Quantum simulation of partially distinguishable boson sampling

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Main Results

- We provide an explicit polynomial time quantum circuit for Boson Sampling with photons of arbitrary distinguishability.
- This is through reducing Boson Sampling to the problem of sampling from irreducible representations of the Unitary group.
- This is solvable through known circuits for the Schur transform [BCH07].

1. Boson Sampling

- Sampling from n indistinguishable single photons on an m-mode interferometer.

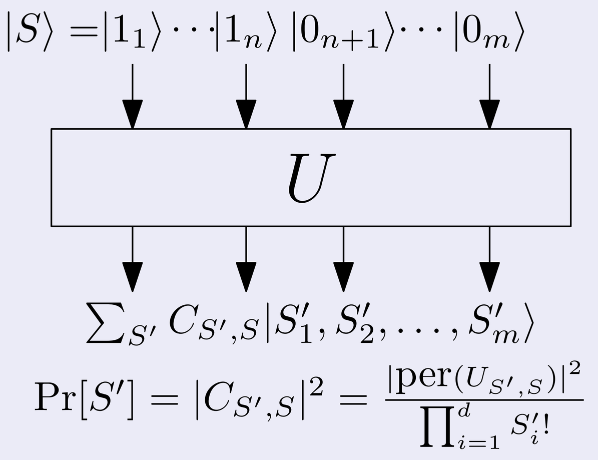

Figure: The Boson Sampling model.

- Efficient classical simulation would imply collapse of the polynomial hierarchy [AA11].
- Practical algorithms for up to 50 photons exist [NSC+17].
- Experimental issues such as loss and distinguishability need to be considered to reach a scale that outperforms classical computation

2. Schur-Weyl duality

- The Hilbert space $(\mathbb{C}^m)^{\otimes n}$ carries dual irreps of $\mathrm{U}(m)$ and S_n.
- An efficient quantum circuit, denoted W, allows us to map between the computational basis and the irrep basis [BCH08].

$$W|\Psi\rangle = \sum_{\lambda} \sum_{q_\lambda, p_\lambda} C^{\lambda}_{q_\lambda, p_\lambda} |\lambda\rangle |q_\lambda\rangle |p_\lambda\rangle$$

- There is also an efficient mapping from occupation numbers to the symmetric $\lambda = (n)$ irrep of $\mathrm{U}(m)$ [RSdG99].
- The fully symmetric irrep of S_n is one state, denoted $|p_{(n)=1}\rangle$.

3. Quantum circuit for Boson Sampling

- Circuit works by creating a single particle representation in terms of qudits via the methods in part 2.
- Interferometer U can be implemented by applying U to each qudit.

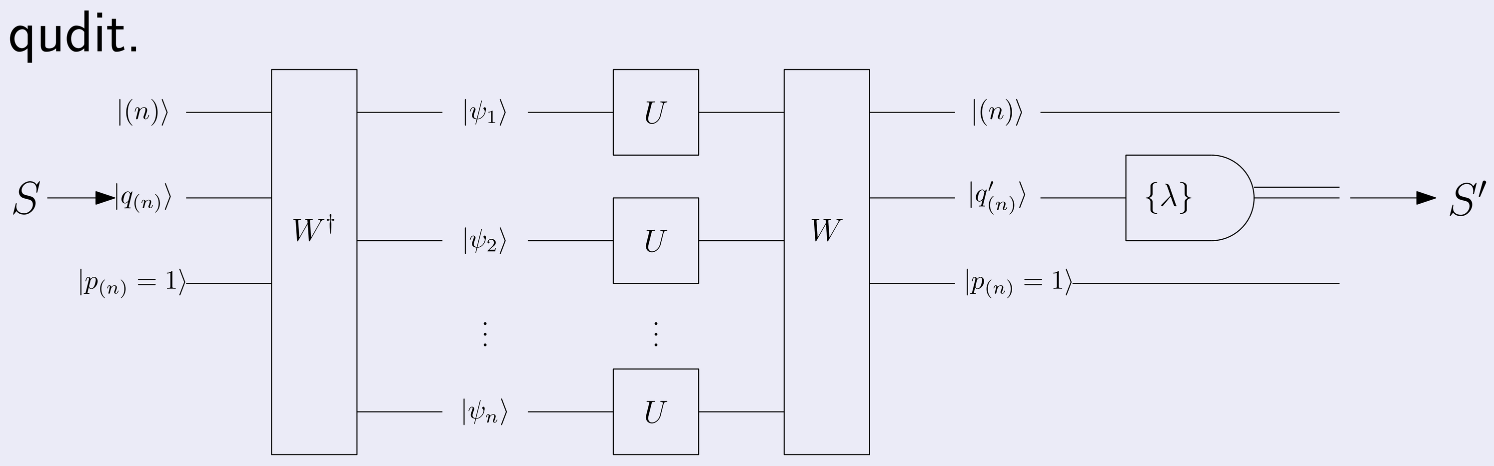

Figure: Circuit for Boson Sampling with indistinguishable photons. This circuit has accuracy $\delta + \epsilon$ due to approximating $U^{\otimes n}$ and W, and runs in polynomial time in terms of $n, m, \log \delta^{-1}$ and $\log \epsilon^{-1}$.

- We also see essentially the same distribution if we remove the second W circuit and measure each qudit in the computational basis.

