCD$

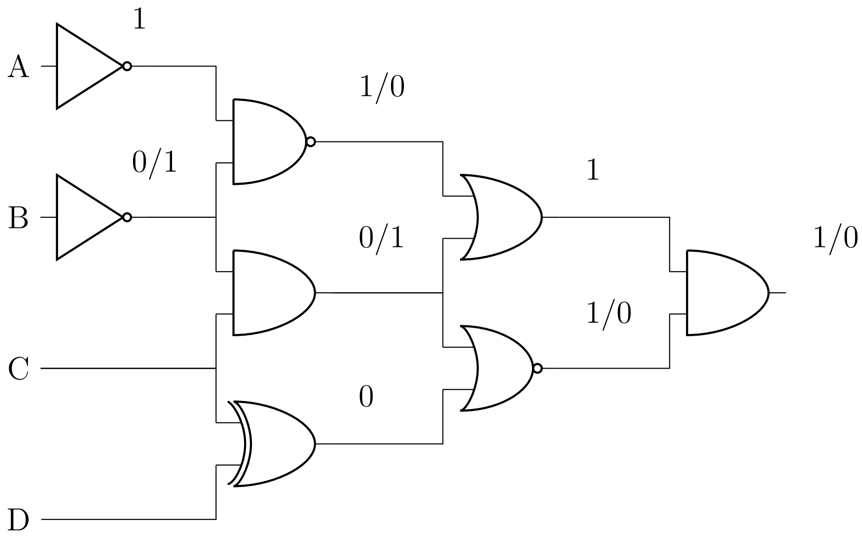
**(i)** 0000




**(ii)** 0100




**(iii)** 0111




3. $FC = 80\% \implies DL = 1 - 0.9^{1-0.8} = 0.020852 = 20852PPM$
   $FC = 90\% \implies DL = 1 - 0.9^{1-0.9} = 0.010481 = 10481PPM$
   $FC = 99\% \implies DL = 1 - 0.9^{1-0.99} = 0.001053 = 1053PPM$
   $DL = 20PPM = 2 * 10^{-5} = 1 - 0.7^{1-FC}$
   $0.6^{1-FC} = 1 - 2 * 10^{-5} = 0.99998$
   $1 - FC = \log_{0.6} 0.99998 = 3.915 * 10^{-5}$
   $FC = 1 - 3.915 * 10^{-5} = 0.99996 = 99.996\%$

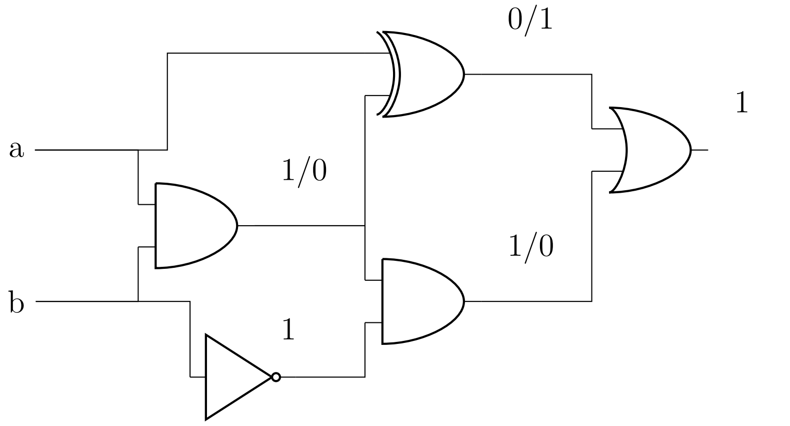
4.

| AB | 00 | 01 | 10 | 11 |
|---|---|---|---|---|
| Z | 1 | 1 | 1 | 0 |
| $P_0$ stuck open | 1 | LastZ | 1 | 0 |
| $P_0$ stuck short | 1 | 1 | 1 | $I_{DDQ}$ |
| $P_1$ stuck open | 1 | 1 | LastZ | 0 |
| $P_1$ stuck short | 1 | 1 | 1 | $I_{DDQ}$ |
| $N_0$ stuck open | 1 | 1 | 1 | LastZ |
| $N_0$ stuck short | 1 | $I_{DDQ}$ | 1 | 0 |
| $N_1$ stuck open | 1 | 1 | 1 | LastZ |
| $N_1$ stuck short | 1 | 1 | $I_{DDQ}$ | 0 |

5. Test patterns are written as *ab*

**(1)** Note that $a = 1$ in order to exercise d sa0, as that is the only input that can set the wire d to 1. Also note that $b = 0$, as otherwise the fault d sa0 will be quenched at the OR-gate.

