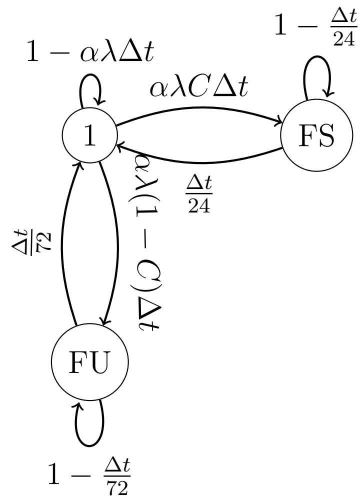
Fault Tolerant Computing and VLSI Testing

Assignment 3

1. The Markov Model for the system with self-diagnostics is:



- 1 is where the system is working successfully.
- FS is where a fault has been detected and the system has been safely deactivated.
- FU is where a fault has not been detected.

The Mean Time To Failure (MTTF) will be the amount of time we expect to stay in the working state. This is the value of t such that $t\alpha\lambda = 1$. This is equal to $t = \frac{1}{\lambda\alpha}$

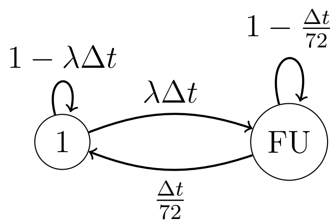
We detect the failure with probability 0.95, and don't with probability 0.05. This gives us a mean time between failures of:

$0.95 * 24 + 0.05 * 72 = 26.4$

Thus, the steady-state availability is:

$A = \frac{1}{\alpha\lambda(\frac{1}{\alpha\lambda}+26.4)} = \frac{1}{1+26.4\alpha\lambda}$

The Markov Model for the system without self-diagnostics is:

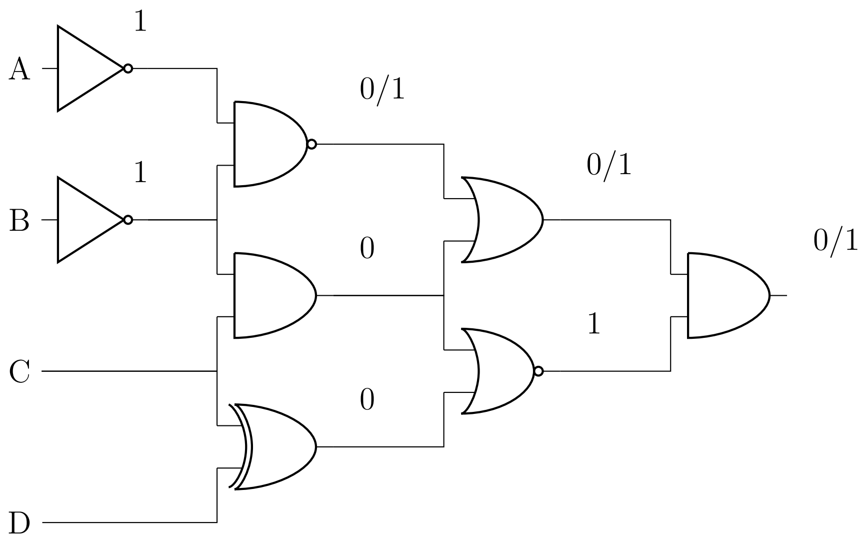


- 1 is where the system is working successfully.
- FU is where a fault has occured.

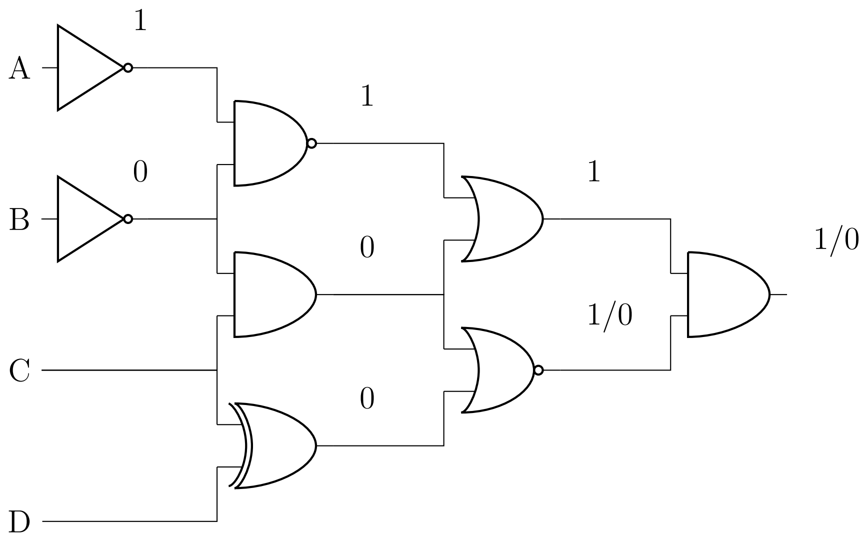
The MTTF is $\frac{1}{\lambda}$ and the steady-state availability is:
$A = \frac{1}{\lambda(\frac{1}{\lambda}+72)} = \frac{1}{1+72\lambda}$
We can find the value of α for which self-diagnostics begins to degrade the steady-state availability by equating these two equations:
$\frac{1}{1+26.4\alpha\lambda} = \frac{1}{1+72\lambda} \implies 1 + 26.4\alpha\lambda = 1 + 72\lambda \implies 26.4\alpha\lambda = 72\lambda$
$\alpha = \frac{72}{26.4} = 2.727$

