$$|S\rangle = |1_1\rangle \cdots |1_n\rangle |0_{n+1}\rangle \cdots |0_m\rangle$$

$$U$$

$$\sum_{S'} C_{S',S} |S'_1, S'_2, \dots, S'_m\rangle$$

$$\Pr[S'] = |C_{S',S}|^2 = \frac{|\operatorname{per}(U_{S',S})|^2}{\prod_{i=1}^d S'_i!}$$