Therefore the only input that could test this fault is 10. Here is what this input produces:




So even with this input, the fault cannot be detected. The fault is therefore redundant.

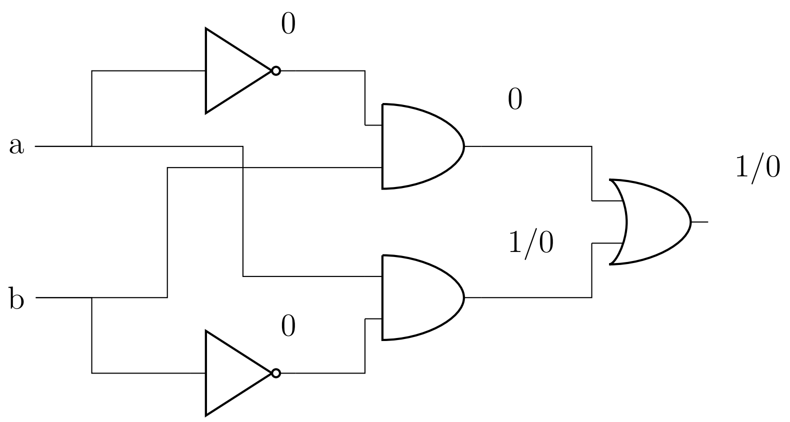
We can remove the redundancy by considering the operation of the circuit:

$a \oplus (a+b) = a\overline{(a+b)} + \overline{a}(a+b) = a\overline{a}\overline{b} + \overline{a}a + \overline{a}b = \overline{a}b$

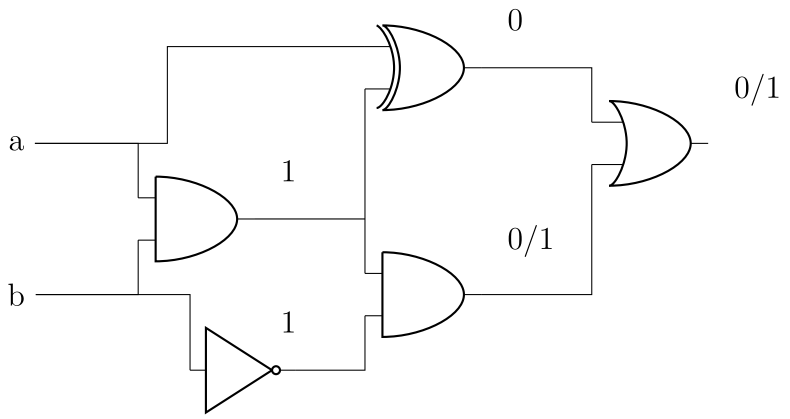
$(a+b)\overline{b} = a\overline{b} + b\overline{b} = a\overline{b}$

$(a \oplus (a+b)) + (a+b)\overline{b} = \overline{a}b + a\overline{b}$

We can now detect the fault using the input 11:



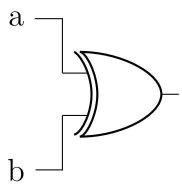

**(2)** 10:




As stated above, the operation is equivalent to:

$\overline{a}b + a\overline{b} = a \oplus b$

Therefore, the boolean operation this circuit is equivalent to is exclusive-OR. The minimum implementation is:




6. Test patterns are written as $abc$.

   Not that a * symbol is used where appropriate to represent the AND operation for clarity, in order to help distinguish between, for example, $\overline{a * c}$ and $\overline{a} * \overline{c}$

   **(i)** $\overline{a}\frac{dz}{da} = \overline{a}(z(a=1) \oplus z(a=0)) = \overline{a}((\overline{c} + cb) \oplus cb)$
   $= \overline{a}(\overline{\overline{c} + cb}cb + (\overline{c} + cb)\overline{cb}) = \overline{a}(\overline{\overline{c}cb}cb + \overline{c}\overline{cb} + cb\overline{cb})$
   $= \overline{a} * \overline{c}\overline{cb} = \overline{a} * \overline{c}(\overline{c} + \overline{b}) = \overline{a} * \overline{c} + \overline{a} * \overline{c}\overline{b} = \overline{a} * \overline{c}(1 + \overline{b}) = \overline{a} * \overline{c} = 1$
   The patterns $\{000, 010\}$ can detect the fault.

