



DEPARTMENT OF COMPUTER SCIENCE

Dictionary Matching with Fingerprints

An Empirical Analysis

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of Master of Engineering in the Faculty of Engineering.

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Declaration

This dissertation is submitted to the University of Bristol in accordance with the requirements of the degree of MEng in the Faculty of Engineering. It has not been submitted for any other degree or diploma of any examining body. Except where specifically acknowledged, it is all the work of the Author.

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Executive Summary

A compulsory section, of at most 1 page

This section should précis the project context, aims and objectives, and main contributions and achievements; the same section may be called an abstract elsewhere. The goal is to ensure the reader is clear about what the topic is, what you have done within this topic, *and* what your view of the outcome is.

The former aspects should be guided by your specification: essentially this section is a (very) short version of what is typically the first chapter. The latter aspects should be presented as a concise, factual bullet point list. The points will of course differ for each project, but an example is as follows:

- I spent 120 hours collecting material on and learning about the Java garbage-collection sub-system.
- I wrote a total of 5000 lines of source code, comprising a Linux device driver for a robot (in C) and a GUI (in Java) that is used to control it.
- I designed a new algorithm for computing the non-linear mapping from A-space to B-space using a genetic algorithm, see page 17.
- I implemented a version of the algorithm proposed by Jones and Smith in [6], see page 12, corrected a mistake in it, and compared the results with several alternatives.

Supporting Technologies

- I used the GNU Multiple Precision Arithmetic Library (GMP) to support my implementation of Karp-Rabin fingerprints.
- I used the C Minimum Perfect Hashing Library (CMPH) for static perfect hashing.
- I used an open-source implementation of Red-Black Trees from [http://en.literateprograms.org/Red-black_tree_\(C\)?oldid=19567](http://en.literateprograms.org/Red-black_tree_(C)?oldid=19567), with some minor adaptations.

Notation and Acronyms

CMPH	:	C Minimum Perfect Hashing Library
GMP	:	GNU Multiple Precision Arithmetic Library
VO	:	A Viable Occurance, a portion of the text which might match a pattern
T	:	A text string of n characters
t_i	:	The i -th character in T
\mathcal{P}	:	A list of k patterns
P_i	:	The i -th pattern in \mathcal{P} , a text string of m_i characters
M	:	A list of the length of each pattern in \mathcal{P} .
$p_{i,j}$:	The j -th character in P_i
$\phi(T)$:	The Karp-Rabin fingerprint of a text string T
ρ_T	:	The period of a text string T

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Chapter 1

Contextual Background

A compulsory chapter, of roughly 10 pages

This chapter should describe the project context, and motivate each of the proposed aims and objectives. Ideally, it is written at a fairly high-level, and easily understood by a reader who is technically competent but not an expert in the topic itself.

In short, the goal is to answer three questions for the reader. First, what is the project topic, or problem being investigated? Second, why is the topic important, or rather why should the reader care about it? For example, why there is a need for this project (e.g., lack of similar software or deficiency in existing software), who will benefit from the project and in what way (e.g., end-users, or software developers) what work does the project build on and why is the selected approach either important and/or interesting (e.g., fills a gap in literature, applies results from another field to a new problem). Finally, what are the central challenges involved and why are they significant?

The chapter should conclude with a concise bullet point list that summarises the aims and objectives. For example:

The high-level objective of this project is to reduce the performance gap between hardware and software implementations of modular arithmetic. More specifically, the concrete aims are:

1. Research and survey literature on public-key cryptography and identify the state of the art in exponentiation algorithms.
2. Improve the state of the art algorithm so that it can be used in an effective and flexible way on constrained devices.
3. Implement a framework for describing exponentiation algorithms and populate it with suitable examples from the literature on an ARM7 platform.
4. Use the framework to perform a study of algorithm performance in terms of time and space, and show the proposed improvements are worthwhile.

Chapter 2

Technical Background

2.1 Pattern Matching: Formal Definitions

Pattern matching with a single pattern is a simple problem to describe intuitively: We have a text and a pattern, and we want to output any indexes where the pattern occurs in the text.

More formally, we refer to the text by T , and define it as a string of n characters $t_0...t_{n-1}$. Likewise, the pattern is referred to as P , and is a string of m characters $p_0...p_{m-1}$. The aim of the text indexing problem is to output indexes $i \in \{m-1, ..., n-1\}$ such that $t_{i-m+1}...t_i = P$.