2. Test patterns are written as $ABCD$

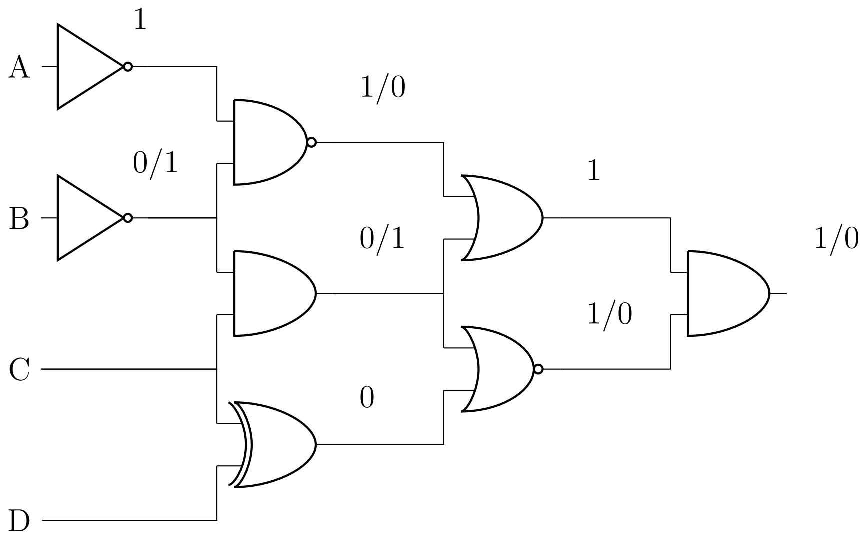
(i) 0000



(ii) 0100



(iii) 0111



3. $FC = 80\% \implies DL = 1 - 0.9^{1-0.8} = 0.020852 = 20852PPM$

   $FC = 90\% \implies DL = 1 - 0.9^{1-0.9} = 0.010481 = 10481PPM$

   $FC = 99\% \implies DL = 1 - 0.9^{1-0.99} = 0.001053 = 1053PPM$

   $DL = 20PPM = 2 * 10^{-5} = 1 - 0.7^{1-FC}$

   $0.6^{1-FC} = 1 - 2 * 10^{-5} = 0.99998$

   $1 - FC = \log_{0.6} 0.99998 = 3.915 * 10^{-5}$

   $FC = 1 - 3.915 * 10^{-5} = 0.99996 = 99.996\%$

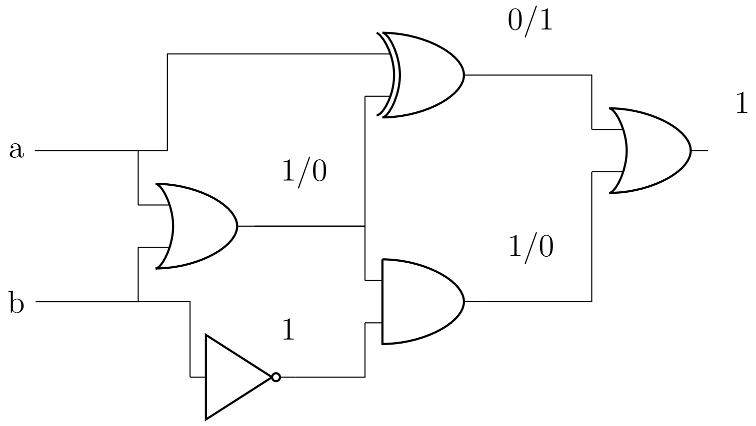
4.