   **(ii)** $h\frac{dz}{dh} = h(z(h=1) \oplus z(h=0)) = h(1 \oplus a\overline{c}) = h\overline{a\overline{c}} = h(\overline{a} + c)$
   $= h\overline{a} + hc$
   Note that $h = cb$.
   $h\frac{dz}{dh} = \overline{a}bc + bc = bc(a+1) = bc = 1$
   The patterns $\{011, 111\}$ can detect the fault.

   **(iii)** $\overline{h}\frac{dz}{dh} = \overline{h}\overline{a} + \overline{h}c = \overline{cb}\overline{a} + \overline{cb}c = (\overline{c} + \overline{b})\overline{a} + (\overline{c} + \overline{b})c$
   $= \overline{a}\overline{b} + \overline{ac} + \overline{b}c + \overline{c}c = \overline{ac} + \overline{ac} + \overline{b}c = 1$
   The patterns $\{000, 001, 010, 101\}$ can detect the fault.

**(iv)** $e\frac{dz}{de} = e(z(e=1) \oplus z(e=0)) = e((a\overline{c}+b) \oplus a\overline{c})$
$= e(\overline{a\overline{c}+b}a\overline{c} + (a\overline{c}+b)\overline{a\overline{c}}) = e(\overline{b} * \overline{a\overline{c}}a\overline{c} + a\overline{c}\overline{a\overline{c}} + b\overline{a\overline{c}}) = eb\overline{a\overline{c}}$
$= be(\overline{a}+c) = \overline{a}be + bec$
Note that $e = c$.
$e\frac{dz}{de} = \overline{a}bc + bc = bc(\overline{a}+1) = bc = 1$
The patterns $\{011, 111\}$ can detect the failure.

**(v)** $\overline{e}\frac{dz}{de} = \overline{a}b\overline{e} + b\overline{e}c = \overline{a}b\overline{c} + bc\overline{c} = \overline{a}b\overline{c}$
The pattern $\{010\}$ can detect the failure.

**(vi)** $c\frac{dz}{dc} = c(z(c=1) \oplus z(c=0)) = c(b \oplus a) = c(\overline{a}b + a\overline{b}) = \overline{a}bc + a\overline{b}c = 1$
The patterns $\{011, 101\}$ can detect the fault.

7. Test patterns are written as $ab$.

   (a) $\overline{a}\frac{di}{da} = \overline{a}(i(a=1) \oplus i(a=0)) = \overline{a}(b \oplus 0) = \overline{a}b = 1$
   The pattern $\{01\}$ can detect the fault.

   (b) $\overline{d}\frac{di}{da} = \overline{d}(i(d=1) \oplus i(d=0)) = \overline{d}(ab \oplus 0) = \overline{d}ab$
   Note that $d = a$.
   $\overline{d}\frac{di}{da} = \overline{a}ab = 0 = 1$
   We have reached a contradiction, therefore this fault cannot be detected.

   (c) $\overline{g}\frac{di}{dg} = \overline{g}(i(g=1) \oplus i(g=0)) = \overline{g}(ab \oplus 0) = \overline{g}ab$
   Note that $g = ab$.
   $\overline{g}\frac{di}{dg} = \overline{ab}ab = 0 = 1$
   We have reached a contradiction, therefore this fault cannot be detected.

8. Below are the signatures calculated for the sequence $M = 10011011$ (fault-free) and the faulty sequence $M' = 11111111$

| $M$ | 1 | $x$ | $x^2$ | $x^3$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| $R$ | 1 | 0 | 1 | 1 |

| $M'$ | 1 | $x$ | $x^2$ | $x^3$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| $R$ | 1 | 0 | 1 | 1 |

The signatures calculated for the faulty sequence $M'$ matches the fault-free sequence $M$, so this fault is not detected.