It is worth noting that there are many other ways of defining this problem. The most notable differences in this paper are that the text and pattern are indexed at zero instead of one, and that the index at the end of the pattern's occurrence is returned instead of the index at the start. Both of these are done to be intentionally to be consistent with the code implemented: The zero-indexing is because the implementations are written in C, which also uses zero indexing, and reporting the index at the end of the occurrence is to cater for the streaming model, on which more information will be provided later.

2.1.1 Dictionary Matching: Formal Definitions

Like pattern matching, dictionary matching is also simple to describe intuitively: We have one text as before, but now we have multiple patterns, and we want to output any indexes where a pattern occurs in the text.

Formally, this is defined as follows: We have a text n characters long $T = t_0...t_{n-1}$, and a set of k patterns $\mathcal{P} = \{P_0, ..., P_k\}$ of respective lengths $M = \{m_0, ..., m_k\}$. Hence a given pattern P_i is a string of m_i characters $p_{i,0}...p_{i,m_i-1}$. We output an index $j \in \{\min(M), ..., n_1\}$ if $\exists i \in \{0, ..., k-1\}$ such that $t_{j-m_i+1}...t_j = P_i$.

Note that for this work, we do not care about what pattern has occurred in the text, only that a pattern has occurred. This is due to a limitation with one of the algorithms, which will be discussed later.

2.2 The Streaming Model

Data streaming is a way of reducing space consumption for certain problems. Under this model, required space is reduced by not processing the entire problem input at once. Instead, the input is provided to the algorithm in portions, delivered via a stream of data. The algorithm processes one portion of the input at a time, and it is required that the algorithm is not allowed to store the entire input.

Under this model, we measure performance by two properties:

- **Space:** The size of the data structure
- **Time:** The time taken to process each portion in the stream

It is easy to see how pattern matching and in turn dictionary matching can be performed in this model. We can process the text by individual characters. During preprocessing we store the pattern and initialise a circular buffer buf which is m characters long. At index j when we receive character t_j we perform the algorithm described in Algorithm 2.1. A dictionary matching variant can be done by storing a circular buffer which is $\max(M)$ characters long and repeating Algorithm 2.1 k times. These algorithms use $O(m)$ and $O(\sum_{i=0}^{k-1} m_i)$ respectively, both in terms of space and time per character.

```

rotate buf by one
append  $t_i$  to buf for  $i = 0$  upto  $m - 1$  do
  if  $buf_i \neq p_i$  then
    return -1
  end
end
return j

```

Algorithm 2.1: A naïve solution to single pattern matching.

```

newstate  $\leftarrow -1$ 
for  $i = 0$  upto  $k - 1$  do
  state  $\leftarrow -1$ 
  j  $\leftarrow 0$ 
  while goto(state,  $p_{i,j}$ )  $\neq fail$  do
    state  $\leftarrow$  goto(state,  $p_{i,j}$ )
    j  $\leftarrow j + 1$ 
  end
  while j  $< m_i$  do
    newstate  $\leftarrow$  newstate + 1
    goto(state,  $p_{i,j}$ )  $\leftarrow$  newstate
    state  $\leftarrow$  newstate
    j  $\leftarrow j + 1$ 
  end
  output(state) =  $\{P_i\}$ 
end
forall the  $a \in \Sigma$  such that goto( $-1, a$ ) = fail do
  goto( $-1, a$ ) =  $-1$ 
end

```

Algorithm 2.2: Constructing the goto function for Aho-Corasick.

Of course, these are poor solutions to both pattern and dictionary matching. We can do much better in terms of both time and space complexity.

2.3 The Aho-Corasick Algorithm for Dictionary Matching

The Aho-Corasick Algorithm for Efficient String Matching[1] – known hereafter as Aho-Corasick – is a deterministic algorithm for dictionary matching. Published in 1975, the algorithm works as a generalisation of Knuth-Morris-Pratt, extending the state machine from single patterns in KMP to multiple patterns.

Preprocessing consists of three algorithms. The first, Algorithm 2.2, produces the goto function, which determines what to do if the next character in the stream is a match. This in essence works by building a suffix tree: We traverse the tree until we either reach the end of the pattern or we hit a leaf, and then append the rest of the pattern to the leaf. Note that Σ refers to the alphabet of the patterns and *fail* is a default fail state for if the goto function cannot find a character for that state.