AB	00	01	10	11
Z	1	1	1	0
P_0 stuck open	1	LastZ	1	0
P_0 stuck short	1	1	1	I_{DDQ}
P_1 stuck open	1	1	LastZ	0
P_1 stuck short	1	1	1	I_{DDQ}
N_0 stuck open	1	1	1	LastZ
N_0 stuck short	1	I_{DDQ}	1	0
N_1 stuck open	1	1	1	LastZ
N_1 stuck short	1	1	I_{DDQ}	0

5. Test patterns are written as ab

(1) Note that $a = 1$ in order to exercise d sa0, as that is the only input that can set the wire d to 1. Also note that $b = 0$, as otherwise the fault d sa0 will be quenched at the OR-gate.

Therefore the only input that could test this fault is 10. Here is what this input produces:



So even with this input, the fault cannot be detected. The fault is therefore redundant.

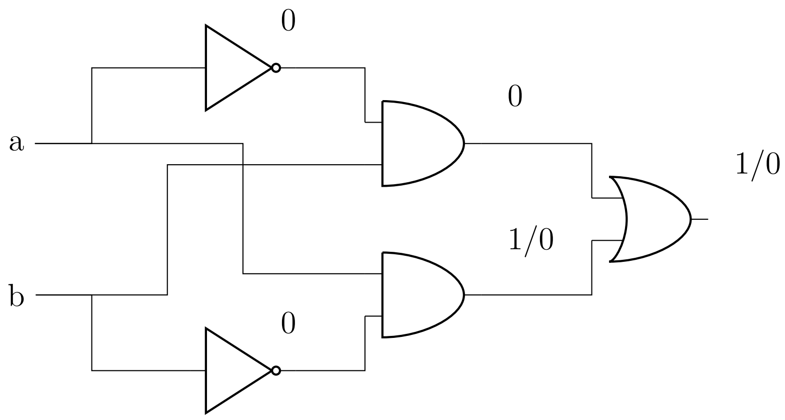
We can remove the redundancy by considering the operation of the circuit:

$a \oplus (a+b) = a\overline{(a+b)} + \overline{a}(a+b) = a\overline{a}\overline{b} + \overline{a}a + \overline{a}b = \overline{a}b$

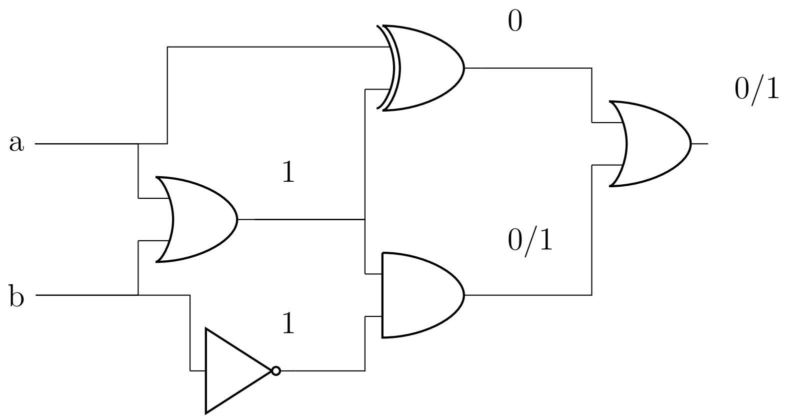
$(a+b)\overline{b} = a\overline{b} + b\overline{b} = a\overline{b}$

$(a \oplus (a+b)) + (a+b)\overline{b} = \overline{a}b + a\overline{b}$

We can now detect the fault using the input 11:



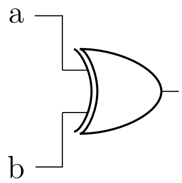
(2) 10:



As stated above, the operation is equivalent to:

$\overline{a}b + a\overline{b} = a \oplus b$

Therefore, the boolean operation this circuit is equivalent to is exclusive-OR. The minimum implementation is:



6. Test patterns are written as abc.

   Not that a * symbol is used where appropriate to represent the AND operation for clarity, in order to help distinguish between, for example, $\overline{a * c}$ and $\overline{a} * \overline{c}$

   (i) $\overline{a}\frac{dz}{da} = \overline{a}(z(a=1) \oplus z(a=0)) = \overline{a}((\overline{c} + cb) \oplus cb)$

$= \overline{a}(\overline{\overline{c} + cb}cb + (\overline{c} + cb)\overline{cb}) = \overline{a}(\overline{\overline{c}}\overline{cb}cb + \overline{\overline{c}}\overline{cb} + cb\overline{cb})$
$= \overline{a} * \overline{c}\overline{cb} = \overline{a} * \overline{c}(\overline{c} + \overline{b}) = \overline{a} * \overline{c} + \overline{a} * \overline{c}\overline{b} = \overline{a} * \overline{c}(1 + \overline{b}) = \overline{a} * \overline{c} = 1$
The patterns $\{000, 010\}$ can detect the fault.