4. Boson Sampling with partially distinguishable photons

- For distinguishability, we introduce a second set of n modes.
- We use 'System' and 'Label' modes to distinguish spatial and, for example, temporal modes.
- Occupation numbers map to symmetric irrep of $\mathrm{U}(m \times n)$.
- This decomposes into irreps of $\mathrm{U}(m) \times \mathrm{U}(n)$ [RCR12].

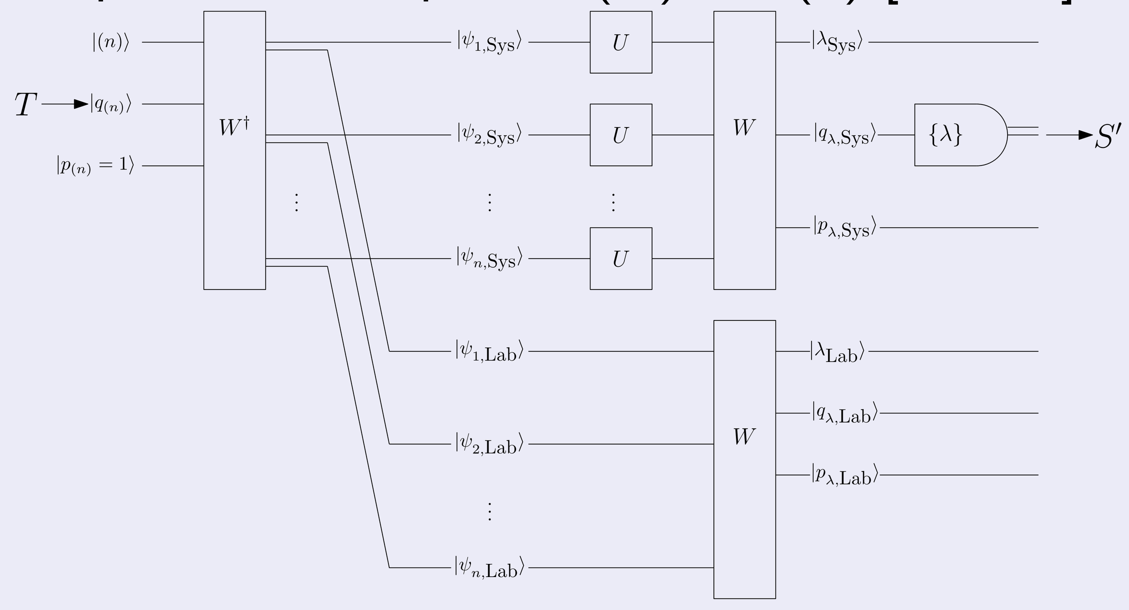

Figure: Circuit for Boson Sampling with photons of arbitrary distinguishability.

- Distinguishability increases entanglement between System and Label registers, creating a more mixed state when the label is traced out.

5. Boson Sampling with loss

- Distribution known for $n + k$ photons with k lost [AB16].
- This can be modelled by simply tracing out k qudits.

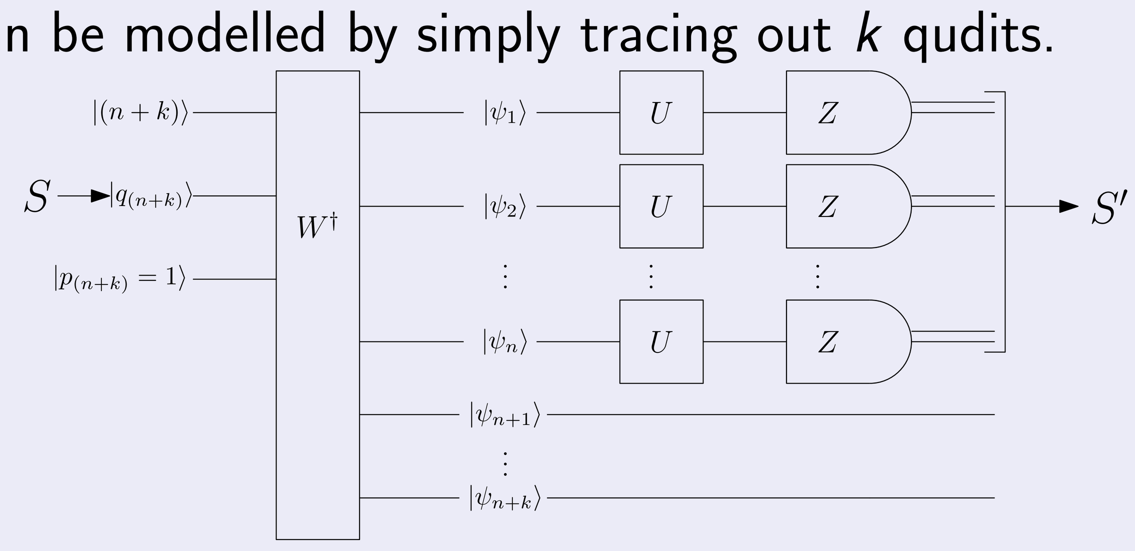

Figure: Circuit for Boson Sampling when k photons are lost.

6. Other results and future work

- Postselecting on $|\lambda_{\mathrm{Sys}}\rangle = |(n)\rangle$ allows us to perform indistinguishable Boson Sampling.
- Even a small probability of indistinguishability can guarantee an entangled system state in the single particle picture.

Questions

- Can we learn how distinguishability affects complexity?
- Are there any applications which irrep sampling can be used for?
- What do more realistic distinguishability models look like?
- Can this circuit simulate other models of loss?

References

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