The second, Algorithm 2.3 constructs the failure function for when the next character cannot be found and the output function for whether or not there is a match. This is similar to how the failure table is computed in Knuth-Morris-Pratt, by using previously computed failure tables to find the longest prefix that is also a suffix of that point in the pattern.

From these two algorithms alone it is possible to perform dictionary matching, using a computation method again similar to Knuth-Morris-Pratt: For each character t_j in the text when we are in state s , we check if $\text{goto}(s, t_j) = \text{fail}$. If that is the case, we call $s \leftarrow \text{failure}(s)$ repeatedly until the previous check no longer holds. We then update our state $s \leftarrow \text{goto}(s, t_j)$, and if $\text{output}(s) \neq \text{empty}$ then we return j , otherwise we return -1 . This runs in amortised $O(|\Sigma|)$ time per character, and worst case $O(|\Sigma| \max(M))$ time per character, as can be seen via Knuth-Morris-Pratt arguments.

To improve on this running time, Algorithm 2.4 is used, which combines the goto and failure functions to produce a next function, which given any state and character returns the next state. Computation now simply becomes as each character t_j comes in when we are in state s , call $s \leftarrow \text{next}(s, t_j)$ and return j if $\text{output}(s) \neq \text{empty}$. This runs in worst case $O(|\Sigma|)$ time per character, where the bottleneck is finding

```

queue ← empty
foreach a ∈ Σ such that goto(-1, a) = s ≠ -1 do
    queue ← queue ∪ {s}
    failure(s) ← -1
end
while queue ≠ empty do
    r ← pop(queue)
    foreach a ∈ Σ such that goto(r, a) = s ≠ fail do
        queue ← queue ∪ s
        state ← failure(r)
        while goto(state, a) = fail do
            state ← failure(state)
        end
        failure(s) ← goto(state, a)
        output(s) ← output(s) ∪ output(failure(s))
    end
end
end

```

Algorithm 2.3: Constructing the failure and output functions for Aho-Corasick.

```

queue ← empty
foreach a ∈ Σ do
    next(-1, a) = goto(-1, a)
    if goto(-1, a) ≠ -1 then
        queue ← queue ∪ {goto(-1, a)}
    end
end
while queue ≠ empty do
    r ← pop(queue)
    foreach a ∈ Σ do
        if goto(r, a) = s ≠ fail then
            queue ← queue ∪ s
            next(r, a) = s
        end
        else
            next(r, a) = next(failure(r), a)
        end
    end
end
end

```

Algorithm 2.4: Constructing the next function for Aho-Corasick.

the value associated with character t_j in the next function. In both the case with the goto and failure functions and the case with only the next function, space complexity is $O(\sum_{i=0}^{k-1} m_i)$.

2.3.1 An Alternative: The Commentz-Walter Algorithm

Much like how Aho-Corasick is an algorithm for dictionary matching based on Knuth-Morris-Pratt, Commentz-Walter[5] is an algorithm based on Boyer-Moore algorithm, using similar techniques to Aho-Corasick to convert the algorithm from single pattern to multiple patterns. While it is interesting to note as an alternative, particularly because of its time improvement on average cases and the fact that a variant of it is used in the GNU command **grep**,¹ it is not implemented in this project. This is because, like Boyer-Moore, the Commentz-Walter algorithm skips indexes in the text, which is not possible in the streaming model.

¹See <http://git.savannah.gnu.org/cgit/grep.git/tree/src/kwset.c>

2.4 Minimal Perfect Hashing

For a universe U , a hash function h is static if it can perform lookups for a pre-defined set of keys $S \subseteq U$ to a set of integers \mathbb{Z}_m . Said hash function is a static *perfect* hash function if $\forall x \in S, h(x)$ is collision-free, and thus takes constant time to look up. Finally, a hash function is a *minimal* perfect hash function if $m = |S|$. In other words, a minimal perfect hash function maps a set of m keys to \mathbb{Z}_m without any collisions.

The implementation of minimal perfect hash functions will not be detailed here, as they are used merely as a library and are thus not part of implementation. For further information, I direct the reader to the C Minimal Perfect Hashing Library (CMPH) website: <http://cmph.sourceforge.net/>. Of particular interest is the paper on the Compress, Hash and Digest algorithm by Belazzougui, Botelho and Dietzfelbinger[2], as the algorithm from CMPH used throughout this work.

