Quantum simulation of partially distinguishable boson sampling

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Main Results

- We provide an explicit polynomial time quantum circuit for Boson Sampling with photons of arbitrary distinguishability.
- This is through reducing Boson Sampling to the problem of sampling from irreducible representations of the Unitary group [RSdG99].
- This is solvable through known circuits for the Schur transform [BCH07].

1. Boson Sampling

- Sampling from n photons on an m-mode interferometer.

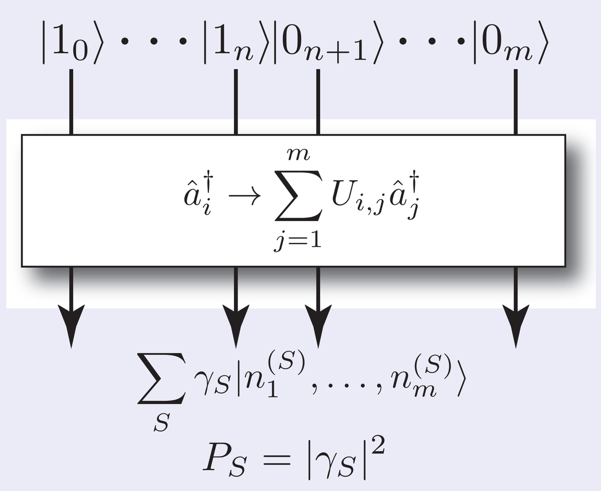

Figure: Boson Sampling model of quantum computation. Reproduced from [GMO+15]

- Efficient classical simulation would imply collapse of the polynomial hierarchy [AA11].
- Though practical algorithms for up to 50 photons exist [NSC+17].

2. Unitary group representations

- The Hilbert space $(\mathbb{C}^m)^{\otimes n}$ carries irreps of $\mathrm{U}(m)$ and S_n.
- An efficient quantum circuit, denoted W, allows up to map between this space and the irreps [BCH08].

$$W|\Psi\rangle = \sum_\lambda \sum_{q_\lambda, p_\lambda} C^\lambda_{q_\lambda, p_\lambda} |\lambda\rangle|q_\lambda\rangle|p_\lambda\rangle$$

- There is also an efficient mapping from occupation numbers to the symmetric $\lambda = (n)$ irrep of $\mathrm{U}(m)$ [RSdG99].
- The fully symmetric irrep of S_n is one state, denoted $|p_{(n)=1}\rangle$.

3. Quantum circuit for Boson Sampling

- Circuit works by creating a single particle representation of our occupation numbers through the methods in part 2.
- Our interferometer U can then be implemented by applying U to each qudit.

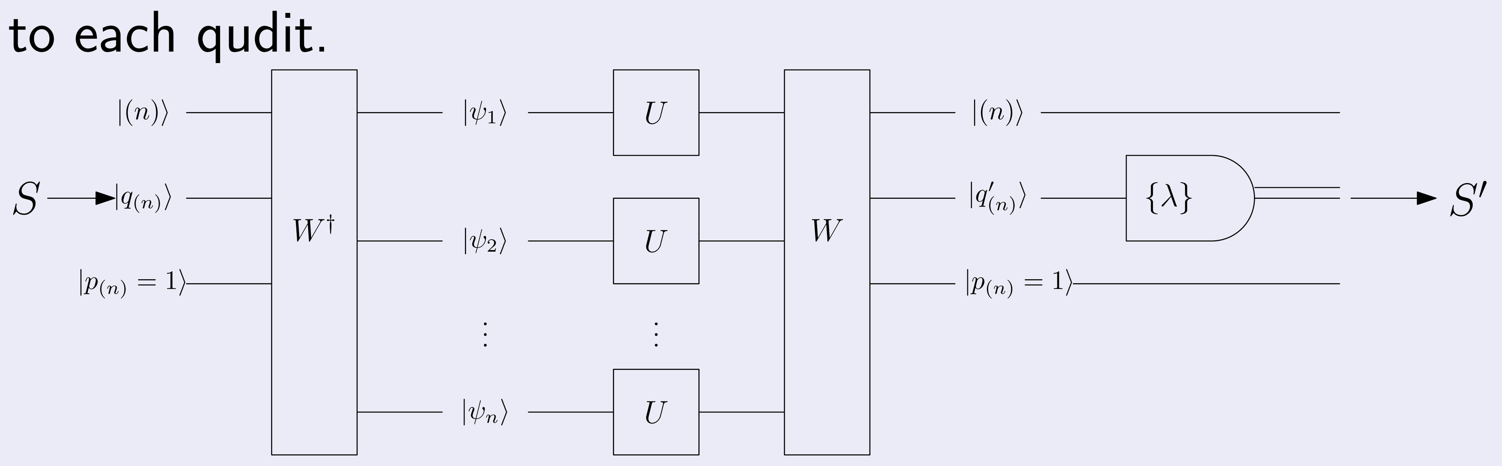

Figure: Circuit for Boson Sampling with indistinguishable photons. This circuit has accuracy $\delta + \epsilon$ due to approximating $U^{\otimes n}$ and W, and runs in polynomial time in terms of $n, m, \log \delta^{-1}$ and $\log \epsilon^{-1}$.