(ii) $h\frac{dz}{dh} = h(z(h = 1) \oplus z(h = 0)) = h(1 \oplus a\overline{c}) = h\overline{a\overline{c}} = h(\overline{a} + c)$
$= h\overline{a} + hc$
Note that $h = cb$.
$h\frac{dz}{dh} = \overline{a}bc + bc = bc(a + 1) = bc = 1$
The patterns $\{011, 111\}$ can detect the fault.

(iii) $\overline{h}\frac{dz}{dh} = \overline{h}\overline{a} + \overline{h}c = \overline{cb}\overline{a} + \overline{cb}c = (\overline{c} + \overline{b})\overline{a} + (\overline{c} + \overline{b})c$
$= \overline{a}\overline{b} + \overline{a}\overline{c} + \overline{b}c + \overline{c}c = \overline{a}\overline{c} + \overline{a}\overline{c} + \overline{b}c = 1$
The patterns $\{000, 001, 010, 101\}$ can detect the fault.

(iv) $e\frac{dz}{de} = e(z(e = 1) \oplus z(e = 0)) = e((a\overline{c} + b) \oplus a\overline{c})$
$= e(\overline{a\overline{c} + b}a\overline{c} + (a\overline{c} + b)\overline{a\overline{c}}) = e(\overline{b} * \overline{a\overline{c}}a\overline{c} + a\overline{c}\overline{a\overline{c}} + b\overline{a\overline{c}}) = eb\overline{a\overline{c}}$
$= be(\overline{a} + c) = \overline{a}be + bec$
Note that $e = c$.
$e\frac{dz}{de} = \overline{a}bc + bc = bc(\overline{a} + 1) = bc = 1$
The patterns $\{011, 111\}$ can detect the failure.

(v) $\overline{e}\frac{dz}{de} = \overline{a}b\overline{e} + b\overline{e}c = \overline{a}b\overline{c} + bc\overline{c} = \overline{a}b\overline{c}$
The pattern $\{010\}$ can detect the failure.

(vi) $c\frac{dz}{dc} = c(z(c = 1) \oplus z(c = 0)) = c(b \oplus a) = c(\overline{a}b + a\overline{b}) = \overline{a}bc + a\overline{b}c = 1$
The patterns $\{011, 101\}$ can detect the fault.

7. Test patterns are written as ab.

(a) $\overline{a}\frac{di}{da} = \overline{a}(i(a = 1) \oplus i(a = 0)) = \overline{a}(b \oplus 0) = \overline{a}b = 1$
The pattern $\{01\}$ can detect the fault.

(b) $\overline{d}\frac{di}{da} = \overline{d}(i(d = 1) \oplus i(d = 0)) = \overline{d}(ab \oplus 0) = \overline{d}ab$
Note that $d = a$.
$\overline{d}\frac{di}{da} = \overline{a}ab = 0 = 1$
We have reached a contradiction, therefore this fault cannot be detected.

(c) $\overline{g}\frac{di}{dg} = \overline{g}(i(g = 1) \oplus i(g = 0)) = \overline{g}(ab \oplus 0) = \overline{g}ab$
Note that $g = ab$.
$\overline{g}\frac{di}{dg} = \overline{ab}ab = 0 = 1$
We have reached a contradiction, therefore this fault cannot be detected.

8. Below are the signatures calculated for the sequence $M = 10011011$ (fault-free) and the faulty sequence $M' = 11111111$

M	1	x	x^2	x^3	M'	1	x	x^2	x^3
1	0	0	0	0	1	0	0	0	0
1	1	0	0	0	1	1	0	0	0
0	1	1	0	0	1	1	1	0	0
1	0	1	1	0	1	1	1	1	0
1	1	0	1	1	1	1	1	1	1
0	0	0	0	1	1	0	0	1	1
0	1	1	0	0	1	0	1	0	1
1	0	1	1	0	1	0	1	1	0
R	1	0	1	1	R	1	0	1	1

The signatures calculated for the faulty sequence M' matches the fault-free sequence M, so this fault is not detected.