2.5 Karp-Rabin Fingerprints

Karp-Rabin fingerprints[6] are a function $\phi : \Sigma^* \rightarrow \mathbb{Z}_p$ for some prime number p . For a text T of length n characters, the Karp-Rabin fingerprint is defined as:

$$\phi(T) = \sum_{i=0}^{n-1} r^i t_i \mod p$$

Where p is a prime number, and r is a random number such that $1 < r < p$. Alongside the fingerprint $\phi(T)$, we store $r^n \mod p$ and $r^{-n} \mod p$ in a tuple. Using these three properties, we can manipulate the fingerprints to affect the underlying strings in three ways[7]. Note that all equations listed below are modulo p .

- **Concatenate:** If we have a fingerprint $\{\phi(u), r^{n_1}, r^{-n_1}\}$ for a string u of length n_1 and another fingerprint $\{\phi(v), r^{n_2}, r^{-n_2}\}$ for a string v of length n_2 , the concatenation of these two strings is $\{\phi(u) + \phi(v) * r^{n_1}, r^{n_1} * r^{n_2}, r^{-n_1} * r^{-n_2}\}$
- **Prefix:** If we have a fingerprint $\{\phi(uv), r^{n_1}, r^{-n_1}\}$ for a string uv of length n_1 and a fingerprint $\{\phi(v), r^{n_2}, r^{-n_2}\}$ for the n_2 suffix of uv , then we can work out the fingerprint of the $n_1 - n_2$ prefix of uv as $\{\phi(uv) - \phi(v) * r^{n_1}, r^{n_1} * r^{-n_2}, r^{-n_1} * r^{n_2}\}$
- **Suffix:** If we have a fingerprint $\{\phi(uv), r^{n_1}, r^{-n_1}\}$ for a string uv of length n_1 and a fingerprint $\{\phi(u), r^{n_2}, r^{-n_2}\}$ for the n_2 prefix of uv , then we can work out the fingerprint of the $n_1 - n_2$ suffix of uv as $\{(\phi(uv) - \phi(u)) * r^{-n_2}, r^{n_1} * r^{-n_2}, r^{-n_1} * r^{n_2}\}$

All of these operations can be completed in constant time.

It is interesting to note that a variant of the Karp-Rabin algorithm can be used for a subset of dictionary matching, where all the patterns are the same length m [4, pp 205-206]. This can be done by storing a fingerprint of the last m characters read from the text, and using static perfect hashing as described in section 2.4 to check if the fingerprint of the text matches any fingerprints of the patterns. Using suffix and concatenation techniques above and storing a circular buffer of the last m characters, we can accomplish this with $O(k + m)$ space and $O(1)$ time per character. However, due to the limitation that all the patterns have to be the same length, this method has not been implemented for this project.

The last point to mention is the probability of a collision. Breslauer and Galil[3] provide a theorem that if u and v are two different strings of length $l \leq n$, $p \in \theta(n^{2+\alpha})$ for some level of accuracy $\alpha \geq 0$ and $r \in \mathbb{Z}_p$ is randomly chosen, then the probability that $\phi(u) = \phi(v)$ is smaller than $\frac{1}{n^{1+\alpha}}$. We will however see later why this does not necessarily hold for the dictionary matching algorithm devised by Clifford et al..

2.6 Porat and Porat: Single Pattern Matching in Sublinear Space

In 2009, Porat and Porat[7] provided the first solution to a pattern matching problem in sublinear space to the size of the pattern. Utilising Karp-Rabin fingerprints as described in the previous section, their randomised algorithm for single pattern matching in the streaming model had $O(\log m)$ complexity both in terms of space and time per character.

Level number	1	2	3
VO locations stored	5	3,4	-1,0,1,2

Figure 2.1: Example state of VO lists after 7 characters, where $T = aaaaaaa$ and $P = aaaaaaa$

Detailed below is not Porat and Porat’s algorithm itself, but a variant of it developed by Breslauer and Galil in 2014[3]. The two algorithms can be seen as computationally equivalent.