- We can also see the same distribution if we remove the second W circuit and just measure each qubit in the computational basis.

4. Boson Sampling with partially distinguishable photons

- For distinguishability, we introduce a second mode.
- Occupation numbers map to the fully symmetric irrep of $\mathrm{U}(m \times n)$.
- Unitary-Unitary duality decomposes this into irreps of $\mathrm{U}(m) \otimes \mathrm{U}(n)$ [RCR12].

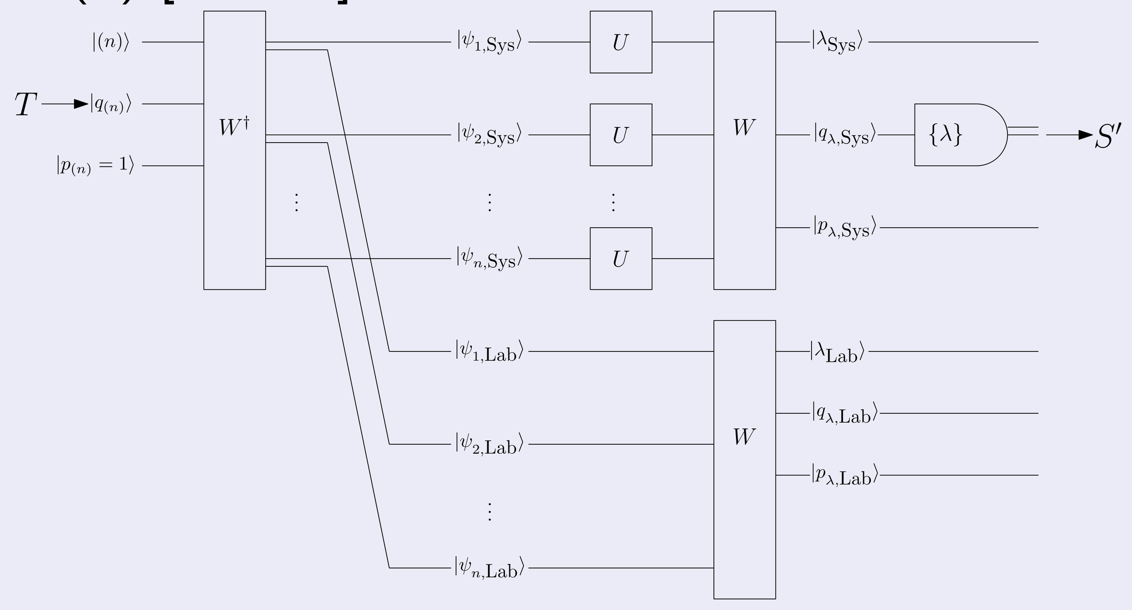

Figure: Circuit for Boson Sampling with photons of arbitrary distinguishability.

5. Boson Sampling with loss

- Distribution known for $n + k$ photons with k lost [AB16].
- This can be modelled by tracing out k qudits.

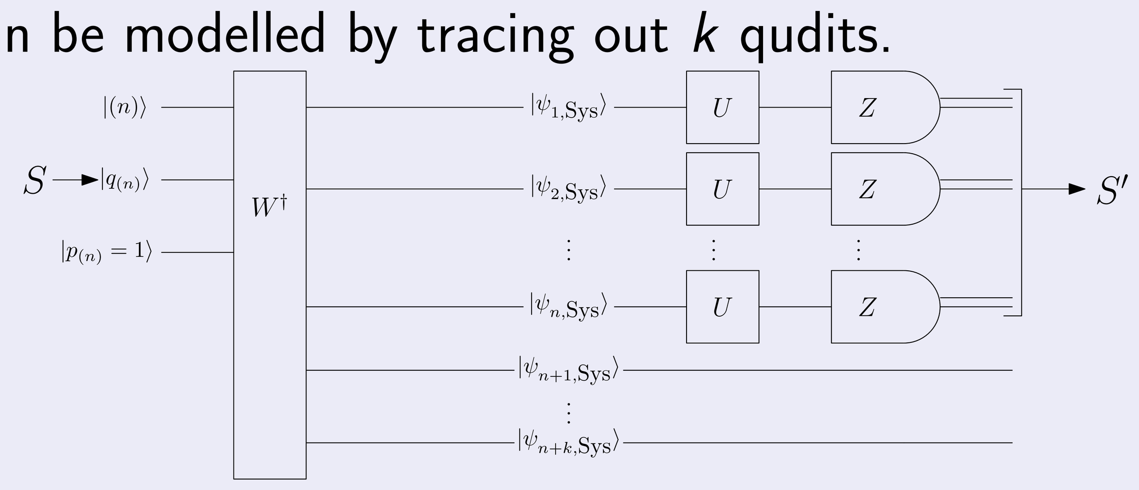

Figure: Circuit for Boson Sampling when k photons are lost.

6. Other results and future work

- Postselecting on $|\lambda_{\mathrm{Sys}}\rangle = |(n)\rangle$ allows us to perform indistinguishable Boson Sampling.
- Even small amounts of indistinguishability might be able to guarantee an entangled qudit state.

Questions

- Can we learn how distinguishability affects complexity?
- Are there any applications which irrep sampling can be used for?
- What do more realistic distinguishability models look like?
- Can we model lossy components?

References

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