Instead of storing the entire pattern in a single fingerprint, the pattern is broken up into $\lfloor \log_2 m \rfloor$ fingerprints, each a power of two prefix of the patter. These fingerprints denoted ϕ_i , are computed as follows:

$$\phi_i = \phi(p_0 \dots p_{2^i-1})$$

If the pattern is not a power of two in length, the remaining characters can be stored either in the fingerprint of the final prefix $\phi_{\lfloor \log_2 m \rfloor}$ or in a new final level, $\phi_{\lceil \log_2 m \rceil}$.

These fingerprints can be created in a streaming fashion, so each character of the pattern only needs to be read once. This can be done via dynamic programming, concatenating the current row with the fingerprint of the already computed previous row:

$$\phi_i = \begin{cases} \phi(p_0), & \text{if } i = 0 \\ \text{Concatenate}(\phi_{i-1}, \phi(p_{2^{i-1}} \dots p_{2^i-1})), & \text{otherwise} \end{cases}$$

With this structure, we can now look at what we compute as each character of the text enters our stream. When t_j enters the stream, we first compute the fingerprint $\phi(t_j)$, update our fingerprint of the text read so far $\phi(t_0 \dots t_j)$ and check if $\phi(t_j) = \phi_0$. If this case is true, we have what is referred to as a viable occurrence (VO) for level 1. When we have a VO at level 1 after character $\phi(t_j)$ has entered the stream, we store two properties: $j - 1$ and $\phi(t_0 \dots t_{j-1})$ ² in a list of viable occurrences for level 1.

After performing the above, we retrieve the oldest VO we have stored at level 1, which has properties j' and $\phi(t_0 \dots t_{j'-1})$. If $j - j' = 2$, we now know that enough characters have passed for us to be able to check if this viable occurrence requires promotion. We remove this occurrence from our list of VOs for level 1 and use the fingerprint suffix operation on our fingerprint of the text and $\phi(t_0 \dots t_{j'-1})$ to retrieve $\phi(t_{j'} \dots t_j)$. We then check if $\phi(t_{j'} \dots t_j) = \phi_1$ and if this is the case, we promote this occurrence by storing j' and $\phi(t_0 \dots t_{j'-1})$ in a list of viable occurrences for level 2. Otherwise, we discard the occurrence.

We repeat the above process $\log_2 m$ times per character. At the i -th level, we check if the oldest VO occurred 2^i characters back and if so, we then check if the fingerprint of the last 2^i characters matches the fingerprint of the 2^i prefix of the pattern. If they match, we promote this occurrence to the $i + 1$ -th level. At the final level, we check if the oldest VO for this level occurred m characters ago. If so, we check if the fingerprint of the last m characters of the text matches the fingerprint of the whole pattern. If they do match, then a match is reported at index j , where t_j was the last character read.

This algorithm gives us $O(\log m)$ time per character, but the space complexity is still linear. This can be easily seen if the text and pattern are both strings of the letter a . After 6 characters, the list of viable occurrences for each level would look like similar to the example given in Figure ?? . Note that level 0 is not included in the aforementioned figure as there are no VOs stored for that level.

At level i , we have to store up to 2^{i-1} viable occurrences. The final row has to store at most $\frac{m}{2}$ viable occurrences. Storing these VOs naïvely will result in $1 + 2 + \dots + 2^{i-1} + \dots + \frac{m}{2} \in O(m)$ space being used overall.

So we are not able to simply store every VO for each level in an array. But there is a way of compressing these VOs.

Consider what has happened when level i receives a promotion from level $i - 1$ at index j . This means that the fingerprint $\phi(t_{j-2^{i-1}} \dots t_j)$ matched ϕ_{i-1} . Now consider if level i receives a promotion at index $j + 1$. Now both the fingerprints $\phi(t_{j-2^{i-1}} \dots t_j)$ and $\phi(t_{j-2^{i-1}+1} \dots t_{j+1})$ matched ϕ_{i-1} . Assuming that a collision did not occur in the Karp-Rabin fingerprinting – an assumption that holds with at least probability $1 - \frac{1}{n^{1+\alpha}}$ since the associated strings are the same length and fingerprinting parameters p and r have been picked correctly – it must hold that $t_{j-2^{i-1}} \dots t_j = t_{j-2^{i-1}+1} \dots t_{j+1}$. In order for this to be the case, it is necessary that the prefix $p_0 \dots p_{2^{i-1}-1}$ repeats itself.

We can see this in the example where the text and pattern are just strings of the letter a . If we consider a more detailed look at where the viable occurrences are promoted to level 3, as shown in Figure 2.2, we

²If $j = 0$ then -1 and the fingerprint of the empty string will be stored as a VO.

j	0	1	2	3	4	5	6
t_j	a	a	a	a	a	a	a
VO for level 3 starting at -1	a	a	a	a			
VO for level 3 starting at 0		a	a	a	a		
VO for level 3 starting at 1			a	a	a	a	
VO for level 3 starting at 2				a	a	a	a

Figure 2.2: Example state of VO list for level 3 after 7 characters, where $T = aaaaaaa$ and $P = aaaaaaa$

can see that the only reason we need to store $2^{3-1} = 4$ VOs is because the 4 character prefix of the pattern is so repetitive.

It is at this point that we shall describe the period of a string. For any string T of length n , the period ρ_T is the shortest prefix of T which we can repeat $\frac{n}{|\rho_T|}$ times in order to re-create T . For the situation shown in Figure 2.2, the period of the pattern prefix $\rho_{p_0 \dots p_3} = a$.

More generally, if level i needs to store more than one VO at a given point, the prefix $p_0 \dots p_{2^i-1-1}$ must be periodic. We can now store the VOs for a given level not as a list, but as an arithmetic progression, with the following properties:

- The location and fingerprint of the oldest VO we need to store
- The location and fingerprint of the newest VO currently stored
- The fingerprint of the period
- The length of the period
- The number of VOs currently stored

The fingerprint and length of the period can both be computed when we need to store two VOs at a given level: The length by taking the second VO location and subtracting the first VO location, and the fingerprint by working out the suffix of the second VO fingerprint and the first VO fingerprint. Both of these are constant time operations.

When we want to remove a VO from a row, we update the oldest location by adding on the length of the period, update the oldest fingerprint by concatenating it with the fingerprint of the period, and decrement our counter. Again, this is a constant time operation.

There is however, a caution about this method. It must be remembered that we are not comparing the strings directly; we are merely comparing fingerprints of them. Thus if there is a collision in the fingerprints, we might have a case where the strings are not periodic.

We can check for this when we insert a new VO into an arithmetic progression. If there are two or more VOs already stored, we compare the difference between the location of the newest VO currently stored and the location of this new VO, and also check the suffix of the new VO's fingerprint with the fingerprint of the newest VO currently stored. If these two values are equal to the length and fingerprint of the period, then we store this new VO by incrementing the number of VOs currently stored and continue as usual.

If the above condition does not hold and these two conditions do not match, there is no clear consensus on how to handle this case of a non-periodic VO. Porat and Porat themselves ignore this case, and simply accept that there is a possibility of both false positives and false negatives. Breslauer and Galil[3] recommend not inserting the occurrence into the pattern, yet reporting the index as a match against the whole pattern anyway to accept some chance of false positives yet still finding all instances of the pattern.

Independent of whether or not the condition holds, inserting and removing VOs can be performed in constant time and the VOs for a given row can be stored compactly in $O(1)$ space. Because there are $\lceil \log_2 m \rceil$ levels, the overall algorithm now uses $O(\log m)$ in both space and time per character.

Chapter 3

Project Execution

A topic-specific chapter, of roughly 20 pages

This chapter is intended to describe what you did: the goal is to explain the main activity or activities, of any type, which constituted your work during the project. The content is highly topic-specific, but for many projects it will make sense to split the chapter into two sections: one will discuss the design of something (e.g., some hardware or software, or an algorithm, or experiment), including any rationale or decisions made, and the other will discuss how this design was realised via some form of implementation.

This is, of course, far from ideal for *many* project topics. Some situations which clearly require a different approach include:

- In a project where asymptotic analysis of some algorithm is the goal, there is no real “design and implementation” in a traditional sense even though the activity of analysis is clearly within the remit of this chapter.
- In a project where analysis of some results is as major, or a more major goal than the implementation that produced them, it might be sensible to merge this chapter with the next one: the main activity is such that discussion of the results cannot be viewed separately.

Note that it is common to include evidence of “best practice” project management (e.g., use of version control, choice of programming language and so on). Rather than simply a rote list, make sure any such content is useful and/or informative in some way: for example, if there was a decision to be made then explain the trade-offs and implications involved.

3.1 Example Section

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foo

Figure 3.1: This is an example figure.

foo	bar	baz
0	0	0
1	1	1
⋮	⋮	⋮
9	9	9

Table 3.1: This is an example table.

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3.1.1 Example Sub-section

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```
for i = 0 upto n do
  | ti ← 0
end
```

Algorithm 3.1: This is an example algorithm.

```
for( i = 0; i < n; i++ ) {
  t[ i ] = 0;
}
```

Listing 3.1: This is an example listing.

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Example paragraph. This is an example paragraph; note the trailing full-stop in the title, which is intended to ensure it does not run into the text.

Chapter 4

Critical Evaluation

A topic-specific chapter, of roughly 10 pages

This chapter is intended to evaluate what you did. The content is highly topic-specific, but for many projects will have flavours of the following:

1. functional testing, including analysis and explanation of failure cases,
2. behavioural testing, often including analysis of any results that draw some form of conclusion wrt. the aims and objectives, and
3. evaluation of options and decisions within the project, and/or a comparison with alternatives.

This chapter often acts to differentiate project quality: even if the work completed is of a high technical quality, critical yet objective evaluation and comparison of the outcomes is crucial. In essence, the reader wants to learn something, so the worst examples amount to simple statements of fact (e.g., “graph X shows the result is Y”); the best examples are analytical and exploratory (e.g., “graph X shows the result is Y, which means Z; this contradicts [1], which may be because I use a different assumption”). As such, both positive *and* negative outcomes are valid *if* presented in a suitable manner.

Chapter 5

Conclusion

A compulsory chapter, of roughly 2 pages

The concluding chapter of a dissertation is often underutilised because it is too often left too close to the deadline: it is important to allocation enough attention. Ideally, the chapter will consist of three parts:

1. (Re)summarise the main contributions and achievements, in essence summing up the content.
2. Clearly state the current project status (e.g., “X is working, Y is not”) and evaluate what has been achieved with respect to the initial aims and objectives (e.g., “I completed aim X outlined previously, the evidence for this is within Chapter Y”). There is no problem including aims which were not completed, but it is important to evaluate and/or justify why this is the case.
3. Outline any open problems or future plans. Rather than treat this only as an exercise in what you *could* have done given more time, try to focus on any unexplored options or interesting outcomes (e.g., “my experiment for X gave counter-intuitive results, this could be because Y and would form an interesting area for further study” or “users found feature Z of my software difficult to use, which is obvious in hindsight but not during at design stage; to resolve this, I could clearly apply the technique of Smith [7]”).

Bibliography

- [1] Alfred V. Aho and Margaret J. Corasick. Efficient string matching: An aid to bibliographic search. *Commun. ACM*, 18(6):333–340, June 1975.
- [2] Djamal Belazzougui, Fabiano C. Botelho, and Martin Dietzfelbinger. Hash, displace, and compress. In Amos Fiat and Peter Sanders, editors, *Algorithms - ESA 2009*, volume 5757 of *Lecture Notes in Computer Science*, pages 682–693. Springer Berlin Heidelberg, 2009.
- [3] Dany Breslauer and Zvi Galil. Real-time streaming string-matching. *ACM Trans. Algorithms*, 10(4):22:1–22:12, August 2014.
- [4] K. Seluk Candan and Maria Luisa Sapino. *Data Management for Multimedia Retrieval*. Cambridge University Press, May 2010.
- [5] Beate Commentz-Walter. A string matching algorithm fast on the average. In Hermann A. Maurer, editor, *Automata, Languages and Programming*, volume 71 of *Lecture Notes in Computer Science*, pages 118–132. Springer Berlin Heidelberg, 1979.
- [6] Richard M. Karp and M.O. Rabin. Efficient randomized pattern-matching algorithms. *IBM Journal of Research and Development*, 31(2):249–260, March 1987.
- [7] B. Porat and E. Porat. Exact and approximate pattern matching in the streaming model. In *Foundations of Computer Science, 2009. FOCS '09. 50th Annual IEEE Symposium on*, pages 315–323, Oct 2009.

Appendix A

An Example Appendix

Content which is not central to, but may enhance the dissertation can be included in one or more appendices; examples include, but are not limited to

- lengthy mathematical proofs, numerical or graphical results which are summarised in the main body,
- sample or example calculations, and
- results of user studies or questionnaires.

Note that in line with most research conferences, the marking panel is not obliged to read